

Sequential trading without perfect foresight: the role of default and collateral*

Wassim Daher[†] V. Filipe Martins-da-Rocha[‡] Mário Páscoa[§]
Yiannis Vailakis[¶]

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Abstract

Temporary equilibrium models replaced the usual assumption that agents can perfectly foresee future prices by weaker informational requirements allowing for inaccurate price forecasts. However, temporary equilibrium models were criticized for requiring some coordination of agents' expectations (i.e., overlapping expectations) and for not providing a mechanism that prevents the economy from collapsing when agents' expectations are not fulfilled. Our paper addresses these two shortcomings. When loans are secured by collateral, we can dispense with restrictions on agents' expectations to get equilibrium at the initial date. Moreover, allowing for default, allows us to restore equilibrium at future dates in the event of expectation errors.

1 Introduction

An important feature of the classical Arrow and Debreu (1954) model is that it imposes very weak informational requirements on the participants of the economy. Decision makers are assumed to know only their own characteristics. No assumptions are imposed on what they know or believe about other agents' characteristics and beliefs. Market prices convey all the relevant information about the economic environment and guide the economic agents to take their decisions.

While keeping the informational structure at a minimum level, the Arrow–Debreu model has often been questioned for its unrealism compared to the actual operation of markets. The approach to model time and uncertainty appears to be too demanding since it requires the presence of markets for the delivery of each good at all conceivable states and dates. Markets

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[†]CES, Université Paris-1 Panthéon–Sorbonne.

[‡]Ceremade, Université Paris–Dauphine.

[§]Faculdade de Economia, Universidade Nova de Lisboa.

[¶]School of Business and Economics, University of Exeter.

open only once. Agents purchase and sell consumption bundles and contingent contracts in the initial period and then just watch the future unfold. However, in reality there is not a complete set of contingent contracts and trade takes place sequentially.

Allowing for a sequence of spot commodities and financial markets, Arrow (1953) addresses some of these issues. Equilibrium allocations can now be implemented by a more realistic structure of markets. However, the transition to a sequential markets model is not without cost, since it requires more stringent informational requirements on the participants of the economy. Traders need to form expectations not only about their own characteristics but also about future prices.

The *perfect foresight approach* to model sequential markets, attributed to Radner (1967) (see also Radner (1972) and Radner (1982)),¹ proposes an equilibrium concept based on the hypothesis that agents are able to forecast correctly the equilibrium prices. Although traders need not agree on the joint probability distribution of future events, they must believe that a unique future price will appear at each event. That is, agents are required to have degenerate and common price expectations (see Radner (1982)). In addition, the expectations are self-fulfilling in the sense that they coincide with the equilibrium prices at each event.

There are two ways to justify the perfect foresight hypothesis. The first one imposes a strong rationality on the participants of the economy: agents are assumed to fully understand the economic environment (including preferences and endowments of other agents). The second explanation proposes that agents can draw on past experience and intuition to correctly predict the evolution of future prices. Although this may be true in a stable world where similar events occur regularly, it is difficult to be justified in environments where agents are exposed to new and unfamiliar events (see Magill and Quinzii (1996, pages 24 and 25)). Moreover, it has been highlighted in the literature (see Blume and Easley (1998)) that learning does not provide a satisfactory foundation for perfect foresight: positive results are delicate and robust results are mostly negative.

These considerations led a sizable literature in economics to propose a *bounded rationality* approach to model competition in uncertain dynamic environments. A common characteristic of bounded rationality models is that agents have no information about their economic environment apart from knowing their own preferences and endowment scenarios. Therefore, the models are consistent with the informational assumptions underlying the Arrow-Debreu model. Agents have beliefs which become now part of the primitives of the economy. These beliefs determine their actions and feed back into the actual evolution of the economic variables.

Temporary equilibrium models, originated in the work of Grandmont (see Grandmont (1970), Grandmont (1977)) and Green (1973), constitute an important part of the bounded rationality approach. The conceptual framework of temporary equilibrium takes into account the possibility that agents may have neither correct nor common and degenerate expectations about the evolution of future prices. The equilibrium achieved at a given moment is only temporary since only the current actions are coordinated by spot prices. No coordination is required for future plans which may turn out to be incompatible if agents have incorrect expectations.

The temporary equilibrium approach has provided a coherent framework as an alternative to the perfect foresight paradigm. Moreover, it has permitted the investigation of a variety of

¹If agents are using equilibrium prices to make inferences about the environment, then perfect foresight equilibrium takes the special form of a so called rational expectations equilibrium (Radner (1979)).

new themes that were difficult to be explained using rational models: the role of money and the existence of liquidity traps, quantity rationing and price stickiness.

Two are the most serious drawbacks of the temporary equilibrium approach. First, the possibility of *arbitrage* in spot markets: in the absence of *borrowing constraints* the size of traded contracts cannot be limited. For some patterns of expectations an agent may find it profitable to take an extreme short position. Existence of equilibrium cannot be guaranteed for all expectation patterns and extra assumptions have been proposed in the literature to overcome this difficulty (see Green (1973), Hart (1974), Grandmont (1977) and Hammond (1983)). Roughly speaking, these extra assumptions require traders to partially agree on a sufficient large set of future prices. These conditions, known as *overlapping expectation conditions*, appear to be rather stringent since they imply an unrealistic uniform perception of future price uncertainty. Subsequent research challenged the role and relevance of overlapping expectations. Milne (1980) argues for exogenous borrowing constraints that reflect lenders' perception for default risk. Alternatively, Stahl (1985a) (see also Stahl (1985b)) emphasizes the need for institutional borrowing constraints.

The second drawback is the possibility of *bankruptcy*: errors in expectations may lead to insolvency and equilibrium at future dates may fail to exist.² Grandmont (1982) points out this problem and argues for the design of *default rules* in temporary equilibrium models.

Our purpose is to provide a framework that keeps the desirable minimal informational requirements of temporary equilibrium models but addresses the shortcomings encountered in these models (overlapping expectations and bankruptcy). To do this we allow for the possibility of default while simultaneously protecting short-sales of assets through collateral requirements. The argument is as follows: if agents are allowed to default, expectations errors can be accommodated, leaving agents still some wealth, so that new equilibrium can be found (under the usual requirement of positivity of agents' wealth). On the other hand, the creditors' willingness to lend depends crucially on their expectations to be repaid. Collateralizing agents' promises will guarantee that creditors will receive at least the garnished collateral as a minimal repayment. This implies that creditors, even if they are pessimistic about delivery rates, are willing to lend and trade takes place in financial markets. In addition, the endogenous nature of collateral requirements has another important effect: it dispenses with overlapping expectations conditions.

The literature on default dates back to the work of Shubik (1972), Shubik and Wilson (1977) and Dubey and Shubik (1979). Default was later introduced in a general equilibrium setting by Dubey, Geanakoplos, and Shubik (1990), and was followed by Kehoe and Levine (1993), Zame (1993), Geanakoplos and Zame (2002), Araujo, Páscua, and Torres-Martinez (2002) and Dubey, Geanakoplos, and Shubik (2005). These papers highlight the important role of default under incomplete opportunities for risk sharing. However, with no exception, all studies rely on a perfect foresight formulation. Therefore, the models can still be questioned on the basis of the informational requirements that justify rationality. In addition, the informational requirements are even more demanding in these models since agents have to be able to predict not only the

²The problem is obviously avoided when the beliefs have full support, i.e. the support of subjective beliefs about future prices coincides with the set of all possible future prices (see Svensson (1981)). But this extreme overlapping expectations condition reduces considerably the financial investment possibilities since agents have to be solvent in real terms and not only in nominal terms.

future prices but also the delivery rates for all assets. This study provides a reappraisal of the implications of default and collateral in a different setting that departs from the rational paradigm by allowing agents to be less sophisticated. We formulate and analyze a two-period model that is in close relation with the traditional temporary equilibrium models, but it deviates from them by allowing for durable goods, collateral and the possibility of default.

We propose a new equilibrium notion that is simultaneously free of stringent informational requirements and consistent with market clearing in both periods. The equilibrium existence result is builded gradually in order to highlight the role played by default and collateral in achieving the equilibrium outcome. It is shown that the reliance on collateral to secure loans, allows us to dispense with the overlapping expectation conditions to get equilibrium in spot markets, while allowing for default is always sufficient to prevent the economy from collapsing due to expectations errors.

The paper is organized as follows. Section 2 presents the model and Section 3 introduces the equilibrium concepts: temporary equilibrium and sequential equilibrium. Section 4 addresses existence of temporary equilibrium at the initial date. Here we argue that collateral can do a better job than overlapping expectations in ruling out arbitrage opportunities. We argue also that overlapping expectations do not avoid arbitrage when default is allowed (since agents may go long and short in the same security). Section 5 addresses existence of temporary equilibrium at the next nodes (that is, existence of a sequence of temporary equilibrium, which we refer to as sequential equilibrium). We start by providing an example of an overlapping expectations economy with a single temporary equilibrium at the initial date but no temporary equilibrium at the second date, due to expectation errors. Then, we show that allowing for default is sufficient to restore equilibrium at the second date. Section 6 discusses alternative approaches where existence of sequential equilibrium is obtained at the cost of either requiring solvency in real terms or perfect foresight. Finally, in the last section, we argue that in the presence of market imperfections, such that asymmetric information, unawareness or time-inconsistent preferences, the perfect foresight approach is still more demanding or can not be clearly formulated, whereas our sequential equilibrium approach remains valid.

2 The Model

We consider an exchange economy which extends over two dates $t \in \{0, 1\}$. We represent exogenous uncertainty about all preferences and initial endowments at $t = 1$ by a finite set S of events.

At every date t there is a finite set L_t of commodities available for trade. Let $X_t := \mathbb{R}_+^{L_t}$ denote the set of commodity bundles and $P_t := \mathbb{R}_+^{L_t}$ denote the set of commodity prices at period t . We depart from the usual intertemporal models by assuming (as in Geanakoplos and Zame (2002) and Araujo, Páscoa, and Torres-Martinez (2002)) that some commodities are durable. The depreciation of goods in state s is captured by a linear and positive mapping $Y_s : X_0 \rightarrow X_1$, in the sense that if $x_0 \in X_0$ is a commodity bundle consumed or used at $t = 0$, then $Y_s(x_0)$ represents what remains for consumption in state s . The depreciation function Y_s links periods $t = 0$ and $t = 1$, in the sense that the depreciated goods consumed or used in period $t = 0$ become part of consumers' endowment in period $t = 1$.

At the first period $t = 0$ there is a finite set J of financial assets available for trade. Assets are assumed to be real, i.e. for each $j \in J$ there exists $A_j(s) \in \mathbb{R}_+^L$ which represents the bundle of commodities (goods or services) to be delivered in state s . In units of account and for any $p_1 \in P_1$, asset returns are given by $V(p_1, s) \in \mathbb{R}^J$ defined by

$$\forall j \in J, \quad V_j(p_1, s) = p_1 \cdot A_j(s).$$

At $t = 0$, each agent chooses to purchase an amount $\theta_j \geq 0$ and to sale an amount $\phi_j \geq 0$ of asset j . Let $\Xi^i \subset \mathbb{R}_+^J \times \mathbb{R}_+^J$ be the space of investment strategies (θ, ϕ) available to agent i and let $Q = \mathbb{R}_+^J$ be the space of asset prices at $t = 0$.

Assumption 2.1. The set Ξ^i is closed convex and non-empty for each agent i .

Each agent i has a utility function $U_0^i : X_0 \rightarrow \mathbb{R}$ for consumption at $t = 0$ and an initial endowment in goods $e_0^i \in X_0$. For each possible realization of exogenous uncertainty s at $t = 1$, agent i has a utility function $U_1^i(\cdot, s) : X_1 \rightarrow \mathbb{R}$ for consumption at $t = 1$ and an initial endowment $e_1^i(s) \in X_1$. At $t = 0$ agent i discounts future consumption with a discount factor $\beta^i > 0$.

Assumption 2.2. For each agent i ,

- (a) initial endowments are strictly positive, i.e. $e_0^i \in \mathbb{R}_{++}^{L_0}$ and $e_1^i(s) \in \mathbb{R}_{++}^{L_1}$ for each $s \in S$,
- (b) the function $x_1 \mapsto U_1^i(x_1, s)$ is continuous, bounded, concave and strictly increasing for each s .

The actions of agents at the initial date $t = 0$ involve commitments for future dates. Agents take into account the consequences of their choices, so decision making has to be described retrospectively, that is, by backward induction.

2.1 Actions at date $t = 1$

If an agent i chooses an action $a = (x_0, \theta, \phi)$ in $A^i = X_0 \times \Xi^i$ at $t = 0$, then at $t = 1$, if the realized state of nature is s , and the commodity price is p_1 , he should receive the amount $V(p_1, s) \cdot \theta$ of units of accounts and deliver the amount $V(p_1, s) \cdot \phi$. Following Dubey, Geanakoplos, and Shubik (2005), default is allowed in the sense that agent i chooses to deliver an amount $d_j \geq 0$ that may be lower than his debt $V_j(p_1, s)\phi_j$ on asset j . As in Dubey, Geanakoplos, and Zame (1995) (see also Geanakoplos and Zame (2002)) we assume that short-sales are backed by exogenous collateral requirements. For each asset j there exists a collateral bundle $C_j \in \mathbb{R}_+^L$ that protects the buyers of this asset (lenders) in the sense that each borrower when he decides to sell at $t = 0$ an amount ϕ_j of asset j , has to constitute the bundle $C_j\phi_j$ at $t = 0$ and has to deliver at $t = 1$, state s and price p_1 , at least the minimum between his promise $V_j(p_1, s)\phi_j$ and the depreciated value of the collateral, i.e. $d_j \geq D_j(p_1, s)\phi_j$ where

$$D_j(p_1, s) := \min\{p_1 \cdot Y_s(C_j), V_j(p_1, s)\phi_j\}. \quad (1)$$

The only reason that agents deliver anything strictly above the minimum between their promise and the depreciated value of the collateral is that they feel a disutility $\lambda_j^i(s) \in [0, +\infty]$ from defaulting.

The payoff from choosing a consumption plan $x_1 \in X_1$ and deliveries $d = (d_j)_{j \in J}$ at state s when the commodity price is p_1 , is given by

$$W^i(p_1, s, a, x_1, d) = U_1^i(x_1, s) - \sum_{j \in J} \lambda_j^i(s) \frac{[V_j(p_1, s)\phi_j - d_j]^+}{p_1 \cdot v(s)} \quad (2)$$

where $v(s) \in X_1$ is exogenously specified with $v(s) \geq \mathbf{1}_{L_1}$.³

This possibility of default implies that for each realization of price/state pair (p_1, s) at $t = 1$, there will be a delivery rate $\kappa_j \in [0, 1]$ such that the units of account delivered to the lenders of one unit of asset j is

$$V_j(\kappa_j, p_1, s) = \kappa_j V_j(p_1, s) + (1 - \kappa_j) D_j(p_1, s). \quad (3)$$

2.1.1 Budget set at date $t = 1$: $B_1^i(p_1, \kappa, s, a)$

At date $t = 1$, given an action $a = (x_0, \theta, \phi)$ chosen at $t = 0$, if the state of nature is s , the commodity price is p_1 and the vector of delivery rates is $\kappa \in K = [0, 1]^J$, then we denote by $B_1^i(p_1, \kappa, s, a)$ the budget set defined by all pairs $(x_1, d) \in X_1 \times D$ where x_1 represents a consumption bundle and d the vector of deliveries in $D = \mathbb{R}_+^J$, satisfying the constraints

$$p_1 \cdot x_1 + \sum_{j \in J} d_j \leq p_1 \cdot [e_1^i(s) + Y_s(x_0 + C\phi)] + \sum_{j \in J} V_j(\kappa_j, p_1, s)\theta_j, \quad (4)$$

$$\forall j \in J, \quad D_j(p_1, s)\phi_j \leq d_j, \quad (5)$$

and

$$W^i(p_1, s, a, x_1, d) > -\infty \quad (6)$$

where $C\phi$ is the commodity bundle defined by

$$C\phi = \sum_{j \in J} C_j \phi_j.$$

Remark 2.1. If $\lambda_j^i(s) = +\infty$ for some asset j and state s , then agent i is not allowed to default in state s with respect to his debt on asset j .

2.1.2 Demand at date $t = 1$: $d_1^i(p_1, \kappa, s, a)$

Assume that at $t = 0$, agent i has chosen an action $a \in A^i$ such that at $t = 1$ and for the realized (p_1, κ, s) the budget $B_1^i(p_1, \kappa, s, a)$ is non-empty. Agent i will choose among the vectors (x_1, d) in $B_1^i(p_1, \kappa, s, a)$ those which maximize his payoff. We denote by $d_1^i(p_1, \kappa, s, a)$ the demand set defined by

$$\operatorname{argmax} \{W^i(p_1, s, a, x_1, d) : (x_1, d) \in B_1^i(p_1, \kappa, s, a)\}. \quad (7)$$

We denote by⁴ $\Gamma^i(p_1, \kappa, s, a)$ the extended real number

$$\sup\{W^i(p_1, s, a, x_1, d) : (x_1, d) \in B_1^i(p_1, \kappa, s, a)\}. \quad (8)$$

³If F is a finite set then $\mathbf{1}_F$ is the vector in \mathbb{R}^F defined by $\mathbf{1}_F(f) = 1$ for every $f \in F$.

⁴If $B_1^i(p_1, \kappa, s, a) = \emptyset$ then we let $\Gamma^i(p_1, \kappa, s, a) = -\infty$.

2.2 Actions at date $t = 0$

At $t = 0$ agent i does not know what will be at $t = 1$ the price p_1 , the vector of rates κ and the state s . We assume that agent i , after observing current prices (p_0, q) , forms expectations on the realization of uncertainty represented by a probability measure

$$\mu^i(p_0, q) \in \text{Prob}(P_1 \times K \times S).$$

The function μ^i is defined on the set $\Delta_0 = \{(p_0, q) \in P_0 \times Q : p_0 \cdot \mathbf{1}_L + q \cdot \mathbf{1}_J = 1\}$. Following Svensson (1981), we distinguish uncertainty about future prices and delivery rates from uncertainty about future states. The former kind of uncertainty is endogenous in nature in the sense that it is transmitted via the individual actions.

Assumption 2.3. The correspondence μ^i is continuous for the weak topology on probability measures.

Remark 2.2. It is proved in Green (1973, Remark 3.1) that the previous assumption implies that the correspondence $(p_0, q) \mapsto \text{supp } \mu^i(p_0, q)$ is lower semi-continuous.

When no default is allowed it is natural to assume that agents anticipate that for each asset the delivery rate will be equal to one.

Assumption 2.4. When no default is allowed, i.e. when $\lambda_j^i(s) = +\infty$ for every agent i , asset j and state s , then

$$\forall (p_0, q), \quad \text{supp } \mu^i(p_0, q) \subset P_1 \times \{\mathbf{1}_J\} \times S. \quad (9)$$

Observe that for every $\beta > 0$ we have

$$(x_1, d) \in B_1^i(p_1, \kappa, s, a) \iff (x_1, \beta d) \in B_1^i(\beta p_1, \kappa, s, a) \quad (10)$$

together with

$$W^i(p_1, s, a, x, d) = W^i(\beta p_1, s, a, x, \beta d) \quad (11)$$

implying that

$$\Gamma^i(\beta p_1, \kappa, s, a) = \Gamma^i(p_1, \kappa, s, a). \quad (12)$$

Therefore only relative prices at $t = 1$ matter. We assume that agents incorporate this in their belief.

Assumption 2.5. For every signal $(p_0, q) \in \Delta_0$ at $t = 0$,

$$\text{supp } \mu^i(p_0, q) \subset \Delta_1 \times K \times S, \quad (13)$$

where $\Delta_1 = \{p_1 \in P_1 : p_1 \cdot \mathbf{1}_{L_1} = 1\}$ is the simplex in \mathbb{R}^{L_1} .

2.2.1 Budget set at date $t = 0$: $B_0^i(p_0, q, \mu^i)$

When agent i chooses his action $a = (x_0, \theta, \phi)$ at $t = 0$, he must check that this action is consistent with the possible future realizations of (p_1, κ, s) with respect to his beliefs μ^i . Therefore, given a commodity/asset price $(p_0, q) \in \Delta_0$, the budget set at $t = 0$, denoted by $B_0^i(p_0, q, \mu^i)$ is the set of all actions $a = (x_0, \theta, \phi) \in A^i$ satisfying the constraints

$$p_0 \cdot [x_0 + C\phi] + q \cdot \theta \leq p_0 \cdot e_0^i + q \cdot \phi \quad (14)$$

and

$$B_1^i(p_1, \kappa, s, a) \neq \emptyset, \quad \forall (p_1, \kappa, s) \in \text{supp } \mu^i(p_0, q). \quad (15)$$

Remark 2.3. Given a first period action a in the budget set $B_0^i(p_0, q, \mu^i)$, the function $\omega \mapsto \Gamma^i(\omega, a)$ takes real values only for those $\omega = (p_1, \kappa, s)$ for which $B_1^i(\omega, a)$ is non-empty, in particular for those ω in $\text{supp } \mu^i(p_0, q)$.

Remark 2.4. When default is allowed for every agent, every asset and at every state, i.e., $\lambda_j^i(s) < +\infty$ for every (i, j, s) in $I \times J \times S$, then the budget set $B_1^i(p_1, \kappa, s, a)$ is always non-empty whatever is the action $a \in A^i$ previously chosen. Indeed the pair (x_1, d) defined by

$$x_1 = e_1^i(s) + Y_s(x_0) \quad \text{and} \quad d_j(s) = D_j(p_1, s)\phi_j$$

always belong to the budget set. It follows that the set $B_0^i(p_0, q, \mu^i)$ coincides with the set of all actions $a = (x_0, \theta, \phi) \in A^i$ satisfying the constraint (14).

2.2.2 Demand at date $t = 0$: $d_0^i(p_0, q, \mu^i)$

Assume that at $t = 0$, agent i has chosen an action a in the budget set $B_0^i(p_0, q, \mu^i)$. A necessary condition for the set $d_1^i(p_1, \kappa, s, a)$ to be non-empty is that p_1 belongs to the relative interior $\text{int } \Delta_1$ of Δ_1 . Agents anticipate this non-arbitrage condition on commodity prices.

Assumption 2.6. For each agent i , we have $\mu^i(p_0, q, [\text{int } \Delta_1] \times K \times S) = 1$.

It follows that, according to his beliefs μ^i , the expected utility at $t = 0$ of the action $a = (x_0, \theta, \phi)$ is given by

$$V^i(p_0, q, a, \mu^i) = U_0^i(x_0) + \beta^i \int_{\Omega} \Gamma^i(\omega, a) \mu^i(p_0, q, d\omega) \quad (16)$$

where $\Omega = (\text{int } \Delta_1) \times K \times S$ represents the relevant uncertainty about $t = 1$. We let $d_0^i(p_0, q, \mu^i)$ be the demand set at $t = 0$ if the commodity/asset price is (p_0, q) , defined by

$$d_0^i(p_0, q, \mu^i) = \text{argmax}\{V^i(p_0, q, a, \mu^i) : a \in B_0^i(p_0, q, \mu^i)\}. \quad (17)$$

3 The equilibrium concept

A family (p_0, q, \mathbf{a}) with $(p_0, q) \in \Delta_0$ and $\mathbf{a} = (a^i)_{i \in I} \in A^I$ is called a *temporary equilibrium* at $t = 0$ given beliefs $\boldsymbol{\mu} = (\mu^i)_{i \in I}$ if

- (i) $a^i \in d_0^i(p_0, q, \mu^i)$ for each $i \in I$,
- (ii) markets clear at $t = 0$, i.e.

$$\sum_{i \in I} x_0^i + C\phi^i = \sum_{i \in I} e_0^i \quad \text{and} \quad \sum_{i \in I} \theta^i = \sum_{i \in I} \phi^i$$

where $a^i = (x_0^i, \theta^i, \phi^i)$.

We denote by $\text{Eq}(\boldsymbol{\mu}, 0)$ the set of temporary equilibria at $t = 0$ given beliefs $\boldsymbol{\mu}$.

Definition 3.1. A family $\boldsymbol{\mu}$ of beliefs is said *temporary viable* if the set $\text{Eq}(\boldsymbol{\mu}, 0)$ is non-empty, i.e. if there exists a temporary equilibrium at $t = 0$ given $\boldsymbol{\mu}$.

A family $(p_1, \kappa, \mathbf{x}_1, \mathbf{d})$ with $(p_1, \kappa) \in \Delta_1 \times K$, $\mathbf{x}_1 = (x_1^i)_{i \in I} \in X^I$ and $\mathbf{d} = (d^i)_{i \in I} \in D^I$ is called a *temporary equilibrium* at $t = 1$ and state s given beliefs $\boldsymbol{\mu}$ and actions \mathbf{a} taken at $t = 0$ if

(i-s) $(x_1^i, d^i) \in d_1^i(p_1, \kappa, s, a^i)$ for each $i \in I$,

(ii-s) commodity markets clear at $t = 1$, i.e.

$$\sum_{i \in I} x_1^i = \sum_{i \in I} e_1^i(s) + Y_s(x_0^i + C\phi^i)$$

(iii-s) financial markets clear at $t = 1$, i.e.

$$\kappa_j = \begin{cases} \frac{\sum_{i \in I} d_j^i}{\sum_{i \in I} V_j(p_1, s)\phi^i} & \text{if } \sum_{i \in I} V_j(p_1, s)\phi^i > 0, \\ \text{arbitrary} & \text{if } \sum_{i \in I} V_j(p_1, s)\phi^i = 0. \end{cases}$$

We denote by $\text{Eq}(\mathbf{a}, s)$ the set of *temporary equilibria* at $t = 1$ and state s given beliefs $\boldsymbol{\mu}$ and actions \mathbf{a} taken at $t = 0$. The set of market clearing prices at state s and time $t = 1$ defined by the projection on $\Delta_1 \times K$ of the set $\text{Eq}(\mathbf{a}, s)$ is denoted by $\mathcal{P}_C(\mathbf{a}, s)$. In other words, (p_1, κ) belongs to $\mathcal{P}_C(\mathbf{a}, s)$ if there exists a pair $(\mathbf{x}_1, \mathbf{d})$ such that $(p_1, \kappa, \mathbf{x}_1, \mathbf{d})$ belongs to $\text{Eq}(\mathbf{a}, s)$.

Our main purpose is to focus on beliefs that are sequentially viable in the following sense.

Definition 3.2. A family $\boldsymbol{\mu} = (\mu^i)_{i \in I}$ of beliefs is *sequentially viable* if there exists a sequence of temporary equilibria, i.e. there exist (p_0, q, \mathbf{a}) a temporary equilibrium at $t = 0$ and a temporary equilibrium at $t = 1$ and each state s , given the previous equilibrium actions \mathbf{a} .

In other words, a family of beliefs $\boldsymbol{\mu}$ is sequentially viable if there exists (p_0, q, \mathbf{a}) satisfying

(a) Equilibrium at $t = 0$, i.e. $(p_0, q, \mathbf{a}) \in \text{Eq}(\boldsymbol{\mu}, 0)$

(b) Equilibrium at $t = 1$, i.e. $\text{Eq}(\mathbf{a}, s) \neq \emptyset$ for every $s \in S$.

4 Arbitrage opportunities

Our purpose in this section is to investigate under which conditions a family $\boldsymbol{\mu}$ of beliefs is temporary viable, i.e. the set $\text{Eq}(\boldsymbol{\mu}, 0)$ is non-empty. At the first period agents consume but also trade assets. A necessary condition for a family of beliefs to be temporary viable is that there are no arbitrage opportunities at the first period. In a different setting (see e.g. Green (1973), Grandmont (1977), Hart (1974) and Hammond (1983)) the non-existence of arbitrage opportunities was proved to be also a sufficient condition. This result is still valid in our framework. We provide hereafter three different conditions on primitives which exclude arbitrage opportunities.

4.1 Bounded short-sales

Naturally if short-sales are bounded, there are no arbitrage opportunities and we can prove that every family of beliefs is temporary viable.

Proposition 4.1. Assume that for each asset, short-sales are bounded, i.e. there exists $\Phi^i \in \mathbb{R}_+^J$ such that $\Xi^i \subset \mathbb{R}_+^J \times [0, \Phi^i]$. Then every family of beliefs is temporary viable.

The proof is based on standard arguments. Details can be found in Appendix.

4.2 Overlapping expectations

When short-sales are not exogenously bounded and if there are no borrowing constraints (i.e. if the collateral C is zero) arbitrage opportunities may occur. When default is not allowed this has already been shown by Green (1973) and Grandmont (1977). Obviously, if default is allowed then the set of arbitrage opportunities is even larger.

We extend the results of Green (1973) and Grandmont (1977), proving that if a family of beliefs satisfies an overlapping expectations condition and if default is not allowed, then it is temporary viable. In order to define the overlapping expectations condition, we need to introduce some notations.

Notation 4.1. Given a family μ of beliefs, we denote by⁵ $\nu^i(p_0, q) \in \text{Prob}(S)$ the marginal probability defined by

$$\forall s \in S, \quad \nu^i(p_0, q, s) = \mu^i(p_0, q, \Delta_1 \times K \times \{s\}) \quad (18)$$

and for every $s \in \text{supp } \nu^i(p_0, q)$, we denote by⁶ $\delta^i(p_0, q, s)$ the conditional probability in $\text{Prob}(\Delta_1 \times K)$ defined by

$$\delta^i(p_0, q, s, dp_1 \times d\kappa) = \frac{1}{\nu^i(p_0, q, s)} \mu^i(p_0, q, dp_1 \times d\kappa \times \{s\}). \quad (19)$$

The probability $\nu^i(p_0, q)$ represents agent i 's subjective beliefs on states and $\delta^i(p_0, q, s)$ represents agent i 's beliefs about the realization of the commodity price and the vector of delivery rates, given that the realized state is s .

Example 4.1. In Dutta and Morris (1997), it is assumed that agents have homogenous beliefs on states, i.e. there exists an objective probability $\nu \in \text{Prob}(S)$ such that

$$\forall (p_0, q) \in P_0 \times Q, \quad \nu^i(p_0, q) = \nu.$$

Assumption 4.1. A family μ of beliefs has overlapping expectations if there exists a subset Σ of S such that

- the family $(B_j, j \in J)$ has rank $\#J$ in $\mathbb{R}^{\Sigma \times L}$ where $B_j(\sigma, \ell) = A_j(\sigma, \ell)$ for each $\sigma \in \Sigma$ and

$$\forall (p_0, q), \quad \Sigma \subset \bigcap_{i \in I} \text{supp } \nu^i(p_0, q); \quad (20)$$

⁵We should write ν_{μ^i} .

⁶We should write δ_{μ^i} .

- for every $\sigma \in \Sigma$, the set $\Lambda_\sigma(p_0, q)$ defined by

$$\Lambda_\sigma(p_0, q) = \bigcap_{i \in I} \text{co Proj}_{\Delta_1} \text{supp } \delta^i(p_0, q, \sigma) \quad (21)$$

has a non-empty relative interior in Δ_1 .

Theorem 4.1. *Consider an economy where default is not allowed. If a family μ of beliefs has overlapping expectations then it is temporary viable.*

Proof. For each $n \in \mathbb{N}$, we let $\Xi_n^i = \{(\theta, \phi) \in \Xi^i : \phi \leq n \mathbf{1}_J\}$. Applying Proposition 4.1, there exists $(p_{0,n}, q_n)$ and \mathbf{a}_n such that $(p_{0,n}, q_n, \mathbf{a}_n)$ is an equilibrium at $t = 0$ when investment strategies are restricted to Ξ_n^i . Since $(p_{0,n}, q_n) \in \Delta_0$, passing to a subsequence if necessary, we can assume that $(p_{0,n}, q_n)$ converges to some $(p_0, q) \in \Delta_0$. Similarly, since for each n we have $\sum_{i \in I} x_{0,n}^i \leq \sum_{i \in I} e_0^i$, passing to a subsequence if necessary, we can assume that $(x_{0,n}^i)$ converges to some x_0^i . The main difficulty resides on proving that the sequence $(z_n^i = \theta_n^i - \phi_n^i)$ is bounded.

Claim 4.1. The sequence (z_n^i) is bounded.

Proof of Claim 4.1. Let α_n be the real number defined by

$$\alpha_n = \sup\{|z_n^i(j)| : (i, j) \in I \times J\}.$$

Assume by way of contradiction that (α_n) is not bounded. Passing to a subsequence if necessary, we can assume that $\lim_n \alpha_n = +\infty$. There exists v^i such that (passing to a subsequence if necessary)

$$\lim_{n \rightarrow \infty} \frac{1}{\alpha_n} z_n^i = v^i.$$

By the choice of (α_n) , there exists $k \in I$ such that $v^k \neq 0$. Moreover since asset markets clear for each n , the family $(v^i)_{i \in I}$ is such that

$$\sum_{i \in I} v^i = 0. \quad (22)$$

Since a_n^i belongs to budget set $B_0^i(p_{0,n}, q_n, \mu^i)$, it must be the case that for every $(p_1, \mathbf{1}_J, s)$ in $\text{supp } \mu^i(p_{0,n}, q_n)$

$$0 \leq p_1 \cdot [e_1^i(s) + \sum_{j \in J} A_j(s) z_{j,n}^i]. \quad (23)$$

In particular for every $\sigma \in \Sigma$, and for every p_1 in $\text{Proj}_{\Delta_1} \text{co supp } \delta^i(p_{0,n}, q_n)$ we have

$$0 \leq p_1 \cdot \frac{1}{\alpha_n} [e_1^i(\sigma) + \sum_{j \in J} A_j(\sigma) z_{j,n}^i]. \quad (24)$$

Passing to the limit and using the lower semicontinuity of $\text{supp } \mu^i$, we get that for each $\sigma \in \Sigma$,

$$\forall p_1 \in \Lambda_\sigma(p_0, q), \quad 0 \leq p_1 \cdot \sum_{j \in J} A_j(\sigma) v_j^i. \quad (25)$$

Since $\Lambda_\sigma(p_0, q)$ has non-empty relative interior in Δ_1 , we can fix a price $\pi \in \text{int}_{\Delta_1} \Lambda_\sigma(p_0, q)$.⁷ It must be the case that for each i ,

$$\sum_{j \in J} A_j(\sigma) v_j^i = 0 \quad \text{or} \quad \pi \cdot \left[\sum_{j \in J} A_j(\sigma) v_j^i \right] > 0. \quad (26)$$

Since $\sum_{i \in I} v_j^i = 0$, we must have for each i ,

$$\forall \sigma \in \Sigma, \quad \sum_{j \in J} A_j(\sigma) v_j^i = 0.$$

Implying that for each i ,

$$\sum_{j \in J} B_j v_j^i = 0.$$

But the family $(B_j, j \in J)$ is of rank $\#J$. Therefore for each i we have $v^i = 0$: contradiction. This ends the proof of the claim. \square

Passing to a subsequence if necessary, we can assume that there exists $(\theta^i, \phi^i) \in \Xi^i$ such that the sequence (z_n^i) converges to $(\theta^i - \phi^i)$. Since no default is allowed, without any loss of generality, we can replace (θ_n^i, ϕ_n^i) by $([z_n^i]^+, [z_n^i]^-)$. Therefore, we can assume that the sequence $(x_{0,n}^i, \theta_n^i, \phi_n^i)$ converges to $a^i = (x_0^i, \theta^i, \phi^i)$. Following Proposition B.1 the budget correspondence is closed, implying that a^i belongs to $B_0^i(p_0, q, \mu^i)$.

To complete the proof of the theorem, it suffices to show that a^i belongs to the demand set $d_0^i(p_0, q, \mu^i)$. We omit the standard arguments based on the lower semi-continuity of the budget set correspondence (Proposition B.2 in Appendix) and the continuity of the utility function $V^i(\cdot, \mu^i)$ (Proposition C.2 in Appendix) on the graph of $B^i(\cdot, \mu^i)$. \square

It is important to notice that when default is allowed and no collateral is required, the assumption of overlapping expectations is no more sufficient. Indeed, agents can purchase and borrow the same quantity of the same asset at the first period. This will be at no cost at $t = 0$ since there are no collateral requirements. At $t = 1$ an agent who defaults, he suffers a penalty on his utility but this may be compensated by the gain of utility from consuming bundles instead of paying his debt. We illustrate this point in the following example. In order to preclude such *self-arbitrage* opportunities, it is needed to impose more restrictions on the family of beliefs than overlapping expectations.

Example 4.2. Consider an economy with one state of nature at $t = 1$, i.e. $S = \{s\}$, one asset, i.e. $J = \{j\}$ that promises to deliver the bundle $\mathbf{1}_L$ (i.e. one unit of each commodity) in state s and no collateral requirements, i.e. $C = 0$. We assume that the utility function $U_1^i(\cdot, s)$ of agent i is differentiable and we denote by $\|\nabla U_1^i(\infty, s)\|$ the real number

$$\lim_{t \rightarrow +\infty} \sum_{\ell \in L} \frac{\partial U_1^i(\cdot, s)}{\partial x(\ell)}(t).$$

⁷If Λ is a subset of Δ_1 , then $\text{int}_{\Delta_1} \Lambda$ is the interior of Λ for the relative topology in Δ_1 .

Beliefs μ^i of agent i are assumed to be defined by

$$\mu^i(p_0, q) = \mathbb{P}_1^i \otimes \lambda \otimes \nu$$

where \mathbb{P}_1^i is any probability on Δ_1 satisfying $\text{supp } \mathbb{P}_1^i = \Delta_1$, ν is the Dirac measure on $\{s\}$ and λ is the restriction of the Lebesgue measure on $K = [0, 1]$. Observe that the family $\boldsymbol{\mu}$ satisfies the overlapping expectations assumption. We claim that if default is allowed and if the default penalty is not large enough with respect to the marginal utility, then agents have incentives to buy and sell the same quantity of asset, leading to a self-arbitrage opportunity.

Claim 4.2. If the default penalty $\lambda_j^i(s)$ satisfies

$$\lambda_j^i(s) < \frac{1}{2} \|\nabla U_1^i(\infty, s)\|$$

then $\boldsymbol{\mu}$ is not temporary viable.

Proof. Assume that there exists (p_0, q, \mathbf{a}) in $\text{Eq}(\boldsymbol{\mu}, 0)$ where $a^i = (x_0^i, \theta^i, \phi^i)$. It follows that there exist Borel measurable functions $\mathbf{r}_1^i : \Omega \rightarrow X_1$ and $\mathfrak{d}^i : \Omega \rightarrow D$ such that for every $\omega \in \text{supp } \mu^i(p_0, q)$,

$$(\mathbf{r}_1^i(\omega), \mathfrak{d}^i(\omega)) \in B_1^i(\omega, a^i) \quad (27)$$

and

$$V^i(p_0, q, a^i, \mu^i) = U_0^i(x_0^i) + \beta^i \int_{\Omega} W^i(\omega, a^i, \mathbf{r}_1^i(\omega), \mathfrak{d}^i(\omega)) \mu^i(p_0, q, d\omega). \quad (28)$$

Fix i , and let \tilde{a}^i be defined by

$$\tilde{x}_0^i = x_0^i, \quad \tilde{\theta}^i = \theta^i + 1 \quad \text{and} \quad \tilde{\phi}^i = \phi^i + 1.$$

We claim that \tilde{a}^i belongs to $B_0^i(p_0, q, \mu^i)$. Indeed, if we pose⁸

$$\tilde{\mathbf{r}}_1^i(\omega) = \mathbf{r}_1^i(\omega) + \kappa \mathbf{1}_L \quad \text{and} \quad \tilde{\mathfrak{d}}^i = \mathfrak{d}^i$$

then⁹ for every $\omega \in \text{supp } \mu^i(p_0, q)$,

$$(\tilde{\mathbf{r}}_1^i(\omega), \tilde{\mathfrak{d}}^i(\omega)) \in B_1^i(\omega, \tilde{a}^i). \quad (29)$$

Lets denote the difference $V^i(p_0, q, \tilde{a}^i, \mu^i) - V^i(p_0, q, a^i, \mu^i)$ by ΔV^i . Since

$$V^i(p_0, q, \tilde{a}^i, \mu^i) \geq U_0^i(\tilde{x}_0^i) + \beta^i \int_{\Omega} W^i(\omega, \tilde{a}^i, \tilde{\mathbf{r}}_1^i(\omega), \tilde{\mathfrak{d}}^i(\omega)) \mu^i(p_0, q, d\omega)$$

we have

$$\begin{aligned} \Delta V^i &\geq \int_{\Omega} U_1^i(\mathbf{r}_1^i(\omega) + \kappa, s) - U_1^i(\mathbf{r}_1^i(\omega), s) \mu^i(p_0, q, d\omega) - \lambda_j^i(s) \\ &\geq \int_K \|\nabla U_1^i(\infty, s)\| \kappa \lambda(d\kappa) - \lambda_j^i(s) > 0. \end{aligned}$$

It implies that

$$V^i(p_0, q, \tilde{a}^i, \mu^i) > V^i(p_0, q, a^i, \mu^i)$$

which contradicts the optimality of a^i in $B_0^i(p_0, q, \mu^i)$. \square

⁸Recall that $\omega = (p_1, \kappa, s)$.

⁹Observe that in this example we have $p_1 \cdot A_j(s) = 1$ since $p_1 \in \Delta_1$ and $A_j(s) = \mathbf{1}_L$.

4.3 Collateral requirements

When no borrowing constraints are imposed, it is proved in Green (1973, Example 5.2) that a temporary equilibrium at $t = 0$ may not exist due to arbitrage opportunities. We have shown that¹⁰ a sufficient condition on a family of beliefs to be temporary viable is that agents' expectations overlap.¹¹ This implies a kind of *coordination* between agents and a costly search for information. As argued in Stahl (1985b) this result is essentially negative since when agents' expectations are too diverse they may not be temporary viable.

To address this issue, Stahl (1985a) and Stahl (1985b) imposed restrictions on trades: an *institutional clearinghouse* restricts each individual to a set of trades that are solvent for all realization of uncertainty in the support of the expectations of the clearing house. The fundamental purpose of the clearing house is to coordinate agents' expectations by reducing the search and information costs. We believe that this model can be challenged in several aspects. First, there is no axiomatic derivation of the clearing house's expectations. Second, interiority conditions on the support of agents' and the clearing house's expectations are needed. Finally, in order to check that the trades of each agent are solvent with respect to each realization of uncertainty in the clearing house's support, it requires a capacity of computation and costs far beyond what is realistic.

We propose an alternative institutional restriction on trades which does not suffer from the previous drawbacks. We claim that if assets are protected by collateral, then any family of beliefs is temporary viable.

Theorem 4.2. *If each asset j is protected by collateral, i.e. if $C_j \neq 0$ for each j , then every family of beliefs is temporary viable.*

Proof. Let Ξ^i be a compact subset of Ξ^i such that the set of all family of actions \mathbf{a} with $a^i = (x_0^i, \theta^i, \phi^i) \in A^i$ satisfying the market clearing constraints

$$\sum_{i \in I} x_0^i + C\phi^i = \sum_{i \in I} e_0^i \quad \text{and} \quad \sum_{i \in I} \theta^i = \sum_{i \in I} \phi^i$$

is a subset of $\prod_{i \in I} [X_0 \times \text{int } \Xi^i]$. The existence of such a set is a direct consequence of the main assumption: $C_j \neq 0$ for each j . This is sufficient to preclude existence of arbitrage opportunities and ensure that $\boldsymbol{\mu}$ is temporary viable. Indeed, we can apply Proposition 4.1 to get the existence of an equilibrium (p_0, q, \mathbf{a}) at $t = 0$ for the economy which investment strategies' set is restricted to Ξ^i . The interiority condition together with the concavity of the utility function $a^i \mapsto V^i(p_0, q, a^i, \mu^i)$ allows us to apply standard arguments to prove that (p_0, q, \mathbf{a}) is also an equilibrium at $t = 0$ for the initial (unconstrained) economy. \square

5 The need for default

Assume that there exists a temporary equilibrium at $t = 0$. When agents can borrow assets and default is not allowed, it may be the case that for some possible state of nature at $t = 1$,

¹⁰For the particular asset structure of contracts for *sure* delivery of each good, this result was proved in Green (1973) (see also Grandmont (1977)).

¹¹See also similar results for securities model in Hart (1974) and Hammond (1983).

there does not exist a market clearing price. In other words, temporary viable beliefs are not sequentially viable. We provide a simple example of an economy having a unique temporary equilibrium at $t = 0$ but for which no temporary equilibrium at $t = 1$ exists.

Example 5.1. Consider an economy with two agents $I = \{i_1, i_2\}$, one good $L_0 = \{\ell_0\}$ at $t = 0$, two goods $L_1 = \{\ell_1, \ell_2\}$ at $t = 1$, one state of nature $S = \{s\}$ and one market for sure delivery of good ℓ_1 at $t = 1$, i.e. there is one asset $J = \{j\}$ with $A_j(s) = \mathbf{1}_{\{\ell_1\}}$. We consider that there is no collateral requirements and that no default is allowed (i.e. $\lambda_j^i(s) = +\infty$). Both agents have the same utility functions, the same discount factor $\beta^i = 1$ and the same initial endowments

$$\forall x_0 \in X_0, \quad U_0^i(x_0) = x_0 \quad \text{and} \quad e_0^i > 0$$

and

$$\forall x \in X, \quad U_1^i(x) = x(\ell_1) + x(\ell_2) \quad \text{and} \quad e_1^i(\ell_1) = e_1^i(\ell_2) > 0.$$

We choose the normalization

$$\Delta_1 = \{p_1 \in P_1 : p_1(\ell_1) + p_1(\ell_2) = 2\}.$$

Beliefs of agent i are defined by the probability¹²

$$\mu^i = \delta^i = P_H \text{Dirac}_{\{p_H^i\}} + P_L \text{Dirac}_{\{p_L^i\}}$$

where $P_H > 0$, $P_L > 0$ and $P_H + P_L = 1$. Observe that $\text{supp } \delta^i = \{p_H^i, p_L^i\}$. Each agent i expects that at $t = 1$ only two prices are possible. We denote by $\pi_H^i = p_H^i(\ell_2)/p_H^i(\ell_1)$ and $\pi_L^i = p_L^i(\ell_2)/p_L^i(\ell_1)$ the price of good ℓ_2 in units of good ℓ_1 . We assume that

$$0 < \pi_L^i < 1 < \pi_H^i < \pi_H^{i_2} < \pi_H^{i_1}. \quad (30)$$

Observe that this last condition implies that expectations overlap in the sense of Green (1973) and Grandmont (1977). Agent i_1 believes that at period $t = 1$ the price of good ℓ_2 will be higher than the price of good ℓ_1 . Although agent i_2 assigns a non-zero probability to this event, he believes that the opposite may also happen.

Assume that agent i has chosen a portfolio $z^i = \theta^i - \phi^i$ at the first period $t = 0$.¹³ At $t = 1$, given a price $p_1 \in \Delta_1$, the demand of agent i is given by

$$d_1^i(p_1, s, z^i) := \text{argmax}\{x(\ell_1) + x(\ell_2) : p_1 \cdot x_1 \leq p_1 \cdot [e_1^i + A_j(s)z^i]\}. \quad (31)$$

In order to have market clearing at $t = 1$, i.e.

$$e_1^{i_1} + e_1^{i_2} \in d_1^{i_1}(p_1, s, z^{i_1}) + d_1^{i_2}(p_1, s, z^{i_2})$$

the price p_1 at $t = 1$ must satisfies $p_1(\ell_1) = p_1(\ell_2)$. We prove that given an appropriate choice of the parameters defining initial endowments of both agents, the optimal choice of portfolio z^{i_1} of agent i_1 at the first period $t = 0$ will lead him to be bankrupt at $t = 1$, i.e.

$$p_1 \cdot [e_1^{i_1} + A_j(s)z^{i_1}] = p_1 \cdot e_1^{i_1} + p_1(\ell_1)z^{i_1} < 0$$

¹²Since no default is allowed, every agent expects the vector of rates to be equal to 1.

¹³When there is no collateral requirements and when no default is allowed, it is no more needed to distinguish asset purchases and sales.

if the commodity price p_1 is $(1, 1)$, implying that μ is not sequentially viable.

Fix a portfolio $z \in \mathbb{R}$, recall that for every $p_1 \in \Delta_1$, the indirect utility $\Gamma^i(p_1, s, z)$ is defined by

$$\sup\{x_1(\ell_1) + x_1(\ell_2) : p_1 \cdot x_1 \leq p_1 \cdot [e_1^i + A_j(s)z]\}.$$

Since $\pi_H^{i_1} > 1$ and $\pi_L^{i_1} > 1$, we have¹⁴

$$\Gamma^{i_1}(p_H^{i_1}, s, z) = z \quad \text{and} \quad \Gamma^{i_1}(p_L^{i_1}, s, z) = z.$$

Since $\pi_L^{i_2} < 1$ and $\pi_H^{i_2} > 1$, we have¹⁵

$$\Gamma^{i_2}(p_L^{i_2}, s, z) = \frac{1}{\pi_L^{i_2}} z \quad \text{and} \quad \Gamma^{i_2}(p_H^{i_2}, s, z) = z.$$

Given a pair $(1, q)$ with $q > 0$ of first period prices, the budget set $B_0^i(q, \mu^i)$ of agent i at the first period $t = 0$ is the set of all consumptions $x_0 \geq 0$ and portfolios $z \in \mathbb{R}$ such that¹⁶

$$x_0 + q \cdot z \leq e_0^i \quad \text{and} \quad z \geq -p_L^i \cdot e_1^i.$$

It is easy to see that the demand $d_0^i(q, \mu^i)$ of agent i at $t = 0$ is given by

$$d_0^i(q, \mu^i) = \operatorname{argmax}\{x_0 + q^i z : x_0 + qz \leq e_0^i, x_0 \geq 0 \quad \text{and} \quad z \geq -p_L^i \cdot e_1^i\}$$

where

$$q^{i_1} = 1 \quad \text{and} \quad q^{i_2} = \frac{P_L}{\pi_L^{i_2}} + P_H.$$

Observe that for each $q > 0$, we have

$$d_0^i(q, \mu^i) \subset \{(x_0, z) \in \mathbb{R}_+ \times \mathbb{R} : x_0 = e_0^i - qz\}$$

therefore we let $f^i(q)$ be the demand for the asset, i.e.

$$f^i(q) = \{z \in \mathbb{R} : \exists x_0 \in X_0, (x_0, z) \in d_0^i(q, \mu^i)\}.$$

It follows that

$$f^i(q) = \operatorname{argmax}\{(q^i - q)z : qz \leq e_0^i \quad \text{and} \quad z \geq -p_L^i \cdot e_1^i\}.$$

Since agent i_2 believes that good ℓ_2 may be more expensive than good ℓ_1 , i.e. $\pi_L^{i_2} < 1$ we have $q^{i_2} > q^{i_1} = 1$. Observe that if q does not belong to (q^{i_1}, q^{i_2}) then both agents will go short or long together on the asset, implying that q cannot be a market clearing price. Consider now a price $q \in (q^{i_1}, q^{i_2})$. The demands on the asset are then given by

$$f^{i_1}(q) = -p_L^{i_1} \cdot e_1^{i_1} \quad \text{and} \quad f^{i_2}(q) = \frac{e_0^{i_2}}{q}.$$

¹⁴Indeed, the optimal choice involves $x_1(\ell_2) = 0$.

¹⁵For the price $p_L^{i_2}$, the optimal choice of agent i_2 involves $x_1(\ell_1) = 0$.

¹⁶Observe that if $z \geq -p_H^i \cdot e^i$ then automatically $z \geq -p_L^i \cdot e^i$.

We claim that choosing appropriately initial endowments, there exists a unique market clearing price. Indeed, if we fix $e_0^{i_2} > 0$ and $e_1^{i_1} \in \mathbb{R}_{++}^L$ such that

$$q^{i_1} = 1 < \frac{e_0^{i_2}}{p_L^{i_1} \cdot e_1^{i_1}} < q^{i_2}$$

then the price $q^* = e_0^{i_2} / (p_L^{i_1} \cdot e_1^{i_1})$ is the unique asset equilibrium price at $t = 0$.

It follows that for each $\pi_1 > 0$, the wealth $w^{i_1}(\pi_1)$ of agent i_1 at $t = 1$ and price $p_1 = (1, \pi_1)$ is given by

$$w^{i_1}(\pi_1) = p_1 \cdot e_1^{i_1} + f^{i_1}(q) = (\pi_1 - \pi_L^{i_1})e_1^{i_1}(\ell_2).$$

Remember that the unique possible market clearing price at $t = 1$ is $p_1^* = (1, 1)$. But for this price agent i_1 's wealth is

$$w^{i_1}(1) = (1 - \pi_L^{i_1})e_1^{i_1} < 0$$

implying that agent i_1 is bankrupt and his demand is not defined.

This example illustrates that even if there is some agreement among consumers about the prices will prevail at the second period,¹⁷ this is not enough to guarantee the existence of an equilibrium in the second period. In the above example, consumer i_1 undertakes an extreme position in the forward market. His position is accommodated by consumer i_2 and an equilibrium always exists at the first period. The problem in the second period arises from the fact that the unique second period price that could clear markets does not belong to the support of consumer i_1 's beliefs. At this price, consumer i_1 goes bankrupt and no equilibrium exists.

Our second main result is that default is always sufficient to guarantee that every temporary viable belief is sequentially viable.

Theorem 5.1. *Assume that each agent i is allowed to default in every state s and on every asset j , i.e. $\lambda_j^i(s) < +\infty$, then every temporary viable belief is sequentially viable. In other words, for every temporary equilibrium at $t = 0$ there will be a temporary equilibrium at $t = 1$ at any possible state s .*

Proof. Let (p_0, q, \mathbf{a}) be a temporary equilibrium with $a^i = (x_0^i, \theta^i, \phi^i)$. We fix a state $s \in S$. Let \mathbf{C} be a compact subset of $X_1 \times D$ such that the set of all families $(x_1^i, d^i)_{i \in I} \in [X \times D]^I$ satisfying the constraints

$$\sum_{i \in I} x_1^i = \sum_{i \in I} e_1^i(s) + Y_s(x_0^i + C\phi^i) \quad \text{and} \quad \sum_{i \in I} d_j^i \leq \sum_{i \in I} \widehat{V}_j(s)\phi_j^i$$

is a subset of $(\text{int } \mathbf{C})^I$ where

$$\widehat{V}_j(s) = \sup\{p_1 \cdot A_j(s) : p_1 \in \Delta_1\}.$$

We let \mathfrak{f} be the correspondence from $\mathbf{C}^I \times \Delta_1 \times K$ to itself defined by

$$\mathfrak{f}(\mathbf{x}_1, \mathbf{d}, p_1, \kappa) = \prod_{i \in I} \gamma^i(p_1, \kappa, s) \times \xi(\mathbf{x}_1) \times \chi(\mathbf{d}, p) \quad (32)$$

¹⁷Which is a sufficient condition to guarantee existence of a temporary equilibrium, see Green (1973).

where

$$\gamma^i(p_1, \kappa, s) = \operatorname{argmax}\{W^i(p_1, s, a^i, x_1, d) : (x_1, d) \in B_1^i(p_1, \kappa, s, a^i) \cap \mathbf{C}\}, \quad (33)$$

$$\xi(\mathbf{x}_1) = \operatorname{argmax}\left\{p_1 \cdot \sum_{i \in I} [x_1^i - e_1^i(s) - Y_s(x_0^i + C\phi)]: p_1 \in \Delta_1\right\} \quad (34)$$

and $\chi(\mathbf{d}, p) = \prod_{j \in J} \chi_j(\mathbf{d}, p)$ where

$$\chi_j(\mathbf{d}, p) = \begin{cases} \left\{ \frac{\sum_{i \in I} d_j^i}{\sum_{i \in I} V_j(p_1, s)\phi^i} \right\} & \text{if } \sum_{i \in I} V_j(p_1, s)\phi^i > 0, \\ [0, 1] & \text{if } \sum_{i \in I} V_j(p_1, s)\phi^i = 0. \end{cases} \quad (35)$$

Claim 5.1. The correspondence \mathfrak{f} is upper semicontinuous with convex compact and non-empty values.

Proof of Claim 5.1. It is obvious that ξ and χ are upper semicontinuous with convex compact non-empty values. In order to apply Berge's Maximum Theorem to the correspondence γ^i , we only have to prove that the budget set correspondence is continuous. Since default is allowed, for any $(p_1, \kappa) \in \Delta_1 \times K$ the set $B_1^i(p_1, \kappa, s, a^i) \cap \mathbf{C}$ is always non-empty. The upper semicontinuity of $(p_1, \kappa) \mapsto B_1^i(p_1, \kappa, s, a^i) \cap \mathbf{C}$ is obvious. The lower semicontinuity follows from the fact that for any $(p_1, \kappa) \in \Delta_1 \times K$, the set of all $(x_1, d) \in \mathbf{C}$ satisfying the constraints

$$p_1 \cdot x_1 + \sum_{j \in J} d_j < p_1 \cdot [e_1^i(s) + Y_s(x_0^i + C\phi^i)] + \sum_{j \in J} V_j(\kappa_j, p_1, s)\theta^i \quad (36)$$

and

$$\forall j \in J, \quad p \cdot Y_s(C_j)\phi_j^i < d_j \quad (37)$$

is always non-empty. Indeed, since $p_1 \in \Delta_1$ and $e_1^i(s) \in \mathbb{R}_{++}^L$, we have $p_1 \cdot e_1^i(s) > 0$, implying that the couple (\hat{x}_1, \hat{d}) defined by

$$\hat{x}_1 = 0 \quad \text{and} \quad \hat{d}_j = p \cdot Y_s(C)\phi_j + \frac{1}{2\#J} p \cdot e_1^i(s)$$

belongs to this set. □

Applying Kakutani's Fixed-Point Theorem, there exist a couple $(p_1, \kappa) \in \Delta_1 \times K$ and a family $(x_1^i, d^i)_{i \in I} \in \mathbf{C}^I$ such that

$$(x_1^i, d^i) \in \gamma^i(p_1, \kappa, s), \quad p \in \xi(\mathbf{x}_1) \quad \text{and} \quad \kappa \in \chi(\mathbf{d}, p). \quad (38)$$

Applying standard arguments we can prove that $(p_1, \kappa, \mathbf{x}_1, \mathbf{d}) \in \operatorname{Eq}(\mathbf{a}, s)$. □

6 Viable beliefs without default

We provide hereafter two frameworks under which it is possible to exhibit sequentially viable beliefs even if default is not allowed. We assume throughout this section that default is not allowed, i.e. $\lambda_j^i(s) = +\infty$, that assets are not backed by collateral, i.e. $C = 0$ and all goods are perishable, i.e. $Y = 0$.

6.1 Real solvency

We already know that if short-sellings are exogenously bounded then any belief is temporary viable. In fact it is possible to choose among the exogenous bounds a specific one which ensures sequential viability of beliefs (even in the absence of default).

Definition 6.1. *We say that real solvency is imposed on investment strategies if for every agent i*

$$\Xi^i \subset \{(\theta, \phi) \in \mathbb{R}_+^J \times \mathbb{R}_+^J : A(s)\phi \leq e_1^i(s) + A(s)\theta, \quad \forall s \in S\}$$

where $A(s)\phi = \sum_{j \in J} A_j(s)\phi_j$.

Remark 6.1. Observe that if agent i is prudent in the sense that the probability $\mu^i(p_0, q)$ has full support for some $(p_0, q) \in P_0 \times Q$, then for every action $(x_0^i, \theta^i, \phi^i) \in B_0^i(p_0, q)$ the investment strategy (θ^i, ϕ^i) satisfies the real solvency constraint

$$A(s)\phi^i \leq e_1^i(s) + A(s)\theta^i, \quad \forall s \in S.$$

In Svensson (1981) it is implicitly assumed that agents are prudent.

Theorem 6.1. *If real solvency is imposed on investment strategies or if agents are prudent, then every family of beliefs is sequentially viable.*

Proof. We only need to prove that if (p_0, q, \mathbf{a}) is temporary equilibrium at $t = 0$, then for each state s there exists a temporary equilibrium at $t = 1$. Observe that whatever are the actions \mathbf{a} chosen at $t = 0$, the new initial endowments of the economy at state s are given by

$$\forall i \in I, \quad \epsilon^i(s, a^i) := e_1^i(s) + A(s)[\theta^i - \phi^i].$$

In particular we have

$$\forall i \in I, \quad \epsilon^i(s, a^i) \geq 0 \quad \text{and} \quad \sum_{i \in I} \epsilon(s, a^i) \in \mathbb{R}_{++}^{L_1}.$$

These conditions are sufficient to prove existence of a competitive equilibrium and therefore the set $\text{Eq}(s, \mathbf{a})$ is non-empty. \square

6.2 Agents with perfect foresight

In this section, we consider that agents have subjective beliefs $\nu^i \in \text{Prob}(S)$ about the realization of exogenous uncertainty.

Definition 6.2. *A family of beliefs μ is called perfect-foresight if it is degenerate, common and self-fulfilling in the sense that there exists a function*

$$\mathbf{p}_1 : P_0 \times Q \times S \longrightarrow P_1$$

such that

- if the price vector (p_0, q) appears in the market at $t = 0$, each agent i believes that contingent to the state s , only the price $\mathbf{p}_1(p_0, q, s)$ will appear at $t = 1$, i.e.,

$$\forall (p_0, q) \in P_0 \times Q, \quad \mu^i(p_0, q, ds \times dp_1) = \nu^i(ds) \text{Dirac}(\mathbf{p}_1(p_0, q, s)),$$

- there exists a temporary equilibrium (p_0, q, \mathbf{a}) at $t = 0$ such that whatever is the realized state s at $t = 1$, the expected price $\mathbf{p}_1(p_0, q, s)$ is a possible market clearing price, i.e. $\mathbf{p}_1(p_0, q, s) \in \mathcal{P}_C(\mathbf{a}, s)$.

Remark 6.2. Observe that any perfect-foresight family of beliefs is sequentially viable.

Theorem 6.2. *Assume that short-sales are bounded.¹⁸ If every agent gives positive probability to each possible state of nature, i.e. $\text{supp } \nu^i = S$ for each i , then a perfect-foresight family of beliefs exists.*

Remark 6.3. If we consider numéraire or nominal assets then the previous result is still valid even without imposing bounds on short-sales.

Proof. For each vector $(p, q) \in P \times Q$ where $P = P_0 \times P_1^S$ and $p = (p_0, p_1)$ with $p_1 = (p_1(s))_{s \in S}$, we denote by $A^i(p, q)$ the *intertemporal* budget set defined by all vectors $(x, \theta, \phi) \in X \times \Xi^i$ where $X = X_0 \times X_1^S$, satisfying the budget constraints

$$p_0 \cdot x_0 + q \cdot \theta \leq p_0 \cdot e_0^i + q \cdot \phi \quad (39)$$

and for each $s \in \text{supp } \nu^i$

$$p_1(s) \cdot x_1(s) + V(p_1(s), s)\phi \leq p_1(s) \cdot e_1^i(s) + V(p_1(s), s)\theta. \quad (40)$$

We denote by $\delta^i(p, q)$ the *intertemporal* demand defined by

$$\delta^i(p, q) = \text{argmax}\{\text{vNM}^i(x) : (x, \theta, \phi) \in A^i(p, q)\} \quad (41)$$

where

$$\text{vNM}^i(x) = U_0^i(x_0) + \beta^i \sum_{s \in S} \nu^i(s) U_1^i(x_1(s), s). \quad (42)$$

Following Radner (1972) we say that a family $(p, q, \mathbf{x}, \boldsymbol{\theta}, \boldsymbol{\phi})$ of prices $(p, q) \in P \times Q$, consumption allocation $\mathbf{x} = (x^i)_{i \in I} \in X^I$ and investment allocations allocation $\boldsymbol{\theta} = (\theta^i, i \in I)$ and $\boldsymbol{\phi} = (\phi^i)_{i \in I}$ is an equilibrium of plans, prices and price expectations if

- for each i , $(x^i, \theta^i, \phi^i) \in \delta^i(p, q)$
- $\sum_{i \in I} (x^i, \theta^i - \phi^i) = \sum_{i \in I} (e^i, 0)$,

¹⁸That is there exists $\Phi^i \in \mathbb{R}_+^I$ such that $\Xi^i \subset \mathbb{R}_+^I \times [0, \Phi^i]$.

where $e^i = (e_0^i, e_1^i) \in X_0 \times X_1^S$.

It was proved in Radner (1972) that such an equilibrium of plans, prices and price expectations exists. We denote by PPE the set of prices and price expectations $(p, q) \in P \times Q$ with $(p_0, q) \in \Delta_0$ and $p_1(s) \in \Delta_1$ for which there exists a consumption allocation $\mathbf{x} = (x^i)_{i \in I} \in X^I$ and investment allocations allocation $\boldsymbol{\theta} = (\theta^i)_{i \in I}$ and $\boldsymbol{\phi} = (\phi^i)_{i \in I}$ such that $(p, q, \mathbf{x}, \boldsymbol{\theta}, \boldsymbol{\phi})$ is an equilibrium of plans, prices and price expectations. And we denote by PPE_0 the projection on Δ_0 of the set PPE. For each $(p_0, q) \in \text{PPE}_0$, we choose $\mathbf{p}_1(p_0, q)$ a vector in P_1^S such that $(p_0, \mathbf{p}_1(p_0, q), q)$ belongs to PPE. We extend this function arbitrarily on the space Δ_0 . We can now construct a family of beliefs $\boldsymbol{\mu}_{\mathbf{p}_1}$ by posing

$$\forall (p_0, q) \in \Delta_0, \quad \mu_{\mathbf{p}_1}^i(p_0, q, ds \times dp_1) = \nu^i(ds) \text{Dirac}(\mathbf{p}_1(p_0, q), s).$$

We claim that $\boldsymbol{\mu}_{\mathbf{p}_1}$ is a perfect-foresight family of beliefs. We in fact prove that any price $(p_0, q) \in \text{PPE}_0$ is a temporary equilibrium price at $t = 0$. Indeed, let $(p, q) \in \text{PPE}$ and fix a consumption allocation $\mathbf{x} = (x^i)_{i \in I} \in X^I$ and investment allocations $\boldsymbol{\theta} = (\theta^i)_{i \in I}$ and $\boldsymbol{\phi} = (\phi^i)_{i \in I}$ such that $(p, q, \mathbf{x}, \boldsymbol{\theta}, \boldsymbol{\phi})$ is an equilibrium of plans, prices and price expectations.

Claim 6.1. The family (p_0, q, \mathbf{a}) defined by $a^i = (x_0^i, \theta^i, \phi^i)$ for each i , is a temporary equilibrium at $t = 0$.

The beliefs are correct in the following sense.

Claim 6.2. For each s , the family $(\mathbf{p}_1(p_0, q, s), \mathbf{x}_1(s))$ is a temporary equilibrium at $t = 1$ given actions \mathbf{a} , i.e. $(\mathbf{p}_1(p_0, q, s), \mathbf{x}_1(s))$ belongs to $\text{Eq}(\mathbf{a}, s)$.

We omit the proofs of Claims 6.1 and 6.2 since they follow from standard arguments. \square

Observe that in order to have perfect-foresight beliefs, each agent i is required to be able to compute the set PPE of equilibria of plans, prices and price expectations. Implicitly, it is assumed that the distribution of agents' characteristics is common knowledge. More problematic is the assumption that agents must coordinate on choosing the same equilibrium price $\mathbf{p}_1(p_0, q, s)$ conditional to the current prices (p_0, q) .

However, there is a particular case where agents do not need to know the distribution of other agents' characteristics nor to coordinate in choosing the same equilibrium price in order to have perfect-foresight beliefs. We fully analyze this case below.

Theorem 6.3. *Assume that there are contingent contracts for each good.¹⁹ If every agent gives positive probability to each possible state of nature and there is no constraint on portfolios,²⁰ then the family of beliefs defined for all $(p_0, q) \in \Delta_0$ by*

$$\mu^i(p_0, q, ds \times dp_1) = \begin{cases} \nu^i(s) \text{Dirac}(q(s)/\|q(s)\|) & \text{if } q(s) \neq 0 \\ \text{arbitrary} & \text{if } q(s) = 0 \end{cases}$$

¹⁹In the sense that $J = S \times L$ and asset (s, ℓ) pays at $t = 1$ the value on the market of one unit of good ℓ if the realized state of nature is s . In other words,

$$\forall s' \in S, \quad A_{(s, \ell)}(s') = \begin{cases} \mathbf{1}_\ell & \text{if } s' = s \\ 0 & \text{if } s' \neq s. \end{cases}$$

²⁰For each $i \in I$, we have $\Xi^i = \mathbb{R}_+^J \times \mathbb{R}_+^J$.

is perfect-foresight, where $q(s) = (q(s, \ell))_{\ell \in L}$.

In other words, if each agent believes at $t = 0$ that the price at $t = 1$ and state s of the good ℓ will coincide with the price $q_{(s, \ell)}$ observed at $t = 0$ of the contract delivering good ℓ at $t = 1$ contingent to state s , then this family of beliefs is self-fulfilling.

Proof. We borrow the notations introduced in the proof of Theorem 6.2. With a contingent contracts for each good, there always exists an equilibrium of plans, prices and price expectations even if no constraints on short-sales are imposed. Observe that by non-arbitrage, if (p, q) belongs to PPE then $q(s)$ is strictly positive for each s , and

$$p_1(s) = \frac{1}{\|q(s)\|} q(s).$$

Now the rest of the proof follows exactly the arguments in the proof of Theorem 6.2. \square

The result described in Theorem 6.3 does not rely only on the completeness of the asset market. The specific asset structure of complete contingent contracts, commodity by commodity, is also crucial. Indeed, if there is a complete set of Arrow securities,²¹ then observing prices at $t = 0$ does not enable agents to forecast a possible equilibrium price for $t = 1$ without knowing the distribution of characteristics of the others and coordinating on selecting a price.

7 Concluding remarks

This paper aims at a reappraisal of the implications of default and collateral in a setting that departs from the traditional rational paradigm by allowing agents to be less sophisticated. We formulate and analyze a two-period model that is in close relation with temporary equilibrium models, but it deviates from them by allowing for durable goods, collateral and the possibility of default. Temporary equilibrium models were previously criticized on the basis of imposing stringent restrictions on agents' expectations patterns as well as for not providing a market mechanism to prevent the economy from collapsing due to expectation errors. This study addresses these shortcomings. Our main message is that the reliance on collateral to secure loans allows us to dispense with restrictions on expectations patterns (i.e. overlapping expectation conditions) to get equilibrium in the initial date, while the possibility of default is always sufficient to ensure equilibrium at future dates.

We previously underlined in Section 6.2 that for the standard model of Radner (1972) the perfect foresight approach is very demanding. We can find in the literature variations of the standard model (asymmetric information, unawareness, and time inconsistency of preferences with naive agents) for which the assumption of perfect foresight becomes more problematic. We briefly highlight that our equilibrium concept is consistent with these models.

²¹That is $J = S$ and the payoff of asset s at $t = 1$ and state s' is 1 if $s' = s$ and 0 elsewhere.

7.1 Asymmetric information

In a recent paper Cornet and de Boisdeffre (2002) incorporate asymmetric information in a sequential asset market model in a very particular way that deviates from the standard modeling. In Radner (1979), each agent knows not only the set of signals he may receive but also the set of signals the other agents may receive.²² In Cornet and de Boisdeffre (2002) (see also de Boisdeffre (2006)) agent i receives a private information signal represented by a subset $T^i \subset S$ about which states will not prevail at the second period $t = 1$. Therefore, when he chooses his action at $t = 0$, he only considers that the states in $S^i = S \setminus T^i$ may realize at $t = 1$. The main difference with the traditional modeling is that agent i is not aware of which signals other agents may receive. But in this case, it is difficult to understand how agents can forecast correctly commodity prices in the second period $t = 1$. Indeed, in Cornet and de Boisdeffre (2002) and de Boisdeffre (2006), it is assumed that agents can forecast correctly prices for states in $\bigcap_{i \in I} S^i$. We believe that the consistency of such an assumption is problematic. Our model encompasses asymmetric information as defined by Cornet and de Boisdeffre (2002) since we can assume that for each agent i there exists a set $S^i \subset S$ such that $\text{supp } \nu^i(p_0, q) = S^i$ for each $(p_0, q) \in \Delta_0$. Given that for our equilibrium concept agents are not required to know the characteristics of the others (they may have beliefs about the characteristics of the others), no inconsistency problem arises.

7.2 Unawareness

In Modica, Rustichini, and Tallon (1998) (see also Kawamura (2005)), agents may not be able to foresee all possible future exogenous contingencies (or states of nature). For each agent i a subset $S^i \subset S$ reflects agent i 's awareness. In contrast to Cornet and de Boisdeffre (2002), they allow for the possibility that the *true state* at $t = 1$ may not belong to the set $\bigcap_{i \in I} S^i$.²³ In Modica, Rustichini, and Tallon (1998) it is assumed that agents are able to correctly forecast delivery rates (or repayment fractions) due to the possibility of bankruptcy of some agents. But to do such a correct anticipation, they should know the characteristics of the other agents, in particular they should know the collection of other agents' foreseen states. This would imply that each agent foresees the same set of states $\bigcup_{i \in I} S^i$. Since in Modica, Rustichini, and Tallon (1998) it is assumed that agents are forced to honor their debts in the states they foresee, this also implies that the expected delivery rate should be equal to one in each collectively foreseen state in $\bigcup_{i \in I} S^i$. For that reason, heterogeneous unforeseen contingencies seems to be rather problematic under the perfect foresight equilibrium concept. Our model allows for unforeseen states.²⁴ As previously, for our equilibrium concept agents are not required to know the characteristics of the others (they may have beliefs about the characteristics of the others). Therefore it is consistent with heterogeneous unforeseen contingencies.

²²This is the reason why they will be able to infer information about others agents' signal through prices.

²³Actually, the state of nature that materializes at $t = 1$ may not even belong to $\bigcup_{i \in I} S^i$.

²⁴In contrary to Modica, Rustichini, and Tallon (1998), we do not make a difference between states that are not foreseen and states with zero probability. Indeed, as Hart (1974), Green (1973), Grandmont (1977) (and many others) we do not assume that agents are forced to honor their debts for states to which they give zero probability.

7.3 Naive agents with time-inconsistent preferences

Let \mathcal{U}_1 denote the space of strictly increasing, concave and continuous functions from X_1 to \mathbb{R} , then the set S of exogenous uncertainty may be interpreted as a subset of the space

$$\mathcal{S} := \mathcal{U}_1^I \times X_1^I \times X_1^J$$

through the mapping

$$\chi : s \mapsto \{\mathbf{U}_1(s, \cdot), \mathbf{e}_1(s), A(s)\}$$

where $\mathbf{U}_1(s, \cdot) = (U_1^i(s, \cdot))_{i \in I}$, $\mathbf{e}_1(s) = (e_1^i(s))_{i \in I}$ and $A(s) = (A_j(s))_{j \in J}$.

Herings and Rhode (2006) allow for naive agents that may have time-inconsistent preferences. At $t = 1$ when s realizes, the utility $V_1^i(s, \cdot)$ that agent i actually has, may differ from the utility $U_1^i(s, \cdot)$ he was expecting at $t = 0$. We can incorporate this framework in our model by considering that the set of exogenous uncertainty is defined by the set²⁵ $S \times \{0, 1\}$ and the mapping ξ defined for all $s \in S$ by

$$\xi(s, 0) = \{\mathbf{U}_1(s, \cdot), \mathbf{e}_1(s), A(s)\} \quad \text{and} \quad \xi(s, 1) = \{\mathbf{V}_1(s, \cdot), \mathbf{e}_1(s), A(s)\}.$$

Recall that $\nu^i(p_0, q)$ represents agent i 's beliefs about exogenous expectations, which is now a probability on $S \times \{0, 1\}$. If the support $\text{supp } \nu^i(p_0, q)$ is a subset of $S \times \{0\}$ then it means that agents are naive and may have time-inconsistent preferences. Observe that in our main existence results (Theorems 4.2 and 5.1) we do not impose any condition on the support of $\nu^i(p_0, q)$. Therefore we allow for time-inconsistent preferences. On the other hand, in order to prove the existence of perfect-foresight beliefs (Theorem 6.3), we assumed that agents' beliefs about exogenous uncertainty have full support. It seems to be difficult to justify time-inconsistent preferences under perfect foresight.

A Proof of Proposition 4.1

Let \mathbf{A}^i be a compact subset of $A^i = X_0 \times \Xi^i$ such that the set of all family of actions $\mathbf{a} = (a^i)_{i \in I}$ with $a^i = (x_0^i, \theta^i, \phi^i) \in A^i$ satisfying the market clearing constraints

$$\sum_{i \in I} x_0^i + C\phi^i = \sum_{i \in I} e_0^i \quad \text{and} \quad \sum_{i \in I} \theta^i = \sum_{i \in I} \phi^i$$

is a subset of $\prod_{i \in I} \text{int } \mathbf{A}^i$. The existence of such a set is a direct consequence of the main assumption: the set Ξ^i is a subset of $\mathbb{R}_+^J \times [0, \Phi^i]$. This is sufficient to preclude existence of arbitrage opportunities and ensure existence of temporary.

We let χ be the correspondence from $\Delta_0 \times \prod_{i \in I} \mathbf{A}^i$ to itself defined by

$$\chi((p_0, q), \mathbf{a}) = \chi^0(\mathbf{a}) \times \prod_{i \in I} \chi^i(p_0, q, a^i) \tag{43}$$

²⁵The symbol 0 represents expectations at $t = 0$ about what can happen at $t = 1$ and the symbol 1 represents realizations at $t = 1$.

where $\chi^0(\mathbf{a})$ is the set defined by

$$\operatorname{argmax} \left\{ \sum_{i \in I} p_0 \cdot [x^i + C\phi^i - e_0^i] + q \cdot (\theta^i - \phi^i) : (p_0, q) \in \Delta_0 \right\} \quad (44)$$

and for each i , $\chi^i(p_0, q, a^i)$ is the set defined by

$$\operatorname{argmax} \{ V^i(p_0, q, a^i, \mu^i) : a^i \in B_0^i(p_0, q, \mu^i) \cap \mathbf{A}^i \}. \quad (45)$$

It is proved in Proposition B.1 that the correspondence $B_0^i(\cdot, \mu^i)$ has a closed graph. Since the set \mathbf{A}^i is compact, it follows that the correspondence $(p_0, q) \mapsto B_0^i(p_0, q, \mu^i) \cap \mathbf{A}^i$ is upper semi-continuous. It is proved in Proposition B.2 that the correspondence $(p_0, q) \mapsto B_0^i(p_0, q, \mu^i)$ is lower semi-continuous. Choosing \mathbf{A}^i large enough²⁶ we obtain that the correspondence $(p_0, q) \mapsto B_0^i(p_0, q, \mu^i) \cap \mathbf{A}^i$ is lower semi-continuous. It is proved in Proposition C.2 that the function $(p_0, q, a) \mapsto V^i(p_0, q, a, \mu^i)$ is continuous on $\operatorname{gph} B^i(\cdot, \mu^i)$. Applying Berge's Maximum Theorem (see Aliprantis and Border (1999, Theorem 16.31)) the correspondence χ^i is upper semi-continuous with non-empty compact values. The concavity of $a \mapsto V^i(p_0, q, a, \mu^i)$ implies that the values of χ^i are convex. It is straightforward to check that χ^0 is upper semi-continuous with compact non-empty and convex values. Applying Kakutani's Fixed Point Theorem, there exists $(p_0, q) \in \Delta_0$ and $a^i \in \mathbf{A}^i$ for each i such that

$$(p_0, q) \in \chi^0(\mathbf{a}) \quad \text{and} \quad a^i \in \chi^i(p_0, q, a^i), \quad \forall i \in I. \quad (46)$$

It is now straightforward to prove that (p_0, q, \mathbf{a}) belongs to $\operatorname{Eq}(\boldsymbol{\mu}, 0)$, i.e. $\boldsymbol{\mu}$ is temporary viable.

B Continuity of the budget correspondence

We fix a family of beliefs $\boldsymbol{\mu}$ and for notational simplicity, the set $B_0^i(p_0, q, \mu^i)$ is denoted by $B_0^i(p_0, q)$. Fix an agent i , the purpose of this section is to prove that the budget correspondence $B_0^i : \Delta_0 \rightarrow A$ has a closed graph and is lower semicontinuous on the simplex Δ_0 .

Proposition B.1. The graph $\operatorname{gph} B_0^i$ of B_0^i defined by

$$\operatorname{gph} B_0^i = \{(p_0, q, a) \in \Delta_0 \times A : a \in B_0^i(p_0, q)\} \quad (47)$$

is closed in $\Delta_0 \times A$.

Proof. Let $(p_{0,n}, q_n)$ be sequence in Δ_0 converging to $(p_0, q) \in \Delta_0$ and (a_n) be a sequence in A^i converging to $a \in A^i$ such that $a_n \in B_0^i(p_{0,n}, q_n)$ for each n , i.e.

$$p_{0,n} \cdot [x_{0,n} + C\phi_n] + q \cdot (\theta_n - \phi_n) \leq p_{0,n} \cdot e_0^i \quad (48)$$

and for every $(p_1, \kappa, s) \in \operatorname{supp} \mu^i(p_{0,n}, q_n)$ there exists $\mathfrak{d}_n(p_1, \kappa, s)$ in D such that

$$0 \leq p_1 \cdot [e_1^i(s) + Y_s(x_{0,n} + C\phi_n)] + \sum_{j \in J} V_j(\kappa_j, p_1, s) \theta_{j,n} - \mathfrak{d}_{j,n}(p_1, \kappa, s), \quad (49)$$

²⁶In order to contain the action \hat{a}^i defined in the proof of Proposition B.2.

$$\forall j \in J, \quad D_j(p_1, s)\phi_{j,n} \leq \mathfrak{d}_{j,n}(p_1, \kappa, s), \quad (50)$$

and

$$\lambda_j^i(s) < +\infty \quad \text{or} \quad \mathfrak{d}_{j,n}(p_1, \kappa, s) \geq p_1 \cdot A_j(s)\phi_{j,n}. \quad (51)$$

We have to prove that a belongs to $B_0^i(p_0, q)$. Passing to the limit on (48) we obtain

$$p_0 \cdot [x_0 + C\phi] + q \cdot (\theta - \phi) \leq p_0 \cdot e_0^i. \quad (52)$$

Now let (p_1, κ, s) in $\text{supp } \mu^i(p_0, q)$. Since the correspondence μ^i is lower semi-continuous, there exists a strictly increasing function $\varphi : \mathbb{N} \rightarrow \mathbb{N}$ and a sequence $(p_{1,n}, \kappa_n, s_n)$ in $\text{supp } \mu^i(p_0, \varphi(n), q_{\varphi(n)})$ converging to (p_1, κ, s) . It follows from (49) and (50) that for each j the sequence

$$(\mathfrak{d}_{j, \varphi(n)}(p_{1,n}, \kappa_n, s_n))_{n \in \mathbb{N}}$$

is bounded. Passing to a subsequence if necessary, there exists $\mathfrak{d}(p_1, \kappa, s) \in D$ such that the previous sequence converges to $\mathfrak{d}(p_1, \kappa, s)$. Observe that the set S is finite. Since (s_n) converges to s , for n large enough we have $s_n = s$. It follows that passing to the limit in (49), (50) and (51) we obtain that

$$0 \leq p_1 \cdot [e_1^i(s) + Y_s(x_0 + C\phi)] + \sum_{j \in J} V_j(\kappa_j, p_1, s)\theta_j - \mathfrak{d}_j(p_1, \kappa, s), \quad (53)$$

$$\forall j \in J, \quad D_j(p_1, s)\phi_j \leq \mathfrak{d}_j(p_1, \kappa, s), \quad (54)$$

and

$$\lambda_j^i(s) < +\infty \quad \text{or} \quad \mathfrak{d}_j(p_1, \kappa, s) \geq p_1 \cdot A_j(s)\phi_j. \quad (55)$$

We have thus proved that a belongs to $B_1^i(p_1, \kappa, s)$. \square

Proposition B.2. The correspondence $(p_0, q) \mapsto B_0^i(p_0, q, \mu^i)$ is lower semi-continuous on Δ_0 .

Proof. We first prove that for every $(p_0, q) \in \Delta_0$, there exists \hat{a} in A^i and a function $\hat{\mathfrak{d}} : \Omega \rightarrow D$ such that

$$p_0 \cdot [\hat{x}_0 + C\hat{\phi}] + q \cdot (\hat{\theta} - \hat{\phi}) < p_0 \cdot e_0^i \quad (56)$$

for every $(p_1, \kappa, s) \in \Delta_1 \times K \times S$,

$$0 < p_1 \cdot [e_1^i(s) + Y_s(\hat{x}_0 + C\hat{\phi})] + \sum_{j \in J} V_j(\kappa_j, p_1, s)\hat{\theta}_j - \hat{\mathfrak{d}}_j(p_1, \kappa, s), \quad (57)$$

$$\forall j \in J, \quad D_j(p_1, s)\hat{\phi}_j < \hat{\mathfrak{d}}_j(p_1, \kappa, s), \quad (58)$$

and

$$\lambda_j^i(s) < +\infty \quad \text{or} \quad \hat{\mathfrak{d}}_j(p_1, \kappa, s) > p_1 \cdot A_j(s)\phi_j. \quad (59)$$

Indeed, let $(p_0, q) \in \Delta_0$. If $p_0 \neq 0$, then we pose

$$(\hat{x}_0, \hat{\theta}, \hat{\phi}) = (0, 0, 0) \quad \text{and} \quad \hat{\mathfrak{d}}(p_1, \kappa, s) = \frac{1}{2}p_1 \cdot e_1^i(s).$$

If $p_0 = 0$ then we have $q \neq 0$ and we pose

$$(\widehat{x}_0, \widehat{\theta}) = (0, 0), \quad \widehat{\theta} = \varepsilon \mathbf{1}_J \quad \text{and} \quad \widehat{\mathfrak{d}}(p_1, \kappa, s) = \varepsilon \left[1 + p_1 \cdot \sum_{j \in J} A_j(s) \right]$$

where $\varepsilon > 0$ is chosen such that

$$\forall s \in S, \quad e^i(s) \geq \varepsilon \left[\mathbf{1}_L + \sum_{j \in J} A_j(s) \right].$$

It then follows from standard arguments that the correspondence B_0^i is lower semi-continuous on Δ_0 . \square

C Continuity of expected utility functions

We recall that $\Omega = (\text{int } \Delta_1) \times K \times S$ represents uncertainty at $t = 1$. Fix an agent i , the purpose of this section is to prove that the indirect utility function

$$F^i : \text{gph } B_0^i \longrightarrow \mathbb{R}$$

defined by

$$F^i(p_0, q, a) = \int_{\Omega} \Gamma^i(\omega, a) \mu^i(p_0, q, d\omega)$$

is continuous on $(\text{gph } B_0^i) \cap [\Delta_0 \times \mathbf{A}]$, where \mathbf{A} is a compact subset of A^i .

Observe that if (p_0, q, a) belongs to $\text{gph } B_0^i$ then for every ω in the support of $\mu^i(p_0, q)$ the budget set $B_1^i(\omega, a)$ is non-empty and

$$\Gamma^i(\omega, a) = \sup\{W^i(\omega, a, x, d) : (x, d) \in B_1^i(\omega, a)\}$$

is well defined. We propose to extend the definition of Γ^i to the pairs (ω, a) for which $B_1^i(\omega, a)$ is empty. For this purpose we introduce the following notation:

$$J^i(s) := \{j \in J : \lambda_j^i(s) = +\infty\}.$$

At state s , agent i is not allowed to default on assets in $J^i(s)$. We extend now the definition of Γ^i to the space $\Omega \times A$ by

$$\Gamma^i(\omega, a) = \begin{cases} \sup\{W^i(\omega, a, x, d) : (x, d) \in B_1^i(\omega, a)\} & \text{if } B_1^i(\omega, a) \neq \emptyset \\ \gamma^i(\omega, a) & \text{if } B_1^i(\omega, a) = \emptyset \end{cases} \quad (60)$$

where for every $\omega = (p_1, \kappa, s)$ and $a = (x_0, \theta, \phi)$

$$\begin{aligned} \gamma^i(\omega, a) &= p_1 \cdot [e_1^i(s) + Y_s(x_0 + C\phi)] + \sum_{j \in J} V_j(\kappa, p_1, s) \theta_j \\ &\quad - \sum_{j \notin J^i(s)} V_j(p_1, s) \phi_j - \sum_{j \in J^i(s)} D_j(p_1, s) \phi_j. \end{aligned} \quad (61)$$

The function γ^i is continuous on $\Omega \times A$ and the budget set $B_1^i(\omega, a)$ is non-empty if and only if $\gamma^i(\omega, a) \geq 0$. In particular we have the following continuity result.

Claim C.1. The function Γ^i is continuous on $\{\gamma^i < 0\}$, i.e. if (ω_n, a_n) is a sequence with $\gamma^i(\omega_n, a_n) < 0$ converging to (ω, a) with $\gamma^i(\omega, a) < 0$ then the sequence $(\Gamma^i(\omega_n, a_n))$ converges to $\Gamma^i(\omega, a)$.

Applying Berge's Maximum Theorem, we obtain the following continuity result.

Claim C.2. The function Γ^i is continuous on $\{\gamma^i \geq 0\}$, i.e. if (ω_n, a_n) is a sequence with $\gamma^i(\omega_n, a_n) \geq 0$ converging to (ω, a) with $\gamma^i(\omega, a) \geq 0$ then the sequence $(\Gamma^i(\omega_n, a_n))$ converges to $\Gamma^i(\omega, a)$.

Proof of Claim C.2. For each $(\omega, a) \in \Omega \times A$ the set $B_1^i(\omega, a)$ is non-empty and convex, but since $p \in \text{int } \Delta_1$, the set $B_1^i(p_1, s, a)$ is also compact. It is obvious that the budget set correspondence B_1^i is upper-semicontinuous on $\Omega \times A$. Moreover, since $e_1^i(s)$ is strictly positive, we can prove following standard arguments that the budget set correspondence B_1^i is also lower-semicontinuous. Applying the Berge's Maximum Theorem, the function Γ^i is continuous on $\Omega \times A$. \square

Proposition C.1. The function Γ^i is bounded and continuous on $\Omega \times \mathbf{A}$.

Proof. The boundedness of Γ^i follows from the boundedness of $(\omega, x, d) \mapsto W^i(\omega, a, x, d)$ on $(\text{int } \Delta_1) \times \mathbf{A} \times X_1 \times D$. We prove now that Γ^i is continuous on $\Omega \times \mathbf{A}$. Let (ω_n, a_n) be a sequence converging to (ω, a) .

If $\gamma^i(\omega, a) > 0$, then for n large enough we have $\gamma^i(\omega_n, a_n) > 0$, implying that the result then follows from Claim C.2.

If $\gamma^i(\omega, a) < 0$, then for n large enough we have $\gamma^i(\omega_n, a_n) < 0$, implying that the result then follows from Claim C.1.

Assume now that $\gamma^i(\omega, a) = 0$. It follows that $B_1^i(\omega, a) \neq \emptyset$, in particular there exists $(x_1, d) \in B_1^i(\omega, a)$ such that

$$\Gamma^i(\omega, a) = W^i(\omega, a, x_1, d).$$

Since $(x_1, d) \in B_1^i(\omega, a)$ we must have

$$p_1 \cdot x_1 \leq p_1 \cdot [e_1^i(s) + Y_s(x_0 + C\phi)] + \sum_{j \in J} V_j(\kappa_j, p_1, s)\theta_j - \sum_{j \in J} d_j \quad (62)$$

and

$$\forall j \in J^i(s), \quad V_j(p_1, s)\phi_j = d_j \quad \text{and} \quad \forall j \notin J^i(s), \quad D_j(p_1, s) \leq d_j. \quad (63)$$

In particular it follows that

$$0 \leq p_1 \cdot x_1 \leq \gamma^i(\omega, a). \quad (64)$$

But since $\gamma^i(\omega, a) = 0$ it implies that

$$\forall j \notin J^i(s), \quad D_j(p_1, s) = d_j \quad \text{and} \quad p_1 \cdot x_1 = 0.$$

Since $p_1 \in \text{int } \Delta_1$ it implies that $x_1 = 0$. Therefore

$$\Gamma^i(\omega, a) = W^i(\omega, a, x_1, d) = \gamma^i(\omega, a) = 0.$$

The sequence $(\Gamma^i(\omega_n, a_n))$ is bounded, passing to a subsequence if necessary we can assume that the sequence $(\Gamma^i(\omega_n, a_n))$ converges to some real number γ_∞ . We propose to prove that $\gamma_\infty = \Gamma^i(\omega, a)$.

We split the set \mathbb{N} in two parts

$$\mathbb{N}_< := \{n \in \mathbb{N} : \gamma^i(\omega_n, a_n) < 0\} \quad \text{and} \quad \mathbb{N}_\geq := \mathbb{N} \setminus \mathbb{N}_<. \quad (65)$$

If the set $\mathbb{N}_<$ is infinite, passing to a subsequence if necessary, we can assume that $\mathbb{N}_< = \mathbb{N}$. It follows that $\Gamma^i(\omega_n, a_n) = \gamma^i(\omega_n, a_n)$. Passing to the limit, we obtain $\gamma_\infty^i = \gamma^i(\omega, a)$. Since $\gamma^i(\omega, a) = 0 = \Gamma^i(\omega, a)$, it implies that $\gamma_\infty = \Gamma^i(\omega, a)$.

Now assume that the set $\mathbb{N}_<$ is finite. For n large enough, $\gamma^i(\omega_n, a_n) \geq 0$. In particular there exists $(x_{1,n}, d_n) \in B_1^i(\omega_n, a_n)$ such that

$$\Gamma^i(\omega_n, a_n) = W^i(\omega_n, a_n, x_{1,n}, d_n).$$

Since \mathbf{A} is compact, passing to a subsequence if necessary, we can assume that $(x_{1,n}, d_n)$ converges to (x_1, d) . The correspondence B_1^i is upper semi-continuous, then passing to the limit we obtain

$$p_1 \cdot x_1 \leq p_1 \cdot [e_1^i(s) + Y_s(x_0 + C\phi)] + \sum_{j \in J} V_j(\kappa_j, p_1, s)\theta_j - \sum_{j \in J} d_j \quad (66)$$

and

$$\forall j \in J^i(s), \quad V_j(p_1, s)\phi_j = d_j \quad \text{and} \quad \forall j \notin J^i(s), \quad D_j(p_1, s) \leq d_j. \quad (67)$$

In particular it follows that

$$0 \leq p_1 \cdot x_1 \leq \gamma^i(\omega, a). \quad (68)$$

But since $\gamma^i(\omega, a) = 0$ it implies that

$$\forall j \notin J^i(s), \quad D_j(p_1, s) = d_j \quad \text{and} \quad p_1 \cdot x_1 = 0.$$

Since $p_1 \in \text{int } \Delta_1$ it implies that $x_1 = 0$. Therefore

$$\gamma_\infty^i = \lim_{n \rightarrow \infty} W^i(\omega, a) = W^i(\omega, a, x_1, d) = \gamma^i(\omega, a) = 0 = \Gamma^i(\omega, a).$$

□

We are now ready to prove the main result of this section.

Proposition C.2. The function F^i is continuous on $(\text{gph } B_0^i) \cap [\Delta_0 \times \mathbf{A}]$.

Proof. Let $(b_n, a_n)_{n \in \mathbb{N}}$ be a sequence in $\Delta_0 \times \mathbf{A}$ converging to $(b, a) \in \Delta_0 \times \mathbf{A}$. Fix $i \in I$ and define by

$$\mu_n := \mu^i(b_n) \quad \text{and} \quad \Gamma_n := \Gamma^i(\cdot, a_n)$$

together with

$$\mu = \mu^i(b) \quad \text{and} \quad \Gamma = \Gamma^i(\cdot, a).$$

For each $n \in \mathbb{N}$ we have

$$F^i(b_n, a_n) = \int_{\Omega} \Gamma_n(\omega) \mu_n(d\omega).$$

We claim that the family $\{\Gamma_n\}_{n \in \mathbb{N}}$ converges continuously to Γ . Indeed, let $(\omega_n)_{n \in \mathbb{N}}$ be a sequence in Ω converging to ω . Since Γ^i is continuous on $\Omega \times \mathbf{A}$, the sequence $\{\Gamma_n(\omega_n)\}$ converges to $\Gamma(\omega)$.

Now we can apply Grandmont (1972, Theorem A.3) to get that the sequence $\{F^i(b_n, a_n)\}$ converges to

$$\int_{\Omega} \Gamma^i(\omega, a) \mu^i(b, d\omega) = F^i(b, a).$$

□

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