Comparative Advantage in Innovation and Production*

Mariano Somale†
Princeton University

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Abstract

This paper develops a multi-country, general equilibrium, semi endogenous growth model of innovation and trade in which specialization in innovation and production are jointly determined. The distinctive element of the model is the ability of the agents to direct their research efforts to specific goods, in a context of heterogeneous innovation capabilities across countries and contemporaneous decreasing returns to R&D. The model features a two-way relationship between trade and technology absent in standard quantitative Ricadian trade models. I calibrate the model using a sample of 29 countries and 18 manufacturing industries and quantify the importance of endogenous adjustments in technology. I find that endogenous adjustments in technology due to directed research can account for up to 52.8% of the observed variance in comparative advantage in production. In addition, the model suggests that standard Ricardian models overestimate the reductions in real income from increases in trade costs and underestimate the increment in real income due to trade liberalizations.

*Visit http://www.princeton.edu/~msomale for the latest version of the paper.

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1 Introduction

Ever since the writings of David Ricardo, the relationship between technology and trade has featured prominently in economic analysis. Traditional Ricardian theories of trade have emphasized the role of technological differences across countries in generating the observed specialization patterns in production and trade. The literature following this tradition typically has been concerned with the *effects* of those technological differences and consequently it has mainly used static models in which technology is exogenous to analyze a myriad of topics such as the patterns of production and trade, the welfare gains from trade, the effects of exogenous technological progress and the effect of the diffusion of technology, among others.[1] Moreover, since the seminal contribution of Eaton and Kortum (2002), we have a rich set of trade models that incorporate the main insights from the Ricardian theory of trade in a context of many goods and many countries and that lend themselves to easy quantitative implementation, allowing the researcher to go beyond the qualitative analysis that previous models permitted.[2]

However, the state of production technology at any given moment in time is hardly exogenous, but a consequence of past innovations. For long, economists have emphasized the economic nature of innovation activity and the role of expected profits in shaping the amount and direction of innovation efforts. In the words of Schmookler: "*...invention is largely an economic activity, which like other economic activities, is pursued for gain.*"[3] In his seminal work on technical change in various capital good industries, *Invention and Economic Growth (1966)*, Schmookler provides evidence about the importance of demand and the expected market size for an innovation as determinants of invention activity. Since then, a large number of empirical studies have provided evidence in support of Smookler's ideas.[4]

The discussion in the previous paragraph implies that in the case of an open economy, not only does technology affect the patterns of trade as in the standard Ricardian model, but also trade can affect innovation and technology through the changes in the expected market size for an innovation that it induces. In particular, in an economy in which research efforts can be *directed* to different goods, trade-induced changes in the expected market size for inventions generate a reallocation of research efforts from those goods in which the market contracts towards those goods in which the market expands, providing an additional margin through which economies can adjust to changes in the trading environment. Although the relationship between trade and innovation has been extensively studied since the pioneering contributions of Grossman and Helpman (1989, 1990, 1991) and Rivera-Batiz and Romer (1991), little attention has been given to the effects of directed research across goods in the quantitative trade literature.[5]

[1] Grossman and Helpman (1995) survey the literature that follows this tradition.

[2] Eaton and Kortum (2012) survey the quantitative trade literature that extends the model in Eaton and Kortum (2002).

[3] Schmookler, Invention and Economic Growth, 1966, p. 206.

[4] Just to mention a few examples, Newell, Jaffee and Stavins (1999) show that the type of innovation on the typical air conditioner (AC) sold at Sears shifted from innovations that reduced the price of ACs in the period 1960-1980 to innovations that made them more energy-efficient in the period 1980-1990, which they argue was the consequence of the rise in energy prices in the later period. Acemoglu and Linn (2004) and Cerda (2007) use changes in demographic trends as a source of exogenous variation in the market size of different types of drugs; they find that the effect on innovation of these changes in market size are economically important.

[5] In a context in which technical change complements factors of production, Acemoglu (1998, 2002, 2003) studies the effects directed technical change on factor rewards.

The purpose of this paper is to study the two-way relationship between trade and technology that emerges in a context of endogenous innovation and directed research, and to assess qualitatively and quantitatively the implications of directed research for innovation, production and trade specialization patterns. One of the challenges in the quantitative assessment of this new margin is that the positive correlations in the innovation, production and export specialization patterns observed in the data are of little help to disentangle the importance of the effects of trade on technology. The reason for this is that such correlations characterize both a world with directed research and a world in which the trade environment does not affect the direction of technical change.[6] A potential approach to deal with this issue is to find a natural experiment in which an exogenous shock affects only the trading conditions of a country, and use that shock to isolate the effects of trade on innovation and technology.[7] Leaving aside the difficulties of finding such natural experiment, this approach cannot be used to evaluate the consequences of other counterfactual shocks on innovation, production and trade patterns, nor does it permit to study the welfare implications of the additional margin of adjustment introduced by directed research.

To overcome these difficulties, I build on Eaton and Kortum (2001) and on recent developments in the static quantitative trade literature to develop a multi-country, general equilibrium, semi-endogenous growth model of innovation and trade in which specialization in innovation and production are jointly determined. The distinctive element of the model is the ability of agents to direct their research efforts to specific goods in a context in which countries differ in their exogenous innovation capabilities; this new element builds into the model the two-way relationship between trade and technology that is the focus of this paper. The semi-endogenous nature of technical change in this model implies that all the effects of directed research are reflected in the levels of the variables of interest in the balanced growth path (BGP), with no effect on BGP-growth rates. These level effects are the focus of this paper. I use the model to disentangle the effects of trade on technology and to study some questions that standard Ricardian quantitative models and reduced-form approaches are not suitable to answer. How important is the feedback from demand and market size to technology? How is comparative advantage in production determined in this context? How are the specialization patterns of innovation, production and trade determined? How does this additional margin affect our conclusions regarding the effects of trade liberalization on production, trade flows and welfare?

The model features contemporaneous decreasing returns in R&D, which are parsimoniously captured by a single parameter common to all countries and goods. Decreasing returns in R&D are an important element of the model since they control the importance of the additional margin of adjustment introduced by directed research; the weaker the decreasing returns in R&D, the stronger the endogenous adjustment in technology in response to changes in the environment. This implies that under the structure of the model, disentangling the importance of each direction in the two-way relationship between technology and trade reduces to the determination of the value of the single parameter capturing the decreasing returns to R&D.

[6] Using R&D expeditures or patents counts as measures of innovation activity, the data shows that countries innovate relatively more in those sectors where they produce and export relatively more.

[7] See Bustos (2008) and Lileeva and Trefler (2010) for studies that use exogenous variations in tariffs to analyze the effect on firms' technology spending decisions.

To estimate the decreasing returns in R&D I use a structural relationship of the model that captures the distinctive element introduced by directed research, i.e., the endogenous and joint determination of manufacturing technology and trade flows. Using cross-sectional production and trade data, I estimate this relationship using two method of moments estimators that yield lower and upper bounds for the degree of decreasing returns in R&D. As an overidentification check for this estimation strategy, I use a dynamic implication of the model that relates the growth rate of income per capita in the BGP to the growth rate of research employment and the degree of decreasing returns in R&D. These alternative approaches yield similar results.

The model presents some convenient features that facilitate its quantitative implementation. First, in the BGP, the model nests the benchmark Ricardian model with no innovation as a special case. More specifically, when R&D possibilities are eliminated, the model reduces at the aggregate level to a multi-industry version of Eaton and Kortum (2002) (henceforth EK model.)[8] In addition, both models share the same structure in the cross-section. This implies that many of the methods developed in the literature to estimate multi-industry versions of the EK model can also be applied to this model. In particular, this feature of the model allows me to use the methods developed in Costinot, Donaldson and Komunjer (2011) to estimate comparative advantage in production, which is a necessary input in the estimation of the degree of decreasing returns in R&D. Second, relative to such a benchmark model with no innovation, the present model adds only one additional parameter to the total number of parameters needed to be estimated to evaluate counterfactual predictions across BGPs. Moreover, armed with an estimate of the parameter capturing the decreasing returns to R&D, any method used to estimate the countries' manufacturing productivities in the model with no innovation can be used to recover countries' innovation capabilities. Consequently, all the additional estimation burden imposed by the introduction of directed research relies on the estimation of decreasing returns to R&D. Third, the fact that the models with and without innovation share the same cross-sectional structure implies that both models perform equally well in matching trade and production data in the cross-section. Moreover, both models can be estimated to match exactly the data and to share all exogenous parameters and manufacturing technologies. Nevertheless, even if the two models are set up in this way, they still differ in their counterfactual predictions regarding the changes in trade flows, manufacturing technology and welfare associated with different shocks, all of which are relevant dimensions for policy analysis.

When performing counterfactual analysis across BGPs, I extend the approach popularized by Deckle, Eaton and Kortum (2007) (henceforth DEK) to include the effect of innovation and directed research and solve the BGP of the model in changes. When the model is solved in changes, only a subset of parameters is needed to perform any given counterfactual evaluation. In particular, this approach does not rely on estimates of unobserved structural parameters such as innovation capabilities or trade costs. In addition, the system of equations of the model in changes encompasses the system of equations corresponding to a multi-industry EK model as a special case. This implies that armed with an estimate of the parameter

[8] More specifically, the model reduces to a multi-industry version of Bernard, Eaton, Jensen and Kortum (2003). However, these models have equivalent reduced-forms at the aggregate level in terms of the determination of wages, production and trade flows.

capturing the decreasing returns in R&D, performing counterfactual analysis in the model with directed research does not impose any additional data requirement over the benchmark model with no innovation.

I calibrate the model using a sample of 29 countries and 18 manufacturing industries and I use it to quantify the importance of directed research on technology, production and trade flows. I decompose comparative advantage in the observed open equilibrium into exogenous and endogenous components. Under the benchmark calibration for the decreasing returns in R&D, I find that the endogenous adjustments in technology due to directed research can account for up to 52.8% of the observed variance in comparative advantage in production. In addition, the estimated endogenous component of comparative advantage in production is tightly connected to relative domestic demand conditions, which according to the theory, is indicative of the presence of high trade frictions.

I use the model to explore quantitatively two counterfactual situations:

(i) I explore the changes in real income as countries move to autarky. The model shows that all countries suffer a reduction in their real income. Relative to the model with directed research, the standard model with no innovation tends to overestimate the reductions in real income, although the differences between models appear to be modest. On average, the reductions in real income predicted by the model with directed research represent 93% of those predicted by the model with no innovation. The main reason for the modest differences in the predicted changes in real income is the presence of high trade frictions in the observed open equilibrium, which reduces the scope for specialization in innovation.

(ii) I examine the effects of a 25% reduction in trade costs. The introduction of directed research has relatively important effects on the model's predictions regarding the changes in trade flows and market shares. The predicted changes in trade flows in the model with no innovation can explain a little more than a third of the variation in the corresponding changes in the model with directed research. In addition, the model with no innovation tends to underestimate the magnitude of the changes in market shares. According to the model, all countries enjoy an increase in their real income. Relative to the model with directed research, the standard model with no innovation tends to underestimate the increases in real income, although the differences are also modest in this case. The increases in real income in the presence of directed research are on average 2% higher than in the case of no innovation.

This paper is related to the large endogenous and semi-endogenous growth literature analyzing the effects of trade on the pace of endogenous technical change. Among these studies we can mention Grossman and Helpman (1989, 1990, 1991), Rivera-Batiz (1991), Taylor (1993), Jones (1995), Eaton and Kortum (2001). The main departure from this literature in this paper is the focus on directed research across goods and the quantitative focus of the analysis.

This paper is also related to Arkolakis, Ramondo, Rodriguez-Clare and Yeaple (2013). They develop a quantitative single-industry model of multinational production and trade in which countries differ in their manufacturing and innovation productivities and in which firms can separate the location of their innovation and production activities. They use the model to study the effects of openness on the specialization in innovation or production patterns and they analyze the role of multinational production as a vehicle through which this international specialization takes place. In contrast, in this paper countries can direct their research efforts to different goods in the economy but cannot separate their innovation and production locations. In a context in which countries differ in their innovation capabilities across the different goods in the economy, I analyze the effects of trade on the innovation and production specialization patterns across the goods in the economy.

Finally, this paper is related to a rapidly growing literature that use static multi-sectors Ricardian quantitative trade models that build on Eaton and Kortum (2002). Among these studies we can mention Arkolakis, Costinot and Rodriguez-Clare (2012), Caliendo and Parro (2014), Chor (2010), Costinot, Donaldson and Komunjer (2012), Donaldson (2012), Levchenko and Zhang (2014a, 2014b), Shikher (2011). The fundamental difference between these studies and this paper is the treatment of technology. While in all the above mentioned studies technology is exogenous, in this paper technology is endogenous and it is affected by trade.

2 The Model

In this section I build on Eaton and Kortum (2001) to develop a multi-country, dynamic, general equilibrium model of innovation and trade in which specialization in innovation and production are jointly determined. The distinctive element of the model is the ability of the agents in any country to direct their research efforts to specific goods in a context in which innovation capabilities vary across goods and countries. The model is a semi-endogenous growth model and as such, aggregate growth rates in the balanced growth path (BGP) are not affected by trade or standard policies such as taxes, R&D subsidies; this implies that all of the new endogenous adjustments in the innovation process induced by directed research are reflected in the levels of manufacturing technology in the BGP.

In the rest of this section I describe the components of the model and I provide a characterization of the market equilibrium, leaving for the next section the analysis of the BGP.

2.1 Basic Environment

Time is continuous and is indexed by $t \in [0, \infty)$. The world consists of N countries. Country i is populated by a continuum of identical and infinitely lived households, each of them with L_{it} members at time t. The mass of households is normalized to one such that L_{it} also represents total population at time t. The representative household in every country grows at the exogenously given rate n, i.e., $L_{it} = L_{i0}e^{nt}$. Labor is the only factor of production and its total inelastic supply at time t is given by the population size L_{it}.

There are two sectors in the economy, a manufacturing sector and a research sector. The manufacturing sector produces a fixed set of final goods taking the level of technology as given, while the research sector invests in R&D to improve the technology of final goods.[9] Labor is perfectly mobile across sectors within a country but is immobile across countries.

R&D and Productivity.— The goal of R&D activity is to obtain new production techniques that improve the efficiency with which final goods are produced. To capture the idea of directed research, I divide the set of final goods in industries and allow countries to direct their research efforts toward

[9] Alternatively, R&D could improve the quality of the product. Both specification are equivalent for the purposes of the present paper.

the different industries. Formally, there is a fixed set Ω of industries with a continuum of goods in each industry. A final good in the economy is identified by the pair $(z, \omega) \in [0,1] \times \Omega$, where ω identifies the industry to which the good belongs and z identifies the good within the industry. Country i can *direct* its research efforts to any industry, but not to any specific good within the industry. I assume that countries differ in their research productivities across industries. Specifically, ideas regarding new techniques arrive to individual firms in country i targeting industry ω as a Poisson process with arrival rate $\iota_i^\omega \left(l_t^{R,\omega}\right)^\upsilon$, where ι_i^ω is a parameter representing the industry specific research productivity of country i, $l_t^{R,\omega}$ is the total number of researchers employed by the representative firm and $\upsilon \in (0,1)$ is a parameter that captures the extent of contemporaneous decreasing returns in research. I assume that there is a fixed set of research firms targeting industry ω and I normalize the mass of this set to one.

The particular functional form assumed for the arrival rate of R&D has two advantages: (i) it captures the contemporaneous decreasing returns in R&D parsimoniously through υ; and (ii) its Inada condition together with no free entry guarantee that all industry are targeted in equilibrium in any country i.[10] Nevertheless, the case of constant returns to scale and free entry can be obtained as the limit $\upsilon \to 1$.

The ability to direct research across industries is the new mechanism introduced in this paper, and it will be responsible for all the endogenous adjustments of technology to shocks that are the focus of the analysis in the present paper. In this context, the parameter υ is of fundamental importance since it determines the relevance of the new mechanism, with higher values of υ associated with stronger adjustments through the new margin.[11]

An idea is the realization of two random variables Z and X. The realization of Z indicates the good z within industry ω to which it applies, while the realization of X indicates the efficiency x of the new technique in the production of the corresponding good. The efficiency of a technique indicates how many units of final output are obtained per unit of labor, i.e., $q = xl$. Throughout the rest of the analysis I assume that Z has a uniform distribution over $[0,1]$ and X has a Pareto distribution with cdf $H(x) = 1 - x^{-\theta}$.

As the result of the R&D process described above, the arrival of ideas regarding new production techniques for any final good (z, ω) in country i has the following characteristics: (i) it follows an inhomogeneous Poisson process $\mathcal{P}(z, \omega, i)$ with arrival rate $\iota_i^\omega \left(L_{it}^{R,\omega}\right)^\upsilon$, where $L_{it}^{R,\omega}$ denotes the total number of researchers targeting industry ω at time t; (ii) for any pair $(z, \omega, i), (z', \omega', i')$ such that $(z, \omega, i) \neq (z', \omega', i')$, $\mathcal{P}(z, \omega, i)$ and $\mathcal{P}(z', \omega', i')$ are independent. The total number of techniques for good (z, ω) discovered up to time t in country i is then a random variable distributed Poisson with parameter

$$T_{it}^\omega \equiv \iota_i^\omega \int_{-\infty}^t \left(L_{is}^{R,\omega}\right)^\upsilon ds. \tag{1}$$

The efficiency of the best and second best techniques up to time t for a good in industry ω are random

[10] No free entry in R&D implies that research firms make positive profits in equilibrium. However, the presence of profits per se does not affect the results of the paper. Alternatively, I could have assumed free entry and that R&D requires the combination of labor and a fixed factor that is specific to each country and industry. With these modifications, the profits in the model presented in the text would become rents of the specific factors in the alternative model.

[11] In the extreme case of $\upsilon = 0$ no innovation takes place and the additional margin of adjusment is not opperational.

variables $X_{it}^{\omega,(1)}, X_{it}^{\omega,(2)}$ with joint cdf given by[12]

$$\begin{aligned} F_{it}^{\omega}(x_1, x_2) &= \Pr\left(X_{it}^{\omega,(1)} \leq x_1, X_{it}^{\omega,(2)} \leq x_2\right) \\ &= \left[1 + T_{it}^{\omega}\left(x_2^{-\theta} - x_1^{-\theta}\right)\right] e^{-T_{it}^{\omega} x_2^{-\theta}} \text{ for } x_1 \geq x_2 \geq 1. \end{aligned} \tag{2}$$

Assuming a law of large numbers across the continuum of goods within each industry, F_{it}^{ω} also represents the cdf of the joint distribution of the best and second best techniques across goods within industry ω in country i.[13]

Given its definition and its role in (2), throughout the paper I will refer to T_{it}^{ω} interchangeably as the stock of ideas or the level of manufacturing technology of country i in industry ω.

Preferences.— The representative household's preferences over streams of per-capita consumption are

$$U_i = \mathbb{E}_0\left[\int_0^{\infty} e^{-\bar{\rho}t} L_{it} \frac{(C_{it}/L_{it})^{1-\eta}}{1-\eta} dt\right] = \mathbb{E}_0\left[\int_0^{\infty} e^{-\rho t} \frac{C_{it}^{1-\eta}}{1-\eta} dt\right] \tag{3}$$

where η is the inverse of the intertemporal elasticity of substitution, $\rho = \bar{\rho} - n\eta$ is the effective rate of time preference,

$$C_{it} = \exp\left\{\int_{\Omega} \alpha_i^{\omega} \log\left[\frac{C_{it}^{\omega}}{\alpha_i^{\omega}}\right] d\omega\right\} \tag{4}$$

and

$$C_{it}^{\omega} = \left[\int_0^1 c_{it}^{\omega}(z)^{\frac{\sigma^{\omega}-1}{\sigma^{\omega}}} dz\right]^{\frac{\sigma^{\omega}}{\sigma^{\omega}-1}} \tag{5}$$

for $\sigma^{\omega} > 0$.

Market Structure and Geography.— Although the details of the market structure in both sectors and its consequences are analyzed later, here I provide an overview that serves as a guide through the exposition. All the assumptions regarding the market structure are completely standard in the literature.

As discussed above, at any given moment in time there are many alternative techniques in each country to produce a given final good (z, ω) that differ in their respective efficiencies. The owners of these techniques around the world engage in price competition in the market of good (z, ω) in each country i. As the result of this competition, the producer with the lowest marginal cost of serving that market becomes the only supplier of the good to that market and charges the minimum between the monopoly price and the maximum price that keeps competitors at bay.[14] In sum, the manufacturing sector is characterized by Bertrand competition in the market of each final good, and monopolistic competition among producers of final goods within an industry.

Entrepreneurs in the research sector finance their R&D activity issuing equity claims that pay nothing

[12] See Appendix C.1.1 for a proof.

[13] Throughout the analysis I assume that T_{i0}^{ω} is sufficiently high for all ω and i, such that we can safely consider that (2) is valid for $x_1 \geq x_2 \geq 0$. All the results in the paper are derived under this assumption. In Appendix C.1.1 I show that the difference between F above and a cdf F' given by (2) but with support $x_1 \geq x_2 \geq 0$ becomes negligible as $T_{it}^{\omega} \to \infty$.

[14] The monopoly price is the optimal price charged by a monopoly that faces the residual iso-elastic demand corresponding to good (z, ω).

if research efforts fail to improve upon the state of the art technique for some good, but entitle their holders to the stream of future monopoly profits if research succeeds. I assume that production techniques generated by the R&D activity can be used only in the manufacturing sector of the country where they were developed, ruling out international licensing and multinational corporations.

In order to abstract from issues regarding intertemporal trade and foreign ownership of domestic firms, I assume that financial assets are not traded. This assumption implies that each country must finance all R&D that takes place within its borders from domestic savings, and that trade is balanced every period.[15] However, consumers of country i can freely borrow and lend at the risk free domestic interest rate r_{it}.

Geographic barriers are modeled in the standard iceberg formulation, whereby τ_{ij}^{ω} units of a good must be shipped from country i in order for 1 unit to arrive to country j, with $\tau_{ij}^{\omega} > 1$ if $i \neq j$ and $\tau_{ii}^{\omega} = 1$.

Throughout the analysis I use w_{it} to denote the wage of country i in period t and set the wage of some country j as the numeraire, $w_{jt} = 1$ for all t.

2.2 Market Equilibrium

2.2.1 Demand

The representative household in country i maximizes its preferences subject to its budget constraint. Given that preferences are additively separable over time, we can divide the consumer's problem into a static and a dynamic problem. The static problem in each period t involves the optimal allocation of total expenditure E_{it} among the different goods in the economy. For a given level of expenditure E_{it}^{ω} allocated to industry ω, the expenditure on individual goods within that industry implied by the CES lower tier utility function is

$$E_{it}^{\omega}(z) = E_{it}^{\omega}\left[p_{it}^{\omega}(z)/P_{it}^{\omega}\right]^{1-\sigma^{\omega}}, \tag{6}$$

where $P_{it}^{\omega} \equiv \left[\int_0^1 p_{it}^{\omega}(z)^{1-\sigma^{\omega}} dz\right]^{\frac{1}{1-\sigma^{\omega}}}$ is the ideal price index of goods in industry ω.

The upper tier Cobb-Douglas utility function implies that the share of total expenditure allocated to industry ω in country i is α_i^{ω}, i.e.,

$$E_{it}^{\omega} = \alpha_i^{\omega} E_{it}. \tag{7}$$

Given the price indices of each industry ω, the aggregate price index is

$$P_{it} = \exp\left\{\int_{\Omega} \alpha_i^{\omega} \log P_{it}^{\omega} d\omega\right\}. \tag{8}$$

From the dual problem of the static problem we get

$$E_{it}^{\omega} = C_{it}^{\omega} P_{it}^{\omega},\ E_{it} = C_{it} P_{it}, \text{ and } E_{it} = \int_{\Omega} E_{it}^{\omega} d\omega. \tag{9}$$

The dynamic problem involves the optimal allocation of expenditure across time subject to an intertem-

[15] Exogenous trade deficits/supluses can be introduced without affecting the qualitative results presented in the paper.

poral budget constraint. The solution to this problem is characterized by the familiar Euler equation

$$\widetilde{C}_{it} = \frac{1}{\eta}\left[r_{it} - \widetilde{P}_{it} - \rho\right], \tag{10}$$

where $\widetilde{X}_t \equiv d\log\left(X_t\right)/dt$, together with transversality conditions on bonds holdings,

2.2.2 Manufacturing Sector

The problem of the firms in the manufacturing sector is completely static: at any time t firms hire labor, produce, set prices and make their export decisions taking as given the level of technology and the demand conditions at the time. At any moment in time, the distribution of manufacturing productivities, the demand and the market structure in each industry of goods are as in Bernard, Eaton, Jensen and Kortum (2003) (henceforth BEJK.) This implies that, in the cross-section, the structure of costs, markups and prices at the industry level are also as in BEJK. For this reason, when describing the structure of the model within an industry, I describe those aspects of the model that are central to the purpose of this paper, relegating nonessential derivations and proofs to the appendix.[16]

Trade Shares and Prices. Bertrand competition implies that country i buys each good from the cheapest source around the world. The cost distributions I.1 in Appendix C.1.2 imply that the fraction of goods from industry ω that country j buys from country i at time t, λ^{ω}_{ijt}, is

$$\lambda^{\omega}_{ijt} = \frac{T^{\omega}_{it}\left(w_i\tau^{\omega}_{ij}\right)^{-\theta}}{\Phi^{\omega}_{jt}}. \tag{11}$$

In the previous expression, Φ^{ω}_{jt} is a cost parameter that summarizes how manufacturing technology around the world, wages around the world and trade costs govern the distribution of the cost of serving market ω in country j; it is given by

$$\Phi^{\omega}_{jt} = \sum_{k=1}^{N} T^{\omega}_{kt}\left(w_k\tau^{\omega}_{kj}\right)^{-\theta}. \tag{12}$$

Given that the distribution of prices in each industry is independent of the source country, (11) also represents the fraction of country j's total expenditure in industry ω that is allocated to goods from country i.

Finally, the exact price index for industry ω in country i is

$$P^{\omega}_{it} = B^{\omega}_{P}\left(\Phi^{\omega}_{it}\right)^{-\frac{1}{\theta}}, \tag{13}$$

where B^{ω}_{P} is a constant that depends only on parameters.[17]

Cost Share in Revenues. In any industry ω, the share of total production costs (including trade

[16] See Appendix C.1.2 for a discussion of the determination of costs, markups and prices.

[17] Specifically, $B^{\omega}_{P} \equiv \left\{\Gamma\left(\frac{1-\sigma^{\omega}+2\theta}{\theta}\right)\left[1+\bar{m}\left(\sigma^{\omega}\right)^{-\theta}\frac{\left(\sigma^{\omega}-1\right)}{\left[\theta-\left(\sigma^{\omega}-1\right)\right]}\right]\right\}^{\frac{1}{1-\sigma^{\omega}}}$. See Appendix C.1.3 for a derivation of (13).

costs) in country j's total expenditure is $\theta/(1+\theta)$.[18] Intuitively, a lower value of θ corresponds to a fatter upper tail of the Pareto distribution from which the efficiency of a new technique is drawn. This implies that, on average, the cost reductions associated with a successful new technique are higher for lower values of θ.[19] Consequently, low values of θ translate into higher average markups and a higher (lower) share of profits (costs) in total revenue.

Given that the distribution of costs and markups is independent of the source country, the share of total production costs in country i's sales to country j is also given by $\theta/(1+\theta)$. Since this is true for any destination country j, we obtain

$$w_{it} L_{it}^{q,\omega} = \frac{\theta}{1+\theta} R_{it}^{\omega}, \tag{14}$$

where $L_{it}^{q,\omega}$, R_{it}^{ω} are the total number of workers employed and the total revenues/sales generated by country i's manufacturing firms in industry ω, respectively.

Finally, market clearing at the industry level implies that the total revenue of country i's firms producing goods in industry ω, R_{it}^{ω}, equals the world expenditure on industry-ω-goods produced in country i,

$$R_{it}^{\omega} = \sum_{j=1}^{N} \lambda_{ijt}^{\omega} E_{jt}^{\omega}. \tag{15}$$

2.2.3 Research Sector

Firms in the research sector invest in R&D to obtain new production techniques that improve the efficiency with which final goods are produced. Research firms finance their R&D activity issuing equity claims that pay nothing if research efforts fail to improve upon the state of the art technique for some good, but entitle their holders to the stream of future profits if research succeeds. Given that no financial assets are traded, the savings of domestic households are the only source of financing for research firms.

Given that there is a continuum $[0,1]$ of identical firms directing their research efforts to industry ω, and that the risks associated with the R&D efforts are independent across firms, well-diversified equity holders can obtain a deterministic return from their equity investment. Consequently, the equilibrium price of the equity claims issued by the research firms equals their expected return.

Let V_{ijt}^{ω} denote the expected present value at time t of the stochastic future stream of profits generated by an idea from country i in country j, conditional on the idea beating the state of the art in country j at time t. Then

$$V_{ijt}^{\omega} = \mathbb{E}_t \left[\int_t^{\infty} e^{-\int_t^s r_{iu} du} \pi_{js}^{\omega}(z)\, ds \right],$$

where $\pi_{js}^{\omega}(z)$ are the profits at time s of the firm producing good (z,ω) in country j. Notice that the expected profits generated in country j at any future time $s > t$ are equal to average profits at s, multiplied by the probability at time t that the idea is still the state of the art in that country at time s, with this probability given by $\Phi_{jt}^{\omega}/\Phi_{js}^{\omega}$.[20] Using this together with the fact that the share of profits in the sales to

[18] See Appendix C.1.4 for a proof.

[19] A new technique is successful if it becomes the most efficient technique to serve the domestic market.

[20] See Appendix C.1.5 for a proof.

country j is $1/(1+\theta)$, the last expression becomes[21]

$$V_{ijt}^{\omega} = \int_t^{\infty} e^{-\int_t^s \left[r_{iu}+\widehat{\Phi}_{ju}^{\omega}\right]du} \frac{E_{js}^{\omega}}{1+\theta} ds. \tag{16}$$

As the last expression clearly shows, in evaluating V_{ijt}^{ω}, future average profits must be discounted at the augmented rate $r_{iu} + \widehat{\Phi}_{ju}^{\omega}$. As more ideas are discovered and the technological frontier in country j grows, some of the firms serving that market at time t are driven out of business by more efficient firms. This endogenous termination probability that reduces the expected value of profits of any firm producing at time t is captured by the additional term $\widehat{\Phi}_{ju}^{\omega}$ in the discount rate.

Given that the price of an equity claim issued by a research firm equals its expected value, any research firm maximizes the expected returns from the R&D activity. On the one hand, for any firm in country i using $l_t^{R,\omega}$ researchers to target industry ω, the expected benefit over the interval dt is $\frac{\iota_i^{\omega}\left(l_t^{R,\omega}\right)^{\upsilon}}{T_{it}^{\omega}}\left[\sum_{j=1}^{N} \lambda_{ijt}^{\omega} V_{ijt}^{\omega}\right] dt$, where $\iota_i^{\omega}\left(l_{it}^{R,\omega}\right)^{\upsilon} dt$ is the probability of having and idea in the interval dt, $1/T_{it}^{\omega}$ is the probability that the idea beats the technological frontier in country i and λ_{ijt}^{ω} is the probability of beating the state of the art technique in country j conditional on beating the frontier in country i. On the other hand, the costs for the firm are simply the wages paid to researchers $w_t l_t^{R,\omega}$. The first-order condition that characterizes the solution to this problem for each individual firm yields the aggregate profit maximization in research condition

$$w_{it} = \frac{\iota_i^{\omega}\upsilon\left(L_{it}^{R,\omega}\right)^{\upsilon-1}}{T_{it}^{\omega}}\left[\sum_{j=1}^{N} \lambda_{ijt}^{\omega} V_{ijt}^{\omega}\right] \tag{17}$$

for all ω, where $L_{it}^{R,\omega}$ is the total number of researchers targeting industry ω.

It is worth mentioning that, in this setting, research firms make positive profits in equilibrium. However, the presence of these profits does not affect the households' saving decisions nor the direction of innovation efforts in the economy.

2.2.4 Closing the Model and Definition of Equilibrium

Balanced Trade. — Since there is no trade in financial assets, households in country i can save only in equity claims issued by the domestic research sector. Consequently, at any moment in time, total income is equal to the sum of total expenditure in final goods and total purchases of domestic equity claims

$$R_{it} + \int_{\Omega}\left[\Pi_{it}^{R,\omega} + w_{it}L_{it}^{R,\omega}\right]d\omega = \int_{\Omega} I_{it}^{\omega} d\omega + E_{it},$$

where $R_{it} \equiv \int_{\Omega} R_{it}^{\omega} d\omega$ is the total revenue generated by manufacturing firms,[22] $\Pi_{it}^{R,\omega}$ are the total profits generated by the research firms in industry ω and I_{it}^{ω} is the total value of household's purchases of equity

[21] From the analysis of the costs share in revenues we know that the total production costs in country i's sales to country j is given by $\theta/(1+\theta)$, which implies that the share of profits is $1/(1+\theta)$.

[22] Recall that $R_{it}^{\omega} \equiv \Pi_{it}^{q,\omega} + w_{it}L_{it}^{q,\omega}$, where $\Pi_{it}^{q,\omega}$ are total profits generated by manufacturing firms in class ω.

claims issued by research firms in industry ω. Noticing that by definition, the total revenue of research firms in industry ω is equal to I_{it}^{ω}, the last expression implies that trade is balanced every period[23]

$$R_{it} = E_{it}. \tag{18}$$

Labor Market Clearing.— The labor market clearing condition requires that the sum of the total number of production workers and researchers allocated to all industries equals the total endowment of labor at each moment in time. Recalling that $L_{it}^{q,\omega}$ represents the production workers employed in industry ω and that $L_{it}^{R,\omega}$ denotes the total number of researchers directing their research efforts to the same industry, the labor market clearing condition can be written as

$$L_{it} = \int_{\Omega} \left[L_{it}^{q,\omega} + L_{it}^{R,\omega} \right] d\omega. \tag{19}$$

In the rest of the paper I use $L_{it}^{\omega}, L_{it}^{q}, L_{it}^{R}$ to denote total workers devoted to industry ω, total workers employed in the manufacturing sector and total workers employed as researchers respectively, i.e.,

$$L_{it}^{\omega} = L_{it}^{q,\omega} + L_{it}^{R,\omega}, \qquad L_{it}^{q} = \int_{\Omega} L_{it}^{q,\omega} d\omega, \qquad L_{it}^{R} = \int_{\Omega} L_{it}^{R,\omega} d\omega.$$

Having described all the components of the economy, we can now state a formal definition of the equilibrium.

Definition 1 *A market equilibrium is a set of functions* r_{it}, w_{it}, P_{it}, R_{it}, E_{it}, C_{it} : $[0, \infty) \to \mathbb{R}_+$ *and* C_{it}^{ω}, P_{it}^{ω}, R_{it}^{ω}, E_{it}^{ω}, V_{it}^{ω}, $L_{it}^{q,\omega}$, $L_{it}^{R,\omega}$, T_{it}^{ω} : $\Omega \times [0, \infty) \to \mathbb{R}_+$ *for each country* $i = 1, ..., N$ *such that conditions (1),(6)-(19) hold.*

3 Balanced Growth Path

In this section I will focus on the balanced growth path of the economy in which P_{it}, R_{it}, E_{it}, C_{it}, C_{it}^{ω}, P_{it}^{ω}, R_{it}^{ω}, E_{it}^{ω}, V_{it}^{ω}, $L_{it}^{q,\omega}$, $L_{it}^{R,\omega}$, T_{it}^{ω} grow at constant rates for all countries i and industries ω. As a general rule, throughout the rest of the paper I omit the subscript t when referring to the BGP-level of variables that are constant in the BGP.

3.1 Solving for the BGP

Growth Rates and Research Intensity.— In the model presented in the last section, growth rates in the BGP depend only on the exogenous rate of population growth and technological parameters. In particular, BGP-growth rates are not affected by trade or standard policies such as taxes, R&D subsidies, etc; a feature that is common to semi-endogenous growth models. This means that the effects of trade through the new margin of adjustment introduced by directed research will only affect the BGP-*levels* of

[23] Trade is not required to be balanced in each industry ω, implying that in general $R_{it}^{\omega} \neq E_{it}^{\omega}$.

variables of interest such as technology, consumption, price level, etc. These level-effects are the focus of the analysis in this paper.

For this reason, I start the analysis of the BGP with Lemma 1, which describes those elements in the model that are not affected by the additional margin of adjustment proposed in this model.

Lemma 1 *In any balanced growth path of this economy the following condition holds:*
(i) wages, trade shares and interest rates are constant;
(ii) the interest rate is the same across countries:

$$r_{it} = r = [n + n\upsilon/\theta]\,\eta + \rho - n\upsilon/\theta$$

(iii) growth rates are given by[24]

$$\widetilde{L}_{it}^{q,\omega} = \widetilde{L}_{it}^{R,\omega} = \widetilde{R}_{it}^{\omega} = \widetilde{V}_{ijt}^{\omega} = n, \qquad \widetilde{T}_{it}^{\omega} = \upsilon n, \qquad \widetilde{P}_{it} = -n\upsilon/\theta, \qquad \widetilde{C}_{it} = n + n\upsilon/\theta$$

(iv) the research intensity $L_{it}^{R,\omega}/L_{it}^{\omega}$ is constant and it is the same for all industries and countries:

$$\kappa\,(n,\rho,\eta,\theta,\upsilon) \equiv \frac{L_{it}^{R,\omega}}{L_{it}^{\omega}} = \frac{\upsilon^2 n}{\upsilon^2 n + \theta\,[r - (1-\upsilon)\,n]}$$

Proof. See Appendix C.2.1.

A higher rate of population growth n raises the expected profits from R&D through a higher expected increase in the size of the market for successful ideas, leading to more innovation and growth as we can see in (iii).

High values of υ and low values of θ are associated with better R&D possibilities since they represent weaker decreasing returns in R&D and a fatter upper tail of the distribution from which the efficiency of an idea is drawn, respectively. These better R&D possibilities are reflected in higher growth rates for the stock of ideas, and the consumption aggregator as we can see in (iii).

The first two terms in the expression for the interest rate in (ii) represent the real interest rate, while the last term is the change in the price level. Notice that the expression in brackets in the first term of (ii) is just the growth rate of the consumption aggregator, $\widetilde{C}_{it}$. The higher $\widetilde{C}_{it}$ is, the steeper is the expected increase in consumption over time, which leads individuals wanting to smooth their consumption to increase their borrowing at any given rate, pushing up the equilibrium real interest rate.

A few things from Lemma 1 are worth emphasizing. First, notice that country size, research productivity and openness have no effect on the BGP-growth rates or the BGP-research intensity. Second, the results of Lemma 1 are valid for any number of industries, including the single industry case; this means that extension of the model to allow for multiple industries and directed research has no effect on BGP's growth rates, interest rates or the research intensity. Consequently, all the additional effects on

[24] For any variable X, $\widetilde{X}_t \equiv \dot{X}_t/X_t$ denotes its instantaneous growth rate.

the innovation process brought about by the additional margin of adjustment emphasized in this paper are reflected on the levels of manufacturing technology, a topic to which I turn next.

Directed Research and Endogenous Manufacturing Technology.— Let us now turn to the analysis of the new distinctive feature of the model with innovation and directed research: the endogeneity of the BGP-distribution of levels of manufacturing technology across industries. R&D investment decisions are made by profit maximizing firms who weigh the costs and the expected benefits from those investments. A fundamental factor affecting the expected benefits of R&D is the expected size of the market for a future innovation. For this reason, a key variable in the study of the determinants of country i's BGP-levels of technology is its market share in industry ω, $\beta_i^{R,\omega}$, defined as the ratio of country i's total sales in industry ω, R_{it}^{ω}, to the world expenditure in the same industry, E_t^{ω}, i.e., $\beta_i^{R,\omega} \equiv R_{it}^{\omega}/E_t^{\omega}$.[25] As shown later, $\beta_i^{R,\omega}$ is constant in the BGP, and consequently it also represents the fraction of present and future profits generated by world expenditure on industry ω accruing to firms in country i.

With this definition in mind, we can combine the definition of $\widetilde{T}_{it}^{\omega}$, Lemma 1.iii, and (17) to get[26]

$$T_{it}^{\omega} = B_T \iota_i^{\omega} \left[\beta_i^{R,\omega} V_t^{\omega}/w_i\right]^{\upsilon} \tag{20}$$

where V_t^{ω} is the present value of profits generated by the stream of world expenditure $\{E_s^{\omega}\}_{s\geq t}$ in the industry; and $B_T \equiv n^{\upsilon-1}\upsilon^{2\upsilon-1}$ is a constant.

The last equation clearly shows all the elements affecting the BGP-level of manufacturing technology in any given industry: the research productivity of the country in the industry, ι_i^{ω}[27]; the size of the market captured by country i's firms, $\beta_i^{R,\omega}V_t^{\omega}$; and the R&D-input costs given by the wage paid to researchers w_i. The effects of ι_i^{ω} and w_i on T_{it}^{ω} are straightforward: everything else equal, a higher research productivity or lower R&D-input costs induce more innovation that is ultimately reflected in a higher manufacturing productivity. To understand the effect of the actual market size captured by country i's firms, notice that in the BGP, $\beta_i^{R,\omega}V_t^{\omega}$ also represents the expected present value of profits for an idea from country i that surpasses the domestic frontier.[28] Consequently a larger market $\beta_i^{R,\omega}V_t^{\omega}$ induces more innovation which is reflected in a higher T_{it}^{ω}.

It should be clear that equation (20) is an equilibrium relationship between endogenous variables and as such, it does not allow to connect directly T_{it}^{ω} to exogenous parameters.[29] Nevertheless, it helps understand how the distribution of the BGP-levels of manufacturing technology across industries is determined. Specifically, it identifies the research productivity ι_i^{ω}, the market size captured by country i's firms $\beta_i^{R,\omega}V_t^{\omega}$, and the cost of researchers w_i, as the only aggregate channels through which all exogenous parameters of the model can affect manufacturing productivity. In particular, factors such as openness, comparative advantage in innovation, country size and home market effects,[30] all affect T_{it}^{ω} through their effect on

[25] $E_t^{\omega} \equiv \sum_{i=1}^{N} E_{it}^{\omega}$.

[26] See Appendix C.2.2 for a derivation.

[27] The relatinship between T_{it}^{ω} and ι_i^{ω} in (20) captures only the direct effect of ι_i^{ω} on T_{it}^{ω}, without considering the potential indirect effects through $\beta_i^{R,\omega}V_t^{\omega}/w_i$.

[28] The domestic frontier is determined by the most efficient techniques discovered in the country of reference.

[29] In fact, no closed form solutions exist for T_{it}^{ω} in terms of primitives except for the autarky case.

[30] See Appendix C.2.9 for a formal discussion of home market effects in this model.

$\beta_i^{R,\omega} V_t^{\omega}$ and w_i.

Notice that the derivation of equation (20) does not depend on the number of industries in the model, implying that it is also valid for the single industry case in which the additional margin of adjustment introduced by directed research is not operational. However, there is a fundamental difference between the single-industry and the multi-industry case that cannot be appreciated clearly from equation (20): when $\Omega = 1$, and no directed research takes places, the degree of openness has no effect on the BGP-level of technology. The reason for this is that for any country, openness brings about two opposing effects on innovation: on the one hand, there is a positive effect on innovation granted by the easier access to foreign markets experienced by domestic firms; on the other hand, there is a negative effect on innovation caused by the increased competition faced by those same firms in their domestic markets. In the single-industry case, these two opposing effects exactly cancel out, leaving the BGP-level of manufacturing technology unchanged.

In contrast, in the multi-industry case, when a country opens up to trade it reallocates its research efforts towards those industries in which it has comparative advantage. This reallocation of resources implies that the two opposing effects on innovation described above do not cancel out at the industry level, and consequently openness has an effect on the distribution of BGP-levels of manufacturing technology.

These effects can be seen more clearly with the help of the next equation that shows how the BGP-level of manufacturing technology relates to the resources allocated to each industry. Letting $\delta_i^{\omega} \equiv L_{it}^{\omega}/L_{it}$ denote the share of the total labor force allocated to industry ω, the definition of $\widehat{T}_{it}^{\omega}$ and points iii and iv of Lemma 1 yield[31]

$$T_{it}^{\omega} = B_T' \iota_i^{\omega} \left[\delta_i^{\omega} L_{it} \right]^{\upsilon}, \tag{21}$$

where $B_T' \equiv (\upsilon n)^{-1} \kappa^{\upsilon}$ is a constant.

The last equation is particularly helpful in showing how openness can have an effect on the level of manufacturing technology in the presence of directed research. To see this consider first the single-industry case, in which there is no directed research by construction. In this case, all the resources must be allocated to the unique existing industry regardless of the degree of openness, i.e., $\delta_i^{\omega} = 1$ for the only industry ω in the economy. Consequently, equation (21) implies that in the single-industry case, the BGP-level of technology of a country depends only on its research productivity and on the size of its labor force, $T_{it}^{\omega} = B_T' \iota_i^{\omega} L_{it}^{\upsilon}$, and it is not affected by trade.

In contrast, in the multi-industry case with directed research, not only does trade affect the allocation of production across industries –a feature present in almost any trade model–, but also it affects the allocation of research efforts across those industries, which is ultimately reflected in the distribution of BGP-levels of manufacturing technology as we can see from (21).

The effects of specialization and trade on the allocation of resources across industries in the BGP can

[31] See Appendix C.2.3 for the derivation.

be seen with the help of the next equation[32]

$$\delta_i^\omega = \frac{\beta_i^{R,\omega}}{\beta_i^R}\alpha^\omega = \frac{\beta_i^{R,\omega}}{\beta_i^{E,\omega}}\alpha_i^\omega, \tag{22}$$

where the variable $\alpha^\omega \equiv E_t^\omega/E_t$ is the share of world expenditure allocated to industry ω, $\beta_i^R \equiv R_{it}/R_t$ is the share of country i in world output[33], and $\beta_i^{E,\omega} \equiv E_{it}^\omega/E_t^\omega$ is the share of country i's expenditure in world expenditure in the industry.

The first equality in (22) clearly shows the effects of specialization and world demand on manufacturing technology. On the one hand, notice that the ratio $\beta_i^{R,\omega}/\beta_i^R$ can be interpreted as a measure of specialization: a value of this ratio above one means that country i contributes more to world output in industry ω than what it does to total world output, reflecting a specialization in production and R&D that is the result of more primitive supply and demand factors such as comparative advantage in innovation, geography and relatively large domestic market in the industry, among others. On the other hand, α^ω reflects the effects of world demand on δ_i^ω: a greater world demand for goods in industry ω -as captured by α^ω- is associated with more production and R&D in that industry in every country, leading to a worldwide higher BGP-level of the manufacturing technology in the industry.

The second equality in (22) is particularly useful in understanding the effects of trade on δ_i^ω and T_{it}^ω. To see this consider first its implications for an autarkic economy. In autarky, whatever a country produces in an industry must be consumed in the domestic market, i.e., production must equal expenditure in every industry or $\beta_i^{R,\omega}/\beta_i^{E,\omega} = 1$ for all ω. In this case, the allocation of resources across industries is driven only by demand conditions in country i as captured by α_i^ω. In contrast, when a country trades with the world, trade does not need to be balanced in each industry and the ratio $\beta_i^{R,\omega}/\beta_i^{E,\omega}$ can differ from one: if $\beta_i^{R,\omega}/\beta_i^{E,\omega} > 1$ (< 1), then country i is a net exporter (importer) in that industry and its level of technology is higher (lower) in that industry relative to an autarkic economy that shares the same fundamental parameters.

Finally, equations (21) and (22) clearly show the fundamental role played by the parameter υ in controlling the relevance of the new margin emphasized in this paper: the weaker the decreasing returns to R&D are (high υ), the stronger the endogenous effects of directed research on manufacturing technology.

Existence of a Balanced Growth Path.— The next proposition gathers all the equations from which the BGP of this economy is obtained and guarantees the existence of a solution. Although the system can be simplified even further, the system of equations presented in the proposition clearly identifies the new elements of the model and facilitates the comparison with a benchmark model with no directed research.

Proposition 1 *The balanced growth path values corresponding to trade shares λ_{ij}^ω; countries' market shares $\beta_i^{R,\omega}$ and β_i^R, countries' expenditure shares $\beta_i^{E,\omega}$ and β_i^E, world-wide industries' expenditure shares*

[32] See Appendix C.2.4 for the derivation.

[33] In the definition of β_i^R, R_{it} denotes country i's total output, $R_{it} \equiv \int_\Omega R_{it}^\omega d\omega$; and R_t denotes world output, $R_t \equiv \sum_{i=1}^N R_{it}$. The letter R in this notation stands for revenue.

α^ω, countries' levels of manufacturing technology T_{it}^ω and wages w_i are obtained as a solution to the following system of equations

$$
\begin{aligned}
\lambda_{ij}^{\omega} &= \frac{T_{it}^{\omega}\left(w_i\tau_{ij}^{\omega}\right)^{-\theta}}{\sum_{k=1}^{N} T_{kt}^{\omega}\left(w_i\tau_{kj}^{\omega}\right)^{-\theta}} \quad (23.1) &\qquad \beta_i^R &= \int_\Omega \alpha^\omega \beta_i^{R,\omega} d\omega \quad (23.5) \\
\beta_i^{R,\omega} &= \sum_{j=1}^{N} \lambda_{ij}^{\omega}\beta_j^{E,\omega} \quad (23.2) & \alpha^\omega &= \sum_{j=1}^{N} \alpha_j^\omega \beta_j^E \quad (23.6) \\
\beta_i^E &= \beta_i^R \quad (23.3) & \beta_i^{E,\omega} &= \frac{\alpha_i^\omega \beta_i^E}{\alpha^\omega} \quad (23.7) \\
T_{it}^{\omega} &= B_T' \iota_i^\omega \left[\frac{\beta_i^{R,\omega}}{\beta_i^R}\alpha^\omega L_{it}\right]^{\upsilon} \quad (23.4) & \beta_i^R &= \frac{w_i L_{it}}{\sum_{j=1}^{N} w_i L_{it}} \quad (23.8)
\end{aligned}
\qquad (23)
$$

for all i,ω. Moreover, for $\upsilon \in (0,1)$ a solution to the system exists with $\beta_i^{R,\omega} > 0$ for all i, ω.

Proof. See Appendix C.2.5.

The presentation of the system of equations (23) in Proposition 1 allows us to distinguish two set of equations. The set of equations in the first column represent structural equations of the model stated previously in the text but now expressed in terms of shares: the first line reproduces the expression for trade shares obtained in (11); the second line corresponds to equation (15) that represents the market clearing condition for the output of each country in each industry; the third line is the balanced trade condition (18); and the fourth line combines the technology relationships (21) and (22). The set of equations in the second column express all the relationships between expenditure shares, $\beta_i^{E,\omega}, \beta_i^E$, market shares $\beta_i^{R,\omega}, \beta_i^R$, industries' shares in world demand α^ω and wages w_i that follow immediately from their definitions.

Proposition 1 clearly shows the two-way relationship between manufacturing technology and market shares in the present model. One direction of this relationship, which is present in any Ricardian model of trade, goes from technology levels to market shares as we can see from equations (23.1) and (23.2). The second direction of this relationship, which is the distinctive element of the model with directed research, goes from market shares to manufacturing technology. This reflects the feedback from underlying supply and demand conditions to innovation as we can appreciate from equation (23.4).

Notice that with the exception of equation (23.4), the rest of the system (23) contains the exact same equilibrium equations corresponding to a multi-industry benchmark model with no endogenous innovation, in which the distribution of firms' productivities within an industry is given by (2) for some exogenous scale parameter T_{it}^ω, and in which the structure of the manufacturing sector is the same as in the present model.[34] Relative to such a benchmark, Proposition 1 shows that the effect of the new

[34] The no innovation benchmark model described corresponds to the case of a multi-industry BEJK (2003) model in which the parameter θ is the same across industries. It also share the same equilibrium equations with a multi-industry EK(2002) model with common θ across industries. Some studies using models with this structure include Costinot, Donaldson and Komunjer (2011), Chor (2010) and Shikher (2011), among others.

margin of adjustment introduced by directed research on the BGP-levels of manufacturing technology is completely captured by equation (23.4).

The observations in the last paragraph have the following implications. First, the parameter υ, that captures the contemporaneous decreasing returns in R&D, controls the relevance of the effects of directed research on manufacturing technology. In particular, when this parameter is set to zero, all the endogenous adjustments of technology are eliminated and we are back to the no innovation benchmark model for some given initial levels of technology. In this sense, the present model nests the benchmark model with no innovation.

Second, notice that relative to the model with no innovation, the introduction of directed research adds only one additional parameter to the total number of parameters that need to be estimated if we are only interested in the BGP-predictions of the model. Moreover, armed with an estimate of υ, any method used to estimate the manufacturing productivities T_{it}^{ω} in the model with no innovation can be used to recover the underlying research productivities ι_i^{ω} through equation (23.4). Consequently, all the additional estimation burden imposed by the present model is related to the estimation of υ.

Third, the fact that the model with and without innovation share the same cross-sectional structure imply that both models perform equally well in matching trade, production and consumption data in the cross-section. Moreover, if we allow for exogenous trade deficits in (23.3), the two models can be estimated to match exactly the aforementioned data and to share all exogenous parameters and T_{it}^{ω}. Nevertheless, even if the two models are set up in this way, they still differ in their counterfactual predictions regarding the changes in trade flows, manufacturing technology and welfare associated with different shocks, all of which are relevant dimensions in policy analysis.

3.2 Trade and Comparative Advantage in Production

In this subsection I study the determinants of manufacturing comparative advantage in the BGP. I show that it can be decomposed into an exogenous and an endogenous component and I analyze the role of trade costs in shaping the later. In the analysis I emphasize the role of the decreasing returns in R&D –captured by υ– in controlling the extent to which trade can affect innovation, production and export specialization patterns. In addition, the analysis in this subsection provides the theoretical foundation for the main estimation strategy of the parameter υ.

Given that comparative advantage is the focus of the analysis that follows, before continuing it is convenient to precisely define it. For any two pairs of countries i, i' and any two pair of industries ω, ω', country i has comparative advantage in production in industry ω if $T_{it}^{\omega}/T_{it}^{\omega'} > T_{i't}^{\omega}/T_{i't}^{\omega'}$. Similarly, country i has comparative advantage in innovation in industry ω if $\iota_i^{\omega}/\iota_i^{\omega'} > \iota_{i'}^{\omega}/\iota_{i'}^{\omega'}$. For this reason, I generally refer to the distribution of the double ratios $\left(T_{it}^{\omega}/T_{it}^{\omega'}\right) / \left(T_{i't}^{\omega}/T_{i't}^{\omega'}\right)$ and $\left(\iota_i^{\omega}/\iota_i^{\omega'}\right) / \left(\iota_{i'}^{\omega}/\iota_{i'}^{\omega'}\right)$ as comparative advantage in production and innovation respectively.

Taking double ratios in equation (20) for any pair of countries i, i' and any pair of industries ω,ω', yields

$$\underbrace{\frac{T_{it}^{\omega}/T_{i't}^{\omega}}{T_{it}^{\omega'}/T_{i't}^{\omega'}}}_{\substack{\text{Comparative Adv.}\\ \text{in Production}}} = \underbrace{\frac{\iota_i^{\omega}/\iota_{i'}^{\omega}}{\iota_i^{\omega'}/\iota_{i'}^{\omega'}}}_{\substack{\text{Exogenous}\\ \text{Comparative Adv.}\\ \text{in Innovation}}} \times \underbrace{\left(\frac{\beta_i^{R,\omega}/\beta_{i'}^{R,\omega}}{\beta_i^{R,\omega'}/\beta_{i'}^{R,\omega'}}\right)^{\upsilon}}_{\substack{\text{Endogenous}\\ \text{Component}}}. \tag{24}$$

The last equation expresses comparative advantage in production (LHS) as the product of the exogenous comparative advantage in innovation, and an endogenous term that reflects the effects of specialization in R&D on manufacturing technology. While the complexity of the interactions among innovation, production and prices precludes any analytic characterization of the endogenous component in (24) in terms of exogenous parameters for the general case of Proposition 1, there are two special cases that yield a closed form characterization of comparative advantage in production: the case of frictionless trade (zero gravity), in which $\tau_{ij}^{\omega} = 1$ for all i, j, ω; and the case in which geographic barriers are prohibitive (autarky), meaning that $\tau_{ij}^{\omega} \to \infty$ for $j \neq i$.[35]

The next lemma characterizes the comparative advantage in production $\left(T_{it}^{\omega}/T_{it}^{\omega'}\right) / \left(T_{i't}^{\omega}/T_{i't}^{\omega'}\right)$ in terms of exogenous parameters of the model for the two extreme cases of autarky and zero gravity.

Lemma 2 *Letting the subscripts a, zg denote autarky and zero gravity respectively, for any pair of countries i,i′ and any pair of industries ω, ω′, the comparative advantage in production is given by*
(i) in autarky

$$\frac{T_{it,a}^{\omega}/T_{it,a}^{\omega'}}{T_{i't,a}^{\omega}/T_{i't,a}^{\omega'}} = \frac{\iota_i^{\omega}/\iota_i^{\omega'}}{\iota_{i'}^{\omega}/\iota_{i'}^{\omega'}} \left[\frac{\alpha_i^{\omega}/\alpha_i^{\omega'}}{\alpha_{i'}^{\omega}/\alpha_{i'}^{\omega'}}\right]^{\upsilon} \tag{25}$$

(ii) in a zero gravity world

$$\frac{T_{it,zg}^{\omega}/T_{it,zg}^{\omega'}}{T_{i't,zg}^{\omega}/T_{i't,zg}^{\omega'}} = \frac{\iota_i^{\omega}/\iota_i^{\omega'}}{\iota_{i'}^{\omega}/\iota_{i'}^{\omega'}} \left[\frac{\iota_i^{\omega}/\iota_i^{\omega'}}{\iota_{i'}^{\omega}/\iota_{i'}^{\omega'}}\right]^{\frac{\upsilon}{1-\upsilon}} \tag{26}$$

Proof. See Appendix C.2.6.

Lemma 2 clearly shows the contrast between the two extreme cases that it considers. On the one hand we have the autarky case portrayed in equation (25) in which the endogenous component of comparative advantage depends on countries' relative expenditure shares allocated to each industry. Given that in autarky countries must produce what they consume, autarkic economies innovate more and produce more in those industries in which their domestic demand is higher. The effect of this demand-induced specialization in innovation on manufacturing comparative advantage is captured in the term in brackets in (25). From the same expression we can see how parameter υ controls the importance of these endogenous

[35] Although the case of autarky allows for a full analytic solution of the model, this is not the case in the case of zero gravity, since there is no closed form solution for relative wages. However, the structure of the model implies that relative wages do not affect manufacturing comparative advantage which is what allows for a closed-form characterization.

adjustments: in the extreme case of $\upsilon = 0$, none of these endogenous adjustments affects the distribution of manufacturing technology which is fixed at some given exogenous level.

In contrast, in a zero gravity world, the *relative* specialization patterns of innovation and production are no longer affected by domestic demand conditions; instead, they reflect fundamental differences in innovation capabilities across industries as captured by comparative advantage in innovation. When countries open up to trade they direct their research efforts towards those industries in which they have comparative advantage in innovation. Over time, as the result of innovation efforts translates into more efficient techniques, the evolution of the distribution of manufacturing technology starts to reflect the underlying specialization in innovation, which ultimately leads to the distribution of comparative advantage in production in Lemma 2.ii.

The characterization of technology in the extreme cases of autarky and zero gravity given in Lemma 2 clearly shows the extent to which trade can affect manufacturing comparative advantage across industries. In particular, Lemma 2 has the following implications.

First, as trade costs falls and countries become more integrated, the observed dispersion in the profile of comparative advantage could rise or fall depending on the relative dispersions of relative local demand conditions and comparative advantage in innovation. In particular, using log standard deviation as a measure of dispersion we get

$$sd\left(\log\left(\frac{T^{\omega}_{it,a}/T^{\omega'}_{it,a}}{T^{\omega}_{i't,a}/T^{\omega'}_{i't,a}}\right)\right) \geq sd\left(\log\left(\frac{T^{\omega}_{it,zg}/T^{\omega'}_{it,zg}}{T^{\omega}_{i't,zg}/T^{\omega'}_{i't,zg}}\right)\right) \Longleftrightarrow sd\left(\log\left(\frac{\alpha^{\omega}_{i}/\alpha^{\omega'}_{i}}{\alpha^{\omega}_{i'}/\alpha^{\omega'}_{i'}}\right)\right) \geq \frac{1}{1-\upsilon} sd\left(\log\left(\frac{\iota^{\omega}_{i}/\iota^{\omega'}_{i}}{\iota^{\omega}_{i'}/\iota^{\omega'}_{i'}}\right)\right)$$

Why should we care about the dispersion in the profile of comparative advantage? Recall that in a standard Ricardian model –corresponding to $\upsilon = 0$–, relative technological differences are the source of the gains from trade. Consequently, analyzing the effects on the gains from trade of observed changes in comparative advantage in production through the lenses of a standard Ricardian model may lead to incorrect conclusions if those changes are themselves the consequence of endogenous changes in innovation induced by changes in the trading environment.

Second, reductions in trade costs could potentially reverse the profile of comparative advantage across countries if the difference between relative domestic demand conditions and relative innovation capabilities across industries is sufficiently large. Moreover, if the contemporaneous decreasing returns to R&D are not too strong, reductions in trade costs can even generate a reversal in the export profile of countries. To see this consider the effects on the trade balance of increasing trade costs from a zero gravity world to autarky in the following context: for some countries i, i' and industries ω, ω', country i has comparative advantage in innovation in industry ω' and at the same time it has a relatively stronger demand for goods in industry ω. When there are no frictions to trade, country i's relatively strong demand for goods in industry ω together with its comparative advantage in the production of goods in industry ω' imply that the country is a net importer in industry ω.[36] As transport costs increase, countries reallocate their production and innovation efforts towards those goods in which they have a strong domestic demand. In the example given, the effects of these reallocations on technology and production improve the trade

[36] Because of balanced trade, this implies that country i is net exporter of goods in industry ω' in a zero gravity world.

balance of country i in industry ω. Whether or not these changes in innovation and production patterns can change the sign of country i's trade balance in industry ω, depends on how strong the endogenous adjustments in manufacturing technology are; if the decreasing returns to R&D are sufficiently week, then the adjustment on manufacturing technology can be strong enough to reverse the export profile of countries. The next lemma formalizes this argument for the case of two mirror symmetric countries and two industries.

Lemma 3 *Consider an economy with two mirror symmetric*[37] *countries $i = 1, 2$ and two industries ω, ω' in which trade cost are symmetric and uniform across industries, i.e. $\tau_{ij}^{\omega} = \tau$ for all ω, i, j such that $i \neq j$. Assume that country 1 has a relative strong demand for goods in industry ω and a comparative advantage in innovation in industry ω' such that the following condition holds*

$$\frac{\alpha_1^{\omega}/\alpha_2^{\omega}}{\alpha_1^{\omega'}/\alpha_2^{\omega'}} > 1 > \frac{\iota_1^{\omega}/\iota_2^{\omega}}{\iota_1^{\omega'}/\iota_2^{\omega'}} \tag{27}$$

Then, countries will display a reversal in their export profile as they move from autarky to frictionless trade if, and only if,[38]

$$\left(\frac{\alpha_1^{\omega}}{\alpha_2^{\omega}}\right)^{\upsilon-\frac{1}{2}} > \frac{\iota_2^{\omega}}{\iota_1^{\omega}} \tag{28}$$

Proof. See Appendix C.2.7.

The possibility of reversals in the export profile of countries is closely connected to the presence of home market effects in the model. In particular, condition (28) implies that a reversal in the export profile of countries never arises if $\upsilon \in [0, 1/2)$, which is the same range of values of υ for which the model does not exhibit home market effects.[39] In this sense, Lemma 3 provides a theoretical threshold for the parameter υ above which the endogenous adjustments in technology are strong enough to allow for the possibility of home market effects and potential reversals in the export profile of countries.

Finally, Lemma 2 implies that in any trading equilibrium that is far from the extreme cases of autarky and zero gravity, we should expect the endogenous component of comparative advantage to be correlated with both comparative advantage in innovation and relative domestic demand. This observation plays an important role in the estimation of parameter υ.

3.3 The BGP in Changes

When performing counterfactual analysis across BGPs, I extend the approach popularized by Deckle, Eaton and Kortum (2007) (henceforth DEK) to include the effect of innovation and directed research and solve the BGP of the model in changes. When the model is solved in changes, only a subset of parameters is needed to perform any given counterfactual evaluation. In particular, this approach does not rely on estimates of unobserved structural parameters such as innovation capabilities or trade costs. In addition,

[37] The countries are mirror images of each other. See the proof of the Lemma in the appendix for a precise definition.
[38] This corresponds to a variation in $\tau^{-\theta}$ from 0 to 1.
[39] See Appendix C.2.9 for a formal discussion of the home market effect in this model.

the system of equations of the model in changes encompasses the system of equations corresponding to a multi-industry EK model as a special case. This implies that armed with an estimate of the parameter capturing the decreasing returns in R&D, performing counterfactual analysis in the model with directed research does not impose any additional data requirement over the benchmark model with no innovation.

The next corollary summarizes the extension of the DEK's method to the present model.

Corollary 1 *Let $\widehat{X} \equiv X'/X$ denote the relative change in variable X from X to X'. Given the constant parameters υ, θ and information about the endogenous trade shares λ_{ij}^{ω}, countries' market shares $\beta_i^{R,\omega}$ and β_i^R, countries' expenditure shares $\beta_i^{E,\omega}$ and β_i^E, and world-wide industries' expenditure shares α^{ω} in the initial BGP; the change in those same endogenous variables between the initial and the new BGP associated with exogenous changes in research productivities $\widehat{\iota}_i^{\omega}$, labor endowments $\widehat{L}_{it}$, preference parameters $\widehat{\alpha}_i^{\omega}$ and trade costs $\widehat{\tau}_{ij}^{\omega}$ can be computed as*

$$\widehat{\lambda}_{ij}^{\omega} = \frac{\widehat{\iota}_i^{\omega}\left(\widehat{L}_{it}/\widehat{\beta}_i^R\right)^{(\upsilon+\theta)}\left(\widehat{\beta}_i^{R,\omega}\right)^{\upsilon}\left(\widehat{\tau}_{ij}^{\omega}\right)^{-\theta}}{\sum_{k=1}^{N}\widehat{\iota}_k^{\omega}\left(\widehat{L}_{kt}/\widehat{\beta}_k^R\right)^{(\upsilon+\theta)}\left(\widehat{\beta}_k^{R,\omega}\right)^{\upsilon}\left(\widehat{\tau}_{kj}^{\omega}\right)^{-\theta}\lambda_{kj}^{\omega}} \quad (29.1) \qquad \widehat{\beta}_i^R = \int_{\Omega}\widehat{\alpha}^{\omega}\widehat{\beta}_i^{R,\omega}\frac{\alpha^{\omega}\beta_i^{R,\omega}}{\beta_i^R}d\omega \quad (29.4)$$

$$\widehat{\beta}_i^{R,\omega} = \sum_{j=1}^{N}\widehat{\lambda}_{ij}^{\omega}\widehat{\beta}_j^{E,\omega}\frac{\lambda_{ij}^{\omega}\beta_j^{E,\omega}}{\beta_i^{R,\omega}} \quad (29.2) \qquad \widehat{\alpha}^{\omega} = \sum_{j=1}^{N}\widehat{\alpha}_j^{\omega}\widehat{\beta}_j^E\beta_j^{E,\omega} \quad (29.5) \qquad (29)$$

$$\widehat{\beta}_i^E = \widehat{\beta}_i^R \quad (29.3) \qquad \widehat{\beta}_i^{E,\omega} = \frac{\widehat{\alpha}_i^{\omega}\widehat{\beta}_i^E}{\widehat{\alpha}^{\omega}} \quad (29.6)$$

for all i, j and ω.

The previous system is obtained directly from the system (23) with a reduction in the total number of equations that is obtained using equations (23.4) and (23.8) in equation (23.1).

A few comments are in order. First, notice that out of the exogenous components in the previous system, the shocks to the parameters $\left\{\widehat{\iota}_i^{\omega}, \widehat{L}_{it}, \widehat{\alpha}_i^{\omega}, \widehat{\tau}_{ij}^{\omega}\right\}$ are provided by the evaluator and the information regarding the initial BGP $\lambda_{ij}^{\omega}, \beta_i^{R,\omega}, \beta_i^R, \beta_i^{E,\omega}, \beta_i^E, \alpha^{\omega}$ is readily obtainable from the data. Consequently, the only exogenous parameters that need to be calibrated/estimated are the decreasing returns parameter υ and the shape parameter θ. All the relevant information regarding the initial distribution of research productivities ι_i^{ω}, labor endowments L_{it}, preference parameters α_i^{ω} and trade costs τ_{ij}^{ω} is summarized in the levels of trade shares λ_{ij}^{ω}; countries' market shares $\beta_i^{R,\omega}$ and β_i^R, countries' expenditure shares $\beta_i^{E,\omega}$ and β_i^E, and world-wide industries' expenditure shares α^{ω} in the initial BGP.

Second, the previous system shows the differences between the counterfactual predictions of the model with directed research and the benchmark model with no innovation, and how the decreasing returns parameter υ controls those differences. To see this more clearly, consider a change in parameters that are exogenous in both models, i.e., changes in labor endowments $\widehat{L}_{it}$, preference parameters $\widehat{\alpha}_i^{\omega}$ and trade costs $\widehat{\tau}_{ij}^{\omega}$. To analyze the effect of those changes in the present model we only need to modify equation (29.1) setting $\widehat{\iota}_i^{\omega} = 1$. The only difference between the system of equations in changes corresponding to a model with no innovation and (29) is equation (29.1), that captures the endogenous change in manufacturing technology. Moreover, setting $\upsilon = 0$ in (29.1) yields the exact same system in changes that is obtained from applying DEK approach to the benchmark model with no innovation.

3.4 Real Income in the Balanced Growth Path

In this subsection I study the effects of changes in trade costs on country i's BGP-level of real income per capita. In particular, I focus on how the additional margin of adjustment introduced by endogenous technical change and directed research modifies the effects that changes in trade costs have on real income per capita relative to a benchmark model with no innovation.

Real income per-capita prevailing at time t in country i in the BGP, W_{it}, can be expressed as

$$W_{it} = B_P \exp\left\{\int_\Omega \log\left(T_{it}^\omega\right)^{\alpha_i^\omega/\theta} d\omega\right\} \exp\left\{\int_\Omega \log\left(\lambda_{ii}^\omega\right)^{-\alpha_i^\omega/\theta} d\omega\right\} \tag{30}$$

The previous expression holds true in both the model with no innovation and the model with directed research. However, in the former the levels of manufacturing technology are fixed while in the later they are endogenous and adjust as innovation responds to shocks. Let $\widehat{X} \equiv X'/X$ denote the relative change in variable X from X before the shock to X' after the shock, and define a foreign shock as a change in foreign research productivities, foreign levels of labor endowment or trade costs that do not affect country i's innovation capabilities or its ability to serve its domestic market. Then, the change in country i's real income associated with a foreign shock can be computed as

$$\begin{aligned} \widehat{W}_{it} &= \exp\left\{\int_\Omega \log\left[\widehat{T}_{it}^\omega\right]^{\alpha_i^\omega/\theta} d\omega\right\} \exp\left\{\int_\Omega \log\left(\widehat{\lambda}_{ii}^\omega\right)^{-\alpha_i^\omega/\theta} d\omega\right\} \\ &= \exp\left\{\int_\Omega \log\left(\widehat{\delta}_i^\omega\right)^{\upsilon\alpha_i^\omega/\theta} d\omega\right\} \exp\left\{\int_\Omega \log\left(\widehat{\lambda}_{ii}^\omega\right)^{-\alpha_i^\omega/\theta} d\omega\right\} \end{aligned} \tag{31}$$

where the second line is obtained using equation (21).

The last expression offers a parsimonious way to evaluate ex-post the change in real income associated with a foreign shock. Notice that we do not need to know the nature of the foreign shock nor its effects on innovation, production and trade flows around the world. It is sufficient to have information on the change in homes shares of expenditure, $\widehat{\lambda}_{ii}^\omega$, the expenditure shares across industries, α_i^ω, the change in the share of each industry in total output, $\widehat{\delta}_i^\omega$ and the parameters θ, υ capturing the dispersion of productivities within each industry and decreasing returns in R&D, respectively. Notice that in the case of the model with no innovation ($\upsilon = 0$), the right hand side of equation (31) reduces to the second term.[40] In this case, we only need information on home shares of expenditure, expenditure shares across industries and the parameter θ. In other words, the set of sufficient statistics needed to evaluate ex-post the changes in real income in the model with no innovation is a strict subset of the corresponding set in the model with directed research ($\upsilon > 0$).

Let us now turn to the analysis of the counterfactual predictions of the model regarding the effect

[40] In this case the formula reduces to the expression found in Arkolakis, Costinot and Rodriguez-Clare (2012) for the case of the multi-industry Eaton and Kortum (2002) model.

of changes in trade costs on real income per-capita in the BGP. In order to evaluate the change in real income according to (31) we first need to use the structure of the model to evaluate the changes in the home share of expenditures, $\widehat{\lambda}_{ii}^{\omega}$, and the share of each industry in total output, $\widehat{\delta}_i^{\omega}$, associated with the change in trade costs. Throughout the analysis this is done as follows: (i) I calibrate the system (29) to some baseline equilibrium using information on endogenous trade shares λ_{ij}^{ω}, countries' market shares $\beta_i^{R,\omega}$ and β_i^R, countries' expenditure shares $\beta_i^{E,\omega}$ and β_i^E, world-wide industries' expenditure shares α^{ω} and the parameter θ; (ii) I solve the system in changes (29) for some value of the parameter υ; and (iii) I compute the change in real income per capita according to (31). In this way, the differences in the predicted changes in real income per capita between the model with no innovation and the model with directed research are those that emerge from setting $\upsilon = 0$ or $\upsilon > 0$ in the system in changes (29). Notice that in general, the differences in the predicted changes in real income per capita between the two models is not only given by the extra term in (31), but also the models have different predictions regarding the changes in trade flows. Proposition 2 summarizes the effects of directed research on the predicted changes in real income associated with a change in trade costs.

Proposition 2 *(i) Consider a world economy of two mirror symmetric countries.*[41] *Starting from an initial open economy equilibrium, a uniform decrease (increase) in trade costs generates a larger increase (lower reduction) in the BGP-level of real income per capita in the model with directed research than in the model with no innovation.*
(ii) For the general asymmetric case, moving to autarky generates lower reductions in the BGP-level of real income per capita in the model with directed research than in the model with no innovation.

Proof. See Appendix C.2.8.

Proposition 2 compares the predictions of the models with and without innovation regarding the changes in real income per-capita *conditional* on observed trade shares and market shares in the original equilibrium. In this sense, this comparison is consistent with the analysis in Arkolakis, Costinot and Rodriguez-Clare (2012). However, Proposition 1 implies that the models with and without innovation can be calibrated to share all exogenous parameters (other than υ) and manufacturing technology in the initial equilibrium. Consequently, the comparison in Proposition 2 is also compatible with the theoretical comparative static exercises in Melitz and Redding (2014).[42] In this way, the changes in real income in the model with no innovation can be interpreted as the changes that arise in the model with innovation when technology is not allowed to adjust. Under this interpretation, the results in Proposition 2 are very intuitive. Directed research introduces a new margin through which economies can adjust to the change in trade costs. For this reason, in the model with directed research, economies can enjoy a higher level of real income after the change in trade costs regardless of the direction of the change.

[41] The countries are mirror images of each other. See the proof in the Appendix for a precise definition.

[42] Although these two alternative approaches yield the same results in the present model, this is not the case in general. See Melitz and Redding (2014) for a detailed discussion of these two alternative approaches.

4 Quantitative Analysis

4.1 Calibration and Data Description

According to Corollary 1, to solve the BGP of the model in changes we need data on endogenous trade shares λ_{ij}^{ω}, countries' market shares $\beta_i^{R,\omega}$ and β_i^R, countries' expenditure shares $\beta_i^{E,\omega}$ and β_i^E, and world-wide industries' expenditure shares α^{ω} in the initial BGP. This implies that the quantitative implementation of the model requires bilateral trade flows and production data disaggregated at the industry level. Consequently, at this point it is necessary to define the mapping between an industry in the model and its empirical counterpart in the data. In this regard, I identify the industries in the model with manufacturing industries corresponding roughly to two-digit ISIC Rev.3 classification, giving a total of $\Omega = 18$ industries.

The data on trade flows is obtained from the OECD STAN (Structural Analysis) Database while production data is sourced form the 2012 UNIDO Industrial Statistic Database INDSTAT2. The sample of countries include 25 OECD countries, 4 non-OECD countries and a constructed rest of the world, yielding a sample of $N = 29$.

There are two key sets of variables that need to be quantified in the model from which we can construct all the shares needed in Corollary 1: the total revenue generated by country i's manufacturing firms in industry ω R_{it}^{ω}, and the total expenditure of country i in each industry ω E_{it}^{ω}. I identify the former with total gross production of country i in the industry, $R_{it}^{\omega} = Y_{it}^{\omega}$; and I identify the later with apparent consumption of country i in the same industry, $E_{it}^{\omega} = AC_{it}^{\omega} \equiv Y_{it}^{\omega} - X_{it}^{\omega} + M_{it}^{\omega}$, where X_{it}^{ω} and M_{it}^{ω} are the total exports and the total imports of country i in the industry in the data.

With the values of R_{it}^{ω} and E_{it}^{ω} quantified as above we can construct the empirical counterpart of the shares corresponding to the initial BGP directly from their definitions: (i) trade shares $\lambda_{ij}^{\omega} = X_{ij}^{\omega}/E_{it}^{\omega}$ for $i \neq j$ and $\lambda_{ii} = 1 - \sum_{j \neq i} \lambda_{ij}^{\omega}$; (ii) market shares $\beta_i^{R,\omega} = R_i^{\omega} / \sum_{j=1}^{N} R_j^{\omega}$ and $\beta_i^R = \sum_{\omega=1}^{\Omega} R_i^{\omega} / \sum_{\omega=1}^{\Omega} \sum_{j=1}^{N} R_j^{\omega}$; (iii) countries' expenditure shares $\beta_i^{E,\omega} = E_i^{\omega} / \sum_{j=1}^{N} E_j^{\omega}$ and $\beta_i^E = \sum_{\omega=1}^{\Omega} E_i^{\omega} / \sum_{\omega=1}^{\Omega} \sum_{j=1}^{N} E_j^{\omega}$; (iv) Cobb-Douglas parameters $\alpha_i^{\omega} = E_{it}^{\omega} / \sum_{\omega=1}^{\Omega} E_i^{\omega}$; (v) world-wide industries' expenditure shares $\alpha^{\omega} = \sum_{j=1}^{N} E_j^{\omega} / \sum_{\omega=1}^{\Omega} \sum_{j=1}^{N} E_j^{\omega}$.

The shape parameter θ, that at any time t captures the dispersion in the efficiency of state of the art techniques within any given industry and country, is central in quantitative applications of a large industry of models that deliver the structural gravity equation (11) and consequently it has been the focus of many empirical studies. Bernard, Eaton, Jensen and Kortum (2003) propose a value of $\theta = 3.6$ based on the calibration of a static single industry model representing the whole manufacturing sector to match the productivity and size advantage of exporters. Simonoska and Waugh (2014), using the same model as in BEJK (2003), obtain estimates of θ in the range $(3, 3.5)$ based on an estimation procedure that matches to the data the implied moments of the model regarding maximal bilateral price differences of manufactured goods, and that accounts for the biases arising from variable markups.[43] Caliendo and Parro (2014) use data on bilateral trade flows and tariffs and obtain values for θ in the range $(3.3, 4.5)$.

[43] For the case of the Eaton and Kortum (2002) model, Simonovska and Waugh find a value of $\theta = 4$ applying a similar procedure.

As a compromise between these values I set $\theta = 4$.[44]

Finally, the calibration of the parameter υ capturing the decreasing returns R&D is discussed next.

4.2 Estimating υ

In this subsection I discuss alternative ways to calibrate the decreasing returns parameter υ that are compatible with the structure of the model.

4.2.1 Comparative Advantage in Production and Market Shares

In this section I estimate the parameter υ using the BGP-implications of the model regarding the cross-sectional relationship between comparative advantage in production, comparative advantage in innovation and relative market shares that is captured in equation (24). Taking logs in equation (24) yields the following log-linear structural relationship

$$\underbrace{\ln\left(\frac{T_{it}^{\omega}/T_{i't}^{\omega}}{T_{it}^{\omega'}/T_{i't}^{\omega'}}\right)}_{\text{Estimable}} = \upsilon \times \underbrace{\ln\left(\frac{\beta_i^{R,\omega}/\beta_{i'}^{R,\omega}}{\beta_i^{R,\omega'}/\beta_{i'}^{R,\omega'}}\right)}_{\text{Observable}} + \underbrace{\ln\left(\frac{\iota_i^{\omega}/\iota_{i'}^{\omega}}{\iota_i^{\omega'}/\iota_{i'}^{\omega'}}\right)}_{\text{Unobservable}} \tag{32}$$

Notice that in the last equation, the only unobservable term corresponds to the log of the comparative advantage in innovation, since figures for comparative advantage in production and market shares can be constructed from production and trade data. In particular, I construct such figures as follows: (i) I obtain markets shares $\beta_i^{R,\omega}$ directly from production data as shown in subsection 4.1; and (ii) I follow Costinot, Donaldson and Komunjer (2011) and estimate comparative advantage in production from trade flows according to a procedure that is consistent with the gravity structure of the model as reflected in equation (23.1).[45] This implies that equation (32) is an estimable equation that can be taken to the data to obtain an estimate of parameter υ.

The estimation of the last equation presents some challenges. The structure of the model implies that estimating equation (32) by OLS treating the unobservable term as an error yields an inconsistent estimator for υ. To see this more clearly it is convenient to look back at the results of Lemma 2 regarding the determinants of the endogenous component of comparative advantage in production in the extreme cases of autarky and zero gravity: in the former case, the endogenous component is completely determined by relative local demand conditions while in the later case it is completely determined by the underlying comparative advantage in innovation. Consequently, in any trading equilibrium that is in between these two extreme cases, we should expect the endogenous component of comparative advantage in production to be correlated with both, relative local demand conditions and the underlying comparative advantage in innovation. For this reason we should expect relative market shares to be positively correlated with the unobservable term in (32), implying that the OLS estimator of υ in equation (32) is biased upwards.

[44] Costinot, Donaldson and Komunjer (2011) obtain a value $\theta = 6.53$ using a static multi-industry model in which θ is common across industries. However, their IV estimation procedure is not valid in the present model because of the two-way relationship between trade flows and R&D that arises as a consequence of directed research.

[45] The details of the estimation procedure can be found in Appendix C.3.1.

To address this endogeneity problem I instrument relative market shares in (32) with exogenous relative local demand conditions that I assume to be orthogonal to relative innovation capabilities. To illustrate this approach I start with the simple case of Cobb-Douglas preferences across industries as in (4) and later I generalized it to the case of CES preferences. As the result of this estimation strategy, I am able to provide an upper bound and a lower bound for the parameter υ, that fit in the $(0,1)$ range predicted by the theory.

In order to avoid issues related to the particular choice of the country and industry relative to which comparative advantage is defined, I define double ratios relative to an "average industry" $\bar{\omega}$ and an "average country" $\bar{\imath}$ as follows. The value of variable X in each country i in the (geometric) average industry $\bar{\omega}$ is defined as

$$X_{it}^{\bar{\omega}} \equiv \prod_{\omega=1}^{\Omega} \left(X_{it}^{\omega}\right)^{\frac{1}{\Omega}} \tag{33}$$

Similarly, the value of variable X in the (geometric) average country $\bar{\imath}$ in each industry is defined as

$$X_{\bar{\imath}t}^{\omega} \equiv \prod_{i=1}^{N} \left(X_{it}^{\omega}\right)^{\frac{1}{N}} \tag{34}$$

For the sake of notation simplicity, in the discussion that follows I use the following notation: for any variable X, I define $X_n \equiv \ln \frac{X_{it}^{\omega}/X_{\bar{\imath}t}^{\omega}}{X_{it}^{\bar{\omega}}/X_{\bar{\imath}t}^{\bar{\omega}}}$ where n is an index of the observations. With this notation, the estimable equation (32) becomes

$$T_n = \upsilon \beta_n^R + \iota_n \tag{35}$$

Cobb-Douglas Preferences.— In this case the instrumental variable approach can be summarized with the following identifying assumptions: (CD.i) I assume that the observed open equilibrium in the data is far from zero gravity.[46] According to Lemma 2 we should expect relative market shares to be correlated with relative domestic demand. (CD.ii) the double ratios of the exogenous Cobb-Douglas demand parameters are uncorrelated with the corresponding double ratios of research productivities.

Under these conditions, it is readily seen that α_n is an instrument for β_n^R in (35), and that the method of moments estimator

$$\widehat{\upsilon}_1 \equiv \frac{\sum_n T_n \alpha_n}{\sum_n \beta_n^R \alpha_n} \tag{36}$$

is a consistent estimator of the parameter υ.

The first two columns of Table 2 show the results of estimating equation (35) by OLS and by the estimator $\widehat{\upsilon}_1$ respectively. As we can see from the table, the OLS estimator yields a value $\widehat{\upsilon}_{OLS} = 1.023$, which is slightly above the upper limit of 1 imposed by the theory, although not statistically different from it. However, this should not be a concern since the theory suggest that the OLS estimator should be biased upwards. When we instrument log-relative market shares β_n^R with Cobb-Douglas demand parameters α_n and estimate υ using the method of moments estimator $\widehat{\upsilon}_1$, the estimated value of υ goes down as expected, yielding a value of $\widehat{\upsilon}_1 = 0.811$ with the corresponding 95% confidence interval included in the $(0,1)$ range predicted by the theory.

[46] Although I am stating this as an assumption, the existence of high trade frictions has been extensively documented. See Eaton and Kortum (2002).

The estimation results discussed in the last paragraph reject the model with no innovation, corresponding to $\upsilon = 0$, in favor of the model with directed research, $\upsilon > 0$. For this reason, it is important to understand what moments in the data are driving these results. Taking probability limits in (36) yields $plim(\widehat{\upsilon}_1) = \mathbb{E}[T\alpha] / \mathbb{E}[\beta^R \alpha]$. The method of moments estimator $\widehat{\upsilon}_1$ uses the differences in the models' implications about $\mathbb{E}[T\alpha]$ to discriminate between them.[47] In a model with no innovation, the Cobb-Douglas preferences and CD.ii imply that log-relative expenditure shares α and log-comparative advantage in production T are exogenous and uncorrelated, i.e. $\mathbb{E}[T\alpha] = 0$. In contrast, Lemma 2 implies a positive correlation between technology and domestic demand conditions in a model with directed research and high trade frictions (CD.i), $\mathbb{E}[T\alpha] > 0$. Consequently, the method of moments estimator $\widehat{\upsilon}_1$ attributes any positive correlation between technology and demand in the data to the effects of directed research.[48]

However, a moment of reflection suggests that the lack of correlation between expenditure shares and technology in the no innovation model is not robust to deviations from the Cobb-Douglas assumption. In particular, if the elasticity of substitution across industries is greater than one, at least some part of the positive correlation between technology and expenditure shares could be driven by the higher demand associated with the lower prices that higher levels of domestic technology induce, i.e. we could have $\mathbb{E}[T\alpha] > 0$ and $\upsilon = 0$. As I explain in more detail below, assumption CD.ii is not satisfied in this case, implying that $\widehat{\upsilon}_1$ is an inconsistent estimator of υ. To deal with this problem I extend the approach to the case of CES preferences. Although I cannot obtain a consistent estimator of υ in this case, I propose two method of moments estimators that provide lower and upper bounds for υ within the range predicted by the theory.

CES Preferences.— In order to account for the endogeneity problems discussed above, in what follows I assume that the consumption aggregator across industries takes the following CES form

$$C_{it} = \left[\int_{\Omega} (\gamma_i^{\omega})^{\frac{1}{\sigma}} \, C_{it}^{\omega \, \frac{\sigma-1}{\sigma}} \, d\omega \right]^{\frac{\sigma}{\sigma-1}} \tag{37}$$

where γ_i^{ω} are exogenous demand parameters and $\sigma > 1$ is the elasticity of substitution across industries. In addition, throughout the analysis I make the following identifying assumptions about the structure of the model: (CES.i) I assume that the observed open equilibrium in the data is far from zero gravity.[49] According to Lemma 2, in any trade equilibrium that is far from zero gravity we should expect market shares to be correlated with domestic demand conditions captured by the exogenous demand parameters γ; (CES.ii) double ratios of exogenous preference parameters γ are uncorrelated with the corresponding double ratios of research productivities.

In what follows I show that under these assumptions, the method of moments estimator $\widehat{\upsilon}_1$ is an

[47] Both models have similar predictions for $\mathbb{E}[\beta^R \alpha]$ under assumption CD.i: in the presence of high trade frictions, the market captured by each country is mainly determined by its domestic market, and as a result, we expect $\mathbb{E}[\beta^R \alpha] > 0$ for all $\upsilon \in [0, 1)$.

[48] In the data $\sum_n \beta_n^R \alpha_n > 0$ and $\sum_n T_n \alpha_n > 0$.

[49] Although I am stating this as an assumption, the existence of high trade frictions has been extensively documented. See Eaton and Kortum (2002).

inconsistent estimator of υ. In order to establish this result it is convenient to first derive the relationship between expenditure shares, prices and the exogenous demand parameters γ that the CES preferences imply: the demand functions corresponding to the CES preferences (37) together with equation (13) that relates the price indices P_{it}^{ω} to the cost parameters Φ_{it}^{ω}, yield the following log-linear structural equation

$$\begin{aligned} \ln\left(\frac{\alpha_{i}^{\omega}/\alpha_{\bar{\imath}}^{\omega}}{\alpha_{i}^{\bar{\omega}}/\alpha_{\bar{\imath}}^{\bar{\omega}}}\right) &= \tfrac{\sigma-1}{\theta}\times\ln\left(\frac{\Phi_{it}^{\omega}/\Phi_{\bar{\imath}t}^{\omega}}{\Phi_{it}^{\bar{\omega}}/\Phi_{\bar{\imath}t}^{\bar{\omega}}}\right) + \ln\left(\frac{\gamma_{i}^{\omega}/\gamma_{\bar{\imath}}^{\omega}}{\gamma_{i}^{\bar{\omega}}/\gamma_{\bar{\imath}}^{\bar{\omega}}}\right) \\ \alpha_n &= \tfrac{\sigma-1}{\theta}\Phi_n+\gamma_n \end{aligned} \tag{38}$$

where in this case α_{it}^{ω} represent endogenous expenditure shares; the second line expresses the same equation using the notation defined before.

After these preliminaries, we are ready to show that if $\sigma>1$, then log-relative expenditure shares α_n are positively correlated with log-comparative advantage in innovation ι_n.[50] First, equation (35) implies that in equilibrium ι_n is positively correlated with log-comparative advantage in production T_n. Second, if trade frictions are high, then equation (12) implies that the main determinant of the cost parameter Φ_{it}^{ω} is the level of domestic manufacturing technology T_{it}^{ω}, implying a positive correlation between the log-double ratios of cost parameters Φ_n and T_n. Figure 2 shows the relationship between Φ_n and T_n in the data, where Φ_n is constructed according to equation (11) using estimates of comparative advantage in production and home trade shares

$$\Phi_n \equiv \ln\left(\frac{\Phi_{it}^{\omega}/\Phi_{\bar{\imath}t}^{\omega}}{\Phi_{it}^{\bar{\omega}}/\Phi_{i't}^{\bar{\omega}}}\right)=\ln\left(\frac{T_{it}^{\omega}/T_{\bar{\imath}t}^{\omega}}{T_{it}^{\bar{\omega}}/T_{\bar{\imath}t}^{\bar{\omega}}}\right)-\ln\left(\frac{\lambda_{ii}^{\omega}/\lambda_{\bar{\imath}\bar{\imath}}^{\omega}}{\lambda_{ii}^{\bar{\omega}}/\lambda_{\bar{\imath}\bar{\imath}}^{\bar{\omega}}}\right)$$

As we can see from the figure, there is a strong positive correlation between Φ_n and T_n, implying that relative price indices across countries are tightly connected to relative domestic technology in the open economy equilibrium. Finally, equation (38) implies that Φ_n is positively correlated with log-relative market shares α_n if $\sigma>1$. These series of links imply that α_n and ι_n are positively correlated when $\sigma>1$, implying that the method of moments estimator $\widehat{\upsilon}_1$ is biased upwards. The previous argument is formalized in the next Lemma.

Lemma 4 *Suppose that the elasticity of substitution across industries is greater that one, $\sigma>1$, and that the assumptions CES.i-ii hold. Then, the method of moments estimator $\widehat{\upsilon}_1$ is an inconsistent estimator of υ. In particular, $\widehat{\upsilon}_1$ is biased upwards.*

Proof: See appendix C.3.2.

The previous Lemma implies that the method of moments estimator $\widehat{\upsilon}_1$ provides an upper bound for υ. To deal with this problem, I propose a second method of moment estimator $\widehat{\upsilon}_2$, that under the structure of the model, is biased downwards. The two estimators together provide an upper bound and a lower bound of υ.

[50] Recall that $X_{nt}\equiv\ln\frac{X_{it}^{\omega}/X_{\bar{\imath}t}^{\omega}}{X_{it}^{\bar{\omega}}/X_{\bar{\imath}t}^{\bar{\omega}}}$

The method of moments estimator $\widehat{\upsilon}_2$ is defined as follows:

$$\widehat{\upsilon}_2 = \frac{\sum_n T_n \widehat{\gamma}_n}{\sum_n \beta_n^R \widehat{\gamma}_n} \tag{39}$$

where $\widehat{\gamma}_n$ are the OLS residuals obtained from estimating (38) taking the unobservable demand parameters γ_n as the error term. The motivation for estimator $\widehat{\upsilon}_2$ is the same general motivation that leads to estimator $\widehat{\upsilon}_1$ in the case of Cobb-Douglas preferences, i.e., to use exogenous local demand conditions to instrument for market shares β^R in (35). If we knew the true value of $(\sigma-1)/\theta$, then we could use (38) to recover the exogenous demand parameters γ_n and use them as instruments for β^R in (35). Given that we do not know with the true value of $(\sigma-1)/\theta$, estimator $\widehat{\upsilon}_2$ replaces the exogenous demand parameters γ_n with the OLS residuals $\widehat{\gamma}_n$. However, the structure of the model implies that Φ_n and γ_n are positively correlated, making the OLS estimator of $(\sigma-1)/\theta$ biased upwards. This bias in the estimation of (38), which affects the construction of $\widehat{\gamma}_n$, introduces a downward bias in the estimator $\widehat{\upsilon}_2$. The intuition for this result is the following. In the model with CES preferences there is a two-way relationship between relative domestic technology and domestic demand. On the one hand, local demand affects innovation and technology in the presence of trade frictions. On the other hand, domestic technology affects domestic demand through its effect on prices. Consequently, the positive correlation between Φ_n and γ_n implies that estimating (38) by OLS over estimates the importance of the second channel at the expense of the first channel, i.e. $(\sigma-1)/\theta$ is over estimated and υ is under estimated. This argument is formalized in the next Lemma.

Lemma 5 *Suppose that assumptions CES.i-ii hold. Then, the method of moments estimator $\widehat{\upsilon}_2$ is an inconsistent estimator of υ. In particular, $\widehat{\upsilon}_2$ is biased downwards.*

Proof: See Appendix C.3.3.

For the case of CES preferences and $\sigma > 1$, the results in Lemma 4 and Lemma 5 imply that the method of moments estimators $\widehat{\upsilon}_1$ and $\widehat{\upsilon}_2$ provide an upper bound and a lower bound for the parameter υ. The second and third column of Table 2 show the estimation results corresponding to these two estimators. As expected from the theory, the estimated values satisfy $\widehat{\upsilon}_2 = 0.706 < 0.811 = \widehat{\upsilon}_1$, providing upper and lower bounds for υ.

Finally, Table 2 summarizes the results of each of the corrections made to the estimation procedures to account for the biases predicted by the theory. In all cases, the estimated values change in the direction suggested by the theory.

For the quantitative exercises that follow, I use a value of 0.758 as the benchmark calibration for υ, corresponding to the center of the interval delimited by the estimated lower and upper bounds for the parameter in the case of CES preferences.

4.2.2 Alternative calibration

An alternative way to calibrate υ is to follow Ramondo and Rodriguez-Clare (2010) and exploit the dynamic implications of the model regarding the rate of growth of income per-capita. Lemma 1 together with the fact that income per-capita is proportional to real wages imply that both variables grow at the common rate $g = n\upsilon/\theta$ in the BGP. Given that R&D is the source of growth in this model, I calibrate n to match the growth rate of research employment, calculated as 0.048 by Jones (2002). Setting the growth rate of income per-capita to $g = 0.01$ (also from Jones (2002)), the values of n and θ calibrated as above imply a value of $\upsilon = 0.83$.

4.3 Results

In this subsection I assess quantitatively the importance of the new margin of adjustment introduced by directed research, emphasizing the effects of trade on innovation and real income per capita. To that end, I study the effects of geography and trade on technology, and real income in the BGP, under alternative assumptions regarding the structure of trade frictions in the model.

Geography and Comparative Advantage.— Here I explore the importance of directed research and geography in the endogenous determination of comparative advantage in production. To that end, I decompose the log of comparative advantage in production into exogenous and endogenous components according to equation (24) for alternative values of υ and for alternative assumptions about the structure of trade frictions. The values of υ are chosen to include the lower and upper bounds estimated in the previous section and the assumptions on trade frictions include the extreme cases of autarky and frictionless trade, as well as the actual observed equilibrium. Then I evaluate the share of the total variance in the log of comparative advantage in production that is driven by the endogenous component under each of these assumptions.

Specifically, taking logs in (24) yields

$$\underbrace{\ln\left(\frac{T_{it}^{\omega}/T_{\bar{\imath}t}^{\omega}}{T_{it}^{\bar{\omega}}/T_{\bar{\imath}t}^{\bar{\omega}}}\right)}_{\text{Comp. Adv. in Prod.}} = \underbrace{\upsilon\ln\left(\frac{\beta_{it}^{R,\omega}/\beta_{\bar{\imath}t}^{R,\omega}}{\beta_{it}^{R,\bar{\omega}}/\beta_{\bar{\imath}t}^{R,\bar{\omega}}}\right)}_{\text{Endogenous}} + \underbrace{\ln\left(\frac{\iota_{i}^{R,\omega}/\iota_{\bar{\imath}}^{R,\omega}}{\iota_{i}^{R,\bar{\omega}}/\iota_{\bar{\imath}}^{R,\bar{\omega}}}\right)}_{\text{Exogenous}}$$

Using data corresponding to the year 2006, I decompose comparative advantage in production in the actual equilibrium according to the last expression as follows: (i) I estimate comparative advantage in production using trade data following Costinot, Donaldson and Komunjer (2012); (ii) I use production data to construct market shares as described in the calibration section; (iii) for a given value of υ, I use the last expression to obtain the exogenous component given by the double ratios of research productivities as a residual. With the values for comparative advantage in innovation obtained in the previous decomposition and the Cobb-Douglas preference parameters obtained in the calibration section, for a given value of υ we can use Lemma 2 to decompose comparative advantage in production in the extreme cases of autarky and frictionless trade.

Table 1 shows the share of the endogenous component in the total variance of comparative advantage in production obtained according to the previous procedure for alternative values of the decreasing returns parameter υ that include the upper and lower bounds estimated in the previous section and the benchmark calibration for υ corresponding to the average the these bounds. First, for the benchmark value of $\upsilon = 0.758$, the share of the endogenous component is 52.8% in the actual equilibrium corresponding to 2006, i.e., the endogenous adjustment induced by directed research account for about half of the total variance in log-comparative advantage. This suggest an important role for directed research in the determination of comparative advantage in production. Second, the importance of the endogenous component of comparative advantage is robust with respect to the value of the parameter υ in the range delimited by the estimated lower and upper bounds for υ. In the actual equilibrium, the share of the endogenous component is 51.2% if we use the lower bound for υ and 54.2% if we use the upper bound. Third, the share of the endogenous component is closer to what it would be in autarky that what it would be in a zero gravity world, indicating that the actual equilibrium is characterized by high trade frictions.

It is important to emphasize that the variance decomposition of comparative advantage presented above is not meant to decompose comparative advantage into its ultimate exogenous determinants, but to evaluate the importance of endogenous adjustments to technology allowed by directed research, whatever the determinants of those adjustments are.

An alternative way to assess the importance of trade frictions in the determination of comparative advantage is to use the measures of the endogenous component of comparative advantage constructed above, to determine how far away the endogenous component in the actual equilibrium is, from its counterparts corresponding to autarky and zero gravity. Figure 3 shows the relationship between the endogenous components in the actual equilibrium and in autarky for the benchmark value of $\upsilon = 0.758$, together with the 45 degree line in red. Each point in the figure corresponds to a country and an industry, since comparative advantage is calculated relative to an "average" country and an "average" industry according to (34) and (33). As we can see, there is a tight connection between the two components. In contrast, Figure 4 shows a much weaker correlation between the endogenous components of comparative in the actual equilibrium and in the zero gravity world. Figures 3 and 4 are compatible with the presence of high impediments to trade in the actual equilibrium.

Moving to Autarky.— Here I consider the effects of raising trade costs to their autarky levels, $\tau_{ij}^{\omega} \to \infty$ for $i \neq j$. Given that in this case the effect on trade flows is trivial, I concentrate on the effects on real income. Table 3 shows the reductions in real income per-capita in the BGP as countries move to autarky, for a range of values of the decreasing returns in R&D parameter υ that include the upper and lower bounds estimated previously. The changes in equilibrium variables needed to compute the changes in real income according to (31) are calculated using the system in changes (29), calibrated to the observed equilibrium in 2006. In the particular case we are analyzing, the change in real income across BGPs can be computed as

$$\ln \frac{W_{it}^a}{W_{it}} = \frac{\upsilon}{\theta} \int_{\Omega} \alpha_i^{\omega} \ln \left(\frac{\alpha_i^{\omega}}{\delta_i^{\omega}} \right) d\omega + \frac{1}{\theta} \int_{\Omega} \alpha_i^{\omega} \ln \left(\lambda_{ii}^{\omega} \right) d\omega \tag{40}$$

where W_{it}^a, W_{it} denote the real income in autarky and in the actual equilibrium respectively.

The second term in the last expression corresponds to the change in real income predicted by the model with no innovation, $\upsilon = 0$. This term is always negative and it is shown in column 1 of Table 3. In contrast, the first term in the last expression is always positive for $\upsilon > 0$, and it reflects the endogenous adjustments in technology due to directed research. As discussed in section 3.4, this implies that the model with no innovation overestimates the reduction in real income associated with moving to autarky. Moreover, the predicted reduction in real income depends negatively on υ; higher values of υ are associated with lower reductions in real income. The other columns in Table 3 show the changes in real income for a range of positive values of υ relative to the changes corresponding to the no innovation model in column 1,

$$\frac{\ln W_{it}^{a,\upsilon}/W_{it}}{\ln W_{it}^{a,0}/W_{it}} = 1 + \upsilon \frac{\int_{\Omega} \alpha_i^{\omega} \ln \left(\alpha_i^{\omega}/\delta_i^{\omega}\right) d\omega}{\int_{\Omega} \alpha_i^{\omega} \ln \left(\lambda_{ii}^{\omega}\right) d\omega} < 1$$

The general picture emerging from Table 3 is that the differences in the predicted changes in real income between the two models appear to be modest. For the case of the benchmark value of $\upsilon = 0.758$, the reductions in real income relative to the model with no innovation range from 72% for Australia, to 98% for Belgium-Luxembourg, with a mean value for the sample of 93%. To understand the reasons behind these modest differences, we have to analyze the determinants of the first term in (40) since, for this particular counterfactual, it is the only driver of the differences between the predictions of the model with no innovation and the model with directed research. Notice that the magnitude of the first term in (40) depends on the difference in the production specialization profile of the country between the actual open equilibrium and autarky. These specialization profiles are captured by δ_i^{ω} and α_i^{ω} in (40). In that equation, δ_i^{ω} represents the share of industry ω in total manufacturing production in the open equilibrium. Recalling that in autarky the allocation of resources is completely driven by domestic demand, the Cobb-Douglas preference parameters α_i^{ω} also represent the share of industry ω in total manufacturing output in the autarky equilibrium.

Figure 5 shows the relationship between α_i^{ω} and δ_i^{ω} in the data. Each point in the figure correspond to a country-industry pair, where the figures for industry shares δ_i^{ω} and demand parameters α_i^{ω} are constructed using production and trade data for the year 2006. As we can see, there is a tight connection between domestic production and domestic demand –the correlation coefficient is 0.81–. As in the case of the analysis of the determinants of the endogenous component of comparative advantage in production, the close relationship between expenditure shares α_i^{ω} and industry shares δ_i^{ω} in the open equilibrium is indicative of the presence of high trade frictions in the open equilibrium. This implies that for the average country, the first term in (40) is small in absolute value. In addition, the first term tends to be important for those countries in which the second term is also important (in absolute terms); the correlation between the terms is -0.76 for the benchmark value of $\upsilon = 0.758$. This two elements are behind the modest relative differences between the predictions of the model with and without innovation.

25% reduction in trade costs— Although the previous exercise helps understand where the actual observed equilibrium stands relative to the autarky equilibrium, moving to autarky is not a policy seriously considered by any country. A more interesting question from the policy perspective is how directed research affects the expected gains from further trade liberalizations. Here I analyze how directed research affects our conclusion regarding the effects of a 25% reduction in trade costs on production, trade flows and real income. As before, the changes in equilibrium variables needed to compute the changes in real income according to (31) are calculated using the system in changes (29), calibrated to the observed equilibrium in 2006.

Let us start with the analysis of the effects of directed research on trade flows. Figure 6 shows the relationship between the predicted log-changes in trade shares in both models, together with the 45 degree line in red. The changes corresponding to the model with directed research were calculated for the benchmark calibration of the decreasing returns parameter $\upsilon = 0.758$. Each point in the graph correspond to an exporter-importer-industry triplet. As we can see, there are important differences between the predictions of the models. A simple regression of the changes in trade shares in the model with directed research on the corresponding changes in trade shares in the model with no innovation yields a slope coefficient of 1.05 This implies that, on average, the direction and magnitude of the changes in trade shares are similar in both models. However, this average hides a lot of variation as we can see from the figure. The R-square corresponding to that regression is 0.378, i.e., only a little more than a third of the variation in trade flows in the model with directed research can be explained by the corresponding changes in trade flows in the model with no innovation. In addition, in 26% of the cases the predicted changes in trade shares in both models go in opposite directions.

Let us now turn to the analysis of the changes in trade shares. Figure 7 shows a scatter plot of the predicted log-changes in market shares $\beta_i^{R,\omega}$ in both models, together with the 45 degree line in red. Each point in the graph corresponds to a country-industry pair. As we can see, there are small differences in the direction of the change in market shares predicted by the models; the correlation coefficient between the two variables is 0.94. However, the standard model underestimates the magnitude of the responses in market shares relative to the model with directed research; a regression of the predicted log-changes in market shares in the model with directed research on its counterpart in the model with no innovation yield a slope coefficient of 3.37.

Let us now turn to the analysis of the effects of the reduction in trade costs on real income per capita. The change in real income across BGPs can be computed as

$$\ln \frac{W_{it}'}{W_{it}} = \frac{\upsilon}{\theta} \int_{\Omega} \alpha_i^{\omega} \ln \left(\frac{\delta_i'^{\omega}}{\delta_i^{\omega}} \right) d\omega - \frac{1}{\theta} \int_{\Omega} \alpha_i^{\omega} \ln \left(\frac{\lambda_{ii}'^{\omega}}{\lambda_{ii}^{\omega}} \right) d\omega \tag{41}$$

where W_{it}' denotes the real income in the new counterfactual equilibrium and W_{it} denotes its counterpart in the baseline open equilibrium in 2006 respectively. Table 4 shows the changes in real income across BGPs associated with a 25% uniform reduction in trade costs across industries and countries. The changes are calculated for the same values of the decreasing returns in R&D parameter υ used in the case of the previous exercise. As in Table 3, columns (2)-(4) present the predicted changes in real income relative to the corresponding predicted change for the case of no innovation ($\upsilon = 0$), shown in column 1.

The first thing to notice is that all countries experience an increase in their real income regardless of the degree of the decreasing returns in innovation. As anticipated, there is a general tendency for the model with no innovation to underestimate the increases in real income. In only 6 cases, the model with directed research predicts a lower increase in real income relative to the model with no innovation.

However, as in the case of moving to autarky, the relative differences in the predicted changes in real income in both models seem to be modest. For the case of the benchmark value of $\upsilon = 0.758$, the changes in real income relative to the model with no innovation range from 83% for China, to 108% for Japan, with a mean value for the sample of 102%.

What is behind these modest effects of directed research in this case? In contrast to the case of moving to autarky, now the predicted change in trade flows depends on the degree of decreasing returns in R&D captured by the parameter υ. This implies that to understand the differences in the predictions of the models we have to analyze both terms in equation (41). Table 5 decomposes the total change in real income into the two terms in (41). The first and second columns show the predicted change in real income corresponding to a model with no innovation and to the model with directed research with the benchmark calibration $\upsilon = 0.758$. The third and fourth columns decompose the change in real income in column 2 into the two terms in (41), such that column 2 is the sum of columns 3 and 4. The first term in (41) is shown in column 4 (industry shares) and the second term is shown in column 3 (home shares).

Two features emerge from Table 5. First, the figures in column 3 are higher than those corresponding to column 1 for all countries. This comparison analyzes the effects of directed research on real income through its effect on trade flows in the second term of (41). Second, the first term of (41) shown in column 4 is negative for all countries. This column shows the effects of directed research on real income through its effect on the specialization patterns of the country in the first term in (41). As we can see, the effect of directed research on the two terms in (41) go in opposite directions. The intuition behind this result is very simple. Let X'' denote the value of variable X after the 25% reduction in trade costs in the model with directed research ($\upsilon > 0$), and let X' denote the value of the same variable after the reduction in trade costs but before any endogenous adjustment in technology takes place. Then, for any country i and industry ω, the relationship between the price levels at these two moments is

$$\frac{P_{it}^{\prime\prime\omega}}{P_{it}^{\prime\omega}} \propto \left(\Phi_{it}^{\prime\prime\omega}/\Phi_{it}^{\prime\omega}\right)^{-\frac{1}{\theta}} = \left(\frac{\lambda_{it}^{\prime\prime\omega}/\lambda_{it}^{\prime\omega}}{T_{it}^{\prime\prime\omega}/T_{it}^{\prime\omega}}\right)^{\frac{1}{\theta}} \tag{42}$$

It should be clear that in the previous expression, the value X' for any variable X corresponds to the predictions of the model with no innovation. Now consider the changes in trade flows and technology when the endogenous adjustments in technology are allowed to operate. These are the effects that are attributable to the new margin of adjustment introduced by directed research. The reduction in trade costs changes the relative expected market size for innovations across different industries. This translates into the expansion of the manufacturing technology in some sectors and the contractions in others. Suppose for a moment that more innovation takes place in industry ω in country i as a consequence of the reduction in trade costs, inducing an increase in the manufacturing technology from $T_{it}^{\prime\omega}$ to $T_{it}^{\prime\prime\omega}$. The same specialization process induces other countries to reduce their innovation in that industry. These two effects together induce an increase in the home share of expenditure of country i in that industry from $\lambda_{it}^{\prime\omega}$ to $\lambda_{ii}^{\prime\prime\omega}$. A similar analysis indicates that manufacturing technology and home shares of expenditure move in the same direction when innovation reallocates away from some industry. This implies that the effects of directed research on manufacturing technology and on the home share of expenditure in (42) work in opposite directions. This explains the modest effects of directed research on real income.

5 Conclusions

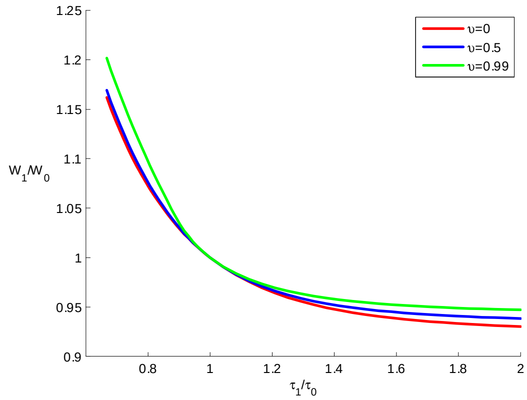
In this paper I develop a multi-country, general equilibrium, semi-endogenous growth model of innovation and trade in which specialization in innovation and production are jointly determined. The distinctive element of the model is the ability of the agents to direct their research efforts to specific industries, in a context of heterogeneous innovation capabilities across countries and contemporaneous decreasing returns to R&D. The model features a two-way relationship between trade and technology absent in standard quantitative Ricardian trade models.

I use the model to disentangle the effects of trade on technology and to study some questions that standard Ricardian quantitative trade models are not suitable to answer. Under the benchmark calibration of the model, I find that the endogenous adjustments in technology due to directed research can account for up to 52.8% of the observed variance in comparative advantage in production in the observed trading equilibrium in 2006. In addition, I find that the differences in the adjustments in trade flows and market shares in response to a 25% reduction in trade costs between the two models can be quantitatively important.

I also show that the standard model with no innovation overestimates the reduction in real income from moving to autarky and tends to underestimate the increases in real income from reductions in trade costs. However, notwithstanding the relevant effects of directed research on technology, production and trade flows, the predicted changes in real income associated with moving to autarky and with a 25% reduction in trade costs do not differ much across models.

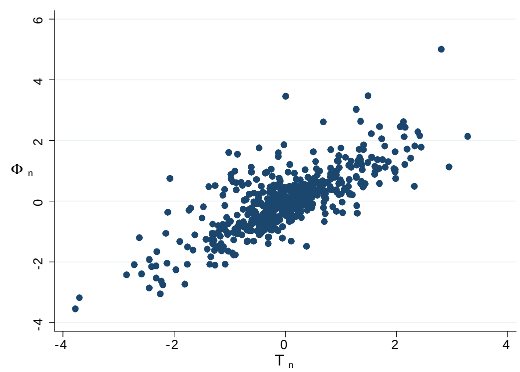
A Figures

Figure 1: Changes in Real Income and Trade Costs



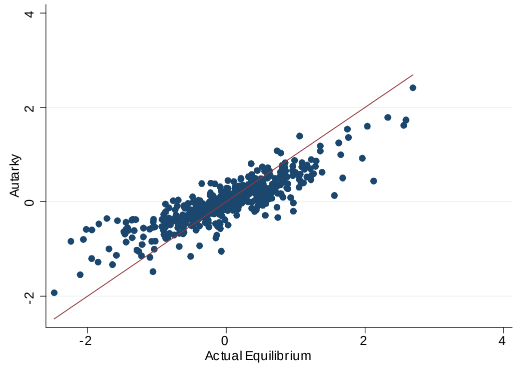
Notes: The figure shows the changes in real income across BGP as functions of changes in trade costs for alternative values of υ. The figure was constructed for the case of two symmetric countries according to system (29) which solves the model in changes conditional on observable variables in the initial equilibrium.

Figure 2: Relationship between Φ_n and T_n



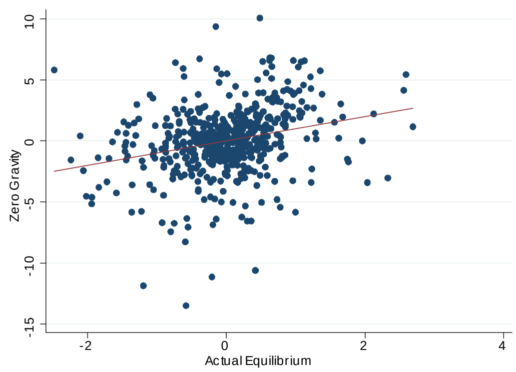
Notes: The figure shows the relationship between log-comparative advantage in production T_n and the log-double ratio of cost parameters Φ_n. Double ratios are taken with respect to the "average" country and "average" industry defined in the text; then each observation correspond to a country-industry pair. Trade and production data correspond to the year 2006.

Figure 3: Endogenous Component of Comparative Advantage: Actual Equilibrium vs. Autarky



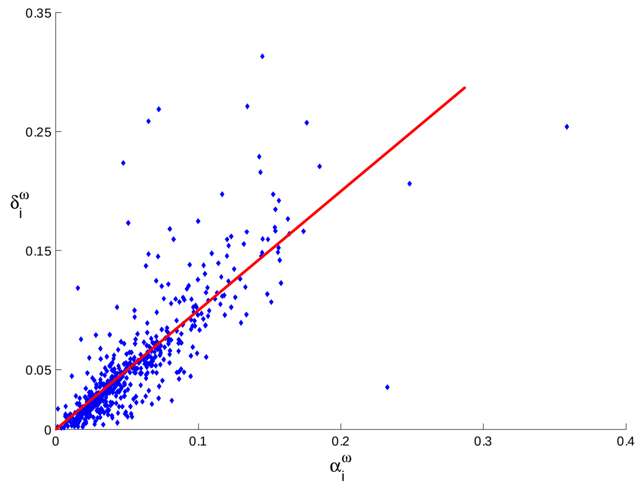
Notes: The figure shows the relationship between the endogenous components of comparative advantage in the actual equilibrium and in autarky for $\upsilon = 0.758$. Double ratios are taken with respect to the "average" country and "average" industry defined in the text; then each observation correspond to a country-industry pair. In red the 45 degree line. Trade and production data correspond to the year 2006.

Figure 4: Endogenous Component of Comparative Advantage: Actual Equilibrium vs. Zero Gravity



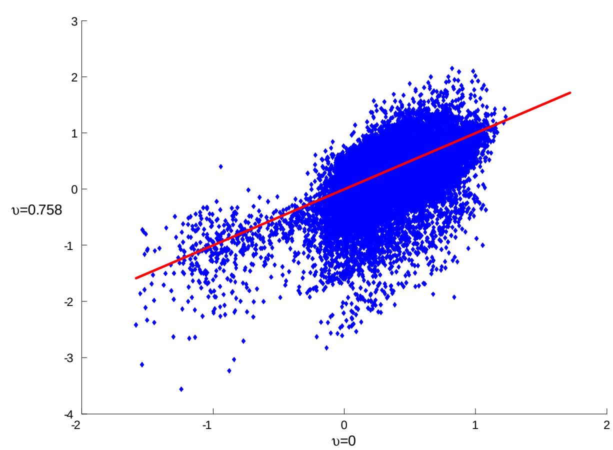
Notes: The figure shows the relationship between the endogenous components of comparative advantage in the actual equilibrium and in zero gravity for $\upsilon = 0.758$. Double ratios are taken with respect to the (geometric) "average" country and "average" industry defined in the text; then each observation correspond to a country-industry pair. In red, the 45 degree line. Trade and production data correspond to the year 2006.

Figure 5: Allocation of Resources. Actual Equilibrium vs. Autarky



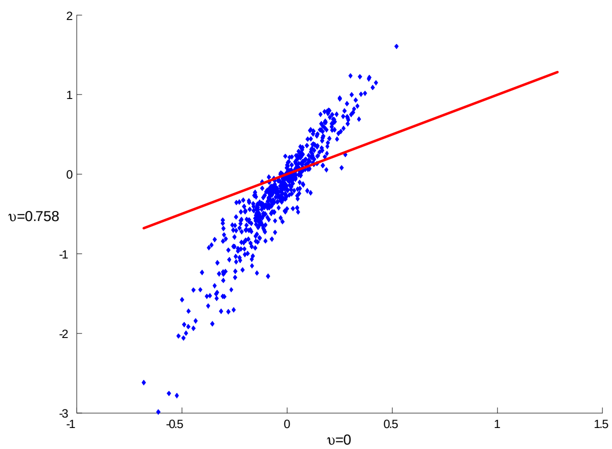
Notes: The figure shows the relationship between the allocation of resources in the actual open equilibrium and in autarky. Each dot in the figure corresponds to a country-industry pair. In red the 45 degree line. The value δ_i^ω represents the share of industry ω in country i's total production in the actual open equilibrium and it is calculated from production data. The corresponding industry shares in autarky are given by the demand parameters α_i^ω, which are obtained from production and trade data as described in the Data Section. Trade and production data correspond to the year 2006.

Figure 6: Log Changes in Trade Shares. No Innovation vs. Directed Research



Notes: The figure shows the relationship between the predicted log-change in trade shares after a 25% reduction in trade costs corresponding to the models with no innovation and directed research. Each dot in the figure represents a exporter-importer-industry triplet, $\log \widehat{\lambda}_{ij}^{\omega}$. Changes in trade shares are calculated according to the system in changes (29). Trade and production data in the baseline equilibrium correspond to the year 2006.

Figure 7: Log Changes in Market Shares. No Innovation vs. Directed Research



Notes: The figure shows the relationship between the predicted log-change in market shares after a 25% reduction in trade costs corresponding to the models with no innovation and directed research. Each dot in the figure represents a exporter-importer-industry triplet, $\log \widehat{\beta}_i^{R,\omega}$. Changes in trade shares are calculated according to the system in changes (29). Trade and production data in the baseline equilibrium correspond to the year 2006.

B Tables

Table 1: Endogenous Component of Comparative Advantage in Production

	$\upsilon = 0.706$	$\upsilon = 0.785$	$\upsilon = 0.811$
	%	%	%
Zero Gravity	91.4	94.1	96.4
Actual Open Equilibrium	51.2	52.8	54.2
Autarky	25.7	26.2	26.3

Notes: The table shows the share of the endogenous component in the total variance of log-comparative advantage in production for different values υ and for alternative assumptions about the structure of trade frictions. The selected values for the decreasing returns parameter υ include the estimated lower and upper bounds estimated in the text and the benchmark calibration for υ corresponding to the average of the bounds.

Table 2: Estimation of υ

D.V.	OLS	IV	IV
log-Comp. Adv. in Prod.		(Expend. Shares)	(Residuals)
	(1)	(2)	(3)
log-Market Shares	1.023***	0.811***	0.706***
	(0.0386)	(0.0472)	(0.0506)
Observations	540	540	540
R-squared	0.566	0.541	0.512

Standard errors in parentheses. ***$p<0.01$

Notes: The table shows the results of estimating equation (35) according to the three different methods discussed above. Columns (1), (2), and (3) show the estimation results corresponding to OLS estimator, the method of moments estimator $\widehat{\upsilon}_1$ and the method of moments estimator $\widehat{\upsilon}_2$. Columns (2) and (3) provide the upper and lower bounds for the parameter υ discussed in the text. An observation corresponds to a country-industry pair, since double ratios are taken with respect to the (geometric) "average" country and industry defined in the text.

Table 3: Changes in Real Income. Moving to Autarky (Losses.) Manufacturing Sector. 2006

	$\upsilon = 0$	$\upsilon = 0.706$	$\upsilon = 0.758$	$\upsilon = 0.811$
	Level %	Rel. to (1)	Rel. to (1)	Rel. to (1)
	(1)	(2)	(3)	(4)
AUS	-16.67	0.74	0.72	0.70
AUT	-36.82	0.98	0.98	0.98
BLX	-41.96	0.99	0.98	0.98
BRA	-6.75	0.90	0.89	0.89
CAN	-30.55	0.96	0.95	0.95
CHE	-26.84	0.93	0.93	0.92
CHN	-12.75	0.83	0.81	0.80
CZE	-26.01	0.95	0.94	0.94
DEU	-20.33	0.98	0.98	0.97
DNK	-30.31	0.92	0.91	0.91
ESP	-14.40	0.97	0.97	0.97
FIN	-21.60	0.94	0.94	0.93
FRA	-15.76	0.98	0.98	0.98
GBR	-17.94	0.98	0.98	0.98
GRC	-28.54	0.85	0.84	0.83
HUN	-37.53	0.98	0.97	0.97
IRL	-30.43	0.87	0.86	0.85
ISR	-33.80	0.85	0.84	0.83
ITA	-13.49	0.91	0.90	0.90
JPN	-6.58	0.96	0.95	0.95
KOR	-11.61	0.97	0.97	0.97
MEX	-28.14	0.96	0.96	0.96
NLD	-31.98	0.97	0.97	0.97
NOR	-20.76	0.92	0.92	0.91
POL	-19.99	0.94	0.94	0.93
PRT	-27.88	0.97	0.96	0.96
SGP	-61.43	0.88	0.88	0.87
SWE	-23.28	0.96	0.96	0.95
USA	-8.24	0.96	0.96	0.95
mean	-24.22	0.93	0.93	0.92
median	-23.28	0.96	0.95	0.95
min	-6.58	0.74	0.72	0.70
max	-61.43	0.99	0.98	0.98

Source: Author's calculation from the OECD STAN Database, UNIDO INDSTAT2 and the model in the text.
Notes: The levels in column (1) are calculated for a value $\theta = 4$. The other columns represent the change in real income relative to the column (1); this relative measure is not affected by the value of θ.

Table 4: Changes in Real Income. 25% Reduction in Trade Costs. Manufacturing Sector. 2006

	$\upsilon = 0$	$\upsilon = 0.706$	$\upsilon = 0.758$	$\upsilon = 0.811$
	Level %	Rel. to (1)	Rel. to (1)	Rel. to (1)
	(1)	(2)	(3)	(4)
AUS	13.71	1.02	1.03	1.05
AUT	24.31	1.02	1.03	1.04
BLX	25.45	1.04	1.05	1.07
BRA	6.32	1.00	1.00	1.00
CAN	27.35	1.01	1.01	1.02
CHE	20.01	0.99	0.98	0.98
CHN	8.23	0.86	0.83	0.79
CZE	19.19	1.01	1.01	1.01
DEU	15.34	1.03	1.04	1.05
DNK	22.03	1.03	1.04	1.06
ESP	12.55	1.01	1.01	1.01
FIN	15.98	1.02	1.03	1.03
FRA	14.18	1.04	1.05	1.06
GBR	15.60	1.02	1.02	1.03
GRC	16.45	1.01	1.01	1.01
HUN	22.10	0.99	0.99	0.99
IRL	19.81	1.02	1.03	1.04
ISR	19.21	0.98	0.97	0.95
ITA	11.39	1.03	1.04	1.05
JPN	6.44	1.07	1.08	1.11
KOR	10.49	1.05	1.06	1.07
MEX	21.64	0.97	0.97	0.97
NLD	23.64	1.05	1.07	1.09
NOR	16.35	0.99	0.99	0.99
POL	15.60	1.01	1.02	1.02
PRT	19.54	1.02	1.02	1.03
SGP	24.69	0.99	1.00	1.00
SWE	18.59	1.02	1.02	1.02
USA	7.59	1.04	1.05	1.07
mean	17.03	1.01	1.02	1.02
median	16.45	1.02	1.02	1.03
min	6.32	0.86	0.83	0.79
max	27.35	1.07	1.08	1.11

Source: Author's calculation from the OECD STAN Database, UNIDO INDSTAT2 and the model in the text.
Notes: The levels in column (1) are calculated for a value $\theta = 4$. The other columns represent the change in real income relative to the column (1).

Table 5: Decomposition of Changes in Real Income. 25% Reduction in Trade Costs. Manufacturing Sector. 2006

	$\upsilon = 0$	$\upsilon = 0.758$		
	Level %	Level %	Home Shares	Industry Shares
	(1)	(2)	(3)	(4)
AUS	13.71	14.13	33.25	-19.12
AUT	24.31	25.07	27.84	-2.76
BLX	25.45	26.75	28.19	-1.44
BRA	6.32	6.32	9.47	-3.15
CAN	27.35	27.68	37.01	-9.34
CHE	20.01	19.65	25.55	-5.89
CHN	8.23	6.85	10.98	-4.13
CZE	19.19	19.36	22.50	-3.14
DEU	15.34	15.94	17.44	-1.50
DNK	22.03	22.99	29.05	-6.06
ESP	12.55	12.67	14.24	-1.58
FIN	15.98	16.42	20.64	-4.22
FRA	14.18	14.82	16.17	-1.34
GBR	15.60	15.93	18.02	-2.10
GRC	16.45	16.56	20.05	-3.49
HUN	22.10	21.91	24.48	-2.57
IRL	19.81	20.34	24.33	-3.99
ISR	19.21	18.58	25.97	-7.39
ITA	11.39	11.83	13.26	-1.42
JPN	6.44	6.99	8.91	-1.92
KOR	10.49	11.13	12.44	-1.31
MEX	21.64	20.96	29.90	-8.93
NLD	23.64	25.25	31.53	-6.29
NOR	16.35	16.14	20.77	-4.63
POL	15.60	15.87	18.82	-2.95
PRT	19.54	19.97	21.22	-1.25
SGP	24.69	24.59	30.25	-5.66
SWE	18.59	18.94	21.98	-3.04
USA	7.59	8.01	10.19	-2.18
mean	17.03	17.30	21.53	-4.23
median	16.45	16.56	21.22	-3.14
min	6.32	6.32	8.91	-19.12
max	27.35	27.68	37.01	-1.25

Source: Author's calculation from the OECD STAN Database, UNIDO INDSTAT2 and the model in the text.
Notes: Columns (1) and (2) show the change in real income associated with a 25% reduction in trade costs for the indicated values of υ. The figures are calculated for a value of $\theta = 4$. Columns (3) and (4) decompose column (2) into the two terms in (41) such that column 2 is the sum of columns 3 and 4.

C Proofs of Theoretical Results

C.1 Section 2

C.1.1 Derivation of (2)

Proof. The analysis that follows applies to any country i, good (z,ω) and time t, and so all references to country, good and time will be drop. Let n be the number of techniques available up to time t for good (z,ω) in country i. As was discussed above, the efficiency of these techniques is obtained as independent draws from the Pareto distribution H. Let $X_1, ..., X_n$ be the random variables corresponding to each of these n draws, and let Y_j represent the j-th best draw among the $X_i s$. We are interested, in the joint distribution of the best and second best draws conditional on n, i.e., the joint distribution of Y_1, Y_2 conditional on n. Following Hogg and Craig (1995) section 4.6, the joint pdf of $Y_1, .., Y_n$ is

$$\begin{aligned} g\left(y_1, ..., y_n|n\right) &= n! h\left(y_1\right) h\left(y_2\right) ... h\left(y_n\right) \text{ for } \infty > y_1 > y_2 > \cdots > y_n > 1 \\ &= 0 \text{ elsewhere} \end{aligned}$$

where h is the pdf corresponding to H. Integrating over $y_3, ..., y_n$, the marginal joint density of Y_1, Y_2 is

$$\begin{aligned} f\left(y_1, y_2|n\right) &= \int_1^{y_2} ... \int_1^{y_{n-2}} \int_1^{y_{n-1}} h\left(y_n\right) ... h\left(y_1\right) dy_n ... dy_1 \\ &= \frac{n!}{(n-2)!} H\left(y_2\right)^{n-2} h\left(y_2\right) h\left(y_1\right) \end{aligned}$$

Once we know $f\left(y_1, y_2\right)$, the joint cdf of Y_1, Y_2 conditional on n can be obtained as

$$\begin{aligned} F\left(x_1, x_2|n\right) &= \Pr\left(Y_1 \leq x_1, Y_2 \leq x_2|n\right) \\ &= \int_1^{x_1} \int_1^{\min\{x_2, y_1\}} g\left(y_1, y_2\right) dy_2 dy_1 \\ &= \int_1^{x_1} \int_1^{x_2} g\left(y_1, y_2\right) dy_2 dy_1 - \int_1^{x_2} \int_1^{y_2} g\left(y_1, y_2\right) dy_1 dy_2 \\ &= nH\left(x_2\right)^{n-1} H\left(x_1\right) - H\left(x_2\right)^n (n-1) \end{aligned}$$

Finally, the unconditional joint distribution of Y_1, Y_2 can be obtain taking expectation over n. Recalling that n is distributed Poisson with parameter T, this yields

$$\begin{aligned} F\left(x_1, x_2\right) &= \sum_{n=0}^{\infty} \frac{F\left(x_1, x_2|n\right) T^n e^{-T}}{n!} \\ &= \sum_{n=0}^{\infty} \frac{nH\left(x_2\right)^{n-1} H\left(x_1\right) T^n e^{-T}}{n!} - \sum_{n=0}^{\infty} \frac{nH\left(x_1\right)^n T^n e^{-T}}{n!} + \sum_{n=0}^{\infty} \frac{H\left(x_1\right)^n T^n e^{-T}}{n!} \\ &= H\left(x_1\right) T e^{-T[1-H(x_2)]} - H\left(x_2\right) T e^{-T[1-H(x_2)]} + e^{-T[1-H(x_2)]} \\ &= \left[1 + T\left(x_2^{-\theta} - x_1^{-\theta}\right)\right] e^{-T x_2^{-\theta}} \end{aligned}$$

and introducing the references to the country, good and time we get the result above.

Recalling that the joint distribution used in BEJK (2003) for the best and second best technique is $K(x_1, x_2) = \left[1 + T\left(x_2^{-\theta} - x_1^{-\theta}\right)\right] e^{-Tx_2^{-\theta}}$ for $x_1 \geq x_2 \geq 0$, notice that F is almost identical to K the only difference being that in this case, F is only valid for $x_2 \geq 1$. The discrepancy arises from the fact hat the minimum efficiency level in the present context is 1, implying that $F(x_1, x_2) = 0$ for $x_2 < 1$. According to K, $\Pr(x_2 \leq 1) = K(\infty, 1) = [1 + T] e^{-T} = \Pr(n \leq 1)$, i.e., the difference is attributable to the fact that at any time t, there is a set of goods with strictly positive mass such that no more than one technique has been discovered. However, notice that this probability approaches zero as $T \to \infty$.

Formally, F_T converges in distribution to K_T as $T \to \infty$, where the subscript T make explicit the dependence of the distributions on the parameter T. Given that T is growing, we need first to normalize the variables and then analyze the convergence of the normalized variables. Let Y_{iT}^d be the random variable representing the i-th best technique according to distribution d and for parameter T, and consider the variables $Z_{iT}^d = Y_i^d T^{1/\theta}$ for $i = 1, 2$ and $d = F_T, K_T$. In what follows I show that $F_T, K_T \to K$, where $K(z_1, z_2) = \left[1 + \left(z_2^{-\theta} - z_1^{-\theta}\right)\right] e^{-z_2^{-\theta}}$ for $z_1 \geq z_2 \geq 0$. Notice that

$$\begin{aligned} F_T^Z(z_1, z_2) &= \Pr\left(Z_{1T}^{F_T} \leq z_1, Z_{2T}^{F_T} \leq z_2\right) \\ &= \Pr\left(Y_{1T}^{F_T} \leq T^{-1/\theta} z_1, Y_{2T}^{F_T} \leq T^{-1/\theta} z_2\right) \\ &= \left[1 + \left(z_2^{-\theta} - z_1^{-\theta}\right)\right] e^{-z_2^{-\theta}} \text{ for } z_1 \geq z_2 \geq T^{-1/\theta} \end{aligned}$$

and so it is clear that $F_T^Z(z_1, z_2) \to K(z_1, z_2)$ for all $z_1 \geq z_2 \geq 0$. Similarly,

$$\begin{aligned} K_T^Z(z_1, z_2) &= \Pr\left(Z_{1T}^{K_T} \leq z_1, Z_{2T}^{K_T} \leq z_2\right) \\ &= \Pr\left(Y_{1T}^{K_T} \leq T^{-1/\theta} z_1, Y_{2T}^{K_T} \leq T^{-1/\theta} z_2\right) \\ &= \left[1 + \left(z_2^{-\theta} - z_1^{-\theta}\right)\right] e^{-z_2^{-\theta}} \text{ for } z_1 \geq z_2 \geq 0 \end{aligned}$$

and so $K_T^Z(z_1, z_2) = K(z_1, z_2)$ for all all $z_1 \geq z_2 \geq 0$. Both results imply that $F_T \to K_T$. ■

C.1.2 Costs, Markups and Prices

In this section As mentioned before, firms in country i produce under constant returns to scale with a unit cost of serving country j given by $w_{it}\tau_{ij}^{\omega}/x(z)$, where $x(z)$ is the efficiency of the firm. The distribution of technologies (2) has the following implications:

I.1. Let $a_{ij}^{\omega(1)}(z)$ be the unit cost of serving market (z, ω) in country j for the most efficient producer of the good in country i. Then the cdf of $a_{ij}^{\omega(1)}(z)$ is

$$G_{ijt}^{\omega}(a) = 1 - e^{-T_{it}^{\omega}\left(w_{it}\tau_{ij}^{\omega}\right)^{-\theta} a^{\theta}}$$

I.2. Let $a_j^{\omega(1)}(z), a_j^{\omega(2)}(z)$ be the costs corresponding to the producers around the world with lowest and the second lowest unit costs of serving market (z,ω) in country j, respectively. The joint cdf of $a_j^{\omega(1)}$ and $a_j^{\omega(2)}$ is

$$G_{jt}^{\omega}(a_1,a_2) = 1 - e^{-\Phi_{jt}^{\omega}a_1^{\theta}} - \Phi_{it}^{\omega}a_1^{\theta}e^{-\Phi_{it}^{\omega}a_2^{\theta}}$$

where Φ_{jt}^{ω} is given in (12). Moreover, G_{jt}^{ω} is also the joint distribution of $a_{jt}^{\omega(1)}(z), a_{jt}^{\omega(2)}(z)$ conditional on country i being the lowest cost supplier.

I.3. At any given moment in time there are many alternative techniques in each country to produce a given final good (z,ω) that differ in their respective efficiencies. As the result of Bertrand competition, the producer with the lowest marginal cost of serving that market becomes the only supplier of the good to that market and charges the minimum between the monopoly price and the maximum price that keeps competitors at bay. Recalling that $a_j^{\omega(1)}(z), a_j^{\omega(2)}(z)$ are lowest and the second lowest unit costs of serving market (z,ω) in country j, the price charged by the unique supplier of good i in that market is

$$p_{jt}^{\omega}(z) = \min\left\{\bar{m}(\sigma^{\omega})\, a_{jt}^{\omega(1)}(z), a_{jt}^{\omega(2)}(z)\right\}$$

where $\bar{m}(\sigma^{\omega})$ is the optimal monopoly markup corresponding to the iso-elastic demand for good (z,ω) which is given by $\bar{m}(\sigma^{\omega}) = \sigma^{\omega}/(\sigma^{\omega}-1)$ if $\sigma^{\omega} > 1$ and $\bar{m}(\sigma^{\omega}) = \infty$ if $\sigma^{\omega} \leq 1$. Moreover, an immediate implication of (I.2) is that the distribution of prices in industry ω and country j does not depend on the source country.

I.4. Let us define the cost gap in country j, $m_{jt}^{\omega}(z) \equiv a_{jt}^{\omega(2)}(z)/a_{jt}^{\omega(1)}(z)$. Then (I.2) implies that $m_j^{\omega}(z)$ is Pareto distributed:

$$M_{jt}^{\omega}(m) = \Pr\left(m_{jt}^{\omega}(z) \leq m\right) = 1 - m^{-\theta}$$

Moreover, the distribution of the cost gap is independent of the source country and of $a_{jt}^{\omega(2)}(z)$.

I.5. With the previous definitions, markups $m_{jt}^{\prime\omega}(z)$ are given by

$$m_{jt}^{\prime\omega}(z) = \min\left\{m_{jt}^{\omega}(z), \bar{m}(\sigma^{\omega})\right\}$$

I.6. The exact price index for industry ω in country i is given by expression (13) in the text.

C.1.3 Derivation of (13)

Proof. In what follows I will drop the subscripts since the analysis applies to any time, country and industry. Starting with the definition of P, we have

$$\begin{aligned} P^{1-\sigma} &= \int_0^1 \left[a^{(1)}(z)\, m'(z)\right]^{1-\sigma} dz \\ &= \int_0^1 \left[a^{(2)}(z)\, \frac{m'(z)}{m(z)}\right]^{1-\sigma} dz \\ &= \mathbb{E}_t\left[\left(a^{(2)}\right)^{1-\sigma}\right] \mathbb{E}_t\left[\left(m'(z)/m(z)\right)^{1-\sigma}\right] \end{aligned}$$

where the first line is obtained using the fact the price is equal to cost times markup, the second line is obtained using the definition of $m(z)$, and the third line is obtained using the fact that m is independent of $a^{(2)}$ (see I.4).

Using the marginal distribution of $a^{(2)}$ we get

$$\mathbb{E}_t\left[\left(a^{(2)}\right)^{1-\sigma}\right]=\Phi^{-(1-\sigma)/\theta}\Gamma\left(\frac{1-\sigma+2\theta}{\theta}\right)$$

Finally, (I.4) and (I.5) imply

$$\begin{aligned}\mathbb{E}_t\left[\left(m'(z)/m(z)\right)^{1-\sigma}\right] &= \int_1^{\bar{m}} dM(m)+\int_{\bar{m}}^{\infty}(\bar{m}/m)^{1-\sigma}dM(m)\\ &= 1+\bar{m}^{-\theta}\frac{(\sigma-1)}{[\theta-(\sigma-1)]}\end{aligned}$$

Using the last two results in the expression above we get the result. ■

C.1.4 Cost Share in Revenues

Proof. I eliminate the subscripts ω, i, t since the analysis that follows is valid for any industry, country and time. Let $Cost(z)\equiv a^{(1)}(z)\,q(z)$ be the total cost of production of country j's demand of good (z,ω). Then

$$\begin{aligned}Cost(z) &= E(z)/m'(z)\\ &= EP^{\sigma-1}p(z)^{1-\sigma}/m'(z)\\ &= EP^{\sigma-1}\left(a^{(1)}(z)\,m'(z)\right)^{1-\sigma}/m'(z)\\ &= EP^{\sigma-1}\left[a^{(2)}(z)\left(m'(z)/m(z)\right)\right]^{1-\sigma}/m'(z)\end{aligned}$$

and integrating over z we get

$$\begin{aligned}Cost &= EP^{\sigma-1}\mathbb{E}_t\left[a^{(2)}(z)\right]\mathbb{E}_t\left[\frac{m'(z)^{-\sigma}}{m(z)^{1-\sigma}}\right]\\ &= \frac{\theta}{1+\theta}E\end{aligned}$$

■

C.1.5 Probability of Staying in the Market

Proof. What is the probability that a lowest cost producer in country j and industry ω in period t is still the state of the art in period $s>t$? Letting $X_{jt}^{\omega}\equiv 1/a_{jt}^{\omega(1)}$, I.2. implies $X_{jt}^{\omega}\sim Fréchet(\Phi_{jt}^{\omega},\theta)$. The quality of the best ideas discovered between t and s in each country k is distributed $F(x)=e^{-T_k' x^{-\theta}}$ for $x\geq 1$ and where $T_k'^{\omega}=T_{ks}^{\omega}-T_{kt}^{\omega}$. Notice that this distribution is independent of the distribution of state of the art ideas at time t. Letting $X_j'(\omega)$ denote the random variable representing the inverse of lowest costs in country j at time s of the ideas generated between t and s, we know $X_j'^{\omega} \sim Fréchet(\Phi_j'^{\omega}, \theta)$, where

$$\begin{aligned} \Phi_j'^{\omega} &= \sum_{k=1}^{N} T_k'^{\omega} \left(w_{ks} \tau_{ij}^{\omega}\right)^{-\theta} \\ &= \Phi_{js}^{\omega} - \Phi_{jt}^{\omega} \end{aligned}$$

Since the ideas generated between period t and s are independent from the ideas up to time t, the distribution of $X_j'^{\omega}$ is also independent from the distribution of X_{jt}^{ω}. Setting $w_{it} = 1$ for all t, the probability of that an idea from country i stays in the market in s conditional on being in the market in period t is just equal to the $\Pr\left(X_{jt}^{\omega} \geq X_j'^{\omega}\right)$. Given the Fréchet distribution of inverse costs we obtain

$$\Pr\left(X_{jt}^{\omega} \geq X_j'^{\omega}\right) = \frac{\Phi_{jt}^{\omega}}{\Phi_{jt}^{\omega} + \Phi_{jt}'^{\omega}} = \frac{\Phi_{jt}^{\omega}}{\Phi_{js}^{\omega}}$$

■

C.2 Section 3

C.2.1 Proof of Lemma 1

Proof of parts (i)-(iii). $L_{it}^{q,\omega}, L_{it}^{R,\omega}$ growing at constant rates together with L_{it} growing at the constant rate n, necessarily implies that there the share of labor allocated to each industry ω is constant in the BGP and so $\widetilde{L}_{it}^{q}(\omega) = \widetilde{L}_{it}^{R}(\omega) = n$. Differentiating (1) with respect to time yields

$$\widetilde{T}_{it}^{\omega} = \iota_i^{\omega} \left(L_{it}^{R,\omega}\right)^{\upsilon} / T_{it}^{\omega}$$

Recalling that $\widetilde{T}_{it}^{\omega}$ is constant in a BGP, log-differentiating the last expression with respect to time yields $\upsilon \widetilde{L}_{it}^{R,\omega} = \widetilde{T}_{it}^{\omega}$ which in turn implies $\widetilde{T}_{it}^{\omega} = \upsilon n$.

Combining equations (11)-(15) and (18) yield

$$w_{it} L_{it}^{q} = \sum_{j=1}^{N} \lambda_{ijt} w_{jt} L_{jt}^{q} \tag{43}$$

with $\lambda_{ijt} = \int_{\Omega} \alpha_j^{\omega} \lambda_{ijt}^{\omega} d\omega$. Given the technology levels T_{kt}^{ω} and labor allocations $L_{kt}^{q,\omega}$ for all countries k and industries ω, the previous equation determines the equilibrium wages at time t, w_{kt}. It should be pretty clear that if $L_{kt}^{q,\omega}$ and T_{kt}^{ω} grow at the same constant rates across countries, then a vector of wages w_k that solves (43) at time t should also solve it at any time $s > t$. Then if we set the w_i as the numeraire, the wages of all countries are constant in the BGP.

Once we know wages are constant then it is easy to see that $\widetilde{R}_{it} = \widetilde{E}_{it} = \widetilde{R}_{it}^{\omega} = n$, from (12) we have $\widetilde{\Phi}_{kt}^{\omega} = \widetilde{T}_{kt}^{\omega} = n\upsilon$ for all k and (11) imply that trade shares are constant.

From (8) and (13) we get $\widetilde{P}_{it} = -n\upsilon/\theta$, which in turn imply that real wages in country i grow at $n\upsilon/\theta$. The relations in (9) imply $\widetilde{C}_{it} = n + n\upsilon/\theta$ and using (10) the interest rate is constant and is given by $r = n\eta + (\eta - 1)\frac{n\upsilon}{\theta} + \rho$. Using this in (16) we get $\widetilde{V}^{\omega}_{ijt} = n$. ■

Proof of part (iv). Using the results of Lemma 1 regarding $\widetilde{T}^{\omega}_{it}$ and r_{it} in the BGP, the expression for the value of an idea (16) yields $V^{\omega}_{ijt} = \frac{E^{\omega}_{tjt}}{(1+\theta)[r-(1-\upsilon)n]}$. Then
$\sum_{j=1}^{N}\lambda^{\omega}_{ijt}V^{\omega}_{ijt} = \frac{\sum_{j=1}^{N}\lambda^{\omega}_{ijt}E^{\omega}_{jt}}{(1+\theta)[r-(1-\upsilon)n]} = \frac{R^{\omega}_{it}}{(1+\theta)[r-(1-\upsilon)n]}$

$$\begin{aligned}\sum_{j=1}^{N}\lambda^{\omega}_{ijt}V^{\omega}_{ijt} &= \frac{\sum_{j=1}^{N}\lambda^{\omega}_{ijt}E^{\omega}_{jt}}{(1+\theta)\left[r-(1-\upsilon)n\right]}\\ &= \frac{R^{\omega}_{it}}{(1+\theta)\left[r-(1-\upsilon)n\right]}\\ &= \frac{w_{it}L^{q,\omega}_{it}}{\theta\left[r-(1-\upsilon)n\right]}\end{aligned}$$

where in the second line I used (15) ($R^{\omega}_{it} = \sum_{j=1}^{N}\lambda^{\omega}_{ijt}E^{\omega}_{jt}$) and in the third equality I used (14) ($w_{it}L^{q,\omega}_{it} = \frac{\theta}{1+\theta}R^{\omega}_{it}$). Using the last expression in the FOC of the firm (17) and solving for T^{ω}_{it} yields

$$T^{\omega}_{it} = \frac{\iota^{\omega}_{i}\upsilon\left(L^{R,\omega}_{it}\right)^{\upsilon-1}L^{q,\omega}_{it}}{\theta\left[r-(1-\upsilon)n\right]}$$

Differentiating (1) with respect to time yields $\widetilde{T}^{\omega}_{it} = \iota^{\omega}_{i}\left(L^{R,\omega}_{it}\right)^{\upsilon}/T^{\omega}_{it}$, and using the result of Lemma 1 regarding $\widetilde{T}^{\omega}_{it}$ we get

$$T^{\omega}_{it} = \frac{\iota^{\omega}_{i}\left(L^{R,\omega}_{it}\right)^{\upsilon}}{\upsilon n}$$

The two previous expressions imply that $L^{q,\omega}_{it}/L^{R,\omega}_{it} = \theta\left[r-(1-\upsilon)n\right]/\upsilon^{2}n$ for all ω and i, i.e., the research intensity is the same across industries and countries, and is given by the expression in the text. ■

C.2.2 Derivation of (20)

Given that in the BGP the interest rate is equalized across countries, Lemma 1 and (16) imply $V^{\omega}_{ijt} = \frac{E^{\omega}_{jt}}{(1+\theta)[r-n(1-\upsilon)]} \equiv V^{\omega}_{jt}$ for any country i. Consequently, V^{ω}_{jt} represents the expected present value of profits generated by country j's stream of expenditure in industry ω.

Differentiating (1) with respect to time yields $\widehat{T}^{\omega}_{it} = \iota^{\omega}_{i}\left(L^{R,\omega}_{it}\right)^{\upsilon}/T^{\omega}_{it}$, and using the result of Lemma 1 regarding $\widehat{T}^{\omega}_{it}$ we get $(T^{\omega}_{it}\upsilon n/\iota^{\omega}_{i})^{\frac{\upsilon-1}{\upsilon}} = \left(L^{R,\omega}_{it}\right)^{\upsilon-1}$, and using this back in the first order condition (17) (FOC of research firms) we get

$$T^{\omega}_{it} = \iota^{\omega}_{i}\upsilon^{\upsilon}\left(\upsilon n\right)^{\upsilon-1}\left[\frac{\sum_{j=1}^{N}\lambda^{\omega}_{ijt}V^{\omega}_{jt}}{w_{i}}\right]^{\upsilon}$$

The previous equation relates the level of the stock of ideas at time t, T_{it}^{ω}, with the expected present value of the profits generated by firms in country i and industry ω. Notice that in the BGP, expression (16) for V_{ijt}^{ω} implies $V_{jt}^{\omega} = E_{jt}^{\omega}\zeta$, where $\zeta = \{(1+\theta)\left[r - n\left(1-\upsilon\right)\right]\}^{-1}$. Then we can write

$$
\begin{aligned}
\sum_{j=1}^{N} \lambda_{ijt}^{\omega} V_{jt}^{\omega} &= \sum_{j=1}^{N} \lambda_{ijt}^{\omega} \alpha_j^{\omega} E_{jt}\zeta \\
&= \left[\sum_{j=1}^{N} \lambda_{ijt}^{\omega} \frac{\alpha_j^{\omega} E_{jt}\zeta}{\sum_{k=1}^{N} \alpha_k^{\omega} E_{kt}\zeta} \right] V_t^{\omega} \\
&= \left[\sum_{j=1}^{N} \lambda_{ijt}^{\omega} \frac{\alpha_j^{\omega} \left(E_{jt}/E_t\right)}{\sum_{k=1}^{N} \alpha_k^{\omega} \left(E_{kt}/E_t\right)} \right] V_t^{\omega} \\
&= \left[\sum_{j=1}^{N} \lambda_{ijt}^{\omega} \frac{\alpha_j^{\omega} \beta_j^{E}}{\sum_{k=1}^{N} \alpha_k^{\omega} \beta_k^{E}} \right] V_t^{\omega} \\
&= \left[\sum_{j=1}^{N} \lambda_{ijt}^{\omega} \beta_j^{E,\omega} \right] V_t^{\omega} \\
&= \beta^{R,\omega} V_t^{\omega}
\end{aligned}
$$

where in the second line I divided and multiply by $V_t^{\omega} = \zeta \sum_{k=1}^{N} \alpha_k^{\omega} E_{kt}$; in the third line I divided numerator and denominator by world expenditure $E_t \equiv \sum_{k=1}^{N} E_{kt}$; in the forth line I used the definition $\beta_j^{E} \equiv E_{jt}/E_t$; in the fifth line I used the fact that $\beta_j^{E,\omega} \equiv E_{jt}^{\omega}/E_t^{\omega} = \alpha_j^{\omega}\beta_j^{E} / \left(\sum_{k=1}^{N} \alpha_k^{\omega}\beta_k^{E}\right)$; and in the fifth line I use the definition of $\beta^{R,\omega}$ together with (15). Using the last result in the previous expression yields the result in the text.

C.2.3 Derivation of (21)

Proof. Notice that $\widetilde{T}_{it}^{\omega} = \iota_i^{\omega} \left(L_{it}^{R,\omega}\right)^{\upsilon} / T_{it}$ and Lemma 1 imply that in the BGP we have

$$
T_{it}^{\omega} = \iota_i^{\omega} \left(L_{it}^{R,\omega}\right)^{\upsilon} / \upsilon n
$$

Dividing and multiplying the RHS of the last expression by $L_{it}^{\upsilon}\left(L_{it}^{\omega}\right)^{\upsilon}$ and using the fact that the research intensity is constant and the same for every industry in the BGP we get

$$
T_{it}^{\omega} = B_T' \iota_i^{\omega} \left[\delta_{it}^{\omega} L_{it}\right]^{\upsilon}
$$

where $\delta_i^{\omega} \equiv L_{it}^{\omega}/L_{it}$ denotes the share of resources allocated to industry ω and $B_T' \equiv (\upsilon n)^{-1} \kappa^{\upsilon}$ is a constant. ■

C.2.4 Derivation of (22)

Proof. Notice that $\widetilde{T}_{it}^{\omega}=\iota_i^{\omega}\left(L_{it}^{R,\omega}\right)^{\upsilon}/T_{it}$ and Lemma 1 imply that in the BGP, $T_{it}^{\omega}=\iota_i^{\omega}\left(L_{it}^{R,\omega}\right)^{\upsilon}/\upsilon n$. Combining this with (21) we get

$$\left(L_{it}^{R,\omega}\right)^{\upsilon}=\upsilon n B_T\left[\beta_i^{R,\omega}V_t^{\omega}/w_i\right]^{\upsilon}$$

Recalling that $\alpha^{\omega}\equiv E_t^{\omega}/E_t$ denotes the share of world expenditure allocated to industry ω, the Cobb-Douglas upper tier utility function implies that we can write $\alpha^{\omega}=\sum_{k=1}^{N}\alpha_k^{\omega}\beta_{kt}^{E}$. Then from the definition of V_t^{ω} we get $V_t^{\omega}=\zeta\alpha^{\omega}E_t$, where ζ is the same constant defined above. With this last relationship in mind, we can take the ratio of the last equation for two industries ω and ω' to get

$$\frac{L_{it}^{R,\omega'}}{L_{it}^{R,\omega}}=\frac{\beta_i^{R,\omega'}\alpha^{\omega'}}{\beta_i^{R,\omega}\alpha^{\omega}}$$

This immediately implies that

$$\frac{L_{it}^{R,\omega}}{L_{it}^{R}}=\frac{\beta_i^{R,\omega}\alpha^{\omega}}{\int_{\Omega}\beta_i^{R,\omega}\alpha^{\omega}d\omega}=\frac{\beta_i^{R,\omega}\alpha^{\omega}}{\beta_i^{R}}$$

Recalling that the research intensity is constant in the BGP we get

$$\delta_t^{\omega}\equiv\frac{L_{it}^{\omega}}{L_{it}}=\frac{\beta_i^{R,\omega}\alpha^{\omega}}{\beta_i^{R}}=\frac{\beta_i^{R,\omega}}{\beta_i^{E,\omega}}\alpha_i^{\omega}$$

where the last equality is obtained using the balanced trade condition, $\beta_i^{E,\omega}=\alpha_i^{\omega}\beta_i^{E}/\sum_{k=1}^{N}\alpha_k^{\omega}\beta_k^{E}$ and $\alpha^{\omega}=\sum_{k=1}^{N}\alpha_k^{\omega}\beta_k^{E}$. ■

C.2.5 Proof of Proposition 1

The description of the equations (23.1)-(23.4) can be found in the text. Here I concentrate on the description of the remaining equations and on the proof of the existence of a BGP in which all market shares are strictly positive.

Derivation of equation (23.5). Starting from

$$R_{it}=\int_{\Omega}R_{it}^{\omega}d\omega,^{51}$$

we obtain (23.1) dividing both sides of the world output $E_t=R_t$ and usin the fact that $R_{it}^{\omega}=\beta_i^{R,\omega}\alpha^{\omega}E_t$.

Derivation of equation (23.6). Starting from the definition of world expenditure in industry ω,

$$E_t^{\omega}=\sum_{j=1}^{N}\alpha_j^{\omega}E_{jt}^{\omega},$$

[51] Total manufacturing output is the sum of output across all industries.

we obtain (23.6) dividing both sides of the last expression by total world expenditure E_t.

Derivation of equation (23.7). Starting from $E_{it}^{\omega} = \alpha_i^{\omega} E_{it}$, we obtain (23.7) dividing both sides by world expenditure in industry ω.

Finally, equation (23.8) is obtained using the fact that labor income is proportional to total output.

Let us now turn to proof of the existence of a solution to system 23.

C.2.6 Proof of Lemma 2

Proof. The characterization under autarky is obtained combining equations (21) and (22) together with the fact that in autarky $\beta_i^{R,\omega}/\beta_i^{E,\omega} = 1$ for all countries and industries.

Let us turn now to the characterization corresponding to a zero gravity world. Using equations (23.4) and (23.8) in (23.1) and specializing it for the zero gravity case yield

$$\lambda_{ij}^{\omega} = \frac{\iota_i^{\omega} \left(\beta_i^{R,\omega}\right)^{\upsilon} w_i^{-(\theta+\upsilon)}}{\sum_k \iota_k^{\omega} \left(\beta_k^{R,\omega}\right)^{\upsilon} w_k^{-(\theta+\upsilon)}}$$

As we can see from the last expression, λ_{ij}^{ω} does not depend on the importer j, implying a zero gravity world the exports of a country represent the same share of any importer's expenditure, i.e., $\lambda_{ij}^{\omega} = \lambda_{ij'}^{\omega}$ for all i, j, j'. This together with equation (23.2) imply that $\beta_i^{R,\omega} = \lambda_{ii}^{\omega}$ for all country i. Consequently, the ratio of the market shares of two countries i, j is given by

$$\frac{\beta_i^{R,\omega}}{\beta_j^{R,\omega}} = \frac{\lambda_{ii}^{\omega}}{\lambda_{jj}^{\omega}} = \frac{\iota_i^{\omega} \left(\beta_i^{R,\omega}\right)^{\upsilon} w_i^{-(\theta+\upsilon)}}{\iota_j^{\omega} \left(\beta_j^{R,\omega}\right)^{\upsilon} w_j^{-(\theta+\upsilon)}}$$

and solving for $\beta_i^{R,\omega}/\beta_j^{R,\omega}$ yields

$$\frac{\beta_i^{R,\omega}}{\beta_j^{R,\omega}} = \left[\frac{\iota_i^{\omega}}{\iota_j^{\omega}}\right]^{\frac{1}{1-\upsilon}} \left[\frac{w_i}{w_j}\right]^{-\frac{(\theta+\upsilon)}{1-\upsilon}}$$

Finally, taking double ratios in equation (23.4) and using the last expression yields the result in the text

$$\begin{aligned} \frac{T_{it}^{\omega}/T_{it}^{\omega'}}{T_{jt}^{\omega}/T_{jt}^{\omega'}} &= \left[\frac{\iota_i^{\omega}/\iota_i^{\omega'}}{\iota_j^{\omega}/\iota_j^{\omega'}}\right] \left[\frac{\beta_i^{R,\omega}/\beta_i^{R,\omega'}}{\beta_j^{R,\omega}/\beta_j^{R,\omega'}}\right]^{\upsilon} \\ &= \left[\frac{\iota_i^{\omega}/\iota_i^{\omega'}}{\iota_j^{\omega}/\iota_j^{\omega'}}\right]^{1+\frac{\upsilon}{1-\upsilon}} \end{aligned}$$

■

C.2.7 Proof of Lemma 3

Proof. Before turning to the proof of Lemma, it is convenient to first find an expression for the market shares in the case of frictionless trade, i.e. $\tau = 1$. In this case, equation (23.1) implies that $\lambda_{ij}^{\omega} = \lambda_{ij'}^{\omega}$ for all i, j, j'. Combining this with equation (23.2) we get $\lambda_{ij}^{\omega} = \lambda_{ij'}^{\omega} = \beta_i^{R,\omega}$ for all i. Using this together with expression resulting from using equation (23.4) in (23.1) we get

$$\begin{aligned} \beta_i^{R,\omega} &= \frac{\iota_i^{\omega} \left(\beta_i^{R,\omega}\right)^{\upsilon} w_i^{-(\theta+\upsilon)}}{\sum_{k=1}^{2} \iota_k^{\omega} \left(\beta_k^{R,\omega}\right)^{\upsilon} w_k^{-(\theta+\upsilon)}} \\ &= \frac{\left(\iota_i^{\omega}\right)^{\frac{1}{1-\upsilon}} w_i^{-\frac{\theta+\upsilon}{1-\upsilon}}}{\left[\sum_{k=1}^{2} \iota_k^{\omega} \left(\beta_k^{R,\omega}\right)^{\upsilon} w_k^{-(\theta+\upsilon)}\right]^{\frac{1}{1-\upsilon}}} \end{aligned}$$

The last condition implies $\beta_i^{R,\omega}/\beta_j^{R,\omega} = \left(\iota_i^{\omega}/\iota_j^{\omega}\right)^{\frac{1}{1-\upsilon}} (w_i/w_j)^{-\frac{\theta+\upsilon}{1-\upsilon}}$, and using this together with the fact that $\sum_k \beta_k^{R,\omega} = 1$ we get

$$\beta_i^{R,\omega} = \frac{\left(\iota_i^{\omega}\right)^{\frac{1}{1-\upsilon}} w_i^{-\frac{\theta+\upsilon}{1-\upsilon}}}{\sum_{k=1}^{N} \left(\iota_k^{\omega}\right)^{\frac{1}{1-\upsilon}} w_k^{-\frac{\theta+\upsilon}{1-\upsilon}}} \tag{44}$$

Let us now turn to the derivation of the results in the Lemma which I prove with the next three results.

Condition (28) implies that country i is a net importer in industry ω when $\tau = 1$— Letting NX_i^{ω} denote country i's net exports in industry ω, we have that $NX_1^{\omega} < 0$ if and only if $X_{12}^{\omega}/X_{21}^{\omega} < 1$, where X_{ij}^{ω} denote the total exports from country i to country j in industry ω. When $\tau = 1$, we have

$$\frac{X_{12}^{\omega}}{X_{21}^{\omega}} = \frac{\lambda_{12}^{\omega}\alpha_2^{\omega}E_2}{\lambda_{21}^{\omega}\alpha_1^{\omega}E_1} = \left(\frac{\iota_1^{\omega}}{\iota_2^{\omega}}\right)^{\frac{1}{1-\upsilon}} \frac{\alpha_2^{\omega}}{\alpha_1^{\omega}}$$

where the second equality is obtain using the fact that with frictionless trade $\lambda_{ij}^{\omega} = \beta_i^{R,\omega}$, $E_1 = E_2$ from the symmetry assumption and equation (44). Consequently,

$$NX_1^{\omega} < 0 \Longleftrightarrow \left(\frac{\iota_1^{\omega}}{\iota_2^{\omega}}\right)^{\frac{1}{1-\upsilon}} < \frac{\alpha_1^{\omega}}{\alpha_2^{\omega}} \tag{45}$$

Due to the fact that the expenditure shares of each country across industries must add up to one, condition (27) implies $\alpha_1^{\omega}/\alpha_2^{\omega} > 1$ and $\alpha_1^{\omega'}/\alpha_2^{\omega'} < 1$. In a similar way, the assumption about symmetric countries and condition (27) imply $(\iota_1^{\omega}/\iota_2^{\omega}) < 1$ and $\left(\iota_1^{\omega'}/\iota_2^{\omega'}\right) > 1$. The last two conditions together with condition (45) yield $NX_1^{\omega} < 0$, i.e., country 1 is a net importer in industry ω when there are no frictions to trade.

Condition (27) implies that country i is a net exporter in industry ω for a sufficiently high value of τ— As before, $NX_1^{\omega} > 0$ if and only if $X_{12}^{\omega}/X_{21}^{\omega} > 1$. For the general case of $\tau > 1$, we have

$$\frac{X_{12}^{\omega}}{X_{21}^{\omega}} = \frac{\lambda_{12}^{\omega}\alpha_2^{\omega}}{\lambda_{21}^{\omega}\alpha_1^{\omega}} = \frac{\alpha_2^{\omega}}{\alpha_1^{\omega}}\frac{\iota_1^{\omega}}{\iota_2^{\omega}}\left(\frac{\beta_1^{R,\omega}}{\beta_2^{R,\omega}}\right)^{\upsilon} \frac{\left[\iota_1^{\omega}\left(\beta_1^{R,\omega}\right)^{\upsilon} + \iota_2^{\omega}\left(\beta_2^{R,\omega}\right)^{\upsilon}\tau^{-\theta}\right]}{\left[\iota_1^{\omega}\left(\beta_1^{R,\omega}\right)^{\upsilon}\tau^{-\theta} + \iota_2^{\omega}\left(\beta_2^{R,\omega}\right)^{\upsilon}\right]}, \tag{46}$$

where the second equality is obtained using the definitions of λ_{ij}^{ω} given in equation (23.1) together with equation (23.4). In autartky $\beta_i^{R,\omega} = \beta_i^{E,\omega}$, which in turn implies that $\beta_1^{R,\omega}/\beta_2^{R,\omega} = \alpha_1^{\omega}/\alpha_2^{\omega}$. Taking the limit as $\tau \to \infty$ in the last expression and we get

$$\lim_{\tau\to\infty} \frac{X_{12}^{\omega}}{X_{21}^{\omega}} > 1 \Longleftrightarrow \left(\frac{\alpha_1^{\omega}}{\alpha_2^{\omega}}\right)^{\upsilon-\frac{1}{2}} > \frac{\iota_2^{\omega}}{\iota_1^{\omega}}. \tag{47}$$

Notice that the RHS of the last equivalence is just condition (27). This implies that if condition (27) holds, then there is a value $\bar{\tau}$ such that for all $\tau > \bar{\tau}$, country 1 is a net exporter in industry ω.

There is only one level of transport costs $\tau \in (0,1)$ such that trade is balanced in each industry— Notice that if trade is balanced in industry ω, then $\beta_i^{R,\omega} = \beta_i^{E,\omega}$, which in turn implies that $\beta_1^{R,\omega}/\beta_2^{R,\omega} = \alpha_1^{\omega}/\alpha_2^{\omega}$. Using this in (46), and letting $z \equiv \tau^{-\theta}$, trade is balanced for some $z \in (0,1)$ if, and only if,

$$f(z) \equiv c\frac{[a+bz]}{[az+b]} = 1,$$

where $c \equiv \left(\frac{\alpha_2^{\omega}}{\alpha_1^{\omega}}\right)^{1-\upsilon} \frac{\iota_1^{\omega}}{\iota_2^{\omega}}$, $a \equiv \iota_1^{\omega}\left(\alpha_1^{\omega}/\alpha_2^{\omega}\right)^{\upsilon}$, and $b \equiv \iota_2^{\omega}$. Let us now analyze the behaviour of the function $f(z)$. First, condition (47) implies that $f(0) > 1$. Second, $f(1) = c < 1$ since $\alpha_2^{\omega}/\alpha_1^{\omega} < 1$ and $\iota_1^{\omega}/\iota_2^{\omega} < 1$. Finally, notice that

$$f'(z) = c\frac{\left[b^2 - a^2\right]}{[az+b]^2} < 0$$

for all $z \in (0,1)$, since condition (28) implies $b \equiv \iota_2^{\omega} < \iota_1^{\omega}\left(\alpha_1^{\omega}/\alpha_2^{\omega}\right)^{\upsilon} \equiv a$. Consequently, there is at most one $z \in (0,1)$ such that trade is balanced in industry ω.

The results proved above imply that as countries move from autarky to trade, they display a unique reversal in their export profile. ■

C.2.8 Proof of Proposition 2

Proof of part (i). To simplify the exposition I consider the case of two symmetric countries that have the same preferences and two industries. However, nothing in the proof depends on countries having the same preferences or in the number of industries. The only requirement is that countries are mirror images of each other.

Countries differ in their research productivities across two industries $\omega = 1, 2$. The mirror symmetry assumption for the two countries imply that research productivities satisfy $\iota_i^{\omega} = \iota_j^{\omega'}$ and for $i \neq j$. Let us now consider the BGP of this world economy. The system of equations (23) to obtain the BGP of the economy reduces to

$$\lambda_{ij}^{\omega} = \frac{\iota_i^{\omega}\left(\beta_i^{R,\omega}\right)^{\upsilon}\left(\tau_{ij}^{\omega}\right)^{-\theta}}{\sum_{k=1}^{2}\iota_k^{\omega}\left(\beta_k^{R,\omega}\right)^{\upsilon}\left(\tau_{kj}^{\omega}\right)^{-\theta}}; \quad \beta_i^{R,\omega} = \sum_{j=1}^{N}\lambda_{ij}^{\omega}\frac{1}{2} \tag{48}$$

where $\tau_{kj}^{\omega} = \tau$ for $k \neq j$, and $\beta_j^{E,\omega} = 1/2$ due to symmetry and equal preferences. Given that $\beta_i^{R,\omega} = 1 - \beta_j^{R,\omega}$, with the previous set of equations we can solve for $\beta_i^{R,\omega}$. The symmetry assumption together with $\tau_{kj}^{\omega} = \tau$ for $k \neq j$ imply that market shares in the other industry satisfy $\beta_i^{R,\omega} = \beta_j^{R,\omega'}$. In addition, symmetry implies that this solution satisfies the balanced trade condition. From the solution to this system, we can back out the manufacturing technology levels

$$T_i^{\omega} = B_T' \iota_i^{\omega} \left[\beta_i^{R,\omega} L\right]^{\upsilon} \tag{49}$$

where L denotes the labor endowment in both countries.

Now consider the following maximization problem (P1),

$$U(\tau) = \max \frac{1}{2}\frac{1}{\theta} \sum_{i=1}^{2} \sum_{\omega=1}^{2} \ln \Phi_i^{\omega}$$

subject to

$$\begin{aligned} &\Phi_i^{\omega} = T_i^{\omega} + T_j^{\omega}\tau^{-\theta} && \lambda_{ij}^{\omega} = \frac{T_i^{\omega}\tau^{-\theta}}{\Phi_j^{\omega}} \\ &T_i^{\omega} = B_T' \iota_i^{\omega} \left(L_i^{R,\omega}\right)^{\upsilon} && \beta_i^{R,\omega} = \sum_{j=1}^{2} \lambda_{ij}^{\omega} \tfrac{1}{2} \\ &\textstyle\sum_{\omega=1}^{2} L_i^{R,\omega} = L \end{aligned} \tag{P1}$$

for all ω, i. The objective function in this problem is proportional to the geometric average of the inverse of the price levels in each country. The proof of the Lemma relies on showing that the system of equations (48) gives a solution to the previous problem.

Claim 1 *The solution to the equations in (48) and (49) are a solution to problem P1.*

Proof. The first order condition with respect to $L_i^{R,\omega}$ yields

$$L_i^{R,\omega} = \frac{\upsilon}{\mu_i \theta} \left[\frac{1}{2}\lambda_{ii}^{\omega} + \frac{1}{2}\lambda_{ij}^{\omega}\right] = \frac{\upsilon}{\mu_i \theta} \beta_i^{R,\omega}$$

where μ_i is the Lagrange multiplier associated with the labor feasibility constraint, and the second equality is obtained using constraint the definition of T_i^{ω} in the constraints. Using this back in the definition of T_i^{ω} and in the expression for λ_{ij}^{ω}, and recalling that symmetry implies $\mu_i = \mu_j$, we arrive to system (48) above. All the symmetry assumptions made imply that for any variable X, the solution to the previous problem satisfies $X_i^{\omega} = X_j^{\omega'}$. Consequently, from the solution of the system we can obtain the rest of the variables corresponding to the solution on the previous problem. In particular, the technology levels are given by

$$T_i^{\omega} = B_T' \iota_i^{\omega} \left[\frac{\beta_i^{R,\omega}}{\beta_i^{R,\omega} + \beta_i^{R,\omega'}} L\right]^{\upsilon} = B_T' \iota_i^{\omega} \left[\beta_i^{R,\omega} L\right]^{\upsilon}$$

since $\beta_i^{R,\omega} + \beta_i^{R,\omega'} = 1$. ■

Armed with the last claim, we are ready to prove the Lemma. Consider a change in real income associated with a change in trade cots $\hat{\tau}$. We are interested in comparing the predicted changes in real income between the model with innovation ($\upsilon > 0$) and the model with no innovation ($\upsilon = 0$), as predicted by solving the system in changes (29) specialized to the symmetric case under consideration, i.e., we are interested in the predicted changes in real income conditional on observed trade shares and market shares in the original equilibrium. These conditional changes are in line with the analysis in Arkolakis, Costinot and Rodriguez-Clare (2012). However, given that the model with and without innovation share the same cross-section structure, we can always assume that the set of exogenous parameters and manufacturing technologies generating the observed initial equilibrium is the same in both models. In this way, this comparison is also compatible with the comparative static exercises in Melitz and Redding (2014). When the two models are set up in this way, real income is also the same across models in the original equilibrium.

Claim 1 implies that the changes in real income associated with the change in trade costs in the model with directed research, $\widehat{W}_{\upsilon>0}$, corresponds to the change in the objective function in problem P1, i.e., $\widehat{W}_{\upsilon>0} = U(\tau')/U(\tau)$. In addition, the change in real income in the model with no innovation, $\widehat{W}_{\upsilon=0}$, corresponds to the change in the objective function in problem P1 when technology levels are kept at their original levels. A straight forward revealed preference argument implies that $\widehat{W}_{\upsilon>0}(\widehat{\tau}) > \widehat{W}_{\upsilon=0}(\widehat{\tau})$ for all $\widehat{\tau} \neq 1$.

Proof of part (ii). Consider the effects of raising trade costs to their autarky levels, $\tau_{ij}^{\omega} \to \infty$ for $i \neq j$. For this particular shock, evaluating (31) is straight forward. In autarky, the home share of expenditure must be equal to one in every industry, while the share of each industry in total output must be equal to the share of consumers' total expenditure allocated to the industry i.e., $\lambda_{ii}^{\omega} = 1$ and $\delta_i^{\omega} = \alpha_i^{\omega}$.[52] Consequently, for any $\upsilon \in [0,1)$, the change in real income associated with moving to autarky, can be computed as

$$\frac{W_{it}}{W_{it}^{a}} = \exp\left\{\int_{\Omega} \log\left(\frac{\delta_i^{\omega}}{\alpha_i^{\omega}}\right)^{\upsilon\alpha_i^{\omega}/\theta} d\omega\right\} \exp\left\{\int_{\Omega} \log\left(\lambda_{ii}^{\omega}\right)^{-\alpha_i^{\omega}/\theta} d\omega\right\} \tag{50}$$

where W_{it}^{a} denotes the real income per capita in autarky. Noticing that Jensen's inequality implies

$$\int_{\Omega} \alpha_i^{\omega} \log\left(\frac{\delta_i^{\omega}}{\alpha_i^{\omega}}\right) d\omega < \log\left(\int_{\Omega} \alpha_i^{\omega} \frac{\delta_i^{\omega}}{\alpha^{\omega}} d\omega\right) = \log\left(\int_{\Omega} \delta_i^{\omega} d\omega\right) = 0$$

we can write (50) as follows

$$W_{it}/W_{it}^{a} = A_i^{\frac{\upsilon}{\theta}} \exp\left\{\int_{\Omega} \log\left(\lambda_{ii}^{\omega}\right)^{-\alpha_i^{\omega}/\theta} d\omega\right\}$$

where $A_i = \exp\left\{\int_{\Omega} \alpha^{\omega} \log\left(\frac{\delta_i^{\omega}}{\alpha_i^{\omega}}\right) d\omega\right\} < 1$. In other words, the benchmark model with no innovation overestimates the reductions in real income per capita from moving to autarky relative to the model with directed research.

[52] Recall that $\delta_i^{\omega} = \frac{\beta_i^{R,\omega}}{\beta_i^{E,\omega}} \alpha_i^{\omega}$, which can differ from α_i^{ω} only if $\beta_i^{R,\omega}/\beta_i^{E,\omega} \neq 1$, i.e., only if trade is not balanced in the class.

C.2.9 Home Market Effect

In this section, I follow Krugman (1980) and define the home market effect as the situation in which the country with the relatively larger domestic market in an industry becomes the net exporter in that industry.

To analyze the home market effect, it is convenient to consider a world with only two countries and two industries, $\Omega = 2$. Countries are mirror images of each other and they only differ in their preferences. In particular, (i) countries have the same research productivity across industries, eliminating any specialization due to comparative advantage; (ii) countries have the same size as captured by population size, eliminating weak scale effects on technology; (iii) $\tau_{ij}^{\omega} = \tau_{ji}^{\omega} = \tau$. Their preferences satisfy $\alpha_i^{\omega} = \alpha_j^{\omega'}$, and of course $\alpha_i^{\omega'} = (1 - \alpha_i^{\omega})$.

In what follows I show how the difference in preferences stated above affects the trade patterns for different values of the decreasing returns parameter υ. The previous conditions guarantee that in equilibrium both countries have the same wage -which I normalize to one- which implies that both countries also have the same total expenditure. Moreover, we only need to focus on one industry since the other industry will just be mirror image of it.

Under these conditions we have

$$\begin{aligned}\beta_i^{R,\omega} &= \lambda_{ii}^{\omega}\beta_i^{E,\omega} + \lambda_{ij}^{\omega}\beta_j^{E,\omega} \\ &= \frac{\left(\beta_i^{R,\omega}\right)^{\upsilon}}{\left(\beta_i^{R,\omega}\right)^{\upsilon}+\left(\beta_j^{R,\omega}\right)^{\upsilon}\tau^{-\theta}}\frac{\alpha_i^{\omega}}{\alpha_i^{\omega}+\alpha_j^{\omega}} + \frac{\left(\beta_i^{R,\omega}\right)^{\upsilon}\tau^{-\theta}}{\left(\beta_i^{R,\omega}\right)^{\upsilon}\tau^{-\theta}+\left(\beta_j^{R,\omega}\right)^{\upsilon}}\frac{\alpha_j^{\omega}}{\alpha_i^{\omega}+\alpha_j^{\omega}}\end{aligned}$$

for $i = 1, 2$ and $j \neq i$, and where in the second line I used the definition of $\beta_1^{E,\omega}$ and the fact that $E_1 = E_2$.

Dividing dividing each side of the last equation by $\beta_i^{R,\omega}$, substracting the equation corresponding to $i = 2$ from the one corresponding to $i = 1$, and solving for $\alpha_2^{\omega}/\alpha_1^{\omega}$ we obtain

$$\frac{\alpha_2^{\omega}}{\alpha_1^{\omega}} = \frac{\left[\left(\beta_1^{R,\omega}/\beta_2^{R,\omega}\right)^{\upsilon}\tau^{-\theta}+1\right]}{\left[\left(\beta_1^{R,\omega}/\beta_2^{R,\omega}\right)^{\upsilon}+\tau^{-\theta}\right]}\frac{\left[1-\left(\beta_1^{R,\omega}/\beta_2^{R,\omega}\right)^{1-\upsilon}\tau^{-\theta}\right]}{\left[\left(\beta_1^{R,\omega}/\beta_2^{R,\omega}\right)^{1-\upsilon}-\tau^{-\theta}\right]} \tag{51}$$

Defining $\beta \equiv \beta_1^{R,\omega}/\beta_2^{R,\omega}$ and $\alpha \equiv \alpha_2^{\omega}/\alpha_1^{\omega}$, the last equation defines β as an implicit function of α, $\beta(\alpha)$. Noticing that in the range $\left(\tau^{-\frac{\theta}{1-\upsilon}}, \tau^{\frac{\theta}{1-\upsilon}}\right)$, the right hand side of (51) is strictly decreasing in β and varies from infinity to zero, the function $\beta(\alpha)$ satisfies

(51.i) $d\beta/d\alpha < 0$ for any value of $\upsilon \in [0, 1]$ and $\tau > 1$.

(51.ii) $\beta = 1$ for $\alpha = 1$ for any value of $\upsilon \in [0, 1]$ and $\tau > 1$

(51.iii) $\beta(\alpha) \in \left(\tau^{-\frac{\theta}{1-\upsilon}}, \tau^{\frac{\theta}{1-\upsilon}}\right)$ for any $\alpha \in [0, \infty)$.

Now let us consider the trade balance (net exports) of country 2 in industry ω. We have

$$TB_2^{\omega} = \alpha_1^{\omega}E_1\lambda_{21}^{\omega} - \alpha_2^{\omega}E_2\lambda_{12}^{\omega}$$

where the first term are country 2's exports to country 1 and the second term are country 2's imports from country 1. Using the definitions of λ_{ij}^{ω}, (51) and the fact that in equilibrium we must have $E_1 = E_2$, we can write the above equation

$$TB_2^{\omega} = \frac{\alpha_1^{\omega} E_1 \tau^{-\theta}}{\left[\beta^{\upsilon}+\tau^{-\theta}\right]} \left[1 - \beta^{\upsilon} \frac{\left[1-\beta^{1-\upsilon}\tau^{-\theta}\right]}{\left[\beta^{1-\upsilon}-\tau^{-\theta}\right]}\right] \tag{52}$$

To analyze the effect of the domestic market size on trade patterns it is instructive to consider first the extreme cases of $\upsilon = 0$ and $\upsilon = 1$. When $\upsilon = 0$ there are no R&D possibilities in the model and the model becomes essentially a two industry version of Eaton and Kortum (2002) (specifically, a two industry version of BEJK). In this case (52) becomes

$$TB_2^{\omega} = \frac{\alpha_1^{\omega} E_1 \tau^{-\theta}}{\left[1+\tau^{-\theta}\right]} \left[1 - \frac{\left[1-\beta\tau^{-\theta}\right]}{\left[\beta-\tau^{-\theta}\right]}\right]$$

Now consider the case in which country 2 has a larger domestic market for industry ω, i.e., $\alpha > 1$. Conditions (51.i) and (51.ii) above imply that $\beta < 1$ if $\alpha > 1$, and using this in the last expression we get $TB_2^{\omega} < 0$. In other words, when $\upsilon = 0$, the country with the larger home market in a given industry is a net importer in that industry.

When $\upsilon = 1$ there are constant returns to R&D. In this case (52) becomes

$$TB_2^{\omega} = \frac{\alpha_1^{\omega} E_1 \tau^{-\theta}}{\left[\beta+\tau^{-\theta}\right]} \left[1 - \beta\right]$$

As before, if country 2 has a larger home country for industry ω, then $\alpha > 1$ and $\beta < 1$, which in turn imply $TB_2^{\omega} > 0$. As in Krugman (1980), the country with the larger home market in a given industry is a net exporter in that industry.

Let us now turn to the intermediate cases when $\upsilon \in (0, 1)$. As before, I consider differences in domestic market size in which country 2 has a relatively larger domestic market in industry ω –which correspond to values of α in the range $[0, 1)$– and I focus on how this difference affects country 2's net exports in that industry. Equations (51) and (52) define the balance of trade as a function of α, $TB_2^{\omega}(\alpha)$, and according to our definition, a home market effect is present if $TB_2^{\omega}(\alpha) > 0$ for $\alpha \in [0, 1)$. Consequently, to analyze the home market effect, we need to study the sign of $TB_2^{\omega}(\alpha)$ for values of α in the range $[0, 1)$.

It is convenient to start with the analysis of the effects of small deviations from the benchmark case of no differences in home market size, $\alpha = 1$. In this case, (51) and (52) imply trade is balanced at the industry level, $TB_2^{\omega}(1) = 0$. A home market effect is present for small differences in market size if $dTB_2^{\omega}/d\alpha|_{\alpha=1} > 0$, i.e., a small relative increase in the size of country 2's domestic market in industry ω induces a trade surplus in that industry. Recalling that

$$\frac{dTB_2^{\omega}}{d\alpha} = \frac{\partial TB_2^{\omega}}{\partial\beta}\frac{\partial\beta}{\partial\alpha}$$

(51.i) and (51.ii) imply that $dTB_2^{\omega}/d\alpha|_{\alpha=1} > 0$ if , and only if, $\partial TB_2^{\omega}/\partial\beta|_{\beta=1} < 0$. Deriving (52) with respect to β and evaluating at $\beta = 1$ we get that $TB_2^{\omega}/\partial\beta|_{\beta=1} < 0$ if, and only if, $\upsilon > \left(1+\tau^{-\theta}\right)/2$. In other words, if the decreasing returns to R&D are sufficiently weak (υ sufficiently high), then there is a home market effect for small differences in the relative size of the home market.

However, from (51) we can see that as the relative size of the domestic market in country 2 approaches infinity, $\alpha \to \infty$, the relative market share of country 1 approaches its lower bound, $\beta \to \tau^{-\frac{\theta}{1-\upsilon}}$. This implies that the denominator of the second term of the expression in brackets in (52) approaches zero, which in turn implies that $TB_2^{\omega} < 0$ for sufficiently large α. This means that even for those values of υ at which there is a home market effect for small differences in domestic markets' sizes, a sufficiently (relative) large domestic market eventually translates into a trade deficit in the corresponding industry.

The intuition of this result is simple. When a country has a relatively larger domestic market in industry ω and $\tau > 1$, relatively more domestic resources are allocated to that industry for any value of υ. When there are no R&D possibilities ($\upsilon = 0$) those additional resources allocated to production do not compensate the larger domestic demand in the industry, and as a result there is a trade deficit in the industry.

When $\upsilon > 0$, the reallocation of resources also involves the redirection of R&D efforts towards industry ω, which endogenously increases the level of technology in that industry giving the country a comparative advantage in production that industry. Notice that the greater domestic demand and the endogenous increase in technology generated by a large domestic market have opposite effects on the trade balance, and consequently, the net effect depends of the relative strength of these two effects.

When $\upsilon = 1$ there are constant returns to R&D and the effect of a larger domestic market on technology is strongest. In this case, the greater technology effect always dominates the greater demand effect, generating a home market effect for any difference in relative domestic market size.

When $\upsilon > \left(1+\tau^{-\theta}\right)/2$, the effect on technology is strong enough to generate a home market effect for small differences in relative domestic market size. However, as the differences in domestic market size increase and more resources are allocated to industry ω in country 2, the decreasing returns in R&D kick in and the endogenous changes in technology cannot compensate the greater domestic demand.

Finally, when $\upsilon \leq \left(1+\tau^{-\theta}\right)/2$ the decreasing returns to R&D are so strong that there is no home market effect for any difference in relative market size. Notice that this means that if $\upsilon \leq 1/2$, then there are no home market affects regardless of the level of trade costs.

With the previous analysis we have proved the following Lemma.

Lemma 6 *Let $\alpha \equiv \alpha_2^{\omega}/\alpha_1^{\omega}$ be the relative market size and let $TB_2^{\omega}(\alpha)$ the net exports of country 2 in industry ω. In the economy described above the following holds:*
(i) If $\upsilon = 1$, $TB_2^{\omega}(\alpha) > 0$ for all $\alpha \in (0,1)$.
(ii) If $\left(1+\tau^{-\theta}\right)/2 < \upsilon < 1$, there is a $\alpha_{\upsilon}^{*} \in (0,1)$ such that (a) $TB_2^{\omega}(\alpha) > 0$ for $\alpha \in (\alpha_{\upsilon}^{*}, 1)$; (b) $TB_2^{\omega}(\alpha_{\upsilon}^{*}) = 0$; and (c) $TB_2^{\omega}(\alpha) < 0$ if $\alpha \in (0, \alpha_{\upsilon}^{*})$.[53]
(iii) If $\upsilon \leq \left(1+\tau^{-\theta}\right)/2$, $TB_2^{\omega}(\alpha) < 0$ for all $\alpha \in (0,1)$.

[53] The subscript in α_{υ}^{*} emphasizes the dependence of the cutoff value on the paramtere υ.

C.3 Section 4

C.3.1 Estimation of Comparative Advantage in Production

I will follow Costionot, Donaldson and Komunjer (2012) to estimate comparative advantage across countries. Equation (23.1) can be express as follows:

$$x_{ijt}^{\omega} = \frac{T_{it}^{\omega} w_{it}^{-\theta} \left(\tau_{ij}^{\omega}\right)^{-\theta}}{\Phi_{jt}^{\omega}} E_{jt}^{\omega}$$

where the only new variable x_{ijt}^{ω} represets country i's total exports of goods in industry ω to country j in period t.

Writing $\ln \tau_{ij}^{\omega} = \mathbb{E}_{\Omega}\left[\ln \tau_{ij}^{\omega}\right] + \left(\ln \tau_{ij}^{\omega} - \mathbb{E}_{\Omega}\left[\ln \tau_{ij}^{\omega}\right]\right)$, we can write the previous expressions as

$$\log x_{ijt}^{\omega} = \log T_{it}^{\omega} w_{it}^{-\theta} + \log \frac{E_{jt}^{\omega}}{\Phi_{jt}^{\omega}} - \theta \log \tau_{ij}^{\omega}$$

Using $\ln \tau_{ij}^{\omega} = \mathbb{E}_{\Omega}\left[\ln \tau_{ij}^{\omega}\right] + \left(\ln \tau_{ij}^{\omega} - \mathbb{E}_{\Omega}\left[\ln \tau_{ij}^{\omega}\right]\right)$ in the last equation, the resulting expression it can be estimated as

$$\log x_{ij}^{\omega} = \xi_i^{\omega} + \psi_j^{\omega} + \xi_{ij} + \varepsilon_{ij}^{\omega}$$

where ξ_i^{ω}, ψ_j^{ω}, ξ_{ij} are exporter-industry, importer-industry and importer-exporter fixed effects, and $\varepsilon_{ij}^{\omega}$ is an error term:

$$\xi_i^{\omega} = \log T_{it}^{\omega} w_{it}^{-\theta}; \qquad \psi_j^{\omega} = \log E_{jt}^{\omega}/\Phi_{jt}^{\omega}$$
$$\xi_{ij} = \mathbb{E}_{\Omega}\left[\ln \tau_{ij}^{\omega}\right]; \qquad \varepsilon_{ij}^{\omega} = \ln \tau_{ij}^{\omega} - \mathbb{E}_{\Omega}\left[\ln \tau_{ij}^{\omega}\right]$$

Given the structure of the fixed effects, the regression can only identify $\left(\xi_i^{\omega'} - \xi_i^{\omega}\right) - \left(\xi_j^{\omega'} - \xi_j^{\omega}\right)$, which can be use to construct measures of revealed comparative advantage

$$CA_{i,i't}^{\omega,\omega'} \equiv \frac{T_{it}^{\omega}/T_{it}^{\omega'}}{T_{i't}^{\omega}/T_{i't}^{\omega'}} = \exp\left\{\left(\xi_i^{\omega} - \xi_i^{\omega'}\right) - \left(\xi_{i'}^{\omega} - \xi_{i'}^{\omega'}\right)\right\}$$

In order to avoid issues related to the particular choice of base year and base industry, I define comparative advantage relative to an "average" industry and country as in the text. Then, starting from a base country i' and a base industry ω'

$$CA_{i,i't}^{\omega,\bar{\omega}} \equiv \frac{CA_{i,i't}^{\omega,\omega'}}{\prod_{\omega=1}^{\Omega}\left[CA_{i,i't}^{\omega,\omega'}\right]^{\frac{1}{\Omega}}}$$

Finally, we can define comparative advantage relative to "average" industry $\bar{\omega}$ and country $\bar{\imath}$ as follows:

$$CA_{it}^{\omega} \equiv \frac{T_{it}^{\omega}/T_{it}^{\bar{\omega}}}{T_{\bar{\imath}t}^{\omega}/T_{\bar{\imath}t}^{\bar{\omega}}} = \frac{CA_{i,i't}^{\omega,\bar{\omega}}}{\prod_{i=1}^{N}\left[CA_{i,i't}^{\omega,\bar{\omega}}\right]^{\frac{1}{N}}}$$

C.3.2 Proof of Lemma 4

Proof. For convenience I rewrite the structural equations (35) and (38) here

$$\begin{aligned} T_n &= \upsilon\beta_n^R + \iota_n \\ \alpha_n &= \frac{\sigma-1}{\theta}\Phi_n + \gamma_n \end{aligned}$$

Notice that

$$\begin{aligned} \mathbb{E}\left[T\alpha\right] &= \upsilon\mathbb{E}\left[\beta^R\alpha\right] + \mathbb{E}\left[\iota\alpha\right] \\ \mathbb{E}\left[\iota\alpha\right] &= \tfrac{\sigma-1}{\theta}\mathbb{E}\left[\iota\Phi\right] \end{aligned}$$

where in the second line I used $\mathbb{E}\left[\iota\gamma\right]=0$ (assumption CES.ii). Taking probability limits in (36), yields

$$plim\left(\widehat{\upsilon}_1\right) = \frac{\mathbb{E}\left[T\alpha\right]}{\mathbb{E}\left[\beta^R\alpha\right]} = \upsilon + \tfrac{\sigma-1}{\theta}\frac{\mathbb{E}\left[\iota\Phi\right]}{\mathbb{E}\left[\beta^R\alpha\right]} > \upsilon$$

where the inequality is obtained from $\sigma>1$, $\mathbb{E}\left[\iota\Phi\right]>0$, and $\mathbb{E}\left[\beta^R\alpha\right]>0$. ∎

C.3.3 Proof of Lemma 5

Proof. For the sake of notation I define $c\equiv\left(\sigma-1\right)/\theta$. The estimator $\widehat{\upsilon}_2$ involves the following steps: ∎

1. Estimate (38) by OLS. Let $\widehat{c}$ be the OLS-estimator of $c\equiv\left(\sigma-1\right)/\theta$ and let $\widehat{\gamma}_n$ be the residuals of the regression.

2. Instrument β_n^R in (35) with the residuals $\widehat{\gamma}_n$, i.e., define the estimator $\widehat{\upsilon}_2$ is in (39)

$$\widehat{\upsilon}_2 \equiv \frac{\sum_n T_n\widehat{\gamma}_n}{\sum_n \beta_n^R\widehat{\gamma}_n}$$

Proof. Assumption CES.i implies that domestic technology is positively correlated with domestic demand conditions. In addition, the data shows a high correlation between log-comparative advantage in production T_n and log-relative cost parameters Φ_n. All of this implies that log-relative demand parameters γ_n are correlated with Φ_n. Consequently, the OLS estimator $\widehat{c}$ is biased upwards:

$$\bar{c} \equiv plim\left(\widehat{c}\right) = c + \frac{\mathbb{E}\left[\Phi_n\gamma_n\right]}{\mathbb{E}\left[\Phi_n\alpha_n\right]} > c$$

Also notice, that by construction, $\widehat{\gamma}_n$ is given by

$$\widehat{\gamma}_n = \alpha_n - \widehat{c}\Phi_n = (c - \widehat{c})\,\Phi_n + \gamma_n$$

and taking probability limits yields

$$plim\,(\widehat{\gamma}_n) = (c - \overline{c})\,\Phi_n + \gamma_n$$

Using the last expression and (35), the probability limit of estimator $\widehat{\upsilon}_2$ is given by

$$\begin{aligned} plim\,(\widehat{\upsilon}_2) &= \frac{\mathbb{E}\,[T_n \times plim\,(\widehat{\gamma}_n)]}{\mathbb{E}\left[\beta_n^R \times plim\,(\widehat{\gamma}_n)\right]} \\ &= \upsilon + (c - \overline{c})\,\frac{\mathbb{E}\,[\iota_n \Phi_n]}{\mathbb{E}\left[\beta_n^R \times plim\,(\widehat{\gamma}_n)\right]} \end{aligned}$$

In the previous expression, the structure of the model implies $(c - \overline{c}) < 0$ and that $\mathbb{E}\,[\iota_n \Phi_n] > 0$. In addition, the sign of $\mathbb{E}\left[\beta_n^R \times plim\,(\widehat{\gamma}_n)\right]$ is obtain from the data since

$$\mathbb{E}\left[\beta_n^R \times plim\,(\widehat{\gamma}_n)\right] = plim \frac{1}{n} \sum_n \beta_n^R \widehat{\gamma}_n$$

where in the last expression, n also represents the total number of observations. In the data we find $\sum_n \beta_n^R \widehat{\gamma}_n > 0$, which implies that

$$plim\,(\widehat{\upsilon}_2) < \upsilon$$

■

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