

# Macroeconomic Announcements, Price Discovery, and Order Flow Effects in the Stock Market: Evidence from Incomplete Data and Multiple Financial Markets\*

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## Abstract

This paper investigates heterogeneity in the market assessment of public macroeconomic announcements by exploring (jointly) two main mechanisms through which macroeconomic news might enter stock prices: instantaneous fundamental news impacts consistent with the asset pricing view of symmetric information, and permanent order flow effects consistent with a microstructure view of asymmetric information related to heterogeneous interpretation of public news. Theoretical motivation and empirical evidence for the operation of both mechanisms are presented. Significant instantaneous news impacts are detected for news related to real activity (including employment), investment, inflation, and monetary policy; however, significant order flow effects are also observed on employment announcement days. A multi-market analysis suggests that these asymmetric information effects come from uncertainty about long term interest rates due to heterogeneous assessments of future Fed responses to employment shocks.

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# 1 Introduction

Asset prices are affected by revisions in expectations about changing economic conditions driven by macroeconomic news, such as output, employment and inflation surprises. Because the objectives of monetary policy are expressed in terms of the same macroeconomic variables, the response of the stock market to macroeconomic news is linked to market assessments of future Fed actions and/or future states of the economy. In this context, it remains an intriguing question the empirical distinction of the link between market beliefs and the mechanisms through which macroeconomic information enters the price process.

The impacts of macroeconomic news on asset prices have been analyzed under two main approaches. The most common, known as the “asset pricing approach”, is based on symmetric information. This approach supports the view that public information is fully and nearly instantaneously incorporated into prices since all agents observe the same piece of information, and interpret any asset pricing implication the same way. Therefore, the implied mechanism suggests that asset prices jump to their new equilibrium values nearly immediately after an announcement is released.<sup>1</sup> The second approach has been less explored. It is based on asymmetric information, which is associated with heterogeneous interpretation of public announcements. The asymmetric information approach points out that although real time macroeconomic information is publicly observed, economic agents might differ in their interpretations of the relevance of specific news for asset prices. This approach recognizes that some agents might have better models or superior information with which they can make more accurate predictions of the economy than the rest of the agents. As a result, the price impact of a macroeconomic surprise might not be instantaneous, but rather it might take some time for the market to aggregate heterogeneous beliefs and learn about the “true”

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<sup>1</sup>A nearly instantaneous adjustment in prices can be interpreted as a case where information affects prices with little or no trading activity, and the fundamental price impact is fully revealed at most a few minutes (or seconds) after the announcement.

price impact of the economic news. The microstructure literature suggests the mechanism in which this learning process occurs is through trading. Therefore, the price formation process is sensitive to the underlying information structure.

Homogeneous assessments imply different reactions than heterogeneous revisions. However, the empirical literature on announcement effects has mostly ignored asymmetric information effects, and the two approaches described above have largely followed two separate lines of research. Combining the two underlying mechanisms is crucial for a complete understanding of news effects in financial markets. Micro effects of macro announcements are only partially understood if one does not take into account relevant issues, such as heterogeneity of beliefs, the process of aggregating heterogeneous information, and news effects on revisions of expectations about long run states of the economy.

In the present paper, I contribute to reduce the gap between these lines of research by combining key elements of the two approaches in a structural microstructure model. Specifically, the goal of this paper is to jointly explore these two mechanisms to evaluate the extent to which heterogeneity in the market assessment of public fundamental information explains the stock market responses to such information. I do so by including two distinct components in the fundamental price. The first component is consistent with the standard “asset pricing view” and describes instantaneous responses (jumps) of the fundamental price to macroeconomic news, while the second component accounts for possible asymmetric information (measured by permanent order flow price impacts) on “announcement” days due to aggregation of heterogeneous private information, or heterogeneous interpretation of public information.

The effects of macroeconomic surprises on asset prices have been analyzed in a number of recent empirical studies. Most of these papers consider the symmetric information view. Based on different data sets and data frequencies, some of these studies address the

conditional mean effects of macroeconomic news using event-study analyses. For instance, Bernanke and Kuttner (2003) and Boyd, Jagannathan, and Hu (2005) analyze the stock market; Balduzzi, Elton, and Green (2001) and Fleming and Remolona (1999) study the bond market; and Andersen, Bollerslev, Diebold, and Vega (2003) analyze exchange rates. Other studies examine news effects on market volatility. For example, Flannery and Protopapadakis (2002) and Bomfim (2003) study stock returns; Andersen and Bollerslev (1998) analyze exchange rates; Li and Engle (1998) and Jones, Lamont, and Lumsdaine (1998) examine bonds. News effects at a multi-market level are investigated in Andersen, Bollerslev, Diebold, and Vega (2004) and Faust, Rogers, Wand, and Wright (2003).

The less explored approach of asymmetric information, under a microstructure framework, has most of its theoretical foundations in the seminal papers of Kyle (1985), and Glosten and Milgrom (1985). Heterogeneous responses to public announcements are theoretically discussed in Kim and Verrecchia (1991a) and Kim and Verrecchia (1991b). Empirical studies accounting for asymmetric information (order flow) effects in asset prices linked to public announcements include Evans and Lyons (2004) (for exchange rates) and Brandt and Kavajecz (2004) (for yields).

Like Andersen, Bollerslev, Diebold, and Vega (2004), this paper analyzes simultaneous reactions of prices to macroeconomic news in several markets. However, I go beyond their analysis by introducing an asymmetric information component described in terms of price impacts of the order flow on announcement days. Indeed, I estimate a structural microstructure model that captures jointly instantaneous news effects and permanent order flow effects, which might have some nontrivial implications for “observed” volatility and market liquidity. In my empirical analysis, I use daily observations to proxy microstructure measures. Specifically, following the Markov chain Monte Carlo (MCMC) approach for microstructure models with unobserved latent variables suggested in Hasbrouck (2004) and Hasbrouck

(2005), I estimate the microstructure parameters from closing prices and trading volume.<sup>2</sup> I extend the Hasbrouck model by introducing news effects on the fundamental (unobserved efficient) price and average differential effects of order flows on announcement regimes versus the non-announcement regime. In addition, I use evidence from other financial markets as a robustness analysis. Specifically, I analyze to what extent the interest rate component, or other primitive components, such as growth rate expectations and risk premia, explain reactions to news in the stock market.

My empirical analysis estimates the effect on the stock market of 19 macroeconomic announcements. In line with the empirical literature, the results suggest instantaneous news effects are important for macroeconomic variables related to real activity, investment, inflation, and monetary policy. In addition, my results support the presence of both instantaneous news impacts and order flow effects on employment announcement days. My theoretical motivation and features of observed trading volume suggest that the combination of asymmetric information with either increases in the volatility of the fundamental price or decreases in the precision of the price implication of the news, are likely reasons for explaining the increment in the asymmetric information component on employment announcement days.

My multi-market analysis suggests that this increase in asymmetric information in the stock market is driven by the interest rate component. In fact, I find evidence of excess sensitivity of long term interest rates to employment news in terms of both news effects and order flow effects. This complements the results of Gurkaynak, Sack, and Swanson (2003) in the sense that, not only do private agents revise their expectations of future Fed policies and/or long run states of the economy, but also their revisions might be heterogeneous. Therefore, the asymmetric information effect comes from uncertainty about long term interest rates due to heterogeneous assessments of future Fed responses to employment shocks. This argument

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<sup>2</sup>The main motivation for implementing Bayesian MCMC methods in these cases is the analytical and computational convenience in dealing with the unobserved trade direction.

is also consistent with the two theoretical reasons discussed above: decreases in the precision of asset price implications of employment news and increases in stock market volatility when employment information arrives.

These results are important for several reasons. On a fundamental level, they contribute to a better understanding of the link between macroeconomic information and the price formation process, which is one of the main functions of financial markets. In addition, the implications for returns, volatility and liquidity are of great relevance not only for portfolio and risk management decisions of market participants, but also for policy decisions, such as government and central bank policies, concerning financial system stability. Indeed, practitioners and policy makers can benefit from new methods for measuring impacts of output and inflation shocks in financial markets. Moreover, the analysis of both mechanisms permits us to evaluate the heterogeneity in the market assessment of economic news and future Fed policies, and provides a more general view of the information structure in financial markets.

This paper is organized as follows. Section 2 presents a theoretical review of the asset pricing approach and a microstructure theoretical framework in the context of public macroeconomic announcements. Section 3 describes the data. Section 4 presents the structural model and the econometric estimation strategy. Section 5 reports my empirical findings. Section 6 presents a robustness analysis based on information from other financial markets, and Section 7 concludes.

## **2 Theoretical Economic Framework**

### **2.1 Macroeconomic Information and the Asset Pricing Approach**

In this subsection, I provide intuition on how macroeconomic fundamental variables affect asset prices under a standard rational expectations equilibrium framework consistent with a

structure in which the interpretation of macroeconomic news is common knowledge. Following the standard consumption-based model with nominal prices, I obtain the familiar pricing equation:

$$m_t = E_t \left[ \left( \beta \frac{u'(c_{t+1})}{u'(c_t)} \frac{\Pi_t}{\Pi_{t+1}} \right) x_{t+1} \right] = E_t [\Lambda_{t+1} x_{t+1}], \quad (1)$$

where  $m_t$  represents the fundamental security price,  $u'(c_t)$  denotes the marginal utility of consumption  $c_t$ ,  $\Pi$  denotes the nominal price level (CPI),  $\beta$  is the discount rate, and  $x_{t+1}$  represents the future payoff (dividends and principal). Under constant relative risk aversion (CRRA) utility with parameter  $\gamma > 0$ , and taking the equilibrium condition  $c_{t+1} = x_{t+1}$ , the stochastic discount factor takes the form:

$$\Lambda_{t+1} = \beta \left( \frac{x_{t+1}}{x_t} \right)^{-\gamma} \frac{\Pi_t}{\Pi_{t+1}} \quad (2)$$

Moreover, equation (1) can be rewritten as follows:

$$m_t = E_t [\Lambda_{t+1}] E_t [x_{t+1}] + cov_t (\Lambda_{t+1}, x_{t+1}) \quad (3)$$

Equations (2) and (3) permit us to analyze the effects of macroeconomic shocks on the different components of the price. For instance, the expected value of the stochastic discount factor, which sets the price of a zero-coupon bond and determines the risk-free rate, decreases with positive output shocks, as well as with positive inflation shocks.<sup>3</sup> In contrast, a positive output shock will increase the current expected payoff (rate of growth), which is represented by  $E_t [x_{t+1}]$ . Therefore, the interest rate and the expected growth rate components react to output (and inflation) shocks in opposite directions. Moreover, the reaction of the covariance term (risk premium) to macroeconomic shocks is also ambiguous. This leaves the overall effect of macroeconomic shocks on stock prices as an important empirical question. Nevertheless, using data from other financial markets, such as bond

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<sup>3</sup>The effect of inflation shocks on  $\Lambda_{t+1}$  is not sensitive to the assumption of power utility.

markets, might give more insights in determining which effect dominates.

## 2.2 A Microstructure Asymmetric Information Model

Under the risk-neutral probability measure, the fundamental asset price in equation (1) satisfies the martingale property, and has a random walk representation. Moreover, even with respect to the actual probability measure, the random walk representation works well on a daily basis due to the little change in consumption and risk aversion at daily or higher frequencies. Thus, under symmetric information and short term horizons, the fundamental price follows a time-series process of the form:

$$m_t = m_{t-1} + u_t \tag{4}$$

where  $u_t$  reflects innovations in the fundamental price over the interval  $(t - 1, t]$  due to the arrival of public information, including macroeconomic announcements. Here, information is instantaneously incorporated in the fundamental price.

Under asymmetric information, an additional component is added to the fundamental price due to revisions in expectations of the market maker(s) conditional on either an order arrival or the aggregated order flow. Glosten and Milgrom (1985) and Kyle (1985) provide formal theoretical derivations of asymmetric information models in a context of insider trading. However, in a context of public announcements, it is convenient to restate the motivation. Indeed, after an announcement release, some trades might be based on superior information associated, for instance, with the presence of better informed agents who are able to process public news in superior ways that lead to better predictions of the asset price impact of such news. Moreover, since this informed trading is based on private models, heterogeneous assessments of the price implication of such public news among informed traders present a natural scenario that describes how information asymmetry might



arise when public announcements are released. As mentioned earlier, the situation of facing informed trading hidden in standard liquidity demands creates an order flow adverse selection problem for market makers, and motivates their revisions in expectations, which make permanent the asymmetric information effect on the fundamental price.<sup>4</sup>

Several specifications that include permanent effects of orders on prices are available in the literature. Hasbrouck (2005) presents one in which the fundamental (efficient) price takes the following form:

$$m_t = m_{t-1} + \varphi Q_t + \varepsilon_t, \quad (5)$$

where  $t$  indexes equally-spaced intervals, and  $Q_t$  is a measure of the cumulative signed order flow over the interval  $(t - 1, t]$ . For instance, if  $\tilde{q}_k$  is defined as an indicator of the trade direction corresponding to the  $k$  trade, where it takes the value of 1 if a transaction was initiated by a buyer (ask), and -1 if a transaction is initiated by a seller (bid), then  $Q_t$  can be defined as  $\sum_{k=1}^{N_t} \tilde{q}_k$ , where  $N_t$  represents the number of trades in the interval. Alternatively, the order flow can be proxied using signed trading volume. In this case,  $Q_t$  can be replaced in equation (5) by  $V_t q_t$ , where  $V_t$  denotes the dollar trading volume (or a function of it), and  $q_t$  represents the sign of  $Q_t$ .<sup>5</sup> Therefore, equation (5) can be replaced by:

$$m_t = m_{t-1} + \varphi V_t q_t + \varepsilon_t \quad (6)$$

The second term in equation (6) characterizes the asymmetric information (spread) component as a function of the order sizes, which are approximated by a function of the volume variable.<sup>6</sup> Other related specifications also account for this size effect. For instance, Glosten

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<sup>4</sup>Appendix A provides a more formal economic intuition by analyzing asymmetric information effects when announcements are released. The theoretical framework is based on an extension of Kyle (1985) in a simple one-period context. In this extension, I accommodate multiple informed traders who receive noisy signals about the price impact of a piece of news.

<sup>5</sup>Hasbrouck (2005) interprets  $q_t$  as the trade direction associated with the last trade of the corresponding interval. This study also finds similar results using either cumulative order flow or signed volume.

<sup>6</sup>See Glosten and Harris (1987) and Hasbrouck (2004) for a complete representation of the components

and Harris (1987) characterize the asymmetric information spread component as an affine function of the order size. Theoretical motivation about why this component should increase with the quantity traded can be found in Kyle (1985) and Easley and O’Hara (1987), in a general context. In the particular context of public releases, Appendix A provides further economic motivation and gives insights to explain possible changes in the marginal effect of order flows.<sup>7</sup> For instance, the marginal order flow impact might increase when any of the following situations occur: the variance of the fundamental price increases, the variance of liquidity demands decreases, the precision of the signal that informed agents obtain about the price implication of an economic news decreases (provided the number of informed agents is sufficiently large), and the number of informed agents decreases (provided the number of informed agents is sufficiently large).

### 3 Data

Jointly analyzing the fields of market microstructure and asset pricing has proven difficult. Hasbrouck (2005) argues that these difficulties arise from differences in the data samples and frequencies favored by each area. Asset pricing models require data at daily or lower frequencies due to various reasons, such as large sample requirements to estimate risk factors, and the fact that daily data might be the highest frequency at which prices keep their convenient martingale properties. On the other hand, microstructure models require intradaily trades and quotes data, which favors analyses of other important issues, such as intraday price dynamics, price discovery, impacts of transaction costs, inventory effects, among others; however, the data samples are small (covering only recent periods), and they are difficult to obtain, particularly in multi-market setups. These issues, combined with the importance of the bid-ask spread.

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<sup>7</sup>In Appendix A, I follow the familiar notation of Kyle (1985), where the parameter associated with the asymmetric information (order flow) effect is denoted by  $\lambda$  (instead of  $\varphi$ ).

of linking the asset pricing and microstructure areas, as well as the possibility of inferring microstructure characteristics from daily data (as suggested by Hasbrouck), motivate the use of daily data in the present study.

In particular, I use daily closing prices and trading volume data for the S&P 500 index, obtained from the Center for Research in Security Prices (CRSP) database. I also use futures daily closing prices and trading volume data for contracts associated with stocks, exchange rates, and bonds at different maturities, obtained from Global Financial Data and Datastream. Specifically, I consider the S&P 500, \$US/Yen, US 5-Year Note and US 10-Year Note futures contracts. My sample period covers from January 1992 through December 2003.

I analyze the effect on asset prices of 19 macroeconomic announcements, classified into seven categories, namely real activity, investment, consumption, trade, price level, forward looking, and monetary policy.<sup>8</sup> The real activity group includes: industrial production, retail sales, nonfarm payroll employment, unemployment rate, capacity utilization, personal income, and consumer credit; the investment group includes: durable goods orders, construction spending, and business inventories; the consumption group includes: new home sales and personal consumption expenditures; the trade group is composed of the goods and services trade balance; the price level group includes: the consumer price index and the producer price index; the forward looking group includes: the index of leading indicators, the National Association of Purchasing Managers (NAPM) index, and housing starts; and finally, the monetary policy category includes only the Federal Open Market Committee (FOMC) announcements. Data on the corresponding macroeconomic releases are obtained from the Bureau of Labor Statistics.<sup>9</sup> Macroeconomic forecasts are obtained from the Money Market Services (MMS) survey, which includes data from telephone surveys conducted normally one

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<sup>8</sup>In this classification, we followed Andersen, Bollerslev, Diebold, and Vega (2004).

<sup>9</sup>These releases are usually made at 8:30 am on regularly scheduled announcement days by the U.S. Department of Labor.

week or less before any macroeconomic news release.<sup>10</sup> Based on this information, a surprise for release  $k$  on day  $t$  is calculated as follows:

$$S_{kt} = \frac{Y_{kt} - \widehat{Y}_{kt}}{\sigma_k} \quad (7)$$

where  $Y_{kt}$  is the realization of variable  $k$ ,  $\widehat{Y}_{kt}$  is the corresponding median forecast, and  $\sigma_k$  is the standard deviation of the forecast error.

Regarding monetary policy shocks, recent literature has pointed out that the federal funds futures dominate all other instruments for predicting near-term changes in the federal funds rate (FFR).<sup>11</sup> Therefore, these instruments can be used to compute monetary policy surprises surrounding FOMC announcements as follows:

$$S_{it} \equiv i_t - E_{t-1}i_t = \left( \frac{D}{D-d} \Delta f f_t \right) \quad (8)$$

where  $i_t$  denotes the federal funds rate,  $\Delta f f_t$  is the change in the rate of the current month's futures contract,  $D$  represents the number of days in the month, and  $d$  indicates the day of the month in which the FOMC meeting occurs.

## 4 Model Specification and Econometric Approach

I extend the version of the augmented Roll (1984) model with trade effects on the fundamental price presented in Hasbrouck (2004, 2005). Although this structural microstructure framework has been motivated in the spirit of sequential trade models, the economic intuition

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<sup>10</sup>This data was kindly provided by Informa Global Markets/MMS. Balduzzi, Elton, and Green (2001) conclude that the MMS survey data is an accurate representation of the consensus expectation in the market. Pearce and Roley (1985) find MMS forecasts unbiased and efficient.

<sup>11</sup>See Gürkaynak, Sack and Swanson (2002, 2003), and Kuttner (2001) for further discussion.

of the strategic trade model presented in Appendix A remains valid, and the estimated asymmetric information spread component, given by the order flow parameters, can be related to the “*Kyle’s lambda*” type parameter discussed in this appendix.<sup>12</sup> The extensions included here are the following. First, I introduce macroeconomic news effects on the fundamental price, which allows for jumps on announcement days. Second, I introduce differentiated (average) order flow effects on the fundamental price associated with different regimes linked to announcement and non-announcement days. In this context, the model can be stated as follows. Let the fundamental price in equation (6) take the following form:

$$m_t = m_{t-1} + (\varphi + I_{k,t}a_k)V_tq_t + u_t, \quad (9)$$

where  $q_t \in \{1, -1\}$  denotes the trade direction (1 if the  $t$  trade is a buy order, -1 if the  $t$  trade is a sell order),  $q_t \sim \text{Bernoulli}(1/2)$ ,  $V_t$  is a function of trading volume, and  $I_{k,t}$  is an indicator of type- $k$  announcement regime. As described in Section 2,  $V_tq_t$  represents signed volume as a proxy of order flow. Moreover, the term  $(\varphi + I_{k,t}a_k)V_tq_t$  characterizes the asymmetric information spread component. In addition,  $u_t$  reflects new public information as follows:

$$u_t = \varepsilon_t + \beta S_{k,t}, \quad (10)$$

where  $\varepsilon_t \sim \text{iid}N(0, \sigma_\varepsilon^2)$ , and  $S_{k,t}$  represents the news variables (or a function of the news variables) defined in equations (7) and (8). The permanent-transitory decomposition is completed by specifying an equation for the observed (log) trade price:

$$p_t = m_t + cq_t \quad (11)$$

Although the original setup of this model refers to intradaily price dynamics, the model

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<sup>12</sup>Back and Baruch (2004) find that the bid-ask spread in the Glosten and Milgrom (1985) model is approximately twice the order size multiplied by the “*Kyle’s lambda*” when the order size is not too big.

has the same form under time aggregation, as pointed out by Hasbrouck (2005). This allows us to infer the microstructure parameters from estimation based on daily data. In terms of interpretation of the structural model, parameters  $\varphi$  and  $a_k$  characterize the order flow effects associated with the asymmetric information aspects discussed in Section 2; parameter  $\beta_k$  measures the direct impact of (*type-k*) public news on the fundamental price, capturing the fundamental news effects discussed in the beginning of Section 2; and parameter  $c$  characterizes the average spread as a measure of aggregated transaction costs (excluding asymmetric information costs). Even though the goal of this study is not the analysis of these transaction costs, as it is in the simplest version of the Roll's model, I maintain the structural form of the model and direct my attention to the asymmetric information parameters and the news effects.

Based on this setup, the returns and their properties can be associated with contemporaneous news effects and market microstructure features. Indeed, from equations (9)-(11), the model implies observed returns:

$$\Delta p_t = c\Delta q_t + (\varphi + I_{k,t}a_k)V_tq_t + \beta_k S_{k,t} + \varepsilon_t, \quad (12)$$

which leads to the following expression for the conditional variance, given some information set  $\Phi_{t-1}$ .<sup>13</sup>

$$\begin{aligned} Var(\Delta p_t | \Phi_{t-1}) &= 2c^2 + (\varphi + I_{k,t}a_k)^2 E(volume_t^2 | \Phi_{t-1}) + c(\varphi + I_{k,t}a_k)E(volume_t | \Phi_{t-1}) \\ &\quad + \beta_k^2 Var(S_{k,t} | \Phi_{t-1}) + \sigma_\varepsilon^2 \end{aligned} \quad (13)$$

To estimate this model, I follow the Bayesian approach of Hasbrouck (2004), which was motivated by the power of MCMC techniques for accommodating latent data. In this

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<sup>13</sup>Recall that  $q_t$  is assumed to be independent of  $V_t$  and  $E(q_t^2 | \Phi_t) = 1$ .

setup, the parameters of the model are included in  $\Theta = \{c, \varphi, a_k, \beta_k, \sigma_\varepsilon^2\}$ . The latent data include the sequence of trade direction indicators  $q = \{q_1, q_2, \dots, q_N\}$  over the sample period.<sup>14</sup> Given the assumptions described above, it is possible to directly sample iteratively from all the complete conditional distributions, which makes the MCMC algorithm a Gibbs sampler. The following steps describe the Gibbs sampler procedure I use to obtain a sequence of draws from the unattainable desired posterior  $F(\Theta, q|\Omega)$  given a vector of initial values  $(\Theta^{(0)}, q^{(0)})$ , where  $\Omega$  denotes the set of observed data, which includes closing prices, trading volume, announcement indicators, and news variables.

Step 1 Draw  $\Theta^{(1)}$  from  $P(\Theta|q^{(0)}, \Omega)$

Step 2 Draw  $q^{(1)}$  from  $P(q|\Theta^{(1)}, \Omega)$

Step 3 Continue in this fashion until generate a sequence of random variables  $\{\Theta^{(j)}, q^{(j)}\}_{j=1}^J$  whose limiting distribution is the desired posterior.<sup>15</sup>

To fully describe the algorithm, the following proposition characterizes the conditional distributions on steps 1 and 2.

**Proposition 1** *Consider the model and underlying assumptions described in equations (9)-(11). Assuming normal/inverted gamma priors on  $\Theta = \{(c, \varphi, a_k, \beta_k), \sigma_\varepsilon^2\}$ , then the conditional posterior  $P(\Theta|q, \Omega) \sim MVN/IG$ , as in the standard Bayesian multivariate regression model.*

*Moreover, the conditional posterior distribution for the latent trade direction at time  $t$  is defined as follows:*

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<sup>14</sup>Although the sequence of efficient prices  $m = \{m_1, m_2, \dots, m_N\}$  represents also a vector of unobserved latent variables, the structural equation (11) pins down its values once  $q$  and  $c$  are known.

<sup>15</sup>The Ergodic Theorem for Markov Chains guarantees the convergence properties of this sequence. This theorem holds under very mild regularity conditions that are satisfied in the microstructure model considered in this paper.

$$P(q_t|q_{-t}, \Theta, \Omega) \propto \phi \left( M_t^*, \sqrt{\frac{\sigma_\varepsilon^2}{2}} \right) \times \frac{\exp \left\{ -\frac{(m_{t-1} - m_{t+1} + \tilde{\varphi}_t V_{t+1} q_{t+1} + \beta S_{k,t+1} + \tilde{\varphi}_t V_t q_t + \beta S_{k,t})^2}{4\sigma_\varepsilon^2} \right\}}{\exp \left\{ -\frac{(m_{t-1} - m_{t+1} + \tilde{\varphi}_t V_{t+1} q_{t+1} + \beta S_{k,t+1} + \tilde{\varphi}_t V_t q_t + \beta S_{k,t})^2}{4\sigma_\varepsilon^2} \right\} + \exp \left\{ -\frac{(m_{t-1} - m_{t+1} + \tilde{\varphi}_t V_{t+1} q_{t+1} + \beta S_{k,t+1} - \tilde{\varphi}_t V_t q_t + \beta S_{k,t})^2}{4\sigma_\varepsilon^2} \right\}},$$

where  $\phi$  denotes the normal pdf,  $q_{-t} = \{q_1, q_2, \dots, q_{t-1}, q_{t+1}, \dots, q_N\}$ ,  $M_t^* = \frac{1}{2}(m_{t-1} + \tilde{\varphi}_t V_t q_t + \beta_k S_{k,t} + m_{t+1} - \tilde{\varphi}_{t+1} V_{t+1} q_{t+1} - \beta_k S_{k,t+1})$ ,  $\tilde{\varphi}_t = (\varphi + I_{k,t} a_k)$  and  $m = p - cq$ .

The first part of Proposition 1 is straightforward from equation (12), which, once  $q$  is known, fits in the standard Bayesian multivariate regression framework where the result is well known. The second part is developed in Appendix B. Proposition 1 is the basis of the Gibbs steps that lead to the sample(s) from the joint posterior used for estimation of the parameters and latent (state) variables in the structural microstructure model. Empirical estimation results are discussed in the next section.

## 5 Estimation Results

I estimate the model described above considering an affine function of volume in the order flow term of equation (9).<sup>16</sup> Thus,  $V_t = (1, \text{trading volume at day } t)'$ ,  $\varphi = (\varphi_0, \varphi_1)'$ , and  $a_k = (a_{0,k}, a_{1,k})'$ . Table 1 presents descriptive statistics for the S&P 500 trading volume on non-announcement and announcement days.<sup>17</sup> In addition, I consider asymmetric news effects by accounting for differential effects associated with positive and negative shocks. Therefore, the model can be summarized from the specifications for fundamental and observed prices

<sup>16</sup>Glosten and Harris (1987) suggest a linear affine function of the number of traded shares to characterize the order size effect on the asymmetric information spread.

<sup>17</sup>The types of announcement days included in this table correspond to the announcements that show significant effects for the index in my estimation results (see Table 2).



in equations (9) and (11):

$$m_t = m_{t-1} + (\varphi + I_{k,t}a_k)V_tq_t + u_t,$$

$$p_t = m_t + cq_t$$

However, in lieu of equation (10), the following equation describes the public information term:

$$u_t = \beta_{1,k}S_{k,t}^+ + \beta_{2,k}S_{k,t}^- + \varepsilon_t, \quad (14)$$

where

$$S_{k,t}^+ = \begin{cases} S_{k,t} & \text{if } S_{k,t} \geq 0 \\ 0 & \text{Otherwise} \end{cases}$$

$$S_{k,t}^- = \begin{cases} |S_{k,t}| & \text{if } S_{k,t} < 0 \\ 0 & \text{Otherwise} \end{cases},$$

and therefore, equation (12) becomes:

$$\Delta p_t = c\Delta q_t + (\varphi + I_{k,t}a_k)V_tq_t + \beta_{1,k}S_{k,t}^+ + \beta_{2,k}S_{k,t}^- + \varepsilon_t \quad (15)$$

Considering these modifications and applying Proposition 1, Table 2 presents estimation results for the S&P500.<sup>18</sup> As mentioned in Section 4, I focus on the interpretation of the asymmetric information coefficient vectors,  $\varphi$  and  $a_k$ , and those corresponding to the direct news impacts,  $\beta_{1,k}$  and  $\beta_{2,k}$ . For reasons of space, I present results only for the announcements that show significance on the coefficients of interest. The results in columns (1) and (2) show that news effects are important for variables regarding real activity, such as non farm employment payrolls, the unemployment index, and retail sales; investment, such as

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<sup>18</sup>I also consider the possibility of structural breaks in the volatility of the efficient price by applying the group of tests presented in Andreou and Ghysels (2002). This procedure indicates two likely breaking points, one corresponding to 10/24/1995 and the other to 3/26/1997.

construction spending; prices, such as the consumer price index; and monetary policy, such as the federal funds rate. In terms of output surprises in real activity and investment, the effects indicate that positive shocks decrease returns and negative shocks increase returns. These results are in line with Andersen, Bollerslev, Diebold, and Vega (2004) and Boyd, Jagannathan, and Hu (2005) given that most of the years included in the sample period correspond to expansions, where bad news has a puzzling positive impact due to a dominant discount rate effect. In contrast, positive inflation surprises show the expected negative effect in the stock index, and lower than expected interest rates show a positive impact.

In terms of asymmetric information effects, although I find that the order flow impact is not significant for most of the announcements, the analysis provides an interesting result in terms of the economic content of employment news. Besides direct news effects, I find significant order flow effects on employment announcement days. Results in columns (5) and (8) of Table 2 show that the average effect of order flow on employment announcement days almost duplicates the average effect of about 0.35 basis points on non-announcement days.<sup>19</sup> Moreover, the size of the average (incremental) order flow effect suggests that the asymmetric information component is of the same order of magnitude as the fundamental news effect on employment announcement days.

Notice also that the intercept coefficient,  $a_{0,k}$ , is driving the incremental asymmetric information effect on employment announcement days since the slope coefficient,  $a_{1,k}$ , is not significant. To illustrate this point, the first panel of Figure 1 shows the cumulative order flow effect,  $\varphi_0 + \varphi_1 volume_t$ , for the full sample; the second panel shows the incremental effect on employment announcement days driven by the significant intercept term; and the third panel shows the total effect on employment announcement days, where the cumulative

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<sup>19</sup>Column (5) of Table 2 shows the average order flow effect on all days given the estimates of  $\varphi$  presented in columns (3) and (5). Similarly, column (8) shows the average incremental order flow effect given the estimates of  $a_k$  presented in columns (6) and (7). The average effect refers to the cumulative impact when the term  $V_t$  in equation (12) is evaluated on the average trading volume.

effect shifts upwards. In addition, Table 3 presents F-tests regarding the coefficients on the specification with employment news. The first column suggests that all of the coefficients in the returns equation are jointly significant; the tests in the second column reject the null of a zero intercept coefficient in the order flow component; the tests in the third column confirm that the slope parameter is not driving the effect; and the tests in the last column indicate that the two employment order flow coefficients are jointly significant.

These results suggest that employment information has particular characteristics that make the mechanism through which it is incorporated into stock prices different. To understand the reason for the increase in the order flow effect on employment announcement days, it is useful to review the motivation provided in Section 2 and Appendix A, where increases in the level of asymmetric information could be associated with one (or a combination) of the following reasons: increases in the volatility of the fundamental price, decreases in the volatility of liquidity demands, decreases in the precision of the signal for a sufficiently large number of informed traders, or decreases in the number of these informed traders. In the context of employment announcement days, some of these reasons seem less likely than others. For instance, it would be difficult to believe that on employment announcement days there is a drop in the number of informed traders compared to days in which other macroeconomic announcements are released. Such a case seems neither economically nor empirically feasible. Regarding liquidity trading, Table 1 reports descriptive statistics of the trading volume on employment and non-employment days. Neither the mean nor the standard deviation are significantly different on employment announcement days (see the last two columns in Table 1). Indeed, based on the variance ratio test presented in Table 1, I cannot reject the null hypothesis of equal variances on employment and non-employment days. Thus, a drop in the volatility of liquidity demands on employment announcement days also seems unlikely. Therefore, the most likely explanations are related to the informative content of the news. In fact, a decrease in the precision of the interpretation of employment

news implications for asset prices, and/or an increase in the volatility of the fundamental price due to the information arrival process, are potential (interrelated) explanations for the incremental order flow impact observed on employment announcement days. Moreover, they are in line with the argument that interpretation of public news might be heterogeneous due to uncertainty about future policy decisions and economic conditions.

These empirical results might be conservative given the daily time aggregation considered in the estimation of the structural model. The fact that most of the announcements are released in the morning, combined with the use of closing prices and a structure that links the order flow component with the last trading activity of a day, indicate that the model is capturing permanent effects in stock prices several hours after the news arrival. Indeed, an important part of the intradaily adjustments is missed with this time aggregation and, therefore, my specification might find difficulties in accounting for other permanent impacts that might be reversed or smoothed during the day due to the arrival of other information. If this is the case, the news effects might be underestimated in my analysis.<sup>20</sup>

The asymmetric information effects observed on employment announcement days are also relevant to explain empirical patterns in conditional volatility. Equation (13) suggests that employment announcement days present larger conditional volatility in terms of observed returns due to the incremental asymmetric information effect. This result is consistent with recent empirical studies that provide evidence of larger stock market volatility on employment announcement days; examples of such studies include Flannery and Protopapadakis (2002) and Rangel (2004).

In the following section, I present a robustness analysis of my empirical results based on a revision of reactions in other financial markets (futures markets), and I provide further discussion of the economic intuition behind my findings for employment announcement days.

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<sup>20</sup>An additional issue is how accurate are the daily proxies of microstructure features. Comparing daily and intradaily estimates for individual stocks, Hasbrouck (2005) finds correlations around .94 for transaction costs and around .75 for price impacts of trades.

## 6 Robustness: A Multi-market analysis

To confirm the empirical evidence obtained in Section 5, I estimate the structural model summarized in equation (15) using data on the S&P 500 futures contract. This is convenient since using this data permits us to avoid possible concerns about the aggregation of trading volume in the actual index. Table 4 presents descriptive statistics for the daily (dollar) volume of the S&P 500 futures contract on announcement and non-announcement days. Table 5 shows the estimation results for this market. The first two columns confirm the results obtained in the previous section in terms of fundamental news impacts. Indeed, sizes and signs of the estimated effects are fully consistent with those obtained for the actual index (see Table 2). Moreover, columns (5) and (8) in Table 5 also confirm the presence of significant incremental order flow effects on employment announcement days.<sup>21</sup> Interestingly, in this market the slope coefficient,  $a_{1,k}$ , is driving the effect (see columns (6) and (7)). This suggests that the order flow impact is stronger in days with large levels of trading volume. Figure 2 illustrates this point by showing the changes in slope between the cumulative asymmetric information effect (on all days) and the incremental asymmetric information effect (on employment announcement days). Despite the difference in the structure of the asymmetric information component showed in Figures 1 and 2, results from the futures market reinforce the empirical findings discussed in the previous section.

To explore further the nature of the reactions in stock markets implied by the structural specification discussed in Section 4, I study reactions to macroeconomic news in other financial markets. For instance, an analysis of bond markets describes the behavior of the interest rate component of asset prices. Moreover, it provides a barometer indicating the market evaluation of future Fed actions when macroeconomic shocks occur. Similarly, for-

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<sup>21</sup>Also, note that, in this market, days of FOMC meetings are also associated with a significant increment in the order flow effect. This new finding might add more support to the argument that reactions in the stock market are linked to revisions in expectations of private agents (that might be heterogeneous) in response to Fed actions. Further discussion is presented at the end of this section.

ign exchange markets can lend insight into the effect of output shocks on expectations of future growth.

An exploration of futures data on these markets suggests that taking into account the seasonality in trading volume is important. Figure 3 presents the pattern of trading (dollar) volume for the 5-Year and 10-Year US Treasury Note contracts. The evident seasonal components are associated with the contract months, namely March, June, September, and December. Given that changes in volume associated with this contract design do not stem from informational asymmetries, I include an indicator variable for these contract months in the order flow term of equation (9), as follows<sup>22</sup>:

$$m_t = m_{t-1} + (\varphi + \varphi_{cm}I_{cm,t} + a_k I_{k,t})V_t q_t + u_t, \quad (16)$$

where  $I_{cm,t}$  is an indicator of the contract months of the Treasury notes.<sup>23</sup> In addition, I perform tests for structural breaks in the variance of the fundamental price innovation in the same fashion as in the previous section.<sup>24</sup>

Table 6 shows the estimation results for the two bond futures contracts, considering employment announcements.<sup>25</sup> The first two columns indicate that instantaneous fundamental news effects are highly significant, and they are consistent with the predictions of the asset pricing view described in Section 2, under power utility. Thus, employment shocks related to positive output shocks affect bond prices negatively (or interest rates positively). In contrast,

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<sup>22</sup>This specification was also estimated for the case of the S&P 500 futures contract. No significant changes in the results were found.

<sup>23</sup> $u_t$  is described in equation (14),  $V_t = (1, volume_t)$ , and the order flow coefficients,  $\varphi$ ,  $\varphi_{cm}$ , and  $a_k$ , are  $2 \times 1$  vectors.

<sup>24</sup>I find three breaks for each contract. One matches in both contracts and corresponds to 09/08/1998. The other occurs in the same year and month, but at a different day: 12/29/2000 for 5-Year notes and 12/19/2000 for 10-Year notes. And the last one does not match: 08/02/96 for 5-Year notes and 08/03/95 for 10-Year notes. However, the results are not highly sensitive to accounting for these changing points, particularly the breaks that do not match in both contracts.

<sup>25</sup>Results on the other 17 announcements are not presented in order to conserve space, but they are available upon request.

employment shocks associated with negative output shocks (like increases in the unemployment rate) impact bond prices positively (or interest rates negatively). More remarkable is the finding that order flow effects are also highly significant for both contracts. In this case, the intercept and slope coefficients drive the asymmetric information effect, whose average value is more than three times that of a non-announcement day.

These results suggest that the reactions observed in the stock market on employment announcement days are driven in an important part by the interest rate component. This is the case not only for the order flow effect, but also for the fundamental news impacts. The finding that bad news for employment is good news for stocks in expansionary periods has been justified in Boyd, Jagannathan, and Hu (2005) and Andersen, Bollerslev, Diebold, and Vega (2004) arguing that information about interest rates dominates during expansions. Nevertheless, the new empirical contribution of the present study consists in pointing out that the unexplored order flow effects in the stock market observed on employment announcement days are also driven by asymmetric information regarding the behavior of long term interest rates.

In addition, the interest rate effect can be described using the argument of Gurkaynak, Sack, and Swanson (2003) about the excess sensitivity of long term interest rates to macroeconomic fundamental news. They find that long term interest rates react significantly to news that would be expected to have only transitory effects on the short-term interest rate. They argue that this phenomenon is due to adjustments in private agents' expectations of the long run inflation target. My findings support that there is, indeed, excess sensitivity of long term interest rates in response to employment news, and complement the results found in Gurkaynak, Sack, and Swanson (2003) by suggesting that, not only do private agents revise their expectations about future Fed actions, but also their revisions are heterogeneous. This result is also consistent with the implications of the theoretical model presented in Appendix A. Indeed, revisions of private agents' expectations about future states of the economy can be

associated with increases in volatility of the fundamental price, and heterogenous revisions can be associated with decreases in the precision of the news implication for stock prices.

Table 7 presents estimation results for a currency market, the \$US/YEN futures contract. In line with Andersen, Bollerslev, Diebold, and Vega (2004), I find that positive employment (output) shocks appreciate the dollar (see columns (1) and (2) in Table 7). However, additional order flow effects on employment announcement days are not present.<sup>26</sup> This suggests that the impact of employment news on the asset pricing component associated with growth rate expectations (future payoffs) is not responsible for the asymmetric information effect. Therefore, my conclusion that this last effect comes from the interest rate component is maintained.<sup>27</sup>

## 7 Concluding Remarks

This paper explores two mechanisms that describe the process through which macroeconomic information enters stock prices, in order to evaluate heterogeneity in the market assessment of public announcements. One mechanism is related to the fundamental price impact of a surprise, which is reflected as a direct instantaneous reaction of stock prices to the news. The second is related to the process of aggregating heterogeneous information in the market stemming from heterogeneous beliefs about the price impact of a macroeconomic surprise. This latter effect is reflected in a permanent post announcement impact of order flow in the fundamental price. Heterogeneity of beliefs, associated with heterogeneous assessments of agents about future Fed policies and/or future states of the economy, makes the news implication for stock prices less precise. I provide theoretical motivation and empirical

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<sup>26</sup>The order flow effect in currency markets on regular days has been studied by Evans and Lyons (2004).

<sup>27</sup>Further analysis would be required to isolate the effect of employment news in the risk premium component. I leave this extension for future work.



support for the presence of both effects on announcement days.

A modified version of the structural microstructure model introduced by Hasbrouck (2004) is the basis of my empirical analysis. Announcement effects, and differential order flow impacts associated with announcement regimes, are allowed to have a permanent effect on the fundamental price. The analysis is based on daily observations of closing prices and trading volume. I follow the econometric approach of Hasbrouck (2004, 2005) to estimate the microstructure parameters from incomplete data using the Gibbs sampler.

In addition to fundamental news impacts associated with announcements on real activity (including employment), investment, inflation and monetary policy, order flow effects also are present on employment announcement days. Moreover, they are the same order of magnitude as fundamental news effects. Futures markets provide further evidence of this incremental asymmetric information effect. From a theoretical perspective, increases in the order flow impact on employment announcement days could more likely be explained by either increases in the dispersion of the news interpretation, or increases in volatility of fundamental prices. Both reasons are consistent with heterogeneity of beliefs.

An analysis of bond and currency markets suggests that the asymmetric information effect observed in the stock market is driven by the interest rate component of stock prices. Excess sensitivity of long-term interest rates to employment news is found in terms of both instantaneous fundamental news impacts and order flow effects. This confirms the argument that the asymmetric information effect observed on employment announcement days is due to heterogeneous beliefs and/or revisions about long-run Fed policies driven by employment surprises. This finding is also consistent with a decrease in the precision of the news implication for stock prices when employment surprises arrive.

The incremental asymmetric information effect observed on employment announcement days also provides an explanation for (at least part of) the excess of returns volatility ob-

served on such days, according to recent empirical evidence. This opens interesting questions regarding the contribution of this effect to the excess of volatility relative to the contribution associated with the volatility of the symmetric information term. In this context, an intradaily analysis will provide a richer dynamic framework to explain this phenomenon. Moreover, introducing specifications that allow more general time varying news and order flow impacts are also appealing to help get a better understanding of the dynamics of these two mechanisms characterizing the information structure on announcement days. I leave these extensions for future research.

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**TABLE 1**  
**Descriptive Statistics for S&P500 Trading Volume**

Type of Announcement Day	Obs	Mean	Std. Dev.	95% Conf. Interval for Mean	T-Test	F-Test
Non-Announcement	2892	695.22	457.01	678.56 711.88		
CPI	132	763.62	488.94	679.43 847.80	-1.68*	0.87
Employment	141	708.96	441.91	635.39 782.54	-0.29	1.02
FFR (FOMC)	102	723.29	556.68	613.95 832.63	-0.56	0.67*
Retail Sales	142	703.65	430.53	632.23 775.08	-0.15	1.14
Construction Spending	141	714.83	455.47	638.99 790.66	-0.44	1.01

Source: Data from The Center for Research in Security Prices (CRSP)

Sample Period: From January 1, 1992 to December 31, 2003

Notes: Volume is given in millions of shares. T-Test denotes a test for equality of means with respect to Non-Announcement days, i.e.,  $H_0: \text{Mean(Non-Announcement Days)} = \text{Mean(Announcement Days)}$ . F-Test denotes a variance ratio test for  $H_0: \text{Std. Dev(Non-Announcement)} = \text{Std. Dev(Announcement)}$  against  $H_a: \text{Std. Dev(Non-Announcement)} > \text{Std. Dev(Announcement)}$ . A size of 5% is used in both tests (\* $p < .05$ ).

**TABLE 2**  
**ESTIMATION RESULTS FOR S&P500**

Announcement	News Effects			Asymmetric Information Coefficients				
	$\beta_{1,k}$	$\beta_{2,k}$	$\varphi_0$	$\varphi$		$\alpha_k$		Average Effect <sup>‡</sup>
<b>REAL ACTIVITY</b>				$\varphi_1$	$\alpha_0$	$\alpha_1$		
Nonfarm Payroll	-0.2711* (0.1233)	-0.0047 (0.1148)	-0.1385 (0.0946)	0.00071* (0.00008)	0.3601* (0.0636)	-0.00007 (0.00023)	0.3996* (0.1632)	0.3514* (0.1106)
Unemployment	0.0350 (0.1288)	0.0202 (0.0969)	-0.1659** (0.0954)	0.00072* (0.00008)	0.3404* (0.0647)	-0.00018 (0.00025)	0.4255* (0.1651)	0.3013* (0.1101)
Retail Sales	-0.0585 (0.1185)	0.3172* (0.1306)	-0.1831** (0.0998)	0.00072* (0.00008)	0.3241* (0.0698)	-0.00055 (0.00083)	0.1215 (0.3715)	-0.2632 (0.4136)
<b>INVESTMENT</b>								
Construction Spending	-0.0122 (0.0994)	0.2165* (0.1088)	-0.1641 (0.1095)	0.00069* (0.00008)	0.3262* (0.0764)	0.00032 (0.00024)	-0.1199 (0.2035)	0.1093 (0.1242)
<b>PRICES</b>								
CPI	-0.4613* (0.1341)	0.1197 (0.0934)	-0.1733** (0.1050)	0.00072* (0.00008)	0.3355* (0.0735)	-0.00051 (0.00066)	0.0905 (0.2984)	-0.3138 (0.3367)
<b>MONETARY POLICY</b>								
FFR	-0.1003 (0.1831)	0.1792** (0.0955)	-0.1772** (0.1032)	0.00070* (0.00008)	0.3150* (0.0735)	0.00027 (0.00035)	-0.0816 (0.4266)	0.1220 (0.2335)

Source: Data from The Center for Research in Security Prices (CRSP)

Sample Period: From January 1, 1992 to December 31, 2003

Notes: Estimates correspond to the fundamental price specification  $m_t = m_{t-1} + (\rho + a_k I_{k,t}) V_t q_t + \beta_{1,k} S_{k,t}^+ + \beta_{2,k} S_{k,t}^- + \varepsilon_t$ , where  $\varphi = (\varphi_0, \varphi_1)$ ,  $a = (a_0, a_1)$ ,  $V_t = (1, volume_t)'$ .

Estimates for transaction costs and volatilities are not reported, but they are available upon request.

Standard errors in parentheses.

\*) 5% Significant.

\*\*\*) 10% Significant.

‡)  $\varphi_0 + \varphi_1 \text{Average (volume)}$

$a_0 + a_1 \text{Average (volume)}$

**TABLE 3**  
Coefficient Tests

Announcement	F Tests			
	B=0 <sup>‡</sup>	$\alpha_{0,k}=0$	$\alpha_{1,k}=0$	F2&F3
REAL ACTIVITY	F1	F2	F3	F4
Nonfarm Payroll	67.37* (12.32)	14.05* (6.95)	0.77 (1.38)	16.04* (5.52)
Unemployment	62.78* (11.35)	15.87** (8.28)	1.73 (2.58)	14.32* (5.84)

Standard errors in parentheses.

\*) 5% significant.

\*\*) 10% significant.

‡)  $B=(c, \varphi_0, \varphi_1, \alpha_{0,k}, \alpha_{1,k}, \beta_{1,k}, \beta_{2,k})$

**TABLE 4**  
Descriptive Statistics for Futures S&P500 Trading Volume

Type of Announcement Days	Obs	Mean	Std. Dev.	95% Conf. Interval for Mean		T-Test	F-Test
Non-Announcement	2865	82.41	41.42	80.89	83.92		
CPI	132	101.82	53.33	92.64	111.00	-5.19*	0.60*
Employment	140	88.10	36.60	81.98	94.21	-1.39	1.34
FFR (FOMC)	101	82.21	44.86	73.35	91.06	0.26	0.88
Retail Sales	142	100.05	55.50	90.85	109.26	-4.88*	0.55*
Construction Spending	140	80.44	34.52	74.67	86.21	0.81	1.52

Source: Thomson Financial Datastream/Chicago Mercantile Exchange.

Sample Period: From January 1, 1992 to December 31, 2003.

Notes: Units are expressed in terms of dollar volume (\$1000). T-Test denotes a test for equality of means with respect to Non-Announcement days, i.e.,  $H_0: \text{Mean(Non-Announcement Days)} = \text{Mean(Announcement Days)}$ . F-Test denotes a variance ratio test for  $H_0: \text{Std. Dev(Non-Announcement)} = \text{Std. Dev(Announcement)}$ . CPI and Retail Sales days favor the alternative  $H_a: \text{Std. Dev(Non-Announcement)} < \text{Std. Dev(Announcement)}$  at the 5% level (\* $p < .05$ ).



**TABLE 5**  
**ESTIMATION RESULTS FOR FUTURES S&P500**

Announcement	News Effects				Adverse Selection Coeff			
	$\beta_{1, \kappa}$	$\beta_{2, \kappa}$	$\phi_0$	$\phi_1$	Average Effect*	$\alpha_0$	$\alpha_1$	Average Effect*
<b>REAL ACTIVITY</b>								
Nonfarm Payroll	-0.3107* (0.1193)	0.0084 (0.1060)	-0.5191* (0.0780)	0.0127* (0.0010)	0.5349* (0.0403)	-0.3226 (0.2696)	0.0066* (0.0027)	0.2614* (0.0973)
Unemployment	-0.0255 (0.1217)	-0.0400 (0.0914)	-0.5280* (0.0793)	0.0127* (0.0010)	0.5281* (0.0419)	-0.3601 (0.2972)	0.0067* (0.0029)	0.2330* (0.1083)
Retail Sales	-0.0229 (0.1057)	0.2892* (0.1306)	-0.5651* (0.0807)	0.0131* (0.0010)	0.5276* (0.0424)	0.3325 (0.3877)	-0.0064 (0.0046)	-0.3091 (0.3106)
<b>INVESTMENT</b>								
Construction Spending	0.2008 (0.1358)	0.2963** (0.1542)	-0.5163* (0.0779)	0.0127* (0.0010)	0.5448* (0.0409)	-0.9472 (0.6363)	0.0131** (0.0077)	0.1095 (0.0995)
<b>PRICES</b>								
CPI	-0.4868* (0.1445)	0.1043 (0.0935)	-0.5393* (0.0790)	0.0131* (0.0009)	0.5493* (0.0409)	0.0620 (0.2534)	-0.0028 (0.0034)	-0.2191 (0.1894)
<b>MONETARY POLICY</b>								
FFR	-0.1729 (0.2075)	0.1744* (0.0846)	-0.5298* (0.0802)	0.0127* (0.0010)	0.5238* (0.0425)	-0.4713* (0.2345)	0.0087* (0.0026)	0.2460* (0.1029)

Source: Thomson Financial Datastream/Chicago Mercantile Exchange

Sample Period: From January 1, 1992 to December 31, 2003.

Notes: Estimates correspond to the fundamental price specification  $m_t = m_{t-1} + (\varphi + a_k I_{k,t}) V_t q_t + \beta_{1,k} S_{k,t}^+ + \beta_{2,k} S_{k,t}^- + \varepsilon_t$ , where  $\varphi = (\varphi_0, \varphi_1)$ ,  $a = (a_0, a_1)$ ,  $V_t = (1, volume_t)'$ .

Estimates for transaction costs and volatilities are not reported, but they are available upon request.

Standard errors in parentheses.

\*) 5% Significant.

\*\*) 10% Significant.

‡)  $\varphi_0 + \varphi_1 Avg(volume)$

$\alpha_0 + \alpha_1 Avg(volume)$

**TABLE 6**  
**Employment Effects on Bond Futures Markets**

Announcement	News Effects			Asymmetric Information Coefficients				
	$\beta_{1, \kappa}$	$\beta_{2, \kappa}$	$\phi_0$	$\phi$	$\alpha_0$	$\alpha_1$	$\alpha_k$	Average Effect <sup>‡</sup>
<b>REAL ACTIVITY</b>								
				5Y Notes				
Nonfarm Payroll	-0.2403* (0.0380)	0.1818* (0.0527)	-0.1196* (0.0207)	0.1273* (0.0088)	0.0518* (0.0131)	0.1626* (0.0448)	-0.0354* (0.0212)	0.1047* (0.0245)
Unemployment	0.1493* (0.0439)	-0.0713* (0.0314)	-0.1244* (0.0204)	0.1277* (0.0088)	0.0475* (0.0129)	0.1768* (0.0435)	-0.0341** (0.0208)	0.1214* (0.0240)
				10Y Notes				
Nonfarm Payroll	-0.3839* (0.0466)	0.1743* (0.0522)	-0.2288 (0.0321)	0.2194 (0.0136)	0.0719* (0.0197)	0.2754* (0.0662)	-0.0782* (0.0323)	0.1465* (0.0341)
Unemployment	0.1739* (0.0645)	-0.1389* (0.0474)	-0.2398* (0.0324)	0.2215* (0.0137)	0.0637* (0.0201)	0.3222* (0.0671)	-0.0844* (0.0331)	0.1840* (0.0344)

Source: Thomson Financial Datastream/Chicago Mercantile Exchange.

Sample Period: From January 1, 1992 to December 31, 2003.

Notes: Estimates correspond to the fundamental price specification  $m_t = m_{t-1} + (\varphi + a_k I_{k,t}) V_t q_t + \beta_{1,k} S_{k,t}^+ + \beta_{2,k} S_{k,t}^- + \varepsilon_t$ , where  $\varphi = (\varphi_0, \varphi_1)$ ,  $a = (a_0, a_1)$ ,  $V_t = (1, volume_t)'$ .

Estimates for transaction costs and volatilities are not reported, but they are available upon request.

Standard errors in parentheses.

\*) 5% Significant.

\*\*\*) 10% Significant.

‡)  $\varphi_0 + \varphi_1 \text{Avg}(\text{volume})$

$a_0 + a_1 \text{Avg}(\text{volume})$

**TABLE 7**  
**ESTIMATION RESULTS FOR FUTURES FX USYEN**

Announcement	News Effects		Asymmetric Information Coefficients					
	$\beta_{1, \kappa}$	$\beta_{2, \kappa}$	$\varphi_0$	$\varphi_1$	Average Effect*	$a_0$	$a_1$	Average Effect*
<b>REAL ACTIVITY</b>								
Nonfarm Payroll	-0.1794** (0.1073)	0.0360 (0.0748)	-0.2324* (0.0372)	0.0220* (0.0010)	0.2494* (0.0270)	0.1924 (0.1394)	-0.0156 (0.0104)	-0.2167 (0.2836)
Unemployment	0.0985 (0.0945)	-0.1183** (0.0700)	-0.2543* (0.0367)	0.0236* (0.0010)	0.2634* (0.0273)	0.2328 (0.1487)	-0.0168** (0.0096)	-0.2076 (0.2661)

Source: Thomson Financial Datastream/Chicago Mercantile Exchange.

Sample Period: From January 1, 1992 to December 31, 2003.

Notes: Estimates correspond to the fundamental price specification  $m_t = m_{t-1} + (\varphi + a_k I_{k,t}) V_t q_t + \beta_{1,k} S_{k,t}^+ + \beta_{2,k} S_{k,t}^- + \varepsilon_t$ , where  $\varphi = (\varphi_0, \varphi_1)$ ,  $a = (a_0, a_1)$ ,  $V_t = (1, volume_t)'$ .

Estimates for transaction costs and volatilities are not reported, but they are available upon request. Standard errors in parentheses.

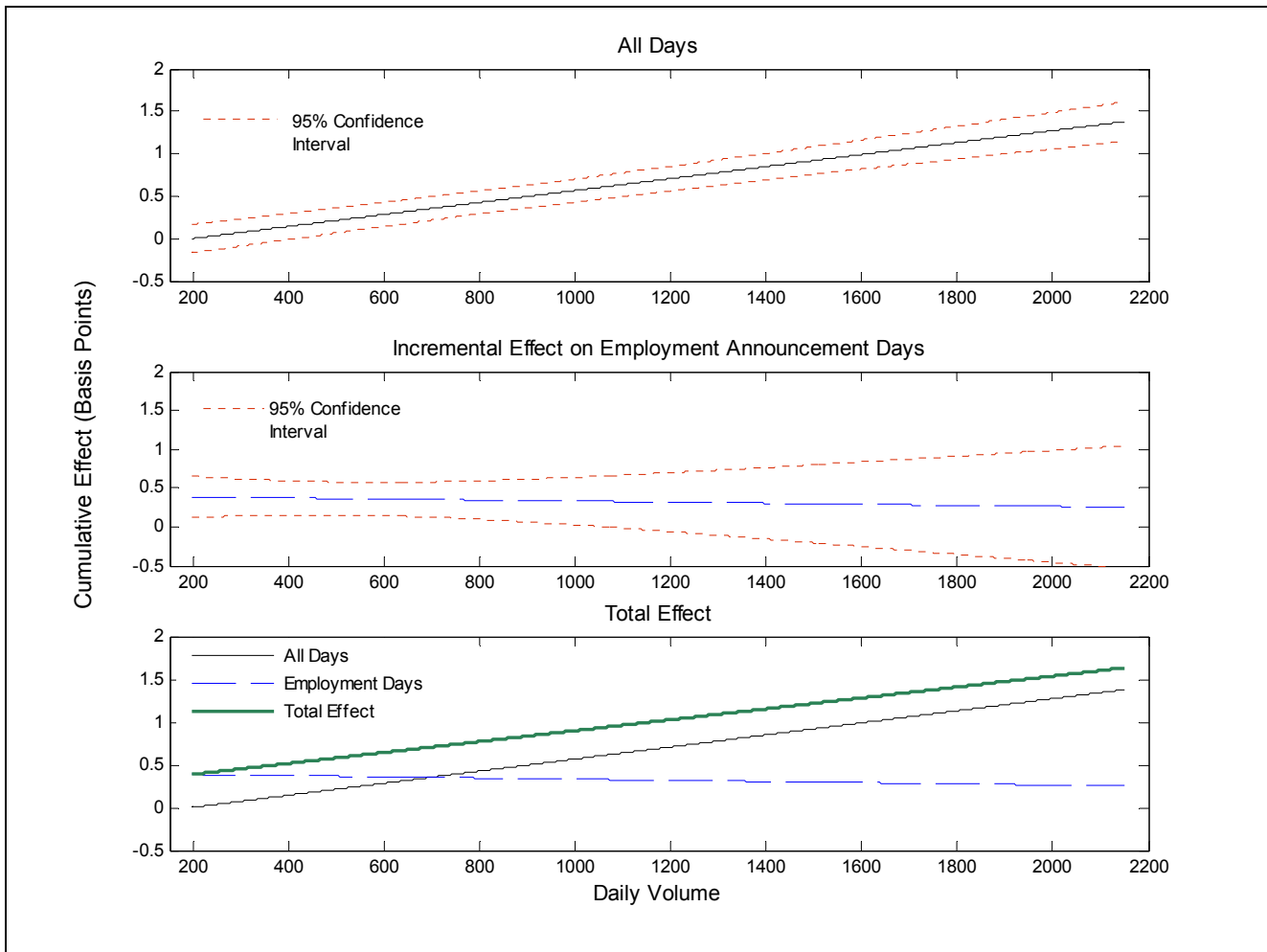
\*) 5% Significant.

\*\*\*) 10% Significant.

‡)  $\varphi_0 + \varphi_1 Avg (volume)$

$a_0 + a_1 Avg (volume)$

**FIGURE 1**

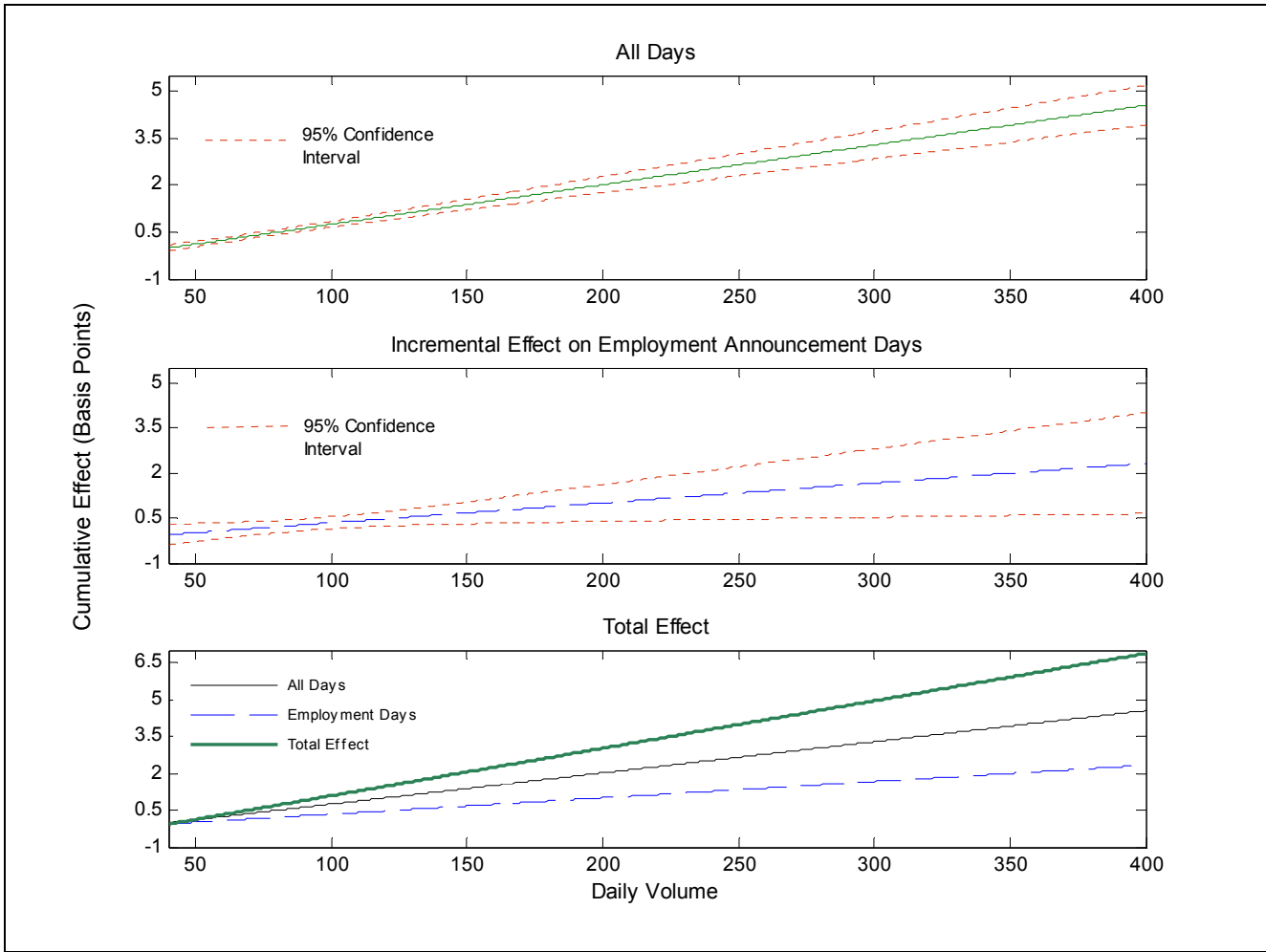


Source: Data from The Center for Research in Security Prices (CRSP)

Sample Period: From January 1, 1992 to December 31, 2003

Notes: The relevant range of volume goes from 197 to 2144.7 (\$1,000,000). Extreme values in the left tail are not shown in the graphs (4.5% of the distribution of volume).

**FIGURE 2**

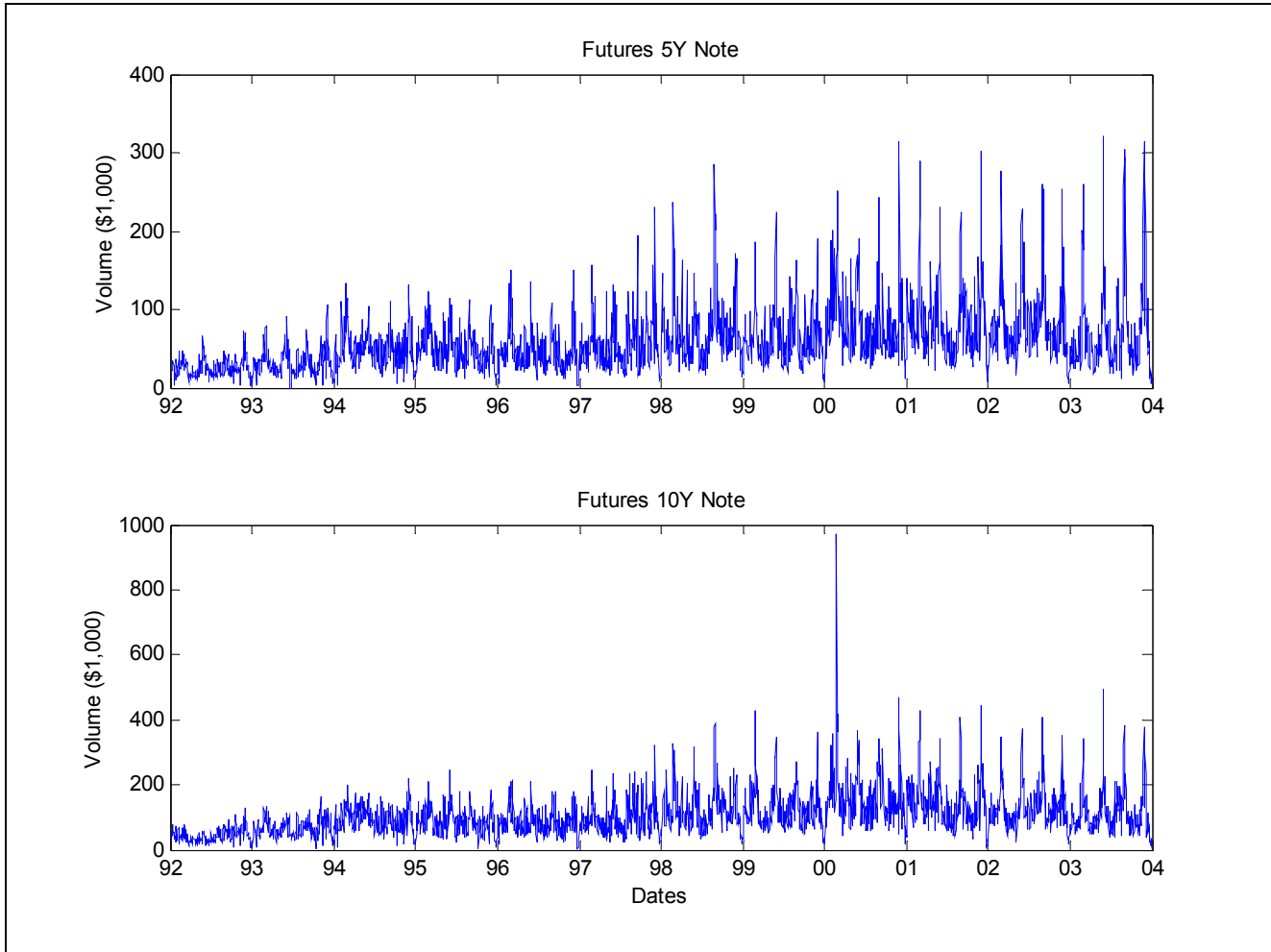


Source: Thomson Financial Datastream/Chicago Mercantile Exchange

Sample Period: From January 1, 1992 to December 31, 2003

Notes: The relevant range of (dollar) volume goes from 39.52 to 397.87 (\$1,000). Extreme values on the left are not shown in the graphs (7% of the distribution of volume).

**FIGURE 3**



Source: Thomson Financial Datastream.  
Sample Period: From January 1, 1992 to December 31, 2003.

# Appendix A

## A Microstructure Model of Price Determination

To provide economic intuition on the impact of trades on fundamental prices due to heterogeneous private information, I present an extension of the Kyle (1985) model that includes noisy signals and more than one agent with superior information.<sup>28</sup> For expositional convenience, I present results for the simple one period model.<sup>29</sup>

Let  $p_0$  represent the starting price of an asset observed before an announcement is released. Now, let  $v$  denote the post announcement fundamental value of the same asset. Assume  $v$  is normally distributed with mean  $p_0$  and variance  $\sigma^2$ . Suppose I have  $M$  informed agents who get noisy signals,  $s_m$  ( $m = 1, 2, \dots, M$ ), about the “true” price impact of a news on a particular announcement day. Specifically,  $s_m = v + \varepsilon_m$ , where  $v$  and  $\varepsilon_m$  are independent, and  $\{\varepsilon_m\}_{m=1}^M$  are *iid* zero mean normal random variables with variance  $\sigma_\varepsilon$ . After obtaining her signal, the informed trader  $m$  demands  $x_m$  units of the asset. In addition, there are noise traders whose aggregated demand  $u$  is normal with mean zero and variance  $\sigma_u$ . There is also a market maker who observes the global order flow and sets prices. I am interested in Nash equilibria with linear pricing. As I show below, linear equilibrium has the advantage that symmetric informed agents will behave in a similar fashion. Accordingly, the informed traders conjecture that the market maker uses a linear pricing rule:

$$p^* = \lambda Q + \mu, \tag{17}$$

where

$$Q = u + \sum_{m=1}^M x_m$$

denotes the global order flow observed by the market maker.

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<sup>28</sup>Holden and Subrahmanyam (1992) extend the Kyle (1985) model allowing for multiple privately informed agents who strategically exploit their long-lived informational advantage.

<sup>29</sup>Although the batch auction nature of Kyle’s model is a simplification, Back and Baruch (2004) have shown that it converges to the more realistic sequential trade models.

Informed trader  $m$  chooses her demand to maximize expected profits given her signal and the conjectured pricing rule. Symmetry implies that, given a linear pricing by the market maker, the only possible equilibrium between the informed traders is one in which they choose identical demands, which are linear in their signals. The market maker makes zero profits and prices satisfy a market efficiency condition:

$$p^* = E(v | Q) = E \left( v \mid u + \sum_{m=1}^M x_m \right) \quad (18)$$

The following proposition characterizes the unique linear equilibrium price.

**Proposition 2** *Given  $v \sim N(p_o, \sigma^2)$ ,  $\{\varepsilon_m\}_{m=1}^M \text{ iid} N(0, \sigma_\varepsilon)$ ,  $\varepsilon_m \perp v$ ,  $u \sim N(0, \sigma_u)$ , and the market efficiency condition in equation (18), there exists a unique linear equilibrium in which the price satisfies equation (17), and the order flow impact on the security price is given by:*

$$\lambda = \frac{\sigma^2 [M(\sigma^2 + \sigma_\varepsilon^2)]^{1/2}}{\sigma_u [(M + 1) \sigma^2 + 2\sigma_\varepsilon^2]} \quad (19)$$

**Proof.** The profits of the informed agent  $m$ , given a linear price conjecture, are:

$$\pi_m = (v - p^*)x_m = \left[ s_m - \varepsilon_m - \lambda \left( u + \sum_{m=1}^M x_m \right) - \mu \right] x_m$$

Moreover, her expected profits given her signal take the following form:

$$E_m [\pi_m | s_m] = [s_m - E_m(\varepsilon_m | s_m) - \lambda x_m - \mu] x_m - \lambda \left( \sum_{k \neq m} E_m(x_k | s_m) \right). \quad (20)$$

She maximizes her expected profits by choosing her optimal demand  $x_m$ . To solve her demand optimization problem, the following two intermediate results are needed:

$$1) E_m(\varepsilon_m | s_m) = \frac{\sigma_\varepsilon^2 (s_m - p_o)}{\sigma_\varepsilon^2 + \sigma^2}$$

This follows from the assumption of normality:  $\begin{pmatrix} \varepsilon_m \\ v \end{pmatrix} \sim N \left[ \begin{pmatrix} 0 \\ p_o \end{pmatrix}, \begin{pmatrix} \sigma_\varepsilon^2 & 0 \\ 0 & \sigma^2 \end{pmatrix} \right]$  implies that  $\begin{pmatrix} \varepsilon_m \\ s_m \end{pmatrix} \sim N \left[ \begin{pmatrix} 0 \\ p_o \end{pmatrix}, \begin{pmatrix} \sigma_\varepsilon^2 & \sigma_\varepsilon^2 \\ \sigma_\varepsilon^2 & \sigma_\varepsilon^2 + \sigma^2 \end{pmatrix} \right]$ , and 1) is the conditional mean associated with this bivariate normal.



$$2) E_m(s_k | s_m) = \frac{\sigma^2(s_m - p_0)}{\sigma_\varepsilon^2 + \sigma^2} + p_0$$

This also follows from the assumption of normality, where  $\begin{pmatrix} s_k \\ s_m \end{pmatrix} \sim N \left[ \begin{pmatrix} p_0 \\ p_0 \end{pmatrix}, \begin{pmatrix} \sigma_\varepsilon^2 + \sigma^2 & \sigma^2 \\ \sigma^2 & \sigma_\varepsilon^2 + \sigma^2 \end{pmatrix} \right]$ .

Given the symmetry among informed traders, they conjecture that the other informed agents have a linear demand in their particular signals,  $x_k = \alpha + \beta s_k$ . Thus,  $E_m(x_k | s_m) = \alpha + \beta E_m(s_k | s_m)$ . Using this relation, and the results in 1) and 2), the FOC of maximizing the expression in equation (20) with respect to  $x_m$  takes the following form:

$$\left\{ \frac{\sigma^2}{\sigma_\varepsilon^2 + \sigma^2} s_m + \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma^2} p_0 - 2\lambda x_m - \lambda \sum_{k \neq m} \left( \alpha + \beta \left[ \frac{\sigma^2(s_m - p_0)}{\sigma_\varepsilon^2 + \sigma^2} + p_0 \right] \right) \right\} = 0$$

Thus, the optimal demand of informed trader  $m$  is:

$$x_m = \left( \frac{\sigma^2 - \lambda(M-1)\beta\sigma^2}{2\lambda(\sigma_\varepsilon^2 + \sigma^2)} \right) s_m + \frac{\sigma_\varepsilon^2}{2(\sigma_\varepsilon^2 + \sigma^2)} \left( \frac{1 - \beta\lambda(M-1)}{\lambda} \right) p_0 - \left( \frac{\mu + \lambda(M-1)\alpha}{2\lambda} \right)$$

To satisfy the linear conjecture,  $x_m = \alpha + \beta s_m$ , the coefficients of  $s_m$  is equalized to  $\beta$ , and the following expression for  $\lambda$  is obtained:

$$\lambda = \frac{\sigma^2}{\beta [(M+1)\sigma^2 + 2\sigma_\varepsilon^2]} \quad (21)$$

On the other hand, the market maker makes zero profits, and prices satisfies the market efficiency condition in equation (18). Given that  $Q = u + \sum_{m=1}^M x_m = u + M\alpha + \beta Mv + \sum_{m=1}^M \varepsilon_m$ , and  $\left\{ v, u, \sum_{m=1}^M \varepsilon_m \right\}$  are mutually independent, the vector  $\begin{pmatrix} v \\ Q \end{pmatrix} \sim N \left( \begin{pmatrix} p_0 \\ M(\alpha + \beta)p_0 \end{pmatrix}, \begin{pmatrix} \sigma^2 & M\beta\sigma^2 \\ M\beta\sigma^2 & \sigma_u^2 + \beta^2 M^2 \sigma^2 + \beta^2 M \sigma_\varepsilon^2 \end{pmatrix} \right)$ .

Thus, equation (18) takes the following form:

$$p^* = E(v | Q) = \frac{M\beta\sigma^2 (Q - M\alpha - M\beta p_0)}{\sigma_u^2 + \beta^2 M^2 \sigma^2 + \beta^2 M \sigma_\varepsilon^2}$$

This is consistent with the linear conjecture in equation (17) provided

$$\lambda = \frac{\beta M \sigma^2}{\sigma_u^2 + \beta^2 M^2 \sigma^2 + \beta^2 M \sigma_\varepsilon^2}. \quad (22)$$

Equalizing equations (21) and (22) implies that  $\beta = \frac{\sigma_u}{\sqrt{(M(\sigma^2 + \sigma_\varepsilon^2))}}$ . Substituting this value in equation (21) leads to the result for  $\lambda$ . ■

A simple exercise of comparative statics suggests that  $\lambda$  increases with the variance of the fundamental price ( $\sigma^2$ ) and decreases the variance of liquidity demands ( $\sigma_u^2$ ). The relation with respect to the other parameters is non-monotonic. For instance, when the precision of the signal ( $\sigma_\varepsilon^{-2}$ ) decreases,  $\lambda$  increases provided  $\sigma_\varepsilon^2 < \frac{(M-3)\sigma^2}{2}$ . This condition is likely to hold when the number of informed agents is not too small. Similarly,  $\lambda$  decreases with the number of informed agents for large enough  $M$  ( $M > 1 + \frac{2\sigma_\varepsilon^2}{\sigma^2}$ ). Given these complicated relations, explaining empirically changes in the  $\lambda$  parameter is not straightforward. There might be many possibilities or interactions. However, some scenarios seem more likely than others in the context of scheduled releases of public information. The empirical part of this study presents further discussion on the interpretation of changes in  $\lambda$  (expressed in terms of changes in  $\varphi$  in equations (5) and (6)), and estimates proxies of this trade price-impact parameter in different regimes associated to “announcement” and “regular” days.

## Appendix B

### Proof. of Proposition 1:

This proof follows the appendix of Hasbrouck (2004). Define the subsequent notation:

$$p = (p_1, p_2, \dots, p_T)$$

$$q_{-t} = (q_1, \dots, q_{t-1}, q_{t+1}, \dots, q_T)$$

Equations (9)-(11) can be summarized as follows:

$$p_t = m_t + cq_t$$

$$m_t = m_{t-1} + \tilde{\varphi}_t V_t q_t + \beta_k S_{k,t} + \varepsilon_t,$$

$$\text{where } \tilde{\varphi}_t = (\varphi + a_k I_{k,t})$$

To conserve space, all densities are conditioned on  $\Theta = \{(c, \varphi, a_k, \beta_k), \sigma_\varepsilon^2\}$  and the observed data  $\{S_{k,t}, I_{k,t}, V_t\}_{t=1}^T$ .

Thus, the conditional distribution of the latent variables is given by:

$$P(q_t | p, q_{-t}) = P(q_t | p_t, m_{t-1}, m_{t+1}, q_{t+1})$$

Moreover, from Bayes rule

$$P(q_t | p_t, m_{t-1}, m_{t+1}, q_{t+1}) = \frac{f(p_t | q_t, m_{t-1}, m_{t+1}, q_{t+1}) P(q_t | m_{t-1}, m_{t+1}, q_{t+1})}{f(p_t | m_{t-1}, m_{t+1}, q_{t+1})},$$

which implies:

$$P(q_t | p_t, m_{t-1}, m_{t+1}, q_{t+1}) \propto f(p_t | q_t, m_{t-1}, m_{t+1}, q_{t+1}) P(q_t | m_{t-1}, m_{t+1}, q_{t+1}) \quad (23)$$

Now, consider the second term in equation (23) and apply Bayes rule:

$$P(q_t | m_{t-1}, m_{t+1}, q_{t+1}) = \frac{f(m_{t+1} | m_{t-1}, q_t, q_{t+1}) P(q_t | m_{t-1}, q_{t+1})}{P(m_{t+1} | m_{t-1}, q_{t+1})}$$

Given that  $P(q_t | m_{t-1}, q_{t+1}) = \frac{1}{2}$ ,

$$P(q_t | m_{t-1}, m_{t+1}, q_{t+1}) \propto f(m_{t+1} | m_{t-1}, q_t, q_{t+1}) = f(\varepsilon_t + \varepsilon_{t+1}), \quad (24)$$

where  $\varepsilon_t = m_t - m_{t-1} - (\tilde{\varphi}_t V_t q_t + \beta_k S_{k,t})$  and  $\varepsilon_{t+1} = m_{t+1} - m_t - (\tilde{\varphi}_{t+1} V_{t+1} q_{t+1} + \beta_k S_{k,t+1})$ .

Now, given the assumption that  $\varepsilon_t \sim iidN(0, \sigma_\varepsilon^2)$ , equation (24) can be written as:

$$P(q_t | m_{t-1}, m_{t+1}, q_{t+1}) \propto \int_{-\infty}^{\infty} \exp \left\{ - \left( \frac{(m_t - m_{t-1} - (\tilde{\varphi}_t V_t q_t + \beta_k S_{k,t}))^2}{2\sigma_\varepsilon^2} + \frac{(m_{t+1} - m_t - (\tilde{\varphi}_{t+1} V_{t+1} q_{t+1} + \beta_k S_{k,t+1}))^2}{2\sigma_\varepsilon^2} \right) \right\} dm_t$$

Solving the integral:

$$P(q_t|m_{t-1}, m_{t+1}, q_{t+1}) \propto \exp \left\{ -\frac{(m_{t-1}-m_{t+1}+\tilde{\varphi}V_{t+1}q_{t+1}+\beta_k S_{k,t+1}+\tilde{\varphi}V_t q_t+\beta_k S_{k,t})^2}{4\sigma_\varepsilon^2} \right\},$$

and defining

$$\Gamma_t = \exp \left\{ -\frac{(m_{t-1}-m_{t+1}+\tilde{\varphi}V_{t+1}q_{t+1}+\beta_k S_{k,t+1}+\tilde{\varphi}V_t q_t+\beta_k S_{k,t})^2}{4\sigma_\varepsilon^2} \right\} + \exp \left\{ -\frac{(m_{t-1}-m_{t+1}+\tilde{\varphi}q_{t+1}+\beta_k S_{k,t+1}-\tilde{\varphi}V_t q_t+\beta_k S_{k,t})^2}{4\sigma_\varepsilon^2} \right\}$$

I obtain the normalized conditional density:

$$P(q_t|m_{t-1}, m_{t+1}, q_{t+1}) = \frac{\exp \left\{ -\frac{(m_{t-1}-m_{t+1}+\tilde{\varphi}V_{t+1}q_{t+1}+\beta_k S_{k,t+1}+\tilde{\varphi}V_t q_t+\beta_k S_{k,t})^2}{4\sigma_\varepsilon^2} \right\}}{\Gamma_t} \quad (25)$$

Now, consider the first term in equation (23),  $f(p_t|q_t, m_{t-1}, m_{t+1}, q_{t+1}) = f(m_t+cq_t|q_t, m_{t-1}, m_{t+1}, q_{t+1})$

$$\begin{aligned} &= f(m_t|q_t, m_{t-1}, m_{t+1}, q_{t+1}) \propto f(m_{t+1}|m_t, q_{t+1})f(m_t|m_{t-1}, q_t) \\ &= \exp - \left\{ \frac{(m_t-m_{t-1}-(\tilde{\varphi}V_t q_t+\beta_k S_{k,t}))^2}{2\sigma_\varepsilon^2} + \frac{(m_{t+1}-m_t-(\tilde{\varphi}V_{t+1}q_{t+1}+\beta_k S_{k,t+1}))^2}{2\sigma_\varepsilon^2} \right\} \end{aligned}$$

Simplifying:

$$f(p_t|q_t, m_{t-1}, m_{t+1}, q_{t+1}) \propto \exp \left\{ \frac{-[m_t-\frac{1}{2}(m_{t-1}+\tilde{\varphi}V_t q_t+\beta_k S_{k,t}+m_{t+1}-\tilde{\varphi}V_{t+1}q_{t+1}-\beta_k S_{k,t+1})]^2}{2\left(\sqrt{\frac{\sigma_\varepsilon^2}{2}}\right)^2} \right\}$$

or,

$$f(p_t|q_t, m_{t-1}, m_{t+1}, q_{t+1}) = \phi_{p_t-cq_t} \left( M_t^*, \sqrt{\frac{\sigma_\varepsilon^2}{2}} \right), \quad (26)$$

where  $M_t^* = \frac{1}{2}(m_{t-1} + \tilde{\varphi}V_t q_t + \beta_k S_{k,t} + m_{t+1} - \tilde{\varphi}V_{t+1}q_{t+1} - \beta_k S_{k,t+1})$ , and  $\phi$  denotes the normal pdf with respect to the random variable  $m_t$ .

Finally, the result follows from substituting equations (25) and (26) into equation (23).<sup>30</sup>

■

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<sup>30</sup>Note that we need some modifications for the endpoints. The following expressions provide the conditional densities for such points.

First point:

$$P(q_1|m_2, q_2, p_1) = \frac{\exp \left\{ \frac{-(p_1-cq_1-(m_2-\tilde{\varphi}V_2q_2-\beta_k S_{k,2}))^2}{2\sigma_\varepsilon^2} \right\}}{\exp \left\{ \frac{-(p_1+c-(m_2-\tilde{\varphi}V_2q_2-\beta_k S_{k,2}))^2}{2\sigma_\varepsilon^2} \right\} + \exp \left\{ \frac{-(p_1-c-(m_2-\tilde{\varphi}V_2q_2-\beta_k S_{k,2}))^2}{2\sigma_\varepsilon^2} \right\}}$$

Last point:

$$P(q_T|m_{T-1}, q_{T-1}, p_T) = \frac{\exp \left\{ \frac{-(p_T-cq_T-(m_{T-1}+\tilde{\varphi}V_T q_T+\beta_k S_{k,T}))^2}{2\sigma_\varepsilon^2} \right\}}{\exp \left\{ \frac{-(p_T+c-(m_{T-1}+\tilde{\varphi}V_T q_T+\beta_k S_{k,T}))^2}{2\sigma_\varepsilon^2} \right\} + \exp \left\{ \frac{-(p_T-c-(m_{T-1}+\tilde{\varphi}V_T q_T+\beta_k S_{k,T}))^2}{2\sigma_\varepsilon^2} \right\}}$$