The Diamond and Dybvig Model With Sequential Service Taken Seriously

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Introduction

- Diamond-Dybvig: simple contracts of demand deposit can achieve optimal risk sharing
- However, it creates possibility of bank runs
- Convertibility suspension rule out this bad equilibrium
- Caveat: it doesn't work in the presence of aggregate risk and sequential service
- Sequential service taken seriously: early withdraw contingent on the acquired information (mechanism design approach)

Diamond-Dybvig Model

Environment

- three periods $t \in \{0, 1, 2\}$ and N agents
- per-capita endowment e
- storage technology: irreversible and $R > 1$
- liquidity shock: $u(c_1 + \varepsilon c_2)A^{1-\varepsilon}$
  - $\varepsilon \in \{0, 1\}$ and $A \geq 1$
  - u is well behaved
- **the queue:** sequential accesses
  - position i with prob. $1/N$
  - **sequential service:** first-come, first-served
- **commitment:** cash machine

Diamond-Dybvig Model

Informational Structure

- **aggregate state:** $\omega \in \Omega \equiv \{0, 1\}^N$

  $$P(\omega) = p^{N-|\omega|}(1-p)^{|\omega|} \text{ (independence assumption}^1\text{)}$$

  where, $|\omega| = \sum_i \omega_i$ is the number of patients

- **private information:** only i knows ω_i

- position: **disclosed** (Green and Lin)
  undisclosed (Peck and Shell)

[1] Independent (across agents) determination of each agents type (impatient or patient)

Diamond-Dybvig Model
Allocation

Allocation (state-dependent)**:** Consumption-plan $\{x_i, y_i\}_{i,\omega}$ such that $x_i, y_i : \Omega \to \mathbb{R}_+$

Assuming specialization (i.e. impatient only consumes early and patient only consumes late), we have that, $x_i(\omega_{-i}, 1) = 0$ and $y_i(\omega_{-i}, 0) = 0$

Feasibility: Consumption-plan (x, y) is *feasible* if

$$\sum_{i=1}^{N} \big((1-\omega_i)\, x_i\,(\omega) + \omega_i R^{-1} y_i(\omega)\big) \leq Y, \quad \forall \omega$$

Diamond-Dybvig Model With Sequential Service

Sequential Service: An allocation (x, y) satisfies sequential service if,

$$x_i(\omega) = x_i(\tilde{\omega}), \qquad \forall \quad \omega, \tilde{\omega} \in \Omega, \text{ such that}, \quad \omega^i = \tilde{\omega}^i$$

where, $\omega^i \in \{0, 1\}^i$ the sequence ω truncated at its first i coordinates

As we assume specialization, $x_i(\omega^{i-1}, 1) = 0$

So, we just need define $x_i(\omega^{i-1}, 0)$

Therefore,

Feasibility with sequential service: Consumption-plan (x, y) is *feasible* if

$$\sum_{i=1}^{N} \left((1-\omega_i)\, x_i \left(\omega^{i-1}, 0\right) + \omega_i R^{-1} y_i(\omega)\right) \leq Y, \quad \forall \omega$$

Diamond-Dybvig Model With Sequential Service

Welfare

Welfare function: The welfare is given by the ex-ante agent utility

$$E_{i,\omega}\left\{u\left[x_i(\omega)+\omega_i y_i(\omega)\right]A^{1-\omega_i}\right\}$$

Given the permutation process, we have,

$$\frac{1}{N}\sum_{i=1}^{N}E_{\omega}\left\{u\left[x_i(\omega)+\omega_i y_i(\omega)\right]A^{1-\omega_i}\right\}$$

Given specialization and sequential service, we have

$$\frac{1}{N}\sum_{i=1}^{N}E_{\omega}\left\{(1-\omega_i)\,Au\left[x_i(\omega^{i-1},0)\right]+\omega_i u\left[y_i(\omega)\right]\right\}$$

Diamond-Dybvig Model With Sequential Service

Incentive Compatibility

Implementability with position disclosure: (x, y) is *incentive-compatible* if it is feasible and

$$E_\omega \left[u \left(y_i \left(\omega_{-i}, 1 \right) \right) / \omega_i = 1 \right] \geq E_\omega \left[u \left(x_i \left(\omega^{i-1}, 0 \right) \right) / \omega_i = 1 \right]$$

for $i = 1, \ldots, N$

Implementability without position disclosure: (x, y) is *incentive-compatible* if it is feasible and

$$\frac{1}{N} \sum_{i=1}^{N} E_\omega \left[u \left(y_i \left(\omega_{-i}, 1 \right) \right) / \omega_i = 1 \right] \geq \frac{1}{N} \sum_{i=1}^{N} E_\omega \left[u \left(x_i \left(\omega^{i-1}, 0 \right) \right) / \omega_i = 1 \right]$$

Diamond-Dybvig Model With Sequential Service

Optimal Allocation

The optimal allocation is given by

$$\max \frac{1}{N} \sum_{i=1}^{N} E_{\omega} \left\{ u \left[x_i(\omega) + \omega_i y_i(\omega) \right] A^{1-\omega_i} \right\}$$

s.t.

$$\sum_{i=1}^{N} \left((1 - \omega_i) \, x_i \left(\omega \right) + \omega_i R^{-1} y_i(\omega) \right) \leq Y, \quad \forall \omega$$

- Solution requires that $x_i(\omega) = x(\omega) = x(\tilde{\omega})$ and $y_i(\omega) = y(\omega) = y(\tilde{\omega})$ for all $\omega, \tilde{\omega}$ s.t. $|\omega| = |\tilde{\omega}|$

Diamond-Dybvig Model With Sequential Service

Optimal Allocation

More generally, the optimal allocation is associated with (include sequential service constraint)

$$\max \frac{1}{N} \sum_{i=1}^{N} E_{\omega} \Big\{ (1-\omega_i)\, Au \left[x_i(\omega^{i-1}, 0) \right] + \omega_i u \left[y_i(\omega) \right] \Big\}$$

s.t.

$$\sum_{i=1}^{N} \left((1-\omega_i)\, x_i \left(\omega^{i-1}, 0 \right) + \omega_i R^{-1} y_i(\omega) \right) \leq Y, \quad \forall \omega$$

- Solution would require partial suspension (contingent early payment)

Diamond-Dybvig Model With Sequential Service

Optimal Mechanism

The optimal allocation with position disclosure (Green and Lin)

$$\max \frac{1}{N} \sum_{i=1}^{N} E_\omega \left\{ (1-\omega_i) \, Au \left[x_i(\omega^{i-1}, 0) \right] + \omega_i u \left[y_i(\omega) \right] \right\}$$

s.t.

$$\sum_{i=1}^{N} \left((1-\omega_i) \, x_i \left(\omega^{i-1}, 0 \right) + \omega_i R^{-1} y_i(\omega) \right) \leq Y, \quad \forall \omega$$

$$E_\omega \left[u \left(y_i \left(\omega_{-i}, 1 \right) \right) / \omega_i = 1 \right] \geq E_\omega \left[u \left(x_i \left(\omega^{i-1}, 0 \right) \right) / \omega_i = 1 \right], i = 1, ..., N$$

- Non binding IC [2]
- No bank run

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[2] A=1

Diamond-Dybvig Model With Sequential Service

Optimal Mechanism

The optimal allocation without position disclosure (Peck and Shell)

$$\max \frac{1}{N} \sum_{i=1}^{N} E_\omega \Big\{ (1-\omega_i)\, Au \left[x_i(\omega^{i-1}, 0)\right] + \omega_i u \left[y_i(\omega)\right] \Big\}$$

s.t.

$$\sum_{i=1}^{N} \left((1-\omega_i)\, x_i \left(\omega^{i-1}, 0\right) + \omega_i R^{-1} y_i(\omega) \right) \leq Y, \quad \forall \omega$$

$$\frac{1}{N} \sum_{i=1}^{N} E_\omega \left[u \left(y_i \left(\omega_{-i}, 1 \right) \right) / \omega_i = 1 \right] \geq \frac{1}{N} \sum_{i=1}^{N} E_\omega \left[u \left(x_i \left(\omega^{i-1}, 0 \right) \right) / \omega_i = 1 \right]$$

- Allow bank runs (Ennis and Keister)

Diamond-Dybvig Model With Sequential Service

Results

Theorem (Green-Lin)

If relative-risk aversion is everywhere greater than one and absolute risk-aversion is nonincreasing then the direct mechanism with disclosure has truth-telling as its unique sequential equilibrium

Example (Peck-Shell)

If $(N, p, R) = (2, \frac{1}{2}, 1.05)$, with utilities $v_0(.) = -\dfrac{10}{c_1}$ and $v_1(.) = -\dfrac{1}{c_1 + c_2}$ then no-disclosure implements runs

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Ennis Keister (2009)

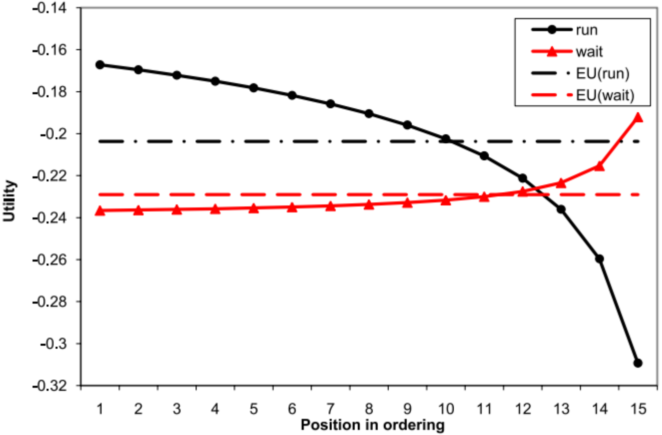

Fig. 2. Expected utility if all other traders run.

Diamond-Dybvig Model With Sequential Service

More Results

- Andolfato et. al (2006) - position disclosed and independence $\Rightarrow$ no bank runs
- Ennis and Keister (2009) - position undisclosed and independence $\Rightarrow$ example of bank run with $A = 1$
- Ennis and Keister (2009) - position disclosed and nonindependence $\Rightarrow$ example of bank run with $A = 1$

Diamond-Dybvig Model With Sequential Service

Final Remarks

- The planner's allocation rule induces a game (mechanism design!)
- IC constraints guarantees that truth-telling strategy is a BNE[3]
- Moreover, optimal allocation rule seeks the best that can be achieved by a direct mechanism[4]
- However, given an (optimal) allocation rule, others equilibria may exist (bank runs)

[3] Weak implementation

[4] Revelation Principle (Myerson, 1981)