

# Non-monotone insurance contracts and their empirical consequences

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## Abstract

The goal of this paper is to show the possibility of a non-monotonic relation between coverage and riskiness which is in contrast with the traditional literature of insurance models since the work of Rothschild and Stiglitz (1976). We present an insurance model where the insured agents have heterogeneity in risk aversion and prevention cost parameter. Risk aversion is a continuous parameter correlated with prevention cost. In the case of positive correlation, more risk averse agents have higher prevention cost leading to a higher demand for coverage. Equivalently, the single crossing property (SCP) holds and implies a positive correlation between coverage and riskiness in equilibrium. On the other hand, if the correlation between risk aversion and prevention cost is negative, not only the SCP is violated, but also the monotonicity of contracts, i.e., the prediction that high (low) risk types choose more (less) comprehensive insurance contracts. In both cases riskiness is monotonic on risk aversion, but when the SCP fails coverage is not monotonic and there is discrete pooling: two different risk averse types (low and high) associated to the same coverage. This implies that the correlation between coverage and riskiness may have any sign. Moreover, we give an implication to disentangle single-crossing and non-single-crossing under zero correlation between coverage and riskiness: coverage is not a monotonic function of riskiness if the SCP fails.

JEL Classification: H41, D82.

Keywords: Insurance contracts, non-single-crossing, non-monotonicity, zero correlation.

# 1 Introduction

The goal of this paper is to show the possibility of a non-monotone relation between coverage and riskiness for insurance models. The seminal paper by Rothschild and Stiglitz (1976) predicts the monotonicity property (MP): “*contracts with more comprehensive coverage are chosen by agents with higher expected accident costs*”. Since then, most of the models predict a positive correlation between insurance contract coverage and riskiness, implying that this claim might be quite robust. Seemingly, it does not require the single crossing property (SCP) and remains true when moral hazard or multidimensional screening are introduced (see Chiappori and Chassagnon (1997) and Villeneuve (1996), for instance).<sup>1</sup>

In the empirical literature, Puelz and Snow (1994) were the pioneering reference in testing asymmetric information on automobile insurance market. Their results were consistent with the presence of asymmetric information. However, as pointed out by Chiappori and Salanié (2000), these results may be spurious because of omitted non-linear effects in the regressions. They proposed a more robust test for asymmetric information using a data base from French automobile insurance contracts. Chiappori and Salanié tested the validity of positive correlation between coverage and riskiness of the contract conditional on all observable variables. They measured *ex-post* riskiness as the occurrence of an accident during one year and reduced the class of contracts to two: partial and full insurance. Their conclusion was that they could not reject this correlation to be different from zero. If MP were valid, this would imply no adverse selection for that data set. More recent empirical works show that there zero correlation result does not necessarily invalidate the presence of adverse selection (see Finkelstein and McGarry (2004) for the

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<sup>1</sup>The exceptions are Smart (2000) and Wambach (2000) where insurance customers have preferences differing in two dimensions that may break down the SCP. In the former these dimensions are degree of risk aversion and accident probability and the last are wealth and risk. Consistent with our finding, when SCP does not hold pooling may occur. However, in both papers types and outcomes have only two realizations and non-monotonicity cannot be analyzed.

long term insurance market). Fang *et al.* (2008) find evidence of advantageous selection in Medigap insurance market. Conditional to the usual sources of this advantageous selection, they show that although risk preferences are not important, cognitive ability is particularly important source.

In this paper we argue that the MP is in fact dependent on the monotonicity assumptions of the coverage demand with respect to unobservable characteristics the insured agents have. Specifically, the MP is dependent on the SCP, i.e., the monotonicity of the marginal utility of coverage with respect to the insurees' characteristics. Since the traditional contract theory literature usually assumes the SCP (which implies a monotonic relation between the contract variables), it is not surprising that the MP naturally emerges in that context. The problem, however, in relaxing the SPC is twofold: mathematically it is very hard to deal with the non-convexities that can appear when the SCP does not hold; and even if we are able to relax this assumption, is it not clear that the MP will not hold. In Araujo and Moreira (2009), this problem is treated in a specific monopolistic setup where the monotonicity does not always hold. Our aim here is to build an insurance model where SCP and MP are not valid and their techniques can be applied (see also Araujo, *et al.* (2007) for an application in labor market with countersignaling).

We propose an insurance model with adverse selection and moral hazard. The insured agents are heterogeneous with respect to risk aversion and prevention cost. Differently from the standard literature, we borrow the Holmstrom and Milgrom (1987) framework of providing incentives for precautionous behavior of an agent with constant risk aversion. We assume that there exists a perfect correlation between risk aversion and prevention cost. We carry out the analysis for both monopolistic and competitive markets and separate the analysis in the usual benchmarks: symmetric information, pure moral hazard and moral hazard with adverse selection. In the symmetric information benchmark, there is full coverage and the correlation between coverage and riskiness is zero. In the pure moral

hazard situation, under some mild regularity assumptions, the MP always holds.

Then, the interesting case is moral hazard with adverse selection where we study the correlation between risk aversion and prevention cost in two stylized cases. In the positive correlation case more risk averse agents have higher marginal cost of prevention. These two heterogeneities go in the same direction, i.e., the more risk averse agents buy more insurance and then have less cautious behavior which reinforces the demand for coverage. Therefore, the MP is a consequence of the self selection constraints and the SCP (i.e., the marginal utility of coverage increases with risk aversion).

In the negative correlation case, more risk averse agents have lower marginal cost of prevention. Thus, these two heterogeneities go in opposite directions, i.e., the marginal utility of coverage increases with risk aversion due to the reduction of risk premium and decreases due to the less costly preventive effort. The SCP holds if and only if one effect always prevails against the other. Otherwise, not only the SCP may be violated but also the MP may not hold because of the self selection constraints.

In the monopolistic setting, breaking the MP down is not a new result. Jullien *et al.* (1999) show that the preventive effort may be non-decreasing with respect to risk aversion (what they call the “non-responsive case”). Since they assume the SCP, coverage is monotonic with risk aversion. Our case is the “responsive case” where coverage may not be monotonic on risk aversion if the SCP fails.

In the competitive setting we use the perfect Bayesian equilibrium concept to sustain the actuarially fair contract (zero profit). Under the SCP, equilibrium is separating, i.e., coverage and riskiness are monotonic on risk aversion, which leads to the MP. If the equilibrium is not separating, there may exist less and more risk averse insureds with the same coverage in equilibrium. Then, there may be cross subsidies in equilibrium: low-risk individuals pay less and high-risk individuals pay more than the actuarial value of their insurance in the pooling set. Excessive premium arises in equilibrium because of non-

single-crossing (see Smart, 2000). We indeed show that our quasi-separable equilibrium<sup>2</sup> may be non-monotonic and then may have discrete pooling.

Although the non-monotonic relation between equilibrium coverage and riskiness is necessary to break the MP down, it is not obvious that it is sufficient. This is because the MP is the positive relation between coverage and expected risk. Therefore, we provide an example where the MP does not hold. More strongly, for this example we show that the correlation between coverage and riskiness can have any sign.

In short, our implications are: (i) under symmetric information, the correlation between coverage and riskiness is zero; (ii) under asymmetric information, if there is a positive correlation between risk aversion and prevention cost, then the MP holds and adverse selection cannot be distinguished from moral hazard; (iii) if there is a negative correlation between risk aversion and prevention cost, then the MP holds in the pure moral hazard case. However, adding adverse selection, the MP may fail and zero correlation between coverage and riskiness may occur whenever the SCP fails.

Under zero correlation between contract coverage and riskiness, our model gives two possibilities: no adverse selection (pure moral hazard or symmetric information) or adverse selection without the SCP, since the non-monotonicity may lead to zero correlation without contradicting the presence of adverse selection. The monotonicity between coverage and riskiness allows us to distinguish between these two possibilities.

The insurance model is presented in Section 2. The perfect correlation between adverse selection parameters is treated in Section 3. The fourth section presents our zero-correlation result and discusses its relation with the presence of single-crossing and adverse selection. The last section gives the concluding remarks.

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<sup>2</sup>Since the SCP does not hold, there does not exist a fully separable equilibrium. We then define the notion of quasi-separable equilibrium: lowest degree of pooling and highest coverage. See Appendix B for the details.

## 2 The model

The insuree's wealth  $\omega$  is normally distributed with mean  $e$  and (exogenous) variance  $\sigma^2$ , where  $e$  is the preventive effort controlled by the insuree. Borrowing the Holmstrom and Milgrom (1987) setup, the insuree's wealth follows a diffusion process whose drift,  $e$ , she can control:

$$d\omega_t = edt + \sigma dW_t,$$

where  $\{W_t\}_{t \in [0,1]}$  is the standard Brownian motion. In this case,  $e$  is a sufficient statistic for the account number of no loss that the insuree has along the total period of insurance. More precisely, we can approximate this diffusion process by a binomial tree which has in each node two outcomes: loss and no loss. So, drift is a proxy for safeness, which is the probability of no accident in the model with only two outcomes (see Hellwig and Schmidt, 2002).<sup>3,4</sup>

The cost of prevention is a function of  $e$  and a parameter  $\eta$ . The insuree's constant absolute risk aversion  $\theta$  and  $\eta$  are adverse selection parameters.

By introducing these two heterogeneities that link the *ex-ante* adverse selection with the *ex-post* moral hazard we have a mixed model<sup>5</sup>. We assume that  $\eta$  is not known by both the insuree and the insurance company, but it is correlated with  $\theta$  such that the insuree's expected precaution given  $\theta$  is  $C(e, \theta)$ . As we will see in the sequel of the paper, the sign of the correlation is not important *per se*. What is important is its effect on the marginal cost of prevention and, in particular, whether the marginal utility of preventive effort is monotonic on  $\theta$  (i.e., whether the SCP is satisfied). We assume standard properties for

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<sup>3</sup>Holmstrom and Milgrom (1987) show that the optimal insurance contract for the whole period must give the agent a bonus that depends linearly on the number of periods in which the wealth increased, and  $\omega$  (final wealth) is a sufficient statistics for this number.

<sup>4</sup>Although this is not a standard insurance model, we adopted this framework in order to benefit from the quasi-linear specification derived from the linearity of optimal and/or equilibrium contracts. Since standard insurance models are not quasi-linear, we would have more technical difficulties to deal with them. See Landsberber and Meilijson (1999) for a more general model.

<sup>5</sup>That is, a model where adverse selection is followed by moral hazard. See Laffont and Martimort (2002), ch. 7 for the definition of mixed models.

$C$ :<sup>6</sup>

**A0.**  $\partial_e C \geq 0 = \partial_e C(0, \cdot)$ ,  $\partial_e C(\theta, \infty) = \infty$ , for all  $\theta$ ,  $\partial_{ee} C > 0$  and  $\partial_{eee} C > 0$ .

The sign of the cross derivatives will play an important role for the validity of the SCP and will be explored in the next section.

The insuree's utility depends on wealth and effort and is represented by the CARA specification:

$$-\exp[-\theta(\omega - C(e, \theta))].$$

The insurance company(ies) is (are) risk-neutral.

The stages of the model are the following: (1) each insurance company chooses the menu of contracts (indexed by the insuree's parameters); (2) the insuree (self) selects his contract; (3) the insuree decides his level of prevention; (4) finally, the state of nature is realized and the contracts are enforced.

We study the following benchmarks: first best, pure moral hazard and moral hazard with adverse selection. In the first case, there is no asymmetric information problem: the insurance company can write contracts on both preventive effort and risk aversion. In the second case, the insurance company can verify only the insuree's risk aversion and in the third it cannot.

## 2.1 First best

The insurance company can control for preventive effort and risk aversion. Therefore, the only important issue here is the risk sharing between the insurance company and the insurees. Since the insurance company is risk neutral and the insurees are risk averse, the optimal allocation of risk satisfies the (Borch's) rule:

$$1 = \partial_e C(e, \theta) \tag{1}$$

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<sup>6</sup>From now on we will use the following notation for derivatives:  $\partial_e C$  is the partial derivative of  $C$  with respect to  $e$  and so forth;  $\dot{e}(\theta)$  is the derivative of  $e$  with respect to  $\theta$ .



i.e., the marginal benefit of risk reduction is equal to its marginal cost. Moreover, the insurance company expropriates all the insuree's rent: the insuree's indirect utility is equal to her reservation utility outside the contract. Let  $e^{FB}(\theta)$  be the solution of (1).

There are many ways to implement this solution under complete information. To make the parallel with the other cases, we use the full insurance implementation (see the next subsection for its description).

## 2.2 Pure moral hazard

Another important benchmark is when insurance companies can control for risk aversion, i.e., there is no adverse selection and the insurance problem is a pure moral hazard one.

From Holmstrom and Milgrom (1987) the optimal contract with moral hazard is linear:

$$(1 - c)\omega + \beta,$$

where  $c$  is the coverage and  $\beta$  is the reimbursement. Observe that  $c$  measures the standard deviation of the amount of wealth to which the insuree is not exposed to (or equivalently, the one that insurance companies face):  $(Var[(1 - c)\omega + \beta - \omega])^{1/2} = c$ .

In terms of the agent's (insuree's) certainty equivalent, her expected utilities in the space of linear contracts  $(c, \beta)$  can be equivalently represented by<sup>7</sup>

$$(1 - c)e + \beta - C(e, \theta) - \frac{\sigma^2}{2}\theta(1 - c)^2.$$

Thus, given a contract  $(c, \beta)$ , the optimal effort is determined by the first order condition of the incentive compatibility constraint, i.e., the equalization of its marginal benefit and cost:

$$1 - c = C_e(e, \theta). \tag{2}$$

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<sup>7</sup>We are using the well known identity:

$$E[\exp(\omega)] = \exp[E(\omega) + .5Var(\omega)]$$

where  $\omega$  is normally distributed,  $E$  is the expectation and  $Var$  the variance operators.

Let  $e(c, \theta)$  be the induced preventive effort by the contract  $(c, \beta)$  for a type  $\theta$ , i.e., the implicit solution of (2). Observe that assumption A0 implies that  $e(c, \theta)$  is well defined and that (2) is necessary and sufficient for optimality. Taking into account (2), the insuree's indirect utility function is quasi-linear

$$v(c, \theta) + \beta,$$

where  $v(c, \theta) = (1 - c)e(c, \theta) - C(e(c, \theta), \theta) - \frac{\sigma^2}{2}\theta(1 - c)^2$ .

The insurance company's expected profit which is the expected social surplus net the insuree's utility:

$$\begin{aligned} U(c, \beta, \theta) &= e(c, \theta) - [(1 - c)e(c, \theta) + \beta] = ce(c, \theta) - \beta \\ &= e(c, \theta) - C(e(c, \theta), \theta) - \frac{\sigma^2}{2}\theta(1 - c)^2 - v(c, \theta) - \beta. \end{aligned}$$

### Monopolistic case

The single insurance company maximizes expected profit subject to the individual rationality (IR) or participation constraint:

$$\begin{aligned} &\max_{(c, \beta)} U(c, \beta, \theta) \\ &\text{s.t. } v(c, \theta) + \beta \geq w_0(\theta), \end{aligned}$$

where the IR constraint guarantees at least the type  $\theta$  agent's the null contract,  $w_0(\theta) = v(0, \theta)$ .

### Competitive case

There are at least two insurance companies competing a la Bertrand. This means that the equilibrium insurance contract is characterized by the maximization of the insuree's utility given the zero profit condition of insurance companies and the insuree participates.

Therefore, the characterizations of the monopolistic optimal or the competitive equilibrium contract are analagous: they are the maximization of the social surplus (i.e., the sum of the insurance company and insuree utilities). The only difference between the two cases is who gets the rent.

In Rothschild and Stiglitz (1976), risk is the probability of bad state of nature and also the insuree' type. In our case, the state space is infinite and the type is the insuree's risk aversion. However, in the Holmstrom and Milgrom (1987) framework the optimal preventive effort,  $e$ , is constant over time (because of the absence of income effects under CARA utility functions) and, therefore,  $\pi = 1 - e$  is a non-ambiguous proxy for riskiness. It is the reciprocal measure of precautious behavior, i.e., the (absolute<sup>8</sup>) decrease in effort of prevention due to risk aversion.

**Definition 1.** *Given a contract  $(c, \beta)$  faced by an agent with coefficient of risk aversion  $\theta$  and induced preventive effort  $e(c, \theta)$ , we define*

$$\pi(c, \theta) := 1 - e(c, \theta)$$

*as her riskiness on  $(c, \beta)$ , respectively.*<sup>9</sup>

**Remark** (*First-best implementation*). *We can describe the first best contract using the linear specification: it has full coverage and the reimbursement is the insuree's opportunity cost, i.e.,*

$$c^{FB}(\theta) = 1$$

$$\beta^{FB}(\theta) = w_0(\theta)$$

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<sup>8</sup>For the purpose of this paper, the choice of this specific absolute measure does not matter, since we are interested in the correlation between riskiness and coverage. Another possibility is a relative measure like the decrease in effort with respect to the first best level.

<sup>9</sup>We can also define a proxy for the contract premium. Let  $\bar{w} = \max_{\theta} w_0(\theta)$  be the maximum opportunity cost of the economy and define such proxy for type  $\theta$  as  $p(\theta) = \bar{w} - w_0(\theta)$ . What is important in this definition is that  $p$  is negative related with the opportunity cost of the agent, i.e., the higher the equivalent certainty is, the lower is the premium.

for each type  $\theta$ . However, since the preventive effort is enforceable, the first-best incentive power is one (see (1)).

Taking the implicit derivative of condition (1) we get the derivative of riskiness:

$$\frac{\cdot}{\pi}{}^{FB}(\theta) = -\frac{\cdot}{e}{}^{FB}(\theta) = \frac{C_{e\theta}(e^{FB}(\theta), \theta)}{C_{ee}(e^{FB}(\theta), \theta)},$$

where  $e^{FB}(\theta) = e(c^{FB}(\theta), \theta)$ . Thus, the relation between first-best riskiness and risk aversion is driven by the monotonicity of the marginal cost of prevention with respect to risk aversion (see section 3 for the discussion in the asymmetric information case).

However, the first-best contract has constant full coverage which leads to a zero correlation between coverage and riskiness, independently of how riskiness is related with risk aversion and of the risk aversion distribution. For future reference, we establish our:

**Implication 0.** *Under symmetric information, the correlation between coverage and riskiness is zero.*

### 2.3 Moral hazard and adverse selection

Now the insurance company cannot control for the insurees' parameters. Taking into account (2) again, the problem now reduces to a one-dimensional screening program.

A direct mechanism (or contract) is a pair of functions that maps the announcement of types to allocations:  $(c(\theta), \beta(\theta))$ . Suppose that  $\theta$  has cumulative distribution  $F$  on  $[\underline{\theta}, \bar{\theta}]$  with density  $f > 0$ . From standard arguments of mechanism design,<sup>10</sup> the insurance company program is restricted by the incentive compatibility and the participation constraints:

$$\mathcal{V}^{(c,\beta)}(\theta) \geq \mathcal{V}^{(c,\beta)}(\hat{\theta}|\theta), \text{ for all } \theta, \hat{\theta} \in \Theta \quad (\text{IC})$$

$$\mathcal{V}^{(c,\beta)}(\theta) \geq w_0(\theta), \quad \text{for all } \theta \in \Theta \quad (\text{IR})$$

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<sup>10</sup>According to the Revelation Principle, we can restrict ourselves to the direct and truthful mechanisms. Sung (2002) shows that linear contracts are optimal for mixed models of adverse selection before moral hazard like the one used here.

where  $\mathcal{V}^{(c,\beta)}(\widehat{\theta}|\theta) = v(c(\widehat{\theta}), \theta) + \beta(\widehat{\theta})$  is type  $\theta$  insuree's indirect utility when she takes type  $\widehat{\theta}$  contract and  $\mathcal{V}^{(c,\beta)}(\theta) = \mathcal{V}^{(c,\beta)}(\theta|\theta)$  is the type  $\theta$  insuree's rent function.

From the envelope theorem, the first order condition of the *IC* constraint is equivalent to

$$\frac{d}{d\theta} \mathcal{V}^{(c,\beta)}(\theta) = v_{\theta}(c(\theta), \theta) \quad (3)$$

which gives the rate at which the insuree's rent must grow to elicit its information.

The analysis of the IC constraint strongly depends on how the risk aversion and the prevention cost are correlated. In standard adverse selection models, the SCP plays an important role for the characterization of implementable contracts. It is the monotonicity of the marginal utility of coverage with respect to risk aversion. An important consequence of the SCP is that implementable contracts are exactly the monotonic ones (see Proposition 1 (ii) below).

In the next section we deal with two distinct cases: the positive and negative correlation between risk aversion and prevention cost. As we shall see, these cases are related to the validity or not of the SCP, which is sufficient for the monotonicity of implementable contracts with respect to risk aversion.

### 3 Risk aversion and prevention cost relation

We will carry out the analysis in the positive correlation (single-crossing) where the marginal cost of prevention is increasing with risk aversion and the negative correlation (non-single-crossing) where the relation is reversed.

#### 3.1 The positive correlation

In this case we assume that more risk averse agents have higher marginal prevention cost:

**Assumption A1:**  $C_{e\theta}(e, \theta) > 0$ .

The cross derivative of  $v$  is

$$v_{c\theta}(c, \theta) = \frac{C_{\theta e}(e(c, \theta), \theta)}{C_{ee}(e(c, \theta), \theta)} + \sigma^2(1 - c) > 0.$$

Then, assumptions A0 and A1 imply that this derivative is always positive, i.e., more risk averse agents have higher marginal utility of coverage. This is the SCP in our quasi-linear utility case.

**Proposition 1** *Suppose that A0 and A1 hold.*

(i) *Pure moral hazard: Suppose that  $\pi(c, \cdot)$  is a non-decreasing function for each  $c$ . Then, the optimal monopolistic and the competitive equilibrium coverage and riskiness are increasing functions of risk aversion.*

(ii) *Moral hazard with adverse selection: A contract  $(c(\theta), \beta(\theta))$  satisfies the IC constraint (i.e., it is implementable) if and only if (3) is satisfied and  $c(\cdot)$  is non-decreasing.*

**Proof.** (i) From the first equation of (4) in the Appendix A and A1 we have that  $e(\cdot, \theta)$  is an increasing function. Since  $\pi(c, \cdot)$  is non-decreasing, we get the result.

(ii) Using standard arguments (see Guesnerie and Laffont, 1984), the proposition is equivalent to the first and second order conditions of the IC constraints. They are necessary and sufficient conditions for implementability under the SCP. In this case, the first order condition is given by (3) and the second order condition by the monotonicity of  $c(\cdot)$ .

■

Proposition 1 (i) is simply a comparative statics on risk aversion in the pure moral hazard situation: the efficient risk sharing is to induce less preventive effort the higher the risk aversion is (since preventive effort is more costly for those types), by providing higher coverage. This leads to a positive relation between coverage and riskiness. The monotonicity condition on  $\pi(c, \cdot)$  means that for any coverage level the relation between risk aversion and riskiness is positive. It is a regularity condition, but it is critical for the

validity of the positive correlation result in the pure moral hazard case (see Julien, *et al.*, 1999). In Appendix C we show that A1 and the constant sign of third derivatives of  $C$  are sufficient for this condition.

Proposition 1 (ii) shows that this positive relation remains true for incentive compatible coverage under moral hazard and adverse selection. Moreover, using (2), the induced riskiness<sup>11</sup>  $\pi(\cdot)$  for a given implementable coverage  $c(\cdot)$  satisfies

$$\dot{\pi}(\theta) = \frac{\dot{c}(\theta) + C_{e\theta}(e(c(\theta), \theta), \theta)}{C_{ee}(e(c(\theta), \theta), \theta)} > 0$$

i.e.,  $\pi(\cdot)$  is also an increasing function.

The next definition gives the monotonicity property (MP) for a given incentive compatible contract with coverage  $c(\theta)$  and riskiness  $\pi(\theta)$ .

**Definition 2.** *The ex-post riskiness is the conditional expectation  $E[\pi|c]$ , taken with respect to the distribution of types. We say that the monotonicity property (MP) is satisfied if and only if  $E[\pi|c]$  is a non-decreasing function of  $c$ .*

Therefore, Proposition 1 implies the MP holds under assumption A1 because it is exactly the monotonicity of the relation between coverage ( $c$ ) and riskiness ( $\pi$ ):

$$\frac{dc}{d\pi} = \frac{\dot{c}(\theta)}{\dot{\pi}(\theta)} \geq 0.$$

In particular, there is a positive correlation between coverage and riskiness and the prediction on the correlation sign is the same with or without adverse selection. This implies also that the MP only cannot distinguish moral hazard from adverse selection. This is consistent with previous works that generalize the MP (see Chiappori, *et al.*, 2005) and tries to distinguish adverse selection from moral hazard in the data set (see Abbring, *et al.*, 2003 and Gardiol, *et al.*, 2003). We then get:

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<sup>11</sup>With some abuse of notation,  $\pi(\cdot)$  denotes  $\pi(c(\cdot), \cdot)$ .

**Implication 1.** *If there is a positive correlation between risk aversion and prevention cost, then the MP holds. Moreover, adverse selection cannot be distinguished from moral hazard.*

### 3.2 The negative correlation

Suppose now a negative correlation between risk aversion and prevention cost. In particular, an *ex-ante* more (less) risk averse agent will be more (less) diligent *ex-post*, decreasing (increasing) his *ex-ante* marginal utility for high coverage.

**Assumption A2:**  $C_{e\theta}(e, \theta) < 0$ .

First let us characterize the optimal monopolistic contract and the competitive equilibrium in the pure moral hazard situation.

**Proposition 2** (*Pure moral hazard*) *Suppose that A0 and A2 holds such that  $\pi(c, \cdot)$  is non-increasing function for each  $c$ . Then the optimal monopolistic and competitive coverage and riskiness in the pure moral hazard case are decreasing functions of risk aversion.*

**Proof.** The proof is analogous to the proof of Proposition 1(i). ■

Proposition 2 is analogous to Proposition 1 (i): the efficient risk sharing in the pure moral hazard is to induce more preventive effort the higher the risk aversion is (since, by assumption A1, preventive effort is less costly for high risk averse type) by giving less coverage. This also leads to a positive correlation between coverage and riskiness, i.e., the MP is also valid for this case.<sup>12</sup> We then get:

**Implication 2.** *Under negative correlation between risk aversion and prevention cost, if there is no adverse selection (pure moral hazard), then the MP holds.*

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<sup>12</sup>An example that fulfills the conditions of Proposition 2 is  $C(e, \theta) = d(e/\theta)$ , where  $d', d'', d''' > 0$ .



Let us move to the case of moral hazard with adverse selection. An important issue here is the validity of the SCP, i.e., the constant sign of  $v_{c\theta}(c, \theta)$ . Using standard arguments, besides the first order condition (3), the second order condition of the *IC* constraint implies the usual up-stream (down-stream) incentives:  $c$  is non-increasing (decreasing) when  $v_{c\theta}(c, \theta) > (<) 0$ . These are the monotonicity conditions.

When the SCP holds the local *IC* constraints are sufficient for implementability (see Guesnerie and Laffont, 1984). However, if the SCP is violated, they are in general only necessary conditions. For instance, suppose that an implementable contract  $(c(\cdot), \beta(\cdot))$  is such that  $c(\cdot)$  crosses two regions where the sign of  $v_{c\theta}$  changes. By the monotonicity conditions, if  $c(\cdot)$  is continuous and not flat at the crossing point, it is non-monotonic and then there may exist types,  $\theta$  and  $\hat{\theta}$ , choosing the same coverage level  $\bar{c}$  (*discrete pooling*). Thus, the first order condition of the incentive compatibility (3) implies that the marginal change of reimbursement with respect to the marginal change of coverage must be the same at these types:

$$-\frac{d\beta}{dc}(\bar{c}) = v_c(\bar{c}, \theta) = v_c(\bar{c}, \hat{\theta})$$

i.e., a cross stream incentive condition must hold. In some particular cases (like the example studied in this paper) these new conditions and the standard monotonicity conditions are sufficient for incentive compatibility. See Araujo and Moreira (2009) and Araujo et al. (2007) for a first treatment of principal-agent models without the SCP for the screening and signaling models, respectively. We will use this approach to our monopolistic and competitive insurance markets.

In what follows we present an example where the ambiguity of the cross derivative sign may lead to the non-monotonicity in the relation between coverage and riskiness.

### 3.3 An example: quadratic cost of prevention

Assume that the cost of preventive effort is quadratic on  $e/\theta$ :

$$C(e, \theta) = \left(\frac{e}{\theta}\right)^2$$

and  $F$  is the uniform distribution on  $[1, 2]$ .

Given a contract  $(c, \beta)$ , the optimal preventive effort is  $e(c, \theta) = \theta^2(1 - c)$  and the pure moral hazard coverage is:

$$c^{pm}(\theta) = \frac{\sigma^2}{\sigma^2 + \theta}.$$

Under moral hazard with adverse selection the insuree's cross derivative of the utility with respect to coverage and risk aversion is

$$v_{c\theta}(c, \theta) = (2\theta - \sigma^2)(1 - c)$$

(see Appendix A). The first term,  $2\theta(1 - c)$ , is the marginal benefit (due to assumption A2) and the second,  $\sigma^2(1 - c)$ , is the marginal cost of risk premium in reducing coverage and raising risk aversion, respectively. Depending on what term prevails, the sign may be positive or negative. Formally,  $v_{c\theta}(c, \theta) \geq 0$  if and only if  $\theta \leq \frac{\sigma^2}{2}$ . Therefore, there are two extreme situations where the SCP holds:  $\sigma^2 \leq 2$  or  $\sigma^2 \geq 4$ . For intermediate values, the SCP is violated.

Let us analyze the monopolistic and competitive cases.

#### Monopolistic case

The only insurance company maximizes the expected profit subject to the incentive compatibility and the participation constraints:

$$\begin{aligned} \max_{(c, \beta)} E[U(c(\cdot), \beta(\cdot), \cdot)] \\ \text{s.t. } IC \text{ and } IR. \end{aligned}$$

See Appendix A for the characterization of the optimal contract. In what follows we only provide a description of it.

- low variance ( $\sigma^2 \leq 2$ ): implementable coverages are non-increasing functions of risk aversion  $\theta$ . The observation of  $\omega$  is very informative about the preventive effort taken by the insuree. The insurance contract reflects more the screening aspect of the incentive scheme, which is driven by the fact that more risk averse types have lower marginal cost of prevention (assumption A2).
- high variance ( $\sigma^2 \geq 4$ ): implementable coverages are non-decreasing functions of  $\theta$ . The cost of the risk premium increases with variance and, for high levels, the total marginal utility of coverage increases with risk aversion. Therefore, the wealth  $\omega$  is less informative about the insuree's preventive effort, also making costly the screening (in particular, there is much pooling).
- intermediate variance ( $\frac{\sigma^2}{2} \in (1, 2)$ ): the SCP does not hold, and then an implementable coverage has to satisfy the up and down stream incentives: it must be non-decreasing (respectively, increasing) for  $\theta \in [1, \frac{\sigma^2}{2}]$  (respectively,  $\theta \in [\frac{\sigma^2}{2}, 2]$ ). There is a combination of the previous two effects: coverage is non-decreasing for low risk averse types and non-increasing for high ones. Moreover, for each  $\theta$  and  $\hat{\theta}$  choosing the same level of coverage  $\bar{c}$ ,  $v_c(\bar{c}, \theta) = v_c(\bar{c}, \hat{\theta})$  or  $\hat{\theta} = \sigma^2 - \theta$ .<sup>13</sup>

For intermediate values of  $\sigma^2$  there exists pooling at the optimal contract. The pooling interval can be of two types: continuous or discrete. We say that the pooling is continuous if there exists an interval of types that pool at the same contract level. Discrete pooling means that the set of types that pool in those levels is a discrete but not singleton set. The interval of types where there exists discrete pooling is called discrete pooling interval.

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<sup>13</sup>Observe that  $\hat{\theta}$  is not a function of  $\bar{c}$ , which greatly simplifies the analysis. For a more general case, the regions of positive and negative sign of  $v_{c\theta}$  are separated by the curve  $v_{c\theta} = 0$ . See Araujo and Moreira (2009) for more details.

The following two figures give the optimal coverage ( $co_1, \dots, co_5$ ) and riskiness ( $\pi_1, \dots, \pi_5$ ) for  $\sigma^2 = 2, 2.5, 3, 3.5$  and  $4$ , respectively. Observe that, differently from the usual case, discontinuities may appear when the SCP does not hold (see, for instance,  $co_2$ ). A more complete discussion of this property can be found in Araujo and Moreira (2009) for the monopolistic case. For the competitive case, see the discussion that follows or Araujo *et al.* (2007) (see also Appendix B for more details).

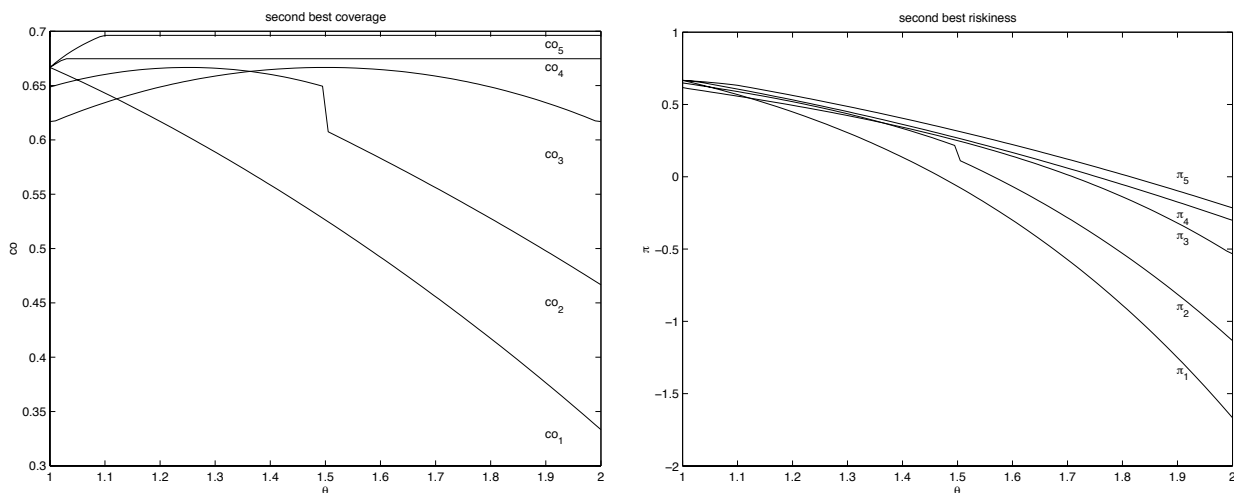


Figure 1: coverage and riskiness for  $\sigma^2 = 2, 2.5, 3, 3.5$  and  $4$ .

## Competitive case

There are at least two identical insurance companies proposing contracts that specify a coverage  $c(\theta)$  and a fixed reimbursement  $\beta(\theta)$  for each reported  $\theta$ . The incentive compatibility and participation constraints remain the same as in the monopolistic case. Moreover, given an equilibrium contract  $(c(\theta), \beta(\theta))$ , the optimal preventive effort of type  $\theta$  is also the same as before:  $e(\theta|\theta)$ , where  $e(\hat{\theta}|\theta) = e(c(\hat{\theta}), \theta)$  is the effort of  $\theta$  when she announces to be type  $\hat{\theta}$ . The main difference from the monopoly is that we impose the actuarially fair or balance constraint on the equilibrium contract due to competition among insurance firms. To support the actuarially fair constraint we will adopt the perfect Bayesian equilibrium (PBE) concept (see Appendix B for formal definition).

**Refinement of equilibria.** Since the concept of PBE leads to indeterminacy an important issue is how to select an equilibrium. Wilson (1977), Rothschild and Stiglitz (1979) and Spence (1979) proposed equilibrium concepts in the context of screening and signaling games. Riley (1979), in a continuous type model, defined the reactive equilibrium that rules out all but a single equilibrium: the Pareto superior separating equilibrium. Since full separability is not possible when the SCP does not hold, we also propose a selection criterion that generalizes the same features of the reactive equilibrium, i.e., separability and Pareto optimality.<sup>14</sup>

**Definition 3.** *A PBE is quasi-separable if the following conditions are satisfied:*

1. *Given a type in a pooling set, there is another pooling type that has the same marginal utilities of coverage.*
2. *There is no other PBE satisfying condition (1) that gives at least the same coverage for every type and a non-zero measure set of types gets higher coverage.*

The first condition of Definition 3 says that if there is pooling, then the marginal utility of coverage must be equalized pairwise. This property is related to the (highest) degree of separability. In particular, only separable or discrete pooling or continuous pooling parts that are degenerated discrete pooling parts are allowed. The second property is the Pareto selection criterion (high coverage). It uniquely pins the equilibrium down by determining the boundary condition of the ordinary differential equation that characterizes the equilibrium. Below we compute the quasi-separable equilibrium for the quadratic cost case.

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<sup>14</sup>Cresta and Laffont (1987) also applied the definition of reactive equilibrium when the SCP holds in the context of insurance market. They discuss the value of information in the classical setup of the Riley (1979) model. The definition that follows is very much inspired in their work.

- low variance ( $0 < \sigma^2 \leq 2$ ): the SCP holds and the quasi-separable equilibrium is decreasing with the highest coverage, i.e., with initial condition  $c(1) = 1$  (full coverage for the lowest type).
- high variance ( $\sigma^2 \geq 4$ ): again the SCP holds and the quasi-separable equilibrium is increasing with final condition  $c(2) = 1$  (full coverage for the highest type).
- intermediate low variance ( $2 < \sigma^2 < 3$ ): the quasi-separable equilibrium presents two parts: discrete pooling on  $[1, \sigma^2 - 1]$  and separation on  $[\sigma^2 - 1, 2]$ ; the highest coverage is the pure moral hazard level at type  $\sigma^2/2$ , i.e.,  $c^{pm}(\sigma^2/2) = 2/3$ . Notice also that the equilibrium coverage has a jump (discontinuity) at the boundary type,  $\sigma^2 - 1$ , between the pooling and separation intervals. The reason is that for each coverage level in the pooling interval the average riskiness in the respective pooling set is greater than the riskiness of the highest risk averse type in this set (since riskiness decreases with risk aversion). Thus, types slightly above  $\sigma^2 - 1$  are fully revealed as high risk averse types having discontinuously lower riskiness and coverage than the average riskiness for types slightly below  $\sigma^2 - 1$ . Another equivalent way to deduce this property is that the coverage must be a function of risk aversion such that the insuree's indirect utility is continuous at  $\sigma^2 - 1$  (see Appendix B).
- intermediate high variance ( $3 \leq \sigma^2 < 4$ ): only low risk averse types are screened out on  $[1, \sigma^2 - 2]$  with final condition  $c(\sigma^2 - 2) = 1$  and constant equal to 1 on  $[\sigma^2 - 2, 2]$  (full coverage for high types).

Under assumption A2 with quadratic cost and uniform distribution, the following figures show the quasi-separable equilibrium coverage  $(c_1, \dots, c_6)$  and riskiness  $(\pi_1, \dots, \pi_6)$

for  $\sigma^2 = 2.25, 2.5, 2.75, 3.25, 3.5$  and  $3.75$ , respectively.

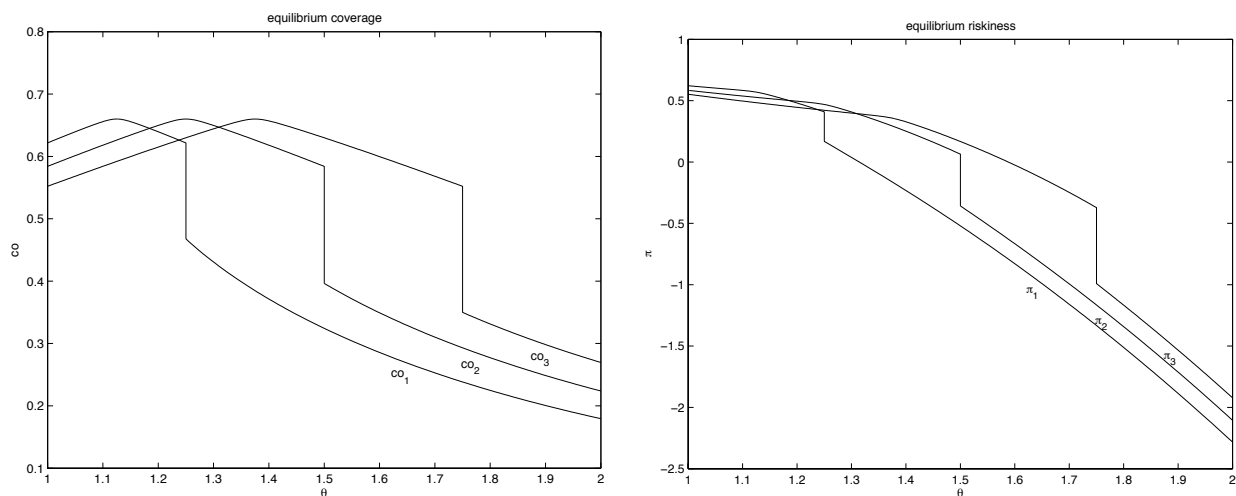


Figure 2a: Equilibrium coverage and riskiness for  $\sigma^2 = 2.25, 2.5$  and  $2.75$ .

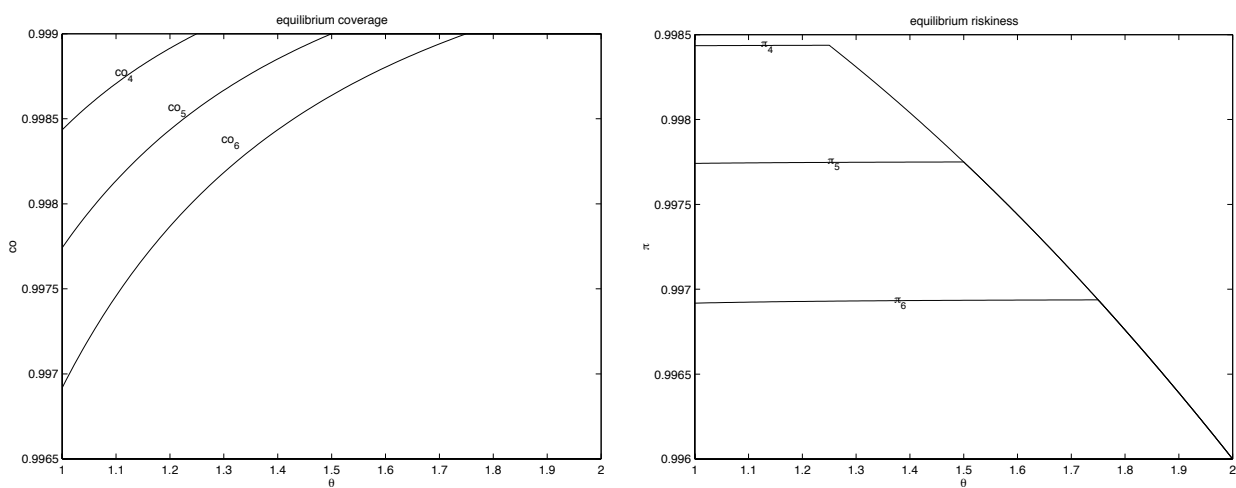


Figure 2b: Equilibrium coverage and riskiness for  $\sigma^2 = 3.25, 3.5$  and  $3.75$ .

Therefore, the equilibrium relation between coverage and riskiness for  $\sigma^2 = 2.25, 2.5$  and  $2.75$  ( $c_1, c_2$  and  $c_3$ , respectively) is given in Figure 3.

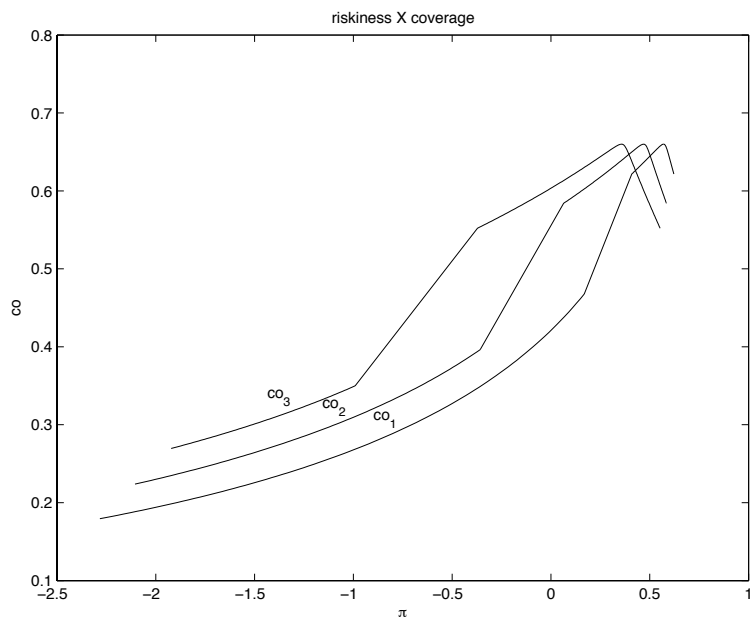


Figure 3: The function  $c(\pi)$  for  $\sigma^2 = 2.25, 2.5$  and  $2.75$ .

We then have the following:

**Implication 2.** *Under negative correlation between risk aversion and prevention cost, if there is moral hazard with adverse selection, then there may be a non-monotonic relation between coverage and riskiness, i.e., the MP may fail.*

Jullien *et al.* (2007) show that under monopoly provision of insurance, the preventive effort can be non-monotonic on risk aversion and in ours preventive effort is always monotonic with risk aversion.<sup>15</sup> We have shown that if there is a negative correlation between risk aversion and prevention cost, coverage may not be a monotonic function of risk aversion although riskiness is always a decreasing function of risk aversion.

De Meza and Webb (2000) are the most close related work. In their paper the less risk-averse agent is risk neutral and does not make any effort, and more risk-averse agents make effort. Due to the non-monetary cost of effort, the SCP does not hold and there may be pooling in equilibrium. However, as their model features only two types of agents, they are

<sup>15</sup>Jullien *et al.* (1999) claim that the monotonicity of the preventive effort is the general case.



unable to consider the emergence of non-monotonicity (there is no variation of contracts in equilibrium). In fact, in their model the presence of (exogenous) administrative costs deters some agents from purchasing (participating) whereas in ours all agents buy positive amounts of insurance. Therefore, the pooling (non-monotonicity result) in our case is due to incentive compatibility whereas in De Meza and Webb's is due to participation constraint.

## 4 Empirical consequences

We now provide the link between the validity of the SCP and the sign of the correlation between coverage and riskiness. The correlation between these variables is

$$\rho = \text{corr}(c, \pi),$$

calculated with the prior distribution of  $\theta$ . By the law of iterated expectations, it is also the correlation of coverage and *ex-post* riskiness:

$$\rho = \text{corr}(c, \pi) = \text{corr}(c, E[\pi|c]).$$

In particular, if the MP is true, then this correlation must be positive. By Implication 1, this the case under a positive relation between risk aversion and prevention cost (assumption A1) whether or not moral hazard follows adverse selection. By Implication 2, this is also true for the pure moral hazard case under negative relation between risk aversion and prevention cost (assumption A2).

Therefore, zero or negative correlation can only happen when there is no asymmetric information (see Implication 0) or there exists moral hazard with adverse selection and the SCP does not hold (see Implication 3). The next proposition show these possibilities are feasible for both the monopolistic and competitive cases.

**Proposition 3** *Assume the presence of moral hazard and adverse selection and A2 with quadratic cost. If risk aversion is uniformly distributed on  $[1, 2]$ , then any sign of the*

correlation between coverage and riskiness is possible for the monopolistic optimal contract and for the competitive quasi-separable equilibrium. In particular, the ex-post riskiness may not be monotonic with respect to coverage.

**Proof.** The figure below gives the correlation between coverage and riskiness,  $\rho_i$ , as a function of the variance  $\sigma^2 \in [2, 4]$  (the interval where the SCP is violated) for the monopolistic optimal contract ( $i = m$ ) and for the competitive quasi-separable equilibrium ( $i = c$ ).<sup>16</sup>

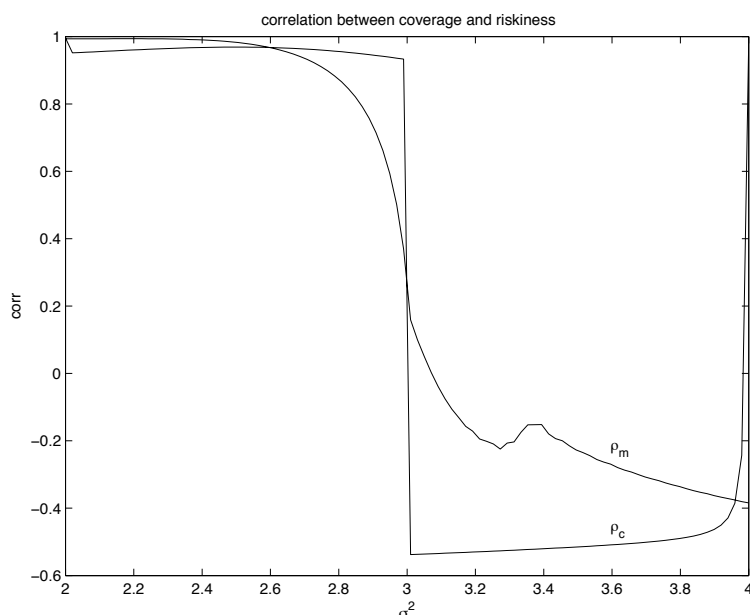


Figure 4: Correlation in the monopolistic and competitive cases.

Under monopoly provision of insurance, this proposition is similar to Jullien *et al.* (1999). However, since coverage is always monotonic on risk aversion in their case, the only way to obtain every sign of correlation between riskiness and coverage is when riskiness is a non-monotonic function of risk aversion. The novelty of our result is for the competitive case.

<sup>16</sup>We did a numerical routine in MATLAB to calculate these functions since there is no close form solution for these correlations.

In the presence of moral hazard and adverse selection, if there is a negative correlation between risk aversion and prevention cost, the SCP cannot hold, leading to (possibly) any sign of correlation between coverage and riskiness. From this perspective, the zero correlation result can be consistent with asymmetric information and non-single-crossing. The following proposition establishes when there is adverse selection in our model under zero correlation result and it is straightforward from the previous discussions.

**Proposition 4** *Suppose that  $A0$ ,  $A1$  or  $A2$  (with the monotonicity assumptions on  $\pi$  of Propositions 1 and 2) hold.*

*(i) Under pure moral hazard, there is a positive relation between coverage and riskiness.*

*(ii) Under moral hazard and adverse selection, if riskiness is monotonic on risk aversion and coverage is non-monotonic on riskiness, then the SCP does not hold. In this case all signs of the correlation between coverage and riskiness may be possible.*

Proposition 4 (i) restates Propositions 1 and 2: if there is full information on risk aversion (pure moral hazard), then there is a monotone relation between coverage and riskiness. Proposition 4 (ii) gives a necessary condition for single-crossing: coverage must be a monotonic function of riskiness.

Therefore, zero correlation can be consistent with the following two cases in our setup: no adverse selection or adverse selection without single-crossing. However, in the case of assumption  $A2$ , the pure moral hazard equilibrium has coverage and riskiness decreasing functions of risk aversion. Thus, in order to disentangle the two cases under zero correlation between coverage and riskiness, it is enough to test the monotonicity of the relation between coverage and riskiness:

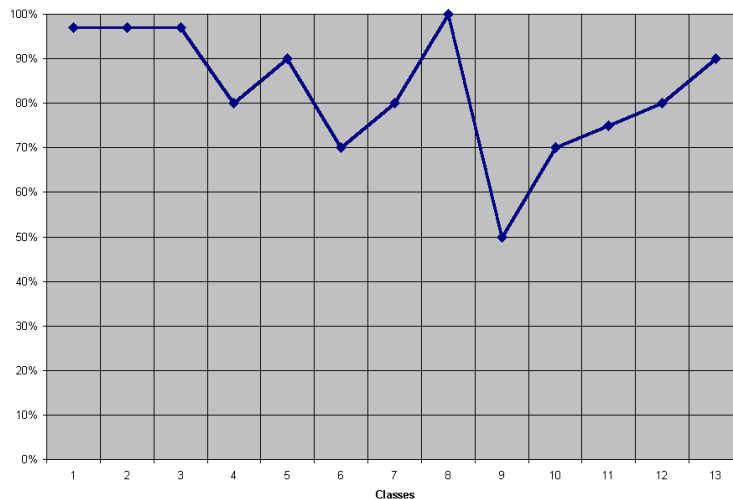
**Proposition 5** *Suppose that  $A0$ ,  $A1$  or  $A2$  (with the monotonicity assumptions on  $\pi$  of Propositions 1 and 2) hold. Under zero correlation between coverage and riskiness, there exist two possibilities: no adverse selection if coverage is monotonic on riskiness*

or adverse selection consistent with non-single-crossing if coverage is non-monotonic on riskiness.

## Casual evidences

As Chiappori and Salanié (2000), Dionne *et al.* (2001) provide a test for MP using data from a large private insurer in Quebec (Canada).<sup>17</sup> They show that when nonlinearity of the risk classification variables are taken into account, the conditional independence of coverage and riskiness cannot be rejected. In a preliminary analysis of the data, they show an histogram of the relation between risk classes and observable deductible choices (contract coverage).

Figure 5 gives the percentage of people that prefer a \$250 deductible (high coverage) instead of a \$500 deductible (low coverage) according to each group of risk (1-13). As we can see, this relation is non-monotonic and there exists zero correlation (conditional independence) between coverage and riskiness.



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<sup>17</sup>Finkelstein and McGarry's (2004) show another empirical evidence that zero correlation between coverage and riskiness does not imply necessarily the absence of adverse selection in the long term insurance market. Fang *et al.* (2008) find evidence of advantageous selection in Medigap insurance market. Conditional to the usual sources of this advantageous selection, they show that although risk preferences are not important, cognitive ability is particularly important source.

Therefore, this relation between coverage and riskiness is non-monotone under zero correlation. Given the model developed in this paper, the casual observation of non-monotonicity can in fact be related with adverse selection and non-single-crossing instead of no adverse selection. However, only a more powerful (statistical) test of the monotonicity property in the relation between coverage and riskiness could distinguish no adverse selection or adverse selection without single-crossing.

## 5 Conclusion

This paper shows, contrary to the standard literature, the possibility of a non-monotonic relation between coverage and riskiness in insurance models. Also, it provides a theoretical insurance model where zero correlation between coverage and riskiness is consistent with asymmetric information. What drives the positive correlation is the SCP. When it is broken, there is a possibility of observing a non-monotonic relationship between coverage and risk aversion and, consequently, between coverage and riskiness when riskiness is monotonic on risk aversion. Moreover, we provide an implication of our model to disentangle single-crossing and non-single-crossing under the zero correlation result: the monotonicity of coverage as a function of riskiness. In particular, as a corollary, we show the necessity of other instruments for controlling the effect of omitted variables when the SCP does not hold. We argue that this procedure can be used as a refinement to check the validity of asymmetric information. Indeed, Dionne *et al.* (2001) is a (potential) casual evidence of the importance of this refinement test.

## Appendix

Using the Envelope Theorem, the derivatives of  $v$  are:

$$\begin{aligned} v_c(c, \theta) &= -e + \theta\sigma^2(1 - c) \\ v_\theta(c, \theta) &= -C_\theta(e, \theta) - \frac{\sigma^2}{2}(1 - c)^2 \\ v_{c\theta}(c, \theta) &= -e_\theta + \sigma^2(1 - c) \end{aligned}$$

where  $e = e(c, \theta)$  and

$$1 - c = C_e(e, \theta).$$

Using the implicit function theorem,

$$\begin{aligned} e_\theta(c, \theta) &= -\frac{C_{\theta e}(e, \theta)}{C_{ee}(e, \theta)}, \\ e_c(c, \theta) &= -\frac{1}{C_{ee}(e, \theta)}. \end{aligned}$$

### Appendix A: monopolistic optimal contract

**Pure moral hazard.** The first order condition of this program gives the well-known characterization of the optimal contract (see Holmstrom and Milgrom, 1987):

$$1 - c(\theta) = \partial_e C(e(c, \theta), \theta) = \frac{1}{1 + \sigma^2 \theta C_{ee}(e(c, \theta), \theta)} \quad (4)$$

$$\beta(\theta) = w_0(\theta) - v(c(\theta), \theta) \quad (5)$$

where (5) is the binding *IR* constraint, i.e., when  $\theta$  is observable the insurance company can condition the contract coverage on risk aversion and make the insuree indifferent between participating or not.

**Moral hazard and adverse selection.** The *IR* constraint is equivalent to  $r(\theta) = \mathcal{V}^{(c, \beta)}(\theta) - w_0(\theta) \geq 0$ , for  $\theta \in [\underline{\theta}, \bar{\theta}]$ . Taking the derivative with respect to  $\theta$  and using (3)

we have:

$$\dot{r}(\theta) = \frac{(1 - c(\theta))e(c(\theta), \theta)}{\theta} - \frac{\sigma^2}{2}(1 - c(\theta))^2 - \frac{e(0, \theta)}{\theta} + \frac{\sigma^2}{2}.$$

Condition (3) and standard arguments of mechanism design literature imply that the monopolist's virtual or relaxed profit function<sup>18</sup> is:

$$\Pi^i(c, \theta) = e(c, \theta) - C(e(c, \theta), \theta) - \frac{\sigma^2}{2} \theta (1-c)^2 + R^i(\theta) \left( C_\theta(e(c, \theta), \theta) + \frac{\sigma^2}{2} (1-c)^2 \right) - \mathcal{V}^{(c, \beta)}(i)$$

where  $R^i(\theta) = \begin{cases} \frac{1-F(\theta)}{f(\theta)} & \text{if } i = 1 \\ -\frac{F(\theta)}{f(\theta)} & \text{if } i = 2 \end{cases}$  is the inverse hazard rate. Depending on the case,  $\mathcal{V}^{(c, \beta)}(\underline{\theta}) = 0$  or  $\mathcal{V}^{(c, \beta)}(\bar{\theta}) = 0$ .<sup>19</sup>

First, let us characterize the relaxed solution, i.e., when we only take into account the first and second order conditions of the *IC* constraint:  $\partial_c \Pi^i(c, \theta) = 0$ , i.e.,

$$1 - c(\theta) = C_e(e(\theta), \theta) = \frac{1 + R^i(\theta) C_{\theta e}(e(\theta), \theta)}{1 + \sigma^2 (\theta - R^i(\theta)) C_{ee}(e(\theta), \theta)}$$

$$\beta(\theta) = \beta(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} v_\theta(c(\tilde{\theta}), \tilde{\theta}) d\tilde{\theta} - v(c(\theta), \theta)$$

where  $e(\theta) = e(c(\theta), \theta)$  and the last equation is obtained solving condition (3).

Under the SCP, only the monotonicity of  $c$  is necessary and sufficient for implementability. If the SCP does not hold, Araujo and Moreira (2009) show another necessary condition for implementability: if  $\theta$  and  $\hat{\theta}$  choose the same coverage level  $\bar{c}$ , then their marginal utilities should coincide, i.e.,  $v_c(\bar{c}, \theta) = v_c(\bar{c}, \hat{\theta})$  or  $e(\bar{c}, \theta) - \theta \sigma^2 (1 - \bar{c}) = e(\bar{c}, \hat{\theta}) - \hat{\theta} \sigma^2 (1 - \bar{c})$ .

Moreover, this implementability condition implies a necessary optimality condition: the ratio between the marginal profit and the marginal informational rent weighted by the density are equalized across pooling types:

$$\frac{\Pi_c(\bar{c}, \theta)}{v_{c\theta}(\bar{c}, \theta)} f = \frac{\Pi_c(\bar{c}, \hat{\theta})}{v_{c\theta}(\bar{c}, \hat{\theta})} \hat{f} \quad (6)$$

where hat means that the function is calculated at  $\hat{\theta}$ .

In some cases these necessary conditions for implementability are also sufficient and we can characterize the optimal contract. This is exactly the case of assumption A2 with

<sup>18</sup>The word 'virtual' means profit discounted by the informational rent. The term 'relaxed' is related to the relaxed program which is the monopolistic insurance problem considering only the first order condition of the IC constraint.

<sup>19</sup>See Jullien (2000) for dealing with countervailing incentives in general.

quadratic cost and uniform distribution of risk aversion. Let us characterize the optimal contract in such case.

First, we have that

$$\dot{r}(\theta) = \left( \frac{\sigma^2}{2} - \theta \right) (1 - (1 - c(\theta))^2).$$

If there is no overinsurance ( $c \leq 1$ ),  $\dot{r}$  changes sign at most once, and then it is only necessary to check the  $IR$  constraint at the boundary values: 1 or 2.

- $\sigma^2 \leq 2$ :  $\dot{r}(\theta) \leq 0$  if and only if  $c(\theta) \geq 0$ . Therefore, since implementable coverages are non-increasing in  $\theta$ ,  $\dot{r}(\theta) \leq 0$  if and only if  $c(2) \geq 0$ . Otherwise,  $r(\cdot)$  is U-shaped with minimum at  $\theta^*$  such that  $c(\theta^*) = 0$ . Using the first order condition of the relaxed program, we have that  $1 - c(2) = \frac{4}{3\sigma^2}$ . Thus,  $c(2) < 0$  if and only if  $\sigma^2 < 3/4$ . The optimal contract is:

$$1 - c^2(\theta) = \frac{1}{-1 + 2\frac{(1+\sigma^2)}{\theta} - \frac{\sigma^2}{\theta^2}}$$

i.e., it is the relaxed solution associated to the hazard function  $R^2$ . Observe that  $c^2$  decreases with  $\theta$  and  $\sigma^2$ .

- $\sigma^2 \geq 4$ :  $\dot{r}(\theta) \geq 0$  if and only if  $c(\theta) \geq 0$ . Therefore, since implementable coverages are non-decreasing in  $\theta$ ,  $\dot{r}(\theta) \geq 0$  if and only if  $c(1) \geq 0$ , which is always the case.

The optimal contract is:

$$1 - c^1(\theta) = \begin{cases} \frac{1}{-1 + 2\frac{(2+\sigma^2)}{\theta} - 2\frac{\sigma^2}{\theta^2}}, & \text{if } 1 \leq \theta \leq \theta_0 \\ 1 - \bar{c}, & \text{if } \theta_0 < \theta \leq 2 \end{cases}$$

where  $\int_{\theta_0}^2 \left[ (1 - C_e(e(\bar{c}, \theta), \theta))e_c(\bar{c}, \theta) + \theta\sigma^2(1 - \bar{c}) + R^1(\theta) (C_{\theta e}(e(\bar{c}, \theta), \theta))e_c(\bar{c}, \theta) - \sigma^2(1 - \bar{c})) \right] f(\theta)d\theta = 0$ ,  $\bar{c} = c^1(\theta_0)$  and  $c^1$  is the relaxed solution associated to the hazard function  $R^1$ . This last equation is the “ironing principle” (see Guesnerie and Laffont, 1984). Also,  $c^1$  non-decreases with  $\theta$  and decreases with  $\sigma^2$ .



For  $\sigma^2 \in [2, 4]$ , Araujo and Moreira (2009) show that  $c$  is implementable if and only if  $c$  is non-decreasing on  $[1, \sigma^2/2]$ , non-increasing on  $[\sigma^2/2, 2]$ , and given  $\theta$  and  $\hat{\theta}$  pooling types (where  $c$  is non flat), then  $\theta$  and  $\hat{\theta}$  are symmetric with respect to  $\sigma^2/2$ . Therefore, the rent function  $r(\cdot)$  is bell shaped and  $r(1) = 0$  if and only if  $c(1) \leq c(2)$ . In this case, the optimal contract is described as follows.

- $\sigma^2 \in [2, 3]$ :

$$c^{SB}(\theta) = \begin{cases} c^u(\theta), & \text{if } 1 \leq \theta \leq \theta_0 \\ c^l(\theta), & \text{if } \theta_0 \leq \theta \leq 2 \end{cases}$$

where  $\theta_0$  is such that  $c^u(1) = c^u(\theta_0)$  and, from (6),  $c^u$  is characterized by

$$\left( \frac{c - \theta\sigma^2(1-c)C_{ee}(e, \theta)}{C_{\theta e}(e, \theta) + \sigma^2(1-c)C_{ee}(e, \theta)} + R^2 \right) f = \left( \frac{c - \hat{\theta}\sigma^2(1-c)C_{ee}(\hat{e}, \hat{\theta})}{C_{\theta e}(\hat{e}, \hat{\theta}) + \sigma^2(1-c)C_{ee}(\hat{e}, \hat{\theta})} + \hat{R}^2 \right) \hat{f}$$

and  $\hat{\theta}$  satisfies  $v_c(c, \theta) = v_c(c, \hat{\theta})$ . For the case we are have that

$$1 - c^u(\theta) = \frac{1 + \left(\frac{\sigma^2}{\theta} - 1\right)^2}{1 + \frac{\sigma^2}{\theta} + \left(\frac{\sigma^2}{\theta} - 1\right)^2 \left(1 + \frac{\sigma^2}{\sigma^2 - \theta}\right) - \frac{4}{\theta^2} \left(\theta - \frac{\sigma^2}{2}\right)}.$$

- $\sigma^2 \in (3, 4]$ :

$$c^{SB}(\theta) = c^1(\theta).$$

## Appendix B: competitive equilibrium contract

Let us first define the concept of competitive equilibrium.

**Definition.** A perfect Bayesian equilibrium (PBE) for the insurance model is a profile of strategies  $\{(c(\theta), \beta(\theta)), e(\cdot|\theta)\}_{\theta \in [\underline{\theta}, \bar{\theta}]}$  and ex-post beliefs  $\mu(\cdot|c)$  such that the following conditions are satisfied:

1. Zero expected profit or actuarially fair constraint:

$$\beta(\theta) = c(\theta) \int e(\theta|\tilde{\theta}) d\mu(\tilde{\theta}|c(\theta)). \quad (7)$$

2. *Maximization of the expected total surplus (best response of the insurees):*

$$\begin{aligned} \theta &\in \arg \max \beta(\widehat{\theta}) + v(c(\widehat{\theta}), \theta) \\ \text{s.t. } &\beta(\widehat{\theta}) + v(c(\widehat{\theta}), \theta) \geq w_0(\theta), \widehat{\theta} \in [\underline{\theta}, \overline{\theta}]. \end{aligned} \quad (8)$$

3. *Consistency of beliefs:  $\mu(\theta|c)$  is the Bayesian updating given the best responses (7) and (8), i.e., it is the posteriori probability of  $\theta$  given  $c$ , whenever it is well defined.*

The following analysis gives all the possible shapes of the equilibria in the competitive case.

- separation:  $\mu(\cdot|c(\theta))$  is a singleton measure concentrated at  $\theta$ . Then, the zero profit condition for the insurance company (7) under full revelation of  $\theta$  for the contract  $(c(\theta), \beta(\theta))$  is

$$\beta(\theta) = c(\theta)e(c(\theta), \theta).$$

The first order condition (8) in the definition of the PBE is:

$$0 = \dot{\beta} + v_c(c, \theta)\dot{c} = (e_c(c, \theta)\dot{c} + e_\theta(c, \theta))c + \theta\sigma^2(1 - c)\dot{c}.$$

Using the previous expression of  $\beta$  we have the following ODE:

$$\dot{c} = \frac{cC_{\theta e}(e(c, \theta), \theta)}{\theta\sigma^2(1 - c)C_{ee}(e(c, \theta), \theta) - c}.$$

- continuous pooling: there exists a non-degenerated interval  $[\theta_1, \theta_2]$  where the equilibrium contract is constant, i.e., every insuree with  $\theta \in [\theta_1, \theta_2]$  chooses the same contract  $(\bar{c}, \bar{\beta})$ . Thus, condition (7) becomes

$$\bar{\beta}(F(\theta_2) - F(\theta_1)) = \int_{\theta_1}^{\theta_2} e(\bar{c}, \tilde{\theta})f(\tilde{\theta})d\tilde{\theta}$$

and (8) and the consistency of beliefs are easily satisfied in the given interval.

- discrete pooling: there are two intervals  $[\theta_1, \theta_2]$  and  $[\widehat{\theta}_1, \widehat{\theta}_2]$  such that for each  $\tilde{\theta} \in [\theta, \theta + d\theta]$ , there is exactly one  $\widehat{\theta} \in [\widehat{\theta} - d\widehat{\theta}, \widehat{\theta}]$  choosing the same coverage  $\bar{c}$ . We have

argued that a necessary condition of the *IC* constraint is  $\partial_c v(\bar{c}, \tilde{\theta}) = \partial_c v(\bar{c}, \hat{\theta})$ . So, let  $\hat{\theta} = \gamma(\bar{c}, \theta)$  be the implicit of the last equation. In other words, given  $\bar{c}$ , there are exactly two types that choose this contract:  $\theta (\leq \sigma^2/2)$  and  $\hat{\theta}$ . Therefore, the updating of  $\theta$  given  $\bar{c}$  is

$$\lambda = \lambda(\bar{c}, \theta) = \Pr(\theta|\bar{c}) = \lim_{d\theta \rightarrow 0} \frac{\Pr(\theta \leq \tilde{\theta} \leq \theta + d\theta)}{\Pr(\theta \leq \tilde{\theta} \leq \theta + d\theta) + \Pr(\hat{\theta} - d\hat{\theta} \leq \tilde{\theta} \leq \hat{\theta})}$$

where  $\hat{\theta} = \gamma(c(\theta), \theta)$  and  $\hat{\theta} - d\hat{\theta} = \gamma(c(\theta + d\theta), \theta + d\theta)$  which implies<sup>20</sup>

$$\lambda = \Pr(\theta|c) = \frac{f(\theta)}{f(\theta) + |\partial_\theta \gamma(c, \theta)| f(\gamma(c, \theta))}.$$

Then, the equilibrium is characterized by  $\beta = c[\lambda e(c, \theta) + (1 - \lambda)e(c, \hat{\theta})]$  and so  $c$  satisfies the following ordinary differential equation (ODE):

$$\dot{\beta} - (e(c, \theta) - \sigma^2 \theta (1 - c)) \dot{c} = 0.$$

Consider the case of A2 with quadratic cost and uniform distribution on  $[1, 2]$ . The separating part of the equilibrium is characterized by the first order condition of (8), i.e., the following ODE - using (7):

$$\frac{\dot{c}}{1 - c} = \frac{2}{h(c, \theta)} \quad (9)$$

where  $h(c, \theta) = \theta - \sigma^2 \frac{1-c}{c}$ .

Observe that the curve implicitly defined by  $h(c, \theta) = 0$  is precisely the pure moral hazard coverage  $c^{pm}$  and the region defined by  $h(c, \theta) > (<) 0$  in the space  $\theta \times c$  is given by  $(\theta, c)$  below (above) the curve  $c^{pm}$ . As we did in the monopolistic model, we analyze several cases according to the variance  $\sigma^2$ :

- low (high) variance, i.e.,  $\sigma^2 \leq 2$  ( $\geq 4$ ): the SCP holds and there is a separating equilibrium if and only if coverage is decreasing (increasing). Therefore, the solution of (9) is a separating equilibrium where its initial (final) condition at  $\theta = 1$  (2) is below (above)  $c^{pm}(1)$  ( $c^{pm}(2)$ ).

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<sup>20</sup>Since  $\gamma(c, \gamma(c, \theta)) = \theta$ , then  $\Pr(\theta|c) + \Pr(\hat{\theta}|c) = 1$ .

- intermediate low variance ( $2 < \sigma^2 < 3$ ): there is no full separation because the SCP does not hold. However, the necessary conditions for incentive compatibility of the monopolistic case are still valid here:  $c$  should be non-decreasing (non-increasing) in the region  $v_{c\theta} > (<) 0$  and if  $\theta$  and  $\hat{\theta}$  are pooling at the same contract  $\bar{c}$  (where it is not flat), then  $v_c(\bar{c}, \theta) = v_c(\bar{c}, \hat{\theta})$  or  $\hat{\theta} = \sigma^2 - \theta$  and the Bayesian updating of  $\theta$  given  $c$  is  $\lambda = 1/2$ . Thus, the first order condition of (8) (using (7)) gives the following ODE that characterizes the discrete pooling part:

$$\frac{\dot{c}}{1-c} = \frac{2}{\frac{\sigma^2}{2\theta - \sigma^2} h(c, \theta) + \theta - \sigma^2}.$$

Since the equilibrium must be bell-shaped, analyzing the sign of this ODE,  $c(\sigma^2/2)$  must be smaller than  $c^{pm}(\sigma^2/2) = 2/3$ . Conversely, if  $c$  is bell-shaped satisfying both ODEs in each case, then  $c$  is an equilibrium.<sup>21</sup>

- intermediate high variance ( $3 \leq \sigma^2 \leq 4$ ): the equilibrium coverage must be separating and then bell-shaped. That is, the equilibrium coverage must be increasing for low risk aversion and this separating part must be in the region below the curve  $c^{pm}$ . However, in this region incentives are overpowered with respect to the pure moral hazard equilibrium. By (9), this separating part would be decreasing which would violate the incentive compatibility. Thus, only equilibria with much (continuous) pooling are possible.
- high variance ( $\sigma^2 > 4$ ): the ODE (9) is not Lipschitzian at the final condition. However, by continuity, the quasi-separable equilibrium can be defined as the limit of a sequence of equilibria with final conditions that converge to 1. Moreover, there is no discontinuity at  $\sigma^2 - 1$  because the average coverage in the pool interval,  $[\sigma^2 - 2, 2]$ , is greater than in the separating interval,  $[1, \sigma^2 - 1]$ .

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<sup>21</sup>A proof of this fact can be adapted from Araujo and Moreira (2001) or Araujo et al. (2007). Observe that it is not a straightforward result because we have to show that the necessary local second order conditions (monotonicity) and the equalization of the marginal utility of coverage across pooling types are sufficient conditions for incentive compatibility.

We assumed uniform distribution on the interval  $[1, 2]$ . However, we can deal with other distributions. Consider for instance the following family of power distributions:

$$f_n(\theta) \cong (\bar{\theta} - \theta)^n$$

where  $n$  is a non-negative real number and  $f_n$  is a density function with support on  $[\underline{\theta}, \bar{\theta}]$ .

The interesting property of this family is that large  $n$  puts more weight on low risk averse types, which reinforces the negative relation between coverage and *ex-post* riskiness.

**Smooth pasting condition:** the continuity of the insuree's indirect utility function at  $\theta_1^* = \sigma^2 - 1$ . There are two possible cases:

- separating to discrete pooling: let us compute the reimbursement for the separating and discrete pooling part at  $\theta_1^*$ , respectively:

$$\begin{aligned}\beta^s(\theta_1^*) &= (1 - c^s(\theta_1^*))c^s(\theta_1^*)\theta_1^{*2} \\ \beta^p(\theta_1^*) &= (1 - c^p(\theta_1^*))c^p(\theta_1^*)\left(\frac{1 + \theta_1^{*2}}{2}\right),\end{aligned}$$

where the last value is the mean between the two pooling types. The smooth pasting condition is then

$$\beta^s(\theta_1^*) + v(c^s(\theta_1^*), \theta_1^*) = \beta^p(\theta_1^*) + v(c^p(\theta_1^*), \theta_1^*).$$

- separating to continuous pooling: let us compute the reimbursement for the continuous pooling part at  $\theta_1^*$  (for the separating part it is the same):

$$\beta^p(\theta_1^*) = (1 - c^p(\theta_1^*))c^p(\theta_1^*) \int_{\theta_1^*}^1 \frac{\theta^2}{3 - \sigma^2} d\theta,$$

where the integral represents the expected value with the uniform distribution. Since  $c^p(\theta_1^*) \simeq 1$ , then  $\beta^p(\theta_1^*) \simeq 0$  which implies that  $c^s(\theta_1^*) = c^p(\theta_1^*)$ .

## Appendix C

We provide a sufficient condition for the monotonicity condition in the statement of Propositions 1(i) and 2.

**Lemma.** *Assume that A0 holds and  $C_{eee} > 0$ .*

*(i) If  $C_{e\theta} > 0$  and  $C_{ee\theta} > 0$ , then  $\pi_\theta > 0$ .*

*(ii) If  $C_{e\theta} < 0$  and  $\theta C_{ee} + C_{ee\theta} < 0$ , then  $\pi_\theta < 0$ .*

**Proof.** (i) Taking the second equality of (4), we have that the left hand side increases with  $e$  and  $\theta$  and the right hand side decreases with them. This immediately implies that the implicit solution of this equation,  $e(c, \theta)$ , is non-increasing in  $\theta$ . Therefore,  $\pi(c, \cdot)$  is a non-decreasing function.

(ii) The proof is analogous to (i).

## References

- Abbring, J., Heckman, J., Chiappori, P.-A., Pinquet, J., 2003. Adverse Selection and Moral Hazard in Insurance: Can Dynamic Data Help to Distinguish? *Journal of the European Economic Association*, 1(2,3): 512-21.
- Abbring, J., Chiappori, P.-A., Pinquet, J., 2003. Moral Hazard and Dynamic Insurance Data. *Journal of the European Economic Association*, 1(4): 767-820.
- Araujo, A., Moreira, H., 2009. Adverse Selection Problems without the Spence-Mirrlees Condition. *Forthcoming Journal of Economic Theory*.
- Araujo, A., Gottlieb, D., Moreira, H., 2007. A Model of Mixed Signals with Applications to Countersignaling. *RAND Journal of Economics*, 38: 1020-43.
- Chassagnon, A., Chiappori, P.-A., 1997. Insurance under Moral Hazard and Adverse Selection: The case of Pure Competition. *Manuscript* Paris: Département et Laboratoire d'Économie Théorique et Appliquée.
- Chiappori, P.-A., 2000. Econometric Models of Insurance under Asymmetric Information. In *Handbook of Insurance*, G. Dionne, ed.

- Chiappori, P.-A., Salanié, B. 2000. Testing Asymmetric Information in Insurance Markets. *Journal of Political Economics*, 108: 56-78.
- Chiappori, P.-A., Jullien, B., Salanié, B., Salanié, F., 2006. Asymmetric Information in Insurance: Some Testable Implications. *RAND Journal of Economics*, 37(4): 783-98.
- Cresta, J.-P., Laffont, J.-J., 1987. Incentive Compatibility of Insurance Contracts and the Value of Information. *The Journal of Risk and Insurance*, 54, 520-40.
- De Meza, D., Webb, D., 2001. Advantageous Selection in Insurance Markets. *RAND Journal of Economics*, 32(2): 249-62.
- Dionne, G., Gouriéroux, C., Vanasse, C., 2001. Testing for Evidence of Adverse Selection in the Automobile Insurance Market: A Comment. *Journal of Political Economics*, 109: 444-453.
- Finkelstein, A., McGarry, K., 2006. Multiple dimensions of private information: evidence from the long-term care insurance market. *American Economic Review*, 96(4): 938-958.
- Gardiol, L., Geoffard, P.-Y., Grandchamp, C., 2006. Separating Selection and Incentive Effects: an Econometric Study of Swiss Health Insurance Claims Data. In *Competitive Failures in Insurance Markets Theory and Policy Implications*, P.-A. Chiappori and C. Gollier, ed.
- Guesnerie, R., Laffont, J.-J., 1984. A Complete Solution to a Class of Principal-Agent Problems with an Application to the Control of Self-Managed Firms. *Journal of Public Economics*, 25: 329-69.
- Hellwig, M., Schmidt, K., 2002. Discrete-Time Approximations of the Holmstrom-Milgrom Brownian-Motion Model of Intertemporal Incentive Provision. *Econometrica*, 70(6): 2225-64.
- Holmstrom, B., Milgrom, P., 1987. Aggregation and Linearity in the Provision of Intertemporal Incentives. *Econometrica*, 55: 303-28.
- Jullien, B., 2000. Participation Constraints in Adverse Selection Models. *Journal of*

*Economic Theory*, 93: 1-47.

Jullien, B., Salanié, B., Salanié, F., 2007. Screening Risk-averse Agents Under Moral Hazard: single-crossing and the CARA case. *Economic Theory*, 30(1): 151-169.

Jullien, B., Salanié, B., Salanié, F., 1999. Should More Risk-Averse Agents Exert More Effort. *The Geneva Papers on Risk and Insurance Theory*, 19: 14-18.

Laffont, J.-C., Martimort, D. 2002. *The Theory of Incentives - The Principal-Agent Model*. Princeton University Press.

Landsberber, M. Meilijson, I., 1999. A General Model of Insurance under Adverse Selection. *Economic Theory*, 14: 331-52.

Puelz, R., Snow, A., 1994. Evidence on Adverse Selection: Equilibrium Signaling and Cross Subsidization in the Insurance Market. *Journal of Political Economics*, 102: 236-57.

Riley, J., 1979. Informational Equilibrium. *Econometrica*, 47(2): 331-59.

Rothschild, M., Stiglitz, J., 1976. Equilibrium in Insurance Markets: An Essay on the Economics of Imperfect Information. *Quarterly Journal of Economics*, 90: 629-49.

Smart, M., 2000. Competitive Insurance Markets with Two Unobservables. *International Economic Review*, 40 (1): 153-69.

Sung, J., 1995. Linearity with project selection and controllable diffusion rate in continuous-time principal-agent problem. *RAND Journal of Economics*, 26(4): 720-43.

Sung, J., 2005. Optimal contracts under moral hazard and adverse selection: a continuous-time approach. *Review of Financial Studies*, 18(3): 1021-73.

Villeneuve, B., 1997. Random Contracts and Positive Profits in an Insurance Market under Adverse Selection. Working Paper, Institut D'Economie Industrielle.

Wilson, C., 1977. A Model of Insurance Markets with Incomplete Information. *Journal of Economic Theory*, 16: 167-207.