Política Econômica — EPGE/FGV — fls

5ª LISTA DE EXERCÍCIOS

(A) O problema do Banco Central, quando adota uma política discricionária resolve seu problema de otimização a cada período de tempo.

Esse problema pode ser apresentado como:

$$\min_{\{\pi_t, x_t, i_t\}} \pi_t^2 + \alpha_x x_t^2 \quad , \forall t$$

s.t. (i) $\pi_t = \beta . E_t\{\pi_{t+1}\} + k . x_t + u_t$

(ii) $x_t = E_t\{x_{t+1}\} - \left(\frac{1}{\sigma}\right)(i_t - E_t\{\pi_{t+1}\} - r_t^e)$

O lagrangeano associado a esse problema será:

$$\mathcal{L} = \pi_t^2 + \alpha_x . x_t^2 + \lambda_t\left[\pi_t - \beta E_t\{\pi_{t+1}\} - k . x_t - u_t\right] + \mu_t\left[x_t - E_t\{x_{t+1}\} + \left(\frac{1}{\sigma}\right)(i_t - E_t\{\pi_{t+1}\} - r_t^e)\right]$$

As condições de primeira ordem associadas serão:

$[\pi_t]$: $2\pi_t + \lambda_t = 0 \Rightarrow \lambda_t = -2\pi_t$ (1)

$[x_t]$: $2 . \alpha_x . x_t - \lambda_t . k + \mu_t = 0$ (2)

$[i_t]$: $\mu_t . \sigma^{-1} = 0 \Rightarrow \mu_t = 0$ (3) ⇒ Note que isso implica que a restrição (ii) não será ativa.

De (3) em (2) ⇒ $2 . \alpha_x . x_t = \lambda_t . k$ (4)

Substituindo (1) em (4) ⇒ $\cancel{2} . \alpha_x . x_t = -\cancel{2} . \pi_t . k$

$$\Rightarrow x_t = -\frac{k}{\alpha_x} . \pi_t \quad (5)$$

Substituindo (5) em (i):

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$$\pi_t = \beta \cdot E_t\{\pi_{t+1}\} + k\left(\frac{-k}{\alpha_x}\cdot \pi_t\right) + u_t = \beta \cdot E_t\{\pi_{t+1}\} - \frac{k^2}{\alpha_x}\pi_t + u_t$$

$$\pi_t \underbrace{\left(1+\frac{k^2}{\alpha_x}\right)}_{\frac{\alpha_x+k^2}{\alpha_x}} = \beta \cdot E_t\{\pi_{t+1}\} + u_t \Rightarrow \pi_t = \left(\frac{\alpha_x \cdot \beta}{\alpha_x + k^2}\right) \cdot E_t\{\pi_{t+1}\} + \left(\frac{\alpha_x}{\alpha_x+k^2}\right) u_t \quad (6)$$

Vamos resolver (6) pelo método dos coeficientes indeterminados:

Como u_t é o choque exógeno, vamos chutar como solução

$$\pi_t = \gamma \cdot u_t \quad (7) \Rightarrow E_t\{\pi_{t+1}\} = E_t\{\gamma \cdot \rho_u u_t + \varepsilon_t^u\} = \gamma \cdot \rho_u \cdot u_t$$

Substituindo (7) em (6), temos:

$$\gamma \cdot u_t = \left(\frac{\alpha_x \cdot \beta}{\alpha_x + k^2}\right)\gamma \cdot \rho_u \cdot u_t + \left(\frac{\alpha_x}{\alpha_x + k^2}\right) \cdot u_t$$

$$\Rightarrow \gamma \underbrace{\left(1 - \frac{\alpha_x \cdot \beta \rho_u}{\alpha_x + k^2}\right)}_{\frac{\alpha_x + k^2 - \alpha_x \beta \rho_u}{\alpha_x + k^2} = \frac{k^2 + \alpha_x(1-\beta \cdot \rho_u)}{\alpha_x + k^2}} = \frac{\alpha_x}{\alpha_x + k^2} \Rightarrow \gamma = \frac{\cancel{\alpha_x + k^2}}{k^2 + \alpha_x(1-\beta\cdot\rho_u)} \cdot \frac{\alpha_x}{\cancel{(\alpha_x + k^2)}}$$

Em (7)

$$\boxed{\pi_t = \frac{\alpha_x}{k^2 + \alpha_x(1-\beta\cdot\rho_u)} \cdot u_t} \quad (8)$$

Usando (8) em (5):

$$\boxed{x_t = -\frac{k}{k^2 + \alpha_x(1-\beta\cdot\rho_u)} \cdot u_t} \quad (9)$$

(B) Analisaremos agora a política ótima sob commitment. Nesse caso, o Banco Central escolhe hoje uma sequência $\{x_t\}_{t\geq 0}$ e $\{\pi_t\}_{t\geq 0}$ com a qual se compromete para todos os períodos futuros. Lembre, do item anterior, que a restrição implicada pela curva IS não é ativa.

O problema resolvido será:

$$\min_{\{x_t,\pi_t\}_{t=0}^{\infty}} \left(\frac{1}{2}\right) E_0\left\{\sum_{t=0}^{\infty} \beta^t\left(\pi_t^2+\alpha_x . x_t^2\right)\right\}$$

$$S.t. \quad (i) \quad \pi_t=\beta . E_t\{\pi_{t+1}\}+k x_t+u_t$$

O lagrangeano será:

$$\mathcal{L}= E_0\left\{\sum_{t=0}^{\infty} \beta^t\left[\left(\frac{1}{2}\right)\left(\pi_t^2+\alpha_x . x_t^2\right)+\mu_t\left(\pi_t-\beta E_t\{\pi_{t+1}\}-k . x_t-u_t\right)\right]\right.$$

Cpo's:

$[\pi_t]$: $\beta^t\left(\frac{1}{\cancel{2}}\right) . \cancel{2}\pi_t+\mu_t+\beta^{t-1} \mu_{t-1} . \beta .=0 \Rightarrow \pi_t+\mu_t-\mu_{t-1}=0$ (1)

$[x_t]$: $\beta^t\left(\frac{1}{\cancel{2}}\right) \alpha_x . \cancel{2} . x_t-\beta^t . \mu_t . k \Rightarrow \alpha_x . x_t=\mu_t . k$ (2)

De (2) em (1) $\Rightarrow \pi_t+\dfrac{\alpha_x . x_t}{k}-\dfrac{\alpha_x . x_{t-1}}{k}=0$

$\rightarrow \pi_t=-\dfrac{\alpha_x}{k}\left(x_t-x_{t-1}\right)$ (3) . Substituindo (3) em (i), temos:

$$-\frac{\alpha_x}{k}\left(x_t-x_{t-1}\right)=\beta E_t\left\{-\frac{\alpha_x}{k} .\left(x_{t+1}-x_t\right)\right\}+k . x_t+u_t$$

$$\underbrace{x_t\left(-\frac{\alpha_x}{k}-\frac{\beta . \alpha_x}{k}-k\right)}_{\frac{-\alpha_x-\beta . \alpha_x-k^2}{k}=\frac{-\alpha_x(1+\beta)-k^2}{k}}=-\frac{\alpha_x}{k} . x_{t-1}-\frac{\alpha_x . \beta}{k} . E_t\left\{x_{t+1}\right\}+u_t$$

$$-x_t\left(\frac{\alpha_x(1+\beta)+k^2}{k}\right) = -\frac{\alpha_x}{k}\cdot x_{t-1} - \frac{\alpha_x.\beta}{k}.E_t\{x_{t+1}\} + u_t$$

$$\Rightarrow \quad x_t = \underbrace{\frac{\alpha_x}{\alpha_x(1+\beta)+k^2}}_{\equiv a} x_{t-1} + \underbrace{\frac{\alpha_x.\beta}{\alpha_x(1+\beta)+k^2}}_{a.\beta} E_t\{x_{t+1}\} - \underbrace{\frac{k}{\alpha_x(1+\beta)+k^2}}_{\frac{k.a}{\alpha_x}} .u_t$$

Vamos propor como solução $x_t = \theta.u_t$ (4)

$\frac{k.a}{\alpha_x}$

Assim $E_t\{x_{t+1}\} = \theta.E_t\{\rho_u.u_t + \varepsilon^u_{t+1}\} = \theta.\rho_u.u_t.$

$$\theta.\rho_u.u_t = a.\theta.u_{t-1} + a.\beta.\theta.\rho_u.u_t - \frac{k.a}{\alpha_x}.u_t$$

$$u_t\left(\theta.\rho_u - \alpha.\beta.\theta.\rho_u + \frac{k.a}{\alpha_x}\right) = a.\theta.u_{t-1}$$

$$(\rho_u.u_{t-1} + \varepsilon^u_t)\left(\theta.\rho_u - \alpha.\beta.\theta.\rho_u + \frac{k.a}{\alpha_x}\right) = a.\theta.u_{t-1}$$

$$u_{t-1}\left(\theta.\rho_u^2 - \alpha.\beta.\theta.\rho_u^2 + \frac{k.a}{\alpha_x}.\rho_u - a.\theta\right) + \left(\theta.\rho_u - \alpha\beta\theta\rho_u + \frac{k.a}{\alpha_x}\right).\varepsilon^u_t = 0$$

tomando $E_{t-1}\{\cdot\}$ da expressão acima, ficamos com:

$$u_{t-1}.\left(\theta.\rho_u^2 - \alpha.\beta.\theta.\rho_u^2 + \frac{k.a}{\alpha_x}.\rho_u - a.\theta\right) = 0$$

Assim: $\theta.\rho_u^2 - \alpha.\beta.\theta.\rho_u^2 + \frac{k.a}{\alpha_x}.\rho_u - a.\theta = 0$

$$\theta\left(\rho_u^2 - \alpha.\beta.\rho_u^2 - a\right) = -\frac{k.a}{\alpha_x}.\rho_u$$

$$\theta\left(\rho_u^2(1-\alpha.\beta) - a\right) = -\frac{k.a}{\alpha_x} \Rightarrow \theta = -\frac{k.a}{\alpha_x}.\frac{1}{[\rho_u^2(1-\alpha\beta)-a]}$$

logo, por (4)
$$\boxed{x_t = \frac{-k.a}{\alpha_x[\rho_u^2(1-\alpha\beta)-a]}.u_t} \quad (5)$$

pela equação 3 :

$$\boxed{\pi_t = -\frac{\alpha_x}{k}\left[\frac{-k.a}{\alpha x[\rho u^2(1-\alpha\beta)-a}(u_t - u_{t-1})\right]} \quad (6)$$

Questão 2)

(A) Vamos usar o metodo dos coeficientes indeterminados para resolver o modelo. Assuma que a solução do modelo segue

$$\begin{cases} \tilde{y}_t = \gamma.v_t \\ \pi_{H,t} = \theta.v_t \end{cases} \Rightarrow \mathbb{E}_t\{\tilde{y}_{t+1}\} = \gamma.\mathbb{E}_t\{\rho v.v_t + \varepsilon^v_{t+1}\} = \gamma.\rho v.v_t$$

$$\mathbb{E}_t\{\pi_{H,t+1}\} = \theta.\rho v.v_t$$

- Substituindo na eq (1):

$$\theta.\cancel{v_t} = \beta.\theta.\rho v.\cancel{v_t} + k_\alpha.\gamma.\cancel{v_t} \Rightarrow \theta(1-\beta\rho v) = k_\alpha.\gamma$$

$$\Rightarrow \boxed{\theta = \frac{k_\alpha.\gamma}{1-\beta\rho v}} \quad (4)$$

- Substituindo em (2) e considerando (3):

$$\gamma.\cancel{v_t} = \gamma\,\rho v.\cancel{v_t} - \left(\frac{1}{\sigma_\alpha}\right)\left(\cancel{\rho} + \phi_\pi.(\theta.\cancel{v_t}) + \phi_y(\gamma.\cancel{v_t}) + \cancel{v_t} - \theta.\rho v.\cancel{v_t} - \cancel{r_t^n}\right)$$

pois $\hat{r}^t_n = r_t^n - \rho = 0 \iff \boxed{r_t^n = \rho}$
↳ hipótese.

$$\gamma = \gamma.\rho v - \frac{1}{\sigma_\alpha}(\phi_\pi.\theta + \phi_y.\gamma - \theta\rho v) - \frac{1}{\sigma_\alpha}$$

$$\gamma\left[1-\rho v + \frac{\phi_y}{\sigma_\alpha}\right] = -\frac{1}{\sigma_\alpha}.\theta(\phi_\pi - \rho v) - \frac{1}{\sigma_\alpha} \quad (5)$$

Substituindo (4) em (5)

$$\gamma\left[\frac{\sigma_\alpha - \sigma_\alpha \rho v + \phi_y}{\sigma_\alpha}\right] = -\frac{1}{\sigma_\alpha}\cdot\frac{k_\alpha \gamma}{(1-\beta \rho v)}\cdot(\phi_\pi - \rho v) - \frac{1}{\sigma_\alpha}$$

$$\gamma\left[\frac{\sigma_\alpha(1-\rho v)+\phi_y}{\sigma_\alpha} + \frac{k_\alpha(\phi_\pi - \rho v)}{\sigma_\alpha(1-\beta\rho v)}\right] = -\frac{1}{\sigma_\alpha}$$

$$\gamma\left[\sigma_\alpha(1-\rho v)+\phi_y + \frac{k_\alpha(\phi_\pi-\rho v)}{(1-\beta\rho v)}\right] = \gamma\left[\frac{\phi_y(1-\beta\rho v)+\sigma_\alpha(1-\rho v)(1-\beta\rho v)+k_\alpha(\phi_\pi-\rho v)}{(1-\beta\rho v)}\right]$$

$$= -1$$

$$\Rightarrow \boxed{\gamma = \frac{-(1-\beta\rho v)}{(1-\beta\rho v)(\sigma_\alpha(1-\rho v)+\phi_y)+k_\alpha(\phi_\pi-\rho v)}} \quad (6)$$

Assim $\boxed{\tilde{y}_t = -(1-\beta\cdot\rho v)\cdot\Delta_v\cdot v_t}$ (7)

onde $$\Delta_v \equiv \frac{1}{(1-\beta\rho v)(\sigma_\alpha(1-\rho v)+\phi_y)+k_\alpha(\phi_\pi-\rho v)}$$

por (4)

$$\theta = \frac{k_\alpha}{\cancel{(1-\beta\rho v)}}\cdot -\cancel{(1-\beta\rho v)}\cdot\Delta_v = -k_\alpha\cdot\Delta_v$$

logo $\boxed{\pi_{H,t} = -k_\alpha\cdot\Delta_v\cdot v_t}$ (8)

(B)

Usando a regra de Taylor (3):

$$i_t = \rho + \phi_\pi . (-K_\alpha . \Lambda_v . v_t) + \phi_y . (-(1-\beta\rho v) . \Lambda_v . v_t) + v_t$$

$$\Rightarrow i_t = \rho + v_t \left[-\phi_\pi K_\alpha . \Lambda_v - \phi_y (1-\beta\rho v) \Lambda_v + 1 \right]$$

$$\Rightarrow i_t = \rho + \left[1 - \Lambda_v \left(\phi_\pi K_\alpha + \phi_y (1-\beta\rho v) \right) \right] . v_t$$

Pela relação de fisher sabemos que:

$$r_t = i_t - \mathbb{E}_t\{\pi_{H,t+1}\} = \rho + v_t \left[1 - \Lambda_v \left(\phi_\pi K_\alpha + \phi_y (1-\beta\rho v) \right) \right] - (-K_\alpha . \Lambda_v) \rho v . v_t$$

$$= \rho + v_t \left[1 - \Lambda_v \left(\phi_\pi . K_\alpha + \phi_y (1-\beta\rho v) + K_\alpha . \Lambda_v \right] \right.$$

$$= \rho + v_t \left[1 - \Lambda_v \left(\phi_\pi . K_\alpha + \phi_y (1-\beta\rho v) + K_\alpha . \rho v \right) \right]$$

Note que $\dfrac{\partial r_t}{\partial v_t} > 0$, Se $\left[1 - \Lambda_v \left(K_\alpha (\phi_\pi - \rho v) + \phi_y (1-\beta\rho v) \right) \right] > 0$

(C)

Seja $y_t = y_t^* + \dfrac{1}{\sigma_\alpha} s_t \Rightarrow s_t = \sigma_\alpha \left(y_t - \overset{\frown 0}{y_t^*} \right)$ hipótese.

$$\Rightarrow s_t = \sigma_\alpha \left(-(1-\beta\rho v) . \Lambda_v . v_t) - y_t^* \right)$$

$$\Rightarrow s_t = - \sigma_\alpha \left[(1-\beta\rho v) . \Lambda_v . v_t + y_t^* \right] .$$

Assim $\dfrac{\partial s_t}{\partial v_t} = -\sigma_\alpha (1-\beta\rho v) < 0$. Assim, um choque positivo de política monetária ($\uparrow v_t$) reduz os termos de troca. ($\downarrow s_t$)

Como $s_t = \dfrac{P_{F,t}}{P_{H,t}}$

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$$\begin{cases} e_t = s_t - \bar{p}_t^* + p_{H,t} \\ e_{t-1} = s_{t-1} - \bar{p}_t^* + p_{H,t-1} \end{cases} \Rightarrow \Delta e_t = \Delta s_t + \Delta p_{H,t}$$

$$\Rightarrow \Delta e_t = -\sigma_\alpha (1-\beta\rho v)\Delta_v \vartheta_t - \cancel{\sigma_\alpha \bar{y}_t^*} + \sigma_\alpha(1-\beta\rho v)\Delta_v \vartheta_{t-1} + \cancel{\sigma_\alpha \bar{y}_t^*} + \pi_{H,t}$$

$$= -\sigma_\alpha (1-\beta\rho v)\Delta_v (\vartheta_t - \vartheta_{t-1}) - K_\alpha . \Delta_v . \vartheta_t$$

$$\Rightarrow \Delta e_t = -\sigma_\alpha(1-\beta\rho v).\Delta_v . \Delta\vartheta_t - K_\alpha . \Delta_v \vartheta_t$$

Assim, $\uparrow \vartheta_t \Rightarrow \downarrow \Delta e_t \Rightarrow$ uma contração na política monetária resulta em apreciação do câmbio nominal.