

Legal Enforcement, Collateral and Heterogeneity of Project Financing Contracts

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Abstract

This paper employs mechanism design to study the effects of imperfect legal enforcement on optimal scale of projects, borrowing interest rates and the probability of default. The analysis departs from an environment that combines asymmetric information about cash flows and limited commitment by borrowers. Incentive for repayment comes from the possibility of liquidation of projects by a court, but courts are costly and may fail to liquidate. The value of liquidated assets can be used as collateral: it is transferred to the lender when courts liquidate. Examples reveal that costly use of courts may be optimal, which contrasts with results from most limited commitment models, where punishments are just threats, never applied in optimal arrangements. I show that when voluntary liquidation is

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allowed, both asymmetric information and uncertainty about courts are necessary conditions for legal punishments ever to be applied. Numerical solutions for several parametric specifications are presented, allowing for heterogeneity on initial wealth and variability of project returns. In all such solutions, wealthier individuals borrow with lower interest rates and run higher scale enterprises, which is consistent with stylized facts. The reliability of courts has a consistently positive effect on the scale of projects. However its effect on interest rates is subtler and depends essentially on the degree of curvature of the production function. Numerical results also show that the possibility of collateral seizing allows comovements of the interest rates and the probability of repayment.

1 Introduction

Both credit markets and the quality of institutions are believed to play a key role in the development of countries. A significant literature argues that their roles in the development process are linked¹. As pointed out by North (1981), good institutions can provide support for private contracts. In particular, an adequate institutional environment enhances the capacity of agents to commit, by enforcing actions determined in contracts. The effects of limited commitment on economic outcomes are widely discussed in the literature: it not only diminishes risk sharing, but also compromises the capacity of agents to invest and thus has a negative impact on growth and social mobility.

The literature that investigates the impact of limited commitment on the capacity to invest typically focus on credit rationing and occupation choice². However there is a widespread acceptance of the idea that the quality of legal enforcement also affects the probability of repayment of loans and interest rates, which is supported by empirical studies³. The incapacity

¹For examples, see Acemoglu and Johnson (2005), LaPorta *et. al.*(1998), Knack and Keefer(1995) and Mauro (1999).

²Examples are Banerjee and Newman (1993), Evans Jovanovic (1993), Holtz-Eakin, Joulfian and Rosen (1994) and Magnac and Robin (1996).

³See Laeven and Madjnoni (2003), Costa and Mello (2006) and Visaria (2005).

of institutions to enforce contracts would stimulate default, and thus increase risks and interest rates. A theoretical investigation of how these two margins - amount of credit and interest rates - respond jointly to the capacity of agents to commit is, to my knowledge, missing in the theoretical development literature. The way in which these two margins interact is not trivial. Is it necessarily the case that bad institutions increase interest rates? Isn't it possible that, with bad institutions, agents anticipate high probabilities of default and credit is rationed so that contracts with high probabilities of default are avoided? These questions can only be answered with a theoretical model that studies these two margins simultaneously. A model that features the impact of limited commitment on interest rates, and not only on credit rationing, is attractive for at least two reasons. First, interest rates have information about credit markets and such a model can be used as a guideline for the interpretation of credit markets data. Second, high interest rates are commonly regarded as a problem in developing countries, and optimal policies to deal with them depend on their causes. If, for instance, they are explained by the lack of competition, price control policies and public provision of credit would be reasonable. On the other hand, if the problem is imperfect enforcement, the emphasis should be on the improvement of legal enforcement mechanisms.

This paper employs mechanism design to evaluate how imperfect legal enforcement affects simultaneously the amount of credit, interest rates and probability of repayment for heterogeneous agents. It studies optimal arrangements between borrowers and lenders conditional on a simple characterization of the environment, that includes the quality of legal enforcement⁴. The analysis is based on a three periods model that combines hidden savings and cash flows with limited commitment. The source of limited commitment is the possibility of strategic default. Individuals borrow from a risk neutral lender in a first period to finance a project that produces two identical cash flows. Incentive for repayment comes from the possibility of liquidation of the project by a court before the last cash flow. When a court liquidates a project, its liquidated

⁴In this sense, this paper differs from Hart's approach of incomplete contracts and residual rights, see Hart (1995).

assets are transferred to the lender. Therefore capital plays the role of collateral. But courts are costly and may fail to liquidate a project even when a contract specifies that it should be liquidated. Lenders are able to commit and are only subject to a zero profits constraint.

I assume that cash flows are revealed to the borrower only in the second period, and that they may be hidden by borrowers. This ex-ante asymmetric information limits the capacity of contracts to condition transfers on cash flow realizations, and the repayment schedule resembles a debt contract: even when there is risk aversion, whenever cash flows exceed a certain threshold level (such that repayment is certain), the amount to be repaid does not depend on cash flows. There is a pooling value of repayments, equal for several realizations of cash flows, that can be used to define borrowing interest rates.

By such definition, borrowing interest rates should not be interpreted simply as an intertemporal price. They determine the amount of transfers requested by contracts to guarantee that borrowers are not subject to legal punishment, but under some contingencies this amount is not paid. When this happens, individuals face the risk of having their projects liquidated. This interpretation of interest rates is consistent with the real behavior of credit markets. In practice, repayment is not always observed. Renegotiation among borrowers and lenders and punishment of defaulters are commonly observed.

An important result follows from this combination of asymmetric information with limited commitment. Optimal arrangements may include the possibility of default, defined as the costly use of courts to request liquidation of projects. This result is shown by numerical examples, and contrasts with most limited commitment models, where punishments are just threats, never applied in optimal arrangements. This happens even when voluntary liquidation is allowed, which means that courts are not the only technology available to liquidate, they are only enforcement tools. I show that when voluntary liquidation is allowed, both asymmetric information and uncertainty about courts are necessary conditions for legal punishments ever to be applied.

The model allows for individual heterogeneity in two ingredients that are commonly believed

to affect the access and risk of credit: the quality of projects – which includes expected return and variance of project outcomes - and wealth of borrower - and thus their capacity to provide collateral. The predictions of the model about how institutions impact on heterogeneous agents are studied by numerical simulations, developed for several parameter specifications. This analysis is useful to confront the model with stylized facts of the credit market, and it also reveals that optimal contracts depend strongly on characteristics of borrowers, which reinforces the importance of using microdata to understand credit markets.

In all the numerical solutions, wealthier individuals borrow with lower interest rates and run higher scale enterprises, which is consistent with stylized facts. The reliability of courts has a consistently positive effect on the scale of projects. However its effect on interest rates is more subtle. A theoretical proposition shows that, when the scale of projects is fixed, interest rates decrease with reliability of courts. This is also observed in numerical solutions with a high curvature on the production function. But for lower curvature in the production function, there is a high response of project scale to legal improvements, but interest rates may increase with more reliable courts. The curvature of the production function also affects the cross sectional relation between initial wealth and borrowing. For high degrees of curvature, borrowing decreases with wealth, while for low curvature the wealth profile of amount of borrowing has an inverted U shape. This suggests that the cross sectional relation between wealth and borrowing could have some information about how improvements in enforcement affects interest rates.

A numerical result that shows the importance of collateral in the model, is the possibility of comovements in the interest rates and probability of repayment. There are numerical examples where, as the initial wealth increase, the interest rates and the probability of repayment both go down. As wealth increases, there is a higher expected transfer of liquidated assets to the lender and this makes it possible that repayment generates a smaller fraction of the lenders revenue, which allows lower interest rates. Those transfers of liquidated assets can be interpreted as collateral seizing.

The theoretical model used in this paper is related to the development literature on limited

commitment and credit rationing. This includes Evans and Jovanovic (1989), where the amount of borrowing available increases with initial wealth, and Lloyd-Ellis and Bernhardt (2001) and Buera (2006), where credit is completely rationed. The model here presented also has credit rationing: there is a maximum amount of borrowing that depends on initial wealth. But in general, the maximum amount of borrowing available is not optimum, even for individuals that are credit constrained.

Another branch of the literature that is related with the current paper is the one on borrowing contracts with ex-post asymmetric information. The idea that asymmetric information may limit the capacity to obtain repayment from agents with positive shocks and produce optimal contracts that resemble debt⁵ was first explored by Townsend (1979), in the context of costly state verification, and originated a large literature⁶. The key idea of costly state verification models is that outputs can be verified by lenders after an audit. In my analysis, I consider the case in which cash flows are unobservable, or there is infinite cost of state verification. This is reasonable specially for small firms, that have a magnitude of cash flows that do not justify auditing costs. With unobservable cash flows, the only tool available for repayment is liquidation. In this framework, collateral is a natural part of borrowing contracts: bigger firms have a higher amount of assets to be used as repayment guarantees.

The threat of liquidation as an incentive for repayment is a common ingredient in the literature of credit contracts. This includes Bolton and Sharfstein(1990) Hart and Moore (1998) and Albuquerque and Hopenhayn (2004). Differently from this literature, this paper interprets default as a result of the incapacity of institutions to enforce contracts, and not simply as liquidation, that may be efficient. The environment considered is similar to DeMarzo and Fishman (2003), with unobserved cash flows and the threat of liquidation as an incentive for repayment. I simplify the dynamic structure to three periods but add heterogeneity of project

⁵Notice that debt contracts are not necessarily the optimal contracts in CSV models. Some optimal contracts that require randomization differ from debt contracts.

⁶e.g. Williamson (1997) in the study of of credit rationing, Gale and Hellwig (1985) focusing on suboptimal capital accumulation and the dynamic extensions by Chang (1990) and Wang (2005).

scale and imperfect enforcement. This inefficiency of institutions to enforce contracts may increase the probability of liquidation. I also allow for risk aversion among investors, a feature that is uncommon in this literature.

The effects of courts on interest rates in an environment that combines asymmetric information with limited commitment is discussed by Krasa, Sharma and Villamil (2004). They depart from an environment in which costly state verification and the possibility of renegotiation makes debt contracts optimal ⁷. Then, they study the effects of two parameters of court quality - the fixed cost of state verification and the fraction of income that can be seized - on repayment and interest rates. There are two fundamental differences between their approach and mine. First, I allow scale of projects to be an endogenous variable that plays the role of collateral. This is crucial in the analysis of heterogeneity of contracts. Second, I interpret courts only as an enforcement tool, while they model them as a technology of state verification.

The paper is organized as follows. Section 2 presents the model, the characterization of the optimal contracts and some propositions describing their properties. Section 3 shows some results from linear programming solutions for the general case where risk aversion is allowed. Section 4 specializes the model to risk neutrality and present some numerical solutions for this case. Section 5 summarizes and discusses the results and confronts them with some findings from the empirical related literature. It also points out directions for future research.

2 The Environment

The economy consists on investors, that live for three periods and receive an endowment (or initial wealth) w in the first period of life, and a risk neutral lender. The utility of the investors is given by :

⁷This framework was first employed by Krasa and Villamil (2000). The idea is similar to costly state verification, but the possibility of renegotiation (incorporated by a Nash Perfect Bayesian Equilibrium) rules out randomization and makes debt contracts optimal.

$$U(c_1) + \beta U(c_2) + \beta^2 U(c_3),$$

where $0 \leq \beta \leq 1$, and c_t is consumption in period t , $U' > 0$ and $U'' \leq 0$. The lender is assumed to be able to commit to a policy specified in a contract, and is willing to offer any contract with nonnegative profits (defined as the present value of transfers to the borrower). At each period investors can save in an outside market with an exogenous interest of r .

In their first period of life, investors can invest in a risky project. When they invest an amount k in the first period, the project produces a cash flow of $\theta f(k)$ in the following two periods, where $\theta \in \Theta$ is a random project quality parameter, with f.d.p $h(\theta)$, that is common knowledge. The parameter θ is unobserved when investment takes place: it is revealed to the investor only after the first cash flow. The parameter distribution $h(\theta)$ is the source of ex ante heterogeneity in the expected quality of projects, while the realization θ is an ex-post (after the contract is defined) shock. The project can be liquidated after the first cash flow, producing an outcome ik in the third period. I assume that $i < (1 + r)^2$, implying that it is not worth to invest in a project just to liquidate it.

Investors can borrow from the lender, that has the capacity to commit. However, the borrowing contracts are constrained by a combination of asymmetric information and limited commitment from the investor. When borrowing takes place, a contract can specify default conditions. These are conditions under which the lender has the right to require the liquidation of the firm by a court. When a court liquidates a project, the outcome of liquidation (or the collateral), ik , is transferred to the lender after one period. But courts are imperfect: they require the payment of a fixed cost $c \geq 0$ to be activated, and when they are activated they liquidate the project with a probability $\lambda \leq 1$.

Savings and cash flows of the borrower cannot be observed by the lender and the courts. After observing the cash flow, a borrower can produce a message about his type, that may be used in contracts. Borrowing contracts must be subject to a zero profits condition: the present value of transfers from the lender to the borrower is not bigger than zero. The interest rate

used to define this present value is r , so the borrower and the lender face the same lending interest rate.

A key feature of the model is the possibility of strategic default. After θ is observed and repayment values are defined, the investors can decide between three options. First repayment, that guarantees that the firm will not be liquidated. Second default, that implies liquidation with a probability λ . Third, voluntary liquidation. The inclusion of a voluntary liquidation option on the contract is important to make it clear that institutions affect credit markets only through their effect on enforcement. The need of courts does not come from them being the only entity with access to a liquidation technology. Their only purpose is to enforce liquidation, something that the agents can do by themselves. Contracts can specify transfers between the lender and the borrower for each of these options. But it cannot specify positive transfers from the borrower to the lender in the case of default. Under default, only the collateral (the liquidation value of the firm) can be transferred to the lender. And this happens with probability λ .

All transfers between the lender and the borrower take place in the first and second periods. This is not restrictive, since in the third period the threat of liquidation cannot be used to extract payments from the borrowers *and*, as the borrower can save in the second period facing the same interest rate as the lender, there is no need for the lender to make transfers to the borrower in the third period.

A contract defines a borrowing amount $b \in B \subseteq \mathfrak{R}$, a scale of project $k \in K \subseteq \mathfrak{R}_+^2$ and a vector $p \equiv (p_r, p_v, p_{d1}, p_{d2}) \in P$ describing the transfers from the borrower to the lender in the second period under repayment (p_r), voluntary liquidation (p_v), default and liquidation by a court (p_{d1}) and default and no liquidation by the court (p_{d2}). As it was argued above, p_{d1} and p_{d2} cannot assume positive values. The assumption that the values of p_{d1} and p_{d2} cannot be positive is the crucial limited commitment constraint.⁸ Therefore, $P \subseteq \mathfrak{R}^2 \times \mathfrak{R}_-^2$. The vector p

⁸Alternative equivalent formulation could characterize as decision variables to be defined in the contracts the discrete choice between default, voluntary liquidation and repayment, d , and the amount of second period transfers p , and impose an alternative limited commitment constraint that any choice for the this decision

may be a random function of the message about the cash flow. Notice that k must be specified in the contract since it plays the role of collateral.

The timing of events is described in the timeline presented in figure 1. In the first period, investors receive a bequest w . Then a borrowing contract defines an amount of borrowing b and a scale of project, k . Finally borrowers save an amount s_1 . In the second period, the borrower receive a cash flow and thus observes θ , issues a message μ about the cash flow, and, based on μ , a vector p is defined. After p is defined, the borrower takes the decision between voluntarily liquidating, repaying or defaulting (decision node D). If the decision is default, the court have a probability λ of liquidating and a probability $(1 - \lambda)$ of not liquidating. Notice that in the second period, optimal savings are different for each branch of the three. I denote savings under repayment, voluntary liquidation, default with liquidation and default without liquidation respectively by s_r , s_v , s_{d1} and s_{d2} .

When investors are taken to court but the court fails to liquidate, they have the possibility to liquidate the project and obtain ik or wait for a cash flow. This is expressed by the binary variable l , that takes value 1 if the decision is for liquidation and 0 if it is not to liquidate. A proposition to be stated below shows that under reasonable conditions, borrowers will never liquidate when courts fail to do so. This means that l will always be equal to zero in optimal contracts. Therefore, throughout the paper, the term voluntary liquidation refers to liquidation before investors are taken to courts. The inclusion of the possibility of voluntary liquidation after courts - that could also be thought as another decision node after court decision in the tree below - makes the model more realistic, and also helps in the derivation of properties of

variable must produce at least the utility of default with zero transfers.

optimal contracts.

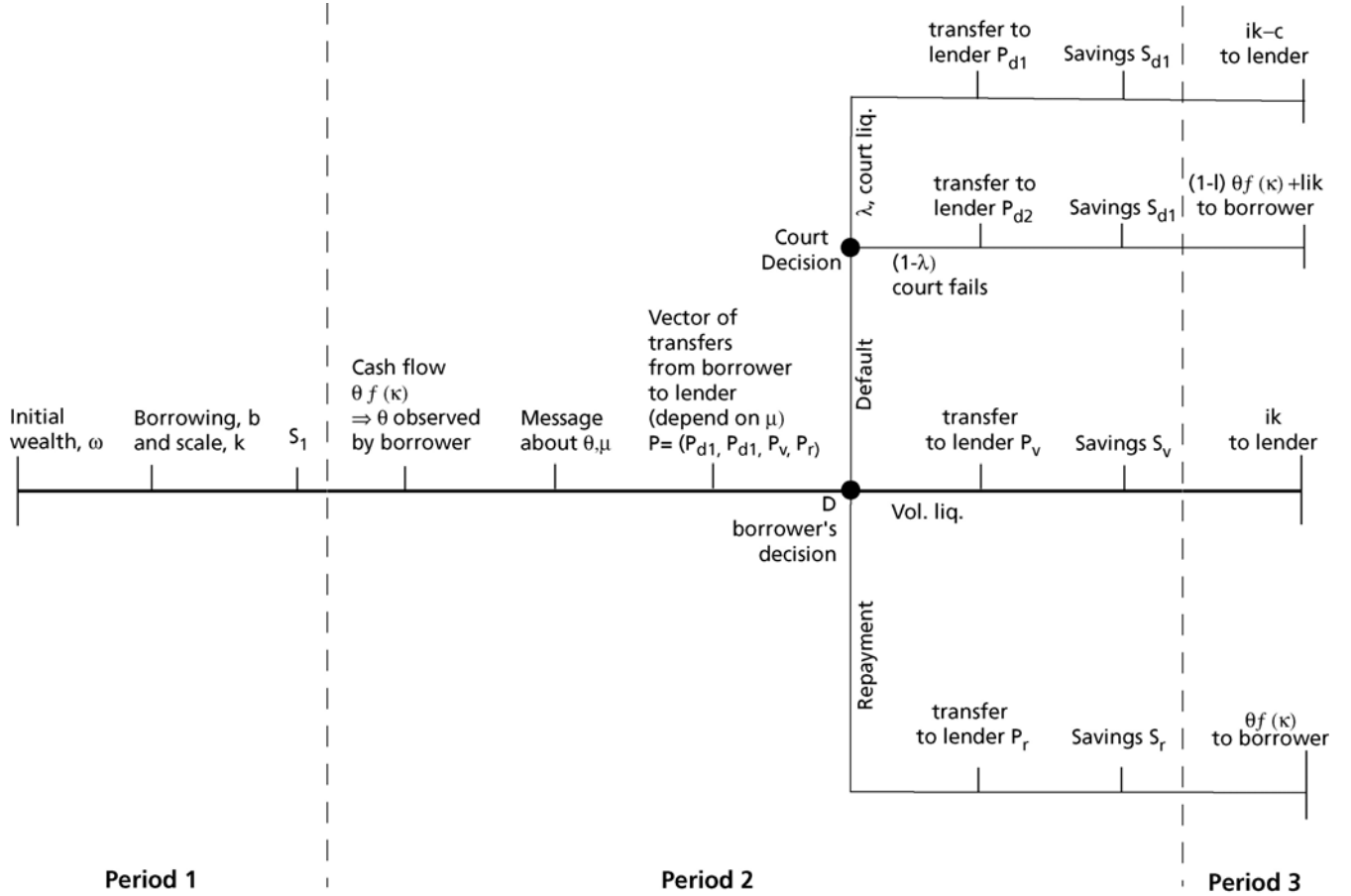


Figure 1 - Timeline and Structure of the Problem

2.1 The Optimal Contract

Definition A borrowing contract is a triple $\{b, k, \pi(p, \theta)\}$, where $b \in B \subseteq \mathfrak{R}$ is the level of borrowing, $k \in K \subseteq \mathfrak{R}^+$ is the scale of the project financed, and $\pi : P \times \Theta \rightarrow (0, 1)$ is the joint probability distribution of p and θ .

Notice that savings in the first period, s_1 , are not part of the contract. In principle, contracts could include s_1 as a choice variable. If this were the case, the contract would need to produce incentives for individuals to adopt the prescribed level of savings. But assuming that s_1 is equal to zero, and including in the contract incentives for people not to save positive amounts does not constrain the problem. As the lender and the borrower face the same saving interest

rate, savings may be implicitly provided by the lender. Indeed, suppose a contract specifies an amount of borrowing b , a saving amount of s_1 and a distribution of p conditional on θ of $\pi(p | \theta)$. With an alternative contract with borrowings $\tilde{b} = b - s_1$, and distribution of transfers $\tilde{\pi}(p + s_1(i + r) | \theta) = \pi(p | \theta)$, and $s_1 = 0$, consumers and the bank would face the same resources in each state as in the first contract. Therefore, if they did not have incentives for hidden savings in the first contract, they would not also have incentives for so in the new one. Notice that this transformation may require a negative value of "borrowing", \tilde{b} . This is not ruled out of the set of possible contracts. From now on, I assume that $s_1 = 0$. But an incentive constraint determining that no hidden savings ($s_1 > 0$) are desired by agents must be added to the contract.

The solution to the problem is constructed backward. After transfers are given in the second period, the borrower defines second period savings. Unlike in the first period, savings in the second period may be bigger than zero, as there are no transfers between the borrower and the lender in the third period. In the case of default and no liquidation by courts, individuals also choose an optimal value of l , defining the decision between liquidating or not after being released by courts. The second period savings decision and the choice of l determine indirect utilities from the second period on conditional on transfers (that includes the utility from third period). The indirect utilities in the second period under repayment, voluntary liquidation, default with liquidation by courts and default without liquidation by courts are respectively denoted by: $V_2^r(\theta f(k), p_r)$, $V_2^v(\theta f(k), p_v)$, $V_2^{d1}(\theta f(k), p_{d1})$, and $V_2^{d2}(\theta f(k), p_{d2})$. Notice that by the concavity of U , all of these functions are concave on both arguments. I denote the indirect utility under default as:

$$V_2^d(\theta f(k), p_{d1}, p_{d2}) \equiv V_2^{d1}(\theta f(k), p_{d1})\lambda + V_2^{d2}(\theta f(k), p_{d2})(1 - \lambda).$$

At the decision node D , investors take the utility maximizing decision. Therefore, the indirect utility in the second period given the vector p is:

$$V_2(\theta f(k), p) = \max\{V_2^d(\theta f(k), p_{d1}, p_{d2}), V_2^v(\theta f(k), p_v), V_2^r(\theta f(k), p_r)\}.$$

The function describing the discrete decision at the node D is $d(A\theta, k, p)$. This function is

defined as follows:

$$\begin{aligned}
d(\theta, k, p) = 2 & \quad \text{if } V_2^v(\theta f(k), p_v) > V_2^r(\theta f(k), p_r) \\
& \quad \text{and } V_2^v(\theta f(k), p_v) \geq V_2^d(\theta f(k), p_{d1}, p_{d2}) \\
d(\theta, k, p) = 1 & \quad \text{if } V_2^r(\theta f(k), p_r) \geq \max\{V_2^d(\theta f(k), p_{d1}, p_{d2}), V_2^v(\theta f(k), p_v)\}, \\
d(\theta, k, p) = 0 & \quad \text{if } V_2^d(\theta f(k), p_{d1}, p_{d2}) > \max\{V_2^v(\theta f(k), p_v), V_2^r(\theta f(k), p_r)\}
\end{aligned} \tag{1}$$

The function d takes value 1 when there is repayment, 2 when there is voluntary liquidation and zero when there is default.

A first step in the characterization of the optimal contract is the definition of the optimal distribution of transfers in the second period conditional on θ , when b , w and k are given. The choice variable in this program is $\pi(\theta, p)$, the joint probability distribution of θ and p . The choice of $\pi(\theta, p)$ is subject to the following conditions. First there is a technological constraint stating that θ is distributed according to $h(\theta)$:

$$\forall \bar{\theta}, \quad \sum_p \pi(p, \bar{\theta}) = h(\bar{\theta}). \tag{2}$$

Second, the transfers policy must be such that investors have no incentives to hide cash flows:

$$\forall \bar{\theta}, \hat{\theta} < \bar{\theta}.$$

$$\sum_p \pi(p | \bar{\theta}) V_2(\bar{\theta} f(k), p) \geq \sum_p \pi(p | \hat{\theta}) V_2(\hat{\theta} f(k), p). \tag{3}$$

Another condition on π is that individuals should have no incentive to make hidden savings. In the case of risk neutrality this condition is innocuous. But if investors are risk averse, the effect of savings on second period utilities depend on θ , p and the decision between repayment, default and voluntary liquidation. With positive savings and risk aversion, (3) may not be a correct characterization of incentives for individuals to report the truth cash flows. The condition for no hidden savings is necessary for (3) to be an accurate incentive constraint. This condition is:

$\forall \mu : \Theta \rightarrow \Theta$ with $\mu(\theta) \leq \theta$ and $\forall s_1 \in (0, b + w - k)$,

$$\begin{aligned} \sum_{p, \theta} \pi(p, \theta) (U(B + w - k) + \beta V_2(\theta f(k), p)) &\geq \\ \sum_{p, \theta} \pi(p, \mu(\theta)) \frac{h(\theta)}{h(\mu(\theta))} (U(B + w - k - s_1) + \beta V_2(\theta f(k), p + (1 + r)s_1)). \end{aligned} \quad (4)$$

This condition states that for any reporting strategy, μ , there is no incentives to hidden savings. The participation condition for the lender, or zero profit condition, is:

$$\begin{aligned} B(1 + r) &\geq \sum_{p, \theta} \pi(p, \theta) \left[d(2 - d)p_r + \frac{(1 - d)(2 - d)}{2} \left(\lambda(p_{d2} + \frac{ik}{1 + r}) \right. \right. \\ &\left. \left. + (1 - \lambda)p_{d1} - c \right) + (d - 1) \frac{d}{2} \left(p_v + \frac{ik}{1 + r} \right) \right], \end{aligned} \quad (5)$$

where d is defined by (1). The conditions for π to be a probability distribution are:

$$\pi \geq 0, \quad \sum_{p, \theta} \pi(p, \theta) = 1. \quad (6)$$

Notice that given w , conditions (2) to (6) cannot be fulfilled for some values of k and b . They can only be defined for a set of feasible borrowing-scale combinations $\Gamma(w)$. This is the set of values of k and b such that:

- (2) to (6) are valid for some π given w, b and k .
- $b + w - k \geq 0$.

Clearly, $\Gamma(w)$ is not empty. Indeed, setting $k = w, b = 0, p_{d1} = p_r = p_v = 0$, and $p_{d2} = -ik$, all constraints of the problem are satisfied. This implies that at least one contract is available to any individual.

Given w , for any $(b, k) \in \Gamma(w)$, the optimal transfers in the second period are defined by the following program:

Program 1

$$\tilde{V}_1(b, k, w) = \max_{\pi} U(b + w - k) + \beta \sum_{p, \theta} \pi(p, \theta) V_2(\theta f(k), p), \quad (7)$$

s.t. (2) to (6).

Given the function $\tilde{V}_1(b, k, w)$, it is possible to define the program determining the choices of b and k given w and A . This program is:

$$\text{Program 2} \quad \tilde{V}(w) = \max_{(b, k) \in \Gamma(w)} \tilde{V}_1(b, k, w).$$

2.2 Some Properties of the Solution

This subsection presents some propositions describing the characteristics of optimal contracts. These propositions depend basically on assumptions on the second period indirect utility functions. Although these assumptions are stated in terms of the indirect utilities, they ultimately depend on the function U . They are all valid for standard specifications of the utility function U such as *CARA* and *CRRA*. Most of them (assumptions (a), (b), (c), (d) and (e)) are valid with linear utility (risk neutrality). The assumptions used in the derivation of the results are:

Assumptions

- (a) $V_2^r(\theta f(k), p)$ is concave on p and $\theta f(k)$.
- (a') $V_2^r(\theta f(k), p)$ is strictly concave on p and $\theta f(k)$.
- (b) $(\partial^2 V_2^r(\theta f(k), p) / \partial^2 p) / (\partial V_2^r(\theta f(k), p) / \partial p)$ is nonincreasing with $\theta f(k)$.
- (c) $-(\partial^2 V_2^r(\theta f(k), p) / \partial^2 p) / (\partial V_2^r(\theta f(k), p) / \partial p)$ is nonincreasing with p .
- (d) $(\partial^2 V_2^r(\theta f(k), p) / \partial p \partial \theta f(k)) \geq 0$.
- (d') $(\partial^2 V_2^r(\theta f(k), p) / \partial p \partial \theta f(k)) > 0$.
- (e) $(\partial^2 V_2^v(\theta f(k), p) / \partial^2 p) / (\partial V_2^v(\theta f(k), p) / \partial p)$ is non increasing with $\theta f(k)$.

Assumption (a) is a consequence of U being concave. Condition (b) states that the absolute risk aversion of repayers with respect to transfers received in the second period is nonincreasing

with θ . Condition (c) that it is nonincreasing with transfers received in the second period. Condition (d) states that the marginal utility of receiving transfers in the second period is non-increasing with θ . Condition (e) states that the indirect utility of liquidators has nonincreasing absolute risk aversion with respect to transfers in the second period. An absolute risk aversion that is not increasing with θ guarantees that if repayment or liquidation is certain, it is possible to substitute lotteries for nonrandom utility equivalents without producing additional incentives for high θ individuals to misreport their cash flows. Nonincreasing absolute risk aversion with transfers received in the second period guarantees that this can be done without additional incentives for hidden savings.

The following proposition states that there is a pooling value of repayment for all types that pay with certainty. This is what makes the optimal contracts similar to debt contracts.

Proposition 1 *Let b, k and w be given. Suppose that p_r can assume any value in \mathfrak{R} , that Assumptions (a) to (d) are valid and that Θ is finite. Then, if an optimal contract implies that types θ_1 and θ_2 chose repayment with probability one, there exists an optimal contract in which both types repay the same amount \hat{p}_r with probability 1. If (a') and (d') are valid, this is a necessary result: an optimal contract where types θ_1 and θ_2 repay with certainty must have both types repaying the same amount with certainty.*

Proof. See appendix 1 ■

This follows from asymmetric information about cash flows. Ideally, with risk aversion, it would be desirable to extract higher payments from individuals with high cash flows. But this is not possible when cash flows can be hidden. Individuals with high cash flow realizations would have incentives to misreport their cash flows. Therefore, the amount of repayment that guarantees zero probability of project liquidation does not depend on θ . This bunching value of repayments can be used to define borrowing interest rates as:

$$r_b = \frac{\hat{p}_r}{b} - 1, \quad (8)$$

where \hat{p}_r is the pooling amount of repayment defined in proposition 1. Notice that this bor-

rowing interest rate differs from r , the outside market saving interest rate.

The presence of a unique value of repayment for types that repay with certainty is directly related to the fact that sometimes randomization is optimal contracts. Randomization may be used to separate high cash flows from medium cash flow investors. Proposition 1 implies that for values of θ such that the probability of repayment is 1, the amount repaid does not depend on θ . Individuals with very high values for θ tend to repay with probability one in order not to lose their future cash flows. But it is possible that some values of θ are not so high to justify repayment at this pooling value, but are high enough to make it worth that a discount is given on some occasions so that there is no liquidation. But these discounts cannot be offered with probability one: a probability of liquidation⁹ must be given as a threat for high θ individuals not to pretend to be one of these intermediate types.

The following lemma determines that, in optimal contracts, defaulters do not liquidate when courts fail to liquidate, or, putting it differently, that l in figure 1 is always equal to zero.

Lemma 2 *Let b, k and w be given. Suppose assumption (e) is valid and $c > 0$. Then, the probability that in the optimal contract $d = 0$ (default) and $l = 1$ (defaulters liquidate after courts fail to liquidate) is zero.*

Proof. See appendix 1 ■

Lemma 3 is useful in the proof of the next proposition.

Proposition 3 *Given the conditions of Lemma 2, both an increase in the reliability of courts, λ and a decrease in the cost of courts, c do not decrease welfare.*

Proof. See appendix 1 ■

This happens because both an increase in λ and a decrease in c increase the set of feasible payoffs. This proposition determines that both an increase in the reliability of courts, λ and a decrease in the cost of courts, c can be interpreted as institutional improvements. The following

⁹Setting a value of p_r that is so high that the choice for liquidation or default is always optimal is equivalent to assigning individuals to liquidation or default (that implies liquidation with a positive probability).

proposition reveals that both asymmetric information and uncertainty about the outcome of courts are essential for default to be a possibility in optimal contracts. Also, it reveals that without asymmetric information, there is no inefficient liquidation of projects. Projects are liquidated only when the cash flows they produce are lower than their liquidation value.

Proposition 4 *If there is no asymmetric information (constraints (4) and (3) are not required) and courts are costly ($c > 0$), there is no default and voluntary liquidation happens if and only if $ik > \theta f(k)$. Also, even if there is asymmetric information, if $\lambda = 1$ and courts are costly, there is no default.*

Proof. I prove both claims by contradiction.

First, suppose that there is no asymmetric information and, when $\theta = \bar{\theta}$, there is a positive probability of default with a pair of transfers $(\bar{p}_{d1}, \bar{p}_{d2})$. In the case where $ik > \bar{\theta}f(k)$, setting $\lambda(\bar{p}_{d1} - c) + (1 - \lambda)\bar{p}_{d2}$ as the transfers from the borrower to the lender with voluntary liquidation would increase the borrowers utility keeping the lenders revenue. If $ik \leq \bar{\theta}f(k)$, setting $\lambda(\bar{p}_{d1} - c) + (1 - \lambda)\bar{p}_{d2}$ as the repayment value (p_r) would allow an increase in utility keeping revenues unchanged. Notice that if there were asymmetric information, this last arrangement might give incentives for higher cash flow individuals to misreport their cash flow and pretend to be type $\bar{\theta}$, thus repaying a lower amount. So the argument above is only valid without asymmetric information.

Second, suppose there is asymmetric information, $\lambda = 1$ and $\theta = \bar{\theta}$, and there is a positive probability of default with the amount of transfers \bar{p}_{d1} (\bar{p}_{d2} is irrelevant since failure by courts have probability zero). Replacing this by voluntary liquidation with transfers \bar{p}_{d1} would keep utility constant, and therefore not affect incentives. But revenues of the lender would increase since the cost of courts would not be paid. ■

3 Linear Programming Solution

As the solution to the optimal contract may involve randomization of transfers between borrowers and lenders, a reasonable approach to solve the problem numerically is the discretization of the choice and state space and the solution by linear programming. This approach has the disadvantage that it lacks some precision because of the discrete gridding, but it has the advantage of being general and allowing for randomization. The functional forms used in the solutions were:

$$U(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}$$

and

$$f(k) = k^\alpha.$$

In order to compute optimal transfers for a given level of A , k , b and w , (program1) I used a grid with 12 values for θ , 50 values for p_r , 30 values for p_v and 5 values for p_{d1} . Five possible values of s_1 were used in constraint (4). The distribution of θ used in the computations is expressed in figure 2¹⁰:

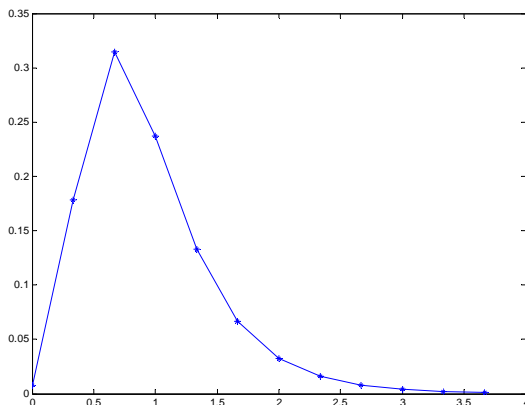


Figure 2 - Distribution of the Shock θ .

¹⁰This distribution was generated from a histogram of a lognormal distribution with mean 1.1, variance 0.3 and median 1.

The following general features emerge from the solution of the Program 1, meaning that both the amount of borrowing and the scale of projects are assumed to be fixed. First, The repayment schedule resembles a debt contract, in the sense that individuals with cash flows above a certain level repay a common amount with certainty.

Borrowers with lower levels of cash flows may have a discount in the repayment value, but they also have a positive probability of being assigned to liquidation or default. This feature of the optimal contract is expressed in figures 3 and 4, computed for the following set of parameters: $c = 0.5$, $\sigma = 0.1$, $\alpha = 0.5$, $i = 0.6$, $\beta = 0.95$, $r = 0.02$, $\lambda = 0.4$, $B = 9$ and $k = 17$. Figure 3 shows the amount repaid conditional on repayment, and figure 4 shows the probability of repayment. These pictures provide an example in which randomization is used in optimal contracts.

Figures 3 to 6 - θ is in the horizontal axis

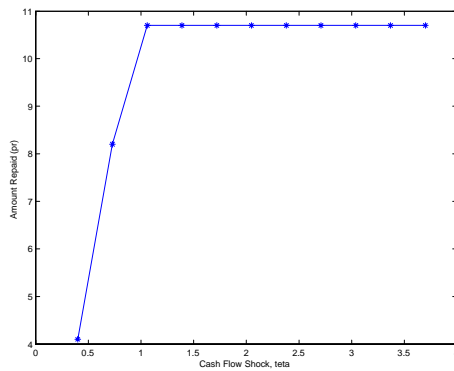


Fig 3 - Amount Repaid

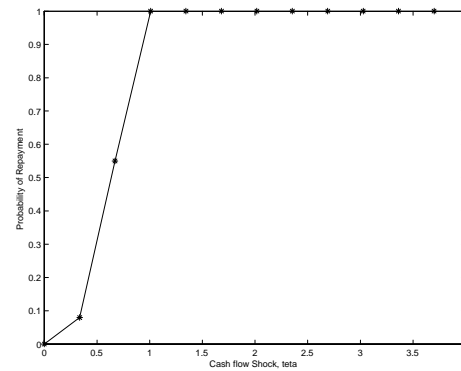


Fig. 4 - Prob. of Repayment

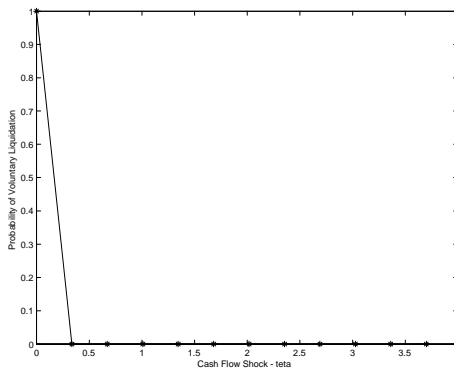


Fig.5 - Prob. of Voluntary Liq.

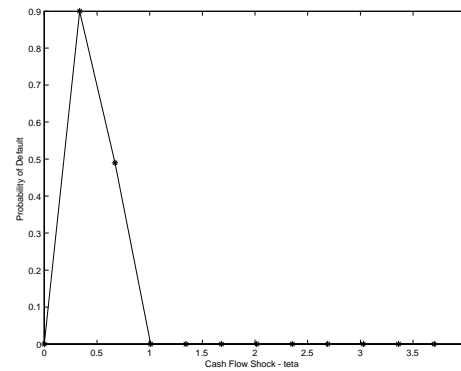


Fig.6 - Prob. of Default

Another feature that is present in the optimal contracts is expressed in figures 5 and 6. Figure 5 shows the probability of voluntary liquidation for each realization of θ . Figure 6 shows the probability of default per value of θ . Notice that despite the fact that there is a positive cost $c = 0.5$ of activating courts, default happens with positive probability in optimal contracts. Individuals with high cash flows tend to be repayers. Individuals with very low cash flows tend to voluntarily liquidate. Those with a intermediary levels of cash flows have a positive probability of default. This pattern is observed in all numerical solutions obtained.

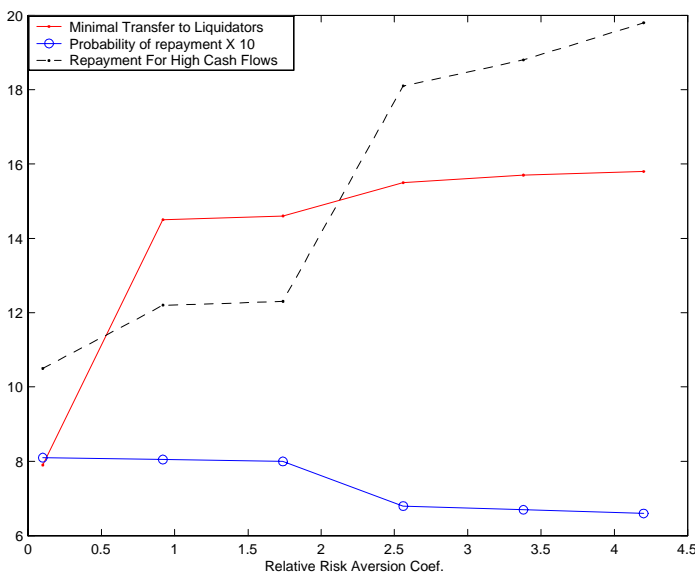


Figure - 7

Another feature that is present in the solution of program 1, is that when borrowers are risk averse, voluntary liquidators normally receive significant positive transfers from the lender. These positive transfers work as an insurance device. Voluntary liquidators tend to be individuals with low cash flows. Positive transfers to these individuals provide some level of risk sharing. In the simulations, both the amount received by voluntary liquidators and the amount paid by successful projects tend to increase with risk aversion. The amount paid by successful investors reflects not only repayment of initial borrowing, but also payments to compensate the transfers received in the contingency of low cash flows. Figure 7, shows, for the parameters used in the example above, how the pooling amount of repayment for high cash flows, the minimal

amount of transfers received by voluntary liquidators and the probability of repayment evolve with risk aversion. Notice that the fact that the probability of repayment decreases with risk aversion also contributes for the values of repayments to be increasing with risk aversion.

4 Risk Neutrality and Continuum of Shocks

The solution with linear programming has the advantage that it is precise, admits lotteries and allows the computation of the general model. However, especially with a fine grid of θ , the curse of dimensionality becomes a serious problem, and solutions demand large amounts of time and computational capacity. At this section, I specialize the analysis to risk neutrality, with $U(x) = x$. For expositional convenience, I assume, without loss of generality, that $\beta = (1 + r) = 1$. Imposing risk neutrality greatly simplifies the analysis. First, constraint (4) is not necessary: hidden savings does not provide any advantage to mitigate incentives, since it contributes equally to utility in all branches of the tree in figure 1. Numerical solutions of the model with risk neutrality show that, as the number of elements in the grid of possible values of θ increases, the fraction of points in which there is randomization tends to vanish. In other words, as the support of θ approaches a continuum, the solution seems to converge to one in which there is no randomization. So, I solve the problem assuming that there is no randomization when there is a continuum of θ and risk neutrality¹¹. For this case, the characterization of the solution is considerably simple, and is provided by proposition 5:

¹¹This could be seen as a solution of the problem with the additional constraint that randomization is not allowed. However, based on numerical exercises, this analysis is intended to be at least a good approximation to the solution with randomization allowed. In general as I increase the number of points in the θ grid, there is randomization for a maximum of 3 values of θ , between the area in which there is repayment with probability 1 and the area where voluntary liquidation or default are chosen with probability 1. The probability of these few points decreases as the grid becomes finer. So I depart from the conjecture that it tends to zero as the support of θ tends to a continuum. I am working on theoretical proof of this result.

Proposition 5 *Suppose borrowers are risk neutral, with utility given by $U(x) = x$, and $\beta = (1 + r) = 1$, and no randomization conditional on θ is allowed. Then, there exists some optimal contracts with the following properties:*

a- *There exists a repayment value \bar{p} such that whenever $\lambda\theta f(k) > \bar{p}$, or $\theta > \bar{\theta}_3(\bar{p}) \equiv \bar{p}/\lambda\theta f(k)$, there is repayment of an amount \bar{p} .*

b- *Whenever $ik > \theta f(k)$, or $\theta < \bar{\theta}_1 \equiv ik/f(k)$, there is voluntary liquidation, and $p_v = -(1 - \lambda)ik$.*

c- *Whenever $\bar{\theta}_1 < \theta < \min(\bar{\theta}_2, \bar{\theta}_3)$, where $\bar{\theta}_2 \equiv (ik + \frac{c}{1-\lambda})/f(k)$, there is voluntary liquidation with and $p_v = -(1 - \lambda)ik$.*

d- *Whenever $\bar{\theta}_2 < \theta < \bar{\theta}_3$, there is default with probability 1, with $p_{d1} = p_{d2} = 0$.*

Proof. See Appendix 1. ■

Proposition 5 determines that whenever there is voluntary liquidation or default, the utility of the borrower is equal to the utility of default with zero transfers. This utility is generated by the values of p_{d1} , p_{d2} and p_v determined in proposition 5. Risk neutrality is essential for that: there is no gains in making transfers to individuals with low cash flows, unless these transfers are needed to rule out that the outside option of default is chosen. Higher transfers from the lender to the borrower in the case of default and voluntary liquidation require higher transfers from repayers. But higher repayment amounts may imply that a lower fraction of cash flow realizations will justify repayment, and thus increase the probability that projects are liquidated by value lower than their cash flow. Since the utility of defaulters and liquidators is always equal to the utility of default with zero transfers, the decision between default and voluntary liquidation depends on the revenue that is obtained with default and $p_{d1} = p_{d2} = 0$ and voluntary liquidation with $p_v = -(1 - \lambda)ik$. Notice from item c of proposition 5 that as λ tends to 1, the probability of default converges to zero, as stated in proposition 4.

From the properties presented in proposition 5, the characterization of the optimal contract is straightforward. We characterize it for a continuum of θ , which can be interpreted as an approximation of the discrete case with a very fine grid.

The repayment amount for individuals that repay, conditional on the size of loans, b and the scale of project, k is such that the expected second period revenue of the borrower is equal to the amount of borrowing in the first period. Therefore, \bar{p} must solve:

$$b = \lambda ikH(\theta_1) + \int_{\theta_1}^{\min(\theta_2, \theta_3(\bar{p}))} (ik - (1 - \lambda)\theta f(k))h(\theta)d\theta \quad (9)$$

$$+ \mathbb{1}(\theta_2 < \theta_3(\bar{p})) \int_{\theta_2}^{\theta_3(\bar{p})} (\lambda ik - c)h(\theta)d\theta + (1 - H(\theta_3(\bar{p})))\bar{p}$$

where $\mathbb{1}(\theta_2 < \theta_3(\bar{p}))$ is an indicator function that has value 1 when $\theta_2 < \theta_3(\bar{p})$ and zero otherwise. The right hand side of equation (9) is the revenue from the borrower given repayment amount \bar{p} . The determination of amount of repayment, \bar{p} , given the amount borrowed, can be seen in figure 3¹²:

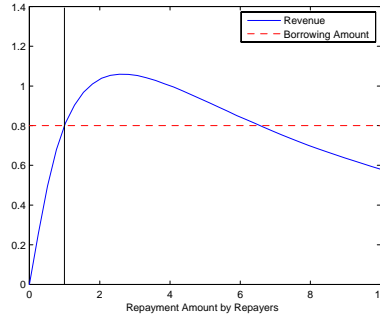


Figure 8

The vertical line shows the value of \bar{p} determined by (9). Notice that there is another higher value of \bar{p} that solves (9), but this implies a higher probability of liquidation of good projects so it produces a lower utility for the borrower. The choice of \bar{p} will always be the smaller value that satisfies (9).

¹²This was generated with $k = 1$, $b = 0.8$, $i = 0.5$, $c = 0.2$, $\lambda = 0.7$, θ has lognormal distribution with $\mu = 1$ and $\sigma = 1$ and $f(k) = k^{0.5}$.

The utility conditional on borrowing and amount of capital is given by:

$$U(k, b) = (w - k + b) + f(k)E(\theta) + (1 - \lambda)ik(H(\theta_1)) + \int_{\theta_1}^{\theta_3(\bar{p}(k,b))} (1 - \lambda)\theta f(k)h(\theta)d\theta + \int_{\theta_3(\bar{p}(k,b))}^{\infty} (\theta f(k) - \bar{p}(k, b))d\theta. \quad (10)$$

The problem specialized for the risk neutral case with $\beta = (1+r) = 1$ and no randomization becomes:

$$\begin{aligned} \max_{(k,b)} U(k, b) \\ \text{s.t. } w - k + b \geq 0 \text{ and (3)}. \end{aligned} \quad (11)$$

The constraint $w - k + b \geq 0$ will always hold with equality when $b > 0$. Indeed, if there savings, diminishing the amount of borrowing and keeping capital constant will not decrease utility. It will reduce the amount to be paid and thus the states in which liquidation happen. Therefore, given w , b can be written as the following function of capital:

$$b = \max\{0, k - w\} \quad (12)$$

Substituting (12) in (10) and using the value of \bar{p} implicitly defined in (9) it is possible to obtain utility as a function of k given w . For the parametric specification used in figure 8, and $w = 1$, the utility of the borrower as a function of the scale of the project is given by:

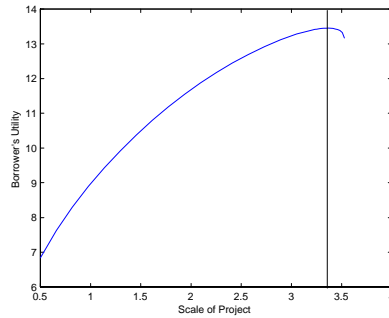


Figure 9

The optimal scale of the project, the one that maximizes utility, is determined by the horizontal line in figure 8.

The next proposition is consistent with the intuitive idea that better enforcement should decrease the risk of no repayment and thus decrease interest rates.

Proposition 6 *In the solution given by equations (9) to (12) (that is the solution for the case with risk neutrality, no randomization conditional on θ and θ defined in a continuum), if projects have a fixed scale and $b > 0$, borrowing interest rates are decreasing with the reliability of courts λ .*

Proof. Define the $\Delta(\lambda, \bar{p})$ as the right hand side of (9). Since scale is fixed, b and k are both fixed. Therefore, $\frac{\partial r_b}{\partial \lambda} = -\frac{1}{b} \frac{\partial \Delta(\lambda, \bar{p}) / \partial \lambda}{\partial \Delta(\lambda, \bar{p}) / \partial \bar{p}}$. It is clear from (9) that $\partial \Delta(\lambda, \bar{p}) / \partial \lambda > 0$. Also, it must be the case that $\partial \Delta(\lambda, \bar{p}) / \partial \bar{p} > 0$, otherwise a decrease in \bar{p} would increase revenue without decreasing utility, implying that \bar{p} is not optimal. Therefore, $\frac{\partial r_b}{\partial \lambda} < 0$. ■

Notice that an essential condition for this to be valid is that the scale of projects is fixed.

4.1 Numerical Results for the Risk Neutral Case

This section presents numerical solutions for the risk neutral version of the model just described. The model is solved for two specifications of the production function, $f(k)$. The first specification is $f(k) = k^{0.5}$. Then I use a production function with a higher curvature. The comparison between this two specification reveals that the curvature of the production function is a key ingredient in the determination of the characteristics of the solution. For the first specification I depart from a baseline specification where $c = 0.3$, $i = 0.5$ and the distribution of θ is lognormal with parameters $\mu = 1.375$ and $\sigma = 0.5$. I compute the solutions for several levels of initial wealth and λ . I also check how the solution respond to different values of c and i and for different specifications of the distribution of θ , $h(\theta)$. The interest rates results presented in this sections refer to the borrowing interest rates, as defined in (8).

The first result concerns credit rationing. Individuals with low levels of wealth need high

amounts of borrowing in order to be able to finance big projects. But the amount of borrowing individuals are able to obtain is limited: for very large loans, the maximum revenue that can be obtained by the lender after the first period is lower than the amount of borrowing. Figure 10 shows, for the baseline specification and $\lambda = 0.7$, the optimal scale of projects and the maximum possible scale achievable by agents as a function of their initial wealth. Notice that there is some credit rationing. The maximum scale available for low wealth individuals is lower than optimal scale for individuals that have very high wealth and therefore are unconstrained in their choice of scale. This is similar to Evans and Jovanovic (1989). But differently from them, even for very low wealth agents, the optimal scale is lower than the maximal scale available.

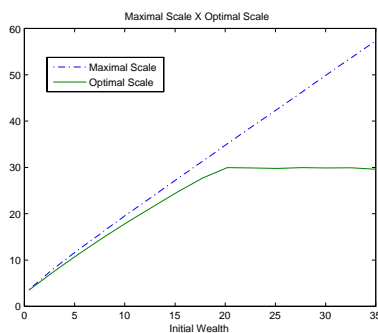


Figure 10

Figure 12, in appendix 2, shows the wealth profile of optimal project scale, size of loans (amount of borrowing), probabilities of default, repayment and voluntary liquidation, and borrowing interest rates, as defined in equation (8). These profiles are shown for 4 different values of the parameter of court reliability, $\lambda : 0.3, 0.5, 0.7$ and 0.9 . The results show that, for all levels of λ investigated, the optimal scale of projects increase with the initial wealth, up to a point where an optimal scale is achieved. The optimal size of loans profile has an inverted U shape: it is increasing for very low levels of wealth, but after a some point it becomes decreasing. After some level of wealth, individuals start to self finance their projects. Both the probability of default and the interest rates are decreasing with initial wealth. Wealthier individuals not only have access to bigger projects, but they also have access to lower interest rates.

Notice that higher reliability of courts implies higher values of loans and capital. But the effect of λ on interest rates and the probability of default is not clear. This is more evident in figure 13, that shows, for 4 levels of initial wealth, the evolution of these same variables with λ . Both the scale of projects and the size of loans (b) are increasing with λ , so the effectiveness of legal enforcement increases scale of borrowing and projects. The effect of λ on the probability of default and the borrowing interest rates is undetermined. For very high values of λ , the probability of default is zero, as stated in proposition 4. But for very low levels of λ the amount of borrowing is extremely low, but these small borrowings have very low probabilities of default. The intermediary values of λ are those that produce high probability of default. Interest rates also have a non monotonic behavior. They tend to be low with very low levels of court reliability and higher for intermediary levels of λ .

Another remarkable feature of figure 12 is that, as the initial wealth increase, the interest rates and the probability of repayment both go down. This implies that interest rates are not only determined by the probability of repayment. Notice that the as wealth increases, the probability of voluntary liquidation also increase. When there is voluntary liquidation, the value of liquidation is transferred to the lender or, putting it differently, there is collateral seizing by the lender. More collateral transfers make it possible that the repayment generates a smaller fraction of the lenders revenue after the first period. Figures 14 and 15, in appendix 2, show a case in which there is no collateral value ($i = 0$). In this case, all revenue of the borrower comes from repayment, and non repayment rates explain almost perfectly the interest rates profile.

Figures 19 and 20 in Appendix 2 show the numerical solutions for the optimal scale, interest rates and the probability of default computed for alternative specifications for c , i and h ¹³. Some remarkable results from this analysis are that the probability of default and the interest rates increase with the variance of θ , and decreases with the cost of courts c . Also, the scale of projects tend to be higher as the liquidation value of projects increase.

¹³In the case of the different specifications of $h(\theta)$, the parameters μ and σ of the lognormal distribution are chosen so that the expected value of θ is constant.

The result that the interest rates may increase as the reliability of courts, λ , increases, contrasts with proposition 6, valid for fixed scale. I recompute the problem using another production function that has a higher curvature, and thus is closer to the case of fixed scale. This production function is:

$$f(k) = (1 + (1 - k)^{-2}).$$

This is a CRRA function with a higher curvature than $f(k) = k^{0.5}$ (or, in the utility context, higher risk aversion) moved one unit to the left and summed by one so that it is always positive and is zero valued at $k = 0$. Figure 11 plots both this production function (production function 2) and the one chosen before (production function 1). This new production function, with higher curvature is closer to a fixed scale case. The gains of scale are initially high, but eventually become very low.

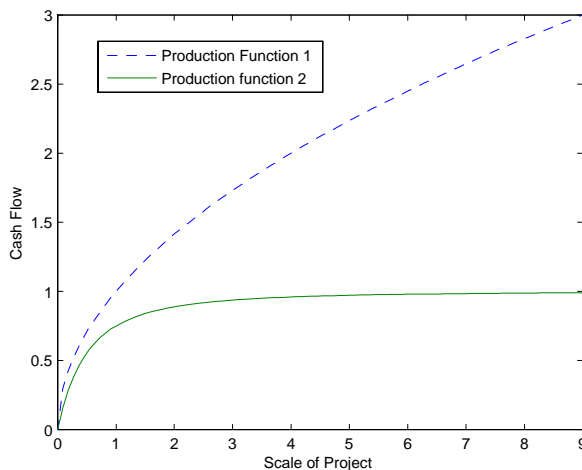


Figure 11

The baseline specification used in the analysis with this production function has $c = 0.3$, $i = 0.5$ and the distribution of θ lognormal with parameters $\mu = 1.5$ and $\sigma = 0.375$. The solutions for a baseline case with this second specification are expressed in figures 16 and 17, in appendix 2. Notice that, as in the previous specification, both interest rates and the probability

of default decrease with wealth. Also, the scale of projects tend to increase both with wealth and λ , although the variation on scale is proportionally smaller than in the first specification. However, a remarkable difference that comes from this specification is that the probability of repayment is tends to be increasing and interest rates are consistently decreasing with λ . Better legal enforcement not only increases the scale of projects, but also decreases interest rates. Further, both effects are higher for low wealth individuals. These numerical results, combined with Proposition 6, indicate that the curvature of the production function is a key ingredient to define how better enforcement affects interest rates.

Another ingredient that is affected by the curvature of the production function is the relation between initial wealth and amount of borrowing. In the high curvature case (as well as in the fixed scale case) the amount of borrowing is always decreasing with wealth. This differs from the solution with low curvature, that has borrowing amounts initially increasing with wealth. This is potentially useful for empirical work: the response of amount of borrowing to initial wealth contains information about the curvature of the production function, and this curvature is a key element to determine wether interest rates respond to quality of enforcement or not.

Figures 20 and 21 in Appendix 2 show the solutions for the optimal scale, interest rates and the probability of default for this second production function and different specifications for i , c and the distribution of θ , $h(\theta)$. The general features of the solution are similar to those presented in Figure 16. A remarkable result, not present in the results for the first specification of the production function is that, not only the interest rates are higher as the variance of θ (risk of project) increases, but also the optimal scale of projects is significantly smaller.

5 Discussion and Concluding Remarks

This paper departs from a model that has debt contracts as optimal project financing arrangements conditional on the environment. It is possible to define a borrowing interest rate from the pooling value of repayment that borrowers must make in other to have no risk of having their

projects liquidated. This definition of borrowing interest rates can be used in an evaluation of how interest rates and scale of projects respond jointly to the reliability of courts to enforce contracts.

The results presented in the preceding sections can be divided in two categories, the theoretical findings and the numerical findings. On the theoretical side, two results can be highlighted. First, an increase in the reliability and a decrease in the cost of courts increase welfare. Second, default may actually be observed in optimal contracts, but this requires both asymmetric information and imperfect enforcement by courts. On the numerical side, there are four main findings. First, wealthier individuals borrow with lower interest rates and run higher scale enterprises. Second, The reliability of courts has a consistently positive effect on the scale of projects. Third, the effect of bad enforcement on interest rates is undetermined when the curvature of the production function is low, and it is negative when this curvature is high or when projects have a fixed scale. Finally, the possibility of collateral seizing makes it possible that interest rates and the probability of default have comovements.

The first theoretical finding has policy implications. The second theoretical finding shows that asymmetric information generates the realistic feature that punishment may actually be observed, which does not happen in most limited commitment models with perfect information and in particular in this model, when asymmetric information is not present. The numerical findings can be confronted with existing empirical work, and they may be used as a baseline for further empirical investigation.

5.1 Evidence from empirical studies and empirical potential

A first result that is related with empirical studies is that the scale of projects increase with the reliability of courts, λ . Higher scale of projects produce higher outputs, so the model generates a theoretical link between development and the quality of institutions. This relation have been explored in empirical studies such as Knack and Keefer(1995) and Mauro (1999) that present evidence that confidence in institutions, including the judicial system, is a predictor of growth

and Acemoglu and Johnson (2005), that present evidence linking property rights institutions and economic growth.

There is also an empirical literature that discusses the impact of institutions on the form of financial intermediation. This includes Acemoglu and Johnson (2005) and LaPorta *et. al.*(1998). Examples of papers that explicitly relate the quality of judicial system and interest rates are Leaven and Madjnoni (2003), that uses cross country data, and Costa and Mello (2006) and Visaria (2005), both of which employ a natural experiment approach. These papers find evidence that bad legal enforcement is connected with high interest rates. This is not generated by all specifications of the model, but is consistent with the results from the second specification of the production function, with high curvature.

Another prediction of the model that have support in empirical studies is that interest rates are higher to low wealth individuals. Karlan and Ziemann (2006) report high interest rates in loans for poor individuals in South Africa. This relation is also found by Araújo and Rodrigues (2003), that use data from credit contracts recorded by the Brazilian Central Bank. They show that the very high interest rates that are prevalent in Brazil affect most strongly small firms. The average interest for firms that are classified by banks as micro-firms is 57%, for those that are classified as small firms it is 44.78%, for medium size firms it is 33.66% and for big firms it is 29.5%. In the model presented, in all parametrizations, wealthier individuals have lower borrowing interest rates, so the model is consistent with these findings.

Another finding by Araújo and Rodrigues that is related with the results of the model is that the average interest rates for large loans are smaller than that for big loans. Again, this is not obtained in any specification of the model, but is consistent with at least two particular cases. First, when the production function has low curvature, in the lower part of the wealth distribution the size of loans increases and the interest rates decrease as the initial wealth increases. If most of the credit contracts have borrowers in this lower part of the wealth distribution, it is possible that higher loans have, on average, lower interest rates. Notice however, that with low curvature of the production function, bad enforcement does not explain

high interest rates. Such specification is consistent with the Brazilian cross-sectional stylized facts but do not to explain why interest rates in Brazil are higher than in most countries. A second possible explanation for interest rates to be decreasing with size of loans comes from difference in risk of projects. For the second specification of the production function, low variance of θ (or low risk on projects) generates simultaneously high scale of projects, and thus large amounts of borrowing, and low interest rates (see Fig. 20, in appendix 2). The variability on the risk of projects could generate a negative correlation between size of loans and interest rates, and also help to explain the negative correlation between interest rates and scale of firms. This second case departs from a specification of the production function that is consistent with imperfect legal enforcement as an explanation for high interest rates.

An important result that comes from numerical analysis, is that the relation between wealth and the amount of borrowing depends strongly on the curvature of the utility function. Therefore, cross-sectional estimates on how the amount of borrowing relates to initial wealth could provide some information about the curvature of the production function. This is important since the curvature is a key ingredient to determine the effect of legal enforcement on interest rates.

In order to evaluate how different specifications of the model fit real credit markets, it is necessary to make a careful empirical analysis with adequate data. But the results just presented illustrate that the model produces several testable implications, that could be useful to confront the model with real credit markets. Also, with adequate data, further research could define the specification of the model that better fit the data through structural estimation of the model or an identifiable version of it. Estimates from different locations and periods, with different legal environments could be compared. An estimated version of the model could also generate estimates of the effects of improvements in legal enforcement.

5.2 Policy Implications

High interest rates and credit rationing, especially for poor individuals, are commonly regarded as a problem in developing countries, and policies suggested to deal with them include subsidy to credit, public provision of credit and interest rate controls. The model discussed presents high interest rates and low amount of credits for low wealth individuals, but those are optimal given the environment. If the model provides a good characterization of credit markets, such policies would have no advantage over the mere redistribution of initial wealth. On the other hand, proposition 3 and the fact that in simulations the contracts respond to the court quality variables, indicate that improvements in legal enforcement could bring welfare gains. This reinforces the importance of evaluating how well the model fits real credit markets data. A research agenda in this direction could include a comparison of this model with other possible explanations for high interest rates and low credit for poor individuals.

The negative result that interest rates may increase even with improvement in legal enforcement also have consequences for policy evaluation. An increase in average interest rates does not necessarily mean a welfare loss. Proposition 4 determines that increases in the reliability of courts never produce welfare losses. However, sometimes interest rates increase with an improvement in enforcement. In those cases, the gain from the possibility of larger investments more than compensates the losses that may come from higher interest rates. In principle, it is possible that legal improvements expand the amount to credit and simultaneously produce an increase in average interest rates. This may help to explain why countries like Brazil, that have an intermediary level of development, have higher borrowing interest rates than less developed and institutionally more unstable countries. In those countries, interest rates are lower, but the amounts of formal borrowing are considerably smaller.

5.3 Theoretical extensions

The model employed is particularly simple in the dynamic structure. Repayment amounts are limited to one period cash flows and the enforcement power of liquidation is greatly limited

by the fact that it affects only one cash flow. Therefore, an extension of the model, either to more periods or infinite periods would certainly increase realism. Also, such an extension could provide insights about the effect of legal enforcements on the term structure of borrowing contracts.

Another possible theoretical extension would be to include ex-ante asymmetric information. Individuals would have better information about their projects than the lender before borrowing contracts are defined. Stiglitz and Weiss (1981) show that credit rationing can emerge in credit markets as a result of adverse selection. But they receive criticism from Bester (1985) that shows that collateral can be used to screen borrowers with different risks and overcome this problem. It might be interesting to evaluate if the notion of collateral employed in the present paper (scale of projects) would be able to produce such screening and, if so, under which conditions.

The current model takes the legal system as exogenous. But, in principle, it is possible to make the parameters related to courts endogenous. One simple way to do so, would be to make the reliability of courts an increasing function of amount invested in legal institutions. This could possibly define the determinants of optimal investment on legal institutions, and serve as a guideline to cross-country comparisons of quality of legal systems.

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7 Appendix 1

7.1 Proofs of the propositions presented

Proposition 1 *Let b, k, A and w be given. Suppose that p_r can assume any value in \mathfrak{R} , that Assumptions (a) to (d) are valid and that Θ is finite. Then, if an optimal contract implies that types θ_1 and θ_2 chose repayment with probability one, there exists an optimal contract in which both types repay a common value \hat{p}_r with probability 1. If (a') and (d') are valid, then an optimal contract where types θ_1 and θ_2 repay with certainty must have both types repaying a common amount with certainty.*

Proof. First, it is always possible in an optimal contract to have individuals of each type with a deterministic value of repayments. Indeed, suppose there is randomization in the repayment amount for some type. By assumption (a), there is a nonrandom amount of repayment that could bring the same utility to this type without a lower revenue for the lender. By assumption (b) this would not give any extra incentives for individuals with higher values of θ to misreport their type. By assumption (c), this would also not give any extra incentive for hidden savings.

Now, suppose that there are different values of repayment for types that repay with certainty $p_{r1} < p_{r2} < \dots < p_{rn}$. (since Θ is finite, there is a finite amount of values). Let Θ_1 be the set of values of θ that pay p_{r1} , and Θ_2 the set of values of θ that pay p_{r2} . By truth telling, all elements in Θ_1 must be bigger than the elements in Θ_2 . By the same reason, individuals with repayment values bigger than p_{r2} have θ lower than those of Θ_1 and Θ_2 , and thus cannot report having a type in Θ_1 or Θ_2 . There is an intermediate level of repayment, p'_r between p_{r1} and p_{r2} that makes the expected utility conditional on being a type in Θ_1 or Θ_2 unchanged. By condition (d), this fixed value of repayment would not decrease the revenue of the lender. By condition (b), this would also not increase the gain of receiving transfers in the second period. Therefore, it would not give additional incentives for hidden savings. Extending this procedure to the other levels of repayment it we can find a unique value of repayment for all types that repay with probability one.

Notice that if (a') is valid, moving from randomization to a unique payment value for each type that repays with certainty increases the revenue of the lender, and thus it must be the case that each type repay one value with certainty (The extra revenue could be transferred for the higher θ individual, increasing ex ante expected utility without generating extra incentives for savings by condition (a)). If (d') is valid, substituting p_{r1} and p_{r2} for a unique value p'_r increases the lenders revenue. Thus, it must be the case that types that repay with certainty repay the same amount. ■

Lemma 2 *Let b, k, A and w be given. Suppose assumption (e) is valid and $c > 0$. Then, the probability that in the optimal contract $d = 0$ (default) and $l = 1$ (defaulters liquidate after courts fail to liquidate) is zero.*

Proof. Suppose an investor with ex-post shock $\bar{\theta}$ defaults and choose $l = 1$. Her utility will be $\widehat{V} = \lambda V_2^v(A\bar{\theta}f(k), p_{d1} + ik) + (1 - \lambda)V_2^v(A\bar{\theta}f(k), p_{d2})$ ¹⁴. If the lender offers a value p_v for voluntary liquidation such that $V_2^v(A\bar{\theta}f(k), p_v) = \widehat{V}$, borrowers would be willing to voluntarily liquidate. As V_2^v is concave, it is clear that $p_v \geq \lambda(p_{d1} + ik) + (1 - \lambda)p_{d2}$, and thus the revenue of the lender is higher than in the contract with default (notice that the cost of default c will not have to be paid). Furthermore, from assumption (e) investors with $\theta > \bar{\theta}$ will not have additional incentives to pretend to be of type $\bar{\theta}$, as the absolute risk aversion for liquidators is nonincreasing with wealth. Also, from (e), additional savings in the first period would not make this new offer more valuable than the previous one: there is no additional incentives for hidden savings. So, the new contract produces the same outcome for all types of investors and increases the amount of resources obtained by the borrower. ■

Proposition 3 *Given the condition on Lemma (2), both an increase in λ and a decrease in c do not decrease welfare.*

¹⁴Notice from the structure presented in figure 1 that $V_2^{d1}(y, p) = V_2^v(y, p + ik)$, and when $l = 1$, $V_2^{d2}(y, p) = V_2^v(y, p)$.

Proof. By Lemma (2), we only have to consider the case with $l = 0$ (no choice of liquidation when courts fail to liquidate). The fact that lower c does not decrease welfare is easily seen: c affects only constraint (5), and lower c relaxes it. Now suppose λ increases from λ' to λ'' . Let C_1 be a contract with $l = 0$ whenever there is default and that is optimal given λ' . I show that there is another contract C_2 , that feasible given λ'' , such that, when $\lambda = \lambda''$, the utility of all types under C_2 is equal to the utility they have when the contract is C_1 and $\lambda = \lambda'$. The new contract C_2 is defined as follows. The values of b and k in C_2 are equal to their values in C_1 . All the probabilities of (θ, p) pairs specified in C_1 that resulted in no default under λ' and are kept unchanged. Suppose that before this change in λ , C_1 implies that an individual with cash flow $\bar{\theta}$ have a probability π of defaulting and facing a transfers vector under default $(\bar{p}_{d1}, \bar{p}_{d2})$. In the new contract this is substituted by a randomization between two scenarios for individuals with type $\bar{\theta}$. With a probability $\pi' = \pi\lambda'/\lambda''$, they are assigned to default and a transfer vector of $(\bar{p}_{d1}, \bar{p}_{d2})$. With probability $\pi'' = \pi(1 - \lambda'/\lambda'')$, p is defined as $p_r = p_v = \bar{p}_{d2}$, $p_{d1} = p_{d2} = 0$. Given the choice for $l = 0$ in the first contract, it is clear that individuals with $\theta = \bar{\theta}$ will decide for repayment in this last scenario. Also, individuals with $\theta > \bar{\theta}$ have no additional incentives to pretend to have a type $\bar{\theta}$. Indeed, by the fact that $p_{d1} = p_{d2} = 0$ and $\bar{p}_{d2} \leq 0$, any individual would prefer either voluntary liquidation or repayment to default in this scenario. So, with this new contract, individuals that declare a type $\bar{\theta}$ have a probability $\pi(1 - \lambda'/\lambda'') + (1 - \lambda'')(\pi\lambda'/\lambda'') = \pi\lambda'$ of facing the choice between liquidation or not with transfers \bar{p}_{d2} and a probability $\pi(1 - \lambda)$ of facing liquidation and seizing of collateral with transfers \bar{p}_{d1} . In terms of utility, the choices available to individuals are unaffected by the change of λ from λ' to λ'' and the change of contract from C_1 to C_2 . And the revenue of the lender for these scenarios under λ'' is $\pi(\lambda(ik + p_{d1}) + (1 - \lambda)p_{d2} - \lambda'/\lambda''c)$, which is bigger than $\pi(\lambda(ik + p_{d1}) + (1 - \lambda)p_{d2} - c)$, the revenue under λ and C_1 for this case. Therefore, the new contract increases welfare. ■

Proposition 5 *Suppose borrowers are risk neutral, with utility given by $U(x) = x$, and $\beta = (1 + r) = 1$, and no randomization conditional on θ is allowed. Then, there exists some*

optimal contracts with the following properties:

- a- There exists a repayment value \bar{p} such that whenever $\lambda\theta f(k) > \bar{p}$, or $\theta < \bar{\theta}_2(\bar{p}) \equiv \bar{p}/\lambda\theta f(k)$, there is repayment of an amount \bar{p} .
- b- Whenever $ik > \theta f(k)$, or $\theta < \bar{\theta}_1 \equiv ik/f(k)$, there is voluntary liquidation, and $p_v = -(1 - \lambda)ik$.
- c- Whenever $\bar{\theta}_1 < \theta < \min(\bar{\theta}_2, \bar{\theta}_3)$, where $\bar{\theta}_2 \equiv (ik + \frac{c}{1-\lambda})/f(k)$, there is voluntary liquidation and $p_v = -(1 - \lambda)ik$.
- d- Whenever $\bar{\theta}_2 < \theta < \bar{\theta}_3$, there is default with probability 1, with $p_{d1} = p_{d2} = 0$.

Proof. Suppose $\bar{\theta}$ is the maximum possible value of θ . It must be the case that $\bar{\theta}f(k) > ik$, otherwise there would be no investment up to the scale k . But no other type can pretend to be $\bar{\theta}$, so any arrangement with default or voluntary liquidation can be replaced by one with the same revenue and higher utility with repayment and therefore no risk of liquidation. For the reasoning presented in proposition 1, all types that repay, repay the same amount. The next step is to show that there is an optimal contract in which whenever there is voluntary liquidation or default the utility of the borrower is equal to that of default with zero transfers in the second period. Let us first order the values of θ as $\theta_1 < \theta_2 < \theta_3 < \dots$. I start with the case where $\theta_1 < \bar{\theta}_1$ (as in statement (b)). Clearly, if $\theta_1 \leq \bar{\theta}_1$ there must be voluntary liquidation with probability 1. Indeed replacing any event with no liquidation by liquidation with additional transfers of ik to the borrower would increase the utility of the borrower without changing the revenue of the lender. If $p_v(\theta_1)$ is higher than $-(1 - \lambda)ik$ the lender would prefer to default and afterwards liquidate by its own. If $p_v(\theta_1) > -(1 - \lambda)ik$, it is possible to write another contract with liquidation and transfers $p'_v(\theta_1) = -(1 - \lambda)ik$ and the repayment for those that repay with certain the amount \bar{p} is reduced by $(p'_v(\theta_1) - p_v(\theta_1)) (h(\theta_1)/\Pr(rep))$, where $\Pr(rep)$ is the probability of the high θ types with probability 1. The change in the objective function is $(p'_v(\theta_1) - p_v(\theta_1)) h(\theta_1) + \Pr(rep)(p'_v(\theta_1) - p_v(\theta_1)) (h(\theta_1)/\Pr(rep)) = 0$. So, if the original contract was optimal, the new one is also optimal. We can make such changes successively for θ_2, θ_3 until we reach the point in which $\theta_n > \bar{\theta}_1$. For θ_n , it is possible that there is default or voluntary liquidation.

If there is voluntary liquidation, it must be the case that $p_v(\theta_n) \leq -(1 - \lambda)\theta_n f(k)$, otherwise the borrower would choose default. If $p_v(\theta_n) < -(1 - \lambda)\theta_n f(k)$ it is possible to change $p_v(\theta_n)$ to $p'_v(\theta_n) = -(1 - \lambda)\theta_n f(k)$. The resulting gain in revenues could be transferred to those high θ 's that repay with probability one. As in the case of θ_1 , this would not decrease the expected value of the objective function and would keep revenues constant. Also, it would keep the utility of θ_n higher than reporting a lower θ . And it would not give extra incentives for misreporting in the contingency of higher values of θ . If, on the other hand, the solution is default with $\lambda p_{d1}(\theta_n) + (1 - \lambda)p_{d2} \equiv p_d < 0$, we could replace p_d by zero and transfer the expected gains from this to the contingency of high θ 's with repayment. As in the case of voluntary liquidation, this would keep the contract optimal. Notice that with these reformulations the utility under default and voluntary liquidation would be the same. So, the key ingredient to determine if it is optimal to voluntarily liquidate or default is the if θ is smaller or bigger than $\bar{\theta}_2$. In the first case, the revenue from voluntary liquidation is higher, and in the second case the revenue from default is higher.

We can proceed with similar reformulations for $\theta_{n+1}, \theta_{n+1}$ and so on until $\bar{\theta}_3$ is reached, after which there is repayment and an utility level that is (except for the treshold case $\bar{\theta}_3$) higher than the utility of liquidation with no transfers. ■

8 Appendix 2 - Numerical solutions

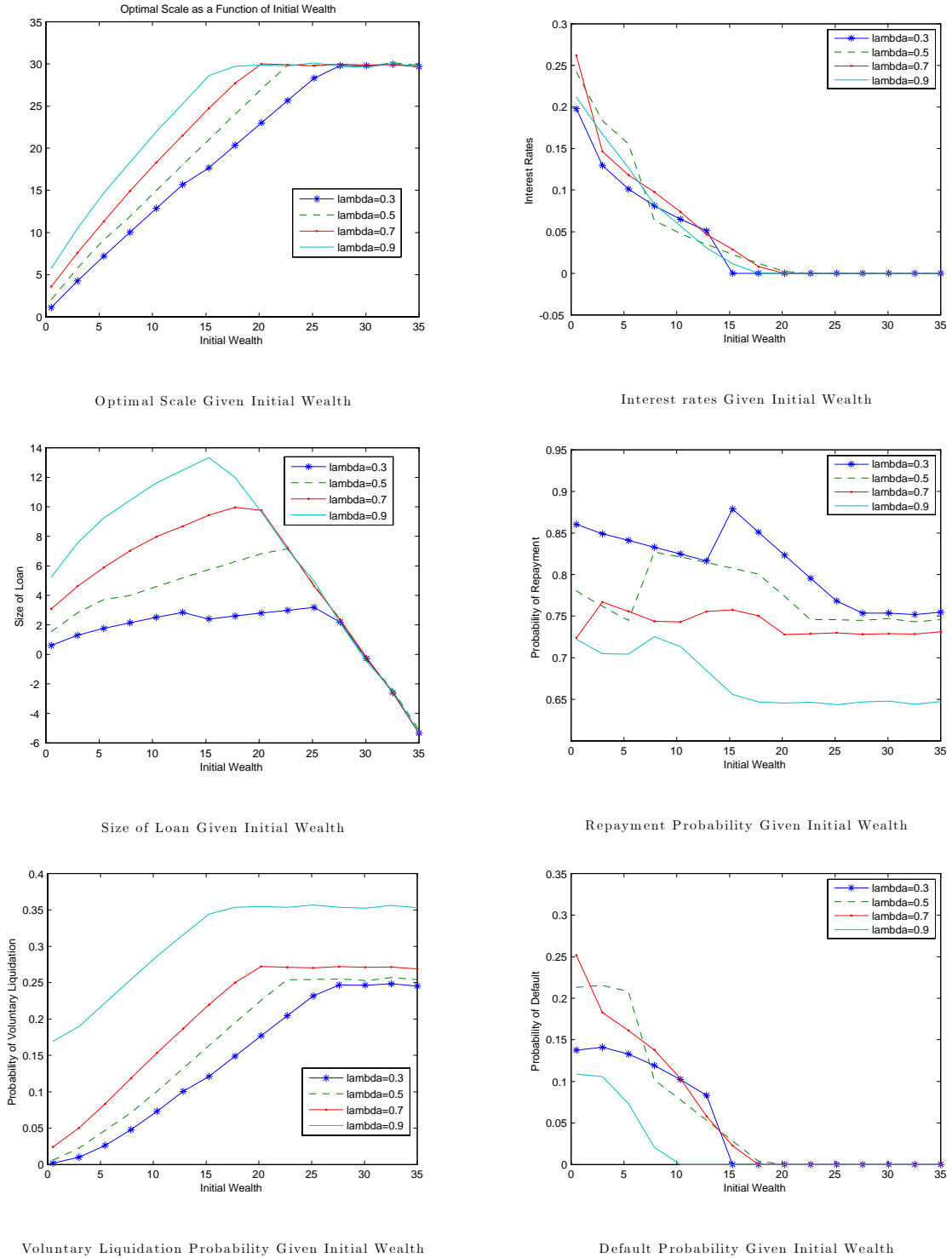
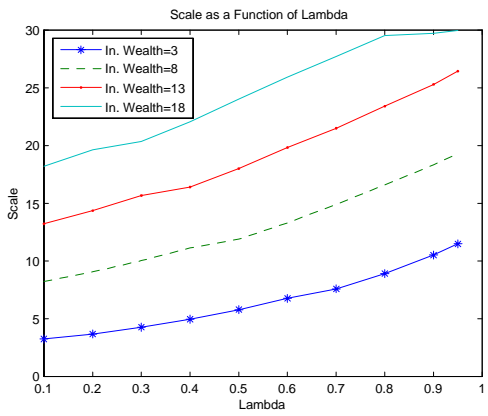
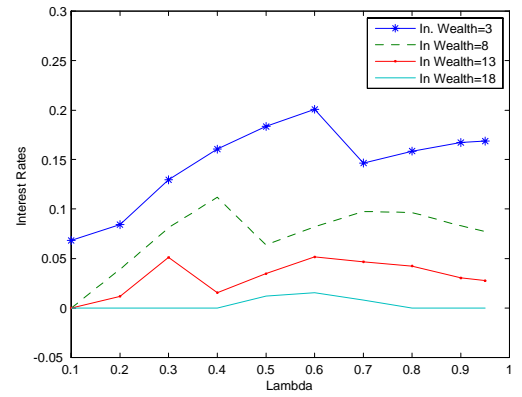


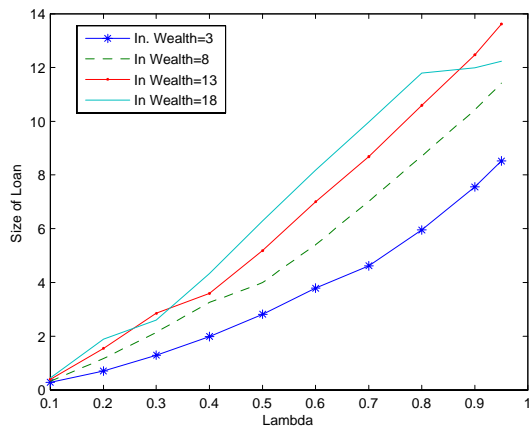
Figure 12 - $f(k) = k^{0.5}$, Baseline Case



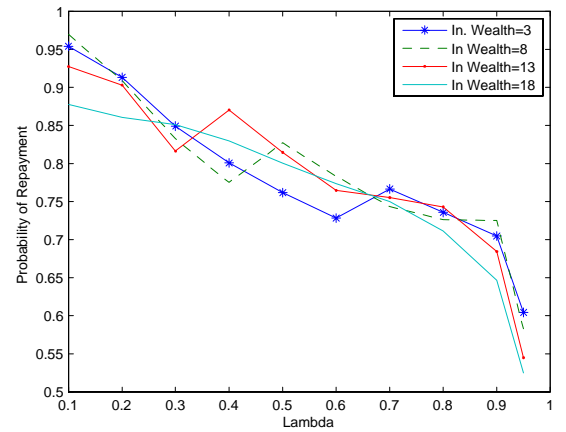
Optimal Scale Given λ



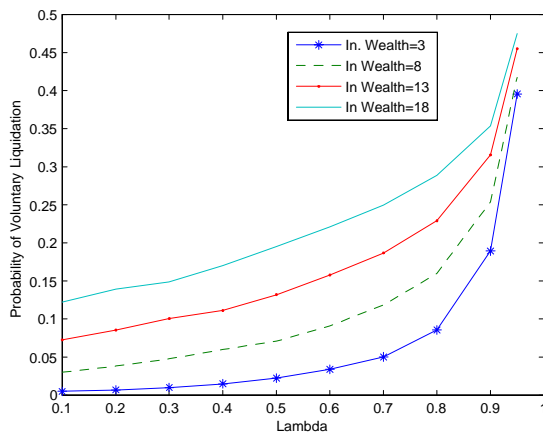
Interest rates Given λ



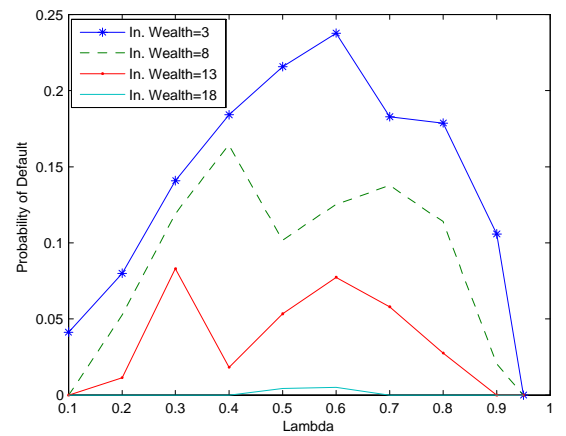
Size of Loan Given λ



Repayment Probability Given λ

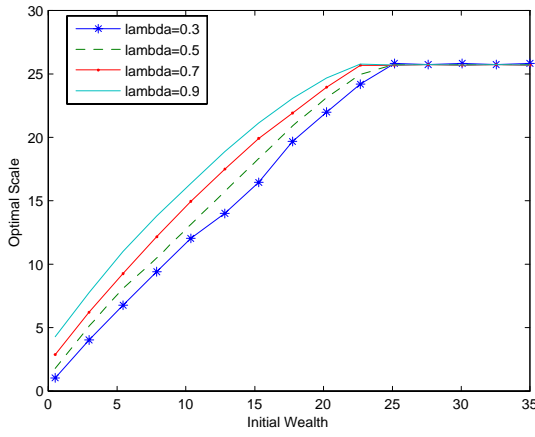


Voluntary Liquidation Probability Given λ

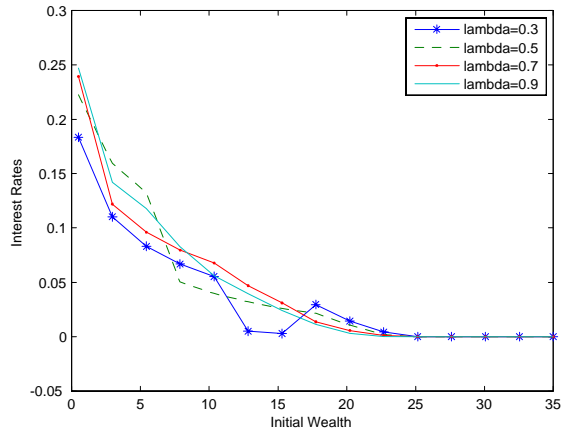


Default Probability Given λ

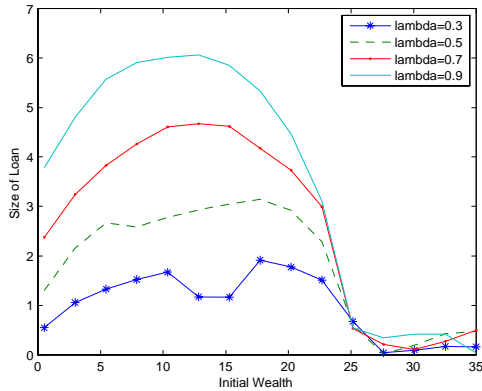
Figure 13 - $f(k) = k^{0.5}$, Baseline Case 45



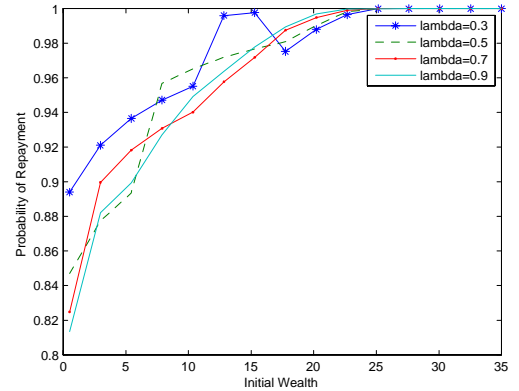
Optimal Scale Given Initial Wealth



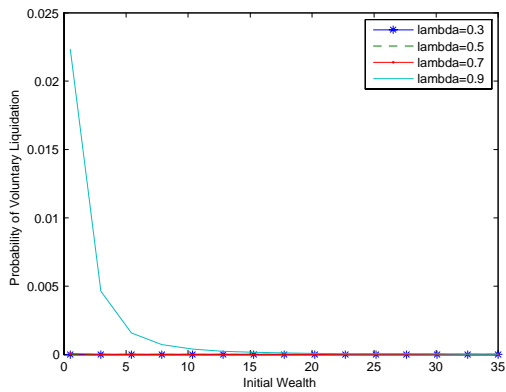
Interest rates Given Initial Wealth



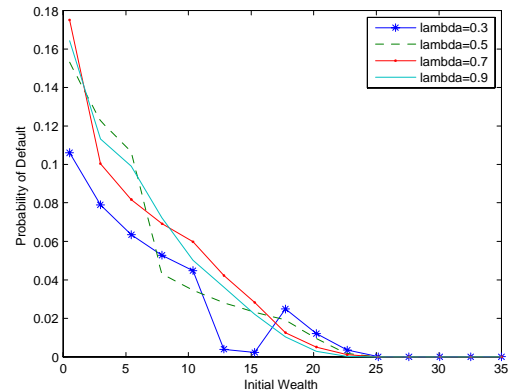
Size of Loan Given Initial Wealth



Repayment Probability Given Initial Wealth

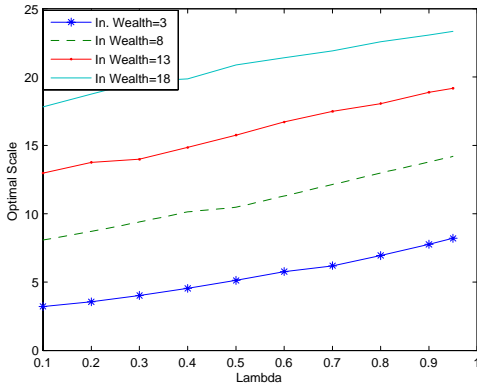


Voluntary Liquidation Probability Given Initial Wealth

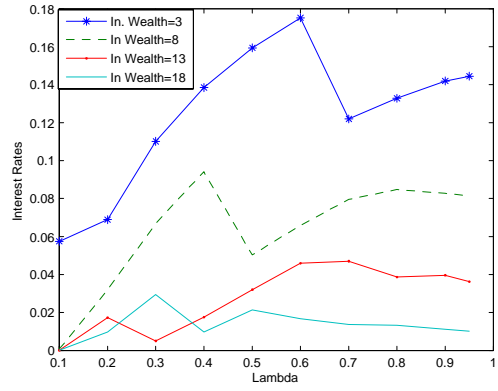


Default Probability Given Initial Wealth

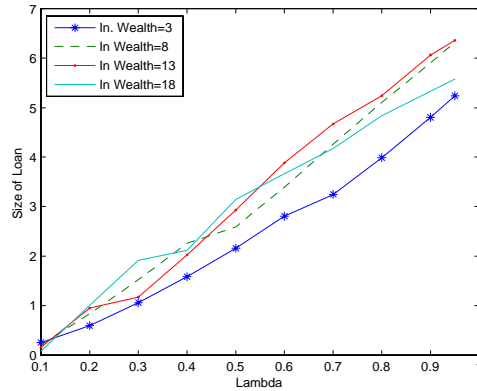
Figure 14 - $f(k) = k^{0.5}$, $i = 0$



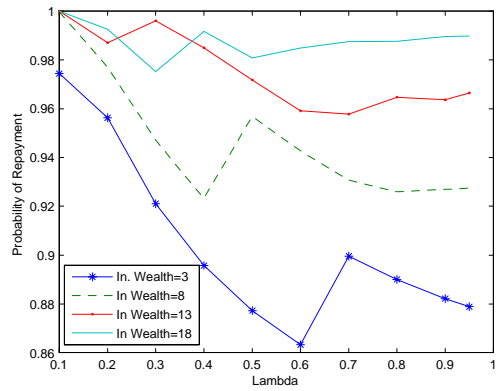
Optimal Scale Given λ



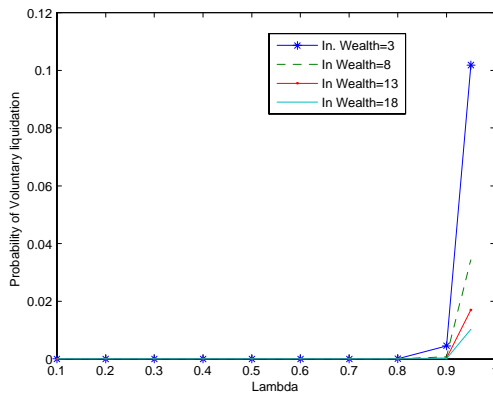
Interest rates Given λ



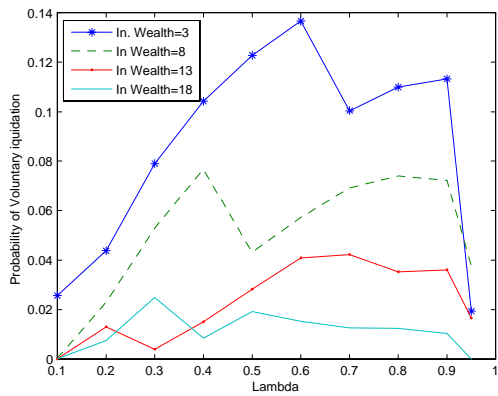
Size of Loan Given λ



Repayment Probability Given λ

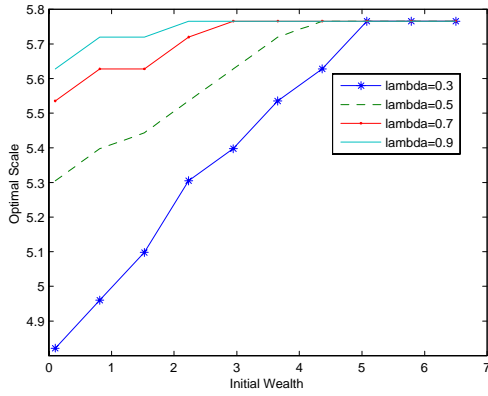


Voluntary Liquidation Probability Given λ

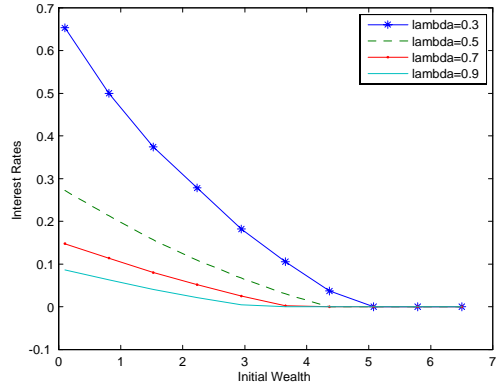


Default Probability Given λ

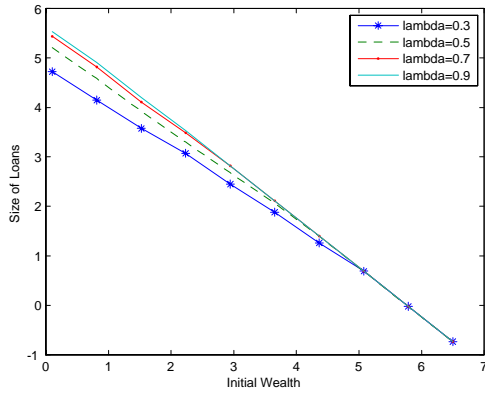
Figure 15 - $f(k) = k^{0.5}$, $i = 0$



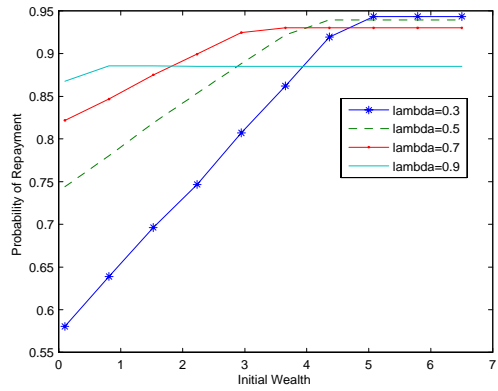
Optimal Scale Given Initial Wealth



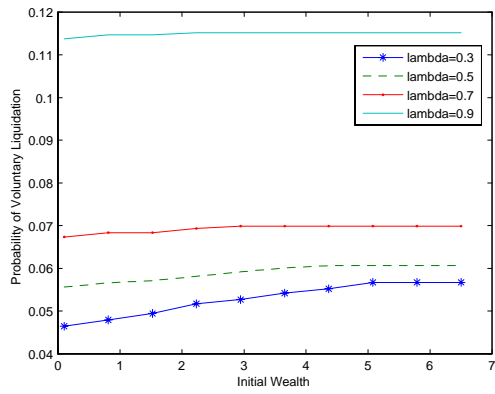
Interest rates Given Initial Wealth



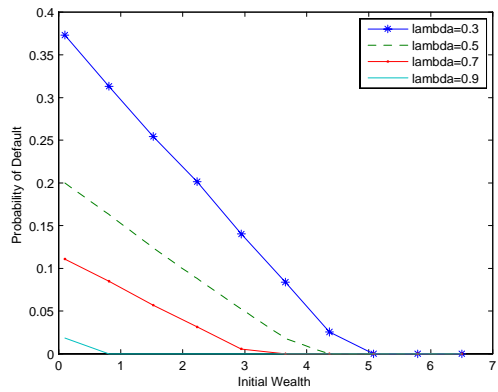
Size of Loan Given Initial Wealth



Repayment Probability Given Initial Wealth

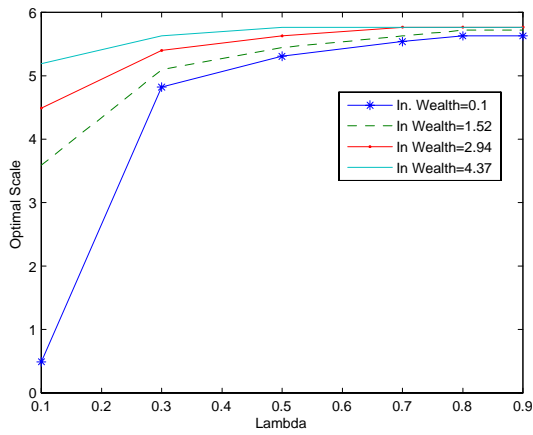


Voluntary Liquidation Probability Given Initial Wealth

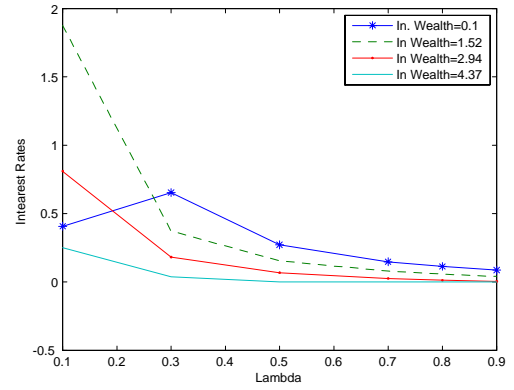


Default Probability Given Initial Wealth

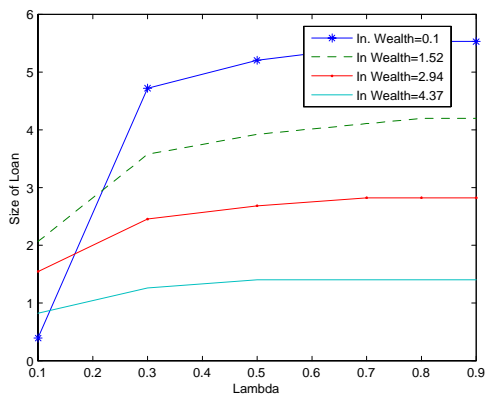
Figure 16 - $f(k) = (1 + (1 - k)^{-2})$ - baseline case



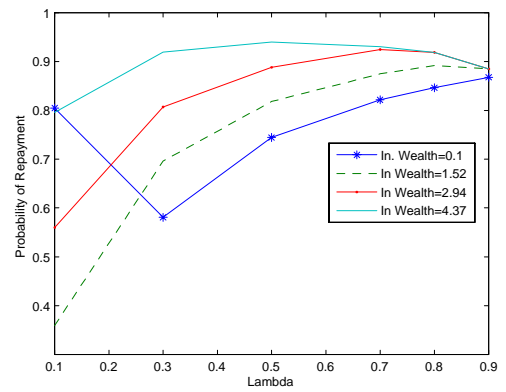
Optimal Scale Given λ



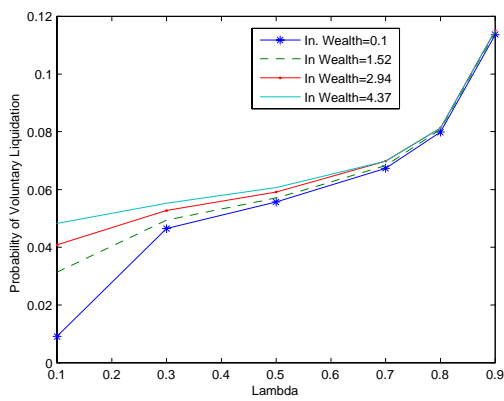
Interest rates Given λ



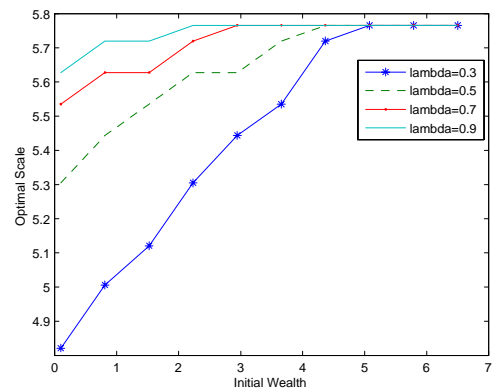
Size of Loan Given λ



Repayment Probability Given Initial Wealth

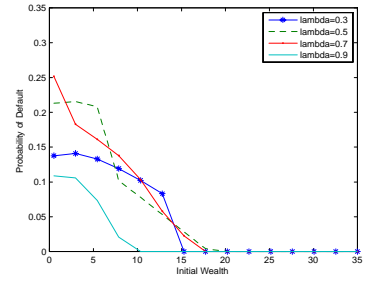
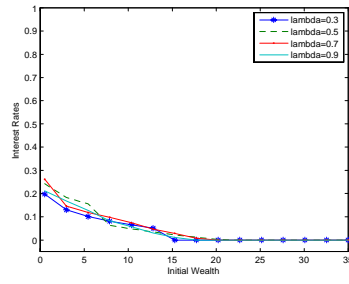
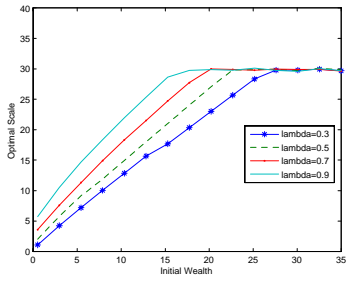


Voluntary Liquidation Probability Given Initial Wealth

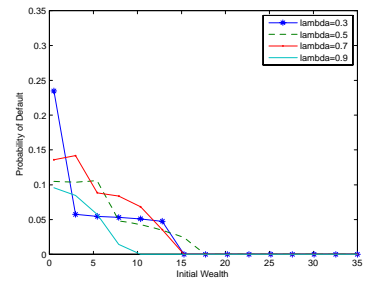
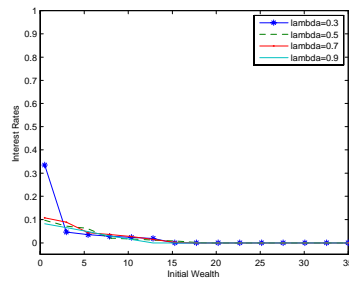
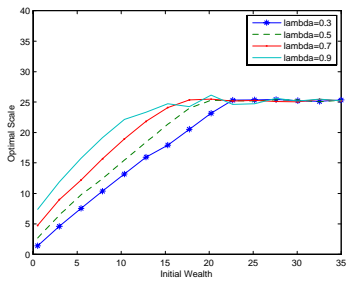


Default Probability Given λ

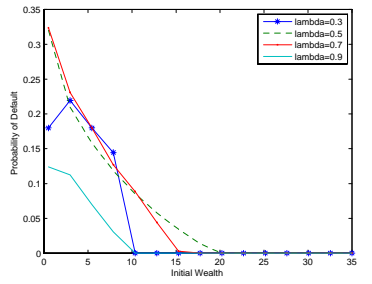
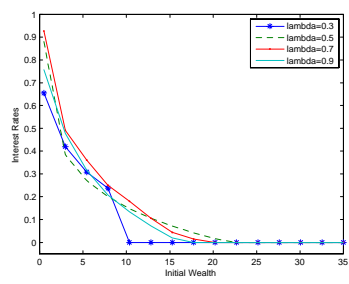
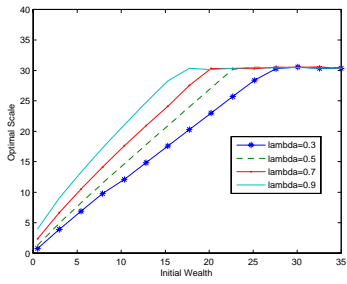
Figure 17 - $f(k) = (1 + (1 - k)^{-2})$ - baseline case
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Baseline Case

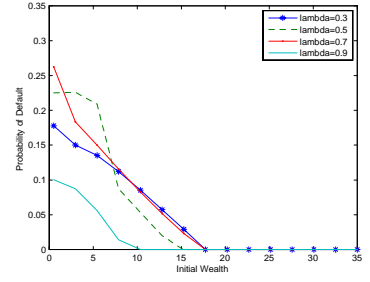
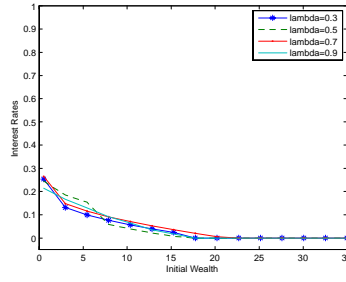
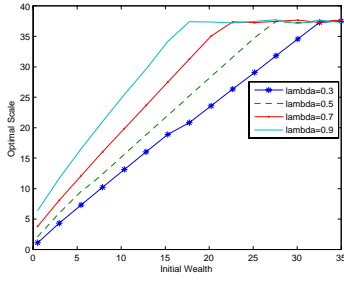


Lower Variance of θ - $\sigma = 1, \mu = 1$

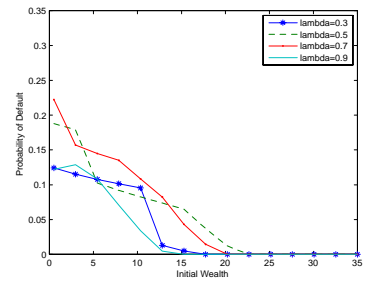
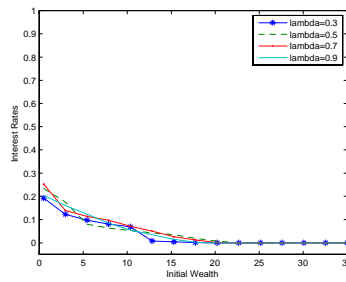
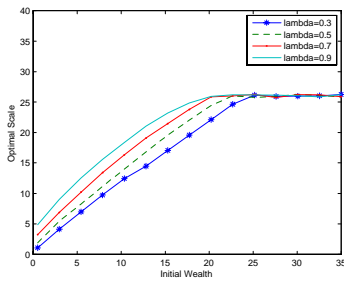


Higher Variance of θ - $\sigma = 0.3, \mu = 1.455$

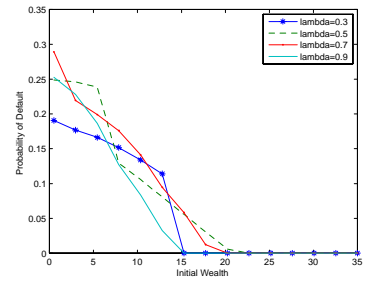
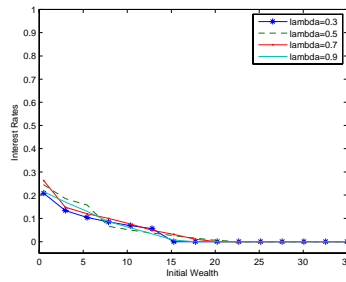
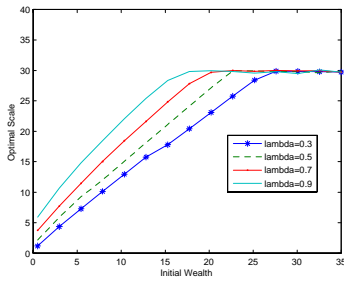
Figure 18 - Baseline case - $f(k) = k^{0.5}$



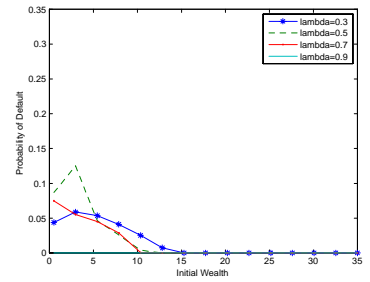
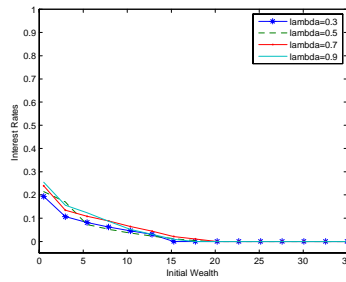
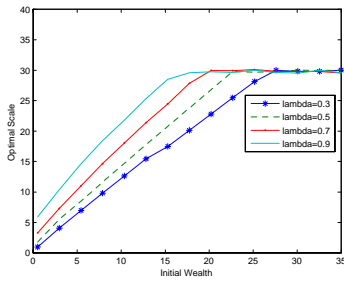
Higher liquidation value - $i = 0.6$



Lower liquidation value - $i = 0.3$

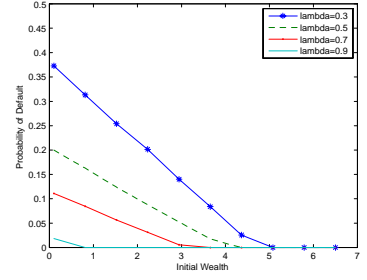
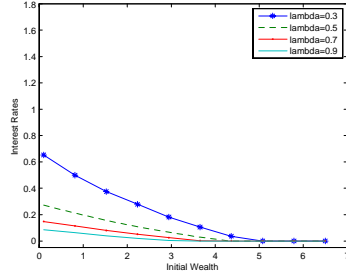
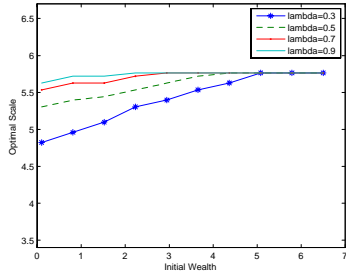


Lower cost of courts - $c = 0.1$

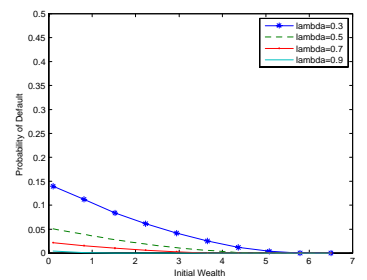
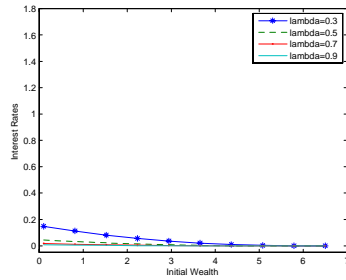
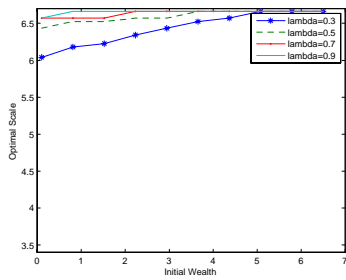


Higher cost of courts - $c = 0.8$

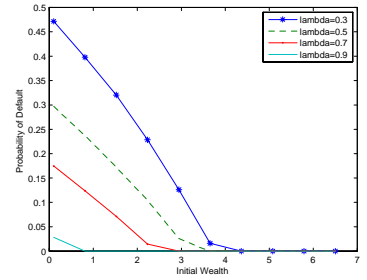
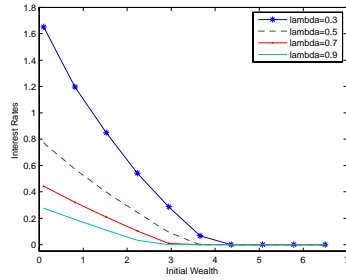
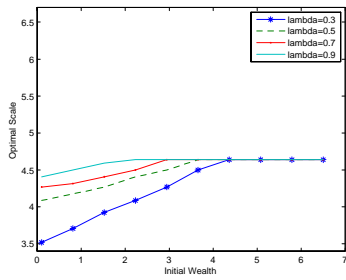
Figure 19 - Baseline case - $f(k) = k^{0.5}$



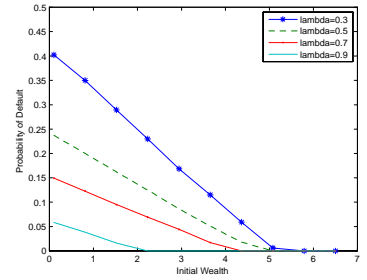
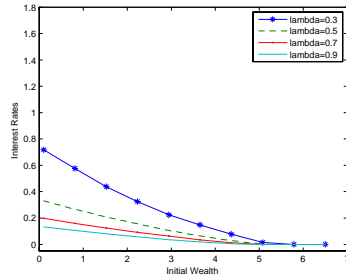
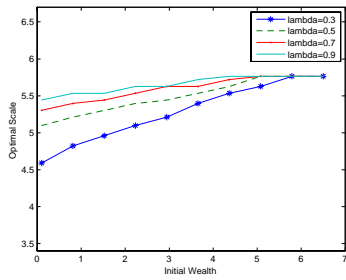
Baseline Case



Lower Variance - $\sigma = 1, \mu = 1$

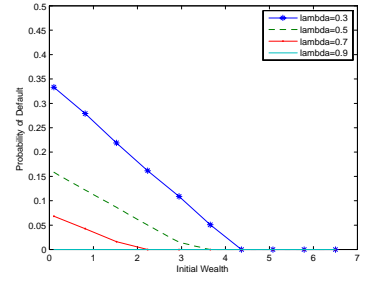
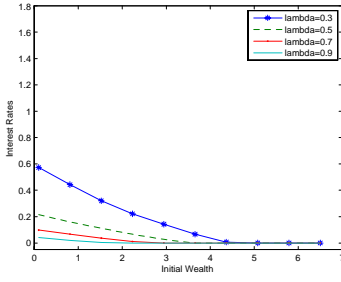
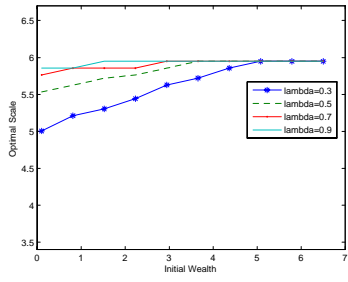


Higher Variance - $\sigma = 2, \mu = -0.5$

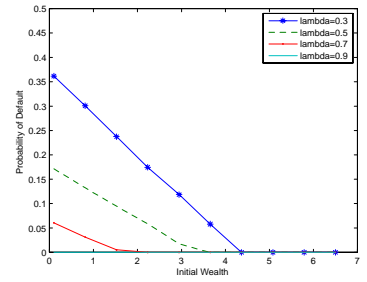
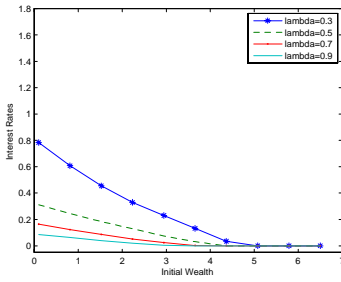
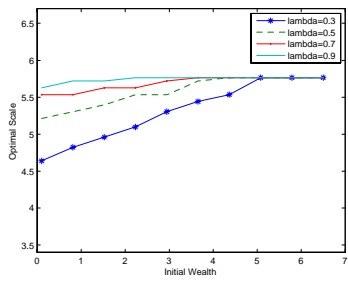


Lower Liquidation Value - $i = 0.2$

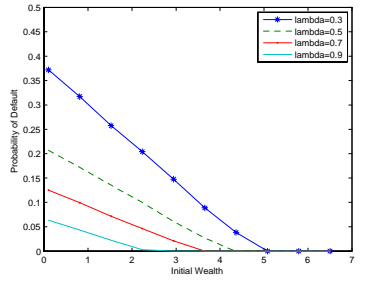
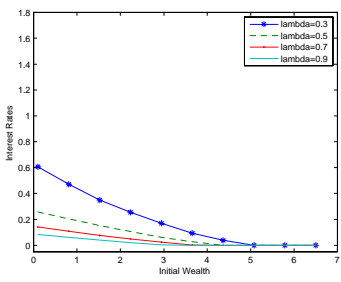
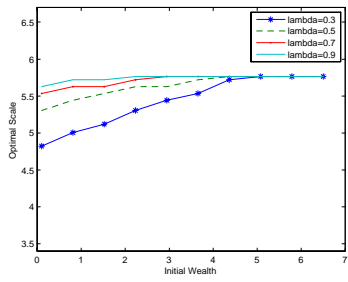
Figure 20 - $f(k) = (1 + (1 - k)^{-2})$ - baseline case



Higher liquidation value - $i = 0.8$



Higher cost of courts - $c = 1$



Lower cost of courts - $c = 0.1$

Figure 21- $f(k) = (1 + (1 - k)^{-2})$ - baseline case