Banking Theory

Diamond-Dybvig and Jacklin

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Introduction

- Objective
  - We seek to explore in depth the role of demand deposit in risk sharing
- Motivation
  - Acquiring a better understanding the functioning of this pervasive mechanism of resource allocation

Outline

- Literature
  - Diamond and Dybvig (1983)
  - Jacklin (1987)

- Similar but with a different approach
  - Diamond and Dybvig: focus on bank runs and their prevention
  - Jacklin: focus on risk sharing role of demand deposits

Main Results (Diamond and Dybvig)

- In the presence of private information, **competitive equilibrium** cannot provide optimal risk sharing
- **Demand deposit contracts** can achieve the social optimum
- However, this arrangement can lead to **bank runs**
- **Suspension of convertibility** or **deposit insurance** can be used to prevent bank runs

Main Results (Jacklin)

- **Dividend-paying equity shares** provide the same risk-sharing opportunities as demand deposits but do not introduce the possibility of a bank run in Diamond Dybvig model
- However, for a **large class of economies**, the demand deposits provide risk-sharing opportunities beyond those supported by equity shares
- **Trading restrictions** are an essential element of the story

Diamond and Dybivig environment

- Three periods $(T = 0, 1, 2)$
- A continuum of agents of measure one
- One perishable consumption good
- Each individual is endowed with one unit of the good at $T = 0$

- Production technology
  - Investment of consumption good is made at $T = 0$
  - If production is interrupted at $T = 1$, it yields zero net return
  - Otherwise, at $T = 2$, it yields a gross return $R \geq 1$

Diamond and Dybivig environment

- Preferences Structure
  - At $T = 0$, agents are ex-ante identical
  - At $T = 1$, individuals suffer an i.i.d. liquidity shock. A person is of type 1 (impatient) with prob. t and of type 2 (patient) with prob. $1 - t$
  - For each type θ, utility is given by
    - $\mu(c_1, c_2; 1) = u(c_1)$, if impatient
    - $\mu(c_1, c_2; 2) = \rho u(c_1 + c_2)$, if patient
  - Thus, at $T = 0$, expected utility is given by

$$\mathbb{E}_\theta[\mu(c_1, c_2; \theta)] = tu(c_{11}) + (1 - t)\rho u(c_{12} + c_{22})$$

where, c_{ik} represents consumption in period i, for an individual of type k

Social Optimal Risk Sharing

The socially optimal allocation is found by solving:

$$\max_{c_{11},c_{12},c_{21},c_{22}} tu(c_{11}) + (1-t)\rho u(c_{12}+c_{22})$$

subject to

$$(c_{11}+c_{12}/R)t + (c_{12}+c_{22}/R)(1-t) = 1$$

and

$$c_{ik} \geq 0$$

for $i = 1,2$ and $k = 1,2$

Social Optimal Risk Sharing

Assuming $\rho R > 1$ and $-cu''(c)/u'(c) > 1$,

the social optimum is characterized by:

$$\begin{aligned} c^*_{12} &= c^*_{21} = 0 \\ c^*_{11} &> 1 \\ c^*_{22} &= R(1 - c^*_{11}t)/(1-t) \\ c^*_{22} &> c^*_{11} \end{aligned}$$

Social Optimal Risk Sharing

By the F.O.C. we have, $u'(c^*_{11}) = R\rho u'(c^*_{22}) > u'(c^*_{22})$.

Thus, since $u'' < 0$, we have $c^*_{22} > c^*_{11}$.

Also,

$$\begin{aligned}
\rho R u'(R) &< R u'(R) \\
&= 1u'(1) + \int_{\gamma=1}^{R} \frac{\partial}{\partial \gamma}[\gamma u'(\gamma)] d\gamma \\
&= u'(1) + \int_{\gamma=1}^{R} [u'(\gamma) + \gamma u''(\gamma)] d\gamma \\
&< u'(1)
\end{aligned}$$

as $-\gamma u''(\gamma)/u'(\gamma) > 1$. Because $u'' < 0$ and the resource constraint trades off c^*_{11} against c^*_{22}, the solution to the F.O.C. must have $c^*_{11} > 1$ and $c^*_{22} < R$.

Competitive Equilibrium

- If types are public information, then, complete markets would achieve the first best allocation (Second Welfare Theorem)
- However, if types are private information, markets are incomplete and optimum may not be achieved
- Incomplete markets:
  - No contingent assets
  - The only market is at $T = 1$, where agents can trade $T = 1$ goods against $T = 2$ goods
  - In any equilibrium, the price of the $T = 2$ good must be equals to $\frac{1}{R}$

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- If $p > 1/R$, then it is always preferable to commit investing in the long-run technology at $T = 0$, but then all early consumers will be trying to sell at $T = 0$ and nobody would buy
- If $p < 1/R$, then it is always preferable to commit investing in the short-run at $T = 0$, but then all late consumers will be trying to buy at $T = 0$ and nobody would sell
- Since the equilibrium price is $p = 1/R$, the market doesn't improve upon autarky

Demand Deposit Contracts

- An initial investment at $T = 0$
- Right to withdraw r_1 per unit of investment in period $T = 1$ (at discretion of depositor and conditional on banks solvency)
- Agents not withdrawing in period 1 get a pro rata share of the bank's assets in period 2 - i.e. receives $[R(1 - r_1 f)/(1 - f)]^+$, where$f$ is the proportion of individuals who withdraw at $T = 1$
- The contract is subject to sequential service constraint

- If r_1 is greater than 1, then the demand deposit provides insurance against liquidity shock
- However, it creates potential for bank runs

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- Take the early withdraw payment as $r_1 = c^*_{11}$
- If only impatients withdraw at $T = 1$, $[R(1 - r_1 t)/(1 - t)]^+ = c^*_{22}$ and optimum would be achieved
- Since $c^*_{11} < c^*_{22}$, the optimal allocation is incentive compatible
- Thus, the symmetric strategy profile "withdraw iff impatient" is a Nash Equilibrium
- Therefore, demand deposit contract could implement the first best!
- However, note that this NE is not unique!

Liquidity Insurance

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Bank Runs

- The symmetric strategy profile "withdraw no matter waht" is also a Nash Equilibrium
- Given $r_1 > 1$, if everyone else tries to withdraw the money will run out - i.e. $[R(1 - r_1 f)/(1 - f)]^+ = 0$
- This justifies early withdrawing causing bank runs
- It's exactly the liquidity service provided by the bank that make it prone to runs!

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Bank Runs (N agents)

- The essential role of a bank is strictly tied to its illiquid nature
- Suppose a bank with N agents, if an individual believe that everyone else will withdraw, she will receive $R[N - r_1(N-1)]^+$ if wait
- If $r_1 \leq 1$, she will receive $R[N - r_1(N-1)]^+ \geq R$ if wait. So, if she is a patient, she will wait no matter what
- If $r_1 > 1$, she will receive $R[N - r_1(N-1)]^+$ if wait. So, if $r_1 > \frac{N}{N-1}$, she will receive nothing
- Thus, for sufficient large N, there is bank run possibility if, only if, a liquidity service is provided (i.e. $r_1 > 1$)

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Suspension of Convertibility

- One variant of the contract can rule out the bad equilibrium
- The contract states that you can withdraw r_1 at $T = 1$ as long as less than t other consumers have withdrawn before you. After that, you are forced to wait
- This guarantees the payment at $T = 2$ and since $c^*_{22} > c^*_{11}$, the patient will always wait
- Problem: you need to know t exactly in order to use these contracts!
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Deposit Insurance

- Government deposit insurance could give the guarantee needed to patients to wait
- In order to provide the insurance, the government must tax
- The capacity to tax in different ways is what will make this policy effective
- Assuming that taxation can be conditional on the amount of withdraws, the government could undo the sequential service constraint
- This way, optimum can be achieved

Diamond and Dybvig results

- In the presence of private information, competitive equilibrium cannot achieve the optimal allocation
- Social optimal risk sharing can be achieved as the superior NE of a bank arrangement
- But, bank runs (all agents withdraw early) is also a NE
- Moreover, it's exactly the liquidity service provided by the bank that make it prone to runs!
- Suspension of convertibility and government deposit insurance could rule out bank runs

Optimal Risk Sharing Using Equity Shares

- Suppose instead of a bank consumers set up a firm
- The firm issues shares (with price 1) and raises an amount C
- At $T = 0$, the shareholder decide the firm production policy and declare a per-share dividend, D, payable at $T = 1$
- Thus, each share represents the right to a dividend D at $T = 1$ and liquidating dividend of $R(C - D)$ at $T = 2$
- At $T = 1$, a market in the ex-dividend shares opens

Optimal Risk Sharing Using Equity Shares

- Suppose all individuals invest in the firm (i.e. $C = 1$)
- If the per share dividend, D, is set at tr_1, and at $T = 1$ trading takes place in the ex dividend shares, the social optimum is achieved

- Impatient: sell all $\Rightarrow c_1 = tr_1 + p$
- Patient: buy all if $p < R(1 - r_1 t) \Rightarrow c_2 = R(1 - r_1 t) + \frac{tr_1}{p} R(1 - r_1 t)$

Optimal Risk Sharing Using Equity Shares

- Market clears at $p^* = (1-t)r_1$
- Note that, $c^*_{22} = R(1-c^*_{11}t)/(1-t) > c^*_{11} \Rightarrow p^* < R(1-r_1t)$, thus patients buy all shares
- Thus, supply of share is given by t and demand is given by $(1-t)tr_1/p^*$
- Therefore p^* clears market, and then, $c^*_1 = r_1$ e $c^*_2 = \dfrac{R(1-r_1t)}{1-t}$
- Optimal allocation is achieved
- Moreover, at $T = 0$, shareholder unanimously agree to declare a dividend of tr_1 payable at $T = 1$

New Environment: Smooth Preferences

- Again, type 1 individuals prefer more consumption in the first period than do type 2 individuals
- Formally, preferences are given by
  - $\mu(c_1, c_2; 1) = U(c_{11}, c_{21})$, if impatient
  - $\mu(c_1, c_2; 2) = V(c_{12}, c_{22})$, if patient
  - with $U_1(c_1, c_2)/U_2(c_1, c_2) > V_1(c_1, c_2)/V_2(c_1, c_2)$
- Thus, at $T = 0$, expected utility is given by

$$\mathbb{E}_\theta[\mu(c_1, c_2; \theta)] = tU(c_{11}, c_{21}) + (1 - t)V(c_{12} + c_{22})$$

Bank's Essentiality

Theorem

If demand deposit cannot be traded and the social optimum subject only to resource constraint is not incentive compatible, demand deposits can be used to achieve the optimal nonstochastic, incentive-efficient allocation, but equity shares cannot

The Need for Trading Restriction

If ex post trading is possible and new assets can be introduced, on the margin individuals have no incentive either to invest in the shares of a dividend-paying firm or to deposit their fund in the bank

Theorem

If demand deposit can be traded costlessly, equity shares and demand deposits are equivalent risk-sharing mechanisms

The Need for Trading Restriction

Example (Diamond Dybvig)

- Suppose that a "rogue" trader can stay outside the conglomerate (bank). Then by investing directly in the production technology it clearly can do better than by staying in the conglomerate
- If the trader is not hit by a liquidity shock, it gets $R > c_2$
- It the trader is hit by a liquidity shock, it can entice a patient consumer in the conglomerate to fetch c_1 and trade for $R > c_2$ (i.e., the patient consumer will be happy to make this trade)

Jacklin Results

- For some environments, dividend-paying equity shares mechanism can be as good as demand deposits
- In order to bank be essential, it's needed to consider more complex environments (preferences)
- Moreover, trading restrictions are needed to support bank's essentiality