FGV EPGE
ESCOLA BRASILEIRA DE ECONOMIA E FINANÇAS

Macroeconomia e Mercados Financeiros

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Asset Pricing

Asset Price Fluctuations

Asset Price Fluctuations

Can asset price fluctuations be explained by changes in fundamental values of the economy?

Are asset price fluctuations a result of market imperfections?

Would fluctuations be compatible with an economy with full information and no frictions?

Asset Price Fluctuations

Fundamental values of the economy:

monetary policy,

expectations about future productivity,

etc

To answer this question, we need to make the model more explicit

We can then set a process for dividends or for other variables and obtain asset prices

Model

Consumers, firms and investors decide how much to invest in each asset

The available wealth to invest today is A_t

Let us say that there are two kinds of assets:

risk-free bonds and

risky equity shares

Risk-Free Bonds

Call bond holdings by L_t

Bonds are negotiated today by the price q_t and they pay \$1 in the following period

The gross return from t to $t+1$ is then $R_t = \frac{1}{q_t}$

Sometimes we write $1 + r_t = \frac{1}{q_t}$

Also, bond prices today are equal to $\frac{1}{R_t}$

Risky Equity Shares

Call equity holdings by N_t

Equity shares are negotiated today by the price p_t

A unit of a share entitles the owner to a time-varying dividend y_t, not known in advance

The return of a share from t to $t + 1$ is $\frac{p_{t+1} + y_{t+1}}{p_t}$.

They are risky because y_{t+1} and p_{t+1} are not know at t

Portfolio Decision

Budget Constraint

$$c_t + \frac{1}{R_t} L_t + p_t N_t \leq A_t$$

Assets in the next period

$$A_{t+1} = L_t + (p_{t+1} + y_{t+1}) N_t$$

Portfolio Decision

L_t and N_t are chosen so that the agents get the most from A_t:

$$E_t \sum_{j=0}^{\infty} \beta^j u(c_{t+j})$$

β: discounting for time

Expectation E_t: there is uncertainty about asset returns

Portfolio Decision

The objective is to find the optimal portfolio of bonds and shares. We find the optimal portfolio by solving

$$\max E_t \sum_{j=0}^{\infty} \beta^j u(c_{t+j})$$

subject to

$$c_{t+j} + \frac{1}{R_{t+j}} L_{t+j} + p_{t+j} N_{t+j} \leq A_{t+j},$$

where $A_{t+j+1} = L_{t+j} + (p_{t+j+1} + y_{t+j+1}) N_{t+j}$

Value Function

The problem above involves finding the solution for all time periods at once

An easier method is to take wealth A_t for one period and solve the portfolio problem for the next period

The portfolio decision affects next period wealth A_{t+1}

Given a starting value for A_t and for the state of the economy, given by y_t and R_t, a function $v(A_t, y_t, R_t)$ summarizes the numerical value of the decisions

The function v is called a value function

Portfolio Decision - Reformulated

The decision problem for the optimal portfolio changes to

$$v(A_t, y_t, R_t) = \max_{c_t, L_t, N_t} u(c_t) + \beta E_t[v(A_{t+1}, y_{t+1}, R_{t+1})]$$

subject to

$$c_t + \frac{1}{R_t} L_t + p_t N_t \leq A_t,$$

Where

$$A_{t+1} = L_t + (p_{t+1} + y_{t+1}) N_t$$

First Order Conditions

The first order conditions of the problem above are:

For L_t:

$$u'(c_t)\frac{1}{R_t} = \beta E_t[v_A(A_{t+1}, y_{t+1}, R_{t+1})]$$

For N_t:

$$u'(c_t)p_t = \beta E_t[v_A(A_{t+1}, y_{t+1}, R_{t+1})(p_{t+1} + y_{t+1})]$$

$v_A(A, y, R)$ is the derivative of v with respect to A

Additional Condition

We need to know the value of $v_A(A_{t+1}, y_{t+1}, R_{t+1})$ to solve the problem

We obtain this value by the following additional condition:

$$v_A(A_t, y_t, R_t) = u'(c_t)$$

Additional Condition

$$v_A(A_t, y_t, R_t) = u'(c_t)$$

This condition is called the envelope condition

It is an additional condition, not an additional assumption

This condition is obtained from the mathematical properties of the maximization problem

Updating one period, we have

$$v_A(A_{t+1}, y_{t+1}, R_{t+1}) = u'(c_{t+1})$$

Solution

Substitute $v_A(A_{t+1}, y_{t+1}, R_{t+1}) = u'(c_{t+1})$ into the first order conditions

It implies

$$u'(c_t)\frac{1}{R_t} = \beta E_t[u'(c_{t+1})]$$

$$u'(c_t)p_t = \beta E_t[u'(c_{t+1})(p_{\mathrm{t+1}} + y_{t+1})]$$

Simplifying

We can simplify the equations by including variables that are known at t inside the expectation

For example, for the first equation, we have

$$u'(c_t)\frac{1}{R_t} = \beta E_t[u'(c_{t+1})] \Rightarrow E_t\left[\frac{\beta u'(c_{t+1})}{u'(c_t)} R_t\right] = 1$$

We can do the same for the second equation

Simplifying

For the second equation, we have

$$u'(c_t)p_t = \beta E_t[u'(c_{t+1})(p_{t+1} + y_{t+1})]$$

$$\Rightarrow E_t\left[\frac{\beta u'(c_{t+1})}{u'(c_t)}\frac{p_{t+1} + y_{t+1}}{p_t}\right] = 1$$

Notice that $\frac{p_{t+1}+y_{t+1}}{p_t}$ is the share return between the two periods

Key Result

The equations above yield

$$E_t\left[\frac{\beta u'(c_{t+1})}{u'(c_t)}R_t\right] = 1$$

$$E_t\left[\frac{\beta u'(c_{t+1})}{u'(c_t)}\frac{p_{t+1}+y_{t+1}}{p_t}\right] = 1$$

Applications

Asset Pricing

The idea is to use the equation

$$E_t\left[\frac{\beta u'(c_{t+1})}{u'(c_t)}\frac{p_{t+1}+y_{t+1}}{p_t}\right]=1$$

together with assumptions about the process for dividends y_t, $y_{t+1}, y_{t+2}, \dots,$

Asset Pricing

As trading clears financial markets, the share price is such that consumption is equal to the dividends available: prices change so that $c_t = y_t$

We obtain

$$E_t\left[\frac{\beta u'(y_{t+1})}{u'(y_t)}\frac{p_{t+1}+y_{t+1}}{p_t}\right] = 1$$

Asset Pricing

So,

$$E_t\left[\frac{\beta u'(y_{t+1})}{u'(y_t)}\frac{p_{t+1}+y_{t+1}}{p_t}\right]=1$$

$$\Rightarrow \frac{1}{p_t u'(y_t)}E_t[\beta u'(y_{t+1})(p_{t+1}+y_{t+1})]=1$$

$$\Rightarrow p_t u'(y_t)=E_t[\beta u'(y_{t+1})(p_{t+1}+y_{t+1})]$$

This is an Equation for Pricing!

The equation

$$p_t u'(y_t) = E_t[\beta u'(y_{t+1})(p_{t+1} + y_{t+1})]$$

gives to us the values that stock prices p_t should follow over time

How To Use It

Substituting for p_{t+1}, p_{t+2} and so on yields that the stock price today is

$$p_t = E_t \sum_{j=1}^{\infty} \beta^j \frac{u'(y_{t+j})}{u'(y_t)} y_{t+j}$$

If we have an idea about the process for y_t, we then have a prediction for the stock price p_t

How It Works: Example

Let us say that the agents use $u(c_t) = \log c_t$ to evaluate dividends over time

That function implies $u'(c_t) = \frac{1}{c_t}$

Substituting in

$$p_t = E_t \sum_{j=1}^{\infty} \beta^j \frac{u'(y_{t+j})}{u'(y_t)} y_{t+j}$$

How It Works: Example

Implies

$$p_t = E_t \sum_{j=1}^{\infty} \beta^j \frac{1/y_{t+j}}{1/y_t} y_{t+j}$$

$$\Rightarrow p_t = E_t \sum_{j=1}^{\infty} \beta^j \frac{y_t}{y_{t+j}} y_{t+j} \Rightarrow p_t = E_t \sum_{j=1}^{\infty} \beta^j y_t$$

$$\boxed{\Rightarrow p_t = \frac{\beta}{1-\beta} y_t}$$

How It Works: Example

The formula

$$p_t = \frac{\beta}{1-\beta} y_t$$

means that prices today are proportional to dividends today

Prices fluctuate in accordance to the fluctuations of dividends

Dividend Process

Let us say that the dividend process depends on past dividends, a growth component, and a shock

$$\log y_{t+1} = \rho \log y_t + g(t+1) + \varepsilon_{t+1}$$

It is in logs to imply changes as a percentage of past values

For example, if $y_0 = 1$ and $\varepsilon_1 = 0$, then $\log y_1 = g$, which implies $y_1 = e^g \approx 1 + g$

Dividend Process

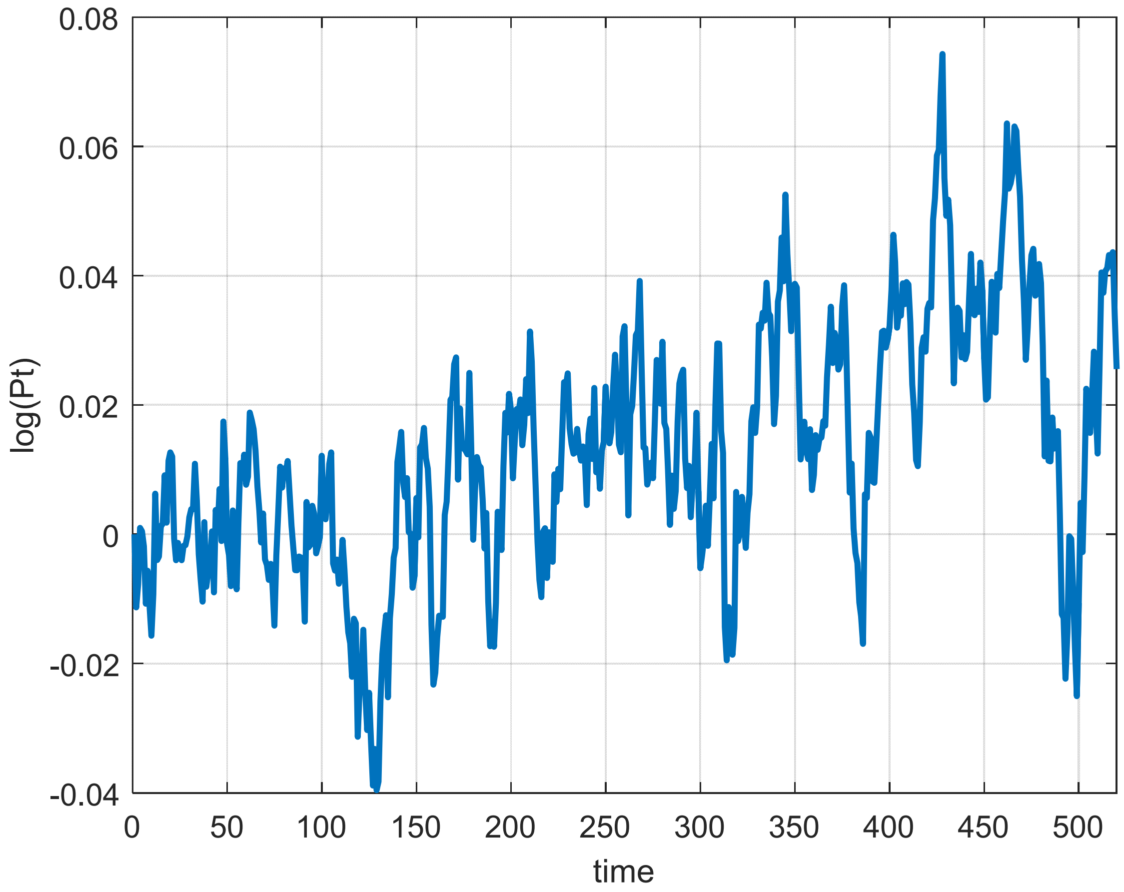
With a random draw for $\varepsilon_1, \varepsilon_2, \ldots$, we obtain the values of dividends over time

Applying the asset pricing formula we obtain the predicted stock price

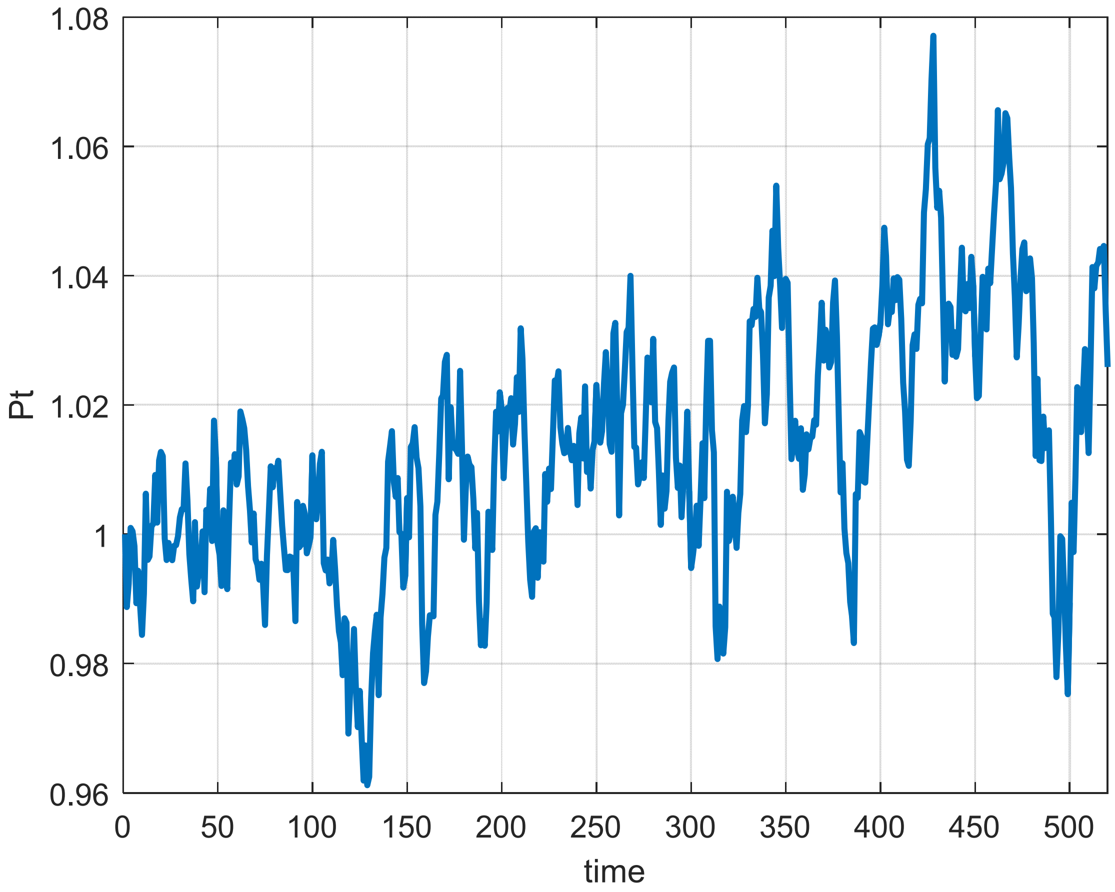
$$\log y_{t+1} = \rho \log y_t + g(t+1) + \varepsilon_{t+1}$$

Set $\rho = 0.90$ and $g = 2\%$ per year

Results, $\log(P_t)$



Results, P_t



Agents are fully rational.

They choose the optimal portfolio at each time.

They take into account the dividend process correctly.

Financial markets have no frictions.

We still have large price fluctuations.

Dividend Process

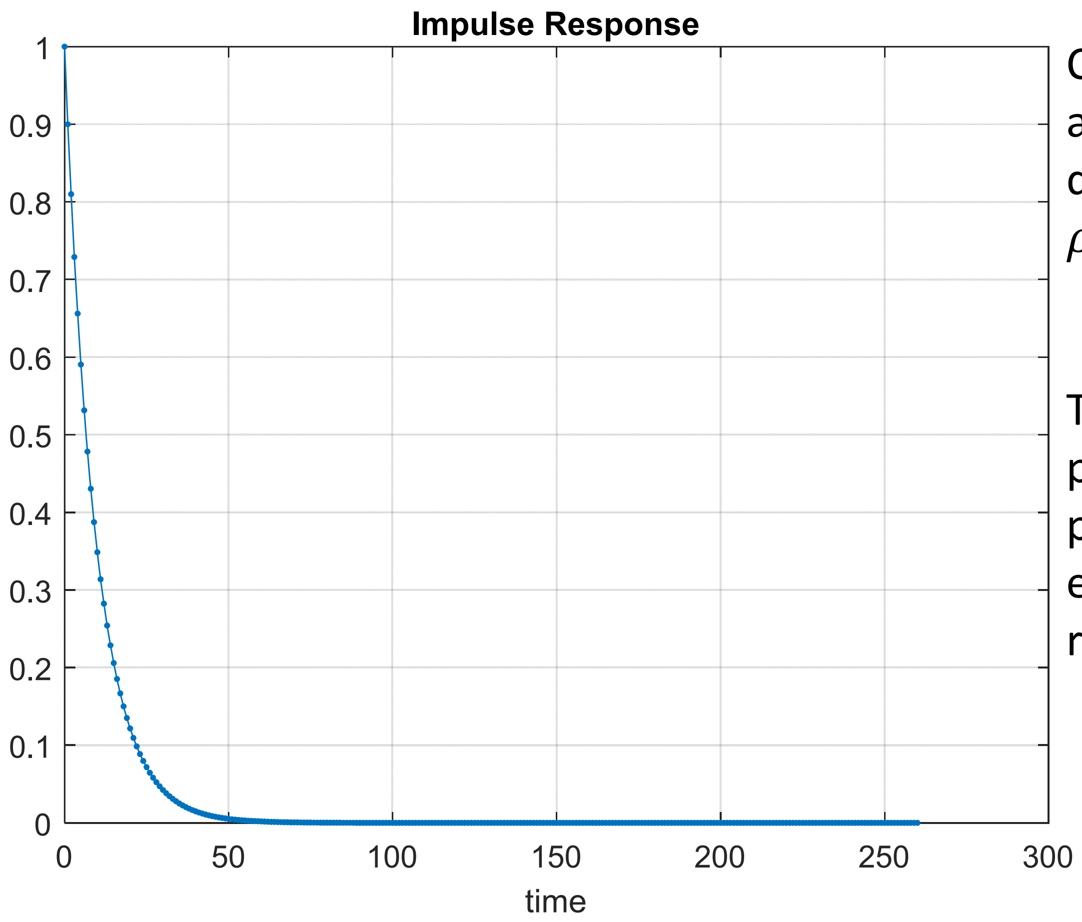
There can be different processes for the dividend

The previous process is such that a shock to the dividend today affects the dividend for many periods

The parameter ρ is large, that is, close to 1, in

$$\log y_{t+1} = \rho \log y_t + g(t+1) + \varepsilon_{t+1}$$

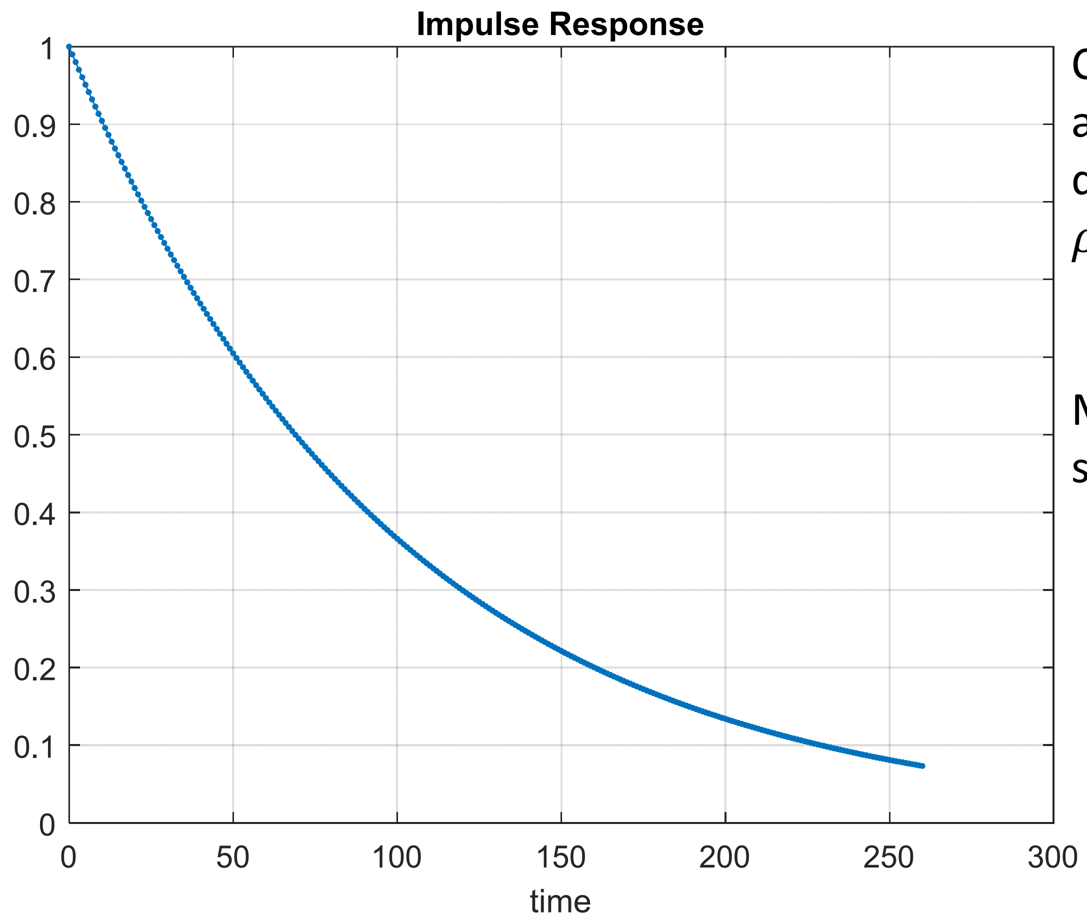
Effect of a Shock to the Dividend



Current response of a shock to dividends, with $\rho = 0.90$

The dividend process has some persistent, but the effects decay relatively fast

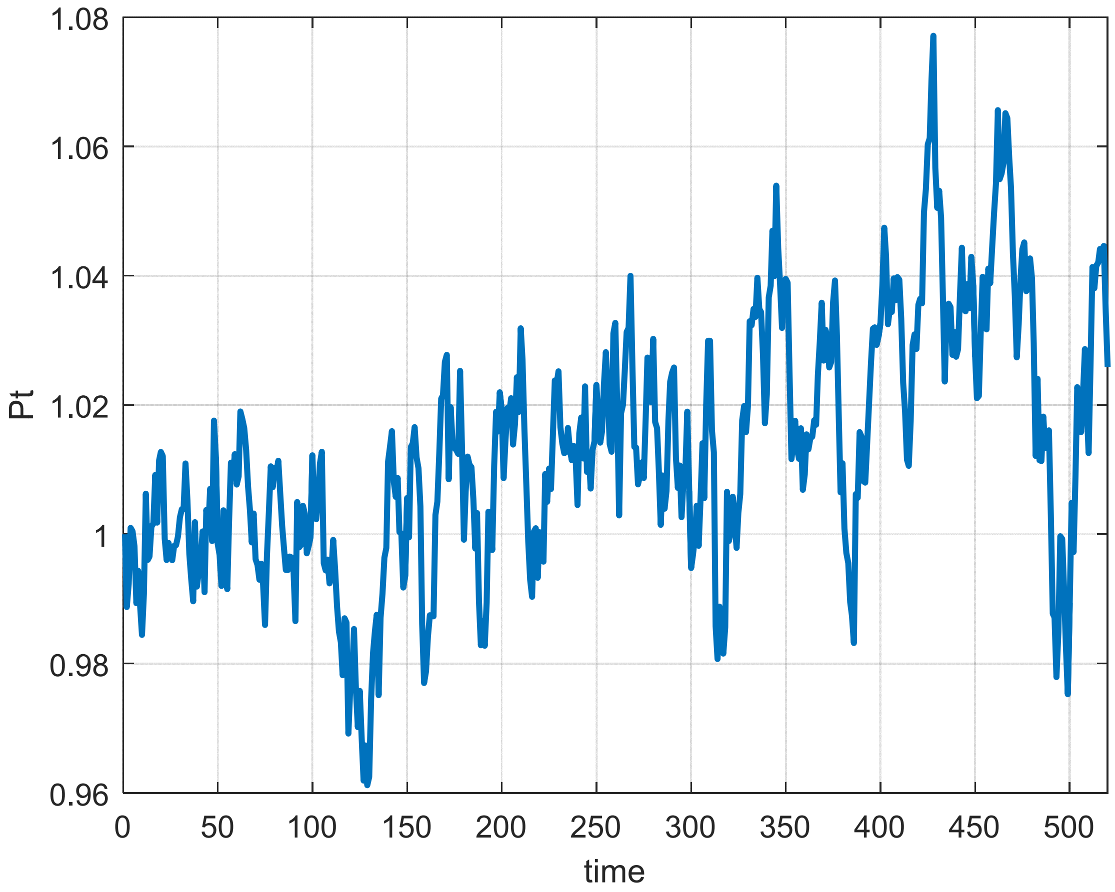
With a Different Dividend Process



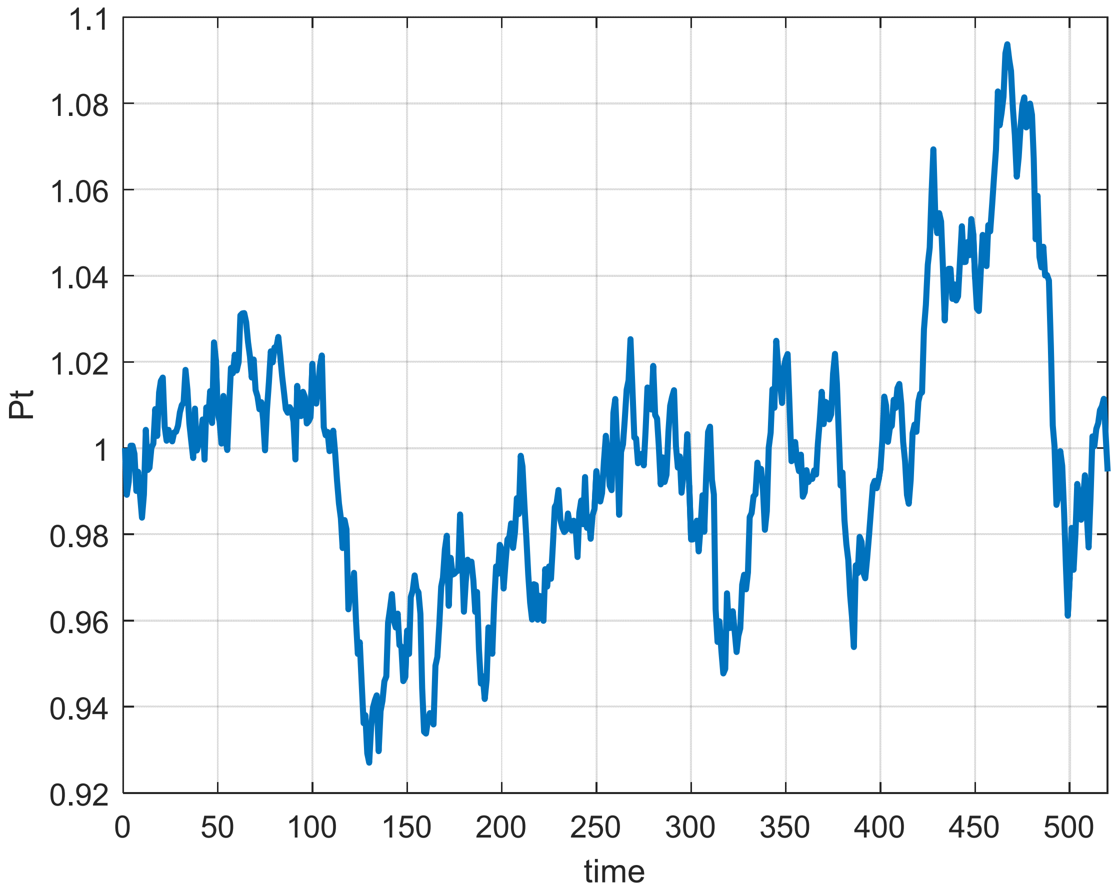
Current response of a shock to dividends, with $\rho = 0.99$

More persistent shock

We Go From Here



To Here



The waves of exhuberance and crisis are even more pronounced.

Dividend processes summarize real changes.

Different processes for y_t imply different prices over time.

The price series are all compatible with rationality.

Conclusions

We derived a pricing equation with the form

$$E_t\left[\frac{\beta u'(c_{t+1})}{u'(c_t)}\frac{p_{t+1}+y_{t+1}}{p_t}\right]=1$$

This equation relates the price today with future prices and future dividends

Conclusions

$$E_t\left[\frac{\beta u'(c_{t+1})}{u'(c_t)}\frac{p_{t+1}+y_{t+1}}{p_t}\right] = 1$$

The expectations are taken into account

In fact, uncertainty, preferences, and changes over long periods are taken into account

Moreover, there are no frictions in financial markets. That is, there are no short-sales constraints, brokerage fees and other transaction costs

Fluctuations

Even with perfect markets, prices fluctuate considerably

We can have financial crises and expansions

Changes in asset prices reflect shocks to dividends

As dividends represent the productivity of the economy, then changes in asset prices reflect shocks to the economy

Asset Price Fluctuations