Monetary Policy, Inflation, and the Business Cycle

Chapter 7

Monetary Policy and the Open Economy *

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*The present chapter is based on Galí and Monacelli (2005), with the notation modified somewhat for consistency with earlier chapters. The section on transmission of monetary policy shocks contains original material.

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All the models analyzed in earlier chapters assumed a closed economy: households and firms were not able to trade in goods or financial assets with agents located in other economies. In the present chapter we relax that assumption by developing an open economy extension of the basic new Keynesian model analyzed in chapter 3. Our framework introduces explicitly the exchange rate, the terms of trade, exports and imports, as well as international financial markets. It also implies a distinction between the consumer price index–which includes the price of imported goods–, and the price index for domestically produced goods. Such a framework can in principle be used to assess the implications of alternative monetary policy strategies in an open economy. Since our framework nests as a limiting case the closed economy model of chapter 3, it allows us to explore the extent to which the opening of the economy affects some of the conclusions regarding monetary policy that we obtained for the closed economy model and, in particular, the desirability of a policy that seeks to stabilize inflation (see chapter 4). We can also analyze what role, if any, does the exchange rate play in the optimal design of monetary policy and/or what is the measure of inflation that the central bank should seek to stabilize. Finally, we can also use our framework to determine the implications of alternative simple rules, as we did in chapter 4 for the closed economy.

The analysis of a monetary open economy raises a number of issues and choices that a modeler needs to be confront, and which are absent from its closed economy counterpart. First, a choice needs to be made between the modelling of a "large" or a "small" economy, i.e. between allowing or not, respectively, for repercussions in the rest of the world of developments (including policy decisions) in the economy being modelled. Secondly, the existence of two or more economies subject to imperfectly correlated shocks generates an incentive to trade in assets between residents of different countries, in order to smooth their consumption over time. Hence, a decision must be made regarding the nature of international asset markets and, more specifically, the set of securities that can be traded in those markets, with possible assumptions ranging from financial autarky to complete markets. Thirdly, one needs to make some assumption about firms' ability to discriminate across countries in the price they charge for the goods they produce ("pricing to market" vs "law of one price"). Furthermore, whenever discrimination is possible and prices are not readjusted continuously, an assumption must be made regarding the currency in which the prices of exported goods are set ("local currency pricing"–when prices are set in the currency of the importing economy– vs "producer currency pricing"–with prices set in the currency of the producer's country–). Other dimensions of open economy modelling that require that some choices include the allowance of not for non-tradable goods, the existence of trading costs, the possibility of international policy coordination, etc.

A comprehensive analysis of those different modeling dimensions and how they may affect the design of monetary policy would require a book of its own, and so it is clearly beyond the scope of the present chapter. Our more modest objective here is to present an example of a monetary open economy model to illustrate some of the issues that emerge in the analysis of such economies and which are absent from their closed economy counterparts. In particular, we develop a model of a small open economy, with complete international financial markets, and where the law of one price holds. Then, in the discussion of our model's policy implications and in the notes on the literature at the end of the chapter, we refer to a number of papers that adopt different assumptions and briefly discuss the extent to which this leads their findings to differ from ours.

The framework below, originally developed in Galí and Monacelli (2005), models a small open economy as one among a continuum of (infinitesimally small) economies making up the world economy. For simplicity, and in order to focus on the issues brought about by the openness of the economy we ignore the possible presence of either cost-push shocks or nominal wage rigidities. The assumptions on preferences and technology, combined with the Calvo price-setting structure and the assumption of complete financial markets, give rise to a highly tractable model and to simple and intuitive log-linearized equilibrium conditions. The latter can be reduced to a two-equation dynamical system consisting of a new Keynesian Phillips curve and a dynamic IS-type equation, whose structure is identical to the one derived in chapter 3 for the closed economy, though its coefficients depend on parameters that are specific to the open economy, while the driving forces are a function or world variables (which are taken as exogenous to the small open economy). As in its closed economy counterpart, the two equations must be complemented with a description of how monetary policy is conducted.

After describing the model and deriving a simple representation of its equilibrium dynamics, in section 3 we analyze the transmission of monetary policy shocks, emphasizing the role played by openness in that transmission. In section 4 we turn to the issue of optimal monetary policy design, focusing on a particular case for which the flexible price allocation is efficient. Under the same assumptions it is straightforward to derive a second order approximation to the consumer's utility, which can be used to evaluate alternative policy rules. We put it to work in section 5, where we assess the merits of two different Taylor-type rules, a policy that fully stabilizes the CPI, and an exchange rate peg. As in previous chapters, the last section concludes with a brief note on the related literature.

1 A Small Open Economy Model

We model the world economy as a *continuum of small open economies* represented by the unit interval. Since each economy is of measure zero, its performance does not have any impact on the rest of the world. Different economies are subject to imperfectly correlated productivity shocks, but we assume they share identical preferences, technology, and market structure.

Next we describe in detail the problem facing households and firms located in one such economy. Before we do so, a brief remark on notation is in order. Since our focus is on the behavior of a single economy and its interaction with the world economy, and in order to lighten the notation, we use variables *without* an i-index to refer to the small open economy being modeled. Variables with an $i \in [0, 1]$ subscript refer to economy i, one among the continuum of economies making up the world economy. Finally, variables with a *star superscript* correspond to the world economy as a whole.

1.1 Households

A typical small open economy is inhabited by a representative household who seeks to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \; U(C_t, N_t) \tag{1}$$

where N_t denotes hours of labor, and C_t is a composite consumption index defined by

$$C_t \equiv \left[(1-\alpha)^{\frac{1}{\eta}} \; (C_{H,t})^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} \; (C_{F,t})^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{2}$$

where $C_{H,t}$ is an index of consumption of domestic goods given by the CES function

$$C_{H,t} \equiv \left(\int_0^1 C_{H,t}(j)^{\frac{\epsilon-1}{\epsilon}} \; dj \right)^{\frac{\epsilon}{\epsilon-1}}$$

where $j \in [0,1]$ denotes the good variety.[1] $C_{F,t}$ is an index of imported goods given by

$$C_{F,t} \equiv \left(\int_0^1 (C_{i,t})^{\frac{\gamma-1}{\gamma}} \, di \right)^{\frac{\gamma}{\gamma-1}}$$

where $C_{i,t}$ is, in turn, an index of the quantity of goods imported from country i and consumed by domestic households. It is given by an analogous CES function:

$$C_{i,t} \equiv \left(\int_0^1 C_{i,t}(j)^{\frac{\epsilon-1}{\epsilon}} \, dj \right)^{\frac{\epsilon}{\epsilon-1}}$$

Note that parameter $\epsilon > 1$ denotes the elasticity of substitution between varieties produced within any given country.[2] Parameter $\alpha \in [0,1]$ can be interpreted as a measure of openness.[3] Parameter $\eta > 0$ measures the substitutability between domestic and foreign goods, from the viewpoint of the domestic consumer, while γ measures the substitutability between goods produced in different foreign countries.

Maximization of (1) is subject to a sequence of budget constraints of the form:

$$\int_0^1 P_{H,t}(j) \, C_{H,t}(j) \, dj + \int_0^1 \int_0^1 P_{i,t}(j) \, C_{i,t}(j) \, dj \, di \, + E_t\{Q_{t,t+1} D_{t+1}\} \leq D_t + W_t N_t + T_t \tag{3}$$

for $t = 0, 1, 2, ...,$ where $P_{H,t}(j)$ is the price of domestic variety j. $P_{i,t}(j)$ is the price of variety j imported from country i. D_{t+1} is the nominal payoff in period $t+1$ of the portfolio held at the end of period t (and which includes shares in firms), W_t is the nominal wage, and T_t denotes lump-sum transfers/taxes. The previous variables are all expressed in units of domestic

[1] As discussed below, each country produces a continuum of differentiated goods, represented by the unit interval.

[2] Notice that it is irrelevant whether we think of integrals like the one in (2) as including or not the corresponding variable for the small economy being modeled, since its presence would have a negligible influence on the integral itself (in fact each individual economy has a zero measure). The previous remark also applies to many other expressions involving integrals over the continuum of economies (i.e., over i) that the reader will encounter below.

[3] Equivalently, $1-\alpha$ is a measure of the degree of home bias. Note that in the absence of some home bias the households in our small open economy would attach an infinitesimally small weight to local goods, and consumption expenditures would be allocated to imported goods (except for a infinitesimally small share allocated to domestic goods).

currency. $Q_{t,t+1}$ is the stochastic discount factor for one-period ahead nominal payoffs relevant to the domestic household. We assume that households have access to a complete set of contingent claims, traded internationally.

The optimal allocation of any given expenditure within each category of goods yields the demand functions:

$$C_{H,t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\epsilon} C_{H,t} \qquad ; \qquad C_{i,t}(j) = \left(\frac{P_{i,t}(j)}{P_{i,t}} \right)^{-\epsilon} C_{i,t} \tag{4}$$

for all $i, j \in [0,1]$, where $P_{H,t} \equiv \left(\int_0^1 P_{H,t}(j)^{1-\epsilon}\, dj \right)^{\frac{1}{1-\epsilon}}$ is the *domestic* price index (i.e., an index of prices of domestically produced goods) and $P_{i,t} \equiv \left(\int_0^1 P_{i,t}(j)^{1-\epsilon}\, dj \right)^{\frac{1}{1-\epsilon}}$ is a price index for goods imported from country i (expressed in domestic currency), for all $i \in [0,1]$. Combining the optimality conditions in (4), with the definitions of price and quantity indexes $P_{H,t}$, $C_{H,t}$, $P_{i,t}$ and $C_{i,t}$ we obtain $\int_0^1 P_{H,t}(j)\ C_{H,t}(j)\ dj = P_{H,t}\ C_{H,t}$ and $\int_0^1 P_{i,t}(j)\ C_{i,t}(j)\ dj = P_{i,t}\ C_{i,t}$.

Furthermore, the optimal allocation of expenditures on imported goods by country of origin implies:

$$C_{i,t} = \left(\frac{P_{i,t}}{P_{F,t}} \right)^{-\gamma} C_{F,t} \tag{5}$$

for all $i \in [0,1]$, and where $P_{F,t} \equiv \left(\int_0^1 {P_{i,t}}^{1-\gamma}\, di \right)^{\frac{1}{1-\gamma}}$ is the price index for *imported* goods, also expressed in domestic currency. Note that (5), together with the definitions of $P_{F,t}$ and $C_{F,t}$ implies that we can write total expenditures on imported goods as $\int_0^1 P_{i,t}\ C_{i,t}\ di = P_{F,t}\ C_{F,t}$.

Finally, the optimal allocation of expenditures between domestic and imported goods is given by:

$$C_{H,t} = (1-\alpha) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t \qquad ; \qquad C_{F,t} = \alpha \left(\frac{P_{F,t}}{P_t} \right)^{-\eta} C_t \tag{6}$$

where $P_t \equiv [(1-\alpha)\ (P_{H,t})^{1-\eta} + \alpha\ (P_{F,t})^{1-\eta}]^{\frac{1}{1-\eta}}$ is the consumer price index (CPI).[4] Note that under the assumption of $\eta = 1$ or, alternatively, when

[4]It is usefult to notice, for future reference, that in the particular case of $\eta = 1$, the CPI takes the form $P_t = (P_{H,t})^{1-\alpha}(P_{F,t})^{\alpha}$, while the consumption index is given by $C_t = \frac{1}{(1-\alpha)^{(1-\alpha)}\alpha^{\alpha}} {C_{H,t}}^{1-\alpha}\ {C_{F,t}}^{\alpha}$

the price indexes for domestic and foreign goods are equal (as in the steady state described below), parameter α corresponds to the share of domestic consumption allocated to imported goods. It is also in this sense that α represents a natural index of openness.

Accordingly, total consumption expenditures by domestic households are given by $P_{H,t}C_{H,t} + P_{F,t}C_{F,t} = P_t C_t$. Thus, the period budget constraint can be rewritten as:

$$P_t \; C_t + E_t\{Q_{t,t+1} \; D_{t+1}\} \leq D_t + W_t \; N_t + T_t \tag{7}$$

As in previous chapters we specialize the period utility function to be of the form $U(C,N) \equiv \frac{C^{1-\sigma}}{1-\sigma} - \frac{N^{1+\varphi}}{1+\varphi}$. Thus we can rewrite the remaining optimality conditions for the household's problem as follows:

$$C_t^{\sigma} \; N_t^{\varphi} = \frac{W_t}{P_t} \tag{8}$$

which is the standard intratemporal optimality condition. In order to derive the relevant intertemporal optimality condition note that the following relation must hold for the optimizing household in our small open economy:

$$\frac{V_{t,t+1}}{P_t} \; C_t^{-\sigma} = \xi_{t,t+1} \; \beta \; C_{t+1}^{-\sigma} \; \frac{1}{P_{t+1}} \tag{9}$$

where $V_{t,t+1}$ is the period t price (in domestic currency) of an Arrow security, i.e. a one-period security that yields one unit of domestic currency if a specific state of nature is realized in period $t+1$, and nothing otherwise, and where $\xi_{t,t+1}$ is the probability of that state of nature being realized in $t+1$ (conditional on the state of nature at t). Variables C_{t+1} and P_{t+1} on the right hand side should be interpreted as representing the values taken by the consumption index and the CPI at $t+1$ conditional on the state of nature to which the Arrow security refers being realized. Thus, the left hand side captures the utility loss resulting from the purchase of an Arrow security considered (with the corresponding reduction in consumption), whereas the right hand side measures the expected one-period-ahead utility gain from the additional consumption made possible by the (eventual) security payoff. If the consumer is optimizing the expected utility gain must exactly offset the current utility loss.

Given that the price of Arrow securities and the one period stochastic discount factor are related by the equation $Q_{t,t+1} \equiv \frac{V_{t,t+1}}{\xi_{t,t+1}}$ we can rewrite (9) as:[5]

$$\beta \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+1}}\right) = Q_{t,t+1} \tag{10}$$

which is assumed to be satisfied for all possible states of nature at t and $t+1$.

Taking conditional expectations on both sides of (10) and rearranging terms we obtain a conventional stochastic Euler equation:

$$Q_t = \beta \; E_t \left\{ \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+1}}\right) \right\} \tag{11}$$

where $Q_t \equiv E_t\{Q_{t,t+1}\}$ denotes the price of a one-period discount bond paying off one unit of domestic currency in $t+1$.

For future reference, we recall that (8) and (11) can be respectively written in log-linearized form as:

$$w_t - p_t = \sigma \; c_t + \varphi \; n_t$$
$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma} \; (i_t - E_t\{\pi_{t+1}\} - \rho) \tag{12}$$

where lower case letters denote the logs of the respective variables, $i_t \equiv -\log Q_t$ is the short-term nominal rate, $\rho \equiv -\log \beta$ is the time discount rate, and $\pi_t \equiv p_t - p_{t-1}$ is CPI inflation (with $p_t \equiv \log P_t$).

1.1.1 Domestic Inflation, CPI Inflation, the Real Exchange Rate, and the Terms of Trade: Some Identities

Next we introduce several assumptions and definitions, and derive a number of identities that are extensively used below. We start by defining the *bilateral terms of trade* between the domestic economy and country i as $\mathcal{S}_{i,t} = \frac{P_{i,t}}{P_{H,t}}$, i.e. the price of country i's goods in terms of home goods. The *effective*

[5] Note that under complete markets a simple no room for arbitrage argument implies that the price of a one-period asset (or portfolio) yielding a random payoff D_{t+1} must be given by $\sum V_{t,t+1} D_{t+1}$ where the sum is over all possible $t+1$ states. Equivalently, we can write that price as $E_t \left\{ \frac{V_{t,t+1}}{\xi_{t,t+1}} D_{t+1} \right\}$. We can thus define the one period stochastic discount factor as $Q_{t,t+1} \equiv \frac{V_{t,t+1}}{\xi_{t,t+1}}$.

terms of trade are thus given by

$$\begin{aligned}\mathcal{S}_t &\equiv \frac{P_{F,t}}{P_{H,t}} \\ &= \left(\int_0^1 \mathcal{S}_{i,t}^{1-\gamma} di\right)^{\frac{1}{1-\gamma}}\end{aligned}$$

which can be approximated (up to first order) around a symmetric steady state satisfying $\mathcal{S}_{i,t} = 1$ for all $i \in [0,1]$ by

$$s_t = \int_0^1 s_{i,t}\ di \tag{13}$$

where $s_t \equiv \log \mathcal{S}_t = p_{F,t} - p_{H,t}$.

Similarly, log-linearization of the CPI formula around the same symmetric steady state yields:

$$\begin{aligned}p_t &\equiv (1-\alpha)\ p_{H,t} + \alpha\ p_{F,t} \\ &= p_{H,t} + \alpha\ s_t \end{aligned}\tag{14}$$

It is useful to note, for future reference, that (13) and (14) hold *exactly* when $\gamma = 1$ and $\eta = 1$, respectively.

It follows that *domestic inflation* – defined as the rate of change in the index of domestic goods prices, i.e., $\pi_{H,t} \equiv p_{H,t+1} - p_{H,t}$ – and *CPI-inflation* are linked according to the relation:

$$\pi_t = \pi_{H,t} + \alpha\ \Delta s_t \tag{15}$$

which makes the gap between our two measures of inflation proportional to the percent change in the terms of trade, with the coefficient of proportionality given by the openness index α.

We assume that the *law of one price* holds for individual goods at all times (both for import and export prices), implying that $P_{i,t}(j) = \mathcal{E}_{i,t}\ P_{i,t}^i(j)$ for all $i, j \in [0,1]$, where $\mathcal{E}_{i,t}$ is the bilateral nominal exchange rate (the price of country i's currency in terms of the domestic currency), and $P_{i,t}^i(j)$ is the price of country i's good j expressed in terms of its own currency. Plugging the previous assumption into the definition of $P_{i,t}$ one obtains $P_{i,t} = \mathcal{E}_{i,t}$ $P_{i,t}^i$, where $P_{i,t}^i \equiv \left(\int_0^1 P_{i,t}^i(j)^{1-\epsilon} dj\right)^{\frac{1}{1-\epsilon}}$ is country i's domestic price index. In turn, by substituting into the definition of $P_{F,t}$ and log-linearizing around the symmetric steady state we obtain:

$$\begin{aligned} p_{F,t} &= \int_0^1 (e_{i,t} + p_{i,t}^i)\ di \\ &= e_t + p_t^* \end{aligned}$$

where $p_{i,t}^i \equiv \int_0^1 p_{i,t}^i(j)\ dj$ is the (log) domestic price index for country i (expressed in terms of its own currency), $e_t \equiv \int_0^1 e_{i,t}\ di$ is the (log) *effective nominal exchange rate*, and $p_t^* \equiv \int_0^1 p_{i,t}^i\ di$ is the (log) *world price index.* Notice that for the world as a whole there is no distinction between CPI and domestic price level, nor between their corresponding inflation rates.

Combining the previous result with the definition of the terms of trade we obtain the following expression:

$$s_t = e_t + p_t^* - p_{H,t} \tag{16}$$

Next, we derive a relationship between the terms of trade and the real exchange rate. First, we define the *bilateral real exchange rate* with country i as $\mathcal{Q}_{i,t} \equiv \frac{\mathcal{E}_{i,t} P_t^i}{P_t}$, i.e., the ratio of the two countries CPIs, both expressed in terms of domestic currency. Let $q_t \equiv \int_0^1 q_{i,t}\ di$ be the (log) *effective real exchange rate*, where $q_{i,t} \equiv \log \mathcal{Q}_{i,t}$. It follows that

$$\begin{aligned} q_t &= \int_0^1 (e_{i,t} + p_t^i - p_t)\ di \\ &= e_t + p_t^* - p_t \\ &= s_t + p_{H,t} - p_t \\ &= (1-\alpha)\ s_t \end{aligned}$$

where the last equality holds only up to a first order approximation when $\eta \neq 1$.[6]

[6]The last equality can be derived by log-linearizing $\frac{P_t}{P_{H,t}} = \left[(1-\alpha) + \alpha\ \mathcal{S}_t^{1-\eta}\right]^{\frac{1}{1-\eta}}$ around a symmetric steady state, which yields

$$p_t - p_{H,t} = \alpha\ s_t$$

1.1.2 International Risk Sharing

Under the assumption of complete markets for securities traded international, a condition analogous to (9) must also hold for the representative household in any other country, say country i:

$$\frac{V_{t,t+1}}{\mathcal{E}_t^i P_t^i} \, (C_t^i)^{-\sigma} = \xi_{t,t+1}\beta \, (C_{t+1}^i)^{-\sigma} \, \frac{1}{\mathcal{E}_{t+1}^i P_{t+1}^i}$$

where the presence of the exchange rate terms reflect the fact that the security purchased by the country i's household has a price $V_{t,t+1}$ and a unit payoff expressed in the currency of the small open economy of reference, and hence need to be converted to country i's currency.

We can write the previous relation in terms of our small open economy's stochastic discount factor as follows:

$$\beta \left(\frac{C_{t+1}^i}{C_t^i}\right)^{-\sigma} \left(\frac{P_t^i}{P_{t+1}^i}\right) \left(\frac{\mathcal{E}_t^i}{\mathcal{E}_{t+1}^i}\right) = Q_{t,t+1} \tag{17}$$

Combining (10) and (17), together with the definition for the real exchange rate definition we have:

$$C_t = \vartheta_i \, C_t^i \, \mathcal{Q}_{i,t}{}^{\frac{1}{\sigma}} \tag{18}$$

for all t, and where ϑ_i is a constant which will generally depend on initial conditions regarding relative net asset positions. Henceforth, and without loss of generality, we assume symmetric initial conditions (i.e., zero net foreign asset holdings and an ex-ante identical environment), in which case we have $\vartheta_i = \vartheta = 1$ for all i.

Taking logs on both sides of (18) and integrating over i we obtain

$$\begin{aligned} c_t &= c_t^* + \frac{1}{\sigma} \, q_t \\ &= c_t^* + \left(\frac{1-\alpha}{\sigma}\right) \, s_t \end{aligned} \tag{19}$$

where $c_t^* \equiv \int_0^1 c_t^i \, di$ is our index for world consumption (in log terms), and where the second equality holds only up to a first order approximation when $\eta \neq 1$. Thus we see that the assumption of complete markets at the international level leads to a simple relationship linking domestic consumption with world consumption and the terms of trade.

1.1.3 A Brief Detour: Uncovered Interest Parity and the Terms of Trade

Under the assumption of complete international financial markets, the equilibrium price (in terms of our small open economy's domestic currency) of a riskless bond denominated in country i's currency is given by $\mathcal{E}_{i,t}\ Q_t^i = E_t\{Q_{t,t+1}\ \mathcal{E}_{i,t+1}\}$, where Q_t^i is the price of the bond in terms of country i's currency. The previous pricing equation can be combined with the domestic bond pricing equation, $Q_t = E_t\{Q_{t,t+1}\}$ to obtain a version of the *uncovered interest parity* condition:

$$E_t\{Q_{t,t+1}\ [\exp\{i_t\} - \exp\{i_t^*\}\ (\mathcal{E}_{i,t+1}/\mathcal{E}_{i,t})]\} = 0$$

Log-linearizing around a perfect-foresight steady state, and aggregating over i, yields the familiar expression:

$$i_t = i_t^* + E_t\{\Delta e_{t+1}\} \tag{20}$$

Combining the definition of the (log) terms of trade with (20) yields the following stochastic difference equation:

$$s_t = (i_t^* - E_t\{\pi_{t+1}^*\}) - (i_t - E_t\{\pi_{H,t+1}\}) + E_t\{s_{t+1}\} \tag{21}$$

As we show in the appendix, the terms of trade are pinned down uniquely in the perfect foresight steady state. That fact, combined with our assumption of stationarity in the model's driving forces and unit relative prices in the steady state, implies that $\lim_{T\to\infty} E_t\{s_T\} = 0$.[7] Hence, we can solve (21) forward to obtain:

$$s_t = E_t\left\{\sum_{k=0}^{\infty}\left[(i_{t+k}^* - \pi_{t+k+1}^*) - (i_{t+k} - \pi_{H,t+k+1})\right]\right\} \tag{22}$$

i.e., the terms of trade are a function of current and anticipated real interest rate differentials.

[7] Our assumption regarding the steady state implies that real interest rate differential will revert to a zero mean. More generally, the real interest rate differential will revert to a constant mean, as long as the terms of trade are stationary in first differences. That would be the case if, say, the technology parameter had a unit root or a different average rate of growth relative to the rest of the world. In those cases we could have persistent real interest rate differentials.

We must point out that while equation (21) (and (22)) provides a convenient (and intuitive) way of representing the connection between terms of trade and interest rate differentials, it does not constitute an additional independent equilibrium condition. In particular, it is easy to check that (21) can be derived by combining the consumption Euler equations for both the domestic and world economies with the risk sharing condition (19) and equation (15).

Next we turn our attention to the supply side of the economy.

1.2 Firms

1.2.1 Technology

A typical firm in the home economy produces a differentiated good with a linear technology represented by the production function

$$Y_t(j) = A_t \; N_t(j)$$

where $a_t \equiv \log A_t$ follows the AR(1) process $a_t = \rho_a \; a_{t-1} + \varepsilon_t$, and where $j \in [0,1]$ is a firm-specific index.[8]

Hence, the real marginal cost (expressed in terms of domestic prices) will be common across domestic firms and given by

$$mc_t = -\nu + w_t - p_{H,t} - a_t$$

where $\nu \equiv -\log(1-\tau)$, with τ being an employment subsidy whose role is discussed later in more detail.

1.2.2 Price Setting

As in the basic model of chapter 3, we assume that firms set prices in a staggered fashion, as in Calvo (1983). Hence, a measure $1-\theta$ of (randomly selected) firms sets new prices each period, with an individual firm's probability of re-optimizing in any given period being independent of the time elapsed since it last reset its price. As shown in chapter 3, the optimal price-setting strategy for the typical firm resetting its price in period t can be approximated by the (log-linear) rule:

[8] An extension of our analysis to the case of decreasing returns considered in shapter 3 is straightforward. In order to keep the notation as simple as possible we restrict the analysis here to the case of constant returns.

$$\overline{p}_{H,t} = \mu + (1-\beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k \; E_t\{mc_{t+k} + p_{H,t}\} \tag{23}$$

where $\overline{p}_{H,t}$ denotes the (log) of newly set domestic prices, and $\mu \equiv \log \frac{\epsilon}{\epsilon-1}$ is the log of the (gross) markup in the steady state (or, equivalently, the equilibrium markup in the flexible price economy).[9]

2 Equilibrium

2.1 Aggregate Demand and Output Determination

2.1.1 Consumption and Output in the Small Open Economy

Goods market clearing in the home economy requires

$$\begin{aligned} Y_t(j) &= C_{H,t}(j) + \int_0^1 C^i_{H,t}(j) \; di \qquad (24) \\ &= \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\epsilon} \left[(1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha \int_0^1 \left(\frac{P_{H,t}}{\mathcal{E}_{i,t} P^i_{F,t}}\right)^{-\gamma} \left(\frac{P^i_{F,t}}{P^i_t}\right)^{-\eta} C^i_t \; di\right] \end{aligned}$$

for all $j \in [0,1]$ and all t, where $C^i_{H,t}(j)$ denotes country i's demand for good j produced in the home economy. Notice that the second equality has made use of (6) and (5) together with our assumption of symmetric preferences across countries, which implies $C^i_{H,t}(j) = \alpha \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\epsilon} \left(\frac{P_{H,t}}{\mathcal{E}_{i,t} P^i_{F,t}}\right)^{-\gamma} \left(\frac{P^i_{F,t}}{P^i_t}\right)^{-\eta} C^i_t$.

Plugging (24) into the definition of aggregate domestic output $Y_t \equiv$

[9] We use $\overline{p}_{H,t}$ to denote newly set prices instead of p^*_t (used in chapter 3) , since in the present chapter we reserve letters with an asterisk to refer to world economy variables.

$\left[\int_0^1 Y_t(j)^{1-\frac{1}{\epsilon}}\, dj\right]^{\frac{\epsilon}{\epsilon-1}}$ we obtain:

$$
\begin{aligned}
Y_t &= (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha \int_0^1 \left(\frac{P_{H,t}}{\mathcal{E}_{i,t} P^i_{F,t}}\right)^{-\gamma} \left(\frac{P^i_{F,t}}{P^i_t}\right)^{-\eta} C^i_t \, di \\
&= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \left[(1-\alpha)\, C_t + \alpha \int_0^1 \left(\frac{\mathcal{E}_{i,t} P^i_{F,t}}{P_{H,t}}\right)^{\gamma-\eta} \mathcal{Q}^{\eta}_{i,t}\, C^i_t \, di\right] \\
&= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \left[(1-\alpha) + \alpha \int_0^1 \left(\mathcal{S}^i_t\, \mathcal{S}_{i,t}\right)^{\gamma-\eta}\; \mathcal{Q}^{\eta-\frac{1}{\sigma}}_{i,t}\, di\right] \qquad (25)
\end{aligned}
$$

where the last equality follows from (18), and where $\mathcal{S}^i_t$ denotes the effective terms of trade of country i, while $\mathcal{S}_{i,t}$ denotes the bilateral terms of trade between the home economy and foreign country i. Notice that in the particular case of $\sigma = \eta = \gamma = 1$ the previous condition can be written exactly as[10]

$$
Y_t = C_t\, \mathcal{S}_t^{\,\alpha} \qquad (26)
$$

More generally, and recalling that $\int_0^1 s^i_t\, di = 0$, we can derive the following first-order log-linear approximation to (25) around the symmetric steady state:

$$
\begin{aligned}
y_t &= c_t + \alpha\gamma\, s_t \;+\; \alpha\left(\eta - \frac{1}{\sigma}\right) q_t \\
&= c_t + \frac{\alpha\omega}{\sigma}\, s_t \qquad (27)
\end{aligned}
$$

where $\omega \equiv \sigma\gamma + (1-\alpha)(\sigma\eta - 1)$. Notice that $\sigma = \eta = \gamma = 1$ implies $\omega = 1$.

A condition analogous to the one above will hold for all countries. Thus, for a *generic country* i it can be rewritten as $y^i_t = c^i_t + \frac{\alpha\omega}{\sigma}\, s^i_t$. By aggregating over all countries we can derive a world market clearing condition as follows

$$
\begin{aligned}
y^*_t &\equiv \int_0^1 y^i_t\, di \qquad (28) \\
&= \int_0^1 c^i_t\, di \equiv c^*_t
\end{aligned}
$$

[10] Here one must use the fact that under the assumption $\eta = 1$, the CPI takes the form $P_t = (P_{H,t})^{1-\alpha}(P_{F,t})^{\alpha}$ thus implying $\frac{P_t}{P_{H,t}} = \left(\frac{P_{F,t}}{P_{H,t}}\right)^{\alpha} = \mathcal{S}_t^{\,\alpha}$.

where y_t^* and c_t^* are indexes for world output and consumption (in log terms), and where the main equality follows, once again, from the fact that $\int_0^1 s_t^i \, di = 0$.

Combining (27) with (19) and (28), we obtain:

$$y_t = y_t^* + \frac{1}{\sigma_\alpha} \; s_t \tag{29}$$

where $\sigma_\alpha \equiv \frac{\sigma}{1+\alpha(\omega-1)} > 0$.

Finally, combining (27) with Euler equation (12), we get:

$$\begin{aligned} y_t &= E_t\{y_{t+1}\} - \frac{1}{\sigma} \; (i_t - E_t\{\pi_{t+1}\} - \rho) - \frac{\alpha\omega}{\sigma} \; E_t\{\Delta s_{t+1}\} \\ &= E_t\{y_{t+1}\} - \frac{1}{\sigma} \; (i_t - E_t\{\pi_{H,t+1}\} - \rho) - \frac{\alpha\Theta}{\sigma} \; E_t\{\Delta s_{t+1}\} \\ &= E_t\{y_{t+1}\} - \frac{1}{\sigma_\alpha} \; (i_t - E_t\{\pi_{H,t+1}\} - \rho) + \alpha\Theta \; E_t\{\Delta y_{t+1}^*\} \end{aligned} \tag{30}$$

where $\Theta \equiv (\sigma\gamma - 1) + (1-\alpha)(\sigma\eta - 1) = \omega - 1$. Not that, in general, the degree of openness influences the sensitivity of output to any given change in the domestic real rate $i_t - E_t\{\pi_{H,t+1}\}$, given world output. In particular, if $\Theta > 0$ (i.e., for relatively high values of η and γ), an increase in openness raises that sensitivity (i.e. σ_α is smaller). The reason is the direct negative effect of an increase in the real rate on aggregate demand and output is amplified by the induced real appreciation (and the consequent switch of expenditure towards foreign goods). This will be partly offset by any increase in CPI inflation relative to domestic inflation induced by the expected real depreciation, which would dampen the change in the consumption-based real rate $i_t - E_t\{\pi_{t+1}\}$–which is the one ultimately relevant for aggregate demand–, relative to $i_t - E_t\{\pi_{H,t+1}\}$.

2.1.2 The Trade Balance

Let $nx_t \equiv \left(\frac{1}{Y}\right)\left(Y_t - \frac{P_t}{P_{H,t}} \; C_t\right)$ denote net exports in terms of domestic output, expressed as a fraction of steady state output Y. In the particular case of $\sigma = \eta = \gamma = 1$, it follows from (25) that $P_{H,t}Y_t = P_tC_t$ for all t, thus implying a balanced trade at all times. More generally, a first-order approximation yields $nx_t = y_t - c_t - \alpha \; s_t$ which combined with (27) implies a simple relation between net exports and the terms of trade:

$$nx_t = \alpha \left(\frac{\omega}{\sigma} - 1 \right) \; s_t \tag{31}$$

Again, in the special case of $\sigma = \eta = \gamma = 1$ we have $nx_t = 0$ for all t, though the latter property will also hold for any configuration of those parameters satisfying $\sigma(\gamma - 1) + (1 - \alpha)(\sigma\eta - 1) = 0$. More generally, the sign of the relationship between the terms of trade and net exports is ambiguous, depending on the relative size of σ, γ, and η.

2.2 The Supply Side: Marginal Cost and Inflation Dynamics

2.2.1 Aggregate Output and Employment

Let $Y_t \equiv \left[\int_0^1 Y_t(j)^{1-\frac{1}{\epsilon}} \; dj \right]^{\frac{\epsilon}{\epsilon-1}}$ represent an index for aggregate domestic output, analogous to the one introduced for consumption. As in chapter 3, we can derive an approximate aggregate production function relating the previous index to aggregate employment. Hence, notice that

$$N_t \equiv \int_0^1 N_t(j) \; dj = \frac{Y_t}{A_t} \int_0^1 \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon} \; dj$$

As shown in chapter 3, however, variations in $d_t \equiv \int_0^1 \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon} \; dj$ around the perfect foresight steady state are of second order. Thus, and up to a first order approximation, the following relationship between aggregate output and employment holds

$$y_t = a_t + n_t \tag{32}$$

2.2.2 Marginal Cost and Inflation Dynamics in the Small Open Economy

As it was shown in chapter 3, the (log-linearized) optimal price-setting condition (23) can be combined with the (log linearized) difference equation describing the evolution of domestic prices (as a function of newly set prices) to yield an equation determining domestic inflation as a function of deviations of marginal cost from its steady state value:

$$\pi_{H,t} = \beta \ E_t\{\pi_{H,t+1}\} + \lambda \ \widehat{mc}_t \tag{33}$$

where $\lambda \equiv \frac{(1-\beta\theta)(1-\theta)}{\theta}$. Thus, relationship (33) does not depend on any of the parameters that characterize the open economy. On the other hand, the determination of real marginal cost as a function of domestic output in the open economy differs somewhat from that in the closed economy, due to the existence of a wedge between output and consumption, and between domestic and consumer prices. Thus, in our model we have

$$\begin{aligned} mc_t &= -\nu + (w_t - p_{H,t}) - a_t \\ &= -\nu + (w_t - p_t) + (p_t - p_{H,t}) - a_t \\ &= -\nu + \sigma \ c_t + \varphi \ n_t + \alpha \ s_t - a_t \\ &= -\nu + \sigma \ y_t^* + \varphi \ y_t + s_t - (1+\varphi) \ a_t \end{aligned} \tag{34}$$

where the last equality makes use of (32) and (19). Thus, we see that marginal cost is increasing in the terms of trade and world output. Both variables end up influencing the real wage, through the wealth effect on labor supply resulting from their impact on domestic consumption. In addition, changes in the terms of trade have a direct effect on the product wage, for any given consumption wage. The influence of technology (through its direct effect on labor productivity) and of domestic output (through its effect on employment and, hence, the real wage–for given output) is analogous to that observed in the closed economy.

Finally, using (29) to substitute for s_t, we can rewrite the previous expression for the real marginal cost in terms of domestic output and productivity, as well as world output:

$$mc_t = -\nu + (\sigma_\alpha + \varphi) \ y_t + (\sigma - \sigma_\alpha) \ y_t^* - (1+\varphi) \ a_t \tag{35}$$

In general, In the open economy, a change in domestic output has an effect on marginal cost through its impact on employment (captured by φ), and the terms of trade (captured by σ_α, which is a function of the degree of openness and the substitutability between domestic and foreign goods). World output, on the other hand, affects marginal cost through its effect on consumption (and, hence, the real wage, as captured by σ) and the terms of trade (captured by σ_α). Note that the sign of its impact on marginal cost is ambiguous. Under the assumption of $\Theta > 0$ (i.e. high substitutability among goods produced in different countries), we have $\sigma > \sigma_\alpha$, implying that an increase in world output raises the marginal cost. This is so because in that case the size of the real appreciation needed to absorb the change in relative supplies is small, with its negative effects on marginal cost more than offset by the positive effect from a higher real wage. Notice that in the special cases $\alpha = 0$ and/or $\sigma = \eta = \gamma = 1$, which imply $\sigma = \sigma_\alpha$, the domestic real marginal cost is completely insulated from movements in foreign output.

How does the degree of openness affect the sensitivity of marginal cost and inflation to changes in domestic and world output? Note also that, under the same assumption of high substitutability ($\Theta > 0$) considered above, an increase in openness reduces the impact of a change in domestic output on marginal cost (and hence on inflation), for it lowers the size of the required adjustment in the terms of trade. By the same token, it raises the positive impact of a change in world output on marginal cost, by limiting the size of the associated variation in the terms of trade and, hence, its countervailing effect.

Finally, and for future reference, we note that under flexible prices $mc_t = -\mu$ for all t. Thus, the natural level of output in our open economy is given by

$$y_t^n = \Gamma_0 + \Gamma_a \; a_t + \Gamma_* \; y_t^* \tag{36}$$

where $\Gamma_0 \equiv \frac{v-\mu}{\sigma_\alpha+\varphi}$, $\Gamma_a \equiv \frac{1+\varphi}{\sigma_\alpha+\varphi} > 0$, and $\Gamma_* \equiv -\frac{\alpha\Theta \; \sigma_\alpha}{\sigma_\alpha+\varphi}$. Note that the sign of the effect of world output on the domestic natural output is ambiguous, depending on the sign of the effect of the former on domestic marginal cost, which in turn depends on the relative importance of the terms of trade effect discussed above.

2.3 Equilibrium Dynamics: A Canonical Representation

In this section we show that the linearized equilibrium dynamics for the small open economy have a representation in terms of output gap and domestic inflation analogous to its closed economy counterpart.

Let $\widetilde{y}_t \equiv y_t - y_t^n$ denote the domestic output gap. Given (35) and the fact that y_t^* is invariant to domestic developments, it follows that the domestic real marginal cost and the output gap are related according to:

$$\widehat{mc}_t = (\sigma_\alpha + \varphi)\ \widetilde{y}_t$$

We can combine the previous expression with (33) to derive a version of the new Keynesian Phillips curve for the open economy:

$$\pi_{H,t} = \beta\ E_t\{\pi_{H,t+1}\} + \kappa_\alpha\ \widetilde{y}_t \tag{37}$$

where $\kappa_\alpha \equiv \lambda\,(\sigma_\alpha + \varphi)$. Notice that for $\alpha = 0$ (or $\sigma = \eta = \gamma = 1$) the slope coefficient is given by $\lambda\,(\sigma + \varphi)$ as in the standard, closed economy new Keynesian Phillips curve. More generally, we see that the form of the inflation equation for the open economy corresponds to that of the closed economy, at least as far as domestic inflation is concerned. The degree of openness α affects the dynamics of inflation only through its influence on the size of the slope of the Phillips curve, i.e., the size of the inflation response to any given variation in the output gap. If $\Theta > 0$ (which obtains for "high" values of η and γ, i.e. under high substitutability of goods produced in different countries), an increase in openness lowers σ_α, dampening the real depreciation induced by an increase in domestic output and, as a result, the effect of the latter on marginal cost and inflation.

Using (30) it is straightforward to derive a version of the so-called dynamic IS equation for the open economy in terms of the output gap:

$$\widetilde{y}_t = E_t\{\widetilde{y}_{t+1}\} - \frac{1}{\sigma_\alpha}\ (i_t - E_t\{\pi_{H,t+1}\} - r_t^n) \tag{38}$$

where

$$r_t^n \equiv \rho - \sigma_\alpha \Gamma_a (1 - \rho_a)\ a_t + \frac{\alpha \Theta \sigma_\alpha \varphi}{\sigma_\alpha + \varphi}\ E_t\{\Delta y_{t+1}^*\} \tag{39}$$

is the small open economy's natural rate of interest.

Thus we see that the small open economy's equilibrium is characterized by a forward looking IS-type equation similar to that found in the closed economy. Two differences can be pointed out, however. First, as discussed above, the degree of openness influences the sensitivity of the output gap to interest rate changes. Secondly, openness generally makes the natural interest rate depend on expected world output growth, in addition to domestic productivity.

3 Equilibrium Dynamics under an Interest Rate Rule

Next we analyze the equilibrium response of our small open economy to a variety of shocks. In doing so we assume that the monetary authority follows an interest rate rule of the form already assumed in chapter 3, namely

$$i_t = \rho + \phi_\pi \, \pi_{H,t} + \phi_y \, \widetilde{y}_t + v_t \tag{40}$$

where v_t is an exogenous component, and where ϕ_π and ϕ_y are non-negative coefficients, chosen by the monetary authority.

Combining (37), (38), and (40) we can represent the equilibrium dynamics for the output gap and domestic inflation by means of the system of difference equations.

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_{H,t} \end{bmatrix} = \mathbf{A}_\alpha \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix} + \mathbf{B}_\alpha \, (\widehat{r}_t^n - v_t) \tag{41}$$

where $\widehat{r}_t^n \equiv r_t^n - \rho$, and

$$\mathbf{A}_\alpha \equiv \Omega_\alpha \begin{bmatrix} \sigma_\alpha & 1 - \beta\phi_\pi \\ \sigma_\alpha\kappa_\alpha & \kappa_\alpha + \beta(\sigma_\alpha + \phi_y) \end{bmatrix} \quad ; \quad \mathbf{B}_T \equiv \Omega_\alpha \begin{bmatrix} 1 \\ \kappa_\alpha \end{bmatrix}$$

with $\Omega_\alpha \equiv \frac{1}{\sigma_\alpha + \phi_y + \kappa_\alpha\phi_\pi}$. Note that the previous system takes the same form as the one analyzed in chapter 3 for the closed economy, with the only difference lying in the fact that some of the coefficients are a function of the "open economy parameters" α, η, and γ, and that $\widehat{r}_t^n$ is now given by (39). In particular, the condition for a locally unique stationary equilibrium under rule (40) takes the same form as shown in chapter 3, namely

$$\kappa_\alpha \, (\phi_\pi - 1) + (1 - \beta) \, \phi_y > 0 \tag{42}$$

which we assume to hold for the remainder of the present section.

The next subsection uses the previous framework to examine the economy's response to an exogenous monetary policy shock, i.e. an exogenous change in v_t. Given the isomorphism with the closed economy model of chapter 4 we can exploit many of the results derived there.

The analysis of the effects of a technology shock (or a change in world output), which we do not pursue below, goes along the same lines as in chapter

3. First, we need to determine the implications of the shock considered for the natural interest rate $\widehat{r}_t^n$ and then we proceed to solve for the equilibrium response of the output gap and domestic inflation exactly as we do below for the case of a monetary policy shock, given the symmetry with which v_t and $\widehat{r}_t^n$ enter the equilibrium conditions.[11]

3.0.1 The Effects of a Monetary Policy Shock

We assume that the exogenous component of the interest rate, v_t, follows an AR(1) process

$$v_t = \rho_v \ v_{t-1} + \varepsilon_t^v$$

where $\rho_v \in [0, 1)$.

The natural rate of interest is not affected by a monetary policy shock so we can set $\widehat{r}_t^n = 0$, for all t for the purposes of the present exercise. As in chapter 3, we guess that the solution takes the form $\widetilde{y}_t = \psi_{yv} \ v_t$ and $\pi_t = \psi_{\pi v} \ v_t$, where ψ_{yv} and $\psi_{\pi v}$ are coefficients to be determined. Imposing the guessed solution on (**??**) and (**??**) and using the method of undetermined coefficients, we find:

$$\begin{aligned} y_t &= \widetilde{y}_t \\ &= -(1 - \beta\rho_v)\Lambda_v \ v_t \end{aligned}$$

and

$$\pi_{H,t} = -\kappa_\alpha \Lambda_v \ v_t$$

where $\Lambda_v \equiv \frac{1}{(1-\beta\rho_v)[\sigma_\alpha(1-\rho_v)+\phi_y]+\kappa_\alpha(\phi_\pi-\rho_v)}$. It can be easily shown that as long as (42) is satisfied we have $\Lambda_v > 0$. Hence, as in the closed economy, an exogenous increase in the interest rate leads to a persistent decline in output and inflation. The size of the effect of the shock relative to the closed economy benchmark depends on the values taken by a number of parameters. More specifically, if the degree of substitutability among goods produced in different countries is high (i.e. if η and γ are high, so that $\omega > 1$) then Λ_v can be shown to be increasing in the degree of openness, thus implying that a given monetary policy shock will have a larger impact in the small open economy than in its closed economy counterpart.

[11] Of course, as in chapter 3, we need to take into account that a technology shock or a shock to world output also lead to a variation in the natural outpt level, thus breaking the identity between output and the output gap.

Using interest rate rule (40) we can determine the response of the nominal rate, taking into account the central bank's endogenous reaction to changes in inflation and the output gap:

$$i_t = \left[1 - \Lambda_v(\phi_\pi \kappa_\alpha + \phi_y(1 - \beta\rho_v))\right] \ v_t$$

Note that as in the closed economy model, the full response of the nominal rate may be positive or negative, depending on parameter values. The response of the real interest rate (expressed in terms of domestic goods) is given by

$$\begin{aligned} r_t &= i_t - E_t\{\pi_{H,t+1}\} \\ &= \left[1 - \Lambda_v((\phi_\pi - \rho_v)\kappa_\alpha + \phi_y(1 - \beta\rho_v))\right] \ v_t \end{aligned}$$

which can be shown to increase when v_t rises (since the term in square brackets is unambiguously positive).

Using (29) we can uncover the response of the terms of trade to the monetary policy shock:

$$\begin{aligned} s_t &= \sigma_\alpha y_t \\ &= -\sigma_\alpha(1 - \beta\rho_v)\Lambda_v \ v_t \end{aligned}$$

The change in the nominal exchange rate is given in turn by

$$\begin{aligned} \Delta e_t &= \Delta s_t + \pi_{H,t} \\ &= -\sigma_\alpha(1 - \beta\rho_v)\Lambda_v \ \Delta v_t - \kappa_\alpha \Lambda_v \ v_t \end{aligned}$$

Thus, a monetary policy contraction leads to an improvement in the terms of trade (i.e. an decrease in the relative price of foreign goods) and a nominal exchange rate appreciation.

Note that, in the long run, the terms of trade revert back to their original level in response to the monetary policy shock, while the (log) levels of both domestic prices and the nominal exchange rate experience a permanent change of size $-\frac{\kappa_\alpha \Lambda_v}{1-\rho_v}$ (given an initial shock of size normalized to unity).

Hence, the exchange rate will overshoot its long-run level in response to the monetary policy shock if and only if

$$\sigma_\alpha(1 - \beta\rho_v)(1 - \rho_v) > \kappa_\alpha \rho_v$$

which requires that the shock is not too persistent. It can be easily shown that the previous condition corresponds to that for an increase in the nominal interest rate in response to a positive v_t shock. Note that, in that case, the subsequent exchange rate depreciation required by the interest parity condition (20) leads to an initial overshooting.

4 Optimal Monetary Policy: A Special Case

In this section we derive and characterize the optimal monetary policy for the small open economy described above, as well as the implications of that policy for a number of macroeconomic variables. The analysis, which follows closely that of Galí and Monacelli (2005), is restricted to a special case for which a second order approximation to the welfare of the representative consumer can be easily derived analytically. Its conclusions should thus not be taken as applying to a more general environment. Instead we present the exercise as an illustration of our approach to optimal monetary design to an open economy.

Let us take as a benchmark the basic new Keynesian model developed in chapter 3. As discussed in that chapter, under the assumption of a constant employment subsidy τ that neutralizes the distortion associated with firms' market power, the optimal monetary policy is the one that replicates the flexible price equilibrium allocation. The intuition for that result is straightforward: with the subsidy in place, there is only one effective distortion left in the economy, namely, sticky prices. By stabilizing markups at their "frictionless" level, nominal rigidities cease to be binding, since firms do not feel any desire to adjust prices. By construction, the resulting equilibrium allocation is efficient, and the price level remains constant.

In an open economy–and as noted, among others, by Corsetti and Pesenti (2001)–there is an additional factor that distorts the incentives of the monetary authority, beyond the presence of market power: the possibility of influencing the terms of trade in a way beneficial to domestic consumers. This possibility is a consequence of the imperfect substitutability between domestic and foreign goods, combined with sticky prices (which render monetary policy non-neutral). As shown below, and as discussed by Benigno and Benigno (2003) in the context of a two-country model, the introduction of an employment subsidy that exactly offsets the market power distortion is not sufficient to render the flexible price equilibrium allocation optimal, for, at the margin, the monetary authority would have an incentive to deviate from it to improve the terms of trade.

For the special parameter configuration $\sigma = \eta = \gamma = 1$ we can derive analytically the employment subsidy that exactly offsets the combined effects of market power and the terms of trade distortions, thus rendering the flexible price equilibrium allocation optimal. That result, in turn, rules out the existence of an average inflation (or deflation) bias, and allows us to focus on policies consistent with zero average inflation, in a way analogous to the analysis for the closed economy found in chapter 4. Perhaps not surprisingly, and as we show below, the policy that maximizes welfare in that case requires that domestic inflation be fully stabilized, while allowing the nominal exchange rate (and, as a result, CPI inflation) to adjust as needed in order to replicate the response of the terms of trade that would obtain under flexible prices.

One may wonder to what extent the optimality of strict domestic inflation targeting is specific to the special case considered here or whether it carries over to a more general case. The optimal policy analysis undertaken in Faia and Monacelli (2007) using a model nearly identical to the one considered here suggests that while the optimal policy involves some variation in the domestic price level, the latter is almost negligible from a quantitatively point of view, thus making strict domestic inflation targeting a good approximation to the optimal policy (at least conditional on the productivity shocks considered here). Using a different approach, De Paoli (2006) reaches a similar conclusion, except when an (implausibly) high elasticity of substitution is assumed.[12] But even in the latter case the losses that arise from following a domestic inflation targeting policy are negligible.[13] More generally, it is clear that there are several channels in the open economy that may potentially render a strict domestic inflation policy suboptimal, including non-unitary elasticity of substitution, local currency pricing, incomplete financial markets, etc. , all of which are unrelated to the sources of policy tradeoffs that may potentially arise in the closed economy. The quantitative significance of

[12] Those results are conditional on productivity shocks being the driving force. Not surprisingly, in the presence of cost-push shocks of the kind considered in chapter 5 stabilizing domestic inflation is not optimal (as in the closed economy).

[13] In solving the optimal policy problem for the general case, de Paoli (2006) adopts the linear-quadratic approach originally developed in Benigno and Woodford (2005), and which replaces the linear terms in the approximation to the households' welfare losses using a second order approximation to the equilibrium conditions.. Faia and Monacelli (2007) solve for the Ramsey policy using the original nonlinear equilibrium conditions as contraints of the policy problem.

the effects of those channels (individually or jointly) still needs to be explored in the literature, and its analysis is clearly beyond the scope of the present chapter.

With that consideration in mind, we next turn to the analysis of the optimal policy in the special case mentioned above.

4.0.2 The Efficient Allocation and its Decentralization

Let us first characterize the optimal allocation from the viewpoint of a social planner facing the same resource constraints to which the small open economy is subject in equilibrium (vis a vis the rest of the world), given our assumption of complete markets. In that case, the optimal allocation must maximize $U(C_t, N_t)$ subject to (i) the technological constraint $Y_t = A_t N_t$, (ii) a consumption/output possibilities set implicit in the international risk sharing conditions (18), and (iii) the market clearing condition (25).

Consider the special case of $\sigma = \eta = \gamma = 1$. In that case, (19) and (26) imply the exact expression $C_t = Y_t^{1-\alpha} \, (Y_t^*)^\alpha$. The optimal allocation (from the viewpoint of the small open economy, which takes world output as given) must satisfy,

$$-\frac{U_n(C_t, N_t)}{U_c(C_t, N_t)} = (1-\alpha) \, \frac{C_t}{N_t}$$

which, under our assumed preferences and given $\sigma = 1$, can be written as

$$C_t \; N_t^\varphi = (1-\alpha) \, \frac{C_t}{N_t}$$

thus implying a constant employment $N = (1-\alpha)^{\frac{1}{1+\varphi}}$.

Notice, on the other hand, that the flexible price equilibrium in the small open economy (with corresponding variables denoted with an n superscript) satisfies:

$$\begin{aligned}
1 - \frac{1}{\epsilon} &= MC_t^n \\
&= -\frac{(1-\tau)}{A_t} \, (S_t^n)^\alpha \, \frac{U_{n,t}^n}{U_{c,t}^n} \\
&= \frac{(1-\tau)}{A_t} \, \frac{Y_t^n}{C_t^n} \, (N_t^n)^\varphi \; C_t^n \\
&= (1-\tau) \, (N_t^n)^{1+\varphi}
\end{aligned}$$

where the term on the right hand side of the second equality corresponds to the real wage (net of the subsidy) normalized by productivity, and where the third equality follows from (26).

Hence, by setting τ such that $(1-\tau)(1-\alpha) = 1 - \frac{1}{\epsilon}$ is satisfied (or, equivalently, $\nu = \mu + \log(1-\alpha)$) we guarantee the optimality of the flexible price equilibrium allocation. As in the closed economy case, the optimal monetary policy requires stabilizing the output gap (i.e., $\widetilde{y}_t = 0$, for all t). Equation (37) then implies that domestic prices are also stabilized under that optimal policy ($\pi_{H,t} = 0$ for all t). Thus, in the special case under consideration, (strict) *domestic inflation targeting* (DIT) is indeed the optimal policy.

4.1 Implementation and Macroeconomic Implications

In this section we discuss the implementation a *domestic inflation targeting* policy (DIT) and characterize some of its equilibrium implications. While that policy has been shown to be optimal only for the special case considered above, we look at the implications of that policy for the general case.

4.1.1 Implementation

As discussed above full stabilization of domestic prices implies

$$\widetilde{y}_t = \pi_{H,t} = 0$$

for all t. This in turn implies that $y_t = y_t^n$ and $i_t = r_t^n$ will hold in equilibrium for all t, with all the remaining variables matching their natural levels at all times.

For the reasons discussed in chapter 4, an interest rate rule of the form $i_t = r_t^n$ is associated with an indeterminate equilibrium, and hence does not guarantee that the outcome of full price stability be attained. That result follows from the equivalence between the dynamical system describing the equilibrium of the small open economy and that of the closed economy of chapter 4. As shown there, the indeterminacy problem can be avoided, and the uniqueness of the price stability outcome restored, by having the central bank follow a rule which makes the interest rate respond with sufficient strength to deviations of domestic inflation and/or the output gap from target. More precisely, we can guarantee that the desired outcome is attained if the central bank commits itself to a rule of the form

$$i_t = r_t^n + \phi_\pi \, \pi_{H,t} + \phi_y \, \widetilde{y}_t \tag{43}$$

where $\kappa_\alpha \, (\phi_\pi - 1) + (1 - \beta) \, \phi_y > 0$. Note that, in equilibrium, the term $\phi_\pi \, \pi_{H,t} + \phi_y \, \widetilde{y}_t$ will vanish (since we will have $\widetilde{y}_t = \pi_{H,t} = 0$), implying that $i_t = r_t^n$ all t.

4.1.2 Macroeconomic Implications

Under strict domestic inflation targeting (DIT), the behavior of real variables in the small open economy corresponds to the one we would observe in the absence of nominal rigidities. Hence, we see from inspection of equation (36) that domestic output always increases in response to a positive technology shock at home. As discussed earlier, the sign of the response to a rise in world output is ambiguous, however, and it depends on the sign of Θ, which in turn depends on the size of the substitutability parameters γ and η and the risk aversion parameter σ.

The natural level of the terms of trade is given by:

$$\begin{aligned} s_t^n &= \sigma_\alpha \, (y_t^n - y_t^*) \\ &= \sigma_\alpha \, (\Gamma_0 + \Gamma_a \, a_t - \Phi \, y_t^*) \end{aligned}$$

where $\Phi \equiv \frac{\sigma+\varphi}{\sigma_\alpha+\varphi} > 0$. Thus, given world output, an improvement in domestic technology always leads to a real depreciation, through its expansionary effect on domestic output. On the other hand, an increase in world output always generates an improvement in the domestic terms of trade (i.e., a real appreciation), given domestic technology.

Given that domestic prices are fully stabilized under DIT, it follows that $e_t^{DIT} = s_t^n - p_t^*$, i.e., the nominal exchange rate moves one-for-one with the (natural) terms of trade and (inversely) with the world price level. Assuming constant world prices, the nominal exchange rate will inherit all the statistical properties of the natural terms of trade. Accordingly, the volatility of the nominal exchange rate under DIT will be proportional to the volatility of the gap between the natural level of domestic output (in turn related to productivity) and world output. In particular, that volatility will tend to be low when domestic natural output displays a strong positive comovement with world output. When that comovement is low (or negative), possibly because of a large idiosyncratic component in domestic productivity, the volatility of the terms of trade and the nominal exchange rate under DIT will be enhanced.

We can also derive the implied equilibrium process for the CPI. Given the constancy of domestic prices it is given by:

$$\begin{aligned} p_t^{DIT} &= \alpha \ (e_t^{DIT} + p_t^*) \\ &= \alpha \ s_t^n \end{aligned}$$

Thus, we see that under the DIT regime, the CPI level will also vary with the (natural) terms of trade and will inherit its statistical properties. If the economy is very open, and if domestic productivity (and hence the natural level of domestic output) is not much synchronized with world output, CPI prices could potentially be highly volatile, even if the domestic price level is constant.

An important lesson emerges from the previous analysis: potentially large and persistent fluctuations in the nominal exchange rate as well as in some inflation measures (like the CPI) are not necessarily undesirable, nor do they require a policy response aimed at dampening such fluctuations. Instead, and especially for an economy that is very open and subject to large idiosyncratic shocks, those fluctuations may be an equilibrium consequence of the adoption of an optimal policy, as illustrated by the model above.

4.2 The Welfare Costs of Deviations from the Optimal Policy

Under the particular assumptions for which strict domestic inflation targeting has been shown to be optimal (i.e., log utility and unit elasticity of substitution between goods of different origin), it is relatively straightforward to derive a second order approximation to the utility losses of the domestic representative consumer resulting from deviations from the optimal policy. Those losses, expressed as a fraction of steady state consumption, can be written as:

$$\mathbb{W} = -\ \frac{(1-\alpha)}{2} \sum_{t=0}^{\infty} \beta^t \ \left[\frac{\epsilon}{\lambda} \ \pi_{H,t}^2 + (1+\varphi) \ \widetilde{y}_t^2 \right] \tag{44}$$

The derivation of (44) goes along the lines of that for the closed economy shown the appendix of chapter 4. The reader is referred to Galí and Monacelli (2005) for the details specific to (44).

The expected period welfare losses of any policy that deviates from strict inflation targeting can be written in terms of the variances of inflation and the output gap:

$$\mathbb{V} = -\frac{(1-\alpha)}{2}\left[\frac{\epsilon}{\lambda}\ var(\pi_{H,t}) + (1+\varphi)\ var(\widetilde{y}_t)\right] \tag{45}$$

Note that the previous expressions for the welfare losses are, up to the proportionality constant $(1-\alpha)$, identical to the ones derived for the closed economy in chapter 4, with domestic inflation (and not CPI inflation) being the relevant inflation variable. Below we make use of (45) to assess the welfare implications of alternative monetary policy rules, and to rank those rules on welfare grounds.

5 Simple Monetary Policy Rules for the Small Open Economy

In the present section we analyze the macroeconomic implications of three alternative monetary policy regimes for the small open economy. Two of the simple rules considered are stylized Taylor-type rules. The first has the domestic interest rate respond systematically to domestic inflation, whereas the second assumes that CPI inflation is the variable the domestic central bank reacts to. The third rule we consider is one that pegs the effective nominal exchange rate. Formally, the *domestic inflation-based Taylor rule* (DITR, for short) is specified as follows:

$$i_t = \rho + \phi_\pi\ \pi_{H,t}$$

The *CPI inflation-based Taylor rule* (CITR, for short) is assumed to take the form:

$$i_t = \rho + \phi_\pi\ \pi_t$$

Finally, the *exchange rate peg* (PEG, for short) implies

$$e_t = 0$$

for all t.

Below we provide a comparison of the equilibrium properties of several macroeconomic variables under the above simple rules for a calibrated version of our model economy. We compare such properties to those associated with a strict domestic inflation targeting (DIT), the policy that is optimal under the conditions discussed above, and which we assume to be satisfied in our baseline calibration. As much of the present chapter the analysis draws directly from Galí and Monacelli (2005).

5.1 A Numerical Analysis of Alternative Rules

5.1.1 Calibration

In this section we present some quantitative results based on a calibrated version of our small open economy. In our baseline calibration we set $\sigma = \eta = \gamma = 1$, in a way consistent with the special case considered above. We assume $\varphi = 3$, which implies a labor supply elasticity of $\frac{1}{3}$. We set ϵ, the elasticity of substitution between differentiated goods (of the same origin) to be equal to 6, thus implying a steady-state markup of 20 percent. Parameter θ is set equal to 0.75, a value consistent with an average period of one year between price adjustments. We assume $\beta = 0.99$, which implies a riskless annual return of about 4 percent in the steady state. We set a baseline value for α (the degree of openness) of 0.4. The latter corresponds roughly to the import/GDP ratio in Canada, which we take as a prototype small open economy. In the calibration of the interest rate rules we follow the original Taylor calibration and set ϕ_π equal to 1.5.

In order to calibrate the stochastic properties of the exogenous driving forces, we fit AR(1) processes to (log) labor productivity in Canada (our proxy for domestic productivity), and (log) U.S. GDP (taken as a proxy for world output), using quarterly, HP-filtered data over the sample period 1963:1 2002:4. We obtain the following estimates (with standard errors in brackets):

$$\begin{aligned} a_t &= \underset{(0.06)}{0.66}\, a_{t-1} + \varepsilon_t^a, \quad \sigma_a = 0.0071 \\ y_t^* &= \underset{(0.04)}{0.86}\, y_{t-1}^* + \varepsilon_t^*, \quad \sigma_{y^*} = 0.0078 \end{aligned}$$

with $corr(\varepsilon_t^a, \varepsilon_t^*) = 0.3$.

5.1.2 Impulse Responses

We start by describing the dynamic effects of a *domestic* productivity shock on a number of macroeconomic variables. Figure *1* displays the impulse responses to a unit innovation in a_t under the four regimes considered. By construction, domestic inflation and the output gap remain unchanged under the optimal policy (DIT). We also see that the shock leads to a persistent reduction in the domestic interest rate, as it is needed in order to support the transitory expansion in consumption and output consistent with the flexible price equilibrium allocation. Given the constancy of the world nominal interest rate, uncovered interest parity implies an initial nominal depreciation followed by expectations of a future appreciation, as reflected in the response of the nominal exchange rate. Given constant world prices and the stationarity of the terms of trade, the constancy of domestic prices implies a mean-reverting response of the nominal exchange rate.

It is interesting to contrast the implied dynamic behavior of the same variables under the optimal policy to the one under the two stylized Taylor rules (DITR and CITR). Notice, at first, that both rules generate, unlike the optimal policy, a permanent fall in both domestic and CPI prices. The unit root in domestic prices is then mirrored, under both rules, by the unit root in the nominal exchange rate.

A key difference between the two Taylor rules concerns the behavior of the terms of trade. While under DITR there is a real depreciation on impact, with the terms of trade reverting gradually to the steady state after that (mirroring closely the response under the optimal policy), under CITR the initial response of the terms of trade is more muted, and is followed by a hump-shaped pattern. The intuition is simple. Under both rules, the rise in domestic productivity and the required real depreciation lead, for *given domestic prices*, to an increase in CPI inflation. However, under CITR the desired stabilization of CPI inflation is partly achieved, relative to DITR, by means of a more muted response of the terms of trade (since the latter affect the CPI), and a fall in domestic prices. The latter, in turn, requires a negative output gap and hence a more contractionary monetary policy (i.e., a higher interest rate). Under our calibration that policy response takes the form of an initial rise in both the nominal and real interest rates, with the subsequent path of the real rate remaining systematically above that implied by the optimal policy or a DITR policy.

Finally, the same figure displays the corresponding impulse responses under the PEG policy. Notice that the responses of output gap and inflation are qualitatively similar to the CITR case. However, the impossibility of lowering the nominal rate and letting the currency depreciate, as would be needed in order to support the expansion in consumption and output required to replicate the flexible price allocation, leads to a very limited response in the terms of trade, and in turn an amplification of the negative response of domestic inflation and the output gap. Interestingly, under a PEG, the complete stabilization of the nominal exchange rate generates stationarity of the domestic price level and, in turn, also of the CPI level (given the stationarity in the terms of trade). This is a property that the PEG regime shares with the optimal policy as specified above. The stationarity in the price level also explains why, in response to the shock, domestic inflation initially falls and then rises persistently above the steady state.

As discussed below the different dynamics of the terms of trade are unambiguously associated with a welfare loss, relative to the optimal policy.

5.1.3 Second Moments and Welfare Losses

In order to complement our quantitative analysis, Table 1 reports business cycle properties of several key variables under alternative monetary policy regimes. The numbers confirm some of the findings that were already evident from visual inspection of the impulse responses. Thus we see that the critical element that distinguishes each simple rule relative to the optimal policy is the excess smoothness of both the terms of trade and the (first-differenced) nominal exchange rate.[14] This in turn is reflected in too high a volatility of the output gap and domestic inflation under the simple rules. In particular, the PEG regime is the one that amplifies both output gap and inflation volatility to the largest extent, with the CITR regime lying somewhere in between. Furthermore, notice that the terms of trade are more stable under an exchange rate peg than under any other policy regime. That finding, which is consistent with the evidence of Mussa (1986), points to the existence of "excess smoothness" in real exchange rates under fixed exchange rates. That feature is a consequence of the inability of prices (which are sticky) to compensate for the constancy of the nominal exchange rate.[15]

Table 2 reports the welfare losses associated with the three simple rules

[14] We report statistics for the nominal *depreciation* rate, as opposed to the level, given that both DITR and CITR imply a unit root in the nominal exchange rate.

[15] See Monacelli (2004) for a detailed analysis of the implications of fixed exchange rates.

analyzed in the previous section: DITR, CITR and PEG. There are four panels in this table. The top panel reports welfare losses in the case of our benchmark parameterization, while the remaining three panels display the effects of lowering the steady-state markup (as implied by an increase in ϵ), the elasticity of labor supply, and both. All entries are to be read as percentage units of steady state consumption, and in deviation from the first best represented by DIT. Under our baseline calibration all rules are suboptimal since they involve nontrivial deviations from full domestic price stability. Also one result stands out clearly: under all the calibrations considered an exchange rate peg implies a substantially larger deviation from the first best than DITR and CITR, as one may have anticipated from the quantitative evaluation of the second moments conducted above. However, and as is usually the case in welfare exercises of this sort found in the literature, the implied welfare losses are quantitatively small for all policy regimes.

We consider next the effect of lowering, respectively, the steady-state markup to 1.1, by setting $\epsilon = 11$ (which implies a larger penalization of inflation variability in the loss function) and the elasticity of labor supply to 0.1 (which implies a larger penalization of output gap variability). This has a general effect of generating a substantial magnification of the welfare losses relative to the benchmark case, especially in the third exercise where both parameters are lowered simultaneously. In the case of low markup and low elasticity of labor supply, the PEG regime leads to non trivial welfare losses relative to the optimum. Notice also that under all scenarios considered here the two stylized Taylor rules, DITR and CITR, imply very similar welfare losses. While this points to a substantial irrelevance in the specification of the inflation index in the monetary authority's interest rate rule, the same result may once again be sensitive to the assumption of complete exchange rate pass-through specified in our context.

6 Notes on the Literature

Earlier work on optimizing open economy models with nominal rigidities focused on the transmission of monetary policy shocks, typically represented as disturbances to an exogenous stochastic process for the money supply.[16] A key contribution in that area is Obstfeld and Rogoff (1995), who develop a two country model where monopolistically competitive firms set prices before

[16] See Lane (1999) for an excellent survey of the early steps in that literature.

the realization of the shocks (i.e. one period in advance). The framework is used to analyze the dynamics of the exchange rate and other variables in response to a change in the money supply (and government spending), and the welfare effects resulting from that intervention. An earlier paper, by Svensson and van Wijnbergen (1989) contains a related analysis, under the assumption of full risk-sharing among consumers from different countries.

Corsetti and Pesenti (2001) develop a version of the Obstfeld-Rogoff model that allows for home-bias in preferences, leading to terms of trade effects in response to shocks that are argued to have potentially important welfare effects. Betts and Devereux (2000) revisit the analysis in Obstfeld and Rogoff (1995) while departing from the assumption of the law of one price found in the latter paper. In particular they allow firms to price discriminate across markets and assuming they set prices (in advance) in terms of the currency of the importing country ("pricing to market").

The effects of money supply shocks on the persistence and volatility of nominal and real exchange rates are analyzed under the assumption of staggered price-setting in Kollmann (2001) and Chari, Kehoe and McGrattan (2002).[17] The assumption of staggered price setting (and staggered wage setting in Kollmann's case) induces much richer and more realistic dynamics than that of price setting one period in advance.

A more recent strand of the literature has attempted to go beyond the analysis of the transmission of exogenous monetary policy shocks, and has focused instead on the implications of sticky price open economy models for the design of optimal monetary policy, using a welfare theoretic approach.[18] Early examples of papers analyzing the properties of alternative monetary policy arrangements in a two-country setting assumed that prices are set one period in advance, They include the work of Obstfeld and Rogoff (2002) and Benigno and Benigno (2003), both using the assumption of producer currency pricing, and Bacchetta and van Wincoop (2001), Sutherland (2002), Devereux and Engel (2003), and Corsetti and Pesenti (2005) in the context of economies with local currency pricing.

More recent frameworks have instead adopted the staggered price-setting structure à la Calvo. Galí and Monacelli (2005), on which the analysis of

[17] Kollmann (2001) assumes prices and wages are set à la Calvo–as in the model of the present chapter–whereas Chari et al. (2002) assume price setting à la Taylor, i.e. with deterministic price durations.

[18] Ball (1999) and Svensson (2000) carry out an analysis similar in spirit, but in the context of non-optimizing models.

the present chapter is based, is an illustration of work along those lines for a small open economy. An extension of that framework incorporating cost-push shocks can be found in Clarida, Galí and Gertler (2001). Kollmann (2002) considers a more general model of a small open economy, with several sources of shocks, and carries out a numerical analysis of the welfare implications of alternative rules. Using a similar framework as a staring point, Monacelli (2005) shows that the introduction of imperfect pass-through generates a trade-off between stabilization of domestic inflation and the output gap, leading to gains from commitment similar to those analyzed in chapter 5 for the closed economy.

Finally, the papers by Clarida, Galí and Gertler (2002), Pappa (2004), and Benigno and Benigno (2006) depart from the assumption of a small open economy and analyze the consequences of alternative monetary policy arrangement in a two-country framework with staggered price setting à la Calvo, and with a special focus on the gains from cooperation.

Appendix. The Perfect Foresight Steady State

In order to show how the home economy's terms of trade are uniquely pinned down in the perfect foresight steady state, we invoke symmetry among all countries (other than the home country), and then show how the terms of trade and output in the home economy are determined. Without loss of generality, we assume a unit value for productivity in all foreign countries, and a productivity level A in the home economy. We show that in the symmetric case (when $A = 1$) the terms of trade for the home economy must necessarily be equal to unity in the steady state, whereas output in the home economy coincides with that in the rest of the world.

First, notice that the goods market clearing condition, when evaluated at the steady state, implies:

$$\begin{aligned}
Y &= (1-\alpha)\left(\frac{P_H}{P}\right)^{-\eta} C + \alpha \int_0^1 \left(\frac{P_H}{\mathcal{E}_i P_F^i}\right)^{-\gamma} \left(\frac{P_F^i}{P^i}\right)^{-\eta} C^i \, di \\
&= \left(\frac{P_H}{P}\right)^{-\eta} \left[(1-\alpha)\ C + \alpha \int_0^1 \left(\frac{\mathcal{E}_i P_F^i}{P_H}\right)^{\gamma-\eta} \mathcal{Q}_i^{\eta}\ C^i \, di\right] \\
&= h(\mathcal{S})^{\eta}\ C\ \left[(1-\alpha) + \alpha \int_0^1 \left(\mathcal{S}^i\ \mathcal{S}_i\right)^{\gamma-\eta}\ \mathcal{Q}_i^{\eta-\frac{1}{\sigma}}\ di\right] \\
&= h(\mathcal{S})^{\eta}\ C\ \left[(1-\alpha) + \alpha\ \mathcal{S}^{\gamma-\eta}\ q(\mathcal{S})^{\eta-\frac{1}{\sigma}}\right]
\end{aligned}$$

where we have made use of (18) and of the relationship

$$\begin{aligned}
\frac{P}{P_H} &= \left[(1-\alpha) + \alpha \int_0^1 (\mathcal{S}_i)^{1-\eta}\, di\right]^{\frac{1}{1-\eta}} \\
&= \left[(1-\alpha) + \alpha\ (\mathcal{S})^{1-\eta}\right]^{\frac{1}{1-\eta}} \equiv h(\mathcal{S})
\end{aligned}$$

and we have substituted $\mathcal{Q} = \frac{\mathcal{S}}{h(\mathcal{S})} \equiv q(\mathcal{S})$. Notice that $q(\mathcal{S})$ is strictly increasing in $\mathcal{S}$.

Under the assumptions above, the international risk sharing condition implies that the relationship

$$\begin{aligned}
C &= C^*\ \mathcal{Q}^{\frac{1}{\sigma}} \\
&= C^*\ q(\mathcal{S})^{\frac{1}{\sigma}}
\end{aligned}$$

must also hold in the steady state.

Hence, combining the two relations above, and imposing the world market clearing condition $C^* = Y^*$ we obtain

$$
\begin{aligned}
Y &= \left[(1-\alpha)\ h(\mathcal{S})^{\eta}\ q(\mathcal{S})^{\frac{1}{\sigma}} + \alpha\ \mathcal{S}^{\gamma-\eta} h(\mathcal{S})^{\eta} q(\mathcal{S})^{\eta}\right]\ Y^* \\
&= \left[(1-\alpha)\ h(\mathcal{S})^{\eta}\ q(\mathcal{S})^{\frac{1}{\sigma}} + \alpha\ h(\mathcal{S})^{\gamma} q(\mathcal{S})^{\gamma}\right]\ Y^* \\
&\equiv v(\mathcal{S})\ Y^* \qquad (46)
\end{aligned}
$$

where $v(\mathcal{S}) > 0$, $v'(\mathcal{S}) > 0$, and $v(1) = 1$.

Furthermore, the clearing of the labor market in steady state implies

$$
\begin{aligned}
C^{\sigma}\left(\frac{Y}{A}\right)^{\varphi} &= \frac{W}{P} \\
&= A\ \frac{1-\frac{1}{\epsilon}}{(1-\tau)}\frac{P_H}{P} \\
&= A\ \frac{1-\frac{1}{\epsilon}}{(1-\tau)}\ \frac{1}{h(\mathcal{S})}
\end{aligned}
$$

which, when combined with the sharing condition above, yields

$$
Y = A^{\frac{1+\varphi}{\varphi}}\ \left(\frac{1-\frac{1}{\epsilon}}{(1-\tau)\ (Y^*)^{\sigma}\ \mathcal{S}}\right)^{\frac{1}{\varphi}} \qquad (47)
$$

Notice that, conditional on A and Y^*, (46) and (47) constitute a system of two equations in Y and S, with a unique solution, given by

$$
Y = Y^* = A^{\frac{1+\varphi}{\sigma+\varphi}}\left(\frac{1-\frac{1}{\epsilon}}{1-\tau}\right)^{\frac{1}{\sigma+\varphi}}
$$

and

$$
\mathcal{S} = 1
$$

which in turn must imply $S_i = 1$ for all i.

References

Bacchetta, Philippe, and Eric van Wincoop (2000): "Does Exchange Rate Stability Increase Trade and Welfare?", *American Economic Review*, 90:5, 1093-1109

Ball, Laurence (1999): "Policy Rules for Open Economies," in J.B. Taylor ed., *Monetary Policy Rules*, University of Chicago Press.

Benigno, Gianluca and Pierpaolo Benigno (2003): "Price Stability in Open Economies", *Review of Economic Studies*

Benigno, Gianluca and Pierpaolo Benigno (2006): "Designing Targeting Rules for International Monetary Policy Cooperation," *Journal of Monetary Economics*, vol. 53 (3), 473-506.

Benigno, Pierpaolo and Michael Woodford (2005): "Inflation Stabilization and Welfare: The Case of a Distorted Steady State," *Journal of the European Economic Association* 3 (6), 1185-1236.

Betts, Caroline and Michael B. Devereux (2000): "Exchange Rate Dynamics in a Model of Pricing-to-Market", *Journal of International Economics* 50,1,215-244.

Chari, V.V., Patrick Kehoe, and Ellen McGrattan (2002): "Monetary Shocks and Real Exchange Rates in Sticky Price Models of International Business Cycles," *Review of Economic Studies* 69, 533-563

Clarida, Richard, Jordi Galí, and Mark Gertler (2001): "Optimal Monetary Policy in Open vs. Closed Economies: An Integrated Approach," *American Economic Review*, vol. 91, no. 2, 248-252.

Clarida, Richard, Jordi Galí, and Mark Gertler (2002): "A Simple Framework for International Monetary Policy Analysis," *Journal of Monetary Economics*, vol. 49, no. 5, 879-904.

Corsetti, Giancarlo and Paolo Pesenti (2001): "Welfare and Macroeconomic Interdependence," *Quarterly Journal of Economics* vol. CXVI,issue 2, 421-446.

Corsetti, Giancarlo and Paolo Pesenti (2005): "International Dimensions of Monetary Policy," *Journal of Monetary Economics*, 52/2 pp 281-305.

de Paoli, Bianca S.C. (2006): "Welfare and Macroeconomic Policy in Small Open Economies," Ph.D. Dissertation, London School of Economics.

Devereux, Michael B. and Charles Engel (2003): "Monetary Policy in the Open Economy Revisited: Exchange Rate Flexibility and Price Setting Behavior", *Review of Economic Studies,* 70, 765-783.

Faia, Ester and Tommaso Monacelli (2007): "Optimal Monetary Policy in a Small Open Economy with Home Bias," mimeo.

Galí, Jordi, and Tommaso Monacelli (2005): "Monetary Policy and Exchange Rate Volatility in a Small Open Economy," *Review of Economic Studies*, vol. 72, issue 3, 2005, 707-734.

Kollmann, Robert (2001): "The Exchange Rate in a Dynamic Optimizing Current Account Model with Nominal Rigidities: A Quantitative Investigation," *Journal of International Economics* vol.*55, 243-262.*

Kollmann, Robert (2002): "Monetary Policy Rules in the Open Economy: Effects on Welfare and Business Cycles", *Journal of Monetary Economics*, Vol.49, pp.989-1015.

Lane, Philip R. (1999): "The New Open Economy Macroeconomics: A Survey," *Journal of International Economics*, vol. 54, 235-266.

Monacelli, Tommaso (2004): "Into the Mussa Puzzle: Monetary Policy Regimes and the Real Exchange Rate in a Small Open Economy," *Journal of International Economics,* 62, 191-217.

Monacelli, Tommaso (2005): "Monetary Policy in a Low Pass-Through Environment," *Jornal of Money, Credit and Banking*, vol 37 (6), 1048-1066.

Mussa, Michael (1986): "Nominal Exchange Rate Regimes and the Behavior of Real Exchange Rates," *Carnegie-Rochester Conference Series on Public Policy*, 117-213.

Obstfeld, Maurice and Kenneth Rogoff (1995): "Exchange Rate Dynamics Redux," *Journal of Political Economy* 103, no. 3, 624-660.

Obstfeld, Maurice and Kenneth Rogoff (2002): "Global Implications of Self-Oriented National Monetary Rules," *Quarterly Journal of Economics*, vol. CXVII, issuue 2, 503-535.

Pappa, Evi (2004): "Should the Fed and the ECB Cooperate ? Optimal Monetary Policy in a Two-Country World," *Journal of Monetary Economics*, vol 51 (4), 753-780.

Sutherland Alan (2003): "International Monetary Policy Coordination and Financial Market Integration", Mimeo University of St. Andrews.

Svensson, Lars E.O. (2000): "Open-Economy Inflation Targeting," *Journal of International Economics*, vol. 50, no. 1.

Svensson, Lars E.O. and Sweder van Wijnbergen (1989): "Excess Capacity, Monopolistic Competition, and International Transmission of Monetary Disturbances," *The Economic Journal*, vol 99, 785-805.

Exercises

1. A Small Open Economy Model

Consider a small open economy where no international trade in assets is allowed (implying that trade is always balanced). Hence,

$$p_t + c_t = p_{H,t} + y_t$$

where c_t denotes consumption, y_t is output, $p_{H,t}$ is the domestic price level, p_t is the CPI (all in logs). Assuming a constant price level in the rest of the world ($p_t^* = 0$), we can write

$$p_t = (1 - \alpha)\ p_{H,t} + \alpha\ e_t$$

where e_t is the nominal exchange rate.

Let $s_t \equiv e_t - p_{H,t}$ denote the terms of trade. Under the assumption of a unit elasticity of substitution between foreign and domestic goods we have

$$s_t = y_t - y_t^*$$

where y_t^* is (log) output in the rest of the world (assumed to evolve exogenously). The domestic aggregate technology can be written as

$$y_t = a_t + n_t$$

where a_t is an exogenous technology process. We assume perfect competition in both goods and labor markets, with flexible prices and wages. The labor supply takes the form:

$$w_t - p_t = \sigma\ c_t + \varphi\ n_t$$

Finally, we assume a money demand function $m_t - p_t = c_t$

a) Determine the equilibrium processes for output, consumption, the terms of trade, and the nominal exchange rate in the small open economy, as a function of productivity a_t, foreign output y_t^*, and the money supply under the assumption that the latter evolves exogenously. Discuss the implications of assuming $\sigma = 1$.

b) How would your answer have to be modified if a fixed nominal exchange rate regime was in place?

c) Discuss, in words, how some of the results in a) and b) would change qualitatively in the presence of imperfect competition and sticky prices.

2. The Effects of Technology Shocks in the Open Economy

Consider the small open economy model described in the present chapter. The equilibrium dynamics for domestic inflation $\pi_{H,t}$ and the output gap $\widetilde{y}_t$ are described by the equations:

$$\pi_{H,t} = \beta \; E_t\{\pi_{H,t+1}\} + \kappa_\alpha \; \widetilde{y}_t$$

$$\widetilde{y}_t = E_t\{\widetilde{y}_{t+1}\} - \frac{1}{\sigma_\alpha} \; (i_t - E_t\{\pi_{H,t+1}\} - r_t^n)$$

and where r_t^n is given by

$$r_t^n = \rho - b \; a_t$$

Natural output is in turn given by

$$y_t^n = d \; a_t$$

The technology parameter folloxs a stationary AR(1) process

$$a_t = \rho_a \; a_{t-1} + \varepsilon_t^a$$

where $\rho_a \in [0, 1)$.

Assume that the monetary authority follows the simple interest rate rule

$$i_t = \rho + \phi_\pi \; \pi_{H,t}$$

where $\phi_\pi > 1$.

a) Determine the response of output, domestic inflation, the terms of trade and the nominal exchange rate to a positive domestic technology shock (note: for the purposes of the present exercise we assume $y_t^* = p_t^* = 0$ all t)

b) Suppose that the central bank pegs the nominal exchange rate, so that $e_t = 0$ for all t. Characterize the economy's response to a technology shock in that case.

TABLE 1

Cyclical properties of alternative policy regimes

	Optimal sd%	DI Taylor sd%	CPI Taylor sd%	Peg sd%
Output	0·95	0·68	0·72	0·86
Domestic inflation	0·00	0·27	0·27	0·36
CPI inflation	0·38	0·41	0·27	0·21
Nominal I. rate	0·32	0·41	0·41	0·21
Terms of trade	1·60	1·53	1·43	1·17
Nominal depr. rate	0·95	0·86	0·53	0·00

Note: Sd denotes standard deviation in %.

TABLE 2

Contribution to welfare losses

	DI Taylor	CPI Taylor	Peg
Benchmark $\mu = 1{\cdot}2, \varphi = 3$			
Var(domestic infl)	0·0157	0·0151	0·0268
Var(output gap)	0·0009	0·0019	0·0053
Total	0·0166	0·0170	0·0321
Low steady state mark-up $\mu = 1{\cdot}1, \varphi = 3$			
Var(Domestic infl)	0·0287	0·0277	0·0491
Var(Output gap)	0·0009	0·0019	0·0053
Total	0·0297	0·0296	0·0544
Low elasticity of labour supply $\mu = 1{\cdot}2, \varphi = 10$			
Var(Domestic infl)	0·0235	0·0240	0·0565
Var(Output gap)	0·0005	0·0020	0·0064
Total	0·0240	0·0261	0·0630
Low mark-up and elasticity of labour supply $\mu = 1{\cdot}1, \varphi = 10$			
Var(Domestic infl)	0·0431	0·0441	0·1036
Var(Output gap)	0·0005	0·0020	0·0064
Total	0·0436	0·0461	0·1101

Note: Entries are percentage units of steady state consumption.

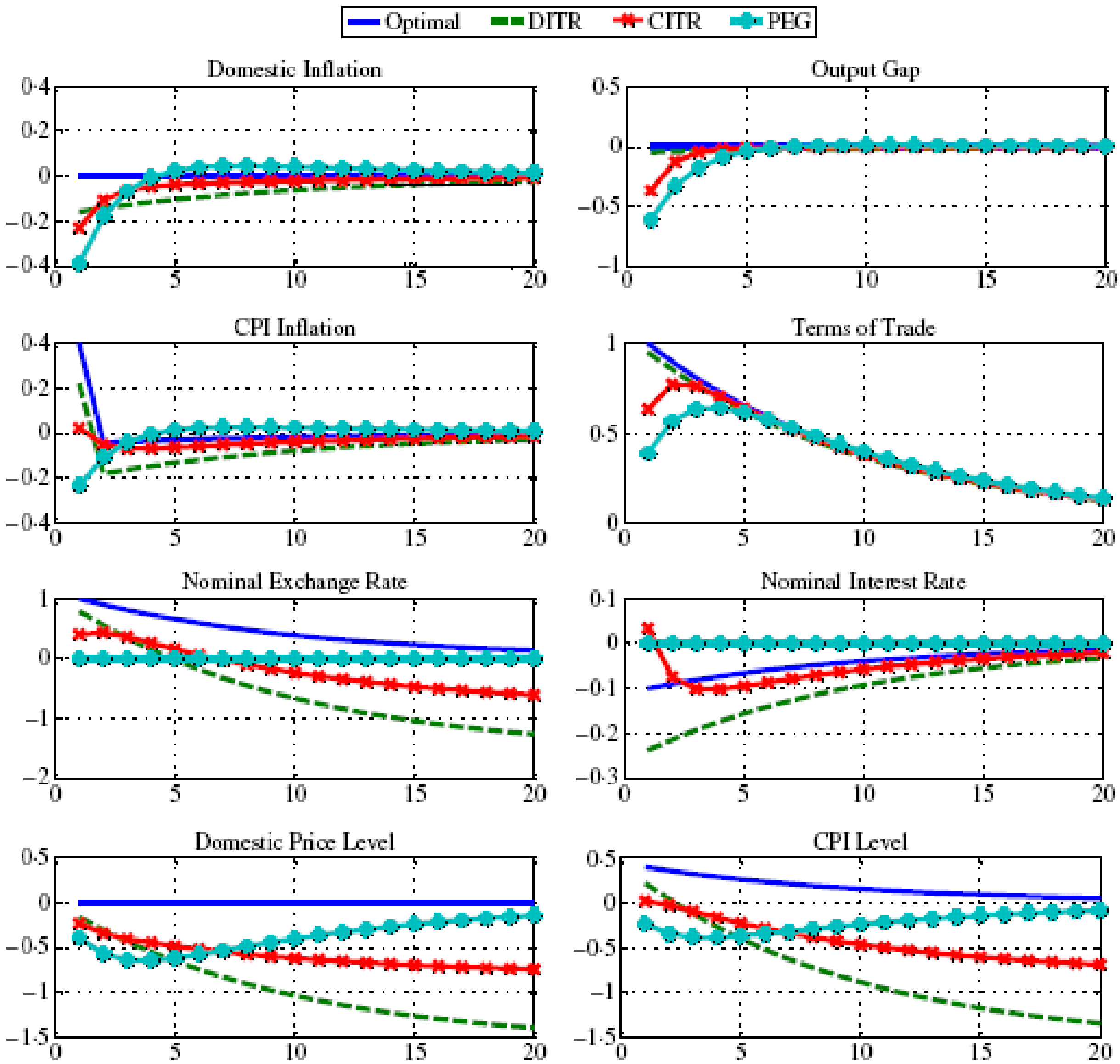

FIGURE 1

Impulse responses to a domestic productivity shock under alternative policy rules