Abstract

Statistical evidence is reported that even outside disaster periods, agents face negative consumption skewness, as well as positive inflation skewness. Quantitative implications of skewness risk for nominal loan contracts in a pure exchange economy are derived. Key modeling assumptions are non-recursive preferences for traders and asymmetric distributions for consumption and inflation innovations. The model is solved using a third-order perturbation and estimated by the Simulated Method of Moments. Results show that skewness risk accounts for 8 percent of the risk premia, reduces bond yields by 20 basis points, and has a price of 0.6 percent per year. Despite the fact that bonds are nominal, most of the priced risk is consumption risk.

JEL Classification: G12, E43, E44

Key Words: Skewness risk; risk premia; bond yields; perturbation; nonlinear dynamic models; simulated method of moments.

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1 Introduction

This paper is concerned with the pricing of financial instruments—in particular, nominal loans—in an environment where traders face skewness risk. By skewness risk, I mean the possibility of extreme realizations from the long tail of an asymmetric shock distribution. To see why skewness risk matters, suppose that inflation innovations are positively skewed so that during the bond holding period, large positive inflation realizations are possible, but large negative realization are unlikely. The more likely, large positive realizations reduce bond payoffs and prices, and their effects are not counterbalanced (in expectation) by those of the less likely, large negative realizations. Hence, given the variance, traders interested in buying nominal bonds would demand compensation for the extra risk that arises from the asymmetry of the inflation distribution.

I explore the relation between skewness risk and bond prices using the simplest possible environment: a consumption-based asset-pricing model where traders receive a stochastic endowment in every period and there are no frictions. Traders have recursive preferences of the form proposed by Epstein and Zin (1989). Crucially, I relax the usual assumption of that shock innovations follow a symmetric distribution (e.g., Normal) and allow instead skewed distributions for the innovations of the inflation and consumption processes. I show that a third-order perturbation of the policy functions that solve the dynamic model explicitly captures the contribution of skewness to bond yields and risk premia, and permits the construction of model-based estimates of the effect of skewness risk on these variables.

The third-order perturbation of the model is estimated by the Simulated Method of Moments (SMM) using quarterly U.S. data. Among the estimated parameters are those of the distributions that generate inflation and consumption innovations. Based on these estimates, I statistically show that the data decisively reject the assumption that shock innovations are drawn from Normal distributions and favor instead the alternative that they are drawn from asymmetric distributions. In particular, the data prefer a specification where inflation innovations are drawn from a positively skewed distribution, and consumption innovations are drawn from a negatively skewed distribution.

I study versions of the model with (asymmetric) Skew normal and Generalized extreme value distributions. These distributions have flexible forms, permit both positive and negative skewness, and are shown to fit the data better than the Normal distribution. Results indicate that skewness risk accounts for 8 percent of the risk premia, reduces bond yields by 20 basis points compared with yields in an economy with only variance risk, and has a price of 0.6 percent per year. Thus, not only is innovation skewness statistically significant, but its effects are economically significant as well. Despite the fact that bonds are nominal and
that inflation variance and skewness are substantial, I find that consumption risk carries a
much larger price than inflation risk and constitutes almost all of the risk premia.

Impulse-response analysis shows that positive (negative) inflation shocks increase (de-
crease) bond yields with quantitatively larger effects on short- than on long-term bonds. As a result, inflation shocks change the slope of the yield curve. In contrast, consumption shocks affect all yields equally and, consequently, push the whole yield curve up or down. Thus, inflation acts like the “slope” factor, and consumption acts like the “level” factor, in factor models of the term structure. However, while the macroeconomic content of statistical factors is unclear, the “factors” in this economy are the state variables of the model, namely aggregate consumption and inflation.

This paper complements the growing literature that studies the effects of consumption
disasters on asset prices, for example Rietz (1988), Barro (2006, 2009), and Barro et al.
(2010). Disasters are extreme, low probability events where output (or consumption) drops by at least 15 percent from peak to trough as, for example, during the Great Depression. Under an empirically plausible parameterization of the disaster probability, disasters can help account for the equity premium puzzle. As that literature, this paper is concerned with asymmetries in consumption growth. However, I provide statistical evidence that, even outside disaster events, consumption innovations are negatively skewed and, hence, agents face the possibility of substantial decreases in consumption. Parameter estimates imply that agents can suffer a reduction in consumption of 5 percentage points below the mean with a probability 2.6 percent. Of course, these decreases are not as dramatic as disasters and are instead associated with recessions, but they are shown to have non-negligible effects on bond prices and risk premia. In this sense, the mechanism highlighted by the disaster literature is also operative at business cycle frequencies and during non-disaster periods. Moreover, while the disaster literature is primarily concerned with consumption, this paper also considers asymmetries in aggregate inflation, which may be important for the pricing of nominal assets.

This paper also complements the finance literature concerned with the role of skewness in asset pricing. Kraus and Litzenberger (1976, 1983) extend the capital asset pricing model (CAPM) to incorporate the effect of skewness on valuations. Harvey and Siddique (2000) study the role of the co-skewness with the aggregate market portfolio and find a negative correlation between co-skewness and mean returns. Using versions of the CAPM, Kapadia (2006) and Chang et al. (2012) find that skewness risk has a negative effect on excess returns in a cross-section of stock returns and option data, respectively. Papers that examine the role of preferences in accounting for the empirical importance of skewness include Golec and Tamarkin (1998) and Brunnermeier et al. (2007). Golec and Tamarkin estimate the
payoff function of bettors in horse tracks and find that it is decreasing in the variance but increasing the skewness, so that at high odds, bettors are willing to accept poor mean returns and variance because the skewness is large. Brunnermeier et al. develop a model of optimal beliefs which predicts that traders will overinvest in right-skewed assets.

The paper is organized as follows. Section 2 describes a bond market subject to asymmetric consumption and inflation shocks. Section 3 describes the data and reports SMM estimates of the parameters and policy functions. Section 4 quantifies the contribution of skewness risk to bond premia and yields, and reports estimates of the price of risk. Section 5 uses impulse-response analysis to study the effects of inflation and consumption shocks on bond yields. Finally, Section 6 concludes.

2 The Market

Traders are identical, infinitely-lived and their number is normalized to 1. The representative trader has recursive preferences over consumption (Epstein and Zin, 1989),

\[
U_t = \left(1 - \beta\right) (c_t)^{1-1/\psi} + \beta \left(E_t \left(U_{t+1}^{1-\gamma}\right)\right)^{(1-1/\psi)/(1-\gamma)}^{1/(1-1/\psi)},
\]

where \(\beta \in (0, 1)\) is the discount factor, \(c_t\) is consumption, \(E_t\) is the expectation conditional on information available at time \(t\), \(\psi\) is the intertemporal elasticity of substitution (IES), and \(\gamma\) is the coefficient of relative risk aversion. As it is well known, recursive preferences decouple elasticity of substitution from risk aversion when \(\gamma = 1/\psi\), and encompass preferences with constant relative risk aversion when \(\gamma = 1/\psi\). Previous literature that employs recursive preferences in consumption-based models of bond pricing include Epstein and Zin (1991), Gregory and Voss (1991), Piazzesi and Schnider (2007), Doh (2008), van Binsbergen et al. (2010), Le and Singleton (2010), and Rudebusch and Swanson (2012).

In every period, the trader receives a non-storable endowment, \(y_t\), that follows the process

\[
\ln(y_t) = (1 - \rho) \ln(y) + \rho \ln(y_{t-1}) + \varsigma_t,
\]

where \(\rho \in (-1, 1)\), \(\ln(y)\) is the unconditional mean of \(\ln(y_t)\), and \(\varsigma_t\) is an independent and identically distributed (i.i.d.) innovation with mean zero, constant conditional variance, and non-zero skewness.\(^1\) The latter assumption relaxes the usual restriction of zero skewness.

\(^1\)The Appendix shows that this model is observationally equivalent to a production economy where labor is the sole, inelastically-supplied input of production. This especial case serves to show that, under certain conditions, this model can be reconciled with a class of asset-pricing models with production. The more involved case where output is a function of elastically-supplied capital and labor is left for future research.
implicit in most of the previous literature (for example, through the assumption of Normal innovations), and allows me to examine the relation between skewness risk and asset prices. Financial assets are zero-coupon nominal bonds with maturities $\ell = 1, \ldots, L$. All bonds are equally liquid regardless of their maturity and can be costlessly traded in the secondary market. The trader’s budget constraint is

$$c_t + \sum_{\ell=1}^{L} \frac{Q_{t}\ell B_{t}\ell}{P_{t}} = y_t + \sum_{\ell=1}^{L} \frac{Q_{t-1}\ell B_{t-1}\ell}{P_{t}},$$

(3)

where $Q_{t}\ell$ and $B_{t}\ell$ are, respectively, the nominal price and quantity of nominal bonds with maturity $\ell$, $Q_{t}^{0} = 1$ because bonds pay one unit of currency at maturity, and $P_{t}$ is the price level. Prices are denominated in terms of a unit called “money”, but the market is cashless otherwise.

The Euler equations that characterize the trader’s utility maximization are

$$\frac{Q_{t}\ell}{P_{t}} = \beta E_{t}\left(\left(\frac{v_{t+1}}{w_{t}}\right)^{1/\gamma} \left(\frac{c_{t+1}}{c_{t}}\right)^{-1/\psi} \left(\frac{Q_{t+1}\ell}{P_{t+1}}\right)^{1}\right), \text{ for } \ell = 1, 2, \ldots L,$$

(4)

where

$$v_t \equiv \max_{\{c_{t}, B_{t}^{1}, \ldots, B_{t}^{L}\}} U_t$$

and

$$w_t \equiv E_t v_{t+1}$$

are the value function and the certainty-equivalent future utility, respectively. As usual, Euler equations compare the marginal cost of acquiring an additional unit of the financial asset with the discounted expected marginal benefit of keeping the asset till next period.

Euler equations of this form have been extensively studied in the asset-pricing literature. One approach involves estimating the parameters using, for example, the Generalized Method of Moments and testing the over-identifying restrictions of the model. Well-known examples are Hansen and Singleton (1982) for expected utility, and Epstein and Zin (1991) for non-expected utility. Another approach involves assuming that the arguments inside the expectations operator are jointly Lognormal and conditionally homoskedastic in order to obtain a tractable linear specification of the pricing function with a constant risk-adjustment factor, as in Hansen and Singleton (1983), Campbell (1986, 1996), and Jerman (1998). Since the adjustment factor is proportional to the variance only, this approach assumes away the contribution of higher-order moments (like the skewness) to the risk premia. In contrast, the

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2An important exception is the disaster literature (e.g., see Barro, 2006) where innovations are drawn from the mixture of a Normal distribution, that describes non-disaster periods, and a Bernoulli distribution, that can generate a disaster with a fixed probability. This combination delivers a negatively skewed distribution.
focus of this paper is on the policy functions that solve the full dynamic model, rather than on the Euler equations alone. As we will see, policy functions are useful characterizations of prices and quantities because they make explicit their dependence on the state variables of the model and on the higher-order moments of the innovations.\(^3\)

The model is closed by assuming that aggregate inflation follows the process\(^4\)

\[
\ln (\Pi_t) = (1 - \mu) \ln(\Pi) + \mu \ln(\Pi_{t-1}) + \eta_t, \quad (5)
\]

where \(\Pi_t = P_t/P_{t-1}\) is the gross rate of inflation between periods \(t - 1\) and \(t\), \(\mu \in (-1, 1)\), \(\ln(\Pi)\) is the unconditional mean of \(\ln(\Pi_t)\), and \(\eta_t\) is an i.i.d. innovation with mean zero, constant conditional variance, and non-zero skewness.\(^5\)

The equilibrium is an allocation for the trader \(C = \left\{ c_t, \left( B^\ell_t \right)_{\ell=1,...,L} \right\}_{t=0}^\infty\) and a price system \(\left\{ (Q^\ell_t)_{\ell=1,...,L} \right\}_{t=0}^\infty\) such that given the price system: (i) the allocation \(C\) solves the trader’s problem; (ii) the goods market clears: \(C_t = Y_t\); and (iii) bonds are in zero net supply: \(B^\ell_t = 0\) for all \(\ell = 1, 2, ..., L\).

The assumption of a representative trader and the normalization of the population size to 1 means that, in equilibrium, aggregate consumption and output are respectively equal to the individual consumption and endowment. That is, \(C_t = c_t\) and \(Y_t = y_t\). With the understanding that \(C_t = y_t\) in equilibrium only, I will refer to (2) as the consumption process.

I now derive the relation between the bond prices determined by the model, and the bond yields and risk premia. These derivations are relatively standard and the intention is simply to be explicit about the variables I will be focusing on in the empirical part of the paper. Following the literature, the gross yield of the \(\ell\)-period bond is

\[
\hat{\nu}_t^\ell = (Q^\ell_t)^{-1/\ell}, \quad (6)
\]

\(^3\)Martin (2012) relaxes the assumption of lognormality and expresses variables as polynomial functions of consumption growth cumulants. Similarly, the perturbation method that I use here to solve the model allows the higher-order moments of consumption, as well as inflation, to affect the model variables.

\(^4\)In preliminary work, I explored another strategy to close the model. That is, by using an autoregressive process for the growth rate of the money supply to represent monetary policy, and specifying real money balances as one of the arguments of the utility function. However, when I estimated this version of the model, I found a relatively low interest rate elasticity of money demand. Coupled with flexible prices, this is equivalent to, but less parsimonious than, directly assuming a time-series process for inflation.

\(^5\)In preliminary work, I considered the case where innovations to inflation and consumption are correlated. This specification has one additional parameter (i.e., the correlation between \(\eta_t\) and \(\varsigma_t\)) which helps account for the contemporaneous correlation of inflation and consumption in the data. However, the other parameter estimates and model implications are basically the same as those reported below. In this sense, results are robust to this generalization of the model. On the other hand, this generalization has the drawback that impulse-responses no longer have a clear interpretation because shocks are not orthogonal.
for $\ell = 1, 2, \ldots, L$. The bond risk premium (or bond premium for short) is the component of the long-term bond price that accounts for the risk involved in holding this bond compared with a sequence of rolled-over shorter-term bonds. The risk arises because future bond prices are not known in advance and, so, the strategy of rolling over shorter-term bonds entails a gamble. I derive the bond premium recursively from the Euler equations

$$Q_t^\ell = Q_t^1 E_t \left( Q_{t+1}^{\ell-1} \right) + \beta \text{cov}_t \left( \left( \frac{V_{t+1}}{W_t} \right)^{\frac{1}{\psi} - \gamma} \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \frac{Q_{t+1}^{\ell-1}}{\Pi_{t+1}} \right),$$

(7)

for $\ell = 2, \ldots, L$, where $V_t$ and $W_t$ are, respectively, the aggregate counterparts of $v_t$ and $w_t$, and I have used the fact that for the one-period bond

$$Q_t^1 = \beta E_t \left( \frac{V_{t+1}}{W_t} \right)^{\frac{1}{\psi} - \gamma} \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \frac{1}{\Pi_{t+1}} .$$

(8)

Following Ljungqvist and Sargent (2004, ch. 13.8), the premium on the $\ell$-period bond is defined as

$$\Gamma_{\ell,t} \equiv \beta \text{cov}_t \left( \left( \frac{V_{t+1}}{W_t} \right)^{\frac{1}{\psi} - \gamma} \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \frac{Q_{t+1}^{\ell-1}}{\Pi_{t+1}} \right).$$

(9)

Notice that the premium depends on the bond maturity and may be positive or negative according to the sign of the covariance between the pricing kernel and $Q_{t+1}^{\ell-1}/\Pi_{t+1}$. This observation highlights the fact that, in general, consumption-based asset-pricing models do not restrict the sign or monotonicity of the bond premia. The same is true in the market-segmentation hypothesis by Culbertson (1957) and the preferred-habitat theory by Modigliani and Sutch (1966), but they rely on a strong preference by investors for particular maturities and, implicitly, rule out arbitrage.

Using definition (9), the price of the $\ell$-period bond can be written as

$$Q_t^{\ell} = Q_t^1 E_t \left( Q_{t+1}^{\ell-1} \right) + \Gamma_{\ell,t} .$$

(10)

From definition (9), the premium is negative when the covariance in (9) is negative. A negative premium is a discount on the longer-term bond in the sense that buying a $\ell$-period bond at time $t$ at price $Q_t^{\ell}$ is cheaper than buying a one-period bond at time $t$ at price $Q_t^1$ and a $(\ell - 1)$-period bond at time $t + 1$ at expected price $Q_{t+1}^{\ell-1}$. Up to an approximation, this implies that the yield of the $\ell$-period bond is larger than the weighted average yield of the $1$- and $(\ell - 1)$-period bonds and the yield curve is, therefore, upward sloping.\(^6\) Conversely, the

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\(^6\)To see this, consider the simplest case where $\ell = 2$. Then, using (6) for $\ell = 1, 2$, it is easy to show that $Q_t^2 < Q_t^1 E_t \left( Q_{t+1}^1 \right)$ implies $\log(i_t^2) > (1/2)(\log(i_t^1) + E_t (\log(i_t^1)))$. This mechanism is different from the one outlined by Hicks (1939, ch. 13) where long-term bonds are “less liquid” than short-term bonds and, hence, a premium is required to induce traders to hold the former. Instead, in this model all bonds are equally liquid and can be traded without penalty before their maturity date. Bansal and Coleman (1996) assume that short-term bonds provide indirect transaction services and the compensation for those services lowers the nominal return of short-term bonds compared with long-term bonds.
premium is positive when the covariance in (9) is positive and the yield curve is downward sloping.

Since this model does not have an exact analytical solution, I use a perturbation method to obtain an approximate solution (see Jin and Judd, 2002). This method involves taking a third-order expansion of the policy functions around the deterministic steady state and characterizing the local dynamics. An expansion of (at least) third-order is necessary to capture the effect of skewness in the policy functions and to generate a time-varying premium.\(^7\) Caldera et al. (2009) show that for models with recursive preferences, a third-order perturbation is as accurate as projection methods (e.g., Chebychev polynomials and value function iteration) in the range of interest while being much faster computationally. The latter is an important advantage for this project because estimation requires solving and simulating the model in each iteration of the routine that optimizes the statistical objective function.

A policy function takes the general form \(f(x_t, \sigma)\) where \(x_t\) is a vector of state variables and \(\sigma\) is a perturbation parameter. The state variables are aggregate consumption and inflation, that is, \(x_t = [C_t \: \Pi_t]'\). The goal is to approximate \(f(x_t, \sigma)\) using a third-order polynomial expansion around the deterministic steady state where \(x_t = x\) and \(\sigma = 0\). Using tensor notation, this approximation can be written as

\[
[f(x_t, \sigma)]^j_i = [f(x, 0)]^j_i + [f_x(x, 0)]^j_i [(x_t - x)]^a_i + (1/2)[f_{xx}(x, 0)]^j_i [(x_t - x)]^b_i + (1/6)[f_{xxx}(x, 0)]^j_i [(x_t - x)]^c_i + (1/2)[f_{\sigma x}(x, 0)]^j_i [\sigma][\sigma] + (1/6)[f_{\sigma x x}(x, 0)]^j_i [(x_t - x)]^{a_i} [\sigma][\sigma][\sigma],
\]

where I have used \([f_{x x}(x, 0)]^j_i = [f_{\sigma x}(x, 0)]^j_i = 0\) (Schmitt-Grohé and Uribe, 2004, p. 763), \([f_{x x x}(x, 0)]^j_i = [f_{\sigma x x}(x, 0)]^j_i = 0\) (Ruge-Murcia, 2012, p. 936), and \([f_{\sigma x x}(x, 0)]^j_i = [f_{x x x}(x, 0)]^j_i = 0\) by Clairaut’s theorem. In this notation, elements like \([f_{x x}(x, 0)]^j_i\) and \([f_{x x x}(x, 0)]^j_i\) are coefficients that depend nonlinearly on structural parameters. As we can see, the policy function includes linear, quadratic, and cubic terms in the state variables, one constant and one time-varying term in the variance, and one constant term in the skewness.\(^8\) In the special case where the distribution of the innovations is symmetric—and,

\(^7\)As it is well known, first-order solutions feature certainty equivalence and imply that traders are indifferent to higher-order moments of the shocks. Second-order solutions capture the effect of the variance, but not of the skewness, on the decision rules, and deliver a constant risk premium. For the implementation of the third-order perturbation, I use the codes described in Ruge-Murcia (2012).

\(^8\)Strictly speaking, \([\sigma][\sigma][\sigma]\) is the third moment of the innovations, rather than the skewness, which is this third moment divided by the cube of the standard deviation.
hence, skewness is zero—the latter term is zero. In the more general case where the distribution is asymmetric, this term may be positive or negative depending on the sign of the skewness and the values of other structural parameters.

The reader familiar with recent contributions to the macro-finance literature will notice that the policy functions for yields (written in the generic form (11)) resemble the basic equations of the canonical factor model of the term structure. However, there are four important differences. First, the factors here are observable macroeconomic variables, rather than latent variables. Latent factor models suggest the existence of two key factors, namely the “level” and “slope” factors, which jointly account for 95 percent of the variance of yields (Rudebusch and Wu, 2008). However, given the purely statistical nature of those models, the macroeconomic content of the factors is unclear. In this model, their macroeconomic interpretation is explicit: the factors are aggregate consumption and inflation. Furthermore, as we will see below, the relation between the level and slope factors and the state variables of the model is empirically straightforward. Second, the function is not affine but non-linear as a result of the quadratic and cubic terms that arise from the third-order perturbation. Le and Singleton (2010) build a factor model where the price/consumption ratio is a quadratic function of the state variables, but a suitable linearization renders the model conditionally affine. Third, since innovations are drawn from an asymmetric distribution, this model is not Gaussian. Finally, while the factor loadings are typically free parameters in statistical factor models, in this model the coefficients of the state variables are non-linear functions of structural parameters.

3 Econometric Analysis

3.1 Data

The third-order approximate solution of the model is estimated using quarterly observations of the growth rate of consumption, the inflation rate, and the three-month, six-month, and twelve-month nominal interest rates. The raw data were taken from the FRED database available at the Web site of the Federal Reserve Bank of St. Louis (www.stls.frb.org). Consumption is measured by personal consumption expenditures on non-durable goods and services, which were converted into real per-capita terms by dividing by the quarterly average of the Consumer Price Index for all urban consumers and by the quarterly average of the mid-month U.S. population estimate produced by the Bureau of Economic Analysis.

Since a period in the model is one quarter, the three-month Treasury Bill rate serves as the empirical counterpart of the one-period nominal interest rate. The six-month and
twelve-month rates are the two-period and four-period interest rates, respectively. Note that the three-, six-, and twelve-month Treasury bills have, in fact, maturities of thirteen, twenty-six, and fifty-two weeks, respectively. Rather than averaging the Treasury Bill rates over the quarter, I used the observations for the first trading day of the second month of each quarter (that is, February, May, August and November). Treasury Bills are ideal for this analysis because, like the bonds in the model, they are zero-coupon bonds with negligible default risk. The original interest rate series, which are quoted as a net annual rate, were transformed into a gross quarterly rate.

Except for the nominal interest rates, all raw data are seasonally adjusted at the source. The sample period is from 1960Q1 to 2001Q2, with the latter date determined by the availability of the twelve-month Treasury Bill rate.9

3.2 Estimation

The model is estimated by the Simulated Method of Moments (SMM). The SMM estimator minimizes the weighted distance between the unconditional moments predicted by the model and those computed from the data, where the former are computed on the basis of artificial data simulated from the model. Lee and Ingram (1991) and Due and Singleton (1993) show that SMM delivers consistent and asymptotically Normal parameter estimates under fairly general regularity conditions. Ruge-Murcia (2012) explains in detail the application of SMM for the estimation of non-linear dynamic models and provides Monte-Carlo evidence on its small-sample properties.

More formally, define $\theta \in \Theta$ to be a $q \times 1$ vector of structural parameters with $\Theta \subset \mathbb{R}^q$, $m_t$ to be a $p \times 1$ vector of empirical observations on variables whose moments are of our interest, and $m_t(\theta)$ to be the synthetic counterpart of $m_t$ whose elements are obtained from

9The U.S. Treasury stopped reporting secondary market yields for this Bill after 27 August 2001. The Board of Governors of the Federal Reserve also publishes “constant maturity” yields for Treasury securities with maturities of one year and more. These yields are constructed by the U.S. Treasury using a polynomial interpolation from the yield curve of actively traded Treasury notes and bonds. In preliminary work, I considered using these yields for the model estimation, but abstained from doing so for two reasons. First, Treasury notes and bonds pay coupons and are, therefore, not directly comparable to the bonds in the model. Second, and more importantly, these yields are constructed, rather than observed, and correspond to securities that were not available for trading at a given date. This concern applies more generally to other constructed series, like the CRSP Fama-Bliss discount bond series. When I compared the 12-month Treasury bill and 1-year constant-maturity rates, I found that although they are highly correlated, the Bill rate is almost always below the constant rate and the average difference is large: about 42 basis points at the annual rate. In certain periods (for example, in mid-May 1981), the difference was as large as 200 basis points.
the stochastic simulation of the model. The SMM estimator, \( \hat{\theta} \), is the value that solves

\[
\min_{\{\theta\}} M(\theta)'WM(\theta),
\]

where

\[
M(\theta) = (1/T) \sum_{t=1}^{T} m_t - (1/\lambda T) \sum_{t=1}^{\lambda T} m_t(\theta),
\]

\( T \) is the sample size, \( \lambda \) is a positive constant, and \( W \) is a \( q \times q \) weighting matrix. Under the regularity conditions in Due and Singleton (1993),

\[
\sqrt{T}(\hat{\theta} - \theta) \to N(0, (1 + 1/\lambda)(J'W^{-1}J)^{-1}J'W^{-1}SW^{-1}J(J'W^{-1}J)^{-1}),
\]

where

\[
S = \lim_{T \to \infty} Var \left( (1/\sqrt{T}) \sum_{t=1}^{T} m_t \right),
\]

and \( J = E(\partial m_t(\theta)/\partial \theta) \) is a finite Jacobian matrix of dimension \( p \times q \) and full column rank.

In this application, the weighting matrix is the diagonal of the inverse of the matrix with the long-run variance of the moments, which was computed using the Newey-West estimator with a Barlett kernel and bandwidth given by the integer of \( 4(T/100)^{2/9} \) where \( T = 166 \) is the sample size. The number of simulated observations is five times larger than the sample size.\(^{10}\) The dynamic simulations of the non-linear model are based on the pruned version of the solution, as suggested by Kim et al. (2008).

An important part of this research project involves relaxing the assumption that shock innovations are drawn from a symmetric distribution. In symmetric distributions, the shape of the right side of the central maximum is a mirror image of the left side. Loosely speaking, this means that positive realizations of a given magnitude are as likely as negative realizations. As pointed out above, symmetric distributions (like the Normal) rule out skewness risk by construction. Instead, I consider here versions of the model where innovations are drawn from a Skew normal distribution and from a Generalized extreme value distribution.

The Skew normal distribution is an asymmetric distribution characterized by three parameters, namely a location, a scale, and a correlation parameters, and it is attractive for several reasons. First, its support is \((-\infty, +\infty)\), while other asymmetric distributions (e.g., the Chi-squared and Rayleigh distributions) have support on \([0, \infty)\) only, which is restrictive for this application. Second, it can accommodate both positive and negative skewness depending on the sign of the correlation parameter. Finally, it nests the Normal distribution as

\(^{10}\)Ruge-Murcia (2012) performs experiments using simulated samples that are five, ten, and twenty times larger than the sample size. Results indicate that the former (five) is much more computationally efficient, and only slightly less statistically efficient, than the larger values (ten and twenty).
a special case when the correlation parameter is zero. This means that it is straightforward to test the hypothesis that innovations are drawn from a (symmetric) Normal distribution against the alternative that they are drawn from an (asymmetric) Skew normal distribution. This simply involves performing the two-sided $t$-test of the hypothesis that the correlation parameter is zero against the alternative that it is different from zero.

However, a drawback of the Skew normal distribution is that its skewness is bounded between $-1$ and $+1$. Although in some empirical applications this constraint may not bind, we will see that in this case it does for consumption innovations. For this reason, I also estimate a version of the model where innovations are drawn from a Generalized extreme value (GEV) distribution. This three-parameter distribution nests the Gumbell, Fréchet and Weibull distributions and permits a wider range for the skewness than the Skew normal distribution. The support of the GEV distribution may be bounded above or below depending on the shape parameter. Although there are values of this parameter for which the mean and variance of the distribution do not exist, this turn out not to be empirically relevant here. Overall, we will see below that the GEV distribution delivers the best model fit and is, therefore, our preferred specification.

Finally, I estimate as benchmark a version of the model where innovation are drawn for a Normal distribution. As discussed above, this version corresponds to the special case of the model with Skew normal innovations where the correlation parameter is constrained to be zero.

The estimated parameters are the discount factor ($\beta$), the intertemporal elasticity of substitution ($\psi$), the coefficient of relative risk aversion ($\gamma$), the rate of inflation in the deterministic steady state ($\Pi$), and the autoregressive coefficients and parameters of the innovation distributions of inflation and consumption. During the estimation procedure, the location parameter was adjusted so that the mean of the innovations is zero, and the unconditional mean of the consumption process was normalized to 1. The moments used to estimate these parameters are the variances, covariances, skewness, and first-order autocovariances of all five data series, plus the unconditional means of inflation and the nominal interest rates.

In terms of identification, it is usually difficult to verify that parameters are globally identified, but local identification simply requires that

$$rank\left\{E\left(\frac{\partial m(\theta)}{\partial \theta}\right)\right\} = q, \quad (15)$$

where (with some abuse of the notation) $\theta$ is the point in the parameter space $\Theta$ where the rank condition is evaluated. I verified that this condition is indeed satisfied at the optimum $\hat{\theta}$ for the three versions of the model.

[11]
3.3 Parameter Estimates

Estimates of the parameters are reported in Table 1. Standard errors are reported in parenthesis and were computed using a block bootstrap with 99 replications. First, notice that estimates of the preference parameters are remarkably similar across innovation distributions. The discount factor is 0.99, which implies an annualized gross real interest rate of 1.04 in the deterministic steady state. The intertemporal elasticity of substitution (IES) is quantitatively low and statistically different from both 0 and 1. Estimates—between 0.13 and 0.16 depending on the distribution—are in line with values reported in earlier literature. For example, Hall (1988) reports estimates between 0.07 and 0.35; Epstein and Zin (1991) report estimates between 0.18 and 0.87 depending on the measure of consumption and instruments used; and Vissing-Jörgensen (2002) reports estimates between 0.30 and 1 depending on the households’ asset holdings. The coefficient of relative risk aversion is about 50, which is of the same order of magnitude as the estimate of 79 reported by van Binsbergen et al. (2010) and the values used by previous calibration studies that use Epstein-Zin preferences (e.g., Tallarini, 2000, and Rudebusch and Swanson, 2012).

The inflation rate is mildly persistent and, since the correlation parameter (Skew normal) and shape parameter (GEV) are positive, inflation innovations are positively skewed. That is, their distribution has a longer tail on the right, than on the left, side, and the median is below the mean. Furthermore, because the correlation parameter is statistically different from zero, the null hypothesis that inflation innovations are drawn from a Normal distribution can be rejected in favor of the alternative that they are drawn from an asymmetric Skew normal distribution with positive skewness (p-value < 0.001). The Lagrange Multiplier (LM) test of this hypothesis (p-value < 0.001) delivers the same conclusion.

Figure 1 plots the estimated cumulative distribution function (CDF) of the inflation innovations under the three models. Note that the CDF of the Skew normal and the GEV distributions have more probability mass in the right tail, and less mass in the left tail, than the Normal distribution. Thus, loosely speaking, large positive inflation surprises can happen sometimes, but large negative ones are unlikely. This means that, for a given variance, the buyer of a nominal bond faces the risk of large realizations from the right tail of the inflation distribution, which reduce the real price of bonds with maturity larger than 2 periods and the real payoff of the bond with maturity equal to 1 period.

The consumption process is very persistent, and the correlation parameter (Skew normal) and shape parameter (GEV) are negative. Thus, consumption innovations are negatively skewed.
skewed: their distribution has a longer tail on the left than on the right side, and the median is above the mean. Since the correlation parameter is on the boundary of the parameter space, I use the LM test rather than the (Wald-type) \( t \)-test to evaluate the hypothesis that innovations are drawn from a Normal distribution against the alternative that they are drawn from a Skew normal distribution. Since the \( p \)-value is below 0.001, the hypothesis can be rejected at standard significance levels.

Figure 2 plots the estimated CDF of the consumption innovations under the three models. In the cases of the Skew normal and GEV distributions, the CDF has more mass in the left tail, and less mass in the right tail, than the Normal distribution. Thus, large negative consumption surprises are more probable than positive ones and a bond buyer faces the risk of unexpected, large declines in consumption during the holding period. By a large decline in consumption, I mean, for example, a reduction in consumption 5 percentage points below the mean and which can take place with probability 2.6 percent in the model with Skew normal innovations. In the quarterly U.S. data, the proportion of observations 5 percentage points below trend is 11.5 percent, if the trend is linear, and 4.2 percent, if the trend is quadratic, and these observations are primarily associated with recessions.

The literature on consumption disasters also features a negatively skewed distribution from the combination of a Normal distribution for non-disaster periods and a Bernoulli distribution for disasters. The results in this section imply that outside disaster episodes, traders still face negative consumption skewness due to business cycle fluctuations—in addition, to positive inflation skewness. More generally, these results support the view of Mandelbrot and Hudson (2004), who advocate relaxing the assumption of normality in favor of more general distributions because it ignores thick tails and skewed realizations in asset returns.

### 3.4 Policy Functions

Using the SMM estimates of the parameters, it is possible to explicitly write the policy functions of the model variables. I focus here on yields of selected bond maturities.

For the version of the model with Normal innovations, the policy functions of yields of the 1-, 2- and 4-period bonds are

\[
i_1^t = 2.05 - 0.179\tilde{C}_t + 0.635\tilde{\Pi}_t - 1.436\sigma^2_t - 0.005\sigma^2_{\Pi} + 0.030\tilde{C}_t\sigma^2_t, \\
i_2^t = 2.05 - 0.177\tilde{C}_t + 0.519\tilde{\Pi}_t - 1.421\sigma^2_t - 0.009\sigma^2_{\Pi} + 0.029\tilde{C}_t\sigma^2_t, \\
i_4^t = 2.05 - 0.171\tilde{C}_t + 0.364\tilde{\Pi}_t - 1.392\sigma^2_t - 0.016\sigma^2_{\Pi} + 0.028\tilde{C}_t\sigma^2_t,
\]

where \( \sigma^2_t \) and \( \sigma^2_{\Pi} \) are the variance of consumption and inflation innovations, respectively, \( \tilde{C}_t \) and \( \tilde{\Pi}_t \) denote deviations with respect to the deterministic steady state, and the units
are percent returns at the quarterly rate. The assumption that the processes of aggregate consumption and inflation are log-linear, and the fact that for the perturbation I take a third-order expansion in logs, imply that the coefficients of quadratic and cubic terms in the state variables of these policy functions are zero.

The coefficients of consumption are negative because a transitory increase in the endowment, and hence in consumption, prompts traders to save, driving equilibrium bond prices up and yields down. Since consumption persistence is high, the effect is quantitatively similar across maturities. The coefficients of inflation are positive because traders demand higher yields after an increase in inflation threatens to reduce ex-post real bond returns. Since inflation is only moderately persistent, the response is larger for shorter- than for longer-term bonds. The coefficients of the conditional variances of consumption and inflation are negative because risk-averse traders react to variance risk by increasing precautionary savings, which, in equilibrium, reduces bond yields.

For the versions of the model where innovations are Skew normal and GEV, the policy functions are respectively

\[
i_1^t = 2.08 - 0.187\tilde{C}_t + 0.565\tilde{\Pi}_t \\
-1.510\sigma^2 + 0.005\sigma^2 + 0.033\tilde{C}_t\sigma^2 + 0.103\sigma^3 + (1.66 \times 10^{-5})\sigma^3,
\]

\[
i_2^t = 2.08 - 0.184\tilde{C}_t + 0.442\tilde{\Pi}_t \\
-1.494\sigma^2 + 0.009\sigma^2 + 0.032\tilde{C}_t\sigma^2 + 0.102\sigma^3 + (4.03 \times 10^{-5})\sigma^3,
\]

\[
i_4^t = 2.08 - 0.179\tilde{C}_t + 0.292\tilde{\Pi}_t \\
-1.463\sigma^2 + 0.014\sigma^2 + 0.031\tilde{C}_t\sigma^2 + 0.099\sigma^3 + (8.47 \times 10^{-5})\sigma^3,
\]

and

\[
i_1^t = 2.27 - 0.221\tilde{C}_t + 0.747\tilde{\Pi}_t \\
-1.944\sigma^2 + 0.005\sigma^2 + 0.047\tilde{C}_t\sigma^2 + 0.150\sigma^3 + (1.66 \times 10^{-5})\sigma^3,
\]

\[
i_2^t = 2.27 - 0.218\tilde{C}_t + 0.653\tilde{\Pi}_t \\
-1.922\sigma^2 + 0.010\sigma^2 + 0.046\tilde{C}_t\sigma^2 + 0.148\sigma^3 + (5.28 \times 10^{-5})\sigma^3,
\]

\[
i_4^t = 2.27 - 0.211\tilde{C}_t + 0.509\tilde{\Pi}_t \\
-1.881\sigma^2 + 0.021\sigma^2 + 0.044\tilde{C}_t\sigma^2 + 0.144\sigma^3 + (16.2 \times 10^{-5})\sigma^3,
\]

where \(\sigma^3\) and \(\sigma^3\) are the third-moment of consumption and inflation innovations. Notice that in these two cases, the asymmetry of the shock innovations affects bond yields and prices directly via the third-order moments. The information contained in the policy functions will
be exploited below to study the price of risk and the quantitative contribution of skewness risk to bond yields and premia.

Let me now compare the approach followed here—i.e., solving the full model by a perturbation method and focusing on the policy functions—with the approach of assuming that arguments inside the expectations operator in the Euler equation are Lognormal in order to derive a pricing function with a risk adjustment factor. Clearly, there are conditions under which both approaches lead to the same results. The reason this is not the case here is that when shocks are asymmetric, the skewness also affect asset prices and returns. Furthermore, although the reduced-form of the pricing function could be as accurate as the third-order perturbation, the structural interpretation of the coefficients would not be the same. For example, the skewness terms would be subsumed in the constant intercept and lead us to over-estimate the risk-free rate.

### 3.5 Fit

Figure 3 reports the fit of the three versions of the model by comparing actual and predicted moments. In all panels, the horizontal axis are the moments computed from U.S. data while the vertical axis and dots are the moments predicted by the model. The straight line is the 45 degree line and, thus, if a model were to perfectly fit the data moments, all dots would be on this line.

We can see in this figure that the models with asymmetric innovations fit the data better than the model with Normal innovations. This impression is statistically confirmed by two measures of fit, namely the Root Mean Squared Error (RMSE) and Mean Absolute Error (MAE), which are also reported in the figure. For example, the RMSE of the models with Skew normal and GEV innovations are, respectively, two-thirds and one-half the RMSE of the model with Normal innovations. Most of the difference comes from the fact that the model with Normal innovations cannot match the unconditional skewness of the data. For instance, the model with Normal innovations predicts that the skewness of consumption growth and inflation are basically zero and the corresponding dots are, therefore, relatively far from the 45 degree line. In contrast, the models with asymmetric innovations predict negative skewness in consumption growth and positive skewness in inflation and interest rates, as in the data.

Among the models with asymmetric innovations, the model with GEV innovations has lower RMSE and MAE than the model with Skew normal innovations. Although both models predict similar (negative) skewness in consumption growth, the former predicts higher (positive) skewness for inflation and interest rates. This difference is partly due to the
constraint in the range of possible values of skewness under the Skew normal distribution (from $-1$ to $+1$). Free from this constraint, the model with GEV innovations delivers predictions about skewness that are much closer to those observed in the data. For this reason, in what follows, I treat this version of the model as the preferred specification.

4 Understanding Skewness Risk

This section shows that there are departures from normality in the data which are better captured by models with asymmetric innovations. This section also quantifies the contribution of skewness risk to the bond premia and its effect on bond yields, and reports the price of skewness risk implied by the model.

4.1 Unconditional Skewness

Skewness is a prominent feature of U.S. economic data. Figure 4 shows that the unconditional distribution of inflation and of the three- and twelve-month Treasury-Bill rates are positively skewed, while that of consumption growth is negatively skewed. All large consumption growth declines are associated with recessions. Table 2 reports estimates of the skewness of these variables which, as we can see, are larger than $+1$ in the case of inflation and interest rates and about $-0.6$ in the case of consumption growth. Finally, Table 3 reports the $p$-values of the Jarque-Bera test of the hypothesis that the data follow a Normal distribution. This goodness-of-fit test is based on sample estimates of the skewness and kurtosis, both of which are should be zero if the data were Normal. Given the plots in Figure 4 and skewness estimates in Table 2, it is not surprising that $p$-values are well below 0.05 for all series and, hence, the hypotheses can be rejected. As a whole, this evidence suggests that the Normal is a poor approximation to the actual distribution of the data.

In what follows, I use an artificial sample with 5000 observations from each version of the model to compute the skewness of the ergodic distribution of consumption growth, inflation and the nominal interest rates, and to test the hypothesis that the artificial data follows a Normal distribution. Consider first the sample generated from the model with Normal innovations. Table 2 shows that the unconditional skewness predicted by this model is basically zero, while Table 3 shows that the hypothesis that the artificial data follow a Normal distribution cannot be rejected at the 5 percent level for any variable. In contrast, for the samples generated from the models with Skew normal and GEV innovations, the unconditional skewness predicted by these models (see Table 2) is quantitatively large and of the same sign as the actual data—though in general the skewness of consumption growth
is larger than the data, while that of inflation and of the interest rates are smaller than the data. Moreover, the hypothesis of normality can be rejected at the 5 percent level in all cases (see Table 3), as it is for the U.S. data.

4.2 Contribution of Skewness Risk to the Bond Premia

Skewness risk arises in this model because, for a given variance, traders face the possibility of extreme realizations from the upper tail of the distribution of inflation innovations and from the lower tail of the distribution of consumption innovations. The former reduce the real payoffs and prices of nominal bonds. The latter increase marginal utility and the kernel used by traders to value financial assets. Since the inflation and consumption processes are serially correlated, these effects are persistent.

As was pointed out above, in the special case where the innovation distribution is symmetric—and skewness is, therefore, zero—the bond premia depends only on the variance of the innovations. In the more general case where the distribution is asymmetric, the bond premia depends on both the variance and skewness of the innovations. In terms of the policy functions (see equation (11)), the key difference is the term \((1/6)[f_{\sigma\sigma}(x,0)]^j[\sigma][\sigma][\sigma]\), which is zero in the former case and non-zero in the latter case.

Figure 5 plots the bond premia predicted by the three versions of the model and maturities ranging from 1 to 8 periods. The figure was constructed using averages over 5000 simulated observations and, thus, correspond to the mean of the ergodic distribution of the bond premium for each maturity and distribution. Since the premia are derived from the Euler equation for a nominal bond that pays one unit of currency (say, one dollar) at maturity, the units of the premia are in units of currency as well.\(^{12}\) In Figure 5, premia are expressed in cents. The thin line is the part of the bond premia due to the variance risk only. The thick line is the total bond premia, including skewness risk. (Since the latter part is zero in the case of the Normal distribution, there is no thick line in this panel: all risk is variance risk.) The distance between the thin and think lines is the contribution of skewness risk to the bond premia.\(^{13}\)

First notice that, for all distributions, the premia is negative, meaning that the \(\ell\)-period bond is sold at a discount \textit{vis a vis} the strategy of buying a 1-period bond today and using the

\(^{12}\)This is also true because for the bond premia (only), I took the Taylor-series expansion in levels, rather than in logs. This was necessary because the expansion was taken around the deterministic steady state, where the bond premia is zero and its logarithm is not defined.

\(^{13}\)In terms of the policy functions, the distance between zero and the thin line is \((1/2)[f_{\sigma\sigma}(x,0)]^j[\sigma][\sigma] + (1/2)[f_{\sigma\sigma}(x,0)]^j(x_t - x)^a[\sigma][\sigma][\sigma]\), while the distance between the thin and think lines is \((1/6)[f_{\sigma\sigma}(x,0)]^j[\sigma][\sigma][\sigma]\). All other terms are zero because the premia depend only on the higher-order moments of the shocks.
proceeds to buy a \( \ell - 1 \)-period bond tomorrow at an uncertain price. As a result, the yield of the \( \ell \)-period bond is larger than the weighted average of the yields of the 1- and \( \ell - 1 \)-period bonds and the yield curve is upward sloping. Overall, the models with asymmetric shocks deliver large premia than the model with Normal shocks, and this difference is amplified by skewness risk.

The estimates of the bond premia used to construct Figure 5 are also reported in Table 4. Using these estimates it is easy to compute the contribution of variance and skewness risk to the premia for each bond maturity. When innovation are drawn from a Normal distribution, 100 percent of the bond premia is due to variance risk. On the other hand, when innovations are drawn from asymmetric distributions, skewness risk contributes to the premia as well. In the case of Skew normal innovations, skewness risk accounts for about 6 percent of the premia, while in the case of GEV innovations, skewness risk accounts for about 8 percent. Both variance- and skewness-risk components increase with the maturity, but the former increases faster and, hence, the contribution of skewness risk is mildly decreasing with the maturity. The finding that skewness risk accounts a relatively large proportion of the bond premia—given that the third moment of the innovations is two-orders of magnitude smaller than the second moment—suggest that the price of skewness risk is substantial.

4.3 Effects of Skewness Risk on Yields

In order to quantify the effect of skewness risk on bond yields, I compute the mean of the ergodic distribution of bond yields for each version of the model. Table 5 reports certainty-equivalent yields, yields when there is only variance risk, and yields when there is both variance and skewness risk. All yields are in percent at the quarterly rate.

Comparing the certainty-equivalent and other yields, we can see that, as expected, risk-averse traders in a stochastic economy induce higher bond prices and lower yields as they attempt to build up precautionary savings. For example in the case with Normal innovations, the yield of the 1-period bond is 0.57 percentage points lower than the certainty-equivalent yield, while the yield of the 8-period is 0.54 points lower.

Comparing yields with both risks and with variance risk only, we can see that skewness risk induces even lower yields compared with the latter. This additional yield decrease primarily reflects the extra risk that a large negative consumption innovation, drawn from the left tail of its negatively-skewed distribution, may unexpectedly increase marginal utility next period. It also reflects the risk that a large positive inflation innovation drawn from the right tail of its positively-skewed distribution may reduce the real payoff/price of this nominal asset. Quantitatively, skewness risk decreases the bond yield of a 1-period bond by
0.026 percentage points (that is, 10.1 annual basis points) in the market with Skew normal innovations, and by 0.051 percentage points (20.4 annual basis points) in the market with GEV innovations. Results for the other maturities are similar.

4.4 The Price of Skewness Risk

Table 6 reports the price of variance and skewness risk for bond maturities from 1 to 8 periods, distinguishing between the risk due to consumption and the risk due to inflation. These prices are taken directly from the estimated policy functions for yields, as the reader can see by comparing the prices in Table 6 and the coefficients in Section 3.4 for the appropriate maturity and distribution.

Three observations follow from Table 6. First, concerning consumption, variance risk carries a negative price or discount, while skewness risk carries a positive price. The former is negative because traders react to variance risk by building up precautionary savings. The latter is positive because in a market with a skewed endowment process, nominal bonds may or may not be helpful for the purpose of consumption smoothing depending on the sign of the skewness. If skewness is positive, the bond payoff may arrive at a time consumption is exceptionally high and, thus, the trader would demand compensation in the form of a higher yield. However, if skewness is negative, the bond payoff may arrive at a time consumption is exceptionally low and, thus, the trader would accept a lower yield. The latter is the empirically relevant case and means that both variance and skewness risk induce lower bond yields. Quantitatively the price of consumption skewness risk decreases slowly with the maturity. In the case with Skew normal innovations, the price is about 0.1 percent return at the quarterly rate (0.4 at the annual rate), while in the case with GEV innovations, it is 0.15 percent at the quarterly rate (0.6 at the annual rate).

Second, concerning inflation, variance risk carries a negative price, while skewness risk carries a positive price. In this case, positive inflation skewness may induce exceptionally large inflation realizations which reduce the real payoff and prices of bonds. In order to make bonds attractive in this market, the yield should be higher than in a market with no or negative inflation skewness. Quantitatively, price of skewness risk is relatively small.

Finally, comparing the prices of consumption and inflation risk shows that the former carries a much larger price than the latter. Since estimates of the second- and third-order moments of consumption and inflation innovations are of the same order of magnitude (see Table 1), we can conclude that, as a rough approximation, all risk priced in this model is consumption risk.

Reasons the price of inflation risk is relatively low in this model are the high degree
of risk aversion in the utility function, which is defined in terms of consumption only, and the assumption of perfectly flexible prices, which insulates the real side of the market from nominal shocks. This result implies that results reported in this paper would carry over to an economy with real bonds.

5 Dynamics

In this section, I use impulse-response analysis to study the effects of inflation and consumption shocks on bond yields. Since the model is non-linear, the effects of a shock depend on its sign, size, and timing (see Gallant et al., 1993, and Koop et al., 1996). For this reason, I consider shocks of different sign and size, and assume that they occur when the system is at the stochastic steady state—i.e., when all variables are equal to the unconditional mean of their ergodic distribution. In particular, I study innovations in the 5th, 25th, 75th and 95th percentiles plus the median innovation. Of course, the size (in absolute value) of innovation in the 5th and 95th (and in the 25th and 75th) percentiles are not same when the distribution is asymmetric, but the point is that the likelihood of these two realizations is the same. Responses are reported in Figures 6 and 7, with the vertical axis denoting percentage deviation from the stochastic steady state and the horizontal axis denoting periods (quarters). Yields are in percent at the quarterly rate.

Figure 6 plots the response of yields to inflation shocks drawn from the each of the distributions. Positive (resp. negative) inflation shocks increase (resp. decrease) bond yields, but the magnitude of the response decreases monotonically with the maturity. In the case of the Normal distribution, the effect of a positive shock is the mirror image of the negative shock of the same magnitude. This result is, of course, due to the symmetry of this distribution. In the case of the Skew normal and GEV distributions, there is an asymmetry in that the positive shock in the 95th percentile delivers a larger response than the equally likely negative shock in the 5th percentile. This asymmetry is reversed for the smaller shocks in the 25th and 75th percentiles. Finally, since the median is less than the mean (zero), the median inflation shock reduces bond yields.

The dynamic effects of inflation shocks are driven by the persistence of inflation: a current, positive shock means higher inflation in future periods, and, as a result, traders rationally expect a lower real bond prices and payoffs in the future. When inflation is serially uncorrelated, a current shock has no effect on yields because the forecasts of future payoffs and prices are unchanged. In this sense, what matters for bond yields in this model is anticipated, rather than unanticipated, inflation.

Figure 7 plots the response of yields to consumption shocks drawn from the each of
the distributions. Positive consumption shocks reduce yields because traders faced with an increased endowment would attempt to intertemporally smooth consumption by saving. Since this is not possible in the aggregate, bond prices must increase and yields decrease to induce traders to optimally consume the current output. As before, responses to Normal shocks are symmetric, but those to Skew normal and GEV shocks are asymmetric. In particular, negative shocks at the 5th and 25th percentiles induce larger effects than the equally-likely, but positive, shocks at the 95th and 75th percentiles. Since the median is larger than the mean (zero), the median consumption shock reduces bond yields.

As in the case of inflation, the dynamic effects depend crucially on the persistence of the consumption process. When consumption is positively autocorrelated, a current positive shock signals above-average consumption in future periods as well, and, so, the effects described above take place in both the current and future periods. In contrast, when consumption is serially uncorrelated, the effects take place in the current period only because, by construction, the economy will return to steady state next period.

From these figures, we can see that, conditional on the shock size, the initial effect of consumption shocks is similar for all yields. Hence, consumption shocks push the whole yield curve up or down, just like the level factor does in factor models of the term structure. In contrast, inflation shocks have a larger effect on short- than on long-term yields. As a result, inflation shocks change the slope of the yield curve. In this way, inflation acts like the slope factor in factor models. However, while statistical factors have no structural interpretation, in this model the level shifts and slope changes of the yield curve are empirically associated with observable macroeconomic variables, respectively aggregate consumption and inflation.

Previous research that attempts to build a tighter link between latent and macroeconomic factors include Ang and Piazzesi (2003), Diebold et al. (2006), Wu (2006) and Rudebusch and Wu (2008). Ang and Piazzesi, and Diebold et al. specify factor models where the state vector consists of a mix of latent variables and macroeconomics variables. In the former case, the macroeconomic variables are the first principal components of various inflation and output measures. In the latter case, the macroeconomic variables are inflation, capacity utilization, and the Federal Funds rate, with possible feedback from latent variables to macro variables. Wu finds that in a calibrated New Keynesian model, monetary policy shocks explain most of the slope changes in the term structure, and technology shocks explain most of the level changes. Rudebusch and Wu build a model where two latent factors are respectively driven by the central bank’s inflation target and reaction function. Among these papers, my results are most directly comparable with Wu (2006). In this model, inflation and consumption directly affect the slope and level of the yield curve, while in Wu’s model, the monetary shocks affect the slope via inflation and technology shock affect the level via output and

[21]
consumption. Jointly, both papers suggest that nominal shocks are important to account for slope changes in the yield curve, while real shocks are important to account for level changes.

6 Conclusions

This paper examines the implications of skewness risk for bond pricing and returns in a pure exchange economy. For a given variance, the possibility of extreme realizations from the long tail of the inflation and consumption distributions affects prices, returns and the pricing kernel used by traders to evaluate payoffs. Quantitative magnitudes of these effects are computed based on parameters estimated from U.S. data. These estimates show that inflation innovations are drawn from a positively skewed distribution, that consumption innovations are drawn from a negatively skewed distribution, and that the hypotheses that they are drawn from Normal distributions are rejected by the data.

Results reported in this paper are relevant for three streams of the literature. First, for the growing literature on consumption disasters, this paper shows that even outside disaster episodes, agents face the possibility of substantial consumption decreases primarily associated with recessions, as well as the possibility of positive inflation surprises. Although the magnitude of these consumption decreases is not as dramatic as disasters, they are shown to have non-negligible implications for bond pricing. Thus, the mechanism highlighted by the disaster literature is also operative at business cycle frequencies and during non-disaster periods. Second, for the finance literature on the role of higher-order moments on valuations, this paper quantitatively shows that in a flexible-price economy, most of the bond premia is driven by consumption risk and that skewness risk is quantitatively important. In particular, skewness risk accounts for 8 percent of the bond premia, has a price of 0.6 percent per year, and implies a reduction of bond yields of 20 basis points compared with an economy with only variance risk. Finally, for the statistical literature on factor models of the term structure, this paper presents a model whose state variables (i.e., aggregate consumption and inflation) behave empirically like “slope” and “level” factors but have a clear macroeconomic interpretation.
A Production Economy

Assume that output is produced by identical firms whose total number is normalized to 1. The representative firm uses the technology

$$y_t = z_t (h_t)^\alpha,$$

(A1)

where $\alpha \in (0, 1]$ is a parameter, $h_t$ is labor input, and $z_t$ is a productivity shock. The time endowment is constant and equal to $H$. The productivity shock follows the process

$$\ln(z_t) = (1 - \phi) \ln(z) + \phi \ln(z_{t-1}) + \varsigma_t,$$

(A2)

where $\phi \in (-1, 1)$, $\ln(z)$ is the unconditional mean of $\ln(z_t)$, and $\varsigma_t$ is an $i.i.d.$ innovation with mean zero, constant conditional variance, and non-zero skewness. Profit maximization implies that

$$\alpha z_t (h_t)^{\alpha - 1} = W_t / P_t,$$

where $W_t$ is the nominal wage. Traders’ preferences and financial markets are as described in Section 2.

The equilibrium is an allocation for the trader $C = \{ c_t, (B^\ell_t)_{\ell = 1, \ldots, L} \}_{t = 0}^\infty$, an allocation for the firm $Y = \{ y_t, h_t \}_{t = 0}^\infty$, and a price system $\{(Q^\ell_t)_{\ell = 1, \ldots, L}, W_t \}_{t = 0}^\infty$ such that given the price system: (i) the allocation $C$ solves the trader’s problem; (ii) the allocation $Y$ solves the firms’s problem; (iii) the goods market clears: $C_t = Y_t$; (iv) the labor market clears: $H_t = H$; and (v) bonds are in zero net supply: $B^\ell_t = 0$ for all $\ell = 1, 2, \ldots, L$. Notice that in equilibrium, hours worked are equal to the time endowment.

Focusing now on the aggregate production function, take logs on both sides of (A1) to obtain

$$\ln(Y_t) = \alpha \ln(H) + \ln(z_t).$$

Quasi-differencing delivers

$$\ln(Y_t) = (1 - \phi) (\alpha \ln(H) + \ln(z)) + \phi \ln(Y_{t-1}) + \varsigma_t.$$

Comparing this equation with the aggregate process for the endowment in Section 2, namely

$$\ln(Y_t) = (1 - \rho) \ln(Y) + \rho \ln(Y_{t-1}) + \varsigma_t,$$

shows that output in this production economy with infinitely-elastic labor supply is observationally equivalent to output in Section 2, with $1 - \rho$ $\ln(Y)$ standing for $(1 - \phi) (\alpha \ln(H) + \ln(z))$. 

[23]


Table 1
Parameter Estimates

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Distribution</th>
<th>Distribution</th>
<th>Distribution</th>
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<tbody>
<tr>
<td></td>
<td>Normal</td>
<td>Skew</td>
<td>GEV</td>
</tr>
<tr>
<td>Preferences</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Discount factor</td>
<td>0.990</td>
<td>0.990</td>
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<tr>
<td></td>
<td>(0.003)</td>
<td>(0.005)</td>
<td>(0.008)</td>
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<td>IES</td>
<td>0.165</td>
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<td>(0.043)</td>
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<td>Risk aversion</td>
<td>49.775</td>
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<td>50.198</td>
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<td></td>
<td>(0.041)</td>
<td>(5.9593)</td>
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<tr>
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</tr>
<tr>
<td>Steady state</td>
<td>1.011</td>
<td>1.011</td>
<td>1.011</td>
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<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
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<td>AR coefficient</td>
<td>0.635</td>
<td>0.565</td>
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<td>Standard deviation</td>
<td>0.006</td>
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<td>(0.002)</td>
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<td>(0.003)</td>
<td>(0.001)</td>
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<td>Standard deviation</td>
<td>0.006</td>
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<td>—</td>
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<td>(0.001)</td>
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<td>(0.001)</td>
<td>(0.001)</td>
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Note: The figures in parenthesis are standard errors computed using a block bootstrap with 99 replications.
## Table 2
### Unconditional Skewness

<table>
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<tr>
<th>Series</th>
<th>Data</th>
<th>Normal</th>
<th>Normal</th>
<th>GEV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption growth</td>
<td>-0.606</td>
<td>-0.007</td>
<td>-0.948</td>
<td>-1.273</td>
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<tr>
<td>Inflation rate</td>
<td>1.226</td>
<td>-0.027</td>
<td>0.629</td>
<td>1.650</td>
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<tr>
<td>3-Month T-Bill rate</td>
<td>1.222</td>
<td>0.024</td>
<td>0.330</td>
<td>0.611</td>
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<tr>
<td>6-Month T-Bill rate</td>
<td>1.146</td>
<td>0.022</td>
<td>0.293</td>
<td>0.512</td>
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<tr>
<td>12-Month T-Bill rate</td>
<td>1.011</td>
<td>0.016</td>
<td>0.270</td>
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</table>

*Note: The sample size of the artificial data is 5000 observations.*
Table III
Jarque-Bera Test

<table>
<thead>
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<th>Data</th>
<th>Normal</th>
<th>Normal</th>
<th>GEV</th>
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<tr>
<td>Consumption growth</td>
<td>&lt; 0.001</td>
<td>0.346</td>
<td>&lt; 0.001</td>
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<td>Inflation rate</td>
<td>&lt; 0.001</td>
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<td>&lt; 0.001</td>
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<td>3-Month T-Bill rate</td>
<td>&lt; 0.001</td>
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<td>6-Month T-Bill rate</td>
<td>&lt; 0.001</td>
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Note: The figures reported are p-values. The sample size of the artificial data is 5000 observations.
### Table 4
Decomposition of the Bond Premia

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<th>5</th>
<th>6</th>
<th>7</th>
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<td><strong>Normal Distribution</strong></td>
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<tr>
<td>Bond premium</td>
<td>-0.0095</td>
<td>-0.0190</td>
<td>-0.0283</td>
<td>-0.0371</td>
<td>-0.0454</td>
<td>-0.0533</td>
<td>-0.0606</td>
</tr>
<tr>
<td>Of which:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Variance risk (%)</td>
<td>-0.0095</td>
<td>-0.0190</td>
<td>-0.0283</td>
<td>-0.0371</td>
<td>-0.0454</td>
<td>-0.0533</td>
<td>-0.0606</td>
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<td>100</td>
<td>100</td>
<td>100</td>
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<td><strong>Skew Normal Distribution</strong></td>
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<td></td>
</tr>
<tr>
<td>Bond premium</td>
<td>-0.0106</td>
<td>-0.0215</td>
<td>-0.0321</td>
<td>-0.0422</td>
<td>-0.0517</td>
<td>-0.0607</td>
<td>-0.0689</td>
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<tr>
<td>Of which:</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Variance risk (%)</td>
<td>-0.0099</td>
<td>-0.0201</td>
<td>-0.0301</td>
<td>-0.0397</td>
<td>-0.0488</td>
<td>-0.0572</td>
<td>-0.0651</td>
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<tr>
<td></td>
<td>93.20</td>
<td>93.58</td>
<td>93.85</td>
<td>94.05</td>
<td>94.21</td>
<td>94.34</td>
<td>94.45</td>
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<tr>
<td>Skewness risk (%)</td>
<td>-0.0007</td>
<td>-0.0014</td>
<td>-0.0020</td>
<td>-0.0025</td>
<td>-0.0030</td>
<td>-0.0034</td>
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<td>6.80</td>
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<td>5.95</td>
<td>5.79</td>
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<td>5.55</td>
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<td><strong>GEV Distribution</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Bond premium</td>
<td>-0.0166</td>
<td>-0.0322</td>
<td>-0.0468</td>
<td>-0.0605</td>
<td>-0.0732</td>
<td>-0.0849</td>
<td>-0.0958</td>
</tr>
<tr>
<td>Of which:</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Variance risk (%)</td>
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<td>-0.0294</td>
<td>-0.0428</td>
<td>-0.0554</td>
<td>-0.0671</td>
<td>-0.0779</td>
<td>-0.0880</td>
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<td></td>
<td>91.10</td>
<td>91.23</td>
<td>91.37</td>
<td>91.50</td>
<td>91.63</td>
<td>91.75</td>
<td>91.86</td>
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<tr>
<td>Skewness risk (%)</td>
<td>-0.0015</td>
<td>-0.0028</td>
<td>-0.0040</td>
<td>-0.0051</td>
<td>-0.0061</td>
<td>-0.0070</td>
<td>-0.0078</td>
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<tr>
<td></td>
<td>8.90</td>
<td>8.77</td>
<td>8.63</td>
<td>8.50</td>
<td>8.37</td>
<td>8.25</td>
<td>8.14</td>
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*Notes:* The premia are expressed in cents. The figures for the variance risk include the part due to time-varying volatility.
## Table 5
### Bond Yields

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<tr>
<th>Maturity</th>
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<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal Distribution</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Certainty-equivalent</td>
<td>2.051</td>
<td>2.051</td>
<td>2.05</td>
<td>2.05</td>
<td>2.051</td>
<td>2.051</td>
<td>2.051</td>
<td>2.051</td>
</tr>
<tr>
<td>With variance risk</td>
<td>1.480</td>
<td>1.485</td>
<td>1.489</td>
<td>1.494</td>
<td>1.498</td>
<td>1.503</td>
<td>1.507</td>
<td>1.512</td>
</tr>
</tbody>
</table>

| Skew Normal Distribution |     |     |     |     |     |     |     |     |
| Certainty-equivalent | 2.083 | 2.083 | 2.083 | 2.083 | 2.083 | 2.083 | 2.083 | 2.083 |
| With variance risk | 1.478 | 1.483 | 1.488 | 1.493 | 1.498 | 1.503 | 1.508 | 1.513 |
| With variance and skewness risk | 1.453 | 1.458 | 1.463 | 1.468 | 1.474 | 1.479 | 1.485 | 1.490 |
| Contribution of skewness risk | -0.026 | -0.026 | -0.025 | -0.025 | -0.024 | -0.024 | -0.024 | -0.024 |

| GEV Distribution |     |     |     |     |     |     |     |     |
| Certainty-equivalent | 2.273 | 2.273 | 2.273 | 2.273 | 2.273 | 2.273 | 2.273 | 2.273 |
| With variance risk | 1.500 | 1.508 | 1.515 | 1.522 | 1.529 | 1.536 | 1.542 | 1.549 |
| With variance and skewness risk | 1.449 | 1.458 | 1.466 | 1.474 | 1.481 | 1.489 | 1.496 | 1.503 |
| Contribution of skewness risk | -0.051 | -0.050 | -0.049 | -0.048 | -0.048 | -0.047 | -0.046 | -0.046 |

*Note: Yields are expressed in percent return at the quarterly rate.*
Table 6

The Price of Risk

| Maturity | Normal Distribution | Skew Normal Distribution | GEV Distribution |
|----------|---------------------|---------------- ---------|-----------------|
|          | Consumption         | Inflation       | Consumption     | Inflation       | Consumption     | Inflation       |
|          | Variance risk       | Variance risk   | Variance risk   | Variance risk   | Variance risk   | Variance risk   |
| 1        | -1.436              | -0.005          | -1.944          | -0.005          | -0.005          | -0.005          |
| 2        | -1.421              | -0.009          | -1.922          | -0.010          | -0.010          | -0.010          |
| 3        | -1.407              | -0.013          | -1.901          | -0.016          | -0.016          | -0.016          |
| 4        | -1.392              | -0.016          | -1.881          | -0.021          | -0.021          | -0.021          |
| 5        | -1.378              | -0.019          | -1.861          | -0.026          | -0.026          | -0.026          |
| 6        | -1.365              | -0.021          | -1.842          | -0.031          | -0.031          | -0.031          |
| 7        | -1.352              | -0.023          | -1.823          | -0.035          | -0.035          | -0.035          |
| 8        | -1.339              | -0.025          | -1.805          | -0.038          | -0.038          | -0.038          |

Skewness risk (×10⁻³)

<table>
<thead>
<tr>
<th></th>
<th>Normal Distribution</th>
<th>Skew Normal Distribution</th>
<th>GEV Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation</td>
<td>-0.005</td>
<td>0.017</td>
<td>-0.005</td>
</tr>
<tr>
<td>Skewness risk</td>
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<td>0.103</td>
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<td>0.102</td>
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<td>-0.013</td>
<td>0.064</td>
<td>0.146</td>
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<td>-0.016</td>
<td>0.085</td>
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<td>0.142</td>
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<td>-0.025</td>
<td>0.135</td>
<td>0.136</td>
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</table>

Notes: Prices are expressed in percent return at the quarterly rate.
References


Figure 1: Estimated CDF Inflation Innovations
Figure 2: Estimated CDF Consumption Innovations

Normal

Skew Normal

GEV
Figure 3: Fit

Normal

Skew Normal

GEV

RMSE = 0.150
MAE = 0.097

RMSE = 0.103
MAE = 0.076

RMSE = 0.075
MAE = 0.055
Figure 4: Skewness in U.S. Data

Inflation

Skewness = 1.23

Consumption Growth

Skewness = -0.61

3-Month Rate

Skewness = 1.22

1-Year Rate

Skewness = 1.01
Figure 5: Bond Premia (in Cents)
Figure 6: Responses to Inflation Shocks

1-Period Bond

4-Period Bond
Normal

8-Period Bond

Skew Normal

GEV

Legend:
- Blue: 5th
- Red: 25th
- Green: 75th
- Cyan: 95th
- Black: Median
Figure 7: Response to Consumption Shocks

- 1-Period Bond
- 4-Period Bond (Normal)
- 8-Period Bond

- Skew Normal
- GEV