On the Limits of Cheap Talk for Public Good Provision

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Abstract

We study a situation where, as a consequence of private information, agents do not have the incentive to invest in a public good if they are unable to have prior discussion with their partners. We show that in a voluntary contribution mechanism with cheap talk, any finite message space does not provide efficiency gain relative to a binary message space when agents truthfully report their type. Using laboratory experiments, we observe that communication is a useful mechanism to enhance efficiency. However, we find that such efficiency gain seems to be very limited, since the simplest communication structure (“Yes” or “No” messages) provides the same incentives to the agents as larger message spaces.

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1 Introduction

A recurrent issue in Economics is understanding when the collection of individual choices is able to generate the socially desired outcome, such as the provision of a public good or the implementation of a joint project. The literature shows that under asymmetric information or coordination failure, individual actions may lead to allocations that yield suboptimal outcomes. However, it has been shown that communication, an inherent aspect to human relations, can be helpful in aligning individual incentives to social ones, i.e., one can enhance efficiency by allowing subjects to use costless and non-binding messages.

Looking at economic relations, it is unlikely that agents would commit resources to a joint relationship without prior discussion with their partners. For example, no entrepreneur would invest in a project when he does not have information about his partner’s intended contributions. Similarly, two parties would not engage in building an alliance without any sign of commitment by the other party. In both these situations, a unilateral decision would be too risky for agents and communication arises as a coordination device to reduce the risk inherent in the relationship and, possibly, to increase efficiency.

This article studies a situation where, as a consequence of private information, agents do not have an incentive to invest in a public good if they are unable to have prior discussions with their partners. It has been shown (Agastya et al. 2007) that by adding a prior stage of communication (binary message space) there exist situations where individuals have incentives to invest and the project is undertaken. This provides some efficiency gain relative to the no communication game. Unless stated otherwise, efficiency is measured by the probability of the project being undertaken. We show, however, that gains from communication are very limited since any finite message space does not increase the probability of the project being implemented relative to the binary message space when agents report their types honestly. The intuition behind this result

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is that, since agents use a threshold rule to contribute, in any situation with more than one contribution threshold, they have incentives to understate their types and thereby free-ride on their partners’ investment. Hence, there is no truthful strategy profile that supports a Pareto dominant equilibrium in a more refined message space. Communication is then a coordination device that signals whether or not players should invest. We use experiments to test all these results.

The model we follow belongs to both the public goods and the strategic communication literature that was developed from Crawford and Sobel’s (1982) seminal paper. The authors showed that even self-interested agents may choose to reveal information through costless and non-binding communication, also known as cheap talk. In a voluntary contribution mechanism environment, we model communication as being a one-period, simultaneous-move game where both players choose their messages from a finite set of possibilities. Our framework allows agents not only to reveal private information but also to declare their intentions. In this sense, our model also is related to the literature that uses communication purely as a coordination device.

Using laboratory experiments, we observe that people do contribute to the joint project even in the no communication game, and that communication does not increase the probability of a project being implemented. However, we observe that communication provides efficiency gain through a relevant variable not directly modeled: the unproductive contribution which is reduced. Also, we find evidence that a larger message space does not provide any efficiency gain relative to the binary one.

The paper is organized as follows, after discussing the related literature, we introduce the model and present the theoretical results in Section 2. Section 3 presents the experimental design and the hypotheses to be tested. In Section 4, we discuss the laboratory results and analyze subjects’ behavior. Concluding remarks are made in Section 5. Proofs are presented in the Appendix.

Related literature

The incentive conflicts behind issues of public interest that are decided by a collection of individual choices intrigue social scientists not only by their practical relevance, but also because of what they reveal about human nature. Holmstrom (1982) showed that a social optimum may not be achieved by an equilibrium in a non-cooperative game, suggesting that people following their individual interests try to free-ride on their partners. Menezes et al. (2001) presented a similar result for discrete public goods in the presence of incomplete information. In a common agency game with private information, Martimort and Moreira (2010) also find that the social optimum is not achieved in equilibrium, however not because of free riding but because of the screening effect between informed principals.

Experiments corroborate these inefficiency results and show that people behave in ways that produce results between the social optimum and the individual interest (Marwell and Ames, 1979), and that repetition reduces the cooperation between individuals (Isaac et al., 1985). Looking at coordination games, van Huyck et al. (1990, 1991) and Cooper et al. (1990) showed that even with perfect information, individuals fail to select the payoff-dominant equilibrium. Since these articles were published, much work has led to the identification of factors, one of which is communication, that can enhance the provision of public goods. Ledyard (1994) provides a good survey of this topic.

Next, we discuss the theoretical models of communication on public good provision and related games, and then we present some empirical findings. After the publication of Crawford and Sobel (1982), a series of extensions and applications of their model were published. Gilligan and Krehbiel (1987) used a similar model to study information theory of legislative rules and Austen-Smith (1990) found that information can be fully aggregated by communication when preferences are sufficiently close, a finding close to Crawford and Sobel’s results.

In contrast to the above models, Palfrey and Rosenthal (1991) used an environment where both players have private information and decision power. In a binary setting, they showed that perfect coordination is not Bayesian-incentive compatible and that agents have weak incentives to free-ride in this kind of game. Doraszelski et al. (2003) used a similar model to analyze the substitutability between communication and voting. The authors found that both are perfect substitutes, or intuitively, that communication works as a pre-play version of the decision game where agents have zero payoff in the first round of the game.

Our basic model departs from a similar structure which uses two privately informed agents who can contribute continuously, as in Agastya et al. (2007). The authors showed that without communication, agents do not have an incentive to contribute to the public good and that binary communication gives them
incentive to provide the public good with positive probability.

These cheap talk results are not limited to the voluntary contribution mechanism (VCM) literature. Farrell (1987) showed that cheap talk can achieve partial coordination in a game similar to the “battle of the sexes”. Forges (1990) used a job market application to show that the set of payoffs found in an equilibrium of the cheap talk game is larger than in the original game. Baliga and Sjostrom (2004) used an arms race context to show that cheap talk helps players to coordinate their action in reaching the payoff dominant equilibrium of a “stag hunt” game.

Given the difficulty of testing these models with field data, several authors had undertaken laboratory experiments to help understand the role of cheap talk. Isaac and Walker (1988) added a communication stage to a VCM model without threshold and observed that communication reduced the free-rider behavior. Closer to our article, Menezes and Saraiva (2005) studied the effects of binary communication and a refund scheme in a threshold VCM model similar to Menezes et al. (2001), and found that only the refund rule increased contribution. Palfrey and Rosenthal (1991) tested their theoretical findings with an experiment where both messages and actions are binary. In this sense, the public good threshold is not on the contributed amount (as in our case) but on the number of contributing subjects. Their experiments supported the no communication equilibrium and suggested that subjects use a cutoff decision rule when it is optimal to do so. However, they observed that communication failed to provide efficiency gain and that players’ behavior in the communication stage was very non systematic.

Cai and Wang (2006) tested the Crawford and Sobel (1982) model in a discrete setting (states, messages, and action). Their results supported the finding that communication leads to efficiency gain and that the closer the preferences of principal and agent, the more the agent transmits information. The authors observed that subjects over-communicate as message senders, in the sense that messages are more informative than the model predicts. Also, they found that messages are more informative to receivers, since they rely on them more than expected. This “truth bias” on communication also was observed by Kawagoe and Takizawa (2009) relative to a level-k model.1

Some papers went beyond the theoretical models and studied how face-to-face communication affects public good provision, such as Cason and Khan (1999), and Belianin and Novarese (2005). As is usual in experiments with this kind of treatment, both found that communication increases cooperation. Bochet et al. (2006) addressed this issue by designing an experiment with three types of communication in a standard VCM model: face-to-face communication, anonymous chat room, and numerical messages. They found that verbal communication (i.e., face-to-face and chat room) strongly increased cooperation, while the numerical messages affected neither efficiency nor contributions.

In the coordination literature, Cooper et al. (1989) did not observe coordination failure on a “battle of the sexes” game with cheap talk. In a later work, Cooper et al. (1992) showed that one-way communication or two-way communication can have different effects depending on whether the game has a Pareto-dominant strategy. Other experiments have studied cheap talk in various contexts such as bargaining. Crawford (1998) is a good survey of experiments on cheap talk.

2 The model

We model the investment choice of two players participating in a joint project, similarly to Agastya et al. (2007).2 Players have an initial endowment \( w_i \), and each player has to decide privately how much to contribute, \( c_i \), to a joint project. This project is undertaken when the sum of the contributions is greater than or equal to the project cost \( k \), i.e., the joint project is undertaken if \( c_1 + c_2 \geq k \).

Each player \( i \) is given his own valuation for the joint project, which is private information. The project value for each player is a random variable with a continuous distribution function \( F_i \) independently distributed in the interval \( [v_{i}, \bar{v}_i] \), where \( 0 \leq v_i < \bar{v}_i \).

An agent’s payoff is a function of his project value, the amount invested by him in the joint project, \( c_i \), and the total amount invested in the project:

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1 The level-k model is a non-equilibrium model of agents’ behavior, reflecting strategic thinking. Its main point is to impose structure on the set of agents’ beliefs.

2 We will use the terms joint project or public good, and the terms investment or contribution interchangeably throughout the text since the model fits both terms in both cases.
\[
U(v_i, c_i, c_{-i}) = \begin{cases} 
 v_i + w_i - c_i & , 	ext{if } c_1 + c_2 \geq k \\
 w_i - c_i & , \text{if } c_1 + c_2 < k.
\end{cases}
\]  

The benefit of not undertaking the project is normalized to zero. We assume that it may be advantageous for players to undertake the project, i.e., \( \pi_1 + \pi_2 > k \) (otherwise the problem would not make sense). On the other hand, if \( \pi_1 + \pi_2 \geq k \), it is always ex post efficient to implement the project, and we have a class of equilibria such that the public good is always provided and the private information is irrelevant. So, in order to make the private information critical, we assume that \( \pi_1 + \pi_2 < k \). Since our interest is in studying the interaction between agents, we want each player to be pivotal for the project, i.e., neither agent should have the incentive to implement the project individually. To avoid equilibria in which a player with a sufficiently high value individually provides the public good, we assume that \( \pi_i < k \) for \( i = 1, 2 \).

2.1 The contribution game

First we analyze the game without communication between players. It is a one stage game where each player observes his assigned project value and decides how much to invest in the joint project. A pair of functions \( C_i : [\pi_i, \pi_i] \rightarrow \mathbb{R}_+ \), \( i = 1, 2 \), where \( C_i(v_i) = \arg \max_{c_i} E_{-i} [U_i(c_i, v_i, C_{-i}(v_{-i})|v_i)] \) is said to be an equilibrium of this game. We define the following special strategies \( C_i(\cdot) \):

**Definition 1** A function \( C_i : [\pi_i, \pi_i] \rightarrow \mathbb{R}_+ \), \( i = 1, 2 \), is said to be a threshold strategy if \( \exists x_i > 0 \) and \( \hat{v}_i \in [\pi_i, \pi_i] \) such that

\[
C_i(v_i) = \begin{cases} 
 x_i & , \text{if } v_i > \hat{v}_i \\
 0 & , \text{otherwise}.
\end{cases}
\]

We say that an equilibrium is a cost sharing equilibrium if both players use a threshold strategy. Agastya et al. (2007) proved that a necessary and sufficient condition for the existence of a cost sharing equilibrium is that a pair of players’ types exists such that the sum of expected benefits exceeds the project’s costs, i.e., there exists \( \hat{v}_i \in [\pi_i, \pi_i] \), \( i = 1, 2 \), such that

\[
(1 - F_2 (\hat{v}_2)) \hat{v}_1 + (1 - F_1 (\hat{v}_1)) \hat{v}_2 \geq k. 
\]

We are interested in understanding how communication can increase the probability of a project being undertaken. So, we look at the extreme case where the only equilibrium of the contribution game is totally inefficient, i.e., the project is never undertaken. In order to do so, we assume that the priors are concave, i.e., \( F_i (\cdot) \) is concave for \( i = 1, 2 \). Agastya et al. (2007) state the following proposition:

**Proposition 1** (Threshold Strategy) If \( F_i (\cdot) \) is concave for \( i = 1, 2 \), then in the unique equilibrium of the contribution game, each player makes a zero contribution. The project is never undertaken.

The intuition of this result is that when \( F_i (\cdot) \) is concave, the left hand side of condition (2) is convex and its maximum is achieved in one of the distribution’s support corners \( (\pi_1, \pi_2) \), \( (\pi_1, \hat{v}_2) \), \( (\pi_1, \pi_2) \) or \( (\pi_1, \pi_2) \), \( (\pi_1, \pi_2) \).

Evaluating (2) in these four cases one can see that this condition is never satisfied.\(^3\)

2.2 The contribution game with cheap talk

One way to address the extreme result of Proposition 1 is by allowing players to have a prior discussion about their intentions. We do this by introducing a prior communication stage to the game. This new stage makes the equilibrium set larger, which allows us to look for the existence of more efficient equilibria. The timing of the game is the following:

1. Nature plays and \( v_i \) is realized, \( i = 1, 2 \);
2. Players observe their type \( v_i \) and send messages \( m_i \), \( i = 1, 2 \);
3. Players observe their partner’s message \( m_{-i} \) and contribute \( c_i \), \( i = 1, 2 \);

\(^3\)See Proposition 6 in Agastya et al. (2007) for the complete proof.
4. Players receive their payoffs.

The communication consists of each player sending a message \( m_i \) to his partner. Differently from Agastya et al. (2007) we do not restrict messages to be binary. We follow Crawford and Sobel (1982) and define the message space as a finite partition of the interval \([v_1, v_2]\). The idea is that each player can send a signal indicating his type or his intention, by communicating an element of the message space. Technically, \( m_i \in \Lambda_i \), where \( \Lambda_i \in \Lambda^i \). The collection of finite partitions \( \Lambda^i, i = 1, 2 \), is defined by:

\[
\Lambda^i \equiv \left\{ \left( (a_{h-1}, a_h) \right)_{h=1}^{n-1} \right\} : n \in \mathbb{N} \text{ and } v_i \equiv a_0 < a_1 < \ldots < a_n \equiv v_i \}
\]

So, we have changed the single-stage contribution game to a two-stage game with cheap talk communication in which players may send any message independent of their assigned project valuation. Since there is no commitment in the first stage, players also can choose freely in the contribution stage despite any exchanged messages. Let this two-stage game be denoted by \( \mathcal{C}(\Lambda_1, \Lambda_2) \), where \( \Lambda_1 \) and \( \Lambda_2 \) are the two message sets. The equilibrium concept of this game is the Perfect Bayesian Equilibrium. We say that an equilibrium is \textit{truthful} if the players reveal themselves truthfully in the communication stage, i.e., a player sends a message \( m_i \) only if his value belongs to this interval.

For example, if both value distributions have support \([0, 100]\), a possible message space is \( \Lambda = \{[0, 50], [50, 100]\} \). This means that players can communicate \( m_i = [0, 50] \) or \( m_i = [50, 100] \). As communication is cheap talk, the player can send either message, independent of their actual type \( v_i \).

As we will show, players will play a relatively simple strategy in all equilibria of interest. It will be helpful to have this strategy in mind throughout our discussion, so we define it in advance:

**Definition 2** (Threshold Strategy Profile of \( \mathcal{C}(\Lambda_1, \Lambda_2) \))\(^4\) Let \( \Lambda_1 \) and \( \Lambda_2 \) be partitions respectively of \( n \) and \( m \) elements. A strategy profile \( \Gamma \) of the game \( \mathcal{C}(\Lambda_1, \Lambda_2) \) is said to be a threshold strategy if

1. At the communication stage, messages are given by functions \( m^i : [v_i, v_i] \rightarrow \Lambda_i, i = 1, 2 \) such that

   \[
   \begin{align*}
   m^1(v_1) &= [a_{i-1}, a_i] \in \Lambda_1, i \leq n, \text{ if } v_1 \in [a_{i-1}, a_i]; \\
   m^2(v_2) &= [b_{j-1}, b_j] \in \Lambda_2, j \leq m, \text{ if } v_2 \in [b_{j-1}, b_j];
   \end{align*}
   \]

2. At the contribution stage, contributions are a function \( C : [v_1, v_2] \times [v_1, v_2] \times \Lambda_1 \times \Lambda_2 \rightarrow \mathbb{R}_{+}^2 \) such that

\[
C(v_1, v_2, m^1, m^2) = \begin{cases} 
(a_{i-1}, b_{j-1}), & \text{if } \exists i, j \text{ s.t. } a_{i-1} + b_{j-1} = k \text{ and } v_1 \geq a_{i-1} \text{ and } v_2 \geq b_{j-1}, \\
(0, 0), & \text{otherwise}
\end{cases}
\]

where we denote \( m_1^i = [a_{i-1}, a_i] \in \Lambda_1 \) and \( m^2_j = [b_{j-1}, b_j] \in \Lambda_2 \) for simplicity.

In other words, in the threshold strategy players truthfully reveal their types at the communication stage and, at the contribution stage, players contribute zero unless it exists \((i, j)\) such that \( a_{i-1} + b_{j-1} = k \) and both announced \((m_1^i, m_2^j)\) such that \( i \geq \hat{i} \) and \( j \geq \hat{j} \). Note that the strategy at the contribution stage is exactly the threshold strategy of Definition 1 with \( \hat{v}_1 = x_1 = a_{\hat{i}-1} \) and \( \hat{v}_2 = x_2 = b_{\hat{j}-1} \). We say that an equilibrium is a \textit{threshold equilibrium} if it is supported by threshold strategy profiles. The following lemma supports our main result.

**Lemma 1** Any truthful equilibrium of \( \mathcal{C}(\Lambda_1, \Lambda_2) \), \( \Lambda_1 \in \Lambda^1 \) and \( \Lambda_2 \in \Lambda^2 \), where players make non-zero contributions is supported by a strategy profile \( \Gamma \) only if \( \Gamma \) is a threshold strategy profile.

Figure 1 help us understand Lemma 1. By backward induction, we first look at the contribution stage assuming that players truthfully revealed their types at the communication stage, and then we determine whether truthful strategies were optimal in the first stage. A strategy profile of the communication game must have a contribution rule that says how much to contribute after every message history. In equilibrium,
the sum of contributions must equal the project cost, otherwise players would be better off reducing their contributions.

Looking at Figure 1, suppose this contribution rule says that player 1 after some message, for example \((m^3_1, m^2_2)\), must contribute \(a_2 + \varepsilon\), where \(\varepsilon > 0\). Since in equilibrium the sum of contributions must be equal \(k\), Player 2 must contribute \(k - (a_2 + \varepsilon) = b_1 - \varepsilon\). Hence, all players with type \((b_1 - \varepsilon, b_1)\) have no incentive to communicate their type truthfully.

Now, suppose the contribution rule says that player 1 must contribute \(a_1 - \varepsilon\), where \(\varepsilon > 0\), following some message history \((m^1_2, m^3_2)\). Hence, players with type in \((a_1 - \varepsilon, a_1)\) will have no incentive to truthfully communicate their type. Consequently, for any contribution rule consistent with truthfulness and positive contribution, we have that \(\varepsilon = 0\) for some message history \((m^1_1, m^2_2)\), i.e., the contribution must follow the partition cutoffs.

Also, we show that any truthful equilibrium can have at most one positive contribution level for each player. If players were to contribute \((a_1, b_2)\) following history \((m^1_1, m^2_2)\) and \((a_2, b_1)\) following history \((m^1_3, m^2_2)\), contributing \(a_2\) and \(b_2\) would give incentive to under-report their types. That is, by truthfully communicating \(m^1_3\), contributing \(a_2\) would have zero expected utility, while he would have strict positive expected utility by communicating \(m^2_3\).

Therefore, there exists a contribution rule consistent with truthful revelation and positive contribution if and only if the message space \((\Lambda_1, \Lambda_2)\) is such that there exists a pair of thresholds such that \(a_{i-1} + b_{j-1} = k\). In this case for each such pairs \((i, j)\), there is only one equilibrium with positive contribution in which \((a_{i-1}, b_{j-1})\) are the contribution threshold. In our example this means that players contribute \((a_1, b_2)\) after histories \((m^1_1, m^2_2)\) or \((m^1_3, m^2_2)\), if cutoffs \((a_1, b_2)\) are used as contribution thresholds, or they contribute \((a_2, b_1)\) after histories \((m^1_3, m^2_2)\) or \((m^1_1, m^2_2)\), if cutoffs \((a_2, b_1)\) are used as contribution thresholds. In all other equilibria, contributions are always zero and the project is never undertaken. So, Lemma 1 allows us to state the following result:

**Proposition 2** Given a pair of partitions \((\Lambda_1, \Lambda_2)\), \(\Lambda_1 \in \Lambda^1\) and \(\Lambda_2 \in \Lambda^2\):

1. There exists a truthful equilibrium of \(\mathcal{C}(\Lambda_1, \Lambda_2)\) where the project is undertaken if and only if there exist \(i \leq n\) and \(j \leq m\) such that \(a_{i-1} + b_{j-1} = k\).

2. For each such pair \((i, j)\) there exists a unique non-trivial truthful equilibrium. The positive contributions in this equilibrium is \((a_{i-1}, b_{j-1})\).

Proposition 2 makes a very strong statement by saying that a truthful equilibrium of the communication game with a binary message space cannot be Pareto dominated by any truthful equilibrium with any finite message space. The intuition is that communication provides efficiency gain only if it allows players to send two clear messages: “Yes, I intend to contribute”, when \(m^1_i\) and \(i\) is greater than a threshold, or “No, I do not intend to contribute”, when \(m^1_i\) and \(i\) is less than a threshold. The proposition is even more restrictive, since it affirms that the “Yes” message is used only when the message sets are such that there exists a pair \((a_{i-1}, b_{j-1})\) where \(a_{i-1} + b_{j-1} = k\). Therefore, cheap talk improves efficiency relative to the game without communication, however this improvement is restricted to the simplest communication form, the binary one.

It is important to highlight that the trivial equilibrium, where players ignore the message stage and contribute zero afterwards, always exists. This implies that when the message sets are such that in a threshold equilibrium the project may be implemented with positive probability, the game has two Pareto ranked equilibria: the threshold and the trivial. This multiplicity of equilibria will be very important when we discuss the laboratory data.

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The simple strategy of Agastya et al. (2007) is a special case of this definition when \(n = m = 2\), \(a_1 = x\), and \(b_1 = k - x\). If \(i = n\) or \(j = m\), we are implicitly assuming that the interval is closed.
3 Experimental design

We ran a total of six sessions, two sessions for each of the following treatments: no communication, cheap talk with a binary message space, and cheap talk with a refined message space. The experiment was programmed and conducted with the software z-Tree, Fischbacher (2007). We used the same two sets of values taken from a uniform distribution and the same group matching in all the sessions.

We had 16 subjects in each session, totaling 96 participants. Subject recruitment was made through the Experimental Economic Center’s website. All participants were inexperienced undergraduate students at Getulio Vargas Foundation majoring in courses such as Economics, Business, Law, Social Science, and History. Each participant received US$4.90 to show up plus the session’s earnings.\(^5\) Sessions lasted approximately 75 minutes each, including the payment time.

Each session consisted of 25 decision rounds with no practice periods. We used the strangers protocol in which, in each period, new groups are formed randomly in a manner such that agents never know the identity of their partners.

During the experiment, the decision to invest in the joint project was presented as an allocation decision. Any reference to social contribution was avoided in order to not bias the agent’s decisions. The subject’s task was to choose the amount of his initial endowment to allocate to a Group Account and how much of his endowment to retain. A detailed payoff table was presented on the subjects’ computer screens in all decision stages to help them estimate their potential earnings as a function of their private information and their possible actions.\(^6\)

At the beginning of each period, the subjects were randomly assigned to new pairs. Each subject received a non-storable endowment of 100 tokens in each period, i.e., \(w = 100\). The subject’s Group Account valuation, \(v_i\), was displayed on his computer screen. This value was private information and the subject only knew that the Group Account value of the other player was uniformly distributed between 0 and 100. A subject’s payoff was defined earlier by equation (1). The Group Account threshold was \(k = 100\).

At the end of each period, after all participants had made their decisions, it was presented in the results screen: (i) if the total allocation to the Group Account was greater than or equal to the threshold, (ii) the amount allocated by the player to the Group Account, (iii) the two messages exchanged, and (iv) his profit in that period. The history of these informations was always available on the computer screen.

3.1 The no communication treatment

This is the baseline treatment, relative to the contribution game presented in Section 2.1. In each period, subjects played a one stage game where they observed their respective Group Account value, \(v_i\), and simultaneously decided the amount, \(c_i\), they wanted to allocate to the Group Account.

The following hypothesis concerns Proposition 1.

**Hypothesis 1** In equilibrium, each subject allocates zero to the Group Account, i.e., the project is never undertaken.

3.2 The binary communication treatment

The participants now played a two-stage game. In the first stage, each subject observed his Group Account value, \(v_i\), and sent one of two possible messages to his partner: “Yes, I intend to allocate tokens to the Group Account” or “No, I do not intend to allocate tokens to the Group Account”. We used the same terminology as Palfrey and Rosenthal (1991). This cheap talk stage allowed players to communicate their type imperfectly, but also it let them mainly communicate their intentions. It is important to emphasize that this message was cheap talk, i.e., it was not binding.

In the second stage, each subject observed the message his partner had sent him and then decided how much to allocate to the Group Account, \(c_i\).

The next hypothesis concerns Proposition 2 when \((A_1, A_2)\) partitions the type space in two intervals.

\(^5\)The total average payment was US$10.50, both values converted by US$1 = R$ 1.62, exchange rate on August 13, 2008.

\(^6\)The use of the detailed payoff table, explained in the instructions, follows Saijo and Nakamura (1995). The instructions are available on request.
Hypothesis 2 The probability that the total amount allocated to the Group Account exceeds the project cost is greater in the cheap talk game than in the no communication game.

3.3 The refined communication treatment

This treatment consisted of the same two stage game as the binary communication treatment, with the only difference being that players could choose to send one of four messages in the first stage: \( m_i \in \Lambda_i \equiv \{(0, 25), [25, 50), [50, 75), [75, 100]\}. In order to follow the binary treatment, the players had to be able to communicate their intentions. When the subjects were asked “How much do you intend to invest?” they had to send one of the four messages:

“\( I \) intend to allocate an amount between 0 and 25 to the Group Account.”
“\( I \) intend to allocate an amount between 25 and 50 to the Group Account.”
“\( I \) intend to allocate an amount between 50 and 75 to the Group Account.”
“\( I \) intend to allocate an amount between 75 and 100 to the Group Account.”

Figure 2 - Partition of the refined communication treatment

Any threshold equilibrium must have contribution partitions that create a rectangle above the negative 45-degree line in Figure 2. But there cannot be multiple such rectangles in a single equilibrium, since this would create incentives for some types to miscommunicate their type, i.e., which rectangle their value is in. Thus, there must be only one rectangle. The single rectangle that maximizes the measure of types on which the project is built is \([50, 100] \times [50, 100]\). But this is exactly the set of types that build the project in the equilibrium of the binary communication game. Thus, the refined communication game cannot improve on the probability of project implementation.

The next hypothesis concerns Proposition 2 when \( \Lambda_i \equiv \{(0, 25), [25, 50), [50, 75), [75, 100]\}.

Hypothesis 3 The probability that the total amount allocated to the Group Account exceeds the project cost in the cheap talk game with a refined message space is not greater than in the cheap talk game with a binary message space.

4 Results

In the no communication treatment and in the refined communication treatment, we observed a negative trend in contributions in the initial periods, an evidence of initial learning. This pattern was not observed in the binary communication treatment. The observations from these initial periods were excluded from the analysis to make the data consistent with the one-shot game being tested, where period is not relevant in explaining decisions. It is worth to highlight that results were not affected by this procedure.

In order to find evidence of learning we looked for any time trend in contributions, for each treatment. In doing so, for each treatment, we regressed contribution on period, value and treatment dummies for different subsamples truncated by periods, clustering errors by session. For all analysis that follows we utilized only the subsample in which the period was not significant at 5% significance to explain contribution. We dropped the 11 and 2 first periods of the no communication and refined communication treatment, respectively.

Excess contribution

The theoretical model discusses only one dimension of efficiency, i.e., the probability of the joint project being implemented. However, inefficiency can also be measured by the unproductive amount invested, or the excess contribution which is defined as:

\[
CE = \begin{cases} 
  c_1 + c_2, & \text{if } c_1 + c_2 < k \\
  c_1 + c_2 - k, & \text{if } c_1 + c_2 \geq k.
\end{cases}
\]
Note pure-strategy equilibrium will have efficiency loss due to coordination failure, since the sum of contributions must equal the cost of the project. Hence, excess contribution can be interpreted as a measure of coordination failure.

In the following subsections we first test the model hypotheses using only the first measure of efficiency, the probability of implementing the joint project. Then, we look at this other dimension of efficiency gain studying the effects of communication on excess contribution. Finally, we look at subject’s strategic behavior.

4.1 Hypotheses test

We first look at efficiency gain as discussed in the model: the probability of the joint project being implemented. The control treatment is the no communication game, where we observed that only 56.3% of subjects played the zero contribution Nash Equilibrium strategy. The positive contribution by the other players resulted in the project being implemented in 16.5% of the cases, what lead us to reject Hypothesis 1 at 5% of significance.\(^7\) Table 1 presents the means and the p-values of the Wilcoxon sign-rank tests performed. This overcontribution pattern is common in public goods experiments and also was observed by Isaac and Walker (1988) and Menezes and Saraiva (2005).

<table>
<thead>
<tr>
<th>Table 1 - Probability of project implementation</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. Observations</td>
</tr>
<tr>
<td>------------------</td>
</tr>
<tr>
<td>No Communication (NC)</td>
</tr>
<tr>
<td>Binary Communication (BC)</td>
</tr>
<tr>
<td>Refined Communication (RC)</td>
</tr>
<tr>
<td>H(_0):BC=NC</td>
</tr>
<tr>
<td>H(_0):RC=NC</td>
</tr>
<tr>
<td>H(_0):RC=BC</td>
</tr>
</tbody>
</table>

The introduction of the communication stage should allow players to coordinate their decisions, making the project more likely to be implemented, as stated in Hypothesis 2. However, the laboratory outcome showed us that both communication schemes failed to increase the probability of the project being undertaken relative to the no communication treatment. In the binary treatment, the project was implemented in only 14% of the cases, while in the refined communication treatment it was implemented in 20% of the cases. Using Wilcoxon signed-rank tests, we see that neither the binary message space nor the refined message space increased the probability of project implementation relative to the no communication treatment. Hence, we reject Hypothesis 2. As will be argued in the Subsection 4.3, this weak effect of the communication stage may be due to the multiplicity of equilibria. I.e., some subjects may be using the babbling strategy and some may be using the threshold strategy, which thereby introduces uncertainty into the game and reduces message credibility. Palfrey and Rosenthal (1991) and Bochet et al. (2006) also obtained similar results.

Hypothesis 3 says that more refined message spaces do not support a truthful equilibrium in which the project is more likely to be undertaken than in any equilibrium supported by a binary message space. This hypothesis is not so intuitive, however the Wilcoxon signed-rank test fails to reject it.

Therefore, we can conclude that the introduction of the cheap talk stage does not increase the probability of the project being implemented.

4.2 Efficiency

In this section we evaluate the efficiency of communication by taking into account its effect on the unproductive contribution. The theoretical model of Section 2 does not include the excess contribution, however we use the three hypotheses derived there as a benchmark for the following analysis.

As already argued, people made blind contributions in the no communication treatment. These blind contributions sustained a higher level of excess contribution, since subjects contributed with no signal about their partner’s valuations or intentions. In the no communication treatment, each group wasted an average of 29 tokens in excess contribution, while in the binary and refined communication treatments we observed an average waste of less than 22 tokens.

\(^7\) All results discussed are significant at 5%.
<table>
<thead>
<tr>
<th>Table 2 - Excess contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>No Communication (NC)</td>
</tr>
<tr>
<td>Binary Communication (BC)</td>
</tr>
<tr>
<td>Refined Communication (RC)</td>
</tr>
<tr>
<td>H₀:BC=NC</td>
</tr>
<tr>
<td>H₀:RC=NC</td>
</tr>
<tr>
<td>H₀:RC=BC</td>
</tr>
</tbody>
</table>

As presented in Table 2, by comparing each communication treatment to the no communication treatment, the Wilcoxon signed-rank test rejects the null hypothesis that communication does not affect excess contribution. Thus, an analogous version of Hypothesis 2, that communication allows efficiency gain relative to the contribution game, is supported by the data.

The same hypothesis test fails to reject the hypothesis that the two communication treatments have the same mean excess contribution. This means that, making an analogy to Hypothesis 3, the refined message space does not provide efficiency gain relative to the binary one in either efficiency measure: probability of project completion or excess contribution.

Therefore, communication indeed seems to increase efficiency in relation to the contribution game. However, this efficiency gain is not obtained by a higher probability of the project being implemented as predicted by the model; the main effect of communication is to reduce the unproductive contribution, or the failure of coordination. With this result in mind, we believe that our findings show that communication is a useful mechanism to enhance efficiency. However, we observed that the efficiency gain due to communication is very limited, since a more refined message space failed to do better than the “Yes” or “No” messages.

4.3 Behavior patterns

We now look at the subjects’ behaviors, attempting to identify the strategies used by them. In the contribution game, the only equilibrium strategy is the zero contribution. However, we observed that slightly more than 56% of the decisions used this strategy, while the others contributed a positive amount. As we can see on Graph 1, the non-zero contributions do not seem to have a clear pattern. As we will argue later, we find the same pattern in contributions following a communication history that contains a “No” message.

The analysis of the communication game is not as straightforward because the two stage game has multiple equilibria. In the binary treatment we have two classes of equilibria: the babbling equilibrium, where players behave freely in the communication stage and contribute zero independently of the message history, and the threshold equilibrium. In the refined communication treatment, we have the babbling equilibrium, three threshold equilibria (one symmetric and two asymmetric) and at least a pair of non-truthful equilibria with positive contribution. Note that the threshold equilibrium Pareto dominates the babbling one for players with values greater than the contribution thresholds. Players with a value smaller than or equal to this threshold are indifferent between playing babbling or threshold strategies, as both strategies give them the same expected payoff. In what follows, we restrict our attention to only two strategies: the babbling and the symmetrical threshold.

The babbling strategy allows players to fool or truthfully reveal their types in the communication stage. However, it states that players contribute zero in the second stage independently of the message they receive. Individuals playing the threshold strategy truthfully reveal their types in the communication stage and their

---

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The babbling strategy allows players to fool or truthfully reveal their types in the communication stage. However, it states that players contribute zero in the second stage independently of the message they receive. Individuals playing the threshold strategy truthfully reveal their types in the communication stage and their

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For example, a strategy where players with values greater than 50 use the message [50, 75) or [75, 100] as “Yes” and contribute 50 on the contribution stage when both players sent message “Yes”.

---
choices in the contribution stage are contingent on the messages exchanged. These players contribute 50 tokens each if the message history is such that both communicated “Yes”, which in the refined treatment means that both said they intend to contribute more than 50 tokens.

We use the following three main different characteristics of these equilibria to identify them in the data: (i) we expect to observe more truthful revelation on the threshold strategy than on the babbling strategy; (ii) positive contribution is consistent only with the threshold strategy; and (iii) the message received should affect contribution only for players with values high enough who are playing the threshold strategy.

If a player is not playing naively, when he sends a message in the first stage he already knows whether there is a positive probability of his contribution being positive in the second stage. That is, if the subject’s value is small enough or if he is playing the babbling strategy, he knows that he will contribute zero in the second stage for sure. On the other hand, someone with high value playing the threshold strategy knows that he will contribute a positive amount only if he receives a positive message from his partner. Note that initially, this latter player does not know for sure whether he will make a positive contribution at the second stage or not, exactly because his contribution is contingent on the message he receives. The histograms below show the communication patterns for each value interval.

Graph 2 - Message histogram by value

We can observe a similar communication pattern in both communication treatments. By looking at the values of players who sent each message, we observed a tendency of under revealing their types in the refined communication treatment. Also, we can see that in some sense the refined message space was used as a binary message space. Without losing information, we bunch the messages \([50, 75)\) and \([75, 100]\) and read them hereinafter as a “Yes” message on the binary treatment, and bunch \([0, 25]\) and \([25, 50]\) hereinafter as a “No” message. Using a Kolmogorov-Smirnov test we cannot reject the hypothesis that the distribution of players’ values who send a “Yes” message is equal on the two communication treatments. Thus, behavior in the communication stage seems to be consistent with the equilibria strategies.

We now turn to the second stage of the communication treatments, i.e., the contribution stage. This stage is a continuation game that follows a communication history, so the analysis of the contribution must be truncated by these messages histories. For each continuation game, Table 3 presents the mean non-zero contribution and the percentage of zero contributions. The communication history seems to have affected marginally the average positive contributions. In fact, using Kolmogorov-Smirnov tests we cannot distinguish the distribution of contributions on the no communication treatment and on the communication treatment when a “No” message was exchanged. On the other hand, the distribution of contributions on the communication treatments following a (Yes,Yes) message history is significantly different from the distribution of contributions on the no communication treatment. Also, we fail to reject the hypothesis that the distribution of contributions following a (Yes,Yes) message history is equal in both communication treatment. All these results are presented in Table 4.

![Graph 2 - Message histogram by value](images1/hist_b.jpg)

![Graph 2 - Message histogram by value](images1/hist_r.jpg)

**Table 3 - Mean positive and zero contributions**

<table>
<thead>
<tr>
<th>Treatment and message history</th>
<th>Mean non-zero contribution</th>
<th>Percentage of zero contributions</th>
<th>Number of observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>No communication</td>
<td>51.5</td>
<td>56.3</td>
<td>448</td>
</tr>
<tr>
<td>Binary</td>
<td>(Yes,Yes)</td>
<td>53.9</td>
<td>37.2</td>
</tr>
<tr>
<td>Binary</td>
<td>not (Yes,Yes)</td>
<td>49.9</td>
<td>81.9</td>
</tr>
<tr>
<td>Refined</td>
<td>(Yes,Yes)</td>
<td>57.1</td>
<td>27.1</td>
</tr>
<tr>
<td>Refined</td>
<td>not (Yes,Yes)</td>
<td>49.8</td>
<td>69.1</td>
</tr>
</tbody>
</table>

9 Precisely, the null hypothesis is \(\text{cdf}_{\text{Binary}}(\text{value}|\text{message sent} = \text{“Yes”}) = \text{cdf}_{\text{Refined}}(\text{value}|\text{message sent} = \text{“Yes”})\). The combined K-S p-value is 0.399, and the number of observations is 799.
However, as can be seen in Table 3, the main effect of communication was to reduce the share of zero contributions. Following a pessimistic message history, i.e., any history different from (Yes,Yes), the vast majority of subjects contributed zero, while after an optimistic message history (Yes,Yes) subjects contributed positive values much often. Also, the evidence that some players made no contributions following optimistic histories suggests that some of them were playing the babbling strategy. The histograms on Graph 3 present the group contributions in the light of the message history, which are helpful in understanding this point.

**Table 4 - Positive Contribution Distribution, Kolmogorov-Smirnov tests**

<table>
<thead>
<tr>
<th>Null Hypothesis</th>
<th>p-value</th>
<th>No. observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_0$:NC = {BC</td>
<td>not (Yes,Yes)}</td>
<td>0.731</td>
</tr>
<tr>
<td>$H_0$:NC = {RC</td>
<td>not (Yes,Yes)}</td>
<td>0.129</td>
</tr>
<tr>
<td>$H_0$:NC = {BC</td>
<td>(Yes,Yes)}</td>
<td>0.000</td>
</tr>
<tr>
<td>$H_0$:NC = {RC</td>
<td>(Yes,Yes)}</td>
<td>0.000</td>
</tr>
<tr>
<td>$H_0$:BC</td>
<td>RC</td>
<td>(Yes,Yes)</td>
</tr>
</tbody>
</table>

**Graph 3 - Contribution histograms given a communication history**

<table>
<thead>
<tr>
<th>Binary treatment</th>
<th>Refined treatment</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="images1/3d_bn.jpg" alt="image1" /></td>
<td><img src="images1/3d_ry.jpg" alt="image1" /></td>
</tr>
<tr>
<td><img src="images1/3d_by.jpg" alt="image1" /></td>
<td><img src="images1/3d_ry.jpg" alt="image1" /></td>
</tr>
</tbody>
</table>

Remember that in the previous subsection, we saw that communication provides efficiency gain by reducing the excess contribution, i.e., reducing the failure of coordination. What Table 3 and Graph 3 suggest is that this efficiency gain is mostly due to a coordination on the zero contribution when the project is not desirable for at least one player. It is also interesting to note how the two communication treatments provide similar incentives in both stages, which is consistent with Proposition 2.

The last remarkable difference between the babbling and the threshold strategies is that a player’s contribution is not affected by the message received whether agents are playing the babbling or the threshold strategy and have a value below a certain threshold. In this scenario, when an agent chooses to send a message “No” in the first stage he is also choosing to contribute zero in the second stage. So, we want to determine whether the message received by the player had different effects on his contribution by looking at players with different values who sent different messages. To do so we regress contribution on value and the received messages across different subsamples truncated by the messages sent and values, using fixed effects.

The threshold strategy would say that the messages received are significant in explaining the contributions only for players with a value over a certain threshold who submit a truthful message. We used a dummy variable that is equal to 1 if a “Yes” message was received and zero otherwise. The coefficient of this variable captures the impact of the message received in the contribution given the type of the player and his announcement on the first stage.

**Table 5 - Coefficient of Dummy “Yes message received”**

| Values subsample | Binary Communication | | | Refined Communication | |
|-----------------|---------------------|-----------------|-------------------|-----------------|-----------------|-----------------|
|                 | [0,50)              | [50,100]        | [0,25)            | [25,50)          | [50,75)         | [75,100]        |
| Message “No” sent | 1.2                 | (2.645, 2.18)  | 5.32             | (6.330, 1.02)   | 0.094           | (1.967, 1.09)   |
| Message “Yes” sent | 4.7                 | (4.373, 162)   | 32.5             | (3.495, 318)    | 7.7             | (11.529, 52)    |

Note: Standard errors and number of observations between parenthesis. Significance of **1% and *5%.

Fixed effects regression of contribution on value and dummy “Yes message received”.

12
As can be seen in Table 5, in both communication treatments, the message received did not affect the contribution decision of subjects with low values, independently of the message they sent in the first stage. This suggests that some of the players who are indifferent between the babbling and the threshold strategies are indeed babbling. Also, in both communication treatments, the message received is not significant to explain the contribution decision of players with value sufficiently high who sent a “No” message. This suggests that subjects used the babbling strategy even in situations where the threshold strategy was dominant.

In both treatments, the message received seemed to be more relevant for players with higher values who sent a “Yes” message in the first stage. This is consistent with the threshold strategy. We did not find any structural change when a “No” message was sent in the first stage. In fact, the message received is relevant in explaining the contribution behavior in those situations where the threshold strategy was dominant. In the refined treatment, the received message was significant for players with intermediary values who sent message “No”. This is consistent with players using an asymmetric threshold strategy, remember that the “No” message bunches \([0, 25]\) and \([25, 50]\).

Thus, we found evidence that subjects played the two possible equilibria: the babbling and the threshold equilibria. It is worth mentioning that the existence of multiple equilibria introduced uncertainty into the game, mostly for the subjects with high value who intended to contribute. When a player with high value who truthfully reported his type received a positive message in the second stage, he was not sure whether his partner was playing the threshold strategy as well or the babbling strategy. His contribution decision at this point is analogous to a “stag hung” game, in which there are two equilibria, one that is payoff-dominant, where both contribute 50 tokens, and another that is risk-dominant, where both contribute zero. This additional uncertainty may have precluded the project being implemented in more situations. At the other extreme, following a pessimistic message history, players did not face any additional uncertainty, and the dominant strategy was the zero contribution. So, the findings in the last two subsections are coherent with the incentives faced by the players.

5 Conclusion

This paper models the behavior of two agents in situations where non-costly communication could lead to welfare gain. We argue that cheap talk can enhance efficiency relative to the no communication game. However, even when players have several (finite) messages possibilities, they cannot do better than when they can use only “Yes” or “No” messages.

Laboratory experiments were used to test these theoretical results. We find evidence that communication indeed seemed to increase efficiency relative to the contribution game. However, this efficiency gain was not obtained by a higher probability of the project being implemented, but by reducing failure if coordination, measured as the amount of unproductive contribution. Communication seemed to work by allowing players to coordinate on the zero contribution when the project was not desirable for at least one of them.

We found evidence that subjects did play the two possible equilibria: the babbling and the threshold equilibria. The existence of multiple equilibria may have been an additional source of uncertainty in the game when players were willing to contribute to the joint project. This may have inhibited the positive effect of communication in the provision of the public good.

We believe that our findings add to the literature on the usefulness of communication to enhance efficiency. Also, the paper shed light on the mechanism that lead to this efficiency gain. However, in this setting where both players have private information and decision power, differently from Crawford and Sobel (1982), the communication gain is very limited because the simplest communication structure ("Yes" or "No" messages) provides the same incentives to agents as any finite partition of the type space.

Appendix

Definition 3 We say that a continuation game \((m_1^1, m_2^2)\) of \(\mathcal{C}(\Lambda_1, \Lambda_2)\) is the contribution game after both players truthfully communicated their type. I.e., it is the contribution game in which the players have prior \(F_1(\cdot), F_2(\cdot)\) and \(v_1 \in m_1^1\) and \(v_2 \in m_2^2\).

Lemma A In a truthful equilibrium of \(\mathcal{C}(\Lambda_1, \Lambda_2)\), in all continuation game \((m_1^1, m_2^2)\) such that \(a_{i-1} + b_{j-1} < k\) the equilibrium contribution is zero.
Proof. After truthful communication \((m^1_i, m^2_j)\) in the first stage, players update their priors to \(v_1 \in m^1_i\) and \(v_2 \in m^2_j\). If \(a_{i-1} + b_{j-1} < k\), by concavity of \(F_1(\cdot), F_2(\cdot)\), Proposition 1 shows that the unique equilibrium contribution is zero. ■

**Lemma B** Any contribution in a truthful equilibrium of \(\mathcal{C}(\Lambda\_1, \Lambda\_2)\) satisfies

\[
C^1(v_1, v_2, m^1_i, m^2_j) \leq a_{i-1} \quad \text{and} \quad C^2(v_1, v_2, m^1_i, m^2_j) \leq b_{j-1}.
\]

**Proof.** Take a positive contribution \(C^1(v_1, m^1_i, m^2_j) > 0\) in a continuation game \((m^1_i, m^2_j)\). By truthful revelation and Lemma A, it must be that

\[
a_{i-1} + b_{j-1} \geq k.
\]

In any equilibria of a continuation game, where contribution is positive we must have that

\[
C^1(v_1, m^1_i, m^2_j) + E[C^2(v_2, m^1_i, m^2_j)|v_2 \in m^2_j] = k,
\]

otherwise player 1 would have incentive to contribute less. So, every type \(v_1 \in m^1_i\) (and \(v_2 \in m^2_j\)) have identical positive contribution in the continuation game \((m^1_i, m^2_j)\).

Clearly, no player have incentive to contribute \(C^1(v_1, v_2, m^1_i, m^2_j) \geq a_i\).

Suppose that \(\exists i\) such that \(C^1(v_1, v_2, m^1_i, m^2_j) \in (a_{i-1}, a_i)\). By (4) we have that \(C^2(v_1, v_2, m^1_i, m^2_j) < b_{j-1}\). By (3), \(m^2_{j'} \in \Lambda_2, j' < j\), such that \(C^1(v_1, v_2, m^1_i, m^2_{j'}) + b_{j'-1} \leq k\). Hence, some players with type \(v_2 \in m^2_{j'}\) will have incentive to over report their type in the communication stage, violating truthful revelation. ■

**Lemma C** Any truthful equilibrium of \(\mathcal{C}(\Lambda\_1, \Lambda\_2)\) can have at most one positive contribution level for each player.

**Proof.** Suppose a truthful equilibrium in which players have more than one positive contribution level. Take the two smallest contribution levels

\[
C^1(v_1, v_2, m^1_i, m^2_j) = c > 0\quad \text{and} \quad C^1(v'_1, v_2, m^1_i, m^2_j) = c' > 0.
\]

where \(c < c'\). By Lemma B we know that \(c \leq a_{i-1}\) and \(c' \leq a_{i'-1}\). The player with type \(a_{i'-1} \in m^1_{i'}\) has incentive to under report his type and communicate \(m^1_i\). Hence, truthful revelation is violated. ■

**Proof of Lemma 1.** By Lemma C, we have that in any truthful equilibrium with positive contribution we may have at most one positive contribution level for each player. By Lemma B we know that this contribution must satisfy \(C^1(v_1, v_2, m^1_i, m^2_j) \leq a_{i-1}\) and \(C^2(v_1, v_2, m^1_i, m^2_j) \leq b_{j-1}\), for all \(i, j\).

By Lemma A we know that the we may have a positive contribution in the continuation game \((m^1_i, m^2_j)\) only if \(a_{i-1} + b_{j-1} \geq k\). Also, by the equilibrium condition of the continuation game (4), then it must be that \(a_{i-1} + b_{j-1} = k\) for some \(i, j\).

Let \(\hat{i}, \hat{j}\) be such that \(a_{\hat{i}-1} + b_{\hat{j}-1} = k\). If \(C^1(v_1, v_2, m^1_{\hat{i}}, m^2_{\hat{j}}) < a_{\hat{i}-1}\), some players with type in \(m^1_{\hat{i}-1}\) would not have incentive to truthfully report their type in the communication stage. Then, the unique positive contribution of the continuation game \((m^1_{\hat{i}}, m^2_{\hat{j}})\) is \(C^1(v_1, v_2, m^1_{\hat{i}}, m^2_{\hat{j}}) = a_{\hat{i}-1}\) and \(C^2(v_1, v_2, m^1_{\hat{i}}, m^2_{\hat{j}}) = b_{\hat{j}-1}\).

Suppose that contribution is zero in any continuation game except \((m^1_i, m^2_j)\). There is no optimal deviation in any continuation game \((m^1_i, m^2_j)\) such that \(i < \hat{i}\) or \(j < \hat{j}\). However, any player with type \(v_1 \geq a_{\hat{i}}\) or \(v_2 \geq b_{\hat{j}}\) would have incentive to communicate \((m^1_i, m^2_j)\). Hence, to be consistent with truthful revelation it must be that \(C^1(v_1, v_2, m^1_i, m^2_j) = a_{\hat{i}-1}\) and \(C^2(v_1, v_2, m^1_i, m^2_j) = b_{\hat{j}-1}\) for every \(i \geq \hat{i}\) and \(j \geq \hat{j}\).

If \(\exists i, j\) such that \(a_{i-1} + b_{j-1} = k\), the unique contribution in a truthful equilibrium of \(\mathcal{C}(\Lambda\_1, \Lambda\_2)\) is zero.
Hence, in any truthful equilibrium with positive contribution, the contribution rule is

\[
C(v_1, v_2, m_1^i, m_2^j) = \begin{cases} 
(a_{i-1}^i, b_{j-1}^j), & \text{if } \exists \hat{i}, \hat{j} \text{ s.t. } a_{\hat{i}-1}^i + b_{\hat{j}-1}^j = k \\
(0, 0), & \text{otherwise.}
\end{cases}
\]

Therefore, given any finite partition \((\Lambda_1, \Lambda_2)\), the threshold strategy profile is the unique strategy that supports a truthful equilibrium where players contribute non-zero values with positive probability.

**Proof of Proposition 2.** To prove necessity, let \(\Gamma\) be an equilibrium strategy profile where players truthfully reveal their types in the communication stage and make non-zero contributions with positive probability. Hence, by Lemma 1, \((\Lambda_1, \Lambda_2)\) must be such that there exists a pair \((i, j)\) with \(a_{i-1}^i + b_{j-1}^j = k\).

To prove sufficiency, one just need to let players use a threshold strategy profile.

The unicity follows directly from Lemma 1 as well.

**References**


