LISTA 6

1)

a) MONÓTONA: $\begin{cases} \text{se } K' > K, & F(K', L) \geq F(K, L) \\ \text{se } L' > L, & F(K, L') \geq F(K, L) \end{cases}$

logo todas são monótonas

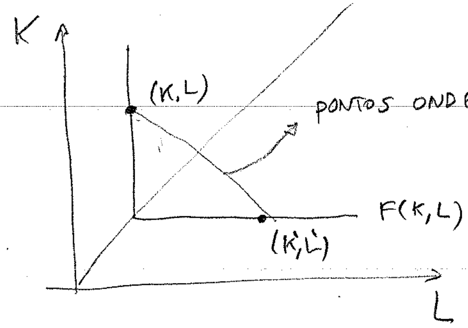
b) F é côncava se para (K,L) e (K',L')

$$F(\alpha K + (1-\alpha)K', \alpha L + (1-\alpha)L') \geq \alpha F(K,L) + (1-\alpha) F(K',L')$$

$\forall \alpha \in [0,1]$

ISOQUANTAS

$F(K,L) = \min\{K, L\}$ é côncava (conjunto de produção convexo)



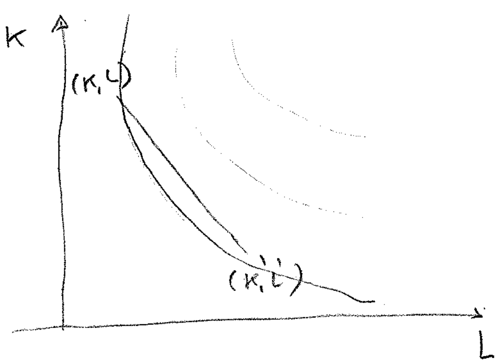
PONTOS ONDE $F(\alpha K + (1-\alpha)K', \alpha L + (1-\alpha)L') \geq \alpha F(K,L) + (1-\alpha)F(K',L')$

$F(K,L) = K^{\alpha} L^{\beta}$

$0 < \alpha \leq 1$

$0 < \beta \leq 1$

é côncava



$F(K,L) = K + L$

é côncava

pois

$F(\alpha K + (1-\alpha)K') =$

$\alpha F(K) + (1-\alpha) F(K')$

$$F(K,L) = K^{1/2} + L^{1/2}$$

$$(\alpha K + (1-\alpha)K')^{1/2} + (\alpha L + (1-\alpha)L')^{1/2}$$

$$\geq \alpha(K^{1/2} + L^{1/2}) + (1-\alpha)K'^{1/2} + (1-\alpha)L'^{1/2}$$

pois

$$(\alpha K + (1-\alpha)K')^{1/2} \geq \alpha K^{1/2} + (1-\alpha)K'^{1/2}$$

e

$$(\alpha L + (1-\alpha)L')^{1/2} \geq \alpha L^{1/2} + (1-\alpha)\cancel{k}^{1/2}$$

pois

$$f : \mathbb{R}_+ \to \mathbb{R}$$

$f(x) = \sqrt{x}$ é côncava.

b) Produtividade Marginal do mesmo $\frac{\partial F}{\partial K}$, $\frac{\partial F}{\partial E}$

Para sabermos se é decrescente tomamos a segunda derivada

$$\frac{\partial}{\partial L}\left(\frac{\partial F}{\partial L}\right) \text{ e } \frac{\partial'}{\partial K}\left(\frac{\partial F}{\partial K}\right)$$

$$F(K,L) = K^{\alpha}L^{\beta}$$

$$\frac{\partial F}{\partial K} = \alpha K^{\alpha-1}L^{\beta} \Rightarrow \frac{\partial^2 F}{\partial K^2} = \alpha(\alpha-1)K^{\alpha-2}L^{\beta} \begin{cases} <0 & \text{se } \alpha<1 \text{ (decrescente)} \\ =0 & \text{se } \alpha=1 \text{ (constante)} \\ > & \text{se } \alpha>1 \text{ (crescente)} \end{cases}$$

$$\frac{\partial F}{\partial L} = \beta K^{\alpha}L^{\beta-1} \Rightarrow \frac{\partial^2 F}{\partial L^2} = \beta(\beta-1)K^{\alpha}L^{\beta-2} \begin{cases} <0 & \text{se } \beta<1 \\ =0 & \beta=1 \\ > & \text{se } \beta>1 \end{cases}$$

$$F(K, L) = \min\{K, L\}$$

$$\frac{\partial F}{\partial L} = \begin{cases} 0, & \text{se } L \geq K \\ 1, & \text{se } L < K \end{cases}$$

logo $\frac{\partial}{\partial L}\left(\frac{\partial F}{\partial L}\right) = 0$ (para K é análogo)

$$F(K, L) = K + L$$

$$\frac{\partial F}{\partial K} = 1, \quad \frac{\partial^2 F}{\partial L^2} = 0$$

$$F(K, L) = K^{1/2} + L^{1/2}$$

$$\frac{\partial F}{\partial K} = \frac{1}{2}\frac{1}{K^{1/2}}, \quad \frac{\partial F}{\partial K} = \frac{-1}{4} K^{-3/2} < 0$$ (decrescentes)

c) Retornos de escala

$\lambda > 0$

$$F(\lambda K, \lambda L) \begin{cases} < \lambda F(K, L), & \text{decrescentes} \\ = \lambda F(K, L), & \text{constantes} \\ > \lambda F(K, L), & \text{crescentes} \end{cases}$$

Funções a) $\begin{cases} \text{constante se } \alpha + \beta = 1 \\ \text{decrescente se } \alpha + \beta < 1 \end{cases}$

b) constante

c) constante

d) $F(K, L) = K^{1/2} + L^{1/2}$

$$F(\lambda K, \lambda L) = \lambda^{1/2} F(K, L) \begin{cases} < \lambda F(K, L) & \text{se } \lambda > 1 \\ > \lambda F(K, L) & \text{se } \lambda < 1 \end{cases}$$

2)

a)

$$\text{máx} \quad P.F(K,L) - rK - wL$$

CPO: $P F_1(K,L) - r = 0$

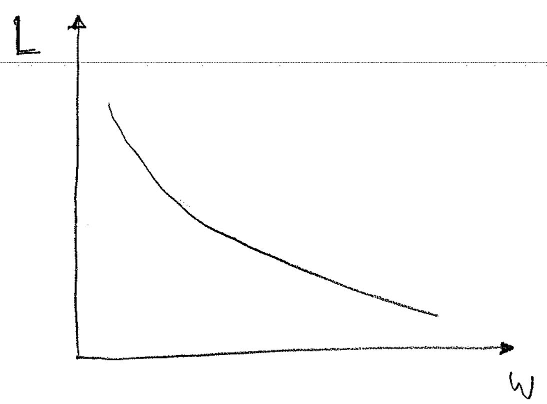
$P F_2(K,L) - w = 0 \qquad (*)$

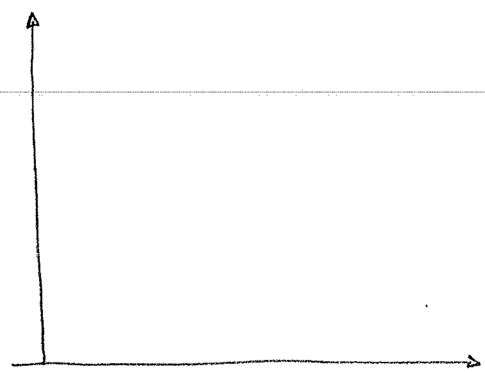
b) $P F_2(\bar{K},L) = w$

$F_2(\bar{K},L) = \frac{w}{p}$

$F_2(1,L) = \beta L^{\beta-1} = \frac{w}{p} \Rightarrow L^{\cancel{\beta-1}} = \left(\frac{w}{p\beta}\right)^{1/\beta-1}$

$L^* = \left(\frac{p\beta}{w}\right)^{\frac{1}{1-\beta}}$





USE $(*)$

c) $\frac{r}{F_1(K,L)} = \frac{w}{F_2(K,L)} \Rightarrow \boxed{\frac{F_1(K,L)}{F_2(K,L)} = \frac{r}{w}}$

Se

2 d) $F(K,L) = K^{\alpha} L^{\beta}$

$F_1(K,L) = \alpha K^{\alpha-1} L^{\beta}$

$F_2(K,L) = \beta K^{\alpha} L^{\beta-1}$

$$\left.\begin{aligned} \alpha K^{\alpha-1} L^{\beta} &= \frac{r}{p} \\ \beta K^{\alpha} L^{\beta-1} &= \frac{w}{p} \end{aligned}\right\} \quad \frac{\alpha K^{\alpha-1} L^{\beta}}{\beta K^{\alpha} L^{\beta-1}} = \frac{r}{w}$$

$$\Downarrow$$

$$K = \frac{\alpha}{\beta} \frac{w}{r} L$$

$$\alpha \left(\frac{\alpha}{\beta} \frac{w}{r} \right)^{-(1-\alpha)} L^{\alpha-1} L^{\beta} = \frac{r}{p}$$

$$\alpha \left(\frac{r}{w} \frac{\beta}{\alpha} \right)^{1-\alpha} L^{-(1-\alpha-\beta)} = \frac{r}{p}$$

$$\alpha \frac{p}{r} \left[\frac{r}{w} \frac{\beta}{\alpha} \right]^{1-\alpha} = L^{1-\alpha-\beta}$$

DEMANDAS
$$\begin{cases} L^{*} = \left[\alpha \frac{p}{r} \left[\frac{r}{w} \frac{\beta}{\alpha} \right]^{1-\alpha} \right]^{\frac{1}{1-\alpha-\beta}} \\ K^{*} = \left[\frac{\alpha}{\beta} \frac{w}{r} \right] \left[\alpha \frac{p}{r} \left[\frac{r}{w} \frac{\beta}{\alpha} \right]^{1-\alpha} \right]^{\frac{1}{1-\alpha-\beta}} \end{cases}$$

LUCRO

$$\pi = P\,F(K^*,L^*) - wL^* - rK^*$$

$$\pi(\alpha,\beta,P,w,r)$$

OFERTA

$$y = F(K^*,L^*)$$

$$y(\alpha,\beta,P,w,r)$$

K^* e L^* são as demandas calculadas anteriormente.

Se $\quad F(K,L) = K^{1/2} + L^{1/2}$

$$F_1(K,L) = \frac{1}{2}K^{-1/2}$$

$$F_2(K,L) = \frac{1}{2}L^{-1/2}$$

DEMANDAS:

$$\frac{1}{2}K^{-1/2} = \frac{r}{P} \Rightarrow \frac{1}{\sqrt{K}} = 2\frac{r}{P}$$

$$K^* = \left(\frac{P}{2r}\right)^2$$

analogamente: $\quad L^* = \left(\frac{P}{2w}\right)^2$

LUCRO

$$\pi^* = P\left(\left(\frac{P}{2r}\right) + \left(\frac{P}{2w}\right)\right) - w\left(\frac{P}{2w}\right)^2 - r\left(\frac{P}{2r}\right)^2$$

$$\frac{P^2}{2r} + \frac{P^2}{2w} - \frac{P^2}{4w} - \frac{P^2}{4r} = \frac{P^2(r+w)}{4(r\cdot w)}$$

$$Y = K^{*1/2} + L^{*1/2}$$

$$Y^* = \frac{P}{2r} + \frac{P}{2w} = \frac{P}{2}\left(\frac{1}{r} + \frac{1}{w}\right) \qquad \text{OFERTA}$$

3)

a) $\min_{L,K \in \mathbb{R}_+^2} w.L + r\bar{K}$

s.a $F(K,L) \geq y$

b) $\min_{L \geq 0} wL + r\bar{K}$

$F(\bar{K}, L) \geq y$

SOL: $F(\bar{K}, L^*) = y$

se $F(K,L) = K^{\alpha}.L^{\beta}$

$F(1,L) = L^{\beta} = y \Rightarrow L^* = y^{1/\beta}$

$C(y) = w y^{1/\beta} + r$

$CME(y) = \frac{w y^{1/\beta} + r}{y}$

$CVME(y) = w y^{\frac{1-\beta}{\beta}}$

$CMg(y) = \frac{w}{\beta} y^{\frac{1-\beta}{\beta}}$

se $F(K,L) = K^{1/2} + L^{1/2}$

$F(1,L) = L^{1/2}$

$C(y) = y^2$

$CME = y$

$CVME = CME$

$CMg(y) = 2y$

c) $\min \quad wL + rK$

$\text{s.a.} \quad F(K,L) \geq y$

CPO: $w - \lambda F_2(K,L) = 0$

$r - \lambda F_1(K,L) = 0$

$F(K,L) = y$

$$\boxed{\frac{F_1(\cdot)}{F_2(\cdot)} = \frac{r}{w}}$$

mesma condições (DUALIDADE)

d) $F(K,L) = K^{\alpha} L^{\beta}$

$F_1(K,L) = \alpha K^{\alpha-1} L^{\beta}$

$F_2(K,L) = \beta K^{\alpha} L^{\beta-1}$

$$\frac{\alpha K^{\alpha-1} L^{\beta}}{\beta K^{\alpha} L^{\beta-1}} = \frac{r}{w}$$

$$\frac{\alpha}{\beta} \frac{K^{-1}}{L^{-1}} = \frac{r}{w}$$

$$\frac{\alpha}{\beta} \frac{L}{K} = \frac{r}{w}$$

$$\frac{\alpha}{\beta} L = \frac{r}{w} K \Rightarrow K = \frac{\alpha}{\beta} \frac{wL}{r}$$

$K^{\alpha} L^{\beta} = y$

$\left(\frac{\alpha}{\beta} \frac{w}{r}\right)^{\alpha} \cdot L^{\alpha+\beta} = y$

$L^* = y^{1/\alpha+\beta} \left(\frac{\alpha}{\beta} \frac{w}{r}\right)^{\frac{-\alpha}{\alpha+\beta}}$

$K^* = \left(\frac{\alpha}{\beta} \frac{w}{r}\right)^{\frac{\beta}{\alpha+\beta}} y^{1/\alpha+\beta}$

$C(y) = y^{1/\alpha+\beta} \left[w \left[\frac{\alpha}{\beta} \frac{w}{r}\right]^{\frac{-\alpha}{\alpha+\beta}} + \right.$

$\left. + r \left[\frac{\alpha}{\beta} \frac{w}{r}\right]^{\frac{\beta}{\alpha+\beta}} \right]$

$CME(y) = \frac{C(y)}{y}$

$CMg(y) = \frac{\partial C(y)}{\partial y}$

$CVME = CME$

Se $F(k,L) = k^{1/2} + L^{1/2}$

$$F_1(k,L) = \frac{1}{2} k^{-1/2}$$

$$F_2(k,L) = \frac{1}{2} L^{-1/2}$$

$$\left(\frac{L}{k}\right)^{1/2} = \frac{r}{w}$$

$$\frac{L}{k} = \left(\frac{r}{w}\right)^2$$

$$L = \left(\frac{r}{w}\right)^2 k$$

$$k^{1/2} + L^{1/2} = y$$

$$k^{1/2} + \frac{r}{w} k^{1/2} = y$$

$$k^{1/2}\left(1 + \frac{r}{w}\right) = y$$

$$k = y^2 \left(1 + \frac{r}{w}\right)^{-2}$$

$$L = \left(\frac{r}{w}\right)^2 \left(1 + \frac{r}{w}\right)^{-2} y^2$$

$$C(y) = r\left(1 + \frac{r}{w}\right)^{-2} y^2 + w\left(\frac{r}{w}\right)^2 \left(1 + \frac{r}{w}\right)^{-2} y^2$$

$$C(y) = y^2 \left(1 + \frac{r}{w}\right)^{-2} \left[r + \frac{r^2}{w} \right]$$

$$C(y) = y^2 \left(\frac{w}{r+w}\right)^2 \left[\frac{(r+w)}{w} \cdot r \right]$$

$$C(y) = y^2 \frac{w \cdot r}{r+w}$$

e)

$$\max_{y \geq 0} \Pi = P y - C(y)$$

CPO: $P = Cmg(y)$

$$F(K,L) = K^{\alpha} L^{\beta}$$

$$C(y) = \theta \, y^{\frac{1}{\alpha+\beta}}$$

$$P = \frac{\theta}{\alpha+\beta} \cdot y^{\frac{1-\alpha-\beta}{\alpha+\beta}}$$

$$\boxed{\left(\frac{\alpha+\beta}{\theta} P\right)^{\frac{\alpha+\beta}{1-\alpha-\beta}} = y(P)}$$

$$F(K,L) = K^{1/2} + L^{1/2}$$

$$C(y) = \gamma y^2$$

$$P = 2\gamma y \Rightarrow \boxed{y = \frac{P}{2\gamma} = \frac{P(r+w)}{2rw}}$$

5)

a) V.

b) F. Não há custos fixos no longo prazo

c) V. Se há retornos decrescentes à partir de certo nível de utilização de insumos o custo médio é crescente à partir de um nível de produto.

d) V. Se o custo marginal é menor do que que o custo médio no nível de produção y, o custo de produzir uma unidade a mais será mais barato do que custou (na média) para produzir as y unidades. Por isso o custo médio deve cair.

e) , O custo médio de curto prazo nunca é inferior ao custo médio de longo prazo. Portanto o custo médio de longo prazo é a envoltória inferior das curvas de custo médio

f) Veja gráfico VARIAN (EDIÇÃO N° 5) pag 398

g) V: $$CV(q) = \int_0^q \frac{\partial CV(q)}{\partial q}\, dq$$

h) $$C_{Me} = \frac{C(q)}{q}$$

custo médio min é atingido em q^*

$$\frac{\partial C_{Me}(q^*)}{\partial q} = 0 \Rightarrow \frac{C'(q)q - C(q)}{q^2} = 0$$

$$\Downarrow$$

$$C'(q) = \frac{C(q)}{q}$$