Portfolios and Business Cycles in an Open Economy
DSGE Model

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Abstract

We present a two country, two good model with nominal rigidities, endogenous capital accumulation and incomplete markets that i) generates substantial international positions in nominal bonds and equity, ii) produces equity home bias of a realistic size, and iii) matches many important features of the international business cycle. Agents use both debt and equity to hedge fluctuations in labour income and consumption expenditures. Since bond positions hedge some of this risk, equity positions depend on the correlation with human capital and expenditure risk, conditional on bond returns. The model implies that the unconditional correlation between equity returns and human capital returns is positive, while the conditional one is negative. This is in line with the data and generates equity home bias. Monetary policy is endogenous in the model and it affects equilibrium portfolios. We confront the predictions to data from G7 countries and find support for our findings.

Keywords: Country portfolios, International business cycles, Equity home bias
JEL Codes: F30, F36, F41, G15

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1 Introduction

This paper is motivated by four facts. Firstly, international trade in assets is large and it involves both debt and equity securities (see e.g. Lane and Milesi Ferretti (2001, 2007)). Thus, total U.S. holdings of foreign securities at the end of 2008 amounted to 4,291 billion U.S. dollars, while foreign holdings of U.S. securities amounted to 10,322 billion of dollars, or 31 and 72% of 2008 U.S. GDP, respectively. Gross U.S. portfolio equity positions (the sum of portfolio equity assets and liabilities) totalled 5,717 billion U.S. dollars, while gross U.S. portfolio debt positions equalled 8,185 billion U.S. dollars (U.S. Treasury (2009a,b)). The large size of these positions means that they can have significant effects on macroeconomic outcomes.

In addition to size, the composition of asset trade also matters. Previous research has shown that fluctuations in asset prices and exchange rates can generate significant wealth redistributions between countries.\(^1\) For example, in 2007 and 2008 U.S. portfolio debt inflows fell much less, both in percentage and absolute terms, than equity inflows, and the valuation effects were much stronger for equity than for debt.\(^2\) Understanding the drivers of both the size and the composition of international trade in assets is therefore of high interest, even for a country as large and developed as the U.S.

Secondly, despite the size and sophistication of international financial markets, countries around the world continue to display a marked tendency to invest disproportionately in domestic equity (see French and Poterba (1991) and Table 1). This tendency is often referred to as ”equity home bias”, as basic models of international risk sharing would imply full diversification across countries (Lucas (1982)). Many explanations have been put forward to account for the observed bias, including frictions in the trading of goods or assets or informational frictions (see Lewis (1999), Karolyi and Stulz (2003) and Sercu and Vanpee (2007) for surveys of the literature). One intuitive set of explanations has focused on the possibility that domestic investors may choose to hold domestic equity, because it insures them against a domestic source of risk, most notably labour income risk (see Heathcote and Perri (2009) amongst others). However, in the data relative equity and human capital returns appear to be positively correlated (Baxter and Jermann (1997)). This is our third fact.

Fourthly, countries tend to exhibit significantly positive cross country correlations across a broad range of macroeconomic variables, including output, consumption, investment and employment (see e.g. Backus et al (1993)).

We propose a two country, two good model with nominal rigidities, endogenous capital accumulation and incomplete markets that i) implies substantial international equity and debt positions, ii) predicts equity home bias of a realistic size, iii) features a positive unconditional correlation between relative equity returns and relative human capital returns,

\(^1\)Lane and Milesi-Ferretti (2001), Ghironi et al. (2005), Gourinchas and Rey (2007), and Tille (2003, 2008) emphasize the quantitative importance of valuation effects on external assets and liabilities.

\(^2\)Based on an updated version of the dataset described in Gourinchas and Rey (2007a), kindly provided by Hélène Rey.
Table 1: Equity Home Bias in G7 Countries

<table>
<thead>
<tr>
<th>Country</th>
<th>Equity Market Cap as % of World Market Cap</th>
<th>Share of domestic stocks in total equity portfolio (in %)</th>
<th>Home Bias</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada</td>
<td>2.4</td>
<td>84.0</td>
<td>81.6</td>
</tr>
<tr>
<td>France</td>
<td>4.3</td>
<td>79.8</td>
<td>75.5</td>
</tr>
<tr>
<td>Germany</td>
<td>4.0</td>
<td>61.3</td>
<td>57.3</td>
</tr>
<tr>
<td>Italy</td>
<td>2.2</td>
<td>67.3</td>
<td>65.1</td>
</tr>
<tr>
<td>Japan</td>
<td>11.3</td>
<td>89.5</td>
<td>88.2</td>
</tr>
<tr>
<td>UK</td>
<td>8.1</td>
<td>77.0</td>
<td>68.9</td>
</tr>
<tr>
<td>US</td>
<td>47.8</td>
<td>88.7</td>
<td>40.9</td>
</tr>
</tbody>
</table>

Home Bias in column (3) is computed as the difference between columns (2) and (1). Source: Coeurdacier et al (2007) based on CPIS data from 2001.

and iv) matches important features of the international business cycle. Our model builds on the extensive literature on business cycles in open economies. The key innovation in our work is the combination of a sophisticated endogenous portfolio decision with a realistic description of the real side of the economy.

In our setting, there is international trade in equities and nominal bonds, the two major types of portfolio assets traded globally, and markets are incomplete. Most previous work in international macroeconomics has featured only very limited portfolio decisions, with the most common settings involving trade in a noncontingent bond or country specific equities. This is because the standard approximation techniques that are used to solve these models only pin down portfolios for a number of restricted cases. We use the recent approximation methods described in Devereux and Sutherland (2009a) to overcome these difficulties and solve for optimal portfolios in a setting with multiple assets and incomplete markets. We show that under very general conditions, steady state positions in equity and bonds can be related to two sources of risk. Agents invest in equities and bonds to insure against fluctuations in the present value of their relative consumption expenditures (“consumption expenditure risk”) and the present value of their labour income (“human capital risk”). Financial income is then more valuable in states of the world where the present value of consumption expenditures is high or when the value of human capital is low. Crucially, however, positions in individual assets have to take into account returns on other assets.

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4 Examples of such specific cases include Coeurdacier et al (2009), Engel and Matsumoto (2009), Heathcote and Perri (2009) and Kollmann (2006).

5 Tille and van Wincoop (2009) present a similar solution method that allows for incorporating portfolio choice in open economy DSGE models. Devereux and Saito (2005), Evans and Hnatkovska (2006) and Judd et al. (2002) describe alternative solution approaches. Other recent papers that featured trade in equities and bonds include Benigno and Nistico (2009), Coeurdacier et al (2007), Coeurdacier and Gourinchas (2009), and Pavlova and Rigobon (2007, 2009), and Bui (2009).
also. Thus, in our framework, equilibrium equity positions depend on the covariance with human capital and consumption expenditure risk, conditional on relative bond returns, and the interactions between different asset returns turn out to be very important both in the model and the data.

Our benchmark model features imperfect substitutability between domestic and foreign goods, endogenous capital accumulation, consumption and investment home bias, nominal price and wage rigidities, endogenous monetary policy and neutral and investment specific technology and monetary shocks. Relatively low substitutability between Home and Foreign goods and home bias in consumption imply that increases in the supply of the Home good induce a large fall in its relative price, and, with nominal rigidities, nominal exchange rates. The behaviour of nominal exchange rates, in turn, has a strong effect in the comovement of many variables in our model, and, inter alia, implies that relative equity returns are positively correlated with relative human capital returns and relative bond returns. Nevertheless, equity home bias is mainly driven by the motive to hedge human capital risk in our model. This is because once we condition on bond returns, equity returns are negatively correlated with human capital returns.

In addition to the response of nominal exchange rates, the key forces are endogenous capital accumulation and nominal price rigidities. Relative equity returns are positively correlated with relative human capital returns in response to productivity and monetary shocks, but move in opposite directions in response to investment specific technology shocks. In response to an investment specific technology shock, firms increase their demand for investment, and, with home bias in investment, their demand for Home labour. Firms, increasing production, face rising marginal costs and would like to raise prices. Since nominal rigidities prevent some from doing so, the overall response in the price level is smaller and the ultimate increase in investment and labour demand is larger than in the case of price flexibility and large enough to imply a rise in relative human capital returns, despite a small nominal depreciation. The Home investment boom lowers payments to shareholders in order to finance investment, thereby generating negative comovement between equity and human capital returns, conditional on the investment shock. Since relative bond returns do not respond strongly to the investment efficiency shock, this also implies that relative equity returns are negatively correlated with relative human capital returns, conditioning on relative bond returns.

Previous work (e.g. Coeurdacier et al (2009) and Engel and Matsumoto (2009)) has noted that in the presence of trade in bonds, it is the conditional covariance between human capital returns and equity returns that drives equity portfolios. There is also some evidence that, while equity returns are positively correlated with human capital returns unconditionally, they are negatively correlated, once we condition on relative bond returns (Coeurdacier and Gourinchas (2009) and below). However, to our knowledge, this is the first paper to present a model that replicates both the conditional and the unconditional empirical pattern of correlations between human capital and equity returns.

Consumption expenditure risk has a much smaller effect on equity positions, as it is mainly hedged using positions in one period nominal bonds. Relative consumption expen-
duities are strongly affected by movements in the nominal exchange rate, as fluctuations in relative consumption volume and relative inflation are relatively modest. Since relative bond returns are highly correlated with the nominal exchange rate, long positions in domestic bonds provide a very good hedge for consumption expenditure risk. While the (conditional and unconditional) positive correlation between relative bond returns and consumption expenditure risk induces Home agents to take long positions in Home bonds, the (conditional and unconditional) positive correlation between relative bond returns and human capital returns induces Home agents to go short in Home bonds. Overall, the former motive prevails quantitatively, implying long positions in Home bonds in our benchmark case.

Monetary policy is represented by a simple rule according to which the central bank reacts strongly to CPI inflation and less strongly to output growth. Engel and Matsumoto (2009) showed that nominal price stickiness can have strong effects on endogenous asset portfolios in a setting with exogenous money supply. We show that this is true still, once we allow for endogenous and realistically calibrated, but not optimal, monetary policy. Monetary policy does affect portfolios through two channels here. Firstly, due to price rigidities, monetary policy has real effects and therefore affects the dynamics of human capital and consumption expenditure risk. Secondly, monetary policy also affects the behaviour of asset returns, and in particular the nominal exchange rate. In our setting, markets are incomplete and risk sharing is imperfect. In this case, monetary policy has welfare effects beyond the conventional effects arising from price dispersion (Devereux and Sutherland (2007, 2008)). This is because by affecting the hedging properties of assets, it affects the degree of overall risk sharing. But since the degree of risk sharing is high under most reasonable specifications of monetary policy, these additional welfare effects are likely to be small.

We confront our model with the data in three ways. Firstly, we calibrate the model and compare its predictions for international asset positions with available evidence. Our benchmark calibration implies that domestic investors hold close to 90 percent of the domestic equity stock, which is virtually identical to the share reported for the US in 2008 (U.S. Treasury (2009a)). The model also predicts that domestic investors take an overall long position in domestic bonds of about 50 percent of GDP, and a corresponding short position in foreign bonds. This is consistent with the evidence presented in Lane and Shambaugh (2007,2009) who show that advanced economies, on average, have negative net foreign currency debt positions. However, a notable exception is the U.S. which has a negative domestic currency debt position.

Secondly, we compare the business cycle implications of our benchmark model with the data and find that the model replicates many important features of the international business cycle. The presence of investment efficiency shocks lower the cross country correlation in consumption, raise the volatility of the real exchange rate and generate countercyclical net exports, as in Coeurdacier et al (2009). However, our model is also able to generate positive cross country correlations in investment and strongly positive correlations between domestic consumption and output, due to the nominal rigidities in price setting and investment adjustment costs. Previous work has noted the difficulty of jointly generating realistic
macroeconomic and asset pricing moments in a domestic context.\textsuperscript{6} Our open economy model is no exception. We find that the equity premium in our benchmark economy is too low and an extension that allows for Epstein-Zin preferences, as in Rudebusch and Swanson (2009), can only generate a realistic level of the equity premium at the expense of excessive volatility in the risk free rate.

Finally, we use data on equity and bond returns, labour income and consumption expenditure for the G7 economies since 1970 to estimate the unconditional and conditional comovement of equity and bond returns with consumption and human capital risk for the US. We find that in the data, as in the model, relative equity returns are unconditionally moderately positively correlated with consumption expenditure and human capital risk, while relative bond returns are more strongly positively correlated with the two sources of risk. Once we condition on bond returns, relative equity returns are negatively correlated with relative human capital returns, while they remain positively correlated with consumption expenditure risk. Our estimation therefore implies equity home bias driven by the motive to hedge human capital risk and, to a lesser extent, consumption expenditure risk. Quantitatively, the position implied by the estimation is very close to the position implied by our model calibration and both are close to data on actual U.S. holdings of U.S. equity. For bond positions, we find that, as in the model, the estimation predicts that the human capital motive should induce U.S. investors to go short in U.S. bonds, while the consumption expenditure motive would induce them to take long positions. Unlike in the model, the consumption expenditure motive dominates in the estimation which therefore predicts an overall short position in U.S. bonds, consistent with actual data on U.S. portfolio positions (e.g. U.S. Treasury (2009a)). However, the size of the bond position predicted by the estimation is too large in comparison with the available data.

This paper is related to a number of recent contributions. Engel and Matsumoto (2009) investigate international trade in equities and currency forward contracts in a model with nominal price stickiness and exogenous money supply, while Devereux and Sutherland (2007, 2008) study the effects of monetary policy on international portfolio choice in a model with nominal price stickiness and incomplete markets. Neither of the two feature endogenous capital accumulation or investigate the business cycle implications of their models. Heathcote and Perri (2009) were the first to show that endogenous capital accumulation can generate a negative correlation between dividends and labour income in a model where equity is the only asset traded and shocks to productivity are the only source of uncertainty. Coeurdacier et al (2009) extend the model of Heathcote and Perri (2009) by adding shocks to investment specific technology and trade in real bonds and investigate its predictions for international portfolio choice and business cycle properties. Unlike their framework, our model features nominal rigidities in price and wage setting, and endogenous monetary policy. What is more, the bonds traded in our setup are nominal and markets are incomplete, while the......
bonds traded in Coeurdacier et al (2009) are real and markets are complete, to a first order approximation.

This paper proceeds as follows. In the next section, we show that in a fairly general class of open economy models, steady state asset positions are related to the comovement of relative asset returns with consumption expenditure and human capital risk, conditional on the returns of other assets. In section 3, we describe our benchmark model. In section 4, we calibrate the model, present quantitative results for its equilibrium portfolios and business cycles predictions and discuss their drivers. Section 5 contains the empirical analysis and section 6 concludes.

2 Asset Positions and Hedging Demands

Below, we will present a two country, two good, production economy model that features endogenous capital accumulation, nominal price and wage rigidities and incomplete markets. A difficulty that arises from the incorporation of these features is that it is, in general, not possible to solve for portfolios in closed form. However, we show that, under a very limited set of assumptions, we can derive a reduced form expression that relates asset positions to fluctuations in (consumption) expenditure and nonfinancial income, at a first order of approximation, that can be used to understand the drivers of asset positions and that can be estimated. Assume that equities and short term bonds are traded between two symmetric countries, generically called “Home” and “Foreign”. Equities are a claim to dividends, while nominal bonds pay one unit of local currency for one period only. The budget constraint of the Home agent is then:

\[
\begin{align*}
S^H_{H,t}P^S_{H,t} + S^H_{F,t}P^S_{F,t}Z_t + B^H_{H,t}P^B_{H,t} + B^H_{F,t}P^B_{H,t}Z_t + P^H_{C,t}C_{H,t} \\
= W_{H,t}L_{H,t} + (D_{H,t} + P^S_{H,t}Z_t) + S^H_{F,t-1}(D_{F,t} + P^S_{F,t})Z_t + B^H_{H,t-1} + B^H_{F,t-1}Z_t,
\end{align*}
\]

where \(S^j_{i,t}\) is the fraction of country \(j\) equity held by country \(i\), \(B^i_{j,t}\) are the number of country \(j\) bonds held by country \(i\), \(P^S_{i,t}\) are the prices of country \(i\) shares in local currency, \(P^i_{C,t}\) is the consumption price index of country \(i\), and \(W_{i,t}, L_{i,t}, D_{i,t}\) are wages, labour demand, and dividends. \(Z_t\) is the nominal exchange rate, defined as the number of units of Home currency per unit of Foreign currency, implying that a rise in \(Z_t\) signifies a depreciation of the Home currency. This expression indicates that nonfinancial income and financial income are used to finance consumption expenditure today or (via financial investment) in the future. To make the dynamic nature of this relationship more precise and to translate it into an international context, it is useful to derive expressions for asset returns and our
definitions of risk. Due to the symmetry of the model, it is without loss of generality to focus on the relative value of all variables, including asset returns, and we will henceforth do so.

Realised relative returns in terms of Home currency between Home and Foreign nominal bonds, $\hat{R}^B_t$, are, linearised around the nonstochastic steady state:

$$\hat{R}^B_t \equiv \hat{R}^B_{H,t} - \hat{R}^B_{F,t} - \hat{Z}_t + \hat{Z}_{t-1} = \bar{E}_t \left[ -\hat{Z}_t \right],$$

(2)

where $R^B_{H,t}$ ($R^B_{F,t}$) are local currency bond returns. $E_t [X_{t+j}]$ is the conditional expectation of $X_{t+j}$ at time $t$, $\bar{E}_t [X_t] \equiv E_t [X_t] - E_{t-1} [X_t]$ denote date $t$ surprises in the value of $X$ and $\hat{X}_t$ is the linearised value of $X_t$, defined as $X_t \equiv \frac{X_t - X}{X}$, where $X$ is the value of $X_t$ in the nonstochastic steady state. Thus, relative returns of Home bonds equal the extent of the unexpected appreciation of the local currency in the same period. Relative linearised equity returns in Home currency, $\hat{R}^S_t$, are determined by the present value of surprises to relative dividends:

$$\hat{R}^S_t \equiv \hat{R}^S_{F,t} - \hat{R}^S_{H,t} - \hat{Z}_t + \hat{Z}_{t-1} = (1 - \beta) \bar{E}_t \left[ \sum_{j=0}^{\infty} \beta^j \left( \hat{D}_{F,t+j} - \hat{D}_{H,t+j} + \hat{Z}_{t+j} \right) \right],$$

(3)

where $R^S_{H,t}$ ($R^S_{F,t}$) are local currency equity returns, $\beta$ is the subjective discount rate that we assume to be common between the two countries and we imposed the appropriate transversality conditions.

Above, we mentioned that agents use financial and nonfinancial income to finance their consumption expenditures. We therefore define two measures, $R^PC_t$ and $R^WL_t$, that we will henceforth call “consumption expenditure risk” and “human capital risk”. In linearised form these are:

$$\hat{R}^{PC}_t \equiv (1 - \beta) \bar{E}_t \sum_{j=0}^{\infty} \beta^j \left( \hat{P}^H_{C,t+j} + \hat{C}_{H,t+j} - \hat{P}^F_{C,t+j} - \hat{C}_{F,t+j} - \hat{Z}_{t+j} \right)$$

(4)

$$\hat{R}^{WL}_t \equiv (1 - \beta) \bar{E}_t \sum_{j=0}^{\infty} \beta^j \left( \hat{W}_{H,t+j} + \hat{L}_{H,t+j} - \hat{W}_{F,t+j} - \hat{L}_{F,t+j} - \hat{Z}_{t+j} \right),$$

(5)

where we again imposed the appropriate transversality conditions. Consumption expenditure risk thus depends on the present value of surprises in relative consumption expenditure, while human capital risk, depends on the present value of surprises in relative labour income.\(^8\)

\(^7\)The complete derivation is given in the appendix.

\(^8\)It is worth noting that the discounted sums in (2) – (5) are formed using the simple nonstochastic discount factor. With incomplete markets, the stochastic discount factors of the Home and Foreign investor are not necessarily equal. However, at the level of approximation that we consider, any time variation in
Using a present value analogue of the period by period budget constraint, (1), in linearised form, deducting the analogous expression for the foreign country, and using expressions (2) – (5), we can then write the relative budget constraint as:

\[
\frac{C}{1 - \beta} \hat{R}_{t}^{PC} = \frac{WL}{1 - \beta} \hat{R}_{t}^{WL} + (2S - 1) \frac{D}{1 - \beta} \hat{R}_{t}^{S} + 2\hat{B}_{t},
\]

(6)

where \( S \) and \( B \) are the steady share of the domestic equity stock and the number of domestic bonds held by domestic investors. \( C, D \) and \( WL \) are the steady state values of consumption, labour income and dividends, respectively. This expression states that innovations in the present value of relative consumption expenditures have to be met by innovations to the value of human capital or the value of equity and bond holdings. Projecting (6) on relative bond returns and deducting the resulting expression from (6), we can then solve for equilibrium equity holdings in the steady state:

\[
S = \frac{1}{2} + \frac{1}{2} \frac{C \text{cov}_{R_t^B} (\hat{R}_{t}^{PC}, \hat{R}_{t}^{S})}{\text{var}_{R_t^B} (\hat{R}_{t}^{S})} - \frac{1}{2} \frac{WL \text{cov}_{R_t^B} (\hat{R}_{t}^{WL}, \hat{R}_{t}^{S})}{\text{var}_{R_t^B} (\hat{R}_{t}^{S})}
\]

(7)

where \( \text{cov}_{R_t^B} (x, y) \) is the covariance between \( x \) and \( y \), conditional on relative bond returns.

Equation (7) is not structural, as equity positions are not expressed in terms of the exogenous shocks of the model, but it nevertheless provides useful intuition about the drivers of equity positions. According to (7), the equity position can be broken into three terms. The first term reflects the pure diversification motive. In our symmetric two country model, agents would hold exactly one half of the equity stock of each country if relative equity returns were uncorrelated with consumption expenditure and human capital risk, conditional on bond returns. The second term arises from a motive to hedge movements in relative consumption expenditures. Domestic equity is more attractive, if relative equity returns are high, on average, when surprises in the present value of future consumption expenditures are high, conditional on relative bond returns. The third term in (7) reflects the motive to hedge human capital returns with equity holdings. This term captures the fact that equity

the discount factors drops out and the discount factor is simply equal to \( \beta \). Note also that, to a first order approximation, the expected stochastic discount factors are equal between Home and Foreign investors, as shown in the appendix.

\[9\]

The market clearing conditions for assets in the steady state imply: \( S_{H}^{F} = S_{F}^{F} = S = 1 - S_{H}^{F} = 1 - S_{F}^{F} \) and \( B_{H}^{F} = B_{F}^{F} = B = -B_{H}^{F} = -B_{F}^{F} \). We also impose that the discounted sum of future net exports is equal to the initial net foreign asset position, a condition that is satisfied at our level of approximation. Full details of the derivation are given in the appendix.
returns are particularly valuable, if other sources of income, in this case human capital returns and relative bond returns, are low. Thus, the domestically held share of the equity stock increases the more negatively correlated relative equity return innovations are with relative human capital return innovations, conditional on relative bond returns. The relative importance of the consumption expenditure and the human capital motive is indicated by the presence of the $C, D$ and $WL$ terms, reflecting the size of steady state consumption expenditure, labour income and dividends. Two features are worth highlighting. Firstly, it is the comovement of relative returns with relative consumption expenditure and labour income that matters. As mentioned before, the symmetry of the model implies that it is without loss of generality to focus on relative moments, but it is worth keeping in mind that relative returns may fall even when absolute returns rise, and vice versa. Secondly, note that the equity positions in (7) are functions of covariances and variances, conditional on relative bond returns. This reflects the fact that equity is not the only financial asset in this economy, and the other financial assets, in this case nominal bonds, are also used to hedge the two sources of risk. The conditional covariance of relative equity return with human capital risk can thus be interpreted as the residual comovement, once some part of this risk has been hedged using the bond positions. It is also worth noting that the expressions for bond and equity positions feature conditional covariance-variance ratios and thus look like multiple regression coefficients, a property which, following Coeurdacier and Gourinchas (2009), we will exploit below, both in the model and in the data.

Starting from (6), but projecting the equation on relative equity returns, we obtain the analogous expression for equilibrium bond holdings:

$$\tilde{B} = \frac{1}{2(1-\beta)} \frac{\beta}{Y} \frac{C \text{cov}_{R^S_t} (\hat{R}^{PC}_t, \hat{R}^B_t)}{\text{var}_{R^S_t} (\hat{R}^B_t)} - \frac{1}{2(1-\beta)} \frac{\beta}{Y} \frac{WL \text{cov}_{R^S_t} (\hat{R}^{WL}_t, \hat{R}^B_t)}{\text{var}_{R^S_t} (\hat{R}^B_t)},$$

(8)

where $\text{cov}_{R^S_t} (x, y)$ is the covariance between $x$ and $y$, conditional on relative equity returns, and $Y$ is annualised steady state output. Since net bond positions are assumed to be zero in this framework, the pure diversification term is absent. Other than that, the bond position in (8) is driven by the same hedging motives as the equity position in (7). The first term reflects the desire to hedge relative consumption expenditure risk using the bond position, while the second term reflects the motive to hedge human capital risk. Thus, positions in domestic bonds are larger, the more positively correlated relative bond returns are with relative consumption expenditure risk and the more negatively correlated relative bond returns are with human capital risk, conditional on relative equity returns. As with equity positions, the fact that the relevant covariances and variances are conditional on relative equity returns reflects that equity returns are also used to hedge some of the consumption expenditure and human capital risk.\(^{10}\)

\(^{10}\)The presence of the $\beta/(1-\beta)$ terms in the expression for bonds results from the fact that the maturity of bonds here is one period, while equity and the sources of risk do not mature.
Equations (7) and (8) will hold in the specific model studied below which features, inter alia, capital accumulation, nominal rigidities in price and wage setting, and incomplete markets. However, the expressions above are independent of the presence of these model features and in fact much more general than the models presented here.\footnote{11}

3 Model

There are two symmetric countries, Home ($H$) and Foreign ($F$), indexed by $i$. Each country is specialised in the production of a composite good using a continuum of country specific intermediate goods. Intermediate goods are produced using labour and capital and there are nominal rigidities in price and wage setting of the Calvo type. The factors of production and the intermediate goods are immobile between countries, but composite goods are traded. In addition to trade in composite goods, countries trade one period nominal bonds and equities and there are three aggregate sources of uncertainty per country: Neutral technology shocks, investment specific technology shocks and monetary shocks.

3.1 Households

Country $i$ is inhabited by a representative consumer with a utility function that is separable in consumption and labour:

$$U_i = \sum_{j=0}^{\infty} \beta_{t+j} \left( \frac{C_{i,t+j}^{1-\sigma} - \tilde{L}_{i,t+j}^{1+\omega}}{1-\sigma} \right),$$

where $\tilde{L}_{i,t}$ is labour supply and the distinction between labour demand and labour supply will be discussed in the context of optimal wage setting below. The intertemporal elasticity of substitution $\sigma$ and inverse of the Frisch labour supply elasticity $\omega$ are common across countries. Following Schmitt-Grohe and Uribe (2003), the discount factor $\beta$ is modelled as:

$$\beta_{t+j+1} = \beta_{t+j} \tilde{\beta} C_{i,t}^\eta; \beta_0 = 1,$$

where $0 < \eta < \sigma$ and $\tilde{\beta} C^\eta < 1$, and individual decision makers take $\eta$ as exogenous. If $\eta = 0$, we obtain the standard case of an exogenous and constant discount factor. By allowing $\eta < 0$, we will ensure that net foreign asset positions are stationary in our approximations.\footnote{12}

\footnote{11}The expressions can easily be adapted to allow for asymmetry between countries, government spending, financial frictions or a more extensive menu of assets. Similar expressions have been derived in a static context by Coeurdacier and Gourinchas (2009) and in a dynamic context with complete markets by Coeurdacier et al (2009), among others.

\footnote{12}We solve our model using linear approximations around the nonstochastic steady state. It is well known that at this level of approximation, net foreign asset positions display a unit root due to market incompleteness. Setting $\eta$ to a low, but positive value removes the unit root, while leaving our results

11
The consumption aggregator for country $i$ is defined as:

$$C_{i,t} = \left[ a^{1/\phi} \left( C_{i,t}^{i} \right)^{-\phi} + (1-a)^{1/\phi} \left( C_{j,t}^{i} \right)^{-\phi} \right]^{\phi}, \quad (11)$$

where $a > 1/2$ is the home bias parameter and $\phi$ is the elasticity of substitution between Home and Foreign goods. These preferences imply the following consumption price indices:

$$P_{C,t}^{i} = \left[ a \left( P_{i,t}^{j} \right)^{1-\phi} + (1-a) \left( P_{j,t}^{i} \right)^{1-\phi} \right]^{\frac{1}{\phi}}, \quad (12)$$

where $P_{i,t}^{j}$ is the price in country $i$ of the composite good produced in country $j$. All prices are quoted in terms of the local currency. The optimal allocation across consumption goods is then given by:

$$C_{i,t}^{i} = a \left( \frac{P_{i,t}^{i}}{P_{i,t}^{C,t}} \right)^{-\phi} C_{i,t}^{i}, \quad C_{j,t}^{i} = (1-a) \left( \frac{P_{j,t}^{i}}{P_{i,t}^{C,t}} \right)^{-\phi} C_{i,t}^{i} \quad (13)$$

where $C_{i,t}^{i}$ denotes consumption of good $j$ by agent $i$. Consumption of the Home and Foreign aggregate consumption goods are given by:

$$C_{j,t}^{i} = \left( \int_{0}^{1} (C_{j,t}^{i} (k))^{\frac{1}{\varepsilon-1}} dk \right)^{\varepsilon-1}, \quad (14)$$

where $C_{j,t}^{i} (k)$ is consumption of the $k$th intermediate good in country $j$ by the agent in country $i$ and $\varepsilon$ is the elasticity of substitution between varieties in consumption. Optimal consumption of Home and Foreign intermediate goods then implies:

$$C_{j,t}^{i} (k) = \left( \frac{P_{j,t}^{i}}{P_{j,t}^{i} (k)} \right)^{\varepsilon} C_{j,t}^{i}, \quad P_{j,t}^{i} = \left( \int_{0}^{1} P_{j,t}^{i} (k)^{1-\varepsilon} dk \right)^{\frac{1}{1-\varepsilon}} \quad (15)$$

where $P_{j,t}^{i}$ is the price in country $i$ of the composite good produced in country $j$. The real exchange rate $Q$ is defined as:

$$Q_{t} = \frac{Z_{t} P_{j,t}^{F}}{P_{i,t}^{C,t}} \quad (16)$$

virtually unaffected.
3.2 Capital Accumulation

At time $t$, each country possesses a capital stock $K_{i,t}$. Country specific capital stocks depreciate at rate $\delta$ and are augmented by country specific investment $I_{i,t}$:

$$K_{i,t+1} = (1 - \delta) K_{i,t} + \chi_{i,t} \Upsilon \left( \frac{I_{i,t}}{I_{i,t-1}} \right) I_{i,t},$$  \hspace{1cm} (17)

where investment in country $i$'s capital stock is a CES aggregate of the Home and Foreign composite good

$$I_{i,t} = \left[ a^{1/\phi_i} \left( \frac{I_{i,t}}{I_{i,t}} \right)^{\phi_i} + (1 - a)^{1/\phi} \left( \frac{I_{i,t}}{I_{i,t}} \right)^{\phi} \right]^{\frac{1}{\phi_i}}.$$ \hspace{1cm} (18)

$\chi_{i,t}$ is a shock to investment specific technology, as in Greenwood et al (1997, 2000) and Fisher (2002, 2006), and the function $\Upsilon \left( \frac{I_{i,t}}{I_{i,t-1}} \right)$ reflects investment adjustment costs, as in Christiano et al (2005) or Smets and Wouters (2008). Investment goods are modelled analogously to consumption goods, implying the following aggregators, investment allocations and price indices:

$$I_{j,t} = \left( \int_0^1 \left( \frac{I_{j,t}(k)}{I_{j,t}} \right)^{\frac{\phi_i}{\phi_i}} \right)^{\frac{1}{\phi_i}} I_{j,t}(k),$$ $I_{j,t} = \left( \frac{P_{i,t}}{P_{i,t}} \right)^{\frac{1}{\phi_i}},$$ \hspace{1cm} (19)

$$P_{i,t} = \left[ a \left( \frac{P_{i,t}}{P_{i,t}} \right)^{1-\phi} + (1 - a) \left( \frac{P_{i,t}}{P_{i,t}} \right)^{1-\phi} \right]^{\frac{1}{1-\phi}}.$$ \hspace{1cm} (20)

where we assume that substitution elasticities between individual varieties for investment and the price indices are equal to those for consumption.

At time $t$, the stock of capital in each country is rented out in a spot market at a rental rate $R_{i,t}$. Investment is then optimally chosen to satisfy the two following conditions:

$$-\lambda_{i,t} = \varpi_{i,t+1} R_{i,t+1} - (1 - \delta) \lambda_{i,t+1}$$ \hspace{1cm} (21)

$$P_{i,t} = -\lambda_{i,t} \chi_{i,t} \left( \frac{I_{i}}{I_{i,t-1}} \right)^{-\frac{1}{\phi_i}} - \varpi_{i,t+1} \chi_{i,t+1} \lambda_{i,t+1} \left( \frac{1}{v - 1} \left( \frac{I_{i,t+1}}{I_{i,t}} \right)^{\frac{\phi_i}{\phi_i}} + \frac{1}{v - 1} \right).$$ \hspace{1cm} (22)

where $v$ is the elasticity of the investment adjustment cost function, $\varpi_{i,t+1}$ is the stochastic discount factor of the country $i$ investor applied at time $t$ to discount date $t+1$ profits and $\lambda_i$ is the Lagrange multiplier on the law of motion for capital. In the case of no adjustment costs ($v \to \infty$), the two expressions collapse into the familiar condition that investment is chosen to equate the expected discounted payoff from investment to its marginal cost at

\[ \text{13The adjustment cost function is chosen to be $\Upsilon \left( \frac{I_{i,t}}{I_{i,t-1}} \right) = \frac{\alpha_1}{1-v} \left( \frac{I_{i,t}}{I_{i,t-1}} \right)^{1-1/v} + \alpha_2$ and the constants $\alpha_1$ and $\alpha_2$ are chosen to ensure that there are no adjustment costs in the steady state $\Upsilon(1) = \Upsilon'(1) = 1.$} \]
$$1 = \beta E_t \left[ \varpi_{t+1}^{i} \frac{P_{i,t+1}^{K}}{P_{i,t}^{L}} \left( R_{i,t}^{K} + (1 - \delta) \frac{P_{i,t+1}^{i}}{\chi_{i,t+1}} \right) \right]. \quad (23)$$

### 3.3 Firms

Each country contains a continuum of firms, each producing a differentiated intermediate good, indexed by $k$, using capital and labour. The production function is given by:

$$Y_{i,t}(k) = A_{i,t} \left( L_{i,t}(k) \right)^{\alpha} \left( K_{i,t}(k) \right)^{1-\alpha}, \quad (24)$$

where $A_{i,t}$ is the exogenous level of productivity in country $i$ and $\alpha$ is the elasticity of output with respect to labour. Firms maximise the present discounted value of profits and choose labour and capital in aggregate country specific spot markets to minimise the cost of production. Firms face the same wages and rental rates of capital and, consequently, all firms in a given country choose the same labour to capital ratio:

$$\frac{K_{i,t}(k)}{L_{i,t}(k)} = \frac{1 - \alpha}{\alpha} \frac{W_{i,t}}{R_{i,t}^{K}}. \quad (25)$$

Total demand for each intermediate good is composed of domestic and foreign demand for consumption and investment. Firms take this demand as given and choose prices in local currency in Home and Foreign to maximise profits. However, with probability $\theta$, they cannot reset prices in the current period (Calvo (1983)). A firm reoptimising in period $t$ will then choose a price $\tilde{P}_{i,t}^{H}$ that maximises the current market value of profits generated while the price remains in effect, taking all other prices as given. Optimal prices for Home firms are then:

$$\tilde{P}_{H,t}^{H}(k) = \frac{\varepsilon}{\varepsilon - 1} \frac{\varpi_{t+1}^{H}(P_{H,t+1}^{H})^{\varepsilon} Y_{H,t+1}^{H}}{\varpi_{t+1}^{H}(P_{H,t+1}^{H})^{\varepsilon} Y_{H,t+1}^{H}} \quad (26)$$

$$\tilde{P}_{H,t}^{F}(k) = \frac{\varepsilon}{\varepsilon - 1} \frac{\varpi_{t+1}^{H}(P_{H,t+1}^{F})^{\varepsilon} \varrho_{H,t} Y_{H,t+1}^{F}}{\varpi_{t+1}^{H}(P_{H,t+1}^{F})^{\varepsilon} \varrho_{H,t} Y_{H,t+1}^{F}} \quad (27)$$

where $Y_{i,t}^{i}(k) = C_{i,t}(k) + I_{i,t}(k), Y_{i,t}^{j}(k) = C_{i,t}(k) + P_{i,t}(k), \varrho_{H,t}$ is the marginal cost function of a firm in country $i$, and foreign prices follow analogously. This expression implies that optimal prices are set as a markup over expected marginal costs over the expected lifetime of the price set, and collapses to the familiar static condition $\tilde{P}_{H,t}^{H}(k) = \frac{\varepsilon}{\varepsilon - 1} \varrho_{H,t}$, if prices are perfectly flexible. The nature of price rigidities implies that all firms will choose the same price when reoptimising. The price indices for Home and Foreign goods then evolve as:

$$P_{j,t}^{i} = \left( \theta \left( P_{j,t-1}^{i} \right)^{1-\varepsilon} + (1 - \theta) \left( \tilde{P}_{j,t}^{i} \right)^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}} \quad (28)$$
3.4 Wage Setting

As in Schmitt Grohe and Uribe (2007), we assume that households supply labour monopolistically to a continuum of labour markets, indexed by \( j \). There is a single union for each labour market and firms, indexed by \( k \), combine labour inputs from each of these different labour markets according to the following aggregator:

\[
L_{i,t}(k) = \int_0^1 L_{i,t}(j,k)^{1-1/\varepsilon_w} \, dj
\]  

Optimal firm decisions imply that the demand by firm \( k \) for labour from sector \( j \) is given by:

\[
L_{i,t}(j,k) = \left( \frac{W_{i,t}(j)}{W_{i,t}} \right)^{-\varepsilon_w} L_{i,t}(k),
\]  

where \( L_{i,t}(k) \) is total labour demand of firm \( k \) and \( W_{i,t}(j) \) are nominal wages in sector \( j \).

Aggregating across all sectors and assuming that unions satisfy demand at the going wage, we have that total labour supply of type \( j \) is:

\[
\tilde{L}_{i,t}(j) = \left( \frac{W_{i,t}(j)}{W_{i,t}} \right)^{-\varepsilon_w} \int_0^1 L_{i,t}(k) \, dk = \left( \frac{W_{i,t}(j)}{W_{i,t}} \right)^{-\varepsilon_w} L_{i,t}.
\]  

Aggregating across all labour markets, we arrive at total labour supply:

\[
\tilde{L}_{i,t} = \int_0^1 \tilde{L}_{i,t}(j) \, dj = L_{i,t} \int_0^1 \left( \frac{W_{i,t}(j)}{W_{i,t}} \right)^{-\varepsilon_w} \, dj^{1/\varepsilon_w}.
\]  

The nominal wage index \( W_{i,t} \) is given by:

\[
W_{i,t} = \left( \int_0^1 (W_{i,t}(j))^{1-\varepsilon_w} \right)^{1/\varepsilon_w}
\]

and has the property that it is the minimum cost of a bundle of intermediate labour inputs yielding one unit of composite labour. Similar to wage setting, we assume that each period the union cannot reset the wage optimally in a random fraction \( \theta_w \) of labour markets. It can then be shown that unions that can reoptimise the wage at time \( t \) will choose the same wage, \( \tilde{W}_{i,t}(j) = \tilde{W}_{i,t} \),

given by:

\[
\tilde{W}_{i,t} = \frac{\sum_{l=0}^\infty \beta_{i,t+l} \theta_w l \left( -\frac{C_{i,t+l}}{p_{i,C,t+l}} \right) W_{i,t}^{\varepsilon_w} L_{i,t+l} \left( -\frac{L_{i,t+l}}{p_{i,C,t+l}} \right) - \tilde{L}_{i,t} \omega_i}{\sum_{l=0}^\infty \beta_{i,t+l} \theta_w l \left( -\frac{C_{i,t+l}}{p_{i,C,t+l}} \right) W_{i,t}^{\varepsilon_w} L_{i,t+l}}.
\]

This expression states that the union chooses the wage as a markup of expected average marginal revenues over the expected average cost of supplying labour effort.\(^{15}\) Aggregate

\(^{15}\)Without nominal wage rigidities, this expression would collapse to: \( W_{i,t} = \frac{\varepsilon_w}{\varepsilon_w - 1} \tilde{L}_{i,t} C_{i,t}^{\varepsilon_w} P_{i,C,t}^L \), implying that wages are set at a markup over the marginal rate of substitution between consumption and labour.
wages then follow the law of motion:

$$W_{i,t}^{1-\varepsilon_w} = \theta_w W_{i,t-1}^{1-\varepsilon_w} + (1 - \theta_w) \tilde{W}_{i,t}^{1-\varepsilon_w}. \quad \text{(34)}$$

### 3.5 Monetary Authority

We assume that the monetary authority responds to CPI inflation and output growth and sets the nominal interest rate, $R_{i,t}^N$, according to:

$$R_{i,t}^N = (R_{i,t}^N)^{\rho_r} \left( \frac{1}{\beta} \left( \frac{P_{C,t}^i}{P_{C,t-1}^n} \right)^{\gamma_\pi} \left( \frac{Y_{i,t}}{Y_{i,t-1}} \right)^{\gamma_y} M_{i,t} \right)^{1-\rho_r}, \quad \text{(35)}$$

where $\rho_r$ is the degree of interest rate smoothing and $\gamma_\pi$ and $\gamma_y$ parametrise the response of the monetary authority to fluctuations in CPI inflation and output, respectively. $M_{i,t}$ is a mean zero shock to interest rates. The monetary rule here is similar to the one used in Del Negro and Schorfheide (2009). It is a generalisation of the rule used in Devereux and Sutherland (2007, 2008) by allowing monetary policy to respond to output growth in addition to inflation and by allowing for interest rate smoothing.\(^\text{16}\) It is worth noting that monetary policy is not optimal here, as it does not necessarily respond in an efficient manner to deviations of inflation and output from their natural levels.

### 3.6 Financial Markets

There is trade in Home and Foreign nominal bonds and Home and Foreign equity. Nominal bonds pay one unit of the domestic currency in the following period and are in net zero supply. Owners of equity of country $i$ receive a claim to country $i$ dividends $D_{i,t}$ which are composed of profits earned by the firm plus the receipts from renting out capital, minus investment spending:

$$D_{i,t} = \Pi_{i,t} + R_{i,t}^K K_{i,t} - P_{I,t}^i I_{i,t}. \quad \text{(36)}$$

Gross nominal returns in domestic currency for bonds and equity are:

$$R_{i,t+1}^S = \frac{D_{i,t} + P_{i,t+1}^S}{P_{i,t}^S}, \quad R_{i,t+1}^B = \frac{1}{P_{i,t}^B} \quad \text{(37)}$$

where $R_{i,t}^S (R_{i,t}^B)$ is the return on holdings of country $i$ equity (bonds) and $P_{i,t}^S (P_{i,t}^B)$ are the prices of country $i$ equity (bonds). The total supply of equity in each country is normalised to unity and it is assumed that in period 0 each household owns the stock of domestic equity. Asset prices are then determined by the usual pricing equations. For the Home investor,\(^\text{16}\) Another difference is that monetary policy responds to PPI inflation in Devereux and Sutherland (2007, 2008), while it responds to CPI inflation here.
these are:

\[
E_t \left[ \varpi_{H,t+1} \left( \frac{P_{C,t}}{P_{H,t+1}} R_{H,t+1}^S \right) \right] = 1 \quad E_t \left[ \varpi_{H,t+1} \left( \frac{P_{C,t}}{P_{H,t+1}} R_{H,t+1}^B \frac{Z_{t+1}}{Z_t} \right) \right] = 1
\]  \quad (38)

and the pricing equations for the Foreign investor are analogous. The budget constraint for the Home investor in period \( t \) is then identical to (1), the budget constraint used to derive our reduced form expression above:

\[
S_{H,t}^H P_{H,t}^S + S_{F,t}^H P_{F,t}^S Z_t + B_{H,t}^H P_{H,t}^B + B_{F,t}^H P_{H,t}^B Z_t = W_{H,t} L_{H,t} + S_{H,t-1}^H (D_{H,t} + P_{H,t}^S) + S_{F,t-1}^H (D_{F,t} + P_{F,t}^S) Z_t + B_{H,t-1}^H + B_{F,t-1}^H Z_t - P_{C,t}^H C_{H,t}
\]  \quad (40)

where \( S_{i,t}^j \) (\( B_{i,t}^j \)) are holdings of country \( j \) equity (bonds) by country \( i \) in period \( t \). The budget constraint of the Foreign agent can be written analogously.

3.7 Market Clearing and Exogenous Processes

The market clearing condition for goods are given by:

\[
C_{H,t}^H + C_{F,t}^H + I_{H,t}^H + I_{F,t}^H = Y_{H,t} \quad C_{F,t}^H + C_{F,t}^F + I_{F,t}^H + I_{F,t}^F = Y_{F,t}
\]  \quad (41)

For assets, we have:

\[
S_{H,t}^H + S_{F,t}^H = 1 \quad S_{H,t}^F + S_{F,t}^F = 1
\]  \quad (42)

\[
B_{H,t}^H + B_{F,t}^H = 0 \quad B_{F,t}^H + B_{F,t}^F = 0
\]  \quad (43)

There are three sources of uncertainty per country, neutral and investment specific technology shocks, and monetary shocks, and each is assumed to follow a first order autoregressive process in logarithms.

\[
\log (A_{i,t+1}) = \rho_A \log (A_{i,t}) + \varepsilon_{A,i,t+1},
\]  \quad (44)

\[
\log (\chi_{i,t+1}) = \rho_\chi \log (\chi_{i,t}) + \varepsilon_{\chi,i,t+1}
\]  \quad (45)

\[
\log (M_{i,t+1}) = \rho_M \log (M_{i,t}) + \varepsilon_{M,i,t+1}
\]  \quad (46)

where \( \varepsilon_{j,i,t+1} \) is a mean zero i.i.d shock and \( \rho_j \) governs the persistence of the autoregressive process.
3.8 Equilibrium and Solution Method

An equilibrium in this economy consists of a set of processes for prices, \( P_{C,t}, P_{F,t}, P_{H,t}, P_{I,t}, P_{F}, P_{H}, P_{F}, P_{H}, P_{F}, P_{H}, P_{F}, P_{H}, P_{F}, P_{H}, \) wages \( W_{H,t}, W_{F,t}, \) rental rates for capital \( R_{K}, R_{K} \), nominal interest rates, \( R_{N}, R_{N} \) and the exogenous processes \( A_{H,t}, A_{F,t}, \chi_{H,t}, \chi_{F,t}, M_{H,t}, M_{F,t} \) such that, taking these as given, the following set of conditions are satisfied for all \( t \geq 0 \):

1. the first order conditions for intermediate goods purchases for consumption and investment, (13) and (19)
2. the first order conditions for capital accumulation, (21) and (22)
3. the optimal labour and capital hiring decisions of firms, (25)
4. the optimal price setting conditions for firms, (26) and (27)
5. the optimal wage setting conditions for unions, (33)
6. the first order conditions for stock purchases by households, (38) and (39)
7. the households’ budget constraints, (40)
8. the market clearing conditions for goods, (41)
9. the market clearing conditions for assets, (42) and (43).

We solve for a linear approximation to this equilibrium around the nonstochastic steady state, applying the methods developed by Devereux and Sutherland (2009a). One key insight of their work is that a first order accurate approximation of the dynamics of this class of models only depends on the non time varying part of portfolios (“steady state portfolios”). Standard linearisation approaches cannot solve for these portfolios, for two reasons. Firstly, optimal portfolios are not uniquely pinned down in a nonstochastic steady state, so there is no natural point around which to approximate. Secondly, optimal portfolios are also not uniquely defined in a first order approximation of a DSGE model, as it satisfies certainty equivalence and implies that all assets are perfect substitutes. The methods described in Devereux and Sutherland (2009a) show how to overcome these problems. Firstly, building on earlier work by Samuelson (1970), and Judd and Guu (2001), the authors demonstrate that the steady state portfolio corresponds to a bifurcation point in the set of nonstochastic equilibria. Secondly, they show that we need to combine a second order accurate approximation of the portfolio euler equations, (38) − (39), with a first order accurate approximation of the rest of the model in order to arrive at a solution for steady state portfolios.
4 Portfolios and Business Cycles in an Open Economy DSGE Model

In this section, we calibrate the model and present optimal portfolios and business cycle statistics. This section also discusses the intuition behind the portfolio positions and in particular, the role of nominal rigidities, endogenous capital accumulation, and monetary policy. We find that optimal equity positions in the steady state feature home bias of a realistic size and that this home bias is driven mainly by the motive to hedge human capital risk, and less so by the motive to consumption expenditure risk. Relative equity returns are unconditionally positively correlated with human capital returns, but negatively, once we condition on relative bond returns. Steady state positions in Home bonds are positive and equal to roughly 50% of GDP and the model generates realistic cross country correlations in output, consumption, investment and labour.

4.1 Calibration

We adopt a benchmark calibration that closely follows the literature on international business cycles and portfolios (e.g. Backus et al. (1994), Chari et al. (2002), Coeurdacier et al. (2007, 2009), Heathcote and Perri (2009)) and present our parameter choices in Table 2. The model is assumed to run at quarterly frequency. We choose $\tilde{\beta}$ and $\eta$ such that the steady state discount rate is equal to 0.99, implying an annual steady state real interest rate of 4.1%. The degrees of consumption and investment home bias are set to $a = 0.85$, implying a steady state import/GDP ratio of 15%. There is great variety in the estimates of the elasticity of substitution between Home and Foreign goods. Macroeconomic studies often estimate values close to, and sometimes below, one, while microeconomic estimates are much larger (see Harrigan (1993) and Hummels (2001) for micro studies and Heathcote and Perri (2002) and Backus and et al (1994) for macro estimates. See also Imbs and Mejean (2009) and the discussion in Coeurdacier (2009)). We follow the macro literature and set the elasticity of substitution between Home and Foreign goods to 1.1. The elasticity of substitution between individual varieties of intermediate goods, $\varepsilon$, and individual labour varieties, $\varepsilon_w$, is set to 10. The former implies an average markup of 11% of prices over marginal costs, while the second implies an average markup of 11% of wages over the the marginal rate of substitution between consumption and labour. The intertemporal elasticity of substitution, $\sigma$, is set to 2, in line with much of the domestic and open economy business cycle literature (e.g. Backus et al (1992, 1994) or Coeurdacier et al (2009)). The nominal stickiness parameter for prices, $\theta$, is set to 0.7, which implies an average duration for prices of 3.3 quarters. The analogous parameter for wages, $\theta_w$, is set to 0.8, as the degree of wage rigidity is often found to be larger than the degree of price rigidity (see e.g. Christiano et al. (2005)). The rate of depreciation is set to $\delta = 0.025$ implying an annual rate of depreciation of around 10%. The elasticity of output with respect to labour, $\alpha$, is set to 0.78

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17The appendix also discusses the role of bond maturity and real bonds.
Table 2: Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Interpretation</th>
<th>Value</th>
</tr>
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<td>$\beta$</td>
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</tbody>
</table>

which, together with the other parameters, implies a steady state labour share of 0.70, the mean value for the US from 1970 – 2008. $\omega$ is set to zero, implying that utility is linear in labour. The investment adjustment cost parameter, $\nu$, is set to $\nu = 0.9$. For the monetary policy rule, we use the estimation results of Canzoneri et al. (2007) for the US which imply $\rho_r = 0.824$, $\gamma_m = 2.02$, $\gamma_Y = 0.184$ and $\sigma_M = 0.00245$ and assume that monetary shocks across countries are not correlated. For the neutral technology process, we rely on Heathcote and Perri (2002) who estimate an AR(1) processes for TFP for US and rest of the world and obtain $\rho_A = 0.91$, $\sigma_A = 0.006$ and a cross country correlation of productivity shocks, $\rho_{A,A}$, of 0.25. For investment specific technology, we follow Fisher (2006) and Coeurdacier et al (2009) and use the ratio of CPI to the investment price deflator as an estimate of the investment shock. Estimating AR(1) processes for the US and rest of the world investment shock, we then obtain $\sigma_\chi = 0.0051$, $\rho_\chi = 0.83$ and $\rho_{\chi,\chi} = 0.21$, where the latter denotes the correlation of investment specific technology shocks between the two countries.18

18See the appendix for further details on data sources and the estimation of investment specific technology.
4.2 International Bond and Equity Portfolios

In Table 3, we present equity and bond portfolios for our benchmark calibration, as well as the contributions of the two motives to hedge risks to these positions, and the unconditional correlations between asset return innovations and sources of risk. The fraction of U.S. equity held by foreign investors is almost perfectly matched by the model. According to the U.S. Treasury Report on Foreign Portfolio Holdings of U.S. Securities (2009), foreign investors were holding 10.4 percent of the U.S. equity stock in 2008 and the report shows that this proportion has been stable in recent years. Our reduced form expressions, (7) and (8), allow us to make some further observations. The covariance variance ratios in these expressions can be recovered by simulating the model and running the following two regressions on the simulated data:

\[
\hat{R}_{WL}^{t+1} = k_1 + \beta_{wl,b} \hat{R}_{B}^{t+1} + \beta_{wl,s} \hat{R}_{S}^{t+1} + \epsilon_{wl}^t
\]

\[
\hat{R}_{PC}^{t+1} = k_2 + \beta_{pc,b} \hat{R}_{B}^{t+1} + \beta_{pc,s} \hat{R}_{S}^{t+1} + \epsilon_{pc}^t
\]

where \(k_1, k_2\) are constants and \(\epsilon_{PC}^t, \epsilon_{WL}^t\) are the error terms of the regression. Substituting the steady state values and regression coefficients into (7) and (8), we can decompose the observed asset positions into the parts induced by the human capital and consumption expenditure motive, respectively. We observe that, in the model, equity home bias is driven mainly by the motive to hedge human capital risk, and to a much smaller degree the motive to hedge consumption expenditure risk. Interestingly, the motive to hedge human capital risk induces U.S. investors to hold U.S. equity despite the fact that the unconditional correlation between relative equity return innovations and human capital risk is positive.

One argument against the hypothesis that equity home bias may arise from a motive to hedge human capital returns has been that the unconditional correlation between equity returns and human capital returns is positive in the data (Baxter and Jermann (1997)). However, as mentioned above, it is the correlation between the two, conditional on relative bond returns, that matters.\(^\text{19}\) Previous work has shown that the conditional correlation between the two may well be negative in the data and we provide some further evidence below. To our knowledge, however, ours is the first contribution that presents a model that can replicate the fact that the unconditional correlation between relative equity returns is positive, while the conditional one is negative.

Steady state holdings of Home bonds are 50% of GDP, with the equivalent short position in Foreign bonds. Available data indicates that, on aggregate, U.S. investors have a short position in U.S. bonds of between 35 and 45% of GDP in 2008.\(^\text{20}\) Our benchmark model

\(^\text{19}\) This point has also been emphasized by, amongst others, Coeurdacier et al (2009), Coeurdacier and Gourinchas (2009) and Engel and Matsumoto (2009).

thus predicts a bond position of a realistic magnitude, but not of the right sign for the U.S. However, Lane and Shambaugh (2007, 2008) show that developed economies, on average, have negative net foreign-currency debt positions, in line with the prediction of our benchmark model. The U.S. is a notable exception and it can plausibly be argued that its outlier status is due to a number of features that are not captured by the model, such as the special role of the dollar as a reserve and vehicle currency. Comparison of the model implied bond positions with their real life counterpart is complicated by several other factors. Firstly, net positions in forward currency markets are equivalent to bond positions in the model and the correct measure of bond positions in the data should therefore net out positions in bond and currency forward markets, but data on the latter is scarce. Secondly, the model features net zero bond positions and firms are fully financed by equity. Coeurdacier et al (2009) show that when firms are partly financed by debt and the Modigliani-Miller theorem applies, agents should hold the same proportion of foreign corporate debt as of foreign equity. With positive foreign equity exposures, this would imply higher gross holdings of foreign debt and thereby reduced net debt positions in Home bonds. Table 3 also indicates the drivers of bond positions. Relative bond returns are positively correlated with human capital and consumption expenditure risk, both unconditionally and conditioning on relative equity returns. Thus, the two motives are offsetting, with the consumption expenditure motive inducing agents to take a long position in Home bonds, while the human capital motive induces Home investors to go short in Home bonds. The consumption expenditure motive prevails, leading to an overall short position in Home bonds. Below, we will provide evidence that the conditional and unconditional correlations between bond returns and human capital and consumption expenditure risk in the model qualitatively match those in the data for the U.S. However, the consumption expenditure motive is stronger in the model, thereby implying the overall long position in U.S. bonds, while the data indicates that the human capital motive dominates and thus leads to an overall short position in U.S. bonds for U.S. investors.

4.3 Discussion and Robustness

In our framework, relative bond and equity returns, and consumption and human capital risk are positively correlated. This is because a relatively low elasticity of substitution and home bias in consumption imply that nominal exchanges respond quite strongly to shifts in supply and demand. Nominal exchange rates in turn contribute to the comovement of asset returns and sources of risk. In Figure 1, we present the impulse responses of relative equity and bond returns and relative human capital and consumption expenditure risk to positive

the Gourinchas and Rey (2007), kindly provided by Hélène Rey, estimates the U.S. short position at 6550 billion U.S. dollars or 46% of GDP.
Table 3: International Equity and Bond Portfolios and Correlations

<table>
<thead>
<tr>
<th>a) Portfolios</th>
<th>Share of Home Equity</th>
<th>Home Bonds/GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>0.90</td>
<td>0.49</td>
</tr>
<tr>
<td>due to:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pure Diversification</td>
<td>0.50</td>
<td>0.00</td>
</tr>
<tr>
<td>Human Capital</td>
<td>0.34</td>
<td>-0.76</td>
</tr>
<tr>
<td>Consumption Expenditure</td>
<td>0.06</td>
<td>1.24</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>b) Correlations</th>
<th>( \hat{R}^S )</th>
<th>( \hat{R}^B )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{R}^{WL} )</td>
<td>0.25</td>
<td>0.96</td>
</tr>
<tr>
<td>( \hat{R}^{PC} )</td>
<td>0.49</td>
<td>0.99</td>
</tr>
</tbody>
</table>

Contributions of hedging motives and correlations were computed relying on 500 simulations of the benchmark model for 500 periods. Contributions of hedging motives were calculated according to the reduced form expressions (7) and (8) and using the coefficients obtained from running the regressions (47) and (48) on the simulated data. Relative bond and equity returns and human capital and consumption expenditure risk were Hodrick Prescott filtered with a smoothing parameter of 1600 before computing correlations.

The first column of the figure indicates that equity and bond returns, as well as human capital and consumption expenditure risk fall in response to a relative productivity shock, while the third columns shows that all four variables increase in response to a contractionary monetary shock. Figure 2, which plots the responses of relative dividends, nominal exchange rates, relative labour income and relative consumption expenditures, indicates why. Comparing the responses in the first column of Figure 1 with the second column of Figure 2, we can see that the asset returns and sources of risk move in the same direction as the nominal exchange.

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21 As noted before, the symmetry of the model implies that we can focus on the response of relative variables without loss of generality. Similarly, we can focus on the response of relative shocks whereby, e.g. a relative Home productivity shock is defined as a positive one standard deviation shock to Home productivity, with a negative shock to Foreign productivity of equal size. Note that a positive shock to the interest rate implies a contractionary monetary policy shock.

22 Note that the fact that all of the variables here are defined as innovations implies that their impulse responses will be zero after one period.

23 Equations (2) – (5) indicated that relative equity returns depend on the present value of relative dividend innovations, relative bond returns depend on the nominal appreciation in the following period, and human capital and consumption expenditure risk depend on the present value of relative labour income and consumption expenditure innovations, respectively. In the appendix, we also provide the impulse responses for output, consumption, investment, labour demand, the real exchange rate and net exports.
rate which depreciates for about three years after a relative productivity shock. Comparing the third column of the two figures, we see that the response of asset returns and sources of risk after a relative shock to investment efficiency again follows the nominal exchange rate, which in this case appreciates, further illustrating its role in driving comovement between asset returns and sources of risk.

While these figures mainly illustrate why many variables are unconditionally positively correlated, they also already include a hint for why equity home bias is driven by the motive to hedge human capital risk in our model. As we can see, the one exception to the general tendency of comovement occurs in response to relative shocks to investment efficiency. Relative equity returns fall strongly, while relative bond returns and consumption expenditure risk decrease only slightly, and human capital returns increase strongly. The responses of relative equity and human capital returns are driven by the combination of endogenous capital accumulation and nominal price rigidities. In response to shocks to investment specific technology, the demand for investment increases which, with home bias in investment, implies an increased demand for Home labour. Firms would like to react to the increase in demand by raising prices, but due to nominal rigidities, not all firms can do so. The limited rise in prices implies that the increase in the demand for investment is higher than it would have been if prices were flexible and the corresponding increase in labour demand raises relative human capital returns. Payouts to shareholders fall, as the rise in investment is paid for by retained earnings, and the fact that the investment boom is even larger with price rigidities implies that relative equity returns fall strongly. Thus, conditional on the investment efficiency shock, relative equity and human capital returns are negatively correlated. The positive comovement in response to productivity and monetary shocks implies an overall positive correlation between equity and human capital returns. But since bond returns only fall slightly in response to investment specific shock, the correlation of human capital returns with bond returns is more strongly positive overall than with equity returns.

Above we stressed several times that it is the comovement of relative equity returns with human capital returns, conditional on relative bond returns, that drives equity positions. One way to investigate this relationship more directly is to consider several combinations of shocks that imply that relative nominal bond returns are unchanged. Since nominal bond returns are by construction zero in these scenarios, there is no distinction between the unconditional comovement of equity and human capital returns and the correlation, conditional on relative bond returns. Since markets are incomplete in our setup, the combination of shocks that implies a zero response of relative bond returns is not unique. However, for each pair of shocks, the combination is unique up to a scalar multiplication, and we present the three combinations in Figure 3. For example, Figure 1 indicates that relative bond returns fall by 0.5 percent in response to a one standard deviation shock to relative productivity, while they fall by 0.014 percent in response to an investment specific technology shock. A combination of a one standard deviation positive relative shock to productivity and a

---

24Note that a rise in the nominal exchange rate here denotes a depreciation.
negative relative shock to investment specific technology of $0.5/0.014 = 36.79$ standard deviations therefore leaves relative bond returns unchanged. The response of relative equity returns and relative human capital returns to such a combination of shocks is depicted in the first column of Figure 3. It clearly shows that the two are negatively correlated. The second and third column present the responses to combinations of investment efficiency and interest rate shocks, and productivity and interest rate shocks, respectively. Both of them also show a negative comovement of relative equity returns with relative human capital returns. Thus, the conditional correlation of human capital returns and equity returns, conditional on relative bond returns, is negative for all three shocks, and therefore overall. Figure 3 only focuses on the conditional comovement of relative equity returns with relative human capital returns. The reason is that for the comovement of relative equity returns with consumption expenditure risk, and for the comovement of relative bond returns with both human capital and consumption expenditure risk, the distinction between the conditional and unconditional moments is not too substantial. The correlation between relative equity returns and consumption expenditure risk is positive, and induces Home investors to take long positions in Home equity beyond those induced by the human capital motive, but the effect is small. The reason is that relative bond returns are highly positively correlated with consumption expenditure risk and a long position in Home bonds therefore hedges consumption expenditure risk effectively. The correlation between relative bond returns and relative human capital returns is also positive, and this motive therefore induces the Home investor to take short positions in Home bonds, and thereby partly offsets the effects of the consumption expenditure motive on bond positions.

4.3.1 The Role of Nominal Rigidities

In order to highlight the effects of nominal rigidities in our framework, the second column of Table 4 presents a version of the model without nominal price rigidities (“Flex Price”)

25 Nominal price rigidities affect portfolios through two mechanisms here. Firstly, the bonds traded are nominal and, as mentioned above, relative bond returns are related to fluctuations in the nominal exchange rate. Higher degrees of nominal rigidity imply that fluctuations in nominal exchange rates are more closely related with fluctuations in real exchange rates which are in turn closely associated with fluctuations in human capital and consumption expenditure risk. The Flex Price model therefore features a lower correlation between relative bond returns and human capital and consumption expenditure risk and lower nominal bond positions.

Secondly, both the human capital motive and the consumption expenditure motive for holding domestic equity are affected. The previous section discussed how nominal price rigidities imply that investment and labour demand respond strongly to shocks to investment specific technology and that the strong investment and labour response induces relative equity and human capital returns to comove negatively in response to this shock. Without

\[ \theta = 0. \]

Note that despite the absence of nominal price rigidities, monetary shocks are not neutral, as there are still nominal rigidities in wages.
price rigidity, the investment and labour response is correspondingly smaller and, relative
human capital returns fall as a result, and relative equity returns also fall. Both the con-
ditional and the unconditional correlation of equity returns with human capital returns are
then positive which implies that the human capital motive induces Home agents to take a
short position in Home equity in this case. It is worth pointing out here that the effect
of nominal rigidities on equity portfolios is different from the one in Engel and Matsumoto
(2009). In their model, there are nominal price rigidities, but there is no endogenous capital
accumulation. As in our model, a sufficient degree of price rigidities implies that relative
human capital returns decrease in response to a productivity shock. This is because a pro-
ductivity shock implies that less workers are needed to produce a given level of output.
Output increases, but nominal price rigidities prevent some firms from lowering prices in
order to stimulate demand. If price rigidities are large enough, relative labour income falls
persistently, and therefore relative human capital returns fall. But unlike in our model,
relative equity returns rise, both due to the increase in productivity, as well as a fall in the
wage bill, and relative equity and human capital returns are therefore negatively correlated.
In our model, relative equity returns fall, because the increase in endogenous investment
implies a lower payout to shareholders. In our setting, equity and human capital returns are
negatively correlated in response to investment specific technology shocks. Another differ-
ence to Engel and Matsumoto (2009) is that monetary policy responds endogenously in our
model, whereas money supply was exogenous in their setting and therefore may magnify
the importance of nominal rigidities. We show here that price rigidities remain effective,
even in an environment where monetary policy is endogenous and responds actively, if not
optimally, to inflation.

Table 4 shows that the equilibrium equity portfolio with flexible prices is nevertheless
strongly home biased and that is because of the effect of price flexibility on the consumption
expenditure motive for holding domestic equity. Since with price flexibility nominal bonds
are no longer as effective in hedging consumption expenditure risk, Home agents now take
a large long position in Home equity in order to hedge this source of risk.

Nominal wage rigidities have less effects on portfolios than nominal price rigidities. The
reason is that the presence of nominal wage rigidities does not translate into markedly
different behaviour of labour income. The limited movement in wages due to wage rigidity
is counteracted by larger variation in the demand for labour, leaving overall labour income
relatively little affected.\textsuperscript{26}

4.3.2 The Role of Capital Accumulation

In the third column of Table 4 we also present portfolios for a model without capital accu-
cumulation (“Fixed Capital”).\textsuperscript{27} In Heathcote and Perri (2009) and Coeurdacier et al (2009),

\textsuperscript{26}Portfolios for a model in which nominal wages are perfectly flexible ($\theta_w = 0$), but that is otherwise
identical to the benchmark model are presented in the appendix.

\textsuperscript{27}In this model, we fix the capital stock by setting investment and the depreciation rate to zero at all
times.
endogenous capital accumulation drives the negative comovement between equity returns and human capital returns. As we can see in the table, this is not the case in our benchmark model. Equity returns are negatively correlated with human capital returns, both conditionally and unconditionally in the Fixed Capital model, despite the absence of endogenous capital accumulation. The reason is that now equity and human capital returns are negatively correlated in response to a productivity shock. The mechanism is essentially the same as in Engel and Matsumoto (2009) which was discussed in the last section. Importantly, this model implies that equity and human capital returns are both conditionally and unconditionally negatively correlated, unlike in the data. The role of endogenous capital accumulation is therefore to generate an unconditionally positive correlation between relative equity and human capital returns, while nominal rigidities generate a negative correlation between relative human capital returns and equity returns, conditional on relative bond returns.

Despite the negative correlation between equity and human capital returns, the overall equity position here exhibits strongly negative equity home bias. This is because equity returns are now strongly negatively correlated with consumption expenditure risk, unless price stickiness is large, and this induces the Home investor to take a large short position in Home equity.\(^{28}\)

Relaxing investment adjustment costs has only small effects on portfolios here. The main effect of adjustment costs on portfolios here is to drive a wedge between equity returns and consumption expenditure returns. Removing investment adjustment costs thus increases the correlation between equity and consumption expenditure risk, but the effects are modest on bond and equity portfolios.\(^{29}\)

### 4.3.3 The Role of Monetary Policy

We model monetary policy using a Taylor type rule, as is standard in the New Keynesian literature (see Gali (2007) and Woodford (2003) for excellent introductions). We already noted that monetary policy is endogenous in our benchmark calibration, and it responds actively, if not optimally, to inflation and output growth. While it does not undo the effects

\(^{28}\) At high degrees of nominal rigidity, we obtain equity home bias of a realistic size in this model also. This is because the correlation of consumption expenditure falls with regard to equity returns and rises with regard to bond returns, therefore reducing the effect of the consumption expenditure motive on equity holdings. Engel and Matsumoto (2009) obtain equity home bias with more modest levels of price rigidity in a model without endogenous capital accumulation. Our results are not directly comparable to theirs, since they assume a different form of price stickiness, but most likely the fact that money supply is exogenous in their framework implies that lower degrees of price rigidity are necessary to generate equity home bias.

\(^{29}\) In the appendix, we also present a model calibration that features neither nominal rigidities in wage and price setting, nor investment adjustment costs \((\theta = \theta_w = 0 \text{ and } \nu \to \infty)\). This calibration can qualitatively produce positive unconditional and negative conditional correlations between equity returns and human capital returns. However, it cannot produce home bias of a reasonable size, as the effect of the human capital motive is too small, while equity returns are highly correlated with consumption expenditure risk. What is more, the lack of investment adjustment costs implies negative cross country correlations in investment.
Table 4: The Role of Nominal Rigidities, Capital Accumulation and Monetary Policy for Portfolios

<table>
<thead>
<tr>
<th>Share of Home Equity</th>
<th>Benchmark (I)</th>
<th>Flex Price (II)</th>
<th>Fixed Capital (III)</th>
<th>Hawk (IV)</th>
<th>Dove (V)</th>
<th>High $\sigma_M$ (VI)</th>
<th>Low $\sigma_M$ (VII)</th>
<th>Currency Union (VIII)</th>
</tr>
</thead>
<tbody>
<tr>
<td>due to:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Human Capital</td>
<td>0.36</td>
<td>-0.21</td>
<td>2.45</td>
<td>0.25</td>
<td>0.61</td>
<td>0.57</td>
<td>0.18</td>
<td>0.99</td>
</tr>
<tr>
<td>Consumption Exp</td>
<td>0.04</td>
<td>1.02</td>
<td>-3.30</td>
<td>0.23</td>
<td>-0.37</td>
<td>-0.31</td>
<td>0.32</td>
<td>-0.98</td>
</tr>
</tbody>
</table>

Correlations:

- $Corr \left( \hat{R}^S, \hat{R}^{WL} \right)$
  - 0.24
- $Corr \left( \hat{R}^S, \hat{R}^{PC} \right)$
  - 0.49

<table>
<thead>
<tr>
<th>Home Bonds/GDP</th>
<th>Benchmark (I)</th>
<th>Flex Price (II)</th>
<th>Fixed Capital (III)</th>
<th>Hawk (IV)</th>
<th>Dove (V)</th>
<th>High $\sigma_M$ (VI)</th>
<th>Low $\sigma_M$ (VII)</th>
<th>Currency Union (VIII)</th>
</tr>
</thead>
<tbody>
<tr>
<td>due to:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Human Capital</td>
<td>-0.76</td>
<td>-0.29</td>
<td>-0.73</td>
<td>-0.73</td>
<td>-0.71</td>
<td>-0.48</td>
<td>-0.95</td>
<td>0.00</td>
</tr>
<tr>
<td>Consumption Exp</td>
<td>1.24</td>
<td>0.59</td>
<td>0.45</td>
<td>1.21</td>
<td>1.17</td>
<td>0.80</td>
<td>1.57</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Correlations:

- $Corr \left( \hat{R}^B, \hat{R}^{WL} \right)$
  - 0.96
- $Corr \left( \hat{R}^B, \hat{R}^{PC} \right)$
  - 0.99

*(I) follows the benchmark calibration described in the text, (II) sets $\theta = 0$, (III) fixes the capital stock by setting both investment and the rate of depreciation, $\delta$, to zero, (IV) sets $\gamma = 1.5$, (V) sets $\gamma = 2.5$, (VI) sets $\sigma_M = 0.012$, (VII) sets $\sigma_M = 0.049$, (VIII) replaces the two monetary rules by one that responds to inflation and output in both countries symmetrically. Contributions of hedging motives and correlations were computed relying on 500 simulations of the benchmark model for 500 periods. Contributions of hedging motives were calculated according to the reduced form expressions (7) and (8) and using the coefficients obtained from running the regressions (47) and (48) on the simulated data. Relative bond and equity returns and human capital and consumption expenditure risk were Hodrick Prescott filtered with a smoothing parameter of 1600 before computing correlations.*
of nominal rigidities entirely, monetary policy does affect equilibrium portfolios, for two reasons. Firstly, monetary policy has real effects in this model, due to nominal rigidities in price and wage setting. Monetary policy will therefore affect the dynamics of labour income and consumption expenditures and therefore our sources of risk. Secondly, monetary policy also affects the dynamics of asset returns. For example, relative nominal bond returns solely depend on nominal exchange rate movements and these are strongly affected by monetary policy. The effects of more or less responsive monetary policy on portfolios are documented in the fourth and fifth column of table 4. There, we present portfolios for monetary policy rules that are more responsive to inflation (“Hawk”) or less responsive (“Dove”) than in our benchmark calibration. More responsive monetary policy lowers the correlation between relative equity returns and human capital risk, and raises its correlation with consumption expenditure risk. As a result, the motive to hedge human capital risk increases equity home bias by less, while the consumption expenditure motive increases it by more than in the benchmark. Less activist monetary policy has the opposite effect. In terms of portfolios, the effects of changes in monetary policy responsiveness are therefore similar to changes in nominal price rigidities and, for example, increases in monetary policy responsiveness have similar effects as reductions in nominal price stickiness on equity portfolios. Increases in the volatility of the monetary shock (“High $\sigma_M$”) have similar effects on equity portfolios as reductions in monetary policy responsiveness. However, their effect on bond positions is larger, with larger values of the monetary shock associated with lower absolute levels of bond holdings. This is because the larger level of volatility of monetary shocks reduces the efficacy of nominal bonds as a risk hedging instrument.30

In standard New Keynesian models, monetary policy has an effect on welfare by modifying the degree of inefficient price dispersion. As noted by Devereux and Sutherland (2007, 2008), in models with endogenous portfolio choice and incomplete markets, there is an additional channel through which monetary policy affects welfare. As monetary policy alters asset returns, it thereby changes the hedging properties of different assets. By changing the hedging properties of the asset menu, monetary policy affects the degree of risk sharing and thereby welfare. In our framework, markets are incomplete, even at the first order of approximation, as the number of shocks per country exceeds the number of assets and monetary policy therefore affects welfare beyond its effects on price dispersion. However, the degree of risk sharing in the model is high under most specifications of monetary policy, for two reasons. Firstly, two assets per country are enough to generate high risk sharing in many open economy settings (see also Benigno and Kucuk-Tuger (2009)), including ours. What is more, the response of relative prices does much to share risk between countries here (Cole and Obstfeld (1991)). The welfare effects of monetary policy through the additional channel of affecting the hedging properties of assets are therefore likely to be small.

In the last column of table 4, we present portfolios for the case of a currency union between the two countries (“Currency Union”). In that case, the two monetary policy rules

30Reductions in the persistence of the interest rate, $\rho_r$, also have similar effects to increases in monetary policy responsiveness, while changes in the responsiveness to output growth, $\gamma_Y$, have only small effects on portfolios.
in (35) are replaced by a single rule:

\[
R_t^N = (R_t^N)^\rho_r \left( \frac{1}{\beta} \left( \frac{P_{C,t}}{P_{C,t-1}} \right)^{\gamma \pi} \left( \frac{Y_t}{Y_{t-1}} \right)^{\gamma Y} M_t \right)^{1-\rho_r},
\]

where \( R_t^N \) is the single nominal interest rate for the currency union, \( P_{C,t}/P_{C,t-1} \) is union wide inflation, \( Y_t/Y_{t-1} \) is union wide output growth and \( M_t \) is a union wide interest rate shock.\(^{31}\)

Thus, we now assume that the monetary policy authority responds to union wide changes in output and inflation. The currency union has the same effect as permanently fixing the nominal exchange here.\(^{32}\) Several implications for portfolios result. The two nominal bonds are now equivalent from the viewpoint of both investors, as they offer the same nominal payout. Bond holdings in the steady state are therefore indeterminate and can be set to zero without loss of generality.\(^{33}\) As a result, only equity portfolios are used to hedge both human capital and consumption expenditure risk. Neumeyer (1998) discusses the existence of a trade off for the effects of a monetary union on risk sharing. On the one hand, the formation of the currency union eliminates nominal exchange rate risk. On the other hand, it reduces the menu of assets available and thereby reduces hedging opportunities and both of these effects are present here. Under our benchmark calibration, a currency union would imply that investors would hold almost perfectly diversified portfolios. The human capital motive still induces domestic investors to take long positions in domestic equity. But the consumption expenditure motive now induces Home agents to take short positions and the two motives offset each other almost perfectly. For our benchmark calibration, currency union thus implies a large degree of diversification of equity holdings, but it is mainly driven by a change in the set of assets traded and not the removal of exchange rate risk.

### 4.4 Business Cycle Moments

In the previous subsection, we illustrated that the benchmark model implies equity home bias and bond holdings of a realistic size. Here, we show that, in addition to matching international asset portfolios, the model matches important features of the international business cycle. In table 5, we present business cycle statistics for our benchmark model, several alternative model configurations, and the data. The table also shows the predictions of two other recent papers that investigate the business cycle implications of international financial asset trade in open economy models. Heathcote and Perri (2002) compare the

\[ \text{Union wide inflation and output growth are given by: } \frac{Y_t}{Y_{t-1}} = \frac{1}{2} \left( \frac{Y_{H,t}}{Y_{H,t-1}} + \frac{Y_{F,t}}{Y_{F,t-1}} \right) \text{ and } \frac{P_{C,t}}{P_{C,t-1}} = \frac{1}{2} \left( \frac{P_{H}^{c,t}}{P_{H,t-1}} + \frac{P_{F}^{c,t}}{P_{F,t-1}} \right). \]

\[ \text{Note, however, that real exchange rates are not constant, due to differences in preferences in the two countries.} \]

\[ \text{Relative bond returns are in any case zero and any bond holding would therefore not provide any hedging benefits for human capital or consumption expenditure risk.} \]
Table 5: Business Cycle Statistics

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Data</th>
<th>Benchmark</th>
<th>HP</th>
<th>CKM</th>
<th>Flex</th>
<th>Free K</th>
<th>Fixed K</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(I)</td>
<td>(II)</td>
<td>(III)</td>
<td>(IV)</td>
<td>(V)</td>
<td>(VI)</td>
<td>(VII)</td>
</tr>
<tr>
<td>Cross Country Correlation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output</td>
<td>0.57</td>
<td>0.43</td>
<td>0.24</td>
<td>0.17</td>
<td>0.27</td>
<td>0.34</td>
<td>0.19</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.26</td>
<td>0.58</td>
<td>0.85</td>
<td>0.58</td>
<td>0.66</td>
<td>0.56</td>
<td>0.63</td>
</tr>
<tr>
<td>Investment</td>
<td>0.31</td>
<td>0.36</td>
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<td>-0.37</td>
<td>0.37</td>
<td>-0.02</td>
<td>0.00</td>
</tr>
<tr>
<td>Hours worked</td>
<td>0.10</td>
<td>0.32</td>
<td>0.14</td>
<td>0.18</td>
<td>0.39</td>
<td>0.31</td>
<td>0.12</td>
</tr>
<tr>
<td>Correlation with GDP</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.85</td>
<td>0.82</td>
<td>0.92</td>
<td>0.38</td>
<td>0.85</td>
<td>0.39</td>
<td>0.96</td>
</tr>
<tr>
<td>Investment</td>
<td>0.94</td>
<td>0.92</td>
<td>0.99</td>
<td>0.71</td>
<td>0.85</td>
<td>0.82</td>
<td>0.00</td>
</tr>
<tr>
<td>Hours worked</td>
<td>0.88</td>
<td>0.46</td>
<td>0.99</td>
<td>0.61</td>
<td>0.79</td>
<td>0.98</td>
<td>-0.71</td>
</tr>
<tr>
<td>Net exports</td>
<td>-0.35</td>
<td>-0.18</td>
<td>0.00</td>
<td>-0.07</td>
<td>0.10</td>
<td>-0.56</td>
<td>0.62</td>
</tr>
<tr>
<td>Real exchange rate</td>
<td>0.07</td>
<td>-0.44</td>
<td>-0.65</td>
<td>-0.22</td>
<td>-0.57</td>
<td>-0.18</td>
<td>-0.61</td>
</tr>
<tr>
<td>Standard Deviation(^1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.82</td>
<td>0.44</td>
<td>0.43</td>
<td>0.43</td>
<td>0.50</td>
<td>0.12</td>
<td>0.86</td>
</tr>
<tr>
<td>Investment</td>
<td>2.88</td>
<td>5.17</td>
<td>1.73</td>
<td>4.42</td>
<td>4.39</td>
<td>12.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Hours worked</td>
<td>0.84</td>
<td>0.77</td>
<td>0.24</td>
<td>0.70</td>
<td>0.48</td>
<td>1.21</td>
<td>0.96</td>
</tr>
<tr>
<td>Net exports</td>
<td>1.50</td>
<td>0.11</td>
<td>0.0</td>
<td>0.57</td>
<td>0.10</td>
<td>0.24</td>
<td>0.14</td>
</tr>
<tr>
<td>Real exchange rate</td>
<td>6.35</td>
<td>0.77</td>
<td>1.00</td>
<td>0.74</td>
<td>0.82</td>
<td>0.23</td>
<td>1.48</td>
</tr>
</tbody>
</table>

\(^I\) presents business cycle statistics for U.S. data from 1970 to 2008, \(^II\) presents the statistics implied by the benchmark calibration, \(^III\) is from Heathcote and Perri (2008), \(^IV\) is from Coeurdacier et al (2009), \(^V\) sets \(\theta = 0\), \(^VI\) sets \(\nu \to \infty\), \(^VII\) sets both investment and the rate of depreciation to zero. Standard deviations and correlations in model and data are for Hodrick Prescott filtered variables with a smoothing parameter of 1600. All variables with the exception of net exports were logged before applying the filter. Model moments result from 500 simulations of 500 periods.

\(^1\) Standard deviations are expressed relative to the standard deviation of GDP.
business cycle implications of models with complete markets, trade in riskless bonds, and financial autarky. They argue that the model with financial autarky matches business cycle statistics most closely and its predictions are presented in the third column of Table 5 (“HP”). In Coeurdacier et al (2009), agents trade real consols and equities, but there are neither nominal rigidities, nor investment adjustment costs, and the business cycle predictions of their model are shown in the fourth column of Table 5 (“CKM”).

Our benchmark calibration is able to match many features of the international business cycle. In particular, it generates positive cross country correlations in output, consumption, investment, and output. Net exports are countercyclical and the correlation between domestic output and consumption and investment is highly positive, as in the data. As in most open economy models, the cross country correlation of consumption is higher than the one with output, while the reverse is true in the data. However, in our model the difference is relatively small, and substantially smaller than in the two other papers. Coeurdacier et al (2009) note that the presence of investment specific technology shocks lowers the cross country correlation of consumption and the correlation between output and the real exchange rate. This is true in our model also, but the table shows that it does not come at the expense of counterfactual behaviour of investment. The fifth and sixth column illustrate that we can trace the difference in predictions to the presence of nominal rigidities and investment adjustment costs. Without investment adjustment costs (“Flex K”), investment responds too strongly to shocks in investment specific technology which implies that the cross country correlation of investment becomes negative, and the correlation between domestic and output low, as in Coeurdacier et al (2009). As we noted above, nominal price rigidities imply that the response of investment and labour to investment efficiency shocks is quite strong. With perfect price flexibility, investment responds less strongly, implying procyclical net exports. The last column illustrates the effect of capital accumulation per se on the cyclicality of net exports. In models without endogenous capital accumulation, such as Engel and Matsumoto (2009), net exports are procyclical, due to the agents’ motive to smooth consumption over time. During booms, agents would like to save in order to consume more in the future also. Without capital accumulation, the only way to do so is by increasing net foreign assets which implies a rise in net exports.

Like many other open economy models, our benchmark model nevertheless has difficulty to match some business cycle properties in the data. Thus, the real exchange rate is still not volatile enough and too highly correlated with GDP. The volatility of net exports is too low, while the volatility of investment is too high. Previous work has noted the difficulty of jointly generating realistic macroeconomic and asset pricing moments in a domestic context. Our

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34 The other two papers that are closely related to our study and that we discussed extensively above, Engel and Matsumoto (2009) and Heathcote and Perri (2009) do not present business cycle statistics.

35 Heathcote and Perri (2002) also report business cycle statistics for models in which countries trade a riskless bond, or where markets are complete. In those cases, the model also predicts negative cross country correlations in investment and also employment.

36 Mehra and Prescott (1985) document the difficulty of generating a realistic equity premium in standard
model is no exception. We find that the equity premium in our benchmark economy is too low. An extension of the model that allows for Epstein-Zin preferences, as in Rudebusch and Swanson (2009), and a more persistent productivity process can generate a realistic level of the equity premium, but at the expense of implying a volatility of the risk free rate that is too high.\footnote{Results are available upon request.}

5 Estimated Moments and Portfolios

In this section, we test the empirical predictions of our model in an additional way. We construct asset return innovations and our measures of risk, \((2) - (5)\), for the US, using quarterly data on G7 countries. Our estimation is closely related to Coeurdacier and Gourinchas (2009) who study international portfolio choice in a very general static setting. The key difference in our estimation is that we focus on a dynamic measure of consumption expenditure risk as the second hedging motive in addition to human capital risk, while their second source of risk is a static measure of real exchange rate risk.\footnote{In open economy models with complete markets, relative consumption expenditures are perfectly correlated with the real exchange rate. With incomplete markets, that is no longer the case, but relative consumption expenditures can be expressed as the sum of relative consumption volume and the real exchange rate, at the first order of approximation.} We find that, as in the model, the unconditional correlation between relative equity returns and relative human capital returns is moderately positive in the data. More importantly, we find that this correlation, conditional on relative bond returns, is negative, implying equity home bias. Relative equity returns are moderately positively correlated with relative consumption expenditure risk, both conditionally and unconditionally, again, as in the model. Relative bond are positively correlated with human capital and consumption expenditure risk, both conditionally and unconditionally. We then use our estimated moments and an adapted version of \((7)\) and \((8)\) to construct predicted equity and bond portfolios. The equity portfolio resulting from our estimation is virtually identical to the one implied by the benchmark model and available data on U.S. equity holdings. The bond portfolio implied by the estimation suggests a short position in U.S. bonds for U.S. investors, as observed in actual data on bond holdings. This is because the human capital motive outweighs the consumption expenditure motive, unlike in the model. However, the size of the predicted bond position is larger than observed bond positions for the U.S.

\footnote{Results are available upon request.}
5.1 Constructing Innovations and Risk Measures

Our methodology follows Campbell and Shiller (1988) and Campbell (1996). We are interested in estimating relative equity and bond return innovations, \((2) - (3)\), and human capital and consumption expenditure risk, \((4) - (5)\), for the U.S. We therefore face two tasks. Firstly, the present value of future relative labour income and relative consumption expenditures are not observable - only current labour income and consumption expenditures are - and we therefore need to construct them using observable data. Secondly, we need to construct expectations of returns and risk in order to retrieve innovations in these variables.

For human capital, denote by \(\tilde{E}_{t+1}[r_{US,t+1}^{WL}]\) the expected log-dividend price ratio of human capital, \(\zeta_{US,t} = \log \left( \frac{W_{US,t}}{V_{US,t}^{WL}} \right)\) is the log-dividend price ratio for human capital, and \(\rho^{-1} = 1 + \exp(\zeta_{US}) = \left( \frac{WL_{t} + V_{US,t}^{WL}}{V_{US,t}^{WL}} \right)\). As in Coeurdacier and Gourinchas (2009), we will use \(\rho = 0.98\), while \(k\) is an unimportant constant. Solving this equation forward and imposing that \(\lim_{t \to \infty} \rho^t (r_{US,t}^{WL} - \Delta \log WL_{US,t}) = 0\), we obtain (up to a constant):

\[
\zeta_{US,t} = E_{t} \sum_{j=0}^{\infty} \rho^j \left( r_{US,t+j}^{WL} - \Delta \log WL_{US,t+j+1} \right).
\]

(51)

This expression states that the ratio of labour income to the value of human capital is high today either when future human capital returns are high, or when future nonfinancial income growth is low. Following Campbell (1996), if we assume that the conditional expected return on financial wealth equals the conditional expected return on human wealth \(E_{t} [r_{US,t+j}^{WL}] = E_{t} [r_{US,t+j}^{S}], \forall j\) and substitute (51) into (50), we obtain:

\[
\tilde{E}_{t+1}[r_{US,t+1}^{WL}] = \tilde{E}_{t+1} \sum_{j=0}^{\infty} \rho^j (\Delta \log WL_{US,t+j+1}) - \tilde{E}_{t+1} \sum_{j=1}^{\infty} \rho^j r_{US,t+j+1}^{WL}.
\]

(52)

\(^{39}\)Baxter and Jermann (1997), Coeurdacier and Gourinchas (2009) and Juillard (2003) have used similar methods.

\(^{40}\)For open economy applications, Coeurdacier and Gourinchas (2009) make the same assumption, while Baxter and Jermann (1997) and Benigno and Nistico (2009) assume that discount rates are constant and thereby omit the second term. See also the discussion in Benigno and Nistico (2009) and Coeurdacier and Gourinchas (2009) and recent empirical work by Lustig and Van Nieuwerburgh (2006) and Lustig et al (2009).
This expression states that the innovation to the return on human capital depends upon innovations to the path of future expected labour income growth, as well as innovations to the path of future expected equity returns proxying for future expected human capital returns. Thus, human capital return innovations today are high, if innovations to expected labour income growth are high, or if innovations to expected human capital returns are low. We then obtain innovations to the relative expected human capital return by subtracting the analogous expression for the rest of the world, converted into dollars, from (52), assuming that \( \rho \) is the same for all countries:

\[
\tilde{E}_{t+1} r_{t+1}^{WL} = \tilde{E}_{t+1} \sum_{j=0}^{\infty} \rho^j (\Delta \log W_{t+1+j}) - \tilde{E}_{t+1} \sum_{j=1}^{\infty} \rho^j r_{t+j+1}^{S},
\]  

(53)

where \( \log W_{t+1+j} \) and \( r_{t+j+1}^{S} \) are relative labour income and relative equity returns, expressed in US dollars. The relative return on human capital thus depends on innovations to expected future relative labour income, as well as innovations to future relative discount rates.

We also need to obtain a measure for relative consumption expenditure risk. Following the same steps as above, we obtain the following expression:

\[
\tilde{E}_{t+1} [r_{t+1}^{PC}] = \tilde{E}_{t+1} \sum_{j=0}^{\infty} \rho^j (\Delta \log PC_{t+1+j}) - \tilde{E}_{t+1} \sum_{j=1}^{\infty} \rho^j r_{US,t+j+1}^{PC},
\]  

(54)

where \( r^{PC} \) is the relevant measure of real exchange rate risk and \( PC_{t} \) are relative consumption expenditures in US dollars. Since we are not aware of any work that suggests the predictability of future returns to the stream of relative consumption expenditures, we set the second term to zero and therefore estimate consumption expenditure risk solely as a function of expected growth in relative consumption expenditures. Equity and bond returns are observable and relative equity and bond return innovations are given by:

\[
\tilde{E}_{t+1} [r_{t+1}^{S}] = \tilde{E}_{t+1} [r_{US,t+1}^{S} - r_{ROW,t+1}^{S}] \quad \tilde{E}_{t+1} [r_{t+1}^{B}] = \tilde{E}_{t+1} [r_{US,t+1}^{B} - r_{ROW,t+1}^{B}],
\]  

(55)

where \( r_{US,t+1}^{S} \) (\( r_{US,t+1}^{B} \)) are log equity (bond) returns in the US, \( r_{ROW,t+1}^{S} \) (\( r_{ROW,t+1}^{B} \)) are log equity (bond) returns in the rest of the world, in dollars.

In order to estimate \( \tilde{E}_{t+1} [r_{t+1}^{S}] \), \( \tilde{E}_{t+1} [r_{t+1}^{B}] \), \( \tilde{E}_{t+1} [r_{t+1}^{WL}] \), \( \tilde{E}_{t+1} [r_{t+1}^{PC}] \), we then run the following first order vector autoregression:

\[
x_{t+1} = A x_{t} + \varepsilon_{t+1},
\]  

(56)

where \( x_{t} = [r_{t}^{S} \Delta l_{t} \ r_{t}^{B} \Delta p_{ct} \ b_{t}] \). \( b_{t} \) represents other controls that help to predict the (relative) growth rate of labour income, bond and equity returns, and consumption expenditures. Here, we use the \( nxa \) variable from Gourinchas and Rey (2007) and the term
Having obtained the estimates \( \hat{A} \) and \( \hat{\varepsilon} \) from the VAR, human capital and consumption expenditure risk are created as:

\[
\tilde{E}_{t+1} \left[ r_{i,t+1}^{WL} \right] = \left( e_2' - \rho e_1' \hat{A} \right) \left( I - \rho \hat{A} \right)^{-1} \tilde{\varepsilon}_{t+1} \quad (57)
\]

\[
\tilde{E}_{t+1} \left[ r_{i,t+1}^{PC} \right] = e_4' \left( I - \rho \hat{A} \right)^{-1} \tilde{\varepsilon}_{t+1} \quad (58)
\]

while equity and bond return innovations are:

\[
\tilde{E}_{t+1} \left[ r_{i,t+1}^S \right] = e_1' \tilde{\varepsilon}_{t+1} \quad \tilde{E}_{t+1} \left[ r_{i,t+1}^B \right] = e_3' \tilde{\varepsilon}_{t+1}. \quad (59)
\]

\( e_i \) is a unit vector whose \( i \)th element is equal to one, while all other elements are equal to zero.

The data is quarterly and runs from the first quarter of 1970 until the first quarter of 2008. The rest of the world is constructed using the G7 economies (Canada, France, Germany, Italy, Japan, UK, US). Further details about data sources and construction are in the appendix. The VAR is estimated in levels, including a constant and one lag.\(^{42}\) In Table 6, we present the unconditional correlations of the asset returns and sources of risk and contrast them with the ones implied by the benchmark model. As we can see, the model does a good job of qualitatively matching the correlations in the data. The correlation of equity return innovations is positive, but quite low, with human capital risk, and somewhat larger, with consumption expenditure risk, as in the model. The correlations of relative bond return innovations with human capital and consumption expenditure risk is positive and quite high in the data, while not quite as high as in our benchmark model.

### 5.1.1 Conditional Measures of Comovement

We are now in a position to estimate the relevant covariance variance ratios. We run the following two regressions:

\[
\tilde{E}_{t+1} \left[ r_{i,t+1}^{WL} \right] = k_1 + \beta_{wl,b} \tilde{E}_{t+1} \left[ r_{i,t+1}^B \right] + \beta_{wl,s} \tilde{E}_{t+1} \left[ r_{i,t+1}^S \right] + \varepsilon_{wl} \quad (60)
\]

\[
\tilde{E}_{t+1} \left[ r_{i,t+1}^{PC} \right] = k_2 + \beta_{pc,b} \tilde{E}_{t+1} \left[ r_{i,t+1}^B \right] + \beta_{pc,s} \tilde{E}_{t+1} \left[ r_{i,t+1}^S \right] + \varepsilon_{pc}, \quad (61)
\]

where the error terms \( \varepsilon_{wl} \) and \( \varepsilon_{pc} \) are attributed both to measurement error in the construction of the sources of risk as well as to fluctuations not spanned by relative bond and equity returns. As noted above, the coefficients obtained from this regression are equal to

\(^{41}\) \( nxa \) is the ratio of net exports to net foreign assets and was found by Gourinchas and Rey (2007) to predict relative equity returns for the US. We rely on an updated version of the original \( nxa \) series that runs until the first quarter of 2008, kindly supplied by Hélène Rey.

\(^{42}\) Akaike and Bayesian information criteria suggest that including one lag is the preferred specification.
Table 6: Unconditional Correlations

<table>
<thead>
<tr>
<th>Correlations</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity Return Innovations with:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Human Capital Risk</td>
<td>0.21</td>
<td>0.24</td>
</tr>
<tr>
<td>Consumption Expenditure Risk</td>
<td>0.34</td>
<td>0.49</td>
</tr>
<tr>
<td>Bond Return Innovations with:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Human Capital Risk</td>
<td>0.77</td>
<td>0.96</td>
</tr>
<tr>
<td>Consumption Expenditure Risk</td>
<td>0.64</td>
<td>0.99</td>
</tr>
</tbody>
</table>

Correlations in the data are based on quarterly data from Q1:1970 to Q3:2008. Model correlations were computed using 500 simulations of 500 periods of 500 periods each.

The loadings that enter in our expression for equilibrium asset portfolios:

\[
\beta_{pc,e} = \frac{\text{cov}_{R_t^p} \left( \hat{R}_{t,PC}^p, \hat{R}_{t}^s \right)}{\text{var}_{R_t^p} \left( \hat{R}_{t}^s \right)}
\]

\[
\beta_{wl,e} = \frac{\text{cov}_{R_t^p} \left( \hat{R}_{t,WL}^w, \hat{R}_{t}^s \right)}{\text{var}_{R_t^p} \left( \hat{R}_{t}^s \right)}
\]

\[
\beta_{pc,b} = \frac{\text{cov}_{R_t^s} \left( \hat{R}_{t,PC}^p, \hat{R}_{t}^b \right)}{\text{var}_{R_t^s} \left( \hat{R}_{t}^b \right)}
\]

\[
\beta_{wl,b} = \frac{\text{cov}_{R_t^s} \left( \hat{R}_{t,WL}^w, \hat{R}_{t}^b \right)}{\text{var}_{R_t^s} \left( \hat{R}_{t}^b \right)}
\]

The results of the regressions are presented in table 7. The first row indicates that the correlation between relative equity return innovations and human capital risk, conditional on relative bond returns, is negative and the regression coefficient is in fact the same as in our benchmark model. The coefficient for consumption expenditure risk is positive and quite low in both model and data, but it is not significant in the data. For bond positions, our model can also qualitatively, if not quantitatively match the regression coefficients. Thus, we find that both the regression coefficients for human capital and consumption expenditure risk are positive and quite large. While they are also positive in the model, they are much smaller.

5.2 Implied Bond and Equity Positions

In our model, we assumed that the two countries are of the same size. Here, we relax that assumption and slightly adapt our reduced form expressions to allow for countries of
Table 7: Regression Results

<table>
<thead>
<tr>
<th></th>
<th>Human Capital Risk</th>
<th>Consumption Expenditure Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data Model</td>
<td>Data Model</td>
</tr>
<tr>
<td>Equity Return Innovations</td>
<td>$-0.16^{**}$</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>(0.06)</td>
<td>(0.07)</td>
</tr>
<tr>
<td>Bond Return Innovations</td>
<td>1.29^{**}</td>
<td>0.93^{**}</td>
</tr>
<tr>
<td></td>
<td>(0.09)</td>
<td>(0.11)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.61</td>
<td>0.41</td>
</tr>
<tr>
<td>No. of Observations</td>
<td>152</td>
<td>152</td>
</tr>
</tbody>
</table>

Data and model regression coefficients were obtained from (60) and (61). Data is from the first quarter of 1970 to the third quarter of 2007. Model regressions were run on data obtained from 500 simulations of 500 periods. ** implies that coefficients are significant at the 95% confidence level. Coefficients for the constants are suppressed.

\[
S = \tilde{\lambda} + (1 - \tilde{\lambda}) \left( \frac{C_{US} \text{cov}_{R_i^B} \left( \hat{R}_{PC}^B, \hat{R}_{i}^S \right)}{D \ \text{var}_{R_i^B} \left( \hat{R}_{i}^S \right)} - \frac{1}{2} \ \frac{W_L \text{cov}_{R_i^B} \left( \hat{R}_{WL}^B, \hat{R}_{i}^S \right)}{D \ \text{var}_{R_i^B} \left( \hat{R}_{i}^S \right)} \right) \quad (64)
\]

\[
\tilde{B} = \frac{1}{2 (1 - \beta) \bar{Y}} \frac{\beta \text{cov}_{R_i^B} \left( \hat{R}_{PC}^B, \hat{R}_{i}^S \right)}{\text{var}_{R_i^S} \left( \hat{R}_{i}^B \right)} - \frac{1}{2 (1 - \beta) \bar{Y}} \frac{W_L \text{cov}_{R_i^S} \left( \hat{R}_{WL}^B, \hat{R}_{i}^B \right)}{\text{var}_{R_i^S} \left( \hat{R}_{i}^B \right)}, \quad (65)
\]

where $\tilde{\lambda} = \frac{C_{US} - C_{ROW}}{D_{US} + D_{ROW}} = \frac{Y_{US}}{Y_{US} + Y_{ROW}}$ is the steady state ratio of consumption, dividends and output between the US and the rest of the world. The average share of US in aggregate GDP in our sample is 45% and we correspondingly set $\tilde{\lambda} = 0.45$. The mean ratios of consumption over dividends, $\frac{C}{D}$, and labour income over dividends, $\frac{W_L}{D}$, are 0.73 and 0.71, respectively. Substituting the regression coefficients obtained in the previous subsection and the steady state coefficients, we obtain the implied portfolios in Table 8. There, we see that the estimated moments imply substantial equity home bias for the US, as the benchmark model does and both are qualitatively and quantitatively similar to the available data on actual equity holdings. The correlation of relative equity returns with relative human capital returns, conditional on relative bond returns, is negative in the data, and it therefore contributes to equity home bias. The consumption expenditure motive also implies long positions in domestic equity for US investors, as the correlation of relative equity returns with consumption expenditure risk, conditional on relative bond returns, is positive, but as in the benchmark model, the contribution of the human capital motive to equity home bias is larger. The model and the data also agree on the directions of the drivers of bond portfolios. Both in the model and in the data, the human capital motive induces Home agents to take short positions in domestic bonds, while the consumption
Table 8: Estimate Equity and Bond Positions

<table>
<thead>
<tr>
<th></th>
<th>Share of Home Equity</th>
<th>Home Bonds/GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>0.88</td>
<td>−1.73</td>
</tr>
<tr>
<td>due to:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pure Diversification</td>
<td>0.50</td>
<td>−</td>
</tr>
<tr>
<td>Human Capital</td>
<td>0.30</td>
<td>−5.78</td>
</tr>
<tr>
<td>Consumption Expenditure</td>
<td>0.12</td>
<td>4.05</td>
</tr>
<tr>
<td>Home Bias</td>
<td>0.43</td>
<td>−</td>
</tr>
</tbody>
</table>

Equilibrium Equity and Bond Positions are calculated using (64) and (65), the regression coefficients estimated in the previous subsection and $\beta = 0.99$, $\lambda = 0.45$, $\zeta = 0.73$ and $\frac{W_L}{Y} = 0.71$.

expenditure motive induces them to take a long position. In our estimation, the former motive dominates, implying overall short positions of U.S. investors in U.S. bonds, while in our benchmark model the consumption expenditure motive is larger and leads to overall long positions in domestic bonds. Finally, while the sign of the bond position is in accord with data on available U.S. holdings, the size is somewhat too large.

6 Conclusion

There is ample evidence on international trade in equities and nominal bonds, equity home bias, and positive cross country correlations in output, consumption, investment, and output. There is also strong evidence that the unconditional correlation between equity returns and human capital returns is positive. We present a two country, two good model with nominal rigidities, endogenous capital accumulation and incomplete markets that matches these facts. Agents trade equity and nominal bonds to hedge fluctuations in labour income and consumption expenditures. Importantly, in the presence of trade in multiple assets, equity holdings depend on the comovement with these sources of risk, conditional on the returns of other assets, as emphasized by, amongst others, Engel and Matsumoto (2009) and Coeurdacier and Gourinchas (2009). We show that in the model the correlation between equity returns and human capital returns is positive, but that once we condition on bond returns, this correlation turns negative. These relationships are confirmed using data on G7 countries and imply equity home bias. Long positions in domestic nominal bonds hedge most of consumption expenditure risk and we show that the model matches the conditional and unconditional correlations of bond returns with the sources of risk also, at least qualitatively. The model is able to match many features of the international business cycle, and in particular, generates positive cross country correlations in investment. We emphasize that both nominal rigidities and endogenous capital accumulation contribute to matching the
patterns of international portfolios and cross country correlations observed in the data.

However, the model shares some of the weaknesses of previous models of international risk sharing. Thus, it predicts a correlation of relative consumption with the real exchange rate that is too high compared to the data (Backus and Smith (1993) and Kollmann (1995)). What is more, it is not able to simultaneously produce a realistic equity premium and risk free rate volatility. While the former raises questions about the appropriate way to model market incompleteness in open economy models, the latter gives reason to translate the lessons learned in the recent asset pricing literature into an open economy context. We regard both of these as worthy endeavours for future research. In addition, the integration of portfolio choice and macroeconomics allows the rigorous analysis of many issues that are at the center of current policy debates in the international policy sphere, but could not be examined systematically before. Among these topics are the origins and effects of global imbalances, the effects of policy decisions on capital flows or the interaction between real and financial globalisation.

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7 Appendix

7.1 Stochastic Discount Factors

The Euler equations for investment in Home equity are:

\[ \beta E_t \left[ \left( \frac{C_{H,t+1}}{C_{H,t}} \right)^{-\sigma} \left( \frac{P_{C,t}^H}{P_{C,t+1}^H} \right) \left( \frac{D_{H,t+1} + P_{H,t+1}^S}{P_{H,t}^S} \right) \right] = 1 \] (66)

\[ \beta E_t \left[ \left( \frac{C_{F,t+1}}{C_{F,t}} \right)^{-\sigma} \left( \frac{P_{C,t}^F}{P_{C,t+1}^F} \right) \left( \frac{D_{H,t+1} + P_{H,t+1}^S}{P_{H,t}^S} \right) \right] = 1 \] (67)

Linearising:

\[ 0 = E_t \left[ -\sigma \left( \hat{C}_{H,t+1} - \hat{C}_{H,t} \right) + \left( \hat{P}_{C,t}^H - \hat{P}_{C,t+1}^H \right) + \left( 1 - \beta \right) \hat{D}_{H,t+1} + \beta \hat{P}_{H,t+1}^S - \hat{P}_{H,t}^S \right] \] (68)

\[ 0 = E_t \left[ -\sigma \left( \hat{C}_{F,t+1} - \hat{C}_{F,t} \right) + \left( \hat{P}_{C,t}^F - \hat{P}_{C,t+1}^F \right) + \left( 1 - \beta \right) \hat{D}_{H,t+1} + \beta \hat{P}_{H,t+1}^S - \hat{P}_{H,t}^S + \hat{Z}_t - \hat{Z}_{t+1} \right] \] (69)

which imply:

\[ E_t \left[ -\sigma \left( \hat{C}_{H,t+1} - \hat{C}_{H,t} \right) - \left( \hat{P}_{C,t+1}^H - \hat{P}_{C,t}^H \right) \right] \] (70)

\[ = E_t \left[ -\sigma \left( \hat{C}_{F,t+1} - \hat{C}_{F,t} \right) - \left( \hat{P}_{C,t+1}^F - \hat{P}_{C,t}^F \right) \right] + \hat{Z}_t - \hat{Z}_{t+1} \] (71)

Thus, in a first order approximation, the Home and Foreign stochastic discount factors are the same in expectation, once they are expressed in the same units.

7.2 Budget Constraint

The budget constraint for the Home agent is:

\[ S_{H,t}^H P_{H,t}^S + S_{F,t}^H P_{F,t}^S Z_t + B_{H,t}^H P_{H,t}^B + B_{F,t}^H P_{H,t}^B Z_t + P_{C,t}^H C_{H,t} \]

\[ = W_{H,t} L_{H,t} + S_{H,t-1}^H (D_{H,t} + P_{H,t}^S) + S_{F,t-1}^H (D_{H,t} + P_{F,t}^S) Z_t + B_{H,t-1}^H + B_{F,t-1}^H Z_t, \] (72)

The net foreign asset position is then given by assets held abroad minus domestic assets held by foreign agents:

\[ \text{NFA}_{H,t} = S_{F,t}^H P_{F,t}^S Z_t + B_{F,t}^H P_{F,t}^B Z_t - S_{H,t}^H P_{H,t}^S - B_{H,t}^H P_{H,t}^B \] (73)

which using the asset market clearing conditions \((S_{H,t}^F = 1 - S_{H,t}^H, B_{H,t}^F = -B_{H,t}^H)\) becomes:

\[ \text{NFA}_{H,t} = (S_{H,t}^F - 1) P_{H,t}^S + S_{H,t}^H Z_t P_{F,t}^S + B_{F,t}^H P_{F,t}^B Z_t + B_{H,t}^H P_{H,t}^B \] (74)

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Define the portfolio excess return as:

\[ \xi_{H,t} = (S_{H,t-1}^H - 1) (P_{H,t}^S + D_{H,t}) + S_{F,t-1}^H (P_{F,t}^S + D_{F,t}) Z_t 
+ B_{H,t-1}^H + B_{F,t-1}^H Z_t - NFA_{H,t-1} R_{H,t}^S, \] (75)

i.e. the difference between actual net foreign assets at the beginning of period \( t \) and net foreign assets at period \( t \) had all wealth been invested in the home equity. Substituting for net foreign asset and writing in terms of returns, we have:

\[ \xi_{H,t} = S_{F,t-1} P_{F,t-1}^S Z_{t-1} - R_{H,t}^S + B_{H,t-1}^H P_{H,t-1}^B (R_{H,t}^B - R_{H,t}^S) 
+ B_{F,t-1}^H Z_{t-1} P_{F,t-1}^B (R_{F,t}^B - R_{H,t}^S) \] (76)

Note that the left hand side of (72) is equal to \( NFA_{H,t} + P_{H,t}^S - C_{H,t} \). Using the expressions for the portfolio excess returns and net foreign assets, we can then write the budget constraint as:

\[ NFA_{H,t} = NFA_{H,t-1} R_{H,t}^S + P_{H,t-1}^S R_{H,t}^S + \xi_{H,t} + L_{H,t} W_{H,t} - P_{C,t}^H C_{H,t} - P_{C,t}^B C_{H,t}. \] (77)

Now, remembering that next exports were defined as: \( N X_{H,t} = Y_{H,t}^H P_{H,t}^P + Y_{F,t}^F P_{F,t}^P Z_t - I_{H,t} P_{H,t}^P - P_{C,t}^H C_{H,t} \), we have:

\[ NFA_{H,t} = NFA_{H,t-1} R_{H,t}^S + \xi_{H,t} + N X_{H,t} \] (78)

or, in linear form:

\[ \hat{NFA}_{H,t} = \hat{NFA}_{H,t-1} \frac{1}{\beta} + \hat{\xi}_{H,t} + \hat{N X}_{H,t}, \] (79)

Linearising the expression for the portfolio excess return and using (79), we have:

\[ \hat{\xi}_{H,t} = (S_{H}^H - 1) \frac{P_{S}^S}{\beta} (\hat{R}_{H,t}^S - \hat{R}_{F,t}^S - \hat{Z}_t + \hat{Z}_{t-1}) + \frac{P_{B}^B}{\beta} B (\hat{R}_{H,t}^B - \hat{R}_{F,t}^B - \hat{Z}_t + \hat{Z}_{t-1}). \] (80)

We can then write the budget constraint in linearised form as:

\[ \hat{NFA}_{H,t} = \hat{NFA}_{H,t-1} \frac{1}{\beta} + (S - 1) \frac{P_{S}^S}{\beta} (\hat{R}_{H,t}^S - \hat{R}_{F,t}^S - \hat{Z}_t + \hat{Z}_{t-1}) \]
\[ + \frac{P_{B}^B}{\beta} B (\hat{R}_{H,t}^B - \hat{R}_{F,t}^B - \hat{Z}_t + \hat{Z}_{t-1}) + \hat{N X}_{H,t} \] (81)
Taking expectations at time $t$ and rewriting:

$$\frac{1}{\beta} \hat{NFA}_{H,t-1} = E_t \left[ \hat{NFA}_{H,t} - \hat{NX}_{H,t} - \frac{PS}{\beta} (S - 1) \left( \hat{R}_{H,t}^S - \hat{R}_{F,t}^S - \hat{Z}_t + \hat{Z}_{t-1} \right) \right]$$

$$- \frac{PB}{\beta} BE_t \left[ \hat{R}_{F,t}^B - \hat{R}_{H,t}^B - \hat{Z}_t + \hat{Z}_{t-1} \right]$$

(82)

Iterating forward, imposing $T \to \infty$, $\lim_{T \to \infty} E_t [NFA_{t+T}] = 0$ and using $E_t \left[ \hat{R}_{H,t+\tau}^S - \hat{R}_{F,t+\tau}^S + \hat{Z}_{t+\tau} - \hat{Z}_{t+\tau-1} \right] = 0$, $\tau > 1$, we get:

$$\frac{1}{\beta} \hat{NFA}_{H,t-1} = \sum_{j=0}^{T} -\beta^j E_t \left[ \hat{NX}_{H,t+j} \right] - \frac{PS}{\beta} (S - 1) E_t \left[ \hat{R}_{H,t}^S - \hat{R}_{F,t}^S - \hat{Z}_t + \hat{Z}_{t-1} \right]$$

$$- \frac{PB}{\beta} BE_t \left[ \hat{R}_{F,t}^B - \hat{R}_{H,t}^B - \hat{Z}_t + \hat{Z}_{t-1} \right]$$

(83)

Rearranging and using the expressions for linearised relative bond and equity returns, we have:

$$\sum_{j=0}^{T} -\beta^j E_t \left[ NX_{H,t+j} \right] = \frac{1}{\beta} NFA_{H,t-1} + D (S - 1) \tilde{E}_t \left[ \sum_{j=0}^{\infty} \beta^j \left( \hat{D}_{H,t+j} - \hat{D}_{F,t+j} - \hat{Z}_{t+j} \right) \right]$$

$$+ B \tilde{E}_t \left[ -\hat{Z}_{t+j} \right]$$

(84)

where $\tilde{E}_t [X_t] = E_t [X_t] - E_{t-1} [X_t]$.

This budget constraint holds if and only if:

$$\sum_{j=0}^{T} -\beta^j \tilde{E}_t \left[ NX_{H,t+j} \right] = D (S - 1) \tilde{E}_t \left[ \sum_{j=0}^{\infty} \beta^j \left( \hat{D}_{H,t+j} - \hat{D}_{F,t+j} - \hat{Z}_{t+j} \right) \right] + B \tilde{E}_t \left[ -\hat{Z}_{t+j} \right]$$

(85)

and

$$\sum_{j=0}^{T} -\beta^j E_{t-1} \left[ NX_{H,t+j} \right] = \frac{1}{\beta} NFA_{H,t-1}$$

(86)
Using, $\hat{NX}_{H,t} = \hat{D}_{H,t}D + \left(\hat{W}_{H,t} + \hat{L}_{H,t}\right) WL - C \left(\hat{P}_{C,t}^H + \hat{C}_{H,t}\right)$ and rewriting, we then have:

$$C \sum_{j=0}^{T} \beta^j \tilde{E}_t \left[\hat{P}_{C,t+j}^H + \hat{C}_{H,t+j}\right]$$

$$= WL \sum_{j=0}^{T} \beta^j \tilde{E}_t \left[\hat{W}_{H,t+j} + \hat{L}_{H,t+j}\right] + SD \tilde{E}_t \left[\sum_{j=0}^{\infty} \beta^j \hat{D}_{H,t+j}\right]$$

$$+ (1 - S) D \tilde{E}_t \left[\sum_{j=0}^{\infty} \beta^j \left(\hat{D}_{F,t+j} - \hat{Z}_{t+j}\right)\right] + B \tilde{E}_t \left[-\hat{Z}_{t+j}\right]$$

(87)

The analogous expression for the foreign country is, in terms of Home currency:

$$C \sum_{j=0}^{T} \beta^j \tilde{E}_t \left[\hat{P}_{C,t+j}^F + \hat{C}_{F,t+j} + \hat{Z}_{t+j}\right]$$

$$= WL \sum_{j=0}^{T} \beta^j \tilde{E}_t \left[\hat{W}_{H,t+j} + \hat{L}_{H,t+j} + \hat{Z}_{t+j}\right] + SD \tilde{E}_t \left[\sum_{j=0}^{\infty} \beta^j \hat{D}_{H,t+j}\right]$$

$$+ (1 - S) D \tilde{E}_t \left[\sum_{j=0}^{\infty} \beta^j \hat{D}_{H,t+j}\right] - B \tilde{E}_t \left[-\hat{Z}_{t+j}\right]$$

(88)

Deducting the foreign budget constraint from the Home one, we obtain:

$$C \sum_{j=0}^{T} \beta^j \tilde{E}_t \left[(\hat{P}_{C,t}^H + \hat{C}_{H,t}) - (\hat{P}_{C,t+j}^F + \hat{C}_{F,t+j} + \hat{Z}_{t+j})\right]$$

$$= WL \sum_{j=0}^{T} \beta^j \tilde{E}_t \left[\hat{W}_{H,t+j} + \hat{L}_{H,t+j} - \left(\hat{W}_{H,t+j} + \hat{L}_{H,t+j} + \hat{Z}_{t+j}\right)\right]$$

$$+ (2S - 1) D \tilde{E}_t \left[\sum_{j=0}^{\infty} \beta^j \left(\hat{D}_{H,t+j} - \left(\hat{D}_{F,t+j} - \hat{Z}_{t+j}\right)\right)\right]$$

$$+ 2B \tilde{E}_t \left[-\hat{Z}_{t+j}\right]$$

(89)

Now define the return on human capital in country $i$ as

$$R_{H,t}^{WL} = \frac{W_{H,t}L_{H,t} + P_{H,t}^W}{P_{H,t-1}^W},$$

(90)
where $P^W_{H,t} = E_t \sum_{j=1}^{\infty} \beta^j \left( \frac{C_{H,t+j}}{C_{H,t}} \right)^{-\sigma} \frac{p_{H,t+j}^P}{p_{C,t+j}^P} W_{H,t+j} L_{H,t+j}$ is the present value of labour income in Home. Linearising, the expression for returns, we have:

$$R^{WL}_{H,t} = (1 - \beta) \tilde{E}_t \sum_{j=0}^{\infty} \beta^j \left( -\sigma \left( \tilde{C}_{H,t+j} - \tilde{C}_{H,t} \right) + \tilde{P}_{C,t+j}^H - \tilde{P}_{C,t+j}^H + \tilde{W}_{H,t+j} + \tilde{L}_{H,t+j} \right) \quad (91)$$

Deducting the analogous foreign expression in Home currency terms, we have:

$$\tilde{R}^{WL}_{H,t} - \tilde{R}^{WL}_{F,t} - Z_t + Z_{t-1} = (1 - \beta) \tilde{E}_t \sum_{j=0}^{\infty} \beta^j \left( \tilde{W}_{H,t+j} + \tilde{L}_{H,t+j} - \tilde{W}_{F,t+j} - \tilde{L}_{F,t+j} - \tilde{Z}_{t+j} \right) \quad (92)$$

Now define the return on consumption expenditure in country $i$ as

$$R^{PC}_{H,t} = \frac{P_{H,t}^P C_{H,t} + P_{H,t}^{PC}}{P_{H,t-1}^{PC}}, \quad (93)$$

where $P_{H,t}^{PC} = E_t \sum_{j=1}^{\infty} \beta^j \left( \frac{C_{H,t+j}}{C_{H,t}} \right)^{-\sigma} \frac{p_{H,t+j}^P}{p_{C,t+j}^P} P_{C,t+j}^H C_{H,t+j}$ is the present value of consumption expenditure in the Home country. Linearising, the expression for returns, we have:

$$R^{PC}_{H,t} = (1 - \beta) \tilde{E}_t \sum_{j=0}^{\infty} \beta^j \left( -\sigma \left( \tilde{C}_{H,t+j} - \tilde{C}_{H,t} \right) + \tilde{P}_{C,t+j}^H - \tilde{P}_{C,t+j}^H + \tilde{C}_{H,t+j} \right) \quad (94)$$

Deducting the analogous foreign expression in Home currency terms, we have:

$$\tilde{R}^{PC}_{H,t} - \tilde{R}^{PC}_{F,t} - Z_t + Z_{t-1} = (1 - \beta) \tilde{E}_t \sum_{j=0}^{\infty} \beta^j \left( \tilde{P}_{C,t+j}^H + \tilde{C}_{H,t+j} - \tilde{P}_{C,t+j}^H - \tilde{C}_{F,t+j} - \tilde{Z}_{t+j} \right) \quad (95)$$

Using the expressions, we just defined, we can then write the relative budget constraints as:

$$\frac{C}{1 - \beta} \tilde{R}^P_t = \frac{WL}{1 - \beta} \tilde{R}^{WL}_t + (2S - 1) \frac{D}{1 - \beta} \tilde{R}^S_t + 2B \tilde{R}^B_t, \quad (96)$$

where $R^{WL}_t = \tilde{R}^{WL}_{H,t} - \tilde{R}^{WL}_{F,t}$, $R^S_t = \tilde{R}^S_{H,t} - \tilde{R}^S_{F,t}$, $\tilde{R}^B_t = \tilde{R}^B_{H,t} - \tilde{R}^B_{F,t}$ and $R^{PC}_t = \tilde{R}^{PC}_{H,t} - \tilde{R}^{PC}_{F,t}$. Now project this equation on relative bond returns $\tilde{R}^B_t$:

$$\frac{C}{1 - \beta} P \left[ \tilde{R}^{PC}_t \mid \tilde{R}^B_t \right] = \frac{WL}{1 - \beta} P \left[ \tilde{R}^{WL}_t \mid \tilde{R}^B_t \right] + \frac{D}{1 - \beta} (2S - 1) P \left[ \tilde{R}^S_t \mid \tilde{R}^B_t \right] + 2B \tilde{R}^B_t \quad (97)$$

where $P \left[ \tilde{X}_t \mid \tilde{Y}_t \right]$ is the projection of $X_t$ on $Y_t$. Subtracting this equation from the one before,
we have:

\[
C \frac{1}{1-\beta} \left( \tilde{R}^P_{t} - P \left[ \tilde{R}^P_{t} | \tilde{R}^B_t \right] \right) = WL \frac{1}{1-\beta} \left( \tilde{R}^W_{t} - P \left[ \tilde{R}^W_{t} | \tilde{R}^B_t \right] \right) + D \frac{1}{1-\beta} (2S - 1) \left( \tilde{R}^S_{t} - P \left[ \tilde{R}^S_{t} | \tilde{R}^B_t \right] \right)
\]

(98)

Rearranging, we have:

\[
S = \frac{1}{2} \left( 1 + \frac{C}{(1-\beta)} D \frac{\text{cov}_{R^B_t} \left( \tilde{R}^P_{t}, \tilde{R}^S_{t} \right)}{\text{var}_{R^S_t} \left( \tilde{R}^S_{t} \right)} - \frac{WL}{(1-\beta)} D \frac{\text{cov}_{R^B_t} \left( \tilde{R}^W_{t}, \tilde{R}^S_{t} \right)}{\text{var}_{R^S_t} \left( \tilde{R}^S_{t} \right)} \right)
\]

(99)

Multiplying the numerator and the denominator by \( \left( \tilde{R}^S_{t} - \left[ \tilde{R}^S_{t} | \tilde{R}^B_t \right] \right) \), and taking unconditional expectations, we have:

\[
S = \frac{1}{2} \left( 1 + \frac{C}{(1-\beta)} D \frac{\text{cov}_{R^B_t} \left( \tilde{R}^P_{t}, \tilde{R}^S_{t} \right)}{\text{var}_{R^S_t} \left( \tilde{R}^S_{t} \right)} - \frac{WL}{(1-\beta)} D \frac{\text{cov}_{R^B_t} \left( \tilde{R}^W_{t}, \tilde{R}^S_{t} \right)}{\text{var}_{R^S_t} \left( \tilde{R}^S_{t} \right)} \right)
\]

(100)

7.3 Bond Maturity and Real Bonds

In the benchmark model, agents trade equities and one period nominal bonds, in line with recent work on international with nominal rigidities (Devereux and Sutherland (2007, 2009b), Engel and Matsumoto (2009)). In Table A.1, we present portfolios for two additional asset menus. In the first case, nominal consols are traded in addition to equity. Cross border trade in longer maturity debt securities is in fact quite large, but, to our knowledge, it has not previously been investigated in the context of open economy models of portfolio choice. Nominal consols pay one unit of local currency per period and relative bond returns are then given by \( \tilde{R}^{B,c}_{t} = (1-\beta) \tilde{E}_{t} \left[ \sum_{j=0}^{\infty} \beta^j \left( -\tilde{Z}_{t+j} \right) \right] \), and thus depend on surprises to the present value of appreciations in the future. The correlations and equilibrium portfolios are quite different in this case. Figure 6 tells us why. In response to productivity shocks, the nominal exchange rate first depreciates, due to the rise in the supply of the Home good. In the long
run, however, it appreciates. Overall, the effect of the long run appreciation dominates and relative returns of long term bonds rise, while returns of short term bonds fall. As a result, nominal bond returns are now strongly negatively correlated with human capital risk and consumption expenditure risk, both conditionally and unconditionally. The human capital motive then induces Home investors to take long positions in Home bonds, while the consumption expenditure motive calls for short positions. Since the latter prevails, the sign of the bond position is thus more in line with the available U.S. evidence, though the size is too large. Equity positions are also quite different. The relationship between equity returns and human capital is qualitatively unchanged, but the size of the contribution of the human capital motive is much lower. Equity home bias is much larger overall, however, due to the consumption expenditure motive. This is mainly due to the fact that consumption expenditure risk is now less correlated with bond returns, increasing the conditional correlation between equity returns and consumption expenditure risk, conditional on bond returns.

Secondly, we consider the asset menu of Coeurdacier et al (2009). There, agents trade equities and real consols. Real consols have a payoff equal to the local price level each period. Relative returns are then given by $\hat{R}_{t}^{B,real} = (1 - \beta) \hat{E}_{t} \left[ \sum_{j=0}^{\infty} \beta^{t+j} (-\hat{Q}_{t+j}) \right]$, implying that relative bond returns now depend on the path of real, not nominal exchange rates. In our model, markets are incomplete, and relative consumption expenditures are not perfectly correlated with the real exchange rate, even at first order approximation (see also Backus and Smith (1993) and Kollmann (1995)). However, the correlation between the two is still very high. As a result, long positions in real bonds provide an almost perfect hedge for consumption expenditure risk and Home agents consequently take a large long position in them. The fact that real bonds can provide an almost perfect hedge for relative consumption expenditure risk then also explains why the effect of the consumption expenditure motive on equity positions is virtually zero in that case.

7.4 Data Sources

The data is quarterly from the first quarter of 1970 to the third quarter of 2008. We take the U.S. as the Home country and a GDP weighted aggregate of the G7 countries with

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43 The long run appreciation of the nominal exchange rate is based on our assumptions about the monetary policy rule. Monetary policy here stabilises inflation, not the price level. In response to productivity shocks, relative inflation falls, leading over time to a fall in the relative price level. Since real exchange rates are stationary, this implies that the nominal exchange rate has to appreciate in the long run in order for the real exchange rate to return to its steady state value (of one).

44 The fact that relative consumption expenditure risk is perfectly hedged through bond bond positions is also at the heart of the result in Coeurdacier et al (2009) that equity positions do not depend on preference parameters, and in particular, the elasticity of substitution between Home and Foreign goods. In our benchmark case, bond positions do not provide a perfect hedge for consumption expenditure risk and as a result, preference parameters do have an effect on the equilibrium portfolios. In general, relative equity returns are a good hedge for relative human capital returns only for a sufficiently low value of $\phi$. 

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the exception of the U.S. (Canada, France, Germany, Italy, Japan, UK) as the Foreign country. National account data is from the OECD quarterly national accounts (QNA), Economic Outlook (EO) and Main Economic Indicators (MEI) databases, while data on equity and bond returns and nominal exchange rates is from the Global Financial Data (GFD) database.

**Output:** Gross domestic product, volume, market prices, EO  
**Consumption:** Private final consumption expenditure, volume, EO  
**Investment:** Gross capital formation, total, volume, EO  
**Labour:** Total employment times Hours worked per employee, Total Economy, EO  
**Real exchange rate:** Ratio of Consumer Prices, all items, MEI, converted into U.S. dollars  
**Net exports:** Net exports of goods and services, USD, EO  
**Labour income:** Compensation of employees (QNA) plus share of mixed surplus (QNA), as described in Coeurdacier and Gourinchas (2009) and based on Gollin  
**Consumption expenditures:** Private Final Consumption Expenditures, QNA  
**Equity returns:** S&P/TSX-300 Total Return Index (Canada), SBF-250 Total Return Index (France), CDAX Total Return Index (Germany), BCI Global Return Index (Italy), Topix Total Return Index (Japan), FTSE All-Share Return Index (UK), S&P 500 Total Return Index (US), all from GFD  
**Bond returns:** 3 month bill rates, GFD

### 7.4.1 Estimation of the Process for Investment Efficiency Shocks

We follow the literature on investment specific technology shocks and used real investment prices as a measure of the investment specific technology shock (e.g. Fisher (2002, 2006)). The real investment price index is computed as the total capital formation deflator divided by the CPI deflator, both from the OECD Economic Outlook database. We estimate an AR(1) process for real investment prices for the U.S. and the Foreign country, where we aggregate the Foreign country, as above. The series are logged and Hodrick-Prescott filtered with a smoothing parameter of 1600 before estimating. Fisher (2006) notes that prior to 1982, investment deflators for the U.S. included little adjustments for quality changes. Consequently, the sample period chosen for our regressions is from the first quarter of 1982 until the third quarter of 2008.
Table A.1: Portfolios for additional model variants

<table>
<thead>
<tr>
<th>Share of Home Equity</th>
<th>Benchmark (I)</th>
<th>Consols (II)</th>
<th>Real Consols (III)</th>
<th>Flex Wage (IV)</th>
<th>No Inv Adj Cost (V)</th>
<th>No Frictions (VI)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.90</td>
<td>1.52</td>
<td>0.88</td>
<td>0.89</td>
<td>0.98</td>
<td>1.12</td>
</tr>
<tr>
<td>due to:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Human Capital</td>
<td>0.36</td>
<td>0.02</td>
<td>0.38</td>
<td>0.36</td>
<td>0.22</td>
<td>−0.01</td>
</tr>
<tr>
<td>Consumption Exp</td>
<td>0.04</td>
<td>0.99</td>
<td>0.00</td>
<td>0.03</td>
<td>0.26</td>
<td>0.63</td>
</tr>
</tbody>
</table>

Correlations:

\[ \text{Corr} \left( \hat{R}_S, \hat{R}_{WL} \right) \] = 0.24  
\[ \text{Corr} \left( \hat{R}_S, \hat{R}_{PC} \right) \] = 0.49

<table>
<thead>
<tr>
<th>Home Bonds/GDP</th>
<th>Benchmark (I)</th>
<th>Consols (II)</th>
<th>Real Consols (III)</th>
<th>Flex Wage (IV)</th>
<th>No Inv Adj Cost (V)</th>
<th>No Frictions (VI)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.49</td>
<td>−1.99</td>
<td>2.76</td>
<td>0.50</td>
<td>0.45</td>
<td>0.39</td>
</tr>
<tr>
<td>due to:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Human Capital</td>
<td>−0.76</td>
<td>1.04</td>
<td>−4.32</td>
<td>−0.72</td>
<td>−0.68</td>
<td>−0.41</td>
</tr>
<tr>
<td>Consumption Exp</td>
<td>1.24</td>
<td>−3.02</td>
<td>7.07</td>
<td>1.22</td>
<td>1.13</td>
<td>0.82</td>
</tr>
</tbody>
</table>

Correlations:

\[ \text{Corr} \left( \hat{R}_B, \hat{R}_{WL} \right) \] = 0.96  
\[ \text{Corr} \left( \hat{R}_B, \hat{R}_{PC} \right) \] = 0.99

(I) follows the benchmark calibration described in the text, (II) features trade in nominal consols and equities, (III) has trade in equities and real consols, (IV) sets \( \theta_w = 0 \), (V) sets \( v \to \infty \), (VI) sets \( \theta = \theta_w = 0 \) and \( v \to \infty \). Unconditional correlations between the asset return innovation and source of risk are shown in brackets. Contributions of hedging motives and correlations were computed using 500 simulations of 500 periods. Contributions of hedging motives were calculated from regressions of the sources of risk on relative bond and equity return innovations and expressions (7) and (8). Asset returns and sources of risk were Hodrick-Prescott filtered with a smoothing parameter of 1600 before computing correlations.
Figure 1: Impulse Responses of Asset Returns and Sources of Risk. This figure presents the response of relative equity returns, relative bond returns, relative human capital returns and consumption expenditure risk to relative shocks to neutral and investment specific technology and the interest rate. Relative shocks are defined as a positive shock to the Home variable of one standard deviation, with a negative shock to the foreign variable of the same size. Note that a relative monetary shock here is a positive shock to nominal interest rates and is therefore contractionary.
Figure 2: Impulse Responses of Portfolio Relevant Variables. This figure presents the response of relative dividends, nominal exchange rates, relative labour income and relative consumption expenditures to relative shocks to neutral and investment specific technology and the interest rate. Relative shocks are defined as a positive shock to the Home variable of one standard deviation, with a negative shock to the foreign variable of the same size. Note that a rise in the nominal exchange rate signifies a depreciation of the Home nominal currency.
Figure 3: Impulse Responses of Relative Equity Returns and Relative Human Capital Returns, conditional on Relative Bond Returns. This figure presents the response of relative equity returns and relative human capital returns for combinations of shocks that leave relative bond returns unchanged. The first graph presents responses to a combination of neutral and investment specific technology shocks, the second to a combination of monetary and investment specific technology shocks and the third to a combination of neutral technology and monetary shocks. Responses of relative equity returns are given by the solid line, while responses of relative human capital returns are given by the dashed line.
Figure 4: Impulse Responses of Other Macro Variables I. This figure presents the response of relative output, consumption, investment and labour demand to relative shocks to neutral and investment specific technology and the interest rate. Relative shocks are defined as a positive shock to the Home variable of one standard deviation, with a negative shock to the foreign variable of the same size. Note that a relative monetary shock here is a positive shock to nominal interest rates and is therefore contractionary.
Figure 5: Impulse Responses of Other Macro Variables II. This figure presents the response of the real exchange rate and net exports to relative shocks to neutral and investment specific technology and the interest rate. Relative shocks are defined as a positive shock to the Home variable of one standard deviation, with a negative shock to the foreign variable of the same size. Note that a relative monetary shock here is a positive shock to nominal interest rates and is therefore contractionary.

Figure 6: Impulse Responses of Nominal Exchange Rates to a Relative Productivity Shock.