Inflation Persistence

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# INFLATION PERSISTENCE*

JEFF FUHRER AND GEORGE MOORE

This paper demonstrates that the behavior of the conventional Phelps-Taylor model of overlapping wage contracts stands in stark contrast with important features of U. S. macro data for inflation and output. In particular, the Phelps-Taylor specification implies far too little inflation persistence. We present a new contracting model, in which agents are concerned with relative real wages, that is data-consistent. In a specification that nests both models, we resoundingly reject the conventional contracting model, but cannot reject the new contracting model.

The most popular model of sticky prices today is the overlapping wage contract model of Phelps [1978] and Taylor [1980]. While that model implies that prices are sticky, it also implies that the inflation rate is so flexible that monetary policy can drive a positive rate of inflation to zero with virtually no loss of output. Phelps recognized this property in his 1978 article, "Disinflation without Recession," but the overlapping wage contracting model delivers even more optimistic policy prescriptions than he realized.[1] As Ball [1991] has shown, monetary policy can create a disinflationary boom in the Phelps-Taylor specification.

Our purpose is to show that the Phelps-Taylor model is not consistent with the dynamic interaction of inflation and output that we find in the data; to present a new contracting model that is data-consistent; and to analyze some monetary policy implications of the new contracting model. In particular, we will show that the standard model implies that the persistence in inflation derives almost exclusively from the persistence in the driving output process. As a result, the standard specification is incapable of imparting the persistence to inflation that we find in the data. That implication does not depend on parameter values or other particulars of the empirical implementation of the model. We will also show that a model of nominal wage contracts in which agents care about *relative* real wages implies additional persistence for inflation, beyond that imparted by the driving output process. As a result, the relative contracting specification can impart to inflation

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1. Taylor [1983] also demonstrates this result, concluding that simulations of a staggered contracting model show "that it is possible in principle for such a disinflation to occur without any increase in unemployment."

the significant persistence that we find in the data. We will show further that the lack of inflation persistence in the standard specification implies unrealistically low sacrifice ratios for typical disinflation experiments. In contrast, the new specification performs quite well by this criterion.

In addition, we show that the two contracting specifications differ in their ability to match the pronounced positive correlation between inflation and excess demand (the Phillips curve) observed in postwar U. S. data. We will show that the difference arises because the standard contracting model relates a two-sided average of the *price level* to an excess demand term, while the relative contracting model relates a two-sided average of the *inflation rate* to an excess demand term. As a result, the relative contracting model is better able to match this important correlation in the data.

Next we contrast the essential elements of the two contracting specifications, drawing out in simplified theoretical versions of the models their different implications for inflation persistence. We then present the stylized facts about inflation and output that the structural contracting models must explain. We estimate and test the two models of overlapping contracts. After rejecting the standard contracting model as inconsistent with the data, we perform a set of policy experiments designed to explore the properties of the relative contracting model.

## I. Contrasting the Contracting Specifications

In this section we consider two models of overlapping wage contracts. In the original work of Taylor [1980], wages are linked to prices with a simple price markup equation, so that the implications for wages and wage inflation are the same as those for prices and price inflation. We follow a similar tack here. The underlying contracts that we have in mind are nominal wage contracts. Because prices are a fixed markup over wages, there is little to distinguish wages from prices in the model.

### A. The Simplified Theoretical Specifications

In order to highlight the behavioral differences between the two contracting specifications, we consider simple two-period contracting versions of each specification. For convenience we assume a markup factor from wages to prices of unity. Thus, for both models, the log wage and the log price index, $p_t$, are defined as the simple average of the contract wage, $x_t$, negotiated in periods $t$ and $t - 1$:

(1) $$p_t = \tfrac{1}{2}(x_t + x_{t-1}).$$

The two-period contracting equation of Taylor [1980] makes the current wage contract an average of the lagged and the expected future wage contracts, adjusted for excess demand, $y_t$:

(2) $$x_t = \tfrac{1}{2}(x_{t-1} + E_t x_{t+1}) + \gamma y_t.$$

Rearranging equation (2) and substituting out the definition of $x_t$ in equation (1), we obtain

(3) $$p_t = \tfrac{1}{2}(p_{t-1} + E_t p_{t+1}) + (\gamma/2)(y_t + y_{t-1}).$$

Thus, the Taylor model clearly imparts considerable inertia to the level of wages and to the price level, as intended. However, the equation bears less desirable implications for the inflation rate. Defining $\pi_t$ as $p_t - p_{t-1}$ and rearranging equation (3),

(4) $$\pi_t = E_t \pi_{t-1} + \gamma y_t.$$

All of the persistence in inflation derives from the persistence in the driving term $y_t$. Thus, a one-period shock to output will affect inflation for one period only; the contracting specification adds no inflation persistence of its own. Similarly, a one-period shock to inflation that does not alter $y_t$ affects inflation for a single period, after which inflation returns to its expectation. Unless the shock itself persists, the effect on inflation will not persist.

Note in addition that this specification bears strong implications for the correlation between inflation and *lagged* output: if current and expected $y_t$ are zero, then inflation must be at its steady state, regardless of the value of past $y_t$. This strong implication will be borne out in the analysis of richer contracting specifications in the empirical work discussed below.

Under the proposed new contracting specification, agents care about *relative* real wages over the life of the wage contract. Thus, the contracting equation that parallels equation (2) is[2]

(5) $$x_t - p_t = \tfrac{1}{2}(x_{t-1} - p_{t-1} + E_t(x_{t+1} - p_{t+1})) + \gamma y_t.$$

We wish to emphasize that the new contracting specification is *not* a model of real wage contracts or perfect indexation. The contracts

2. Each real contract price should be strictly defined relative to the prices in effect over the life of the contract. See Appendices B and C for the effect of this simplification theoretically and empirically.

are still negotiated in *nominal* terms. Thus, the model does not impose any real rigidities.

Substituting the definition of $x_t$ in equation (5) into the price index equation (1), we obtain

$$\pi_t = \tfrac{1}{2}(\pi_{t-1} + E_t\pi_{t+1}) + \gamma\hat{y}_t, \tag{6}$$

where $\hat{y}_t$ is a moving average of current and past output. Inflation now depends on its past (and thus on past output). Thus, this specification imparts significant inertia to the inflation rate (as well as the price level) beyond the inertia in the driving term. One-time shocks to output and inflation will have persistent effects on inflation that last well beyond the lifetime of the initial shock. As we will demonstrate below, the data also exhibit this kind of persistence, so that the inability of the standard contracting specification to generate this persistence constitutes a significant empirical failure.

In one sense, this is the fundamental distinction between the two specifications: the Taylor model of wage contracts implies that inflation exhibits no persistence beyond the persistence in the driving term. The relative wage model implies that inflation exhibits considerable persistence of its own, independent of the driving term.

It is important to note that the fundamental difference in the persistence imparted to inflation by the two specifications does not depend on the size of $\gamma$, the sensitivity of the current nominal wage contract to excess demand. Neither does it depend on the time-series properties of excess demand. It is an inherent property of the contracting specifications.

There are other implications of this difference in specification. First, the two specifications imply that agents have different objectives in negotiating their wage contracts. Second, the specifications imply that disinflations will proceed quite differently. These two points are expanded below.

In the original version of Taylor's [1980] model, contracting agents compare the current wage contract with other wage contracts previously negotiated and still in effect, and with wage contracts expected to be negotiated over the duration of the contract. This is our equation (2), and Taylor's [1980] equation (1). We can rearrange equation (2) to show that the original casting of Taylor's model also implies that the nominal wage is set so that the expected real wage over the life of the contract is consistent with expected excess demand conditions:

$$x_t = (\tfrac{1}{2})(p_t + E_t p_{t+1}) + (\gamma/2)y_t. \tag{7}$$

In the relative wage specification, however, agents compare the real value of their wage contracts with the real value of wage contracts previously negotiated and still in effect, and with contracts expected to be negotiated over the duration of the contract, equation (5). We suggest that it is a priori more plausible that agents care more about the price-level-adjusted value of neighboring wage contracts than about their nominal value. This does not motivate an optimizing foundation for overlapping nominal contracts any more than the original specification does, but it suggests that agents negotiate wage contracts to keep up with contracting neighbors in real, rather than nominal, terms.

Finally, the two contracting specifications bear markedly different implications for the way disinflations proceed in the economy. According to equation (4), if excess demand is negative, then expected inflation must *exceed* current inflation. But the only way for this to happen and for inflation to fall is for inflation to immediately jump *below* its new equilibrium upon commencement of the disinflationary program, and then rise to its new, lower equilibrium from below. A simulation of the model confirms that this is exactly how the standard contracting specification behaves. This extremely flexible "overshooting" behavior of inflation stands in stark contrast to the costly and slow disinflations documented in Ball [1993].

Rearranging equation (6), the implications for a disinflation under the relative wage model are quite different:

$$(\pi_{t+1} - \pi_t) - (\pi_t - \pi_{t-1}) = \gamma \hat{y}_t. \tag{8}$$

When excess demand is negative, equation (6) implies that the level of inflation will be falling, while equation (8) implies that the change in inflation must jump below zero upon commencement of a credible disinflation. This qualitative behavior corresponds much more closely to the dynamic response of inflation observed during disinflationary contractions in the United States and elsewhere.

## II. The Empirical Specifications

We now develop empirical specifications of the two models of nominal contracts. We begin by presenting reduced-form evidence of inflation's persistent autocorrelation and of its persistent cross-correlation with output. We take the evidence presented in this section to be the set of stylized facts that the contracting specifications must explain. We then develop, estimate, and compare the empirical counterparts with the simplified theoretical contracting specifications discussed above. Note that from here on, we refer to Taylor's original specification as the "standard" contracting specification, and to the new specification as the "relative" contracting specification.

### *A. The Stylized Facts*

The key variables in our analysis are quarterly series for the log price level and its inflation rate and the deviation of output from trend. It is as appropriate to use price data as wage data because, in the models discussed above, wages and prices are linked by a simple markup equation. Thus, the models' implications for wages and wage inflation are virtually identical to those for prices and price inflation. We argue below that, as documented in Bernanke and Blinder [1992], the short-term nominal rate is closely linked to real output, and is thus essential to forming expectations of output. Our short rate is the three-month Treasury bill rate. Mnemonics and series definitions are listed in Table I. Note that we do not employ a segmented trend here: for our per capita nonfarm business output series, the best estimate of the trend breakpoint appears to be 1968:4. Because our estimation sample begins in 1965, there is little difference between the output gap generated using the segmented trend and the gap implied by a

TABLE I
QUARTERLY DATA, 1965:1–1993:3

| Mnemonic | Definition |
|---|---|
| $y_t$ | log of per capita nonfarm business output |
| $p_t$ | log of the implicit deflator for nonfarm business output |
| $r_t$ | 3-month Treasury bill rate |
| $\pi_t$ | inflation rate, $4\,\Delta p_t$ |
| $\tilde{y}_t$ | deviation of $y_t$ from linear trend |

single trend. Thus, we present results for the single trend series here.[3]

Table II presents augmented Dickey-Fuller tests for the various series.[4] The initial test regressions are estimated with six lags. Then we reduce the lag lengths until the last lag remains statistically significant and the residuals appear to be uncorrelated. At conventional significance levels, we cannot reject the hypotheses that the inflation rate and the interest rate series are integrated of order one. The log of per capita output, on the other hand, appears to be trend stationary over the sample period.[5]

Tables III and IV show test statistics for a Johansen multivariate test regression of a model that includes the inflation rate, the bill rate, and the output gap.[6] The estimation strategy is similar to that used in the Dickey-Fuller test regressions. We begin with a model that includes six lags of each variable. Then we reduce the lag length of each variable until the last lag of each variable is jointly significant in all three equations and the residuals are uncorrelated.

The maximum eigenvalue and trace statistics in Table IV are consistent with the univariate Dickey-Fuller tests. We can reject the hypothesis that the vector autoregression contains three unit roots in favor of two unit roots at the 1 percent significance level. But we can reject two unit roots in favor of one, and one unit root in favor of zero unit roots, only at the 20 percent significance level.

It is difficult to interpret the model with a single cointegrating vector and two unit roots. Table V displays the estimated cointegrating vector, $\beta$, and the error-correction coefficients, $\alpha$, together with their $p$-values, for the model with two unit roots. The coefficient on the inflation rate is not significantly different from zero, and the deviation of output from trend moves one-for-one with the short-term *nominal* rate.[7] Nonetheless, this constrained vector autore-

3. Parallel results for a segmented trend, using both the 1968:4 breakpoint and the more conventional 1973 breakpoint, have been computed. None of the qualitative results described below are at all sensitive to this assumption.

4. Table 8.5.2 in Fuller [1976] gives critical values for the $\tau_\mu$ and $\tau_\tau$ statistics.

5. Trend stationarity serves as an adequate description of the behavior of output. We do not claim that the results presented here strongly favor trend stationarity over difference stationarity, only that we could not reject either hypothesis in favor of the other. See Christiano and Eichenbaum [1990] for detailed discussion.

6. Table A.3 in Johansen and Juselius [1990] gives critical values for the maximum eigenvalue and trace statistics.

7. See Fuhrer and Moore [1995] for a structural interpretation of the output/short nominal rate correlation.

TABLE II
AUGMENTED DICKEY-FULLER TESTS
$\Delta x_t = \beta_0 x_{t-1} + \Sigma_{i=1}^{n} \beta_i \Delta x_{t-i} + \mu + \gamma t + \epsilon_t$

| Series | $n$ | $Q(12)$ | $\beta_0$ | $\tau_\mu$ | $\tau_\tau$ |
|---|---|---|---|---|---|
| $\pi_t$ | 2 | 8.2 | −0.15 | −2.12 | |
| $r_t$ | 3 | 22.4 | −0.07 | −2.26 | |
| $y_t$ | 3 | 20.1 | −0.14 | | −4.20 |
| $\tilde{y}_t$ | 3 | 18.7 | −0.09 | −3.12 | |

TABLE III
JOHANSEN TEST REGRESSION
$\Delta x_t = \Pi x_{t-1} + \Sigma_{i=1}^{n} \Gamma_i \Delta x_{t-i} + \mu + \epsilon_t$

| Series | Maximum lag | Q(12) |
|---|---|---|
| $\Delta \pi_t$ | 3 | 6.6 |
| $\Delta r_t$ | 2 | 13.0 |
| $\Delta \tilde{y}_t$ | 3 | 15.8 |

gression provides evidence that the output gap, inflation, and the bill rate are significantly linked at low and higher frequencies, and we thus take these three as a minimal set of variables required to model inflation and output in a vector autoregression. The strong link between the bill rate and the output gap argues for its inclusion in the estimation steps below, as it appears that its omission would significantly worsen forecasts of the output gap that enter the price specifications.

The unconstrained vector autoregression, estimated by ordinary least squares, is actually quite stable. Table VI displays the nonzero roots of the companion matrix of the stationary vector autoregression. The dominant roots are a complex pair with a modulus of 0.95, well within the unit circle.

We prefer to characterize the operating characteristics of the stationary vector autoregression with its vector autocorrelation function rather than its impulse-response function. Deriving the autocorrelation function requires no identifying assumptions, while deriving the impulse-response function does.[8] The autocorrelation function nicely summarizes our intuition about the dynamic interaction of inflation and output.

8. In subsection II.B we present the impulse-response functions of the structural models, where we believe that we have made plausible identifying restrictions.

TABLE IV
JOHANSEN TEST STATISTICS

| Number of unit roots | Maximum eigenvalue | Trace |
|---|---|---|
| 1 | 7.28 | 7.28 |
| 2 | 11.75 | 19.03 |
| 3 | 26.84 | 45.87 |

Figure I displays the vector autocorrelation function implied by the stationary vector autoregression. The diagonal elements show the univariate autocorrelation functions of inflation and the output gap, and the off-diagonal elements show their lagged cross correlations. Inflation is quite persistent, with positive autocorrelations out to lags of about four years, while the output gap is somewhat less persistent. Much of the conventional wisdom about the dynamic interaction of inflation and output can be found in the off-diagonal elements of the vector autocorrelation function. In the off-diagonal element of the first row, for example, a high level of output is followed by a high level of inflation about six quarters later. In the off-diagonal element of the second row, a high level of inflation is negatively correlated with a low level of output about ten quarters later.[9]

While we cannot reject the hypotheses that the data contain one or two unit roots, we choose a stationary representation of the data for two reasons. First, it is difficult to interpret the data under the assumption of nonstationarity. In contrast, the stationary VAR's estimated vector autocorrelation function, which does not exist for integrated series, conforms well to our intuition (and to conventional wisdom) about the dynamic interaction of these series. Second, we want to show that the orthodox overlapping contracts model cannot capture the persistence that is inherent in the inflation process. By viewing inflation as an I(0) process instead of an I(1) process, we bias downward our estimate of inflation persistence, and we strengthen the argument that the standard contracting model cannot adequately explain inflation persistence.

We take the vector autocorrelation function in Figure I, especially its first row and column, to be the stylized facts that the wage contracting models must explain.

9. The responses of the bill rate, not presented in Figure I, also conform to intuition. For example, a high level of the bill rate is followed by a low level of output about six quarters later.

TABLE V
COINTEGRATING VECTOR AND ERROR-CORRECTION COEFFICIENTS

| | $\beta$ | | $\alpha$ | |
|---|---|---|---|---|
| Variable | Coefficient | $p$-value | Coefficient | $p$-value |
| $\pi_t$ | 6.7 | 0.65 | −0.0037 | 0.067 |
| $r_t$ | −22.9 | 0.04 | −0.0019 | 0.035 |
| $\tilde{y}_t$ | −36.7 | $2.5 \times 10^{-4}$ | 0.0020 | 0.030 |

## *B. The Models*

The definition of the aggregate price index in terms of lagged nominal contracts is common to both specifications. The models differ in the mechanism that determines the nominal wage contract. The first model is Taylor's wage contracting specification: agents negotiate nominal wages with a concern for the real wage over the contracting period. In the second model, the nominal contract wage is negotiated with a concern for the relative real wage over the contracting period. As noted above, while both Taylor's and our specifications are motivated by discussions of *wage* contracting, with a simple markup from wages to prices, the implications for prices and price inflation are virtually identical. In the estimation below, we use the price data described in subsection II.A in place of wage data. From hereon in we refer to "contract prices" rather than to "contract wages."

In both specifications, agents negotiate nominal contracts that remain in effect for four quarters. Recall that the vector autoregression of subsection II.A indicates that three lags of inflation, and thus four lags of the price level, are sufficient to model the data.

TABLE VI
ROOTS OF THE STATIONARY VECTOR AUTOREGRESSION

| Modulus | Period |
|---|---|
| 0.95 | 70.0 |
| 0.87 | 22.8 |
| 0.65 | 2.8 |
| 0.63 | 3.4 |
| 0.53 | 5.0 |
| 0.30 | 2.0 |

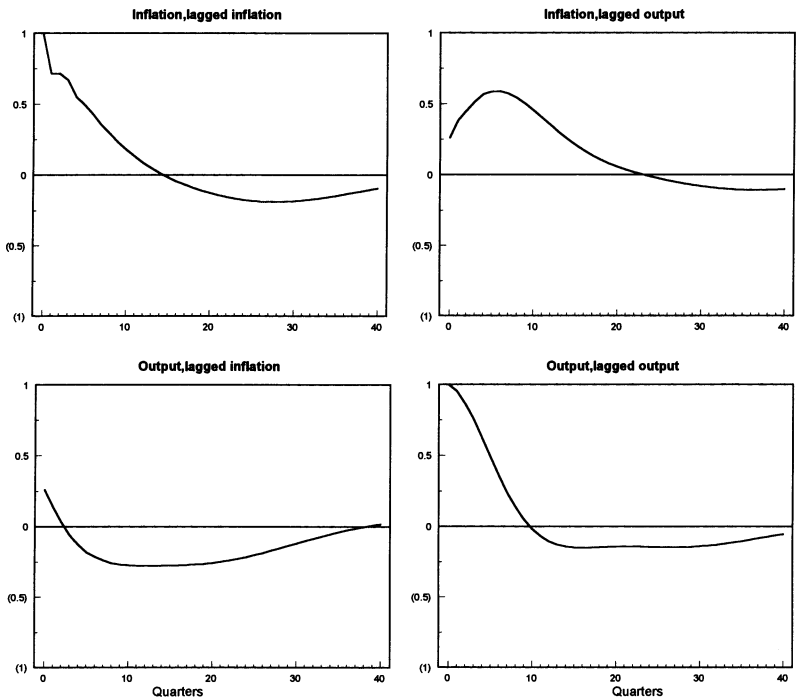


FIGURE I
Autocorrelation Function, Vector Autoregression

Because the solutions to all the models presented here are restricted vector autoregressions, the unconstrained vector autoregression estimates imply a structural model with no more than four lags of prices. Thus, we need not consider contracts that exceed four quarters in length.[10]

The aggregate log price index in quarter $t$, $p_t$, is a weighted average of the log contract prices $x_{t-i}$ that were negotiated in the current and the previous three quarters and are still in effect. The weights $f_i$ are the proportions of the outstanding contracts that

10. In a regression of the residuals from the estimated contract equation for this specification on lags of inflation, output, and the bill rate, lags of the price index beyond lag 3 enter insignificantly. In a specification that imposes six-quarter contracts, using the same downward-sloping contract distribution assumed here, the autocorrelation properties of the estimated model again fail to match those of the unconstrained vector autoregression.

were negotiated in quarters $t - i$,

$$p_t = \sum_{i=0}^{3} f_i x_{t-i}, \tag{9}$$

where $f_i \geq 0$ and $\Sigma f_i = 1$. The sticky price index, $p_t$, is directly observable, while the flexible contract price, $x_t$, is not. To recover the realization of the contract price from the realization of the price index, the lag operator $f(L) = f_0 + f_1 L + f_2 L^2 + f_3 L^3$ must be invertible.

The precise shape of the contract distribution is not well determined by the data. Unconstrained estimation of the contract distribution yields a downward-sloping function with imprecisely estimated weights. Taylor's contract distribution sets all of the $f_i$ to 0.25, but that lag operator is not invertible. We use a downward-sloping linear function of contract length,

$$f_i = 0.25 + (1.5 - i)s, \qquad 0 < s \leq \tfrac{1}{6}, \qquad i = 0, \ldots, 3. \tag{10}$$

This distribution characterizes the contract distribution with a single slope parameter $s$, and it is invertible. When $s = 0$, it is the rectangular distribution of Taylor; and when $s = 1/6$, it is the triangular distribution.

The distribution of contract lengths determines the steady-state real contract price in terms of the steady-state inflation rate, $\bar{\pi}$. With a constant inflation rate, the contract price satisfies $\bar{x}_{t-i} = \bar{x}_t - i\bar{\pi}$, and the log of the equilibrium real contract price is the product of the inflation rate and the mean contract length, $\Sigma i f_i$:

$$\bar{x}_t - \bar{p}_t = \bar{\pi} \sum_{i=1}^{3} i f_i. \tag{11}$$

Apart from the downward-sloping distribution of contract lengths, our standard contracting specification is identical to Taylor's. The current contract price depends upon the price level expected to prevail over the life of the contract, adjusted for excess demand conditions, $\gamma \tilde{y}_t$:

$$x_t = \sum_{i=0}^{3} f_i E_t(p_{t+i} + \gamma \tilde{y}_{t+i}) + \epsilon_t. \tag{12}$$

Equivalently, substitute equation (9) into equation (12) to obtain the two-sided representation,[11]

$$(13)\qquad x_t = \sum_{i=1}^{3} \beta_i x_{t-i} + \sum_{i=1}^{3} \beta_i E_t(x_{t+i}) + \gamma^* \sum_{i=0}^{3} f_i E_t(\tilde{y}_{t+i}) + \epsilon_t,$$

where

$$\beta_i = \sum_j f_j f_{i+j} \Big/ \left(1 - \sum_j f_j^2\right), \qquad \text{and} \qquad \gamma^* = \gamma \Big/ \left(1 - \sum_j f_j^2\right).$$

In their contract price decisions, agents compare the current nominal contract price with an average of the nominal contract prices that were negotiated in the recent past and those that are expected to be negotiated in the near future. The weights in the average measure the extent to which the past and future contracts overlap the current one. When output is expected to be high, the current nominal contract price is high relative to the nominal contract prices on overlapping contracts.

For purposes of estimation our version of the standard contracting model comprises equations (9), (12), and two unconstrained equations for the bill rate and the output gap from the vector autoregression of Section I. Note that there is no direct feedback between the bill rate and the price equations. The bill rate is included only because it is essential in forecasting the output gap, which of course feeds directly into the contracting equations. Thus, expectations of the output gap are consistent with the VAR equation for the output gap. Its expectations depend in turn on expectations of the bill rate, which are consistent with the VAR equation for the bill rate. Any contemporaneous response of output to price shocks is subsumed in the error covariance. The only structural shock that is identified in the model is the shock to the contracting equation. The model jointly determines the observable price index, the output gap, and the bill rate. The unobservable contract price is determined by the identity in equation (9).

Taking the estimated parameters in the VAR equations as given, there are only two undetermined parameters in the standard contracting model: the slope of the contract distribution, $s$, and the coefficient of the output gap, $\gamma$, in equation (12). We assume rational expectations, so that the expectations for prices and excess demand are consistent with the contracting equations and the

11. When the $f_i$ are constant at 0.25, equation (13) is identical to equation 1 in Taylor [1980].

vector autoregression equations for the output gap and the bill rate underlying Figure I. We estimate the two undetermined parameters by maximum likelihood.[12]

The standard contracting model cannot determine the slope of the contract distribution with much precision. In the estimate reported here, the slope of the contract distribution is constrained to 0.08, the midpoint of its admissible region (and its estimated value in the relative contracting model). As shown in Table VII, the maximum likelihood estimate of $\gamma$ is extremely small, although it is not estimated with much precision.[13]

The residuals of the standard contracting equation are very strongly autocorrelated. The Q(12) statistic is 72.8, and the partial autocorrelation function falls within two standard deviations of zero only after lag 4.

The vector autocorrelation function implied by the standard contracting model for the inflation rate and nominal output is displayed in the dashed lines in Figure II. Comparing the stylized facts in Figure I with the operating characteristics of the structural model in Figure II illustrates clearly our basic point: the standard contracting model implies no persistence for the inflation rate beyond that in the driving process. Thus, inflation's autocorrelation and its cross correlation with output, shown in Figure II, are radically different from those of the vector autoregression. The autocorrelation function of the inflation rate dies out much more rapidly in the standard contracting model than it does in the reduced-form vector autoregression. Furthermore, while the reduced-form cross correlations between inflation and real output are substantial in magnitude and plausible in sign, the structural correlations between inflation and output are estimated to be virtually zero.

The dashed lines in Figure III represent the response of the inflation rate to a unit standard deviation shock to inflation and real output. Consistent with the results in Figure II for the autocorrelation function, the response of inflation to an inflation shock dies out within a year. The response of inflation to a shock in real output cannot be detected in the plots.

12. Appendix A outlines the computation of the likelihood function.

13. A line search over eight orders of magnitude for $\gamma$ reveals that this estimate is a global maximum. The likelihood deteriorates markedly when we set $\gamma$ to the estimate for the relative contracting model. The $p$-value for the likelihood ratio test that $\gamma$ is equal to the relative contracting model's estimate is $5.9 \times 10^{-4}$.

TABLE VII
STANDARD CONTRACTING SPECIFICATION ESTIMATES

| Coefficient | Value | Standard error |
|---|---|---|
| $\gamma$ | $3.24 \times 10^{-7}$ | $5.39 \times 10^{-6}$ |
| Residual s.e. | 0.0069 | |
| Q(12) | 72.8 | |
| Log likelihood | 1727.0 | |

These small responses depend almost entirely on the functional form of the standard contracting equation, not on the estimated parameter values in the contracting specification. The small estimated value of the output-gap coefficient in the standard contracting equation, $\gamma$, effectively decouples the contracting equation from output. But even when $\gamma$ is increased by a factor of one thousand, the response of inflation to inflation shocks is unchanged, and the response of inflation to bill rate and output shocks remains almost undetectable.

The relative contracting specification can far better mimic the stylized facts of the inflation and output processes.[14]

Let $v_t$ be the index of real contract prices that were negotiated on the contracts currently in effect:

$$v_t = \sum_{i=0}^{3} f_i(x_{t-i} - p_{t-i}). \tag{14}$$

Note that we have implicitly defined the real contract price as the difference between the current nominal contract price and the current price index, $x_t - p_t$. This is a convenient simplification from the theoretically preferable specification that defines the real contract price as the difference between the nominal contract price and the weighted average of price indexes that are expected to prevail over the life of the contract. The simplification yields an algebraically more straightforward model and affects none of the empirical conclusions. See Appendices B and C for details on the alternative specification and associated empirical results.

Now suppose that agents set nominal contract prices so that the current real contract price equals the average real contract

14. Buiter and Jewett [1981] analyzed a similar model, but they did not explore its implications for inflation persistence.

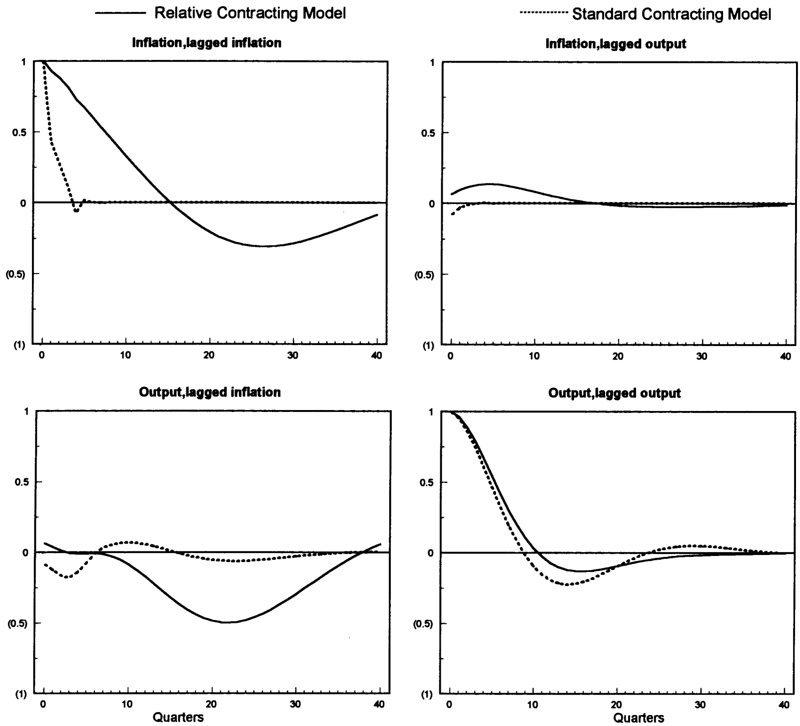


FIGURE II
Autocorrelation Functions

price index expected to prevail over the life of the contract, adjusted for excess demand conditions. Equation (12) becomes

$$x_t - p_t = \sum_{i=0}^{3} f_i E_t(v_{t+i} + \gamma \tilde{y}_{t+i}) + \epsilon_t. \tag{15}$$

Substituting equation (14) into equation (15) yields the relative version of Taylor's contracting equation,

$$x_t - p_t = \sum_{i=1}^{3} \beta_i(x_{t-i} - p_{t-i}) + \sum_{i=1}^{3} \beta_i E_t(x_{t+i} - p_{t+i}) + \gamma^* \sum_{i=0}^{3} f_i E_t(\tilde{y}_{t+i}) + \epsilon_t, \tag{16}$$

where the $\beta_i$ and $\gamma^*$ are defined as in equation (13).

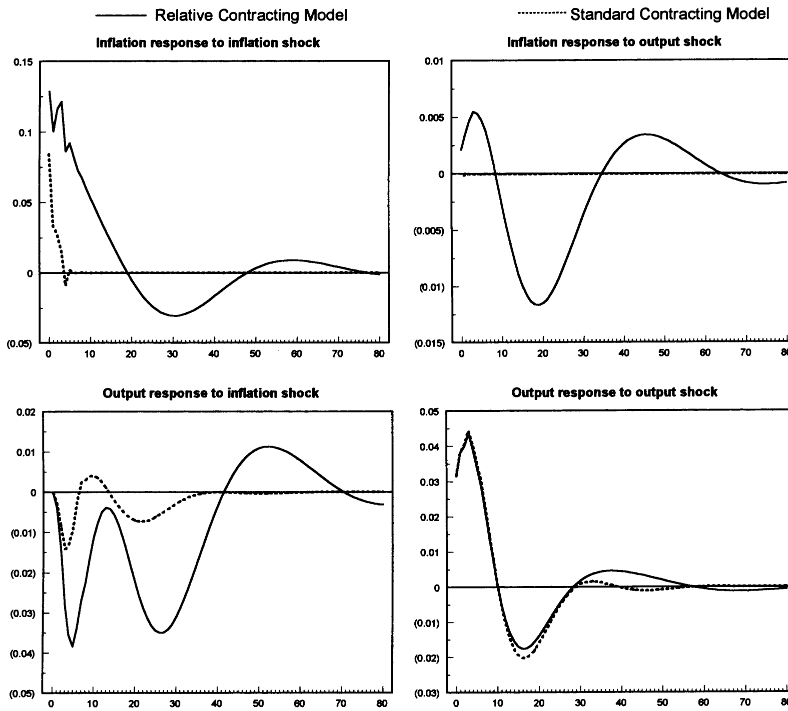


FIGURE III
Impulse Response Functions

In their contract price decisions, agents compare the current real contract price with an average of the real contract prices that were negotiated in the recent past and those that are expected to be negotiated in the near future. The weights in the average measure the extent to which the past and future contracts overlap the current one. When output is expected to be high, the current real contract price is high relative to the real contract prices on overlapping contracts.

We estimate the slope and excess demand parameters in equation (15) by maximum likelihood, again including reduced-form equations for the bill rate and real output gap from the vector autoregression of Section I. Table VIII displays the parameter estimates.

Some residual autocorrelation remains in the contracting equation. The error has an MA(1) component with an autocorrelation coefficient of −0.33 at lag 1. The standard error of the relative

TABLE VIII
RELATIVE CONTRACTING SPECIFICATION ESTIMATES

| Coefficient | Value | Standard error |
|---|---|---|
| s | 0.08225 | 0.01262 |
| $\gamma$ | 0.00435 | 0.00197 |
| Residual s.e. | 0.0043 | |
| Q(12) | 28.4 | |
| Log likelihood | 1737.4 | |

contracting equation is about half that of the nominal contracting equation, and the estimated effect of aggregate demand on contract prices is four orders of magnitude larger and precisely estimated in the relative contracting model.

It is hard to discuss the history of inflation in the 1970s without considering supply shocks, particularly the large changes in the real price of oil. While both contracting specifications allow for supply shocks ($\epsilon_t$ in equations (12) and (15)), they do not attempt to identify the source of the shocks, and in particular the real price of oil is not broken out in the model. Because our focus on prices rather than on wages allows us to ignore the effects of supply shocks on productivity, and because we do not articulate monetary policy responses to shocks, the omission of oil prices has essentially no effect on the conclusions drawn here. In essence, we have assumed that different types of supply shocks affect inflation similarly. Judging from the empirical success of the specification, this appears to be a tenable assumption for our purposes.[15]

The empirical implications of the relative contracting specification are striking. The solid lines in Figure II present the vector autocorrelation function for inflation and real output implied by the relative contracting model. Comparing Figures I and II, the vector autocorrelation function of the relative contracting model mimics the stylized facts much more closely than the autocorrelation function of the nominal contracting model does. The inflation autocorrelation function dies out slowly in the relative model, and the cross correlations between inflation and output have the appropriate signs and magnitudes.

The solid lines in Figure III present the impulse-response function of the relative contracting model. Again, the contrast

15. A measure of the real price of oil appears to be completely uncorrelated with the errors made by the contract specification, so that its omission is unlikely to have damaged any inferences drawn from the model.

between the two contracting specifications—the solid and dashed lines—is remarkable. The magnitude and persistence of the response of inflation and output to an inflation shock, the first row of the figures, is much greater in the relative contracting model. The response of both variables to an output shock, the first column of the figures, is also larger and more persistent.

*C. A Formal Hypothesis Test*

Although the qualitative results shown in Figures I–III are compelling evidence that the standard contracting model cannot generate enough inflation persistence to be consistent with the data—while the relative contracting model can—a formal test of the relative versus the standard model is desirable. Such a test can be constructed by embedding both models in a more general framework. A variation on equations (14) and (15) that encompasses both the relative and the standard contracting models is

$$v_t = \sum_{i=0}^{3} f_i(x_{t-i} - \delta p_{t-i}) \tag{17}$$

$$x_t - \delta p_t = \sum_{i=0}^{3} f_i E_t(v_{t+i} + \gamma \tilde{y}_{t+i}) + \epsilon_t, \tag{18}$$

where $0 \leq \delta \leq 1$. The parameter $\delta$ indexes the degree to which contracts are negotiated in relative terms. When $\delta = 0$, equations (17) and (18) are the standard contracting model. When $\delta = 1$, equations (17) and (18) are the relative contracting model.

We estimate $\delta$, $\gamma$, and $s$ by maximum likelihood, again holding fixed the VAR equations for the other variables. The estimated value of $\delta$ is 0.94; the $p$-value of the likelihood-ratio test of the single restriction imposed by the relative price contracting model, $\delta = 1$, is 0.5308.

The likelihood ratio test statistic for the single restriction imposed by the nominal contracting model, $\delta = 0$, is 21.3, with $\chi^2(1)$ probability of $3.9 \times 10^{-6}$. The data decisively reject the standard contracting model, and they fail to reject the relative contracting model.[16] Given the informal evidence presented in the vector

16. While it is natural to think of testing the contracting models by comparing their likelihood values with that of the unconstrained vector autoregression in Section I, neither of the structural contracting models is nested within the vector autoregression. The fundamental price series in the vector autoregression is the inflation rate, while the fundamental price series in the structural models is the log of the price level.

autocorrelation functions and the impulse responses, this formal result is perhaps not surprising.

### D. The Phillips Curve and the Contracting Specifications

As shown in Section I above, the standard contracting model relates a two-sided average of the *price level* to an excess demand term, while the relative contracting model relates a two-sided average of the *inflation rate* to an excess demand term. As a result, the relative contracting model will behave much more like a price-price Phillips curve, albeit a two-sided Phillips curve. As such, it will be much better able to match the well-documented positive correlation between the inflation rate and the level of excess demand. The standard contracting model will have much more difficulty in matching this important correlation. The rest of this section demonstrates this point algebraically for the empirical specifications of subsection II.B.

Let $\theta_t = x_t - x_{t-1}$ be the contract price inflation rate. Taking the first difference of equation (9), we have $\pi_t = f(L)\theta_t$ and $\theta_t = f^{-1}(L)\pi_t$ (recall that $f(L)$ is invertible). Ignoring the error term and the expectation operator, the contracting equation for the standard contracting model (equation (12)) is

$$x_t = f(L^{-1})(p_t + \gamma y_t). \tag{19}$$

Substituting equation (19) into equation (9), the price level is a symmetric moving average of lagged and expected future prices, where the coefficients sum to one, plus an excess demand term:

$$p_t = f(L)f(L^{-1})(p_t + \gamma y_t). \tag{20}$$

The relative contracting model uses the same price index equation, equation (9), but it defines the real contract price index as

$$v_t = f(L)(x_t - p_t) \tag{21}$$

and writes current real contract prices in terms of the expected real contract price index,

$$x_t - p_t = f(L^{-1})(v_t + \gamma y_t). \tag{22}$$

Combining equations (21) and (22), the real contract price index satisfies a two-sided difference equation that resembles equation

(20) from the standard contracting model:

$$v_t = f(L)f(L^{-1})(v_t + \gamma y_t). \tag{23}$$

The key insight is that the real contract price index $v_t$ can be written as a distributed lag of the price index inflation rate, $\pi_t$. Using the fact that $\Sigma f_i = 1$ and substituting the definition of the price index, we can rewrite the real contract price as

$$\begin{aligned} x_t - p_t &= f_0(x_t - x_t) + f_1(x_t - x_{t-1}) + f_2(x_t - x_{t-2}) + f_3(x_t - x_{t-3}) \\ &= f_1\theta_t + f_2(\theta_t + \theta_{t-1}) + f_3(\theta_t + \theta_{t-1} + \theta_{t-2}) \\ &= g(L)\theta_t \qquad (24) \\ &= g(L)f^{-1}(L)\pi_t. \end{aligned}$$

Substituting this definition of $x_t - p_t$ into equation (21), we obtain

$$v_t = g(L)\pi_t.$$

Substituting this expression for $v_t$ into equation (23) and multiplying both sides of the equation by $g^{-1}(L)$ yields a two-sided inflation equation that is analogous to the two-sided price level equation (see equation (20)):

$$\pi_t = f(L)f(L^{-1})[\pi_t + \gamma g^{-1}(L)y_t]. \tag{25}$$

Note that while the lag-lead distribution on the excess demand term differs a bit from the two-sided price level equation, the lag-lead distribution on $\pi_t$ is identical to the lag-lead distribution on $p_t$ in equation (20). This two-sided Phillips curve is a generalized version of the simple two-period example of equation six in Section I.[17]

## III. Policy Experiments

We perform a battery of policy experiments designed to explore the properties of the relative contracting model. We characterize monetary policy as a nominal output growth "reaction function": nominal output growth $\Delta Y_t$ responds with differing intensity to deviations of inflation from target ($\bar{\pi}$) and deviations of the output

17. Note that the real contracting model implies that all the elements of $g(L)$ are positive. Many estimated Phillips curves, however, find a significant role for both the level and the change in the excess demand term, suggesting that some of the coefficients in $g(L)$ in a less restricted version of the model would be significantly negative.

gap from zero:

$$\Delta Y_t = \alpha_\pi(\pi_t - \bar{\pi}) + \alpha_y \tilde{y}_t. \tag{26}$$

Nominal output is the sum of real output and the price index in the model. We abstract completely from how monetary policy is able to control nominal output.[18] We use $[\alpha_\pi, \alpha_y] = [-0.1, -0.1]$ as the baseline of our policy experiments, and we vary the size of the policy parameters $\alpha_\pi$ and $\alpha_y$ over two orders of magnitude. The baseline parameter setting implies that for a one-percentage-point deviation of inflation above its target (or the output gap above zero), monetary policy decreases nominal output growth by one-tenth percentage point per quarter.

For each setting of the policy parameters, we examine three characteristics of the system. The first is associated with a deterministic thought experiment. We start the system in its steady state with a 3 percent rate of inflation. At the begining of the experiment, we lower the target inflation rate to zero, and we compute the output sacrifice ratio, computed as the cumulative annual deviation of output from trend, discounted at 3 percent per year. The other two characteristics are the unconditional variance of inflation and the output gap as the model is repeatedly shocked by disturbances drawn from the estimated distribution of the model residuals.[19]

Figure IV shows model solution trajectories for the disinflation experiment for both the standard (dashed lines) and the relative (solid lines) contracting models when the policy parameters are set at their baseline values. As is evident from this figure, disinflation proceeds more rapidly and with far less disruption according to the standard model. The sacrifice ratio in the baseline simulation in Figure IV is six times smaller for the standard model than it is for the relative contracting model. As noted by Phelps [1978] and Ball [1991], the standard model can even generate a disinflationary boom.

Figure V displays the paths of inflation and the output gap for similar disinflation experiments for the relative contracting model. The policy parameters are set at plus or minus an order of magnitude around the baseline. The simulation path for each experiment is labeled by its $(\alpha_\pi, \alpha_y)$ pair. Table IX shows the system characteristics for the same settings of the policy parameters.

18. This simplification yields considerably simplified dynamics in the policy experiments, because the lags and simultaneities in the linkage from the instrument of monetary policy to the economy are absent in this simple policy rule.

19. Appendix A details the method for computing the unconditional variances.

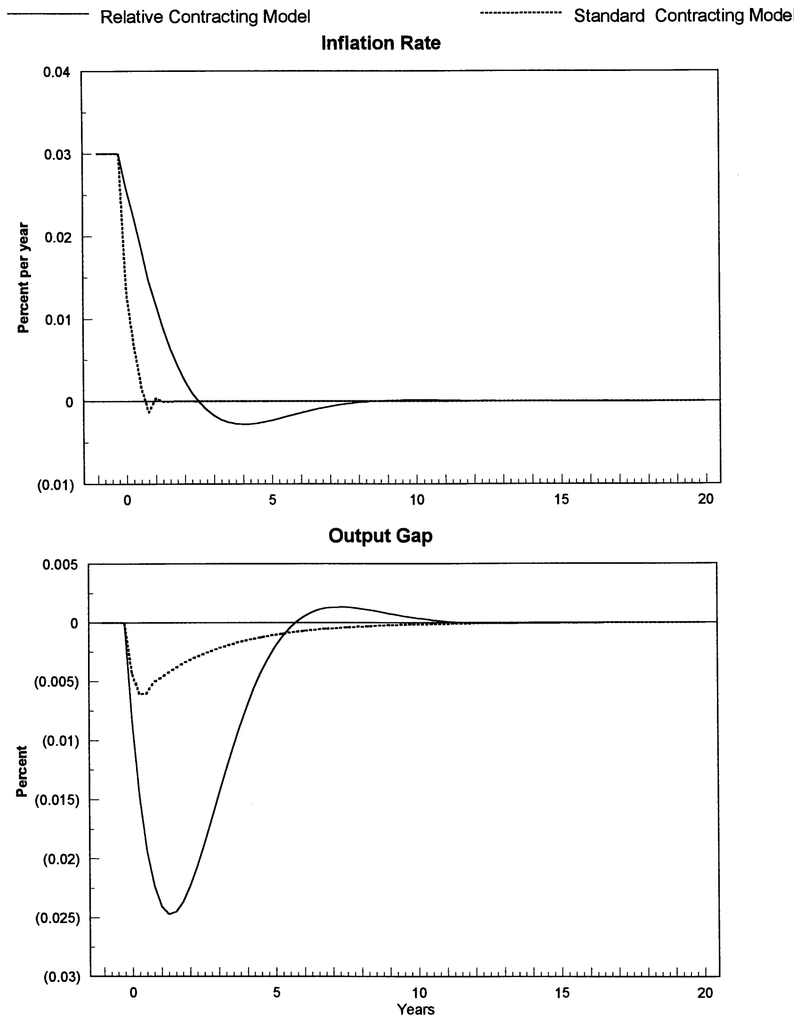


FIGURE IV
Disinflation $\alpha_\pi = \alpha_y = -0.1$

As measured by the disinflation simulations and the three system characteristics, the model behaves sensibly as the policy parameters vary over two orders of magnitude. At the baseline parameter settings, the sacrifice ratio is 2.3, a bit lower than some of the estimates in Gordon [1985], but not outside the traditionally accepted range of estimates. Inflation drops to its new target after

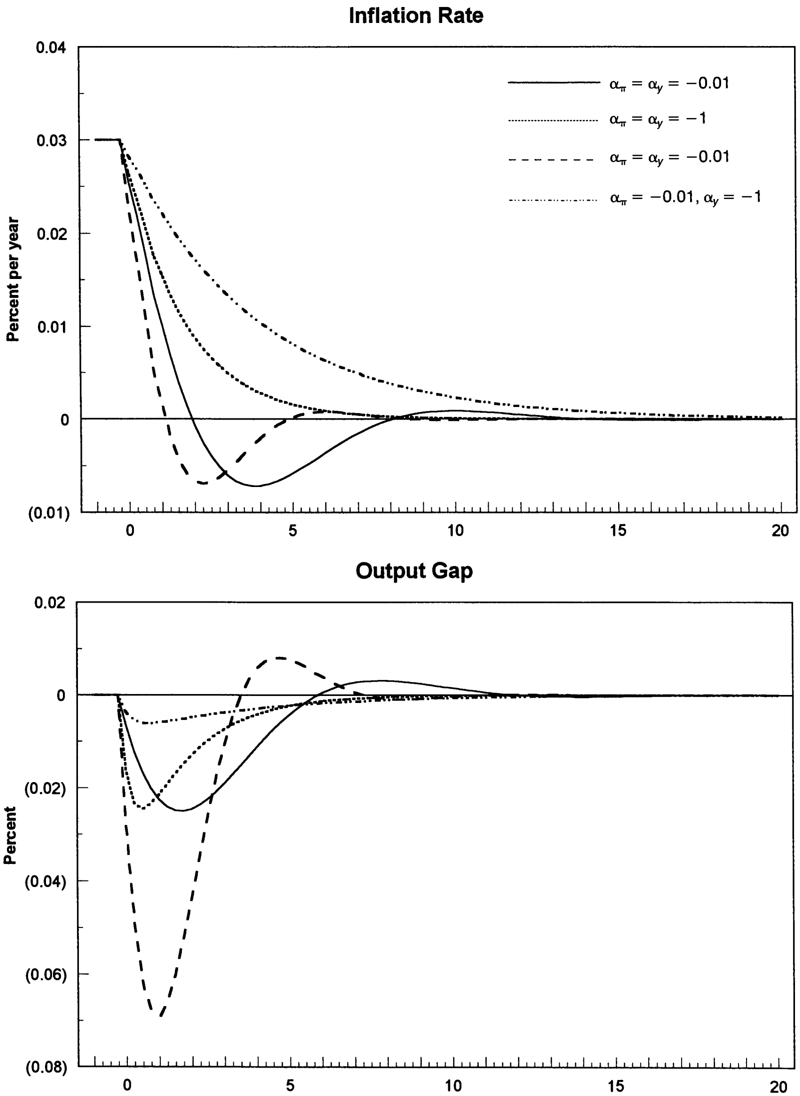


FIGURE V
Disinflation, Relative Contracting Model

about two and one-half years, while the output gap bottoms out at $-2.5$ percent.

When nominal output responds overwhelmingly to inflation ($\alpha_\pi = -1, \alpha_y = -0.01$), inflation drops toward its new target most

TABLE IX
SYSTEM CHARACTERISTICS

| Policy | | | Variance | |
|---|---|---|---|---|
| $\alpha_\pi$ | $\alpha_y$ | Sacrifice ratio | Inflation | Output |
| −0.10 | −0.10 | 2.33 | 0.00104 | 0.00290 |
| −0.01 | −0.01 | 2.47 | 0.00117 | 0.00342 |
| −1.00 | −1.00 | 2.03 | 0.00150 | 0.00199 |
| −1.00 | −0.01 | 4.30 | 0.00060 | 0.01544 |
| −0.01 | −1.00 | 0.90 | 0.00355 | 0.00022 |

quickly, and consequently output loss is greatest. The sacrifice ratio doubles, as the unconditional variance of inflation halves and output variance quintuples. Thus, according to this model, and not surprisingly, a policy that focuses exclusively on inflation pays a hefty price in lost output, measured either by the sacrifice ratio or by the unconditional variance of the output gap.

When nominal output responds in a balanced way to inflation and output deviations, either vigorously or weakly ($\alpha_\pi = \alpha_y = -1$ or $\alpha_\pi = \alpha_y = -0.01$), the implications for output loss are about the same. The sacrifice ratio remains between 2 and 2.5, and the unconditional variances are about the same as the baseline. Thus, a balanced increase or decrease in policy vigor from the baseline does relatively little to alter the sacrifice ratio or the variances of the targets of monetary policy.

When policy concerns itself primarily with output deviations ($\alpha_\pi = -0.01, \alpha_y = -1$), the implications for the sacrifice ratio seem appealing. By very gradually lowering the inflation rate over ten or more years and aggressively stabilizing output, policy appears able to minimize the sacrifice ratio: it drops to 0.9. While the unconditional variance of inflation is more than three times as large as the baseline simulation, output variance is an order of magnitude lower than the baseline. We find this optimistic prediction to be highly implausible. After all, it suggests that the monetary authority could credibly disinflate by lowering the output gap by one basis point for each percentage point deviation of inflation from its target over a span of ten years. It is more likely that a single basis point change in the output gap would go completely unnoticed, and such a policy would be interpreted as not pursuing an inflation target. While the model treats this policy as credible, it is unlikely that market participants would.

## IV. Conclusion

The overlapping wage contracting model of Taylor [1980] imparts considerable persistence to the level of wages and prices. However, it implies that the persistence in the inflation rate derives solely from the excess demand term. Equivalently, the specification implies that both the autocorrelation function of inflation and its cross correlation with output will die out very rapidly.

By way of contrast, the relative contracting model presented above imparts persistence to both the level and the inflation rate of wages and prices. The model implies persistence in the inflation rate in addition to that derived from the excess demand term. Equivalently, the new specification implies that both the autocorrelation function of inflation and its cross correlation with output decay quite slowly.

The data favor the relative contracting model. As measured by its autocorrelation function and its cross correlations with real output, the inflation rate is quite persistent. The inflation autocorrelation function reaches zero only after a lag of about four years, and inflation is strongly correlated with output at lags and leads of two to four years.

The standard contracting model of Phelps and Taylor cannot replicate these prominent features of the data. The inflation autocorrelation function dies out within a year in that model, and the cross correlation of inflation with lagged output is virtually zero. These properties are implied by the functional form of the model, and they are insensitive to our particular parameter estimates. When we increase the response of the contract price to excess demand by a factor of ten thousand times its estimated value, the inflation autocorrelation function remains the same, and the cross correlations of inflation with output remain very small.

When contracting agents negotiate wage contracts relative to real wage contracts in effect during the life of the contract, these properties change dramatically. The relative contracting model can mimic the correlation properties of inflation and output quite nicely. In addition to this graphical evidence of the superiority of the relative contracting model, a likelihood ratio test in a model that nests both specifications decisively rejects the standard contracting model; the $p$-value of the test is $3.9 \times 10^{-6}$. By contrast, the test of the restrictions imposed by the relative contracting model returns a $p$-value of 0.53.

In conjunction with a simple nominal output reaction function, the relative contracting model implies significant trade-offs among monetary policy goals—the output sacrifice ratio and the variance of inflation and output. The model implies that an aggressive disinflationary policy would yield a marked increase in lost output. The model also implies that an extremely gradual disinflation would significantly lower output loss, but we suggest that such a policy would face severe credibility problems.

## APPENDIX A: COMPUTATIONS

Each of our forward-looking models can be cast in the format,

$$\sum_{i=-\tau}^{0} H_i x_{t+i} + \sum_{i=1}^{\theta} H_i E_t(x_{t+i}) = \epsilon_t, \tag{27}$$

where $\tau$ and $\theta$ are positive integers, $x_t$ is a vector of variables, and the $H_i$ are conformable square coefficient matrices. The expectation operator $E_t(\cdot)$ denotes mathematical expectation conditioned on the process history through period $t$:

$$E_t(x_{t+i}) = E(x_{t+i} | x_t, x_{t-1}, \ldots).$$

The random shock $\epsilon_t$ is independently and identically distributed $N(0,\Omega)$. The covariance matrix $\Omega$ is singular whenever equation (27) includes identities.

We represent constants in the model of equation (27) with a device analogous to the column of ones that represents the constant in an ordinary least squares data matrix. Each model includes an identity for the number one:

$$\mathrm{ONE}_t = \mathrm{ONE}_{t-1}.$$

Constants are coded as a coefficient times the variable ONE, and ONE is initialized at unity in the estimation and simulation exercises.

Because $\epsilon_t$ is white noise, $E_t(\epsilon_{t+k}) = 0$ for $k > 0$. Leading equation (27) by one or more periods and taking expectations conditioned on period-$t$ information yields a deterministic forward-looking equation in expectations,

$$\sum_{i=-\tau}^{\theta} H_i E_t(x_{t+k+i}) = 0, \qquad k > 0. \tag{28}$$

We use the generalized saddlepath procedure of Anderson and Moore [1985] to solve equation (28) for expectations of the future in terms of expectations of the present and the past. For a given set of initial conditions, $\{E_t(x_{t+k+i}):k > 0, i = -\tau, \ldots, -1\}$, if equation (28) has a unique solution that grows no faster than a given upper bound, that procedure computes the vector autoregressive representation of the solution path,

$$E_t(x_{t+k}) = \sum_{i=-\tau}^{-1} B_i E_t(x_{t+k+i}), \qquad k > 0. \tag{29}$$

In the models we consider here, the roots of equation (29) lie on or inside the unit circle.

Using the fact that $E_t(x_{t-k}) = x_{t-k}$ for $k > 0$, equation (29) is used to derive expectations of the future in terms of the realization of the present and the past. These expectations are then substituted into equation (27) to derive a representation of the model that we call the *observable structure,*

$$\sum_{i=-\tau}^{0} S_i x_{t+i} = \epsilon_t. \tag{30}$$

Equation (30) is a structural representation of the model because it is driven by the structural disturbance $\epsilon_t$; the coefficient matrix $S_0$ contains the contemporaneous relationships among the elements of $x_t$. It is an observable representation of the model because it does not contain unobservable expectations.

For maximum likelihood estimation of the contracting equations, the reduced-form equations from the VAR for the bill rate and the output gap are combined with the contracting equations in the format of equation (27). For a given set of contracting parameters, the likelihood function of the model is evaluated using the observable structure and the realization of the data. The likelihood function is maximized with a sequential quadratic programming algorithm using numerical derivatives.

Impulse-response functions of the estimated models are computed by simulating the observable structure with appropriate settings for the exogenous shock term $\epsilon_t$.

Computing the vector autocorrelation function of the various models requires a few more steps. Premultiplying the observable structure by $-S_0^{-1}$, we have the *reduced form* of the structural model,

$$x_t = \sum_{i=-\tau}^{-1} B_i x_{t+i} + B_0 \epsilon_t. \tag{31}$$

The coefficient matrices $\{B_i{:}i = -\tau, \cdots, -1\}$ in equation (31) are identical to those in equation (29), while $B_0$ is simply $S_0^{-1}$.

The companion system of the reduced form is

$$\begin{bmatrix} x_t \\ x_{t-1} \\ \vdots \\ x_{t-\tau+1} \end{bmatrix} = \begin{bmatrix} B_{-1} & B_{-2} & \cdots & B_{-\tau} \\ I & & & 0 \\ & \ddots & & \vdots \\ & & I & 0 \end{bmatrix} \begin{bmatrix} x_{t-1} \\ x_{t-2} \\ \vdots \\ x_{t-\tau} \end{bmatrix} + \begin{bmatrix} B_0\epsilon_t \\ 0 \\ \vdots \\ 0 \end{bmatrix}. \tag{32}$$

In a more compact notation the companion system is

$$y_t = Ay_{t-1} + \eta_t, \tag{33}$$

where $y_t = [x_t, \ldots, x_{t-\tau+1}]'$, and $\eta_t = [B_0\epsilon_t, 0, \ldots, 0]'$. Recursively substituting equation (33) into itself,

$$y_{t+k} = A^k y_t + \sum_{i=1}^{k} A^{k-i}\eta_{t+i}. \tag{34}$$

Because $\eta_t$ is uncorrelated over time, the covariance matrix of the $k$-period ahead forecasts of $y_t$ is

$$V_t(y_{t+k}) = \sum_{i=0}^{k-1} A^i\Psi(A^i)', \tag{35}$$

where $\Psi$ is the covariance matrix of $\eta_t$. In a stationary model, as $k$ goes to infinity, the conditional covariance matrix $V_t(y_{t+k})$ converges to $\Gamma_0$, the unconditional covariance matrix of $y_t$.

While the vector autoregressive model in Section II is stationary, the structural models include the log price level, an I(1) variable. In the structural models we compute successive terms of $V_t(y_{t+k})$ until the conditional variances of the I(0) variables converge to constants. At this point the conditional variances of the I(1) variables are increasing at a linear rate. When the conditional variances of the stationary variables converge, we treat the sum in equation (35) as if it were $\Gamma_0$, the unconditional covariance matrix of $y_t$. The vector autocovariance function of $y_t$ is then computed recursively according to

$$\Gamma_k = A\Gamma_{k-1}, \qquad k > 0. \tag{36}$$

This procedure correctly computes the autocovariance function of the stationary variables.

Finally, dividing each row and column of $\Gamma_k$, $k \geq 0$, by the square root of the corresponding diagonal element of $\Gamma_0$ yields the model's vector autocorrelation function.

## Appendix B: An Alternative Definition of the Real Contract Price

As discussed in subsection II.B, our definition of the real contract price, $x_t - p_t$, is a convenient simplification from a theoretically preferable definition. Here we explore the theoretical and empirical implications of defining the real contract price as the difference between the nominal contract price and the weighted average of index prices expected to prevail over the life of the contract.

As in Section I, we use two-period contracts to simplify the exposition, and we cast the discussion in terms of wage rather than price contracts. Denote the weighted average of price indexes expected to prevail over the life of the contract as $\bar{p}$:

$$\bar{p}_t = (\tfrac{1}{2})(p_t + E_t p_{t+1}).$$

Defining the (expected) real contract wage as the difference between the nominal contract wage and $\bar{p}$, the index of real contract wages, $v_t$ becomes

$$v_t = (\tfrac{1}{2})(x_t - \bar{p}_t + x_{t-1} - \bar{p}_{t-1}).$$

In the definition of $v_t$ we assume that the index prices that appear in $\bar{p}$ that are dated $t$ and earlier are *realized,* not expected, price indexes. Thus, agents negotiating nominal contracts in period $t$ compare the current real contract wage with *realized* real contract wages that were negotiated in the past and are still in effect.[20] For example, from the viewpoint of period $t$, the realized real contract wage that was negotiated in period $t - 2$ is

$$x_{t-2} - (\tfrac{1}{2})(p_{t-2} + p_{t-1}).$$

Remembering the distinction between realized and expected real contract wages, the contracting specification that corresponds to equation (15) is[21]

$$x_t - \bar{p}_t = (\tfrac{1}{2})(v_t + E_t v_{t+1} + \gamma[\tilde{y}_t + \tilde{y}_{t+1}]) + \epsilon_t. \tag{37}$$

20. Defining the relative contract price index in terms of *expected* price indexes from different viewpoint dates does not alter the qualitative behavior of the model.

21. This specification may also be represented as a two-sided Phillips curve, although the lag structure is no longer symmetric. Using the new definitions of the real contract price index, $v_t$, and the contracting equation, we can derive

$$\pi_t = [g^{-1}(L)h(L^{-1}) + f(L)f(L^{-1})(I - g^{-1}(L)h(L^{-1})]\pi_t + f(L^{-1})g^{-1}(L)\gamma y_t,$$

where $f(L)$ is defined in equation (9), $g(L)$ is defined in equation (24), and

$$h(L^{-1}) = (f_1 + f_2 + f_3)L^{-1} + (f_2 + f_3)L^{-2} + f_3L^{-3}.$$

## APPENDIX C: EMPIRICAL ESTIMATES OF THE $\bar{p}$ SPECIFICATION

We estimate the slope and excess demand parameters for the four-quarter contract version of equation (37) by maximum likelihood, taking as given the reduced-form equations for the output gap and the bill rate. Table X displays the parameter estimates. The autocorrelation function for the alternative specification, computed at the parameter estimates in Table X, preserves all the qualitative features of the autocorrelation function for the relative contracting model presented in the text.

### 1. A Formal Hypothesis Test of the p and $\bar{p}$ Specifications

Both the $\bar{p}$ and $p$ specifications may be nested within a more general model. As in subsection II.C the general model uses one parameter, $0 \leq \delta_1 \leq 1$, to index the definition of real contract prices between $p$ ($\delta_1 = 1$) and $\bar{p}$ ($\delta_1 = 0$). The following three equations, combined with the rest of the equations common to the $p$ and $\bar{p}$ models, comprise the model within which the two specifications may be nested:

$$\bar{p}_t = \delta_1 p_t + (1 - \delta_1) \sum_{i=0}^{3} f_i E_t p_{t+i}$$

(38) $$v_t = \sum_{i=0}^{3} f_i (x_{t-i} - \bar{p}_{t-i})$$

$$x_t - \bar{p}_t = \sum_{i=0}^{3} f_i E_t (v_{t+i} + \gamma \tilde{y}_{t+i}) + \epsilon_t.$$

While the economic interpretation of the hybrid $p/\bar{p}$ model is not particularly appealing, this nesting specification allows us to assess the empirical importance of the simplification that we use in our relative contracting specification.

TABLE X
REAL CONTRACTING ESTIMATE, "$\bar{p}$" DEFINITION OF REAL CONTRACT PRICE

| Coefficient | Value | Standard error |
|---|---|---|
| s | 0.07746 | 0.01411 |
| $\gamma$ | 0.00109 | 0.00071 |
| Residual s.e. | 0.0039 | |
| Q(12) | 28.1 | |
| Log likelihood | 1729.2 | |

To test the restrictions imposed by the $p$ and the $\bar{p}$ versions of the relative contracting model compared with the model that nests them both, we freely estimate the slope parameter $s$, the excess demand parameter $\gamma$, and the $p/\bar{p}$ parameter $\delta_1$ in the nesting model. The estimated values of $[s,\gamma,\delta_1]$ are [0.08,0.004,1.0], with a converged log-likelihood value of 1737.4. The converged values for the nesting model are nearly identical to those for the $p$ real contracting model, and the converged log-likelihood value is virtually the same. Thus, one cannot reject the restriction imposed by the $p$ real contracting model. The likelihood ratio test for the restriction imposed by the $\bar{p}$ model takes the value 9.24 with one degree of freedom. The data reject the theoretically preferable $\bar{p}$ model with a $p$-value of less than $1.0 \times 1.0^{-5}$.

Finally, one may consider a model that nests both the standard/relative contracting model and the $p/\bar{p}$ models:

$$\bar{p}_t = \delta_1 p_t + (1 - \delta_1) \sum_{i=0}^{3} f_i E_t p_{t+i}$$

$$(39) \qquad v_t = \sum_{i=0}^{3} f_i(x_{t-i} - \delta_2 \bar{p}_{t-i})$$

$$x_t - \delta_2 \bar{p}_t = \sum_{i=0}^{3} f_i E_t(v_{t+i} + \gamma \tilde{y}_{t+i}) + \epsilon_t,$$

where $\delta_2$ indexes between relative and standard contracting models ($\delta_2 = 1$ for real contracts; $\delta_2 = 0$ for nominal contracts). Unfortunately, attempts to freely estimate this model were unsuccessful, due to numerical difficulties. The slope, excess demand, relative/standard, and $p/\bar{p}$ parameters appear to be jointly unidentified in the data. However, taking the nested hypotheses one at a time, we can decisively reject the standard contracting model in favor of the relative contracting model, and reject the $\bar{p}$ in favor of the $p$ model.

FEDERAL RESERVE BANK OF BOSTON

## REFERENCES

Anderson, Gary, and George Moore, "A Linear Algebraic Procedure for Solving Linear Perfect Foresight Models," *Economics Letters*, XVII (1985), 247–52.

Ball, Laurence, "The Genesis of Inflation and the Costs of Disinflation," *Journal of Money, Credit and Banking*, XXIII (1991), 439–52.

——, "What Determines the Sacrifice Ratio?" NBER Working Paper No. 4306, March 1993.

Bernanke, Ben S., and Alan S. Blinder, "The Federal Funds Rate and the Channels of Monetary Transmission," *American Economic Review*, LXXXII (1992), 901–21.

Buiter, Willem, and Ian Jewett, "Staggered Wage Setting with Real Wage Relativities: Variations on a Theme of Taylor," *The Manchester School,* XLIX (1981), 211–28.
Christiano, Lawrence J., and Martin Eichenbaum, "Unit Roots in Real GNP: Do We Know, and Do We Care?" *Carnegie-Rochester Conference Series on Public Policy,* XXXII (1990), 7–62.
Fuhrer, Jeffrey C., and George R. Moore, "Monetary Policy Trade-offs and the Correlation between Nominal Interest Rates and Real Output," *American Economic Review,* LXXXV (March 1995), forthcoming.
Fuller, Wayne A., *Introduction to Statistical Time Series* (New York: John Wiley & Sons, Inc, 1976).
Gordon, Robert J., "Understanding Inflation in the 1980s," *Brookings Papers on Economic Activity* (1985), 263–99.
Johansen, Søren, and Katerina Juselius, "Maximum Likelihood Estimation and Inference on Cointegration—With Applications to the Demand for Money," *Oxford Bulletin of Economics and Statistics,* LII (1990), 169–210.
Phelps, Edmund S., "Disinflation without Recession: Adaptive Guideposts and Monetary Policy," *Weltwirtschaftliches Archiv,* C (1978), 239–65.
Taylor, John B., "Aggregate Dynamics and Staggered Contracts," *Journal of Political Economy,* LXXXVIII (1980), 1–24.
——, "Union Wage Settlements during a Disinflation," *American Economic Review,* LXXIII (1983), 981–93.