(Some theoretical aspects of) Corporate Finance

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Outline: the financial structure of the firm

- Irrelevance results: the Modigliani–Miller’s theorems
- Taxation
- Limited liability and bankruptcy
- Irrelevance of the payout policy
The financial structure of the firm: literature

- Modigliani, F. and Miller, M.
  The cost of capital, corporation finance, and the theory of investment.

- Stiglitz, J.
  A re-examination of the Modigliani–Miller theorem.

- Stiglitz, J.
  On the irrelevance of corporation financial policy.
  *American Economic Review* 64, 1974
The financial structure of the firm: literature

DeMarzo, P. M.

DeMarzo, P. M. and Duffie, D.

Purnanandam, A.
The financial structure of the firm

- From now on, we fix a production plan $y$ and we focus on the relevance of the financial plan $\beta$
- Assume the objective of the manager is to maximize the value

\[ V = E + D + y_0 \]

- Recall that
  - the (present value of the) debt is given by $D = q \cdot \beta$
  - the equity price $E$ (and the security price $q$) is determined at equilibrium and may depend on $\beta$
- The manager maximizes the function $\beta \mapsto E(\beta) + q \cdot \beta$
- Assume for simplicity that $y_0 = 0$
- There is a natural question: Is there an optimal choice $\beta^*$?
- The answer is: the financial policy of the firm is irrelevant
A simple case

- Consider that there is a riskless asset $a_0$ promising 1, i.e.,

$$\forall s \in S, \quad \kappa_s(a_0) = 1$$

We denote by $r_0$ the risk free interest rate, i.e.,

$$q(a_0) = \frac{1}{1 + r_0}$$

- Assume that the firm can only issue (sell) the riskless bond $a_0$, i.e.,

$$\exists Z \geq 0, \quad \beta = Z 1_{a_0}$$

In other words, the firm commits to deliver at $t = 1$ the amount $Z$ independent of the state of nature.
A simple case

The value of the firm is equity plus debt, i.e.,

\[ V = E + D \quad \text{where} \quad D = \frac{1}{1 + r_0} Z \]

- the firm is said levered if \( D \neq 0 \)
- the firm is said unlevered if \( D = 0 \)
- the debt-equity ratio is defined by

\[ \frac{D}{E} \]

- a specific choice of \( \beta \) leads to a specific debt equity ratio

\[ D(\beta)/E(\beta) \]
The conventional approach before 1958

Assume the firm starts without debt

- the dividend of the firm $\delta_1 = (y_s)_{s \in S}$ is risky
- debt is not risky, issuing debt may attract investors and increase the firm’s value
  - no debt: investors can buy the risky cash flow $(y_s)_{s \in S}$
  - with debt: investors can still buy a risky cash flow $(y_s - Z)_{s \in S}$ but now they can also buy a riskless cash flow $Z 1_S$

- a low level of debt reduces the contingencies where the firm will be unable to meet its obligations
- however, for a high level of debt, the possibility for bankruptcy becomes non-negligible and the benefits of leverage (issuing non-risky asset) disappears
- it seems that there is an optimal level of debt
Weak irrelevance result: Modigliani–Miller (1958)

**Theorem**

Consider a stock market equilibrium

\[
\{(y(j), \beta(j))_{j \in J}, (q, E), (c^i, \theta^i, \alpha^i)_{i \in I}\}
\]

Assume that

- the two firms \( j_1 \) and \( j_2 \) have the same production plan, i.e.,
  \( y(j_1) = y(j_2) \)
- but may have different financial policies \( \beta(j_1) \neq \beta(j_2) \)

Then the two firms have the same value, i.e.,

\[
E(j_1) + q \cdot \beta(j_1) = E(j_2) + q \cdot \beta(j_2)
\]
Weak irrelevance result: Modigliani–Miller (1958)

Proof.

Assume there exists an agent $i$ having shares of both firms, i.e.,

$$\alpha^i(j_1) > 0 \quad \text{and} \quad \alpha^i(j_2) > 0$$

Then there exists a family of state price deflators $(\lambda_s)_{s \in S}$ in $\mathbb{R}_{++}^S$ such that

$$q = \sum_{s \in S} \lambda_s \kappa_s$$

and

$$\forall j \in \{j_1, j_2\}, \quad E(j) = \sum_{s \in S} \lambda_s \delta_s(j)$$

This implies that

$$\forall j \in \{j_1, j_2\}, \quad V(j) \equiv E(j) + q \cdot \beta(j) = \sum_{s \in S} \lambda_s y_s$$
Weak irrelevance result: Modigliani–Miller (1958)

What about if no agent has shares of both firms?

Proof.

...
Strong irrelevance result: Stiglitz (1969, 1974)

**Theorem**

Consider a stock market equilibrium

\[
\{(y(j), \beta(j))_{j \in J}, (q, E), (c^i, \theta^i, \alpha^i)_{i \in I}\}
\]

For any other financial plan \((\hat{\beta}(j))_{j \in J}\) there exists another stock market equilibrium such that firms’ values remain unchanged.
Strong irrelevance result: Stiglitz (1969, 1974)

More precisely, the following family

\[ \{(y(j), \hat{\beta}(j))_{j \in J}, (q, \hat{E}), (c^i, \hat{\theta}^i, \alpha^i)_{i \in I}\} \]

is a stock market equilibrium where

\[ \hat{E}(j) = E(j) + q \cdot [\beta(j) - \hat{\beta}(j)] \]

\[ \hat{\theta}^i = \theta^i + \sum_{j \in J} \alpha^i(j) [\hat{\beta}(j) - \beta(j)] \]
Conditions for the validity of irrelevance results

- Frictionless capital markets:
  - no transaction costs
  - no institutional restrictions on security trades
- No taxes
- Investors and firms have access to the same security (bond) markets
- Unlimited liability of shareholders
- Investor do not try to infer information about the likelihood of states of nature by observing firm’s financial policy
Conditions for the validity of irrelevance results

- Linear effect: the change in the return on equity, induced by a modification of the firm’s capital structure, is a linear combination of the returns on the assets traded by the firm.
- Consider there are derivative securities (like options) whose payoff is a function (possibly non-linear) of the future value of another asset.
- When the firm’s capital structure changes, not only the dividend of the stock changes but also that of the derivative securities written on the firm’s shares.
- In general, the firm’s valuation will not be independent of the firm’s capital structure.

Gottardi, P.
An analysis of the conditions for the validity of Modigliani–Miller theorem with incomplete markets.
*Economic Theory* 5, 1995
The role of taxes

- Taxes are paid at $t = 1$
- We do not explain (model) the endogenous formation of tax rates
- We do not explain the purpose of taxes (intermediaries, public projects)

### Three tax rates

- Corporate tax rate $t_c$ on corporate income $\delta_1 = y_1 - \kappa_1 \cdot \beta$
- Personal tax rate $t_s$ on equity income
- Personal tax rate $t_b$ on financial assets (bonds) income

### Assumption

- All tax payers face the same tax rates
- Tax rates are constant and independent of the level of income to which they apply
Assumptions

Since taxes apply to income, we assume that

- every security \(a\) is claim to a non-negative amount, i.e.,
  \[\forall a \in A, \ \forall s \in S, \ \kappa_s(a) \geq 0\]

- every firm \(j\) never takes the risk to be bankrupt, i.e., the production-finance plan \((y, \beta)\) belong to \(D\) where
  \[D \equiv \{(y, \beta) \in Y \times \mathbb{R}^A : \ \forall s \in S, \ y_s \geq \kappa_s \cdot \beta\}\]
Investors’ budget sets

Each investor $i$ takes as given:

- each firm $j$’s production-finance plan $(y(j), \beta(j))$
- the security price vector $q$ and the equity price vector $E$

and chooses

- a consumption plan $(c_0, c_1) \in \mathbb{R}_+ \times \mathbb{R}_+^S$
- a security portfolio $\theta \in \mathbb{R}_+^A$ and shareholdings $\alpha \in \mathbb{R}_+^J$

satisfying budget restrictions
Investors’ budget sets

- solvency restriction at $t = 0$

$$c_0 + q \cdot \theta + E \cdot \alpha \leq \omega^i_0 + (E + \delta_0) \cdot \xi^i$$ (1)

- solvency restriction at every $s \in S$ at $t = 1$

$$c_s \leq \omega^i_s + (1 - \tau_b) \kappa_s \cdot \theta + (1 - \tau_s)(1 - \tau_c)\delta_s \cdot \alpha$$ (2)

The set of actions $(c, \theta, \alpha)$ satisfying (1) and (2) is denoted by $B^i(q, E, \tau)$.
A stock market equilibrium with taxes is a list

\[ \{(y(j), \beta(j))_{j \in J}, (q, E), (c^i, \theta^i, \alpha^i)_{i \in I}\} \]

of

- production-finance plan \((y(j), \beta(j))\) for each firm \(j\)
- security and equity prices \((q, E)\)
- action \((c^i, \theta^i, \alpha^i)\) of each investor \(i\)

such that
Stock market Equilibrium

(a) investor \(i\)'s action is optimal, i.e.,

\[
(c^i, \theta^i, \alpha^i) \in \arg\max \{U^i(c) : \ c \in B^i(q, E, \tau)\}
\]

(b) security and stock markets clear, i.e.,

\[
\forall a \in A, \ \sum_{j \in J} \beta(j, a) = \sum_{i \in I} \theta^i(a) \quad \text{and} \quad \forall j \in J, \ \sum_{i \in I} \alpha^i(j) = 1
\]

(c) commodity markets clear, i.e.,

\[
c_p^0 + \sum_{i \in I} c^i = \sum_{i \in I} \omega^i + \sum_{j \in J} y_s(j)
\]

where \(c_0^p = 0\) and

\[
c_s^p \equiv \sum_{j \in J} \left\{\tau_c + (1 - \tau_c)\tau_s\right\} y_s(j) + \left\{\tau_b - \tau_c - (1 - \tau_c)\tau_s\right\} \kappa_s \cdot \beta(j)
\]
Gains from corporate leverage

Assume that there exist two firms $j_u$ and $j_\ell$ such that

- both firms have the same production plan, i.e., there exists $y \in Y(j_u) \cap Y(j_\ell)$ such that

$$y = y(j_u) = y(j_\ell)$$

- firm $j_u$ is unlevered, i.e., $\beta(j_u) = 0$
- firm $j_\ell$ is levered, i.e., $\beta(j_\ell) > 0$
- firm $j_\ell$ is never bankrupt, i.e.,

$$\forall s \in S, \quad y_s \geq \kappa_s \cdot \beta(j_\ell)$$
Gains from corporate leverage

Proposition

Assume that there is a stock market equilibrium

\[ \{(y(j), \beta(j))_{j \in J}, (q, E), (c^i, \theta^i, \alpha^i)_{i \in I}\} \]

Then

\[ V(j_\ell) = V(j_u) + \left[ 1 - \frac{(1 - \tau_c)(1 - \tau_s)}{1 - \tau_b} \right] D(j_\ell) \]
Role of corporate taxes

Assume that personal income taxes coincide, i.e.,

\[ \tau_s = \tau_b \]

then

\[ V(j_\ell) = V(j_u) + \tau_c D(j) \]

- Firms have an incentive to raise capital via bond issues until the bankruptcy limit
- Does not fit with data
Role of personal tax system

Assume that taxes matter, i.e.,

\[(1 - \tau_c)(1 - \tau_s) \neq 1 - \tau_b\]

then

- if

  \[(1 - \tau_c)(1 - \tau_s) < 1 - \tau_b\]

  then firms still have an incentive to raise capital via bond issues until the bankruptcy limit

- if

  \[(1 - \tau_c)(1 - \tau_s) > 1 - \tau_b\]

  then firms have an incentive to fully finance their capital through equity

- these predictions do not fit with data
Do taxes may provide an optimal debt-to-equity ratio?

Theory

Miller, M. H.
Debt and taxes.
*Journal of Finance* 32, 1977

Empirical Literature

Mackie-Mason, Jeffrey K.
Do taxes affect corporate financing decisions?
*Journal of Finance* 45, 1990

Desai, M. A., Foley, C. F., Hines Jr., J. R.
A multinational perspective on capital structure choice and internal capital markets.
*Journal of Finance* 59, 2004
Limited liability and bankruptcy

- Assume that only the riskless bond is available, i.e., $A = \{a_0\}$
- Fix a firm and its production-finance plan $(y, Z)$
- It may be the case that for some state $s$, we have
  \[ \delta_s < 0 \quad \text{or equivalently} \quad y_s < Z \]
- In order to protect (and attract) investors, shareholders have limited liability in the sense that a new shareholder at $t = 1$ is not called upon to make payments when $\delta_s < 0$
Limited liability and bankruptcy

- If $\delta_s < 0$ the firm is said to be bankrupt:
  - all the firm’s production $y_s$ is used to pay bondholders
  - shareholders receive nothing
- It is as if the firm is issuing a bond which yields
  \[ \forall s \in S, \quad b_s = \min\{y_s, Z\} \]
- One share of the firm yields
  \[ \forall s \in S, \quad \hat{\delta}_s = \max\{\delta_s, 0\} = \max\{y_s - Z, 0\} \]
What are the markets available to investors

Irrelevance of the financial policy

Every investor can purchase or sell any bond issued by firms

Possible relevance of the financial policy

1. Investors can purchase or sell only a “pooled bond” based on firms’ bonds
2. Investors can only purchase bonds issued by firms
Empirical Literature

- Smith, C. and Warner, J.

- Weiss, L. A.
  Bankruptcy resolution: Direct costs and violation of priority of claims.

- Opler, T. C. and Titman, S.
  Financial distress and corporate performance.
  *Journal of Finance* 49, 1994
Firms objectives

- We focused our attention on the effects of the firm’s decisions on the assets’ rates of return.
- Other factors may guide the firms’ financial decisions. The capital structure may be viewed as:
  - a signaling device: Ross (1977), Leland and Pyle (1977)
  - a way of reducing agency costs: Jensen and Meckling (1976)
  - a way to allocate control rights between various claimants: Aghion and Bolton (1988), Harris and Raviv (1989)
Irrelevance of the payout policy: A simple multi-period model

- A multi period stock market: \( t \in \{0, 1, \ldots, T\} \)
- There is no uncertainty at every period
- Bonds (or financial assets) are long-lived and pay dividends at every period \( t \)
- There is only one firm
Payout policies

The firm chooses:

- An investment-production plan \((x, y) \in Z\), where

\[ x = (x_0, x_1, \ldots, x_{T-1}) \in \mathbb{R}_+^T \]

and

\[ y = (y_1, \ldots, y_T) \in \mathbb{R}_+^T \]

- \(x_t\) represents the units of the good invested (inputs) at period \(t\)
- \(y_{t+1}\) represents the units of the good produced (outputs) at period \(t + 1\)
- The technological restrictions are represented by the set \(Z\)
Payout policies

The firm chooses:

- A financial plan \( \beta = (\beta_0, \beta_1, \ldots, \beta_{T-1}) \) where
  \[
  \beta_t \in \mathbb{R}^A
  \]

- A dividend policy \( n = (n_0, n_1, \ldots, n_{T-1}) \) where \( n_t \) represents the number of shares at
  - the end of period \( t \), after payment of dividends \( n_{t-1}\delta_t \)
  - beginning of period \( t + 1 \) before payments of dividends \( n_t\delta_{t+1} \)
Interpretation and notations

Denote by $p_t$ the ex-dividend price of one stock of firm $j$ at period $t$ after announcing the payout policy.

- If $n_t > n_{t-1}$ then firm $j$ is issuing shares and obtains the revenue $p_t[n_t - n_{t-1}]$.

- If $n_t < n_{t-1}$ then firm $j$ repurchases shares and spends $-p_t[n_t - n_{t-1}]$. 
Interpretation and notations

Consider the firm’s production and payout policy

\((x, y, \beta, n)\)

- each investor \(i\) chooses the stockholding \(\alpha_t \geq 0\) in units of stocks of the firm
- one stock of the firm purchased at date \(t\) is a claim to the dividend \(\delta_{t+1}\) paid at period \(t+1\)

\[n_t \delta_{t+1} = y_{t+1} - x_{t+1} - \kappa_{t+1} \beta_t + p_{t+1} [n_{t+1} - n_t] + q_{t+1} [\beta_{t+1} - \beta_t]\]

- for notational convenience, we let

\[y_0 = 0, \quad x_T = 0, \quad n_{-1} = 1 \quad \text{and} \quad \beta_{-1} = 0\]
Investors’ budget sets

Each investor $i$ takes as given:

- the firm’s production-investment plan $(y, x)$ and payout policy $(\beta, n)$
- the securities’ prices $q = (q_t)_t$ and the stock prices $p = (p_t)_t$

and chooses

- a consumption plan $c = (c_t)_t \in \mathbb{R}^{T+1}_+$
- a security portfolio $\theta \in \mathbb{R}^{A \times T}$ and a stock-holdings vector $\alpha \in \mathbb{R}^T_+$ satisfying ...
Investors’ budget sets

initially at \( t = 0 \)

\[ c_0 + q_0 \cdot \theta_0 + p_0 \cdot \alpha_0 \leq \omega^i_0 + (p_0 + \delta_0) \cdot \xi^i \quad (3) \]

for each \( t \in \{1, \ldots, T - 1\} \),

\[ c_t + q_t \cdot \theta_t + p_t \cdot \alpha_t \leq \omega^i_t + (\kappa_t + q_t) \cdot \theta_{t-1} + (\delta_t + p_t) \cdot \alpha_t \quad (4) \]

and finally

\[ c_T \leq \omega^i_T + \kappa_T \cdot \theta_{T-1} + \delta_T \cdot \alpha_{T-1} \]

The set of actions \((c, \theta, \alpha)\) satisfying the budget constraints is denoted by \(B^i(q, p)\)
Investors’ budget sets

initially at $t = 0$

$$c_0 + q_0 \cdot (\theta_0 - \xi^i) + p_0 \cdot (\alpha_0 - 0) \leq \omega^i_0 + \delta_0 \cdot \xi^i$$

(5)

for each $t \in \{1, \ldots, T - 1\}$,

$$c_t + q_t \cdot (\theta_t - \theta_{t-1}) + p_t \cdot (\alpha_t - \alpha_{t-1}) \leq \omega^i_t + \kappa_t \cdot \theta_{t-1} + \delta_t \cdot \alpha_{t-1}$$

(6)

and finally

$$c_T \leq \omega^i_T + \kappa_T \cdot \theta_{T-1} + \delta_T \cdot \alpha_{T-1}$$

The set of actions $(c, \theta, \alpha)$ satisfying the budget constraints is denoted by $B^i(q, p)$
Stock market Equilibrium

Given the firm’s capital budgeting and payout policy

$$(x, y, \beta, n)$$

a stock market equilibrium is a list of

$$\{(q, p), (c^i, \theta^i, \alpha^i)_{i \in I}\}$$

of

- security and share prices \((q, p)\)
- action \((c^i, \theta^i, \alpha^i)\) of each investor \(i\)

such that
Stock market Equilibrium

(a) investor \( i \)'s action is optimal, i.e.,

\[ (c^i, \theta^i, \alpha^i) \in \arg\max \{ U^i(c) : (c, \theta, \alpha) \in B^i(q, p) \} \]

(b) security and stock markets clear, i.e., for every \( 0 \leq t < T - 1 \)

\[ \forall a \in A, \quad \beta_t(a) = \sum_{i \in I} \theta^i_t(a) \quad \text{and} \quad \sum_{i \in I} \alpha^i_t = n_t \]

(c) commodity markets clear, i.e.,

\[ \sum_{i \in I} c^i = [y - x] + \sum_{i \in I} \omega^i \]
No-arbitrage and state price deflator

The definition is the same apart from stock markets clearing condition:

\[ \forall t \in \{0, \ldots, T - 1\}, \quad \sum_{i \in I} \alpha^i_t = n_t \]

The value \( V_t \) of the firm at period \( t \) is defined by

\[ V_t = p_t n_t + q_t \cdot \beta_t + (y_t - x_t) \]
Irrelevance of the payout policy

By no-arbitrage, there exists a vector of state prices

$$\lambda = (\lambda_0, \ldots, \lambda_T) \in \mathbb{R}_{++}^{T+1}$$

such that for every \(t \in \{0, \ldots, T - 1\}\)

$$\lambda_t q_t = \lambda_{t+1}(\kappa_{t+1} + q_{t+1}) \quad \text{and} \quad \lambda_t p_t = \lambda_{t+1}(p_{t+1} + \delta_{t+1})$$

Observe that

$$\lambda_0 V_0 = \sum_{t=0}^{T} \lambda_t (y_t - x_t)$$

and

$$\lambda_t V_t = \sum_{\tau=t}^{T} \lambda_{\tau} (y_{\tau} - x_{\tau})$$

The values of the firm are independent of the payout policy.
How to explain the existence of an optimal capital structure?

Costs of agency

These costs are due to conflict of interests among agents involved in the financial decisions:

- debtholders, new shareholders, old shareholders and managers all enter into negotiations for different reasons
- bringing them into agreement is costly, concessions are required to achieve at least a second-best solution
How to explain the existence of an optimal capital structure?

Costs of asymmetric information

Managers (insiders) may have a private information about the investment opportunities of the firm and related cash flows

- the information asymmetry may lead to some inefficiencies in the financing decisions of the firms
- managers may use their choice of capital structure as a credible signal to the market of their private information