Common Features and Business Cycle in Mercosur

Abstract

This paper studies the business cycles of the Mercosur’s members in order to investigate their degree of synchronization, which is a necessary condition to harmonize the economic policies among them. To estimate the business cycles, the Beveridge-Nelson-Stock-Watson multivariate trend-cycle decomposition is applied, testing the presence of cointegration and serial correlation common feature. Inasmuch as the cycles are estimated, their synchronization is analyzed by means of coherence and phase in frequency domain. The results suggest that, despite the evidence of common components, the business cycles are not synchronized. That may generate an enormous difficulty conciliating further commercial agreements into Mercosur.

Key-words: Mercosur, business cycles, trend-cycle decomposition, common features, spectral analysis.

Jel Codes: C32, E32, F02, F23.
1 Introduction

The design of economic blocks, like the European Union, has the purpose to amplifying the society welfare through unification of economic policies and commercial agreements. According to Backus and Kehoe (1992) and Chistodoulakis and Dimelis (1995), the success of these politics depends on the similarities of the business cycles of the economic block members. In this sense, this paper is aimed to analyze the degree of synchronization among the Mercosur members.

The Mercosur or southern common market is a regional trade agreement founded in 1991 by the treaty of Asunción. Presently, the Mercosur’s members are Argentina, Brazil, Paraguay, Uruguay and Venezuela who has joined in 2006. These countries differ in their institutions, economic policies and industrial structures, creating an enormous internal asymmetry in Mercosur (Flores, 2005). However, despite these differences, we can still investigate if their business cycles are similar. As mentioned Mercosur was created in 1991; however, our data set ranges from 1951 to 2003. Therefore, if we find evidence in favor of similarity we can safely assume this cannot be attributed only to Mercosur1. In fact, an inverse causality is investigated: if the similarities among the countries provoke their commercial integration.

A business cycle is a periodic but irregular up-and-down movement in economic activity, measured by fluctuations in real GDP and other macroeconomic variables. However, in compliance with Lucas (1977), we focus our analysis on GDP, defining business cycles as the difference between the effective product of an economy and its long-run trend. Once the long-run trend is not observable, it is necessary to estimate it to obtain the business cycles.

In the empirical literature there is no consensus about how to estimate the trend-cycle components of economic time series and how to analyze the so-called co-movements2 in their business cycles. During the past few decades a rich debate about the abilities of different statistical methods to decompose time series in long-term and short-term fluctuations has taken place (Baxter and King, 1995; Guay and St-Amant, 1996). Among the more common univariate methodologies are the Hodrick-Prescott (HP) filter and the linear detrending. However, these methodologies do not take in account the existence of common features among the economics series. In addition, as showed by Harvey and Jaeger (1993) the HP filter can induce spurious cyclicality when applied to integrated data. Therefore, in order to obtain a measure of the business cycles, the Beveridge-Nelson-Stock-Watson (BNSW) multivariate trend-cycle decomposition is employed, considering evidences of cointegration and serial correlation common feature.

Common features may be seen as restrictions over the dynamics of the countries and, consequently, over the dynamic of their business cycle components. While cointegration refers to relations among variables in the long-run, the common cyclical restrictions refer to relations in the short-run. Engle and Kozicki

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1Besides, there is not a consensus that Mercosur led to an increase in the flow of commerce among its integrated parts.
2Two countries present comovements when their real GDP expansions and downturns are simultaneous.
(1993) and Vahid and Engle (1993) proposed the serial correlation common feature (SCCF) as a measure of common cyclical feature in the short-run, which is applied in many empirical works. For example, Gouriéroux and Peaucelle (1993) analyzed some questions about purchase power parity; Campbell and Mankiw (1990) found a common cycle between consumption and income for most G-7 countries; Engle and Kozicki (1993) found common international cycles in GNP data for OECD countries; Engle and Issler (2001) found common cycles among sectorial output for US; and Candelon and Hecq (2000) tested the Okun’s law.

It is worth noting that the existence of a common cyclical feature neither implies nor is implied by the existence of similar business cycles as observed by Quah (Engle and Kozicki, 1993-comment) and Cubadda (1999). Therefore, to investigate the degree of synchronization or co-movement of their business cycles an extra effort is necessary. In this sense, many authors have used the linear correlations across cycles; however, this analysis gives a static measure of the co-movements since it is not a simultaneous analysis of the persistence of co-movement (Engle and Kozick, 1993). To avoid this critique, the measures of coherence and phase in frequency domain are applied in order to investigate how synchronized the business cycles are (Wang, 2003). These frequency domain techniques constitute a straightforward way to represent economic cycles, once they provide information for all frequencies.

Finally, the results indicate the existence of common trends and common cycles among the economies under study. According to Beine et al. (2000), the existence of such co-movements provides support for some types of convergence and for sustainability of an optimal currency area. Thus, we confirm the necessity to use a multivariate approach, which is a contribution of this work. Frequency domain results identified synchronization in two sub-groups: Argentina-Venezuela and Brazil-Paraguay. In general, countries of the economic block are not synchronized.

Beyond this introduction, the paper is organized in the following form. Section 2 presents the econometric methodology. Section 3 reports estimation and test and in section 4 the results. Finally, the conclusions are summarized in the last section.

2 Econometric Model with common feature

In order to implement the BNSW decomposition taking in account the common features restrictions a VAR model is estimated and the existence of long-run and short-run common dynamics is tested. Then, a Gaussian Vector Autoregression of finite order \( p \), VAR(\( p \)), such that:

\[
y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \ldots + \phi_p y_{t-p} + \varepsilon_t
\]  

(1)

where \( y_t \) is a vector of \( n \) first order integrated series, \( I(1) \), and \( \phi_i, i = 1, \ldots, p \) are matrices of dimension \( n \times n \) and \( \varepsilon_t \sim Normal (0, \Omega) \), \( E(\varepsilon_t) = 0 \) and \( E(\varepsilon_t \varepsilon_{t}) = \{\Omega, \text{ se } t = \tau \text{ and } 0_{n \times n}, \text{ se } t \neq \tau\} \); where \( \Omega \) is
no singular. The model (1) can be written equivalently as:

$$\Pi (L) y_t = \varepsilon_t$$  \hspace{1cm} (2)$$

where $$\Pi (L) = I_n - \sum_{i=1}^{p} \phi_i L^i$$ and $$L$$ represents the lag operator. $$\Pi (1) = I_n - \sum_{i=1}^{p} \phi_i$$ when $$L = 1$$.

### 2.1 Long run restrictions (Cointegration)

The following assumptions are assumed:

**Assumption 1**: The $$(n \times n)$$ matrix $$\Pi (\cdot)$$ satisfies:

1. Rank $$(\Pi (1)) = r$$, $$0 < r < n$$, such that $$\Pi (1)$$ can be expressed as $$\Pi (1) = -\alpha \beta'$$, where $$\alpha$$ and $$\beta$$ are $$(n \times r)$$ matrices with full column rank $$r$$.

2. The characteristic equation $$|\Pi (L)| = 0$$ has $$n - r$$ roots equal to 1 and all other are outside the unit circle.

The assumption 1 implies that $$y_t$$ is cointegrated of order $$(1, 1)$$. The elements of $$\alpha$$ are the adjustment coefficients and the column of $$\beta$$ span the cointegration space. Decomposing the polynomial matrix

$$\Pi (L) = \Pi (1) L + \Pi^* (L) \Delta$$,

where $$\Delta \equiv (1 - L)$$ is the difference operator, a Vector Error Correction Model (VECM) is obtained:

$$\Delta y_t = \alpha \beta' y_{t-1} + \sum_{j=1}^{p-1} \Gamma_j \Delta y_{t-j} + \varepsilon_t$$ \hspace{1cm} (3)$$

where $$\alpha \beta' = -\Pi (1), \Gamma_j = -\sum_{k=j+1}^{p} \phi_k (j = 1, \ldots, p - 1)$$ and $$\Gamma_0 = I_n$$.

### 2.2 Common cycles restrictions

The VAR(p) model can present additional short-run restrictions as showed by Vahid and Engel (1993).

**Definition 1** *Serial Correlation Common Feature* hold in (3) if exist a $$(n \times s)$$ matrix $$\bar{\beta}$$ of rank $$s$$, whose column span the cofeature space, such as $$\bar{\beta}' \Delta y_t = \bar{\beta}' \varepsilon_t$$, where $$\bar{\beta}' \varepsilon_t$$ is a $$s$$-dimensional vector that constitute an innovation process with respect to all information prior to period $$t$$.

Consequently, the SCCF restrictions occur if there is a cofeature matrix $$\bar{\beta}$$ that satisfies the following assumption:

**Assumption 2** $$(\bar{\beta}' \Gamma_j) = 0_{s \times n} \quad j = 1, \ldots, p - 1$$

**Assumption 3** $$(\bar{\beta}' \alpha \beta') = 0_{s \times n}$$
2.3 Trend - cycle decomposition

The BNSW trend-cycle decomposition can be introduced by means of the Wold representation of the stationary vector $\Delta y_t$ given by:

$$\Delta y_t = C(L)\varepsilon_t$$  \hspace{1cm} (4)

where $C(L) = \sum_{i=0}^{\infty} C_i L^i$ is matrix polynomial in the lag operator, $C_0 = I_n$ and $\sum_{i=1}^{\infty} j |C_j| < \infty$. Using the following polynomial factorization $C(L) = C(1) + \Delta C^*(L)$, it is possible to decompose $\Delta y_t$ such that:

$$\Delta y_t = C(1) \varepsilon_t + \Delta C^*(L) \varepsilon_t$$  \hspace{1cm} (5)

where $C_i^* = \sum_{j>1}^{\infty} (-C_j)$, $i \geq 0$, and $C_0^* = I_n - C(1)$. Ignoring the initial value $y_0$ and integrating both sides of (5), we obtain:

$$y_t = C(1) \sum_{j=1}^{T} \varepsilon_t + C^*(L)\varepsilon_t = \tau_t + c_t$$  \hspace{1cm} (6)

Equation (6) represents the BNSW decomposition where $n$ variables that compound $y_t$ are decomposed in $n$ random walk process called “stochastic trend” and $n$ stationary process named “cycles”. Thus, $\tau_t = C(1) \sum_{j=1}^{T} \varepsilon_t$ and $c_t = C^*(L)\varepsilon_t$ represent the trend and cycle components, respectively. Assuming that long-run restrictions exist, then $r$ cointegration vectors exist $(r < n)$. These vectors eliminate the trend component which implies that $\beta' C(1) = 0$. Thus, $C(1)$ has dimension $n - r$ which means that exist $n - r$ common trends. Analogously, assuming short-run restrictions, there are $s$ cofeature vectors that eliminate the cycles, $\tilde{\beta}' C^*(L) = 0$, which implies that $C^*(L)$ has dimension $n - s$, which is the number of common cycles. It is worth noting that $r + s \leq n$ and the cointegration and cofeature vectors are linearly independent (Vahid and Engle, 1993). To obtain the common trends it is necessary (and sufficient) to multiply equation (6) by $\tilde{\beta}'$, such that

$$\tilde{\beta}' y_t = \tilde{\beta}' C(1) \sum_{j=1}^{T} \varepsilon_t = \tilde{\beta}' \tau_c$$

This linear combination doesn’t contain cycles because the cofeatures vectors eliminate the cycle. In the same way, to get the common cycles it is necessary to multiply equation (6) by $\beta'$, and so

$$\beta' y_t = \beta' C^*(L)\varepsilon_t = \beta' c_t$$

This linear combination doesn’t contain the stochastic trend, because the cointegration vectors eliminate the trend component. A special case emerges when $r + s = n$. In this case the estimate of trend and cycle components of $y_t$ becomes extremely easy. Once $\tilde{\beta}'$ and $\beta'$ are linearly independent matrices, it is possible to build a matrix $A$, such as $A_{n \times n} = (\tilde{\beta}', \beta')'$ has full rank and therefore is invertible. Partition the columns
of its inverse according as $A^{-1} = (\tilde{\beta}^- - \beta^-)$ and recover the trend and cycle components by pre-multiplying $y_t$ by $A^{-1}$:

$$y_t = A^{-1} Ay_t = \tilde{\beta}^- (\tilde{\beta}' y_t) + \beta^- (\beta' y_t)$$

$$= \tau_t + c_t$$

(7)

This implies that $\tau_t = \tilde{\beta}^- \tilde{\beta}' y_t$ and $c_t = \beta^- \beta' y_t$. Therefore, trend and cycle are linear combinations of $y_t$. Note that $\tau_t$ is generated by a linear combination of the cofeature vectors, containing the long-run component (because $\tilde{\beta}' y_t$ is a random walk component) while $c_t$ is generated by a linear combination of cointegration vectors, containing the short-run component (because $\beta' y_t$ is $I(0)$ and serially correlated).

### 2.4 Estimation and testing

It is possible to represent a pseudo-structural model using the $s$ cofeatures vectors such as

$$\begin{bmatrix} I_s & \beta' s \\ 0 & I_{n-s} \end{bmatrix} \Delta y_t = \begin{bmatrix} 0 & 0 & \ldots & 0 \\ \tilde{\alpha} \tilde{\Gamma}_1 \ldots \tilde{\Gamma}_{p-1} \end{bmatrix} \begin{bmatrix} \beta' y_{t-1} \\ \Delta y_{t-1} \\ \vdots \\ \Delta y_{t-p+1} \end{bmatrix} + w_t$$

(8)

where there are $n - s$ unrestricted equations, the cofeature matrix $\tilde{\beta}' = (I_s \beta' s)$ has been normalized on the first $s$ variables using the identity matrix; $\tilde{\alpha}$, $\tilde{\Gamma}_i$ are the parameter matrices for the $n - s$ remaining equations of the VECM. We can rewrite the VECM with SCCF restrictions as a model of reduced-rank structure. In (3) we define a vector $X_{t-1} = [y_{t-1}\beta', \Delta y_{t-1}, \ldots, \Delta y_{t-p+1}]'$ of dimension $(n(p-1)+r) \times 1$ and a $n \times (n(p-1)+r)$ matrix $\Phi = [\alpha, \Gamma_1, \ldots, \Gamma_{p-1}]$, therefore (3) is written as:

$$\Delta y_t = \Phi X_{t-1} + \varepsilon_t$$

(9)

If assumptions (1) and (2) hold, then the matrices $\Gamma_i$, $i = 1, \ldots, p$ are all of reduced rank $(n-s)$ and they can be written as $\Phi = A[\Psi_0, \Psi_1, \ldots, \Psi_{p-1}] = A\Psi$, where $A$ is $n \times (n-s)$ full column rank matrix and $\Psi$ has dimension $(n-s) \times (n(p-1)+r)$ and $\beta' A \Psi = 0$, that is, $\tilde{\beta} \in sp(A_{\perp})$ where $A_{\perp}$ is the orthogonal complement of $A$. Therefore, let $A = \tilde{\beta}_{\perp}$.

Hence, the pseudo-structural model (8) can be expressed as a dynamic factor model with $n - s$ factor $F_m = \Psi X_{t-1}$ which are linear combinations of the right hand side

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3The space is denoted by $sp$. The orthogonal complement of the $n \times s$ matrix $\Pi$, $n > s$ and rank$(\Pi) = s$, is the $n \times (n-s)$ matrix $\Pi_{\perp}$ such that $\Pi' \Pi_{\perp} = 0$ and rank$(\Pi : \Pi_{\perp}) = n$. Furthermore, $\Pi_{\perp}$ spans the null space of $\Pi$ and $\Pi'$ spans the left null space of $\Pi_{\perp}$.
variables in (3).

\[ \Delta y_t = \tilde{\beta}_\perp (\Psi_1, \Psi_2, ..., \Psi_{p-1}) X_{t-1} + \varepsilon_t \]  

(10)

\[ = \tilde{\beta}_\perp \Psi X_{t-1} + \varepsilon_t \]  

(11)

\[ = \tilde{\beta}_\perp F_m + \varepsilon_t \]  

(12)

To estimate the coefficient matrices \( \tilde{\beta}_\perp \) and \( \Psi \) in the reduced rank model (11) we use the procedure of Anderson (1951) (see additionally Anderson, 1988, Johansen, 1995). This procedure is based in canonical analysis. The use of canonical analysis may be regarded as a special case of reduced-rank regression. More specifically, the maximum-likelihood estimation of the parameters of the reduced-rank regression model may result in solving a problem of canonical analysis\(^4\). Therefore, we can use the expression \( \text{CanCorr}\{X_t, Z_t | W_t\} \) that denotes the partial canonical correlations between \( X_t \) and \( Z_t \): both sets concentrate out the effect of \( W_t \) that allows us to obtain canonical correlation, represented by the eigenvalues \( \hat{\lambda}_1 > \hat{\lambda}_2 > \hat{\lambda}_3 > ... > \hat{\lambda}_n \). The Johansen test statistic is based on canonical correlation. In model (3) we can use the expression \( \text{CanCorr}\{\Delta y_t, y_{t-1} | W_t\} \) where \( W_t = [\Delta y_{t-1}, \Delta y_{t-2}, ..., \Delta y_{t+p-1}] \) that summarizes the reduced-rank regression procedure used in the Johansen approach. It means that one extracts the canonical correlations between \( \Delta y_t \) and \( y_{t-1} \): both sets concentrated out the effect of lags of \( W_t \). Moreover, we could also use a canonical correlation approach to determine the rank of the common features space due to SCCF restrictions. It is a test for the existence of cofeatures in the form of linear combinations of the variables in the first differences which are white noise (i.e., \( \tilde{\beta}' \Delta y_t = \tilde{\beta}' \varepsilon_t \) where \( \tilde{\beta}' \varepsilon_t \) is a white noise). In other words, in equation (3) we observe that all serial correlation of \( \Delta y_t \) are captured by \( \alpha \beta' y_{t-1} + \sum_{j=1}^{p-1} \Delta \Gamma_j y_{t-j} \) once \( \varepsilon_t \) is an innovation. Simplifying, we called \( Q_t \) a conditional set given by \( Q_t = \{\beta' y_{t-1}, \Delta y_{t-1}, ... , \Delta y_{t-p+1}\} \). The idea is simple: the canonical correlation find the linear combination of the elements \( \Delta y_t \) that will be orthogonal to set \( Q_t \). Therefore, this linear combination is such that doesn’t exist any structure between \( \Delta y_t \) and \( Q_t \) beyond an innovation. An expression \( \text{CanonCorr} (X_t, Z_t | W_t) \) denote a canonical correlation between \( X_t \) and \( Z_t \) conditional on \( W_t \), such that \( W_t \) can contain deterministic terms as constants, deterministic trend, seasonal dummies etc. Therefore, \( \text{CanonCorr} \left[ \Delta y_{t}, (y_{t-1}' \beta, \Delta y_{t-1}', ..., \Delta y_{t-p+1}'))) W_t \right] \) allow to obtain canonical correlations, called eigenvalues, that are used to test the presence of a reduced rank model. Based on Tiao and Tsay (1985), Vahid and Engle (1993) propose a sequential test for SCCF, assuming that the rank of \( \beta \) is known. The sequence of hypotheses to be tested are: \( H_0 : \text{rank} \left( \tilde{\beta} \right) \geq s \) against \( H_a : \text{rank} \left( \tilde{\beta} \right) < s \) (see Lütkepohl, 1991; Velu et al, 1986) starting with \( s = 1 \) against the alternative model with \( s = 0 \) (doesn’t exist common cycle)\(^5\). If the null hypotheses is not rejected we implement the test for \( s = 2 \), and so on.

\(^4\)This estimation is referred as Full Information Maximum Likelihood - FIML

\(^5\)Equivalently a sequence of common feature Gaussian likelihood ratio test statistics for \( H_0 : \text{rank} \left( \Phi \right) \leq n - s \) against \( H_a : \text{rank} \left( \Phi \right) > n - s \), where is defined in (9).
In the VECM the significance of the $s$ smallest eigenvalues is determinate through the following statistic:

$$
\xi_s = -T \sum_{i=1}^{s} \ln(1 - \lambda_i^2) \sim \chi_{(v)}^2, \quad s = 1, \ldots, n - r
$$

(13)

$\lambda_1 < \lambda_2, \ldots, < \lambda_{n-r} < 1$, with $v = s[n(p-1)+r] - s(n-s)$ degrees of freedom, where $n$ is the dimension of the system and $p$ the lag order of the VAR model. Suppose that the statistical test (13) has found $s$ independent linear combinations of the elements of $\Delta y_t$ orthogonal to $Q_t$, this implies that exist a $n \times s$ matrix $\tilde{\beta}$ of full rank $s$ with $s$ eigenvectors associate with the $s$ smallest eigenvalues. Reinsel and Ahn (1992) propose a correction in statistic (13) in small samples $\xi_{s,corr} = \frac{T-n(p-1)-r}{T} \xi_s$, where $T$ is the real number of observations after the deduction of initial points in regressions containing lags.

3 Empirical results

3.1 Database

The database used was extracted from Penn World Table, corresponding to Real GDP per capita series of Mercosur countries. The frequency is annual, ranging from 1951 to 2003. We consider the model $Y_t = T_t \times C_t \times I_t$ whereas $C_t$ is the cycle, $T_t$ the trend of the series and $I_t$ the irregular components. Taking log in $Y_t$ we obtained $y_t = \log Y_t = \log(T_t) + \log(C_t) + \log(I_t) = \tau_t + c_t + i_t$. The Figure I reports the GDP in log terms. After 1975, in general, the series become closer - a behavior that may be generate by a common trend. Figure II displays the growth rates of real gross domestic products. It is possible to see the recession in the growth rate of the Argentina in 1989-1990. Decomposing the series in trend and cycle components allow us to investigate if increases or decreases of the GDP growth rate are due to changes in long-run or short-run components. For example, it is possible to investigate if the depression in Argentina was caused by a reduction in business cycle or by a reduction in the long-run trend. In the next section we estimate the cycle and trend components and so their growth rates.

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6For $p = 1$ the degrees of freedom is $(r+s)^2$. Notice in the model $\Delta y_t = \alpha \beta' y_{t-1} + \varepsilon_t$, the rank($\alpha \beta'$) = $\tilde{r} = n - s - r$, hence $v = (n - \tilde{r}) \times (np - \tilde{r}) = (n - (n-s-r))^2 = (r+s)^2$.

7Heston, Robert Summers and Bettina Aten, Penn World Table Version 6.1, Center for International Comparisons at the University of Pennsylvania (CICUP), October 2002. Real GDP per capita (Constant Prices: Chain series) http://put.econ.upenn.edu/php_site/put_index.php
3.2 Common Features results

To implement the methodology above, the hierarchical procedure due to Vahid and Engle (1993) is used. First, the VAR order is estimated, followed by the number of cointegration and cofeature vectors. The lag
order, $p$, is estimated by means of information criteria: Akaike (AIC), Schwarz (SC) and Hannan-Quinn (HQ) (Lütkepohl, 1993). To identify the number of long-run restrictions, $r$, it is used Johansen cointegration test. The number of short-run restrictions due to SCCF, $s$, is estimated using $\chi^2$ test. Lastly, the parameters are estimated using the FIML procedure (see Vahid and Issler (1993)).

Since BNSW decomposition assumes that the series are I(1), we begin the analysis testing if series have stochastic trend using the augmented Dickey-Fuller (ADF), Phillips-Perron (PP) and DF-GLS unit root tests. In all cases, the null hypothesis is the presence of unit root. The results for all countries are reported in Table I. The three tests do not reject the unit root null hypothesis, at 5% level of significance, for all countries$^8$.

Table I. Augmented Dickey-Fuller (ADF), Phillips-Perron (PP) and Elliott-Rothenberg-Stock (DF-GLS) unit root tests

<table>
<thead>
<tr>
<th>Country</th>
<th>ADF Statistic</th>
<th>Critic value (5%)</th>
<th>PP Statistic</th>
<th>Critic value (5%)</th>
<th>DF-GLS Statistic</th>
<th>Critic value (5%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina</td>
<td>-1.869127</td>
<td>-3.498692</td>
<td>-1.927693</td>
<td>-3.498692</td>
<td>-1.944709</td>
<td>-3.183600</td>
</tr>
<tr>
<td>Brazil</td>
<td>-0.240450</td>
<td>-3.498692</td>
<td>-0.430874</td>
<td>-3.498692</td>
<td>-0.599848</td>
<td>-3.186800</td>
</tr>
<tr>
<td>Paraguay</td>
<td>-0.575794</td>
<td>-3.500495</td>
<td>-0.739247</td>
<td>-3.498692</td>
<td>-1.080536</td>
<td>-3.186800</td>
</tr>
<tr>
<td>Venezuela</td>
<td>-1.097299</td>
<td>-3.498692</td>
<td>-1.078037</td>
<td>-3.498692</td>
<td>-0.809421</td>
<td>-3.183600</td>
</tr>
</tbody>
</table>

To estimate the order of the VAR, the AIC, HQ and SC information criteria are used. Table II shows the results for $p \in \{1, 2, 3, 4, 5\}$. As the data are annual we consider that an upper bound of 5 lags is sufficient. We observe that the three criteria suggest $p = 1$, indicating a VAR(1) model. Although $p$ selected by the criteria was one, to check the robustness of the results, we additionally test the model for $p = 2$ and $p = 3$.

Table II. Identification of the VAR order

<table>
<thead>
<tr>
<th>Criteria</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>HQ</td>
<td>-17.98521*</td>
<td>-17.51349</td>
<td>-17.10059</td>
<td>-16.77532</td>
<td>-16.38107</td>
</tr>
</tbody>
</table>

Note: * indicate lag suggested by information criteria

$^8$In the case of ADF and DF-GLS tests, the choice of lags of the dependent variable in the right side of the test equation is based on the Schwarz criterion. In PP test we use the nucleus of Bartlett and the window of Newey-West. In all tests the test equation has a constant and a linear trend. In any case, the results are robust to exclusion of the deterministic components.
Considering $p = 1$, $p = 2$ and $p = 3$ the usual diagnostic tests are applied in order to verify if these specifications are suitable. For $p = 1$ and $p = 2$ the LM test of serial autocorrelation does not indicate the presence of autocorrelation in the residuals, at 5% level of significance, while for $p = 3$ the opposite result is obtained. The White heteroskedasticity test (without cross terms) does not find evidence of heteroskedasticity, at 5% level of significance, for $p = 1, 2, 3$. Lastly, using the Jarque-Bera test, we do not reject the null hypothesis that residuals have normal distribution, at 5% level of significance, only for $p = 1$. Thus, the better is obtained when $p = 1$.

In order to investigate if there are long-run common components, we use the procedure of Johansen (1988) to test if the series are cointegrated. For each value of $p$ we consider two cases. First, we introduce a constant in the cointegration equation and, after, we add a linear trend. As the linear trend in the cointegration equation is significant (at 5% level), we consider this case in all subsequent analysis. In Table III the results for the cointegration test are shown for the case with constant and trend in the cointegration vector. The trace test indicate $r = 2$ for $p = 1, 2, 3$ while the maximum eigenvalue test suggest $r = 2$ for $p = 1, 2$ and $r = 1$ for $p = 3$. Thus, for $p = 1, 2$ both tests generate the same result and for $p = 3$ we opt for $r = 2$.

Table IV shows the SCCF test for $p = 1, 2$ using the correction given by Reinsel and Ahn (1992). For $p = 1$ the test indicates that $s = 4$, at 5% level of significance, but as the p-value is close to 5% we may assume $s = 3$ without trouble (see Table IV (a)). For $p = 2, 3$ the test indicates $s = 3$ (see Table IV (b) e (c)). Therefore, in all cases $s + r = n$. These results confirm the necessity to use a multivariate approach to identify the business cycles. In the next section we analyze the economic cycles obtained from the BNSW decomposition, considering the common cycles and the common trend restrictions. Once $s + r = n$ it is possible to find the trend and cycle components as shown above. Figure III show the common cycles for each value of $p$. We observe that for $p = 1, 2$ common cycles are very similar.

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9 The null hypothesis of the LM test is the absence of serial correlation until the lag $h$. We consider $h$ from 1 to 5.

10 The normality test uses the orthogonalization of Cholesky.
Table III. Johansen’s cointegration test

<table>
<thead>
<tr>
<th>Null hypothesis</th>
<th>Trace Test</th>
<th>Maximum Eigenvalue Test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Statistic</td>
<td>Critical value 5%</td>
</tr>
<tr>
<td>$r = 0$</td>
<td>130.0234*</td>
<td>88.80380</td>
</tr>
<tr>
<td>$r \leq 1$</td>
<td>76.15980*</td>
<td>63.87610</td>
</tr>
<tr>
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<td>37.16161</td>
<td>42.91525</td>
</tr>
<tr>
<td>$r \leq 3$</td>
<td>15.71202</td>
<td>25.87211</td>
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<tr>
<td>$r \leq 4$</td>
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<td>12.51798</td>
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</tbody>
</table>

Note: *indicating rejection of null hypothesis, at 5% level of significance

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<th>Maximum Eigenvalue Test</th>
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<td>Critical value 5%</td>
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Note: *indicating rejection of null hypothesis, at 5% level of significance

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<th>Trace Test</th>
<th>Maximum Eigenvalue Test</th>
</tr>
</thead>
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<td>$r \leq 4$</td>
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<td>12.51798</td>
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</table>

Note: *indicating rejection of null hypothesis, at 5% level of significance
Table IV. Common cycle test.

<table>
<thead>
<tr>
<th>Hipótese nula</th>
<th>Corr. Quad.</th>
<th>$\xi_{(p,s)}$</th>
<th>$[r + s]^2$</th>
<th>$p - \text{valor}$</th>
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<tr>
<td>$s &gt; 0$</td>
<td>0.0246</td>
<td>1.2971</td>
<td>9</td>
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</tr>
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<td>$s &gt; 1$</td>
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<td>5.3875</td>
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<tr>
<td>$s &gt; 2$</td>
<td>0.2025</td>
<td>17.1553</td>
<td>25</td>
<td>0.8761</td>
</tr>
<tr>
<td>$s &gt; 3$</td>
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<td>50.4638</td>
<td>36</td>
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</tr>
<tr>
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<td>0.6373</td>
<td>103.2064</td>
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<td>0.0000</td>
</tr>
</tbody>
</table>

Note: *indicating rejection of null hypothesis, at 5% level of significance

<table>
<thead>
<tr>
<th>Hipótese nula</th>
<th>Corr. Quad.</th>
<th>$\xi_{corr}^s$</th>
<th>$s \left[ (p - 1) + r \right] - s \left( n - s \right)$</th>
<th>$p - \text{valor}$</th>
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</thead>
<tbody>
<tr>
<td>$s &gt; 0$</td>
<td>0.1513</td>
<td>0.6797</td>
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<td>0.6596</td>
</tr>
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<td>14.7231</td>
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<td>0.4715</td>
</tr>
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<td>106.2583</td>
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</tbody>
</table>

Note: *indicating rejection of null hypothesis, at 5% level of significance

<table>
<thead>
<tr>
<th>Hipótese nula</th>
<th>Corr. Quad.</th>
<th>$\xi_{corr}^s$</th>
<th>$s \left[ (p - 1) + r \right] + s^2 - sn$</th>
<th>$p - \text{valor}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s &gt; 0$</td>
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<td>5.5138</td>
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<td>0.7015</td>
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<td>0.7506</td>
<td>129.2546</td>
<td>60</td>
<td>0.0000</td>
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</tbody>
</table>

Note: *indicating rejection of null hypothesis, at 5% level of significance
Figure III. Common Cycles.

Figure IV. Cyclical components for $p = 1$, $s = 3$ and $r = 2$. 
Figure 4 shows the business cycle components for our best estimative; $p = 1$, $s = 3$ and $r = 2$. We note an enormous contraction in Argentina in 90’s, as expected. Moreover, in the case of Brazil the period of the economic miracle is apparent. To analyze the robustness of the results we estimate business cycles for each country for $p=1,2,3$. Figure V shows the business cycle for each country. It is possible to see that business cycles obtained from different $p$ are similar.

Figure V. Cyclical components in each country for $p = 1$, $p = 2$ and $p = 3$.

4  Business cycle analysis

The degree of association among the contemporaneous movements may be measured through the pairwise linear correlation as reported in Table VI (see appendix A) for $p = 1, 2, 3$. We can observe that for $p = 1$ and $p = 2$ Brazil and Argentina have low positive correlation and for $p=3$ there is a negative correlation. Paraguay and Uruguay present a negative correlation while Brazil and Uruguay show a positive correlation. About the common cycles, it is possible to see that the economic cycle of Argentina is more influenced by
common cycle 1, whereas the Venezuela is influenced more negatively by common cycle 2. Lastly, notice that for some values of \( p \) there is a positive correlation and for other a negative correlation between countries, which exemplify why the linear correlation is not an ideal measure to identify co-movements.

Hence, analysis through linear correlation coefficient gives a static measure of the co-movements as noted by Engle and Kozick (1993). For capturing the simultaneous persistence of co-movement we use techniques based on the frequency domain. Analysis in frequency domain does not bring additional information, but it is an alternative method to analyze the data and a straightforward way to represent the economic cycles. A measure that corresponds to correlations in the time domain is the "coherence"\(^{11}\) in the frequency domain. The coherence between two variables is a measure of the degree to which these variables are jointly influenced by cycles of specific frequency. Another measure that we will employ is the "phase". The phase of the cross spectrum indicates if cycles in specific frequency are synchronized or not. When a phase is null, it means that exist synchronized cycles in that frequency. Figures VII to X show the coherence and phase between pairs of the business cycles in members’ Mercosur\(^{12}\). These pictures show values of coherence varying between zero and one (vertical axis). Values of phase are calculated to each value of frequency and it varies on the vertical axis. At the final point of the horizontal axis, the frequency 0.5 correspond to period of two year, the point 0.25 to four years, frequency 0.1 corresponds ten years, and so on.

\(^{11}\)See Appendix B.

\(^{12}\)To estimate coherence it is used a MSCOHERE function of Matlab 7.0 which considers smoothed with Hamming window of 30 with 50% overlap.
As mentioned, in finding robustness in our results we also consider analysis in frequency domain for values $p = 2, 3$. In the first row of Figure VI the ideal values of coherence and phase are shown, that is, coherence one and phase zero in all frequencies. For example, this picture shows results for synchronization of business cycle of Argentina with himself at each value $p$, and, after, the same is made for Brazil and Paraguay. The same results occur for Uruguay and Venezuela that are not reported in pictures. Figure VI also show that Argentina and Paraguay have reasonable values for coherence and phase only for $p = 3$. 

Figure VI. Coherence and Phase
In Figure VII is a reported interesting result for coherence and phase. First, Argentina and Uruguay appear to have high values of coherence in special for $p = 2$ and $p = 3$, but values of phases close to zero only for $p = 3$. Is means that these countries appear to have non-synchronized cycles. After that, Argentina and Venezuela appear to have values of coherence and phase very similar to ideal values of synchronization. Similar results occurs between Brazil and Paraguay, that is, for $p = 1$ and $p = 2$ values of coherence and phase are close to one and zero respectively. Hence, Brazil and Paraguay compose another group with synchronized cycle.
In Figure VIII is observed that Brazil and Uruguay appear to be synchronized only for $p = 3$ and Paraguay and Uruguay only for $p = 1$. Finally, in Figure IX shows Uruguay and Venezuela with values of coherence next to one but their phase are different from zero in almost all frequencies indicating that these couples of countries are not synchronized.

In summary the major evidence of synchronization is given in two group of countries; Argentina-Venezuela and Brazil-Paraguay. In general, all couples of countries present values of coherence less than one and their phase are generally different from zero.
Furthermore, the lack of synchronization among the business cycles confirms that the presence of common cycles does not imply synchronization and corroborates the importance to conduct this analysis in frequency domain.

5 Conclusion

The design of economic blocks is based on the harmonization of economic and commercial policies. However, as argued by Backus and Kehoe (1992) and Chistodoulakis and Dimelis (1995), this harmonization is well succeeded when the members of the block are sufficiently similar. In this direction, it is indispensable to analyze the dynamic of the members, investigating the degree of synchronization of their business cycles. Regarding the Mercosur, it is common to see in the media quarrels on the intensification of this economic block, however it is not common to argue which are the necessary daily pay-conditions for this intensification and if they are valid. Considering the members of Mercosur (Argentina, Brazil, Paraguay, Uruguay and Venezuela), this paper analyze if there exist any common dynamic in their economies and if
their business cycles are synchronized. To implement the analysis we estimate a VAR model and test the presence of common trends and common cycles. The business cycles components are found when BNSW trend-cycle decomposition is employed, taking in account the cointegration and serial correlation common feature restrictions. Measures of coherence and phase in the frequency domain are used to determinate the degree of co-movements in business cycles.

The results suggest that there are three common trends and two common cycles among the countries. According to Beine et al. (2000), the existence of such common dynamics provides support for some types of convergence and for sustainability of an optimal currency area. This result confirms the necessity to use a multivariate approach, a contribution from this work. Frequency domain results identified evidences of synchronization in two sub-groups; Argentina-Venezuela and Brazil-Paraguay but that in general the countries are not synchronized. Hence, the lack of synchronism or symmetry in the business cycle of Mercosur makes difficult a greater integration into this economic block.
References


### Table VI. Linear correlations in business cycles and in common cycle

<table>
<thead>
<tr>
<th>Countries</th>
<th>Argentina</th>
<th>Brazil</th>
<th>Paraguay</th>
<th>Uruguay</th>
<th>Venezuela</th>
<th>C. Cycle 1</th>
<th>C. Cycle 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>VAR order $p = 1$</strong></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
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<td></td>
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APPENDIX B: COHERENCE AND PHASE

Consider a vector of two stationary variables \( y_t = (X_t, Y_t) \). Let \( S_{YY}(w) \) represent the *population spectrum* of \( Y \) and \( S_{YX}(w) \) the *population cross spectrum* between \( X, Y \). The *population cross spectrum* can be written in terms of its real and imaginary components as \( S_{YX}(w) = C_{YX}(w) + i Q_{YX}(w) \), where \( C_{YX}(w) \) and \( Q_{YX}(w) \) are labeled the *population cospectrum* and *population quadrature spectrum* between \( X, Y \) respectively.

The *population coherence* between \( X \) and \( Y \) is a measure of the degree to which \( X \) and \( Y \) are jointly influenced by cycles of frequency \( w \).

\[
\rho_{YX}(w) = \frac{|C_{YX}(w)|^2 + |Q_{YX}(w)|^2}{S_{YY}(w) S_{XX}(w)}
\]

Coherence takes values in \( 0 \leq \rho_{YX}(w) \leq 1 \). A value of one for coherence at a particular point means the two series are altogether in common at that frequency or cycle; if coherence is one over the whole spectrum then the two series are common at all frequencies or cycles.

The *cross spectrum* is in general complex, and may express in its polar form as:

\[
S_{YX}(w) = C_{YX}(w) + i Q_{YX}(w) = R(w) \exp(i \theta(w))
\]

where \( R(w) = \left\{ |C_{YX}(w)|^2 + |Q_{YX}(w)|^2 \right\}^{\frac{1}{2}} \) and \( \theta(w) \) represent the gain and the angle in radians at the frequency \( w \). The angle satisfies \( \tan(\theta(w)) = \frac{Q_{YX}(w)}{C_{YX}(w)} \). More details in Hamilton (1994).