

Robust Monetary Policy with the Consumption-Wealth Channel

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Abstract

This paper studies how a central bank's concerns about model uncertainty affects monetary policy in the Blanchard-Yaari framework with sticky prices. First, I present analytical solutions by assuming that all exogenous disturbances are white noise. It is shown that an increased preference for robustness and stronger wealth effects imply more aggressive policy responses to cost shocks. In addition, under white noise shocks, the monetary policy design problem in the overlapping generations model is isomorphic to the same problem in the standard New Keynesian framework. Then, I use numerical methods to study the case of persistent shocks, in which expectations play a role in the dynamics of the model. Numerically, it is shown that an increased preference for robustness continues to imply more aggressive responses to cost shocks. By contrast, stronger wealth effects lead to less aggressive responses. In some cases, the central bank may even reduce interest rates responding to cost shocks.

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1 Introduction

The lack of agreement on the most plausible model of the economy and the inherent uncertainty about how the economy actually works leads monetary authority to design policies that aim to be effective even in worst-case situations, where the model adopted by the central bank is no longer valid. Robust rules are designed to avoid poor responses of monetary policy in these extreme cases, where misspecification is present.

The literature on the design of such rules is growing rapidly, especially the branch related to the use of control theory, advanced by Hansen & Sargent (2004). Giordani & Söderlind (2004) and Leitimo & Söderstorm (2004, 2005) are examples of some recent contributions on the robust control approach to monetary policy.

Robust policies, though, depend on a *reference model*, i.e., a model that the policymaker believes to be the most likely description of an economic system. The reference model can vary over time and across countries, featuring

additional channels of monetary policy transmission. One potentially important additional channel is the Consumption-Wealth Channel which allows asset prices to impact real activity.

The views on the importance of asset prices for consumption vary widely. For instance, Ludvigson & Steindel (1999) espouse the view that wealth effects on consumption for the US are unstable and measured with a great deal of uncertainty. In addition, Ludvigson et al. (2002) show that the consumption-wealth channel plays a minor role in the propagation of monetary policy in the US, though their results depend on some identifying assumptions and are sensitive to the econometric model used in the analysis.

On the other hand, Bertaut (2002) and Ludwig & Sløk (2002) point to significant impacts of equity and housing prices on consumption. These studies highlight the differences in the size of the wealth effect between market-based financial systems, being the US the leading example, and bank-based financial systems, common in continental Europe. The empirical evidence in these studies show that the stock market effect is stronger in economies with market-based financial systems. Finally, Altissimo et al. (2005) survey the empirical evidence in the literature and conclude that, in general, empirical studies reveal a statistically significant relationship between wealth and consumption. However, the strength of this relationship differ markedly across countries.

In summary, the empirical evidence suggests, though not unanimously, that the wealth effect on consumption is important. Moreover, its size seems to vary over time in a given country, as documented by Ludvigson & Steindel (1999); and across economies, as shown by Bertaut (2002) and Ludwig & Sløk (2002). For this reason, the interaction between robustness and the size of wealth effects is an important issue for the understanding of monetary policy effectiveness across different economic environments.

This paper studies how a central bank's concerns about model uncertainty affects the design of monetary policy in an overlapping generations model in which monetary policy can be transmitted via the consumption-wealth channel. I am able to find analytical solutions to the robust control problem by assuming a white noise structure to all exogenous disturbances.

I find the usual result in the literature concerning the use of robust control in closed economies, according to which increased preference for robustness implies more aggressive policy responses to cost shocks but not to shocks to the natural level of output. In addition, the output gap, interest rate and equity prices become more sensitive to cost shocks.

I also study how the size of wealth effects impacts the aggressiveness result. I find that monetary policy responses to cost shocks are even more aggressive if the consumption-wealth channel has an important role in the transmission mechanism.

Analytically, it is shown that the monetary policy design problem in the Blanchard-Yaari model is isomorphic to the same problem in a simple New Keynesian closed economy with an aggregate demand equation based on the Euler equation of a representative agent with logarithmic preferences, more volatile cost shocks and a steeper output gap-inflation trade-off.

All the results above depend crucially on the assumption that all shocks are white noise. Closed-form solutions for the case of persistent shocks are hard to get; therefore I use numerical methods to solve the model. Numerical results show that, under persistent shocks, an increased preference for robustness also leads to more aggressive responses to cost shocks. In contrast, stronger wealth effects are associated with less aggressive responses. For some parameter values, the central bank may even reduce interest rates responding to cost shocks. This last finding depends on the importance of output stabilization for the central bank. If there is a medium to high degree of preference for output stabilization, the central bank may be forced to reduce interest rates to minimize the effects of a plunge in asset prices, caused by cost shocks, in aggregate demand.

The paper is organized in three additional sections. In section two, I present a version of the Blanchard-Yaari model of overlapping generations, developed by Nisticò (2005), featuring nominal rigidities. Section three, following Leitemo & Söderstorm (2004, 2005), derives closed-form solutions to the optimal robust policy under discretion. I also present, in this section, numerical results, based on procedures outlined in Giordani & Söderlind (2004), concerning the case of persistent shocks. Section four concludes and provides additional discussion on the results.

2 The Blanchard-Yaari Model of Overlapping Generations with Sticky Prices

This section presents a discrete-time version of the perpetual youth model based upon Blanchard(1985) and Yaari(1965) with sticky prices. In the model, monetary policy can be transmitted through the Consumption-Financial Wealth Channel, since the Euler equation for aggregate consumption depends upon aggregate financial wealth.

The Blanchard-Yaari framework has been used to address important issues in Monetary Economics. Piergallini (2003) studies optimal monetary policy in a model with real balance effects. Annicchiarico, Marini and Piergallini(2004) study the interaction between fiscal and monetary policy in a variant of Piergallini(2003). Finally, Nisticò(2005) studies the role of monetary policy for stock-price dynamics in a model without any trade-off between inflation and output gap stabilization, since cost shocks are absent.

In addition to the Blanchard-Yaari model, discrete-time versions of the closely related specification due to Weil(1991) have been used by Ireland(2004) and Benassy(2003) to study liquidity traps and liquidity effects.

The model I am about to present is a simplified version of Nisticò(2005) with cost shocks and shocks to the natural level of output. In the model, there is a trade-off between inflation and output gap stabilization, which makes the monetary policy design problem non-trivial.

2.1 Population Structure and Households

At a given time t , a new generation of consumers, with uncertain lifetimes, is born. Let γ be the probability of dying before next period begins.

The size of cohort s at time t is given by $n_{s,t} = n(1-\gamma)^{t-s}$. In each generation, there is a continuum of consumers, indexed by j , uniformly distributed in the interval $[0, 1]$. So, the aggregate population can be computed as

$$n_t = \sum_{s=-\infty}^t n(1-\gamma)^{t-s} \int_0^1 dj = \frac{n}{\gamma}$$

Assuming zero population growth, n_t can be normalized to 1. Therefore, $n = \gamma$ is the size of a new generation born at time t . Since population is constant, a fraction of equal size is dying.

I consider a cashless economy in which consumer j belonging to a generation born at s faces the following optimization problem:

$$\text{Max } E_0 \sum_{t=0}^{\infty} \beta^t (1-\gamma)^t [\log(C_t(s, j)) + \kappa \log(1 - L_t(s, j))]$$

subject to the following constraints:

$$P_t C_t(s, j) + B_t(s, j) + P_t \int_0^1 Q_t(i) Z_t(s, j, i) di \leq W_t(s, j) L_t(s, j) + \omega_t(s, j) \quad (1)$$

$$L_t(s, j) = \left(\frac{W_t(s, j)}{W_t(s)} \right)^{-\eta_t} L_t(s) \quad (2)$$

The choice variables are consumption $C_t(s, j)$, labor $L_t(s, j)$, a risk free bond whose nominal value is $B_t(s, j)$ and a portfolio of shares issued by a continuum of firms indexed by i , $Z_t(s, j, i)$, whose real price is $Q_t(i)$.

$W_t(s, j)$ is nominal wage and $\omega_t(s, j)$ is the amount of financial wealth belonging to consumer j from generation born at s .

β is the subjective discount factor and κ is a preference parameter.

Households have some market power in the labor market. Under monopolistic competition, there are firms able to bundle a cohort specific labor in order to maximize the following criterion:

$$W_t(s) L_t(s) - \int_0^1 W_t(s, j) L_t(s, j) dj$$

subject to the technology for bundling labor inputs $L_t(s, j)$:

$$L_t(s) = \left[\int_0^1 L_t(s, j)^{\frac{\eta_t-1}{\eta_t}} dj \right]^{\frac{\eta_t}{\eta_t-1}}.$$

Since wages are flexible and labor market is not segmented by cohort, there is just one single wage, therefore $W_t(s) = W_t$.

The aggregate wage is given by : $W_t = [\int_0^1 W_t(s, j)^{1-\eta_t} dj]^{\frac{1}{1-\eta_t}}$.

Equation (2) is the demand for labor provided by consumer j and used as input by the bundler to produce $L_t(s)$.

Since there is no bequest motive and lifetime is uncertain, there is a life insurance market that redistributes among those who survived, in a given cohort, the financial wealth of the members who died. The zero profit condition in the competitive insurance market implies a gross return of $\frac{1}{1-\gamma}$, therefore financial wealth is given by

$$\omega_t(s, j) = \frac{1}{1-\gamma} [B_{t-1}(s, j)R_{t-1} + P_t \int_0^1 (Q_t(i) + D_t(i))Z_{t-1}(s, j, i)di] \quad (3)$$

$D_t(i)$ is the dividend paid by the share and R_{t-1} is a risk-free interest rate. First order conditions give the following equations:

$$\frac{W_t}{P_t} = \kappa(1 + \mu_t^w) \left[\frac{C_t(s, j)}{1 - L_t(s, j)} \right] \quad (4)$$

$$R_t E_t[\Lambda_{t,t+1}(s, j)] = 1 \quad (5)$$

$$Q_t(i)P_t = E_t[\Lambda_{t,t+1}(s, j)P_{t+1}(Q_{t+1}(i) + D_{t+1}(i))] \quad (6)$$

Since all consumers make the same decision concerning labor supply, wages are equalized across individuals and are given by $W_t(s, j) = W_t(s) = W_t$. Therefore, I use aggregate wages W_t in equation (4). Furthermore, following Clarida, Gali and Gertler(2002), $\mu_t^w = \frac{1}{\eta_t - 1}$ is the wage markup, which can vary exogenously and is going to be the source of cost shocks in the Phillips Curve.

In equations (5) and (6), $\Lambda_{t,t+1}(s, j)$ denotes the stochastic discount factor:

$$\Lambda_{t,t+1}(s, j) = \beta \frac{P_t C_t(s, j)}{P_{t+1} C_{t+1}(s, j)} \quad (7)$$

The next equation defines human wealth $H_t(s, j)$, for individual j from cohort s :

$$H_t(s, j) = E_t \sum_{k=0}^{\infty} \Lambda_{t,t+k}(s, j) (1-\gamma)^k W_{t+k}(s, j) L_{t+k}(s, j) \quad (8)$$

Using equations(1),(3),(6), (7), human wealth defined in equation (8) and the transversality condition $\lim_{k \rightarrow \infty} E_t[\Lambda_{t,t+k}(s, j)(1-\gamma)^k W_{t+k}(s, j)] = 0$, it can be shown that nominal consumption is a linear function of financial and human wealth¹:

$$P_t C_t(s, j) = \varphi[\omega_t(s, j) + H_t(s, j)] \quad (9)$$

where $\varphi = 1 - \beta(1 - \gamma)$ is a constant.

¹Equation (9) is derived in details by Piergallini(2003) and Nisticò(2005).

2.2 Aggregation

The aggregate value of any macroeconomic variable is given by a weighted average of each consumer j in each generation s . For the computation of the average, I use the size of each generation and the distribution of agents belonging to generation s .

In fact, if $x_t(s, j)$ stands for any variable associated with individual j in a given generation s , the aggregate variable X_t is given by

$$X_t = \sum_{s=-\infty}^t \gamma(1-\gamma)^{t-s} \int_0^1 x_t(s, j) dj$$

Applying the aggregator above to expressions (4),(5),(6),(1) and (9) yields the following aggregate expressions:

$$\frac{W_t}{P_t} = \kappa(1 + \mu_t^w) \left[\frac{C_t}{1 - L_t} \right] \quad (10)$$

$$R_t E_t[\Lambda_{t,t+1}] = 1 \quad (11)$$

$$Q_t(i)P_t = E_t[\Lambda_{t,t+1}P_{t+1}(Q_{t+1}(i) + D_{t+1}(i))] \quad (12)$$

$$P_t C_t + B_t + P_t \int_0^1 Q_t(i) Z_t(i) di \leq W_t L_t + \omega_t \quad (13)$$

$$P_t C_t = \varphi[\omega_t + H_t] \quad (14)$$

The aggregate wealth is given by

$$\omega_t = B_{t-1}R_{t-1} + P_t \int_0^1 (Q_t(i) + D_t(i)) Z_{t-1}(i) di \quad (15)$$

The next step is to get the Euler equation for aggregate consumption by combining equations (12) to (15).

After some algebra², the Euler equation for aggregate consumption is given by

$$P_t C_t = \frac{1}{\beta} E_t[\Lambda_{t,t+1} P_{t+1} C_{t+1}] + \frac{\varphi\gamma}{(1-\gamma)} E_t[\Lambda_{t,t+1} \omega_{t+1}] \quad (16)$$

Equation (16) has the same form as the Euler equation for individual households except for the correction term due to wealth effects. In fact, the turnover of generations engender distributional effects. Since all generations face the same interest rate, consumption dynamics at the cohort level is the same and is governed by equations (5) and (7). Nevertheless, older generations are wealthier and, because of that, have higher consumption levels than younger generations. Since newborns are continuously replacing members of older generations,

²See Piergallini(2003) and Nisticò(2005) for a step-by-step derivation of equation (16).

aggregate consumption dynamics differ from individual consumption because newborns hold no financial wealth.

2.3 Firms and Price-Setting Behavior

The final good is produced by a representative firm which chooses intermediate goods $Y_t(i)$ as inputs in order to maximize profits given by

$$P_t Y_t - \int_0^1 P_t(i) Y_t(i) di$$

subject to the following production function:

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\nu-1}{\nu}} di \right]^{\frac{\nu}{\nu-1}}.$$

The solution of the profit maximization problem yields expressions for the input demand function and the selling price for the final good.

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\nu} Y_t$$

$$P_t = \left[\int_0^1 P_t(i)^{1-\nu} di \right]^{\frac{1}{1-\nu}}. \quad (17)$$

The economy has a continuum of monopolistic firms, uniformly distributed in the interval $[0, 1]$. Each firm produces a differentiated intermediate product i , using the linear production function:

$$Y_t(i) = A_t L_t(i) \quad (18)$$

where A_t is an aggregate technology shock and $L_t(i)$ stands for labor input.

Each firm participates in a competitive labor market and demands labor input such that total real costs $\frac{W_t}{P_t} L_t(i)$ are minimized subject to (18). Real marginal costs are given by $MC_t = \frac{W_t}{P_t A_t}$.

In addition to the level of labor input, each firm chooses the selling price $P_t(i)$ for product i . Following Calvo(1983), in each period firms adjust their prices with a constant and exogenous probability $(1 - \phi)$. With probability ϕ , a particular firm does not receive the green light to adjust its price, then it will charge the last period price.

If a given firm receives the green light, it will set its price optimally in order to maximize profits, taking into account that the chosen price will not change until the firm receives another green light.

The optimization problem is

$$\text{Max}_{P_t(i)} E_t \left\{ \sum_{k=0}^{\infty} \Lambda_{t,t+k} \phi^k \left[\frac{P_t(i)}{P_{t+k}} Y_{t+k}(i) - \frac{W_{t+k}}{P_{t+k}} L_{t+k}(i) \right] \right\}$$

The first order condition for the solution implies the following expression for the optimal price:

$$P_t^* = \left(\frac{\nu}{\nu-1}\right) E_t \left\{ \sum_{k=0}^{\infty} \frac{\phi^k \Lambda_{t,t+k} Y_{t+k} P_{t+k}^{\nu-1}}{E_t \left[\sum_{k=0}^{\infty} \phi^k \Lambda_{t,t+k} Y_{t+k} P_{t+k}^{\nu-1} \right]} P_{t+k} MC_{t+k} \right\} \quad (19)$$

All firms end up choosing the same price, so I drop the index i . The pricing rule is a constant mark-up over a weighted average of expected future nominal marginal costs.

A fraction $(1-\phi)$ of all firms chooses P_t^* while a fraction ϕ keeps prices constant at the same level charged in the previous period; therefore, the aggregate price level, using equation (17), can be written as:

$$P_t^{1-\nu} = (1-\phi)P_t^{*1-\nu} + \phi P_{t-1}^{1-\nu} \quad (20)$$

2.4 Equilibrium

Market Clearing Conditions for bonds, equities and final goods are

$$B_t = \sum_{s=-\infty}^t \gamma(1-\gamma)^{t-s} \int_0^1 B_t(s, j) dj = 0$$

$$Z_t(i) = \sum_{s=-\infty}^t \gamma(1-\gamma)^{t-s} \int_0^1 Z_t(s, j, i) dj = 1$$

$$Y_t = C_t$$

I define aggregate dividends and the equity price as

$$D_t = \int_0^1 D_t(i) di$$

$$Q_t = \int_0^1 Q_t(i) di$$

Imposing market clearing conditions and remembering that $L_t = \frac{Y_t}{A_t}$, I have the following equations, based on expressions (10), (12), (13) and (15):

$$\frac{W_t}{P_t} = \kappa(1 + \mu_t^w) \left[\frac{Y_t}{1 - \left(\frac{Y_t}{A_t}\right)} \right] \quad (21)$$

$$Q_t P_t = E_t[\Lambda_{t,t+1} P_{t+1} (Q_{t+1} + D_{t+1})] \quad (22)$$

$$\omega_t = P_t Q_t + P_t D_t \quad (23)$$

$$P_t Y_t = W_t \frac{Y_t}{A_t} + P_t D_t \quad (24)$$

Using (21) and (22), $E_t[\Lambda_{t,t+1}\omega_{t+1}] = P_t Q_t$. Accordingly, I have a new version for the aggregate Euler Equation (16), which explicitly links equity prices and aggregate consumption.

$$P_t Y_t = \frac{1}{\beta} E_t[\Lambda_{t,t+1} P_{t+1} Y_{t+1}] + \frac{\varphi\gamma}{(1-\gamma)} Q_t \quad (25)$$

Finally, equilibrium is characterized by a demand and a supply block of equations.

Demand Block: equations (11),(21),(22),(24) and (25).

Supply Block: equations (19) and (20).

2.5 Equilibrium under Flexible Prices

In the flexible price equilibrium there is no distortions affecting prices and wages. Therefore, all firms set prices at the optimal level without nominal rigidity and there is no cyclical distortions in the labor market, so μ_t^w is set at its average level ($\overline{\mu^w}$).

I use the superscript n , which stands for natural level, to denote variables under flexible prices.

Under flexible prices: $P_t(i) = P_t$ and real marginal costs are constant $MC_t^n = \frac{\nu-1}{\nu}$. This result comes from equation (17) and (19).

The natural level of output, using the definition of real marginal costs and equation (21), is:

$$Y_t^n = \frac{A_t}{\frac{\nu}{\nu-1}\kappa(1 + \overline{\mu^w}) + 1}$$

An expression for dividends under flexible prices, using equation (24), is

$$D_t^n = \frac{Y_t^n}{\nu}$$

2.6 The log-linear version of the model

The first step before log-linearization is to find the steady state for the system of equations given by expressions (11),(21),(22),(24) and (25). Steady state magnitudes are written without time subscript.

From equation (25), I have the following steady state condition:

$$\beta = \frac{1}{R} \left[\frac{\beta\gamma\varphi}{1-\varphi} \left(\frac{Q+D}{Y} \right) + 1 \right] \quad (26)$$

Defining the new variable ψ as

$$\psi = \frac{\beta\gamma\varphi}{1-\varphi} \left(\frac{Q+D}{Y} \right)$$

After that, the steady state version of (25) takes the form

$$\beta = \frac{1}{R}(1 + \psi)$$

Defining a new subjective discount factor $\tilde{\beta} = \frac{\beta}{(1+\psi)}$, I get

$$\tilde{\beta} = \frac{\beta}{(1 + \psi)} = \frac{1}{R}$$

From (11) and (22), $\frac{Q+D}{QR} = 1$, or equivalently,

$$\frac{D}{QR} = 1 - \tilde{\beta} = 1 - \frac{1}{R} \quad (27)$$

In steady state, output and dividends are:

$$Y = \frac{\bar{A}}{\frac{\nu}{\nu-1}\kappa(1 + \mu^w) + 1} \quad (28)$$

$$D = \frac{Y}{\nu} \quad (29)$$

Equations (26) to (29) can be solved for Y , D , Q and R . In the sequence, all remaining endogenous variables are computed.

Log-linearizing equations (11),(21),(22),(24) and (25) around the steady state defined by equations (26) to (29) yields

$$w_t - p_t = y_t + \chi(y_t - a_t) + u_t \quad (30)$$

$$q_t = \tilde{\beta}E_t(q_{t+1}) + (1 - \tilde{\beta})E_t(d_{t+1}) - [r_t - E_t(\pi_{t+1})] \quad (31)$$

$$d_t = \frac{Y}{D}y_t - \frac{\nu - 1}{\nu} \frac{Y}{D}(y_t - a_t + w_t - p_t) \quad (32)$$

$$y_t = \frac{\psi}{1 + \psi}q_t + \frac{1}{1 + \psi}E_t(y_{t+1}) - \frac{1}{1 + \psi}[r_t - E_t(\pi_{t+1})] \quad (33)$$

Lowercase letters denote the difference between a given economic variable and its steady state, both measured in logarithms. The variable u_t is the log-linear approximation to $1 + \mu_t^w$ and π_t represents inflation. In addition, $\chi = \frac{L}{1-L}$ where $L = \frac{Y}{A}$.

The log-linear approximation for real marginal costs, $mc_t = w_t - p_t - a_t$, from (30), is

$$mc_t = (1 + \chi)(y_t - a_t) + u_t \quad (34)$$

After defining $\mu = \frac{\nu}{\nu-1}$, combine (29) and (34), to express (32) as a function of real marginal costs:

$$d_t = y_t + \frac{1}{1-\mu} mc_t \quad (35)$$

Using (35) to write (31) as

$$q_t = \tilde{\beta} E_t(q_{t+1}) + (1 - \tilde{\beta}) E_t(y_{t+1}) + \frac{(1 - \tilde{\beta})}{(1 - \mu)} E_t(mc_{t+1}) - [r_t - E_t(\pi_{t+1})] \quad (36)$$

The log-linear approximation to the natural level of output is $y_t^n = a_t$.³

Defining the output gap as $x_t = y_t - y_t^n$, imposing the market clearing condition for final goods $y_t = c_t$ and using expressions (34) and (35) to write (36) and (33) as

$$\begin{aligned} q_t = & \tilde{\beta} E_t(q_{t+1}) + (1 - \tilde{\beta}) \left[1 + \frac{(1 + \chi)}{(1 - \mu)} \right] E_t(x_{t+1}) - [r_t - E_t(\pi_{t+1})] \quad (37) \\ & + (1 - \tilde{\beta}) \left[1 + \frac{1}{(1 - \mu)} \right] E_t(u_{t+1}) \end{aligned}$$

$$x_t = \frac{\psi}{1 + \psi} q_t + \frac{1}{1 + \psi} E_t(x_{t+1}) - \frac{1}{1 + \psi} [r_t - E_t(\pi_{t+1})] + \frac{1}{1 + \psi} E_t(y_{t+1}^n) - y_t^n \quad (38)$$

Log-linearizing (19) and (20) yields the New Keynesian Phillips Curve given by

$$\pi_t = \tilde{\beta} E_t(\pi_{t+1}) + \frac{(1 - \phi)(1 - \phi\tilde{\beta})}{\phi} mc_t$$

Finally, using (34) and the output gap definition, the Phillips Curve can be written as

$$\pi_t = \tilde{\beta} E_t(\pi_{t+1}) + \frac{(1 - \phi)(1 - \phi\tilde{\beta})}{\phi} (1 + \chi) x_t + \frac{(1 - \phi)(1 - \phi\tilde{\beta})}{\phi} u_t \quad (39)$$

Equations (37), (38) and (39) define a linear rational expectations model, describing approximately the Blanchard-Yaari artificial economy.

3 The Robust Monetary Policy Design Problem

The lack of agreement on the most plausible model of the economy and the inherent uncertainty about how the macroeconomy really works have stimulated

³In general, the natural level of output may involve different types of exogenous disturbances as in Nisticò (2005).

a body of research trying to characterize desirable policy rules in the presence of model uncertainty. The robust approach to monetary policy assumes that the central bank has a model believed to be a good description of reality, but has uncertainty aversion. If reality deviates from the model in a way hard to be described by probabilistic statements, the policymaker wants to avoid poor performances. Thus, the monetary policy design problem should take into account the effects of specification errors and try to be robust against them. Robust rules are therefore designed to avoid poor responses of monetary policy in worst-case situations.

Robust control techniques have been applied to design monetary policy rules robust to model uncertainty. The robust control approach is discussed in great detail by Hansen & Sargent (2004) and Giordani & Söderlind (2004). Leitemo & Söderstorm (2004, 2005) study optimal monetary policy, using the robust control approach, in the New Keynesian framework for closed and open economies, obtaining closed-form solutions by assuming that all shocks are white noise.

The central bank, under the robust control approach, aims at minimizing a loss function, considering a *reference model* which is a reasonable representation of the law of motion of the economy. The loss function is assumed to be quadratic and the model linear. The central bank realizes that the true description of the economy may deviate from the benchmark *reference model*. However, it cannot specify a probability distribution associated with these deviations. To model such very unstructured uncertainty, shock terms representing model misspecifications are appended to the equations defining the linear model. These disturbances are controlled by a fictitious *evil agent*.

The initial minimization problem, with the introduction of the evil agent, becomes a min-max problem. Since the evil agent is just a way to model the planner's cautionary behavior, he knows the planner reference model and loss function, which he wants to maximize. This formulation describes a two-person zero sum game. According to its preference for robustness, the central bank allocates a budget to the evil agent, which defines the set of alternative models, around the reference model, that the central bank cares about.

In mathematical terms, using the notation in Giordani & Söderlind (2004), the problem can be formulated as follows.

$$\underset{u_t}{Min} \underset{v_t}{Max} E_0 \sum_{t=0}^{\infty} \beta^t L(x_t, u_t)$$

subject to:

$$E_t x_{t+1} = A x_t + B u_t + C(v_{t+1} + e_{t+1})$$

$$E_0 \sum_{t=0}^{\infty} \beta^t v'_{t+1} v_{t+1} \leq \eta$$

where x_t is the state vector, u_t is the central bank control vector, e_{t+1} is a vector of zero mean, iid shocks with an identity covariance matrix and v_{t+1} is

the evil agent's control vector.

The evil agent's control vector is premultiplied by C , so random noise are masking to some extent the true law of motion of the economy. In addition, η represents the size of the budget allocated to the evil agent. The standard dynamic control problem under rational expectations sets $\eta = 0$, implying $v_{t+1} = 0$.

The equilibrium of the model is found by combining the linear law of motion with policy functions for u_t and v_{t+1} , which are the solution of the min-max problem described above. The literature has focused on two situations: the *worst-case model* and the *approximating model*. The worst-case model defines an equilibrium configuration where the central bank's fears are all materialized and is the equilibrium outcome against which the monetary policy response is designed. The approximating model is the outcome where the central bank sets policy and agents form expectations to reflect the effect of misspecification in the worst-case model, but in reality there is no misspecification, thus $v_{t+1} = 0$.

3.1 Solving the model analytically

This paper applies the robust control approach to an artificial economy in which monetary policy can be transmitted through the consumption-financial wealth channel. In what follows, equations (47) to (49) are cast in the min-max problem in order to derive the solution to the robust control problem. As in Leitimo & Söderstorm (2004, 2005) I search for closed-form solutions.

3.1.1 The Worst-Case Model

Assuming that all shocks are white noise, I can write the linear rational expectations model as:

$$q_t = \tilde{\beta}E_t(q_{t+1}) + (1 - \tilde{\beta})\alpha_1 E_t(x_{t+1}) - [r_t - E_t(\pi_{t+1})] \quad (40)$$

$$x_t = (1 - \alpha_2)q_t + \alpha_2 E_t(x_{t+1}) - \alpha_2[r_t - E_t(\pi_{t+1})] - \sigma_n \varepsilon_t^n \quad (41)$$

$$\pi_t = \tilde{\beta}E_t(\pi_{t+1}) + \alpha_3 x_t + \alpha_4 \sigma_u \varepsilon_t^u \quad (42)$$

where $\alpha_1 = 1 + \frac{(1+\chi)}{(1-\mu)}$, $\alpha_2 = \frac{1}{1+\psi}$, $1 - \alpha_2 = \frac{\psi}{1+\psi}$, $\alpha_3 = \frac{(1-\phi)(1-\phi\tilde{\beta})}{\phi}(1 + \chi)$ and $\alpha_4 = \frac{(1-\phi)(1-\phi\tilde{\beta})}{\phi}$.

I have to rewrite the cost shock and the shock to the natural level of output in such a way that ε_t^n and ε_t^u have unit variance.

The budget constraint for the evil agent is given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t [(v_t^n)^2 + (v_t^u)^2] \leq \eta$$

I assume that the central bank sets the short-term interest rate r_t to minimize a standard quadratic loss function⁴.

$$E_0 \sum_{t=0}^{\infty} \beta^t [\pi_t^2 + \lambda x_t^2]$$

The robust control problem can be formulated as follows.

$$\underset{r_t}{Min} \underset{v_t^n, v_t^u}{Max} E_0 \sum_{t=0}^{\infty} \beta^t [\pi_t^2 + \lambda x_t^2]$$

subject to:

$$q_t = \tilde{\beta} E_t(q_{t+1}) + (1 - \tilde{\beta}) \alpha_1 E_t(x_{t+1}) - [r_t - E_t(\pi_{t+1})] \quad (43)$$

$$x_t = (1 - \alpha_2) q_t + \alpha_2 E_t(x_{t+1}) - \alpha_2 [r_t - E_t(\pi_{t+1})] - \sigma_n (\varepsilon_t^n + v_t^n) \quad (44)$$

$$\pi_t = \tilde{\beta} E_t(\pi_{t+1}) + \alpha_3 x_t + \alpha_4 \sigma_u (\varepsilon_t^u + v_t^u) \quad (45)$$

$$E_0 \sum_{t=0}^{\infty} \beta^t [(v_t^n)^2 + (v_t^u)^2] \leq \eta \quad (46)$$

I assume that neither the central bank nor the evil agent has access to a commitment technology. Therefore, expectations are given and monetary policy is set under discretion.

I also assume that the central bank and the evil agent play a Nash game, consequently the optimal choice of each player is consistent with the optimal choice of his opponent.

Optimality conditions are:

$$x_t = -\frac{\alpha_3}{\lambda} \pi_t \quad (47)$$

$$v_t^u = \frac{\alpha_4 \sigma_u}{\theta} \pi_t \quad (48)$$

$$v_t^n = 0$$

The Lagrange multiplier on (46) is denoted by θ , known as the robustness parameter, which is inversely related to the evil agent's budget η . Therefore, a decreasing θ means an increasing preference for robustness. So $\eta = 0$ implies $\theta = \infty$ and in this case, robust monetary policy coincides with optimal monetary policy under discretion without model uncertainty.

⁴Woodford(2003) and Walsh(2003) show that the loss function can be obtained as a second order approximation of the representative agent's utility function, when $\gamma = 0$.

Based on the optimality conditions, I have the following results:

Result 1: With cost shocks, there exists a trade-off between inflation and output gap variability. This trade-off is not affected by the robustness parameter θ .

Result 2: Since cost shocks introduce a trade-off between inflation and output gap, the evil agent adds noise to these shocks. The amount of noise is an increasing function of inflation, the variance of the disturbance process ε_t^u ($\alpha_4\sigma_u$) and preference for robustness ($\frac{1}{\theta}$).

Result 3: Since shocks to the natural level of output do not introduce any trade-off between inflation and output gap, the evil agent does not add any noise to these shocks.

Since the central bank is able to offset the effects of shocks to the natural level of output and there is no persistence, the only relevant variable that can be used to add uncertainty to the system is ε_t^u .

The importance of cost shocks in generating a trade-off between inflation and output gap is discussed in Clarida, Gali and Gertler (1999). Results (1) to (3) are reported in Leitemo & Söderstorm (2004).

As there is no persistence in the model, the only state variables are the two shocks ε_t^n and ε_t^u and all expectations are zero, so closed-form solutions can be found.

Using (47) and (48) in (45) and imposing $E_t(\pi_{t+1}) = 0$, inflation is given by:

$$\pi_t = \frac{\lambda\sigma_u\alpha_4}{\lambda + \alpha_4^2 A_1} \varepsilon_t^u \quad (49)$$

where $A_1 = (1 + \chi)^2 - \frac{\lambda}{\theta}$

The expression for x_t can be obtained by substituting (49) in (47).

$$x_t = -\frac{\sigma_u(1 + \chi)\alpha_4^2}{\lambda + \alpha_4^2 A_1} \varepsilon_t^u \quad (50)$$

Finally, the noise introduced by the evil agent is $v_t^u = \frac{\lambda\sigma_u^2\alpha_4^2}{\theta\lambda + \alpha_4^2 A_1\theta} \varepsilon_t^u$.

This paper studies the case of small amounts of misspecification (large values for θ , very close to ∞). In this case, $\frac{\lambda\sigma_u\alpha_4}{\lambda + \alpha_4^2 A_1}$ and $\frac{\sigma_u(1 + \chi)\alpha_4^2}{\lambda + \alpha_4^2 A_1}$ are both positives. This consideration leads to the next result:

Result 4: For small amounts of misspecification, inflation increases and output gap declines after a positive cost shock.

Imposing $v_t^n = 0$ and setting expectations to zero in (43) and (44), I can solve for the interest rate and equity price.

$$r_t = \frac{(1 - \alpha_2)}{\alpha_2} q_t - x_t - \sigma_n \varepsilon_t^n = \frac{(1 - \alpha_2)}{\alpha_2} q_t + \frac{\alpha_3}{\lambda} \pi_t - \sigma_n \varepsilon_t^n$$

Using $q_t = -r_t$ from (43), I have expressions written in terms of the underlying shocks:

$$r_t = \frac{\sigma_u(1+\chi)\alpha_4^2}{\lambda + \alpha_4^2 A_1} \varepsilon_t^u - \sigma_n \varepsilon_t^n \quad (51)$$

$$q_t = -r_t = -\frac{\sigma_u(1+\chi)\alpha_4^2}{\lambda + \alpha_4^2 A_1} \varepsilon_t^u + \sigma_n \varepsilon_t^n \quad (52)$$

To summarize, I have

Result 5: The interest rate reacts to equity prices with a coefficient that depends upon α_2 and consequently is a function of the size of the wealth effect. In addition, inflation and shocks to the natural level of output enter the central bank reaction function. The reaction to inflation depends on the inflation-output gap trade-off and the reaction to shocks to the natural level of output is set in order to neutralize its effects on the economy.

Result 6: For small degrees of misspecification, the central bank responds to cost shocks by tightening monetary policy. Monetary Policy responses to cost shocks have a negative full impact on the output gap and equity prices, since

$$\left| \frac{\partial r_t}{\partial \varepsilon_t^u} \right| = \left| \frac{\partial q_t}{\partial \varepsilon_t^u} \right| = \left| \frac{\partial x_t}{\partial \varepsilon_t^u} \right| \text{ implies } \frac{\partial q_t}{\partial r_t} = \frac{\partial x_t}{\partial r_t} = -1.$$

3.1.2 The Approximating Model

The worst-case model discussed so far represents a situation where the central bank's worst fears about model uncertainty happens to be true. In the approximating model, the policy rule and expectations are in line with the central bank's fears about misspecification. However, in practice, misspecification is absent and the evil agent sets his control variables to zero.

The solution combines the optimal robust interest rate rule, equation (51) and the original linear rational expectations model, without the evil agent's control variables, given by equations (40) to (42).

The expressions for the output gap and equity price remain unchanged, since the evil agent does not introduce any noise in these equations and the interest rate rule is the same as in the worst-case model. Thus, equations (50) and (52) also describes the behavior of output gap and equity price under the approximating model.

Using equation (50) in the Phillips Curve (42) and setting $E_t(\pi_{t+1}) = 0$ yields an expression for inflation under the approximating model solution:

$$\pi_t = \left[\sigma_u \alpha_4 - \frac{\sigma_u(1+\chi)^2 \alpha_4^2}{\lambda + \alpha_4^2 A_1} \right] \varepsilon_t^u \quad (53)$$

In summary, I have

Result 7: The approximating model solution has no impact on output gap and equity price sensitivity to cost shocks and to shocks to the natural level of output but changes inflation behavior since the noise added by the evil agent is absent.

3.1.3 An Isomorphic Monetary Policy Problem

Inspecting optimality conditions associated with the robust monetary policy design problem and comparing with the ones obtained by Leitimo & Söderstorm (2004), it is easy to conclude that the robust monetary policy problem is isomorphic to the one related to a representative agent New Keynesian closed economy model, characterized by the following reference model:

$$x_t = E_t(x_{t+1}) - [r_t - E_t(\pi_{t+1})] - \sigma_n \varepsilon_t^n \quad (54)$$

$$\pi_t = \tilde{\beta} E_t(\pi_{t+1}) + \alpha_3 x_t + \alpha_4 \sigma_u \varepsilon_t^u \quad (55)$$

Equation (54) is the aggregate demand equation associated with a representative agent economy with logarithmic preferences. In fact, since all shocks are white noise, making expectations irrelevant for computing the equilibrium, an increase in the interest rate has two effects on the aggregate demand. A direct effect $-\alpha_2 r_t$ and an indirect effect via the wealth effect $-(1 - \alpha_2)r_t$, since $q_t = -r_t$. The IS equation (54) is such that it is taking into account both effects. Therefore, considering equity-price dynamics is not important for its impact has been already fully accounted for.

Equation (55) is a Phillips Curve with more volatile cost shocks and a steeper output gap-inflation trade-off than its counterpart in a representative agent model. To understand that fact, observe that the volatility of cost shocks depends on α_4 and the trade-off between inflation and output gap is a function of α_3 . Additionally, both α_3 and α_4 are increasing in γ . Consequently, they are bigger than their counterparts in the representative agent case, where $\gamma = 0$.

To establish this, just remember that $\tilde{\beta} = \frac{\beta}{(1+\psi)}$, $\alpha_3 = \frac{(1-\phi)(1-\phi\tilde{\beta})}{\phi}(1+\chi)$ and $\alpha_4 = \frac{(1-\phi)(1-\phi\tilde{\beta})}{\phi}$.

First, consider, respectively, the expressions for the size of the wealth effect on the Euler equation (*SWE*) and for ψ :

$$SWE = \frac{\varphi\gamma}{(1-\gamma)} = \frac{\gamma}{1-\gamma} - \beta\gamma \quad (56)$$

$$\psi = \frac{\beta\gamma\varphi}{1-\varphi} \left(\frac{Q+D}{Y} \right) = \beta \left(\frac{Q+D}{Y} \right) \frac{\gamma\varphi}{1-\varphi} = \beta \left(\frac{Q+D}{Y} \right) \left(\frac{\gamma}{\beta(1-\gamma)} - \gamma \right)$$

Combine the above equations to find that

$$\psi = \beta^2 \left(\frac{Q+D}{Y} \right) SWE \quad (57)$$

Taking the following derivative:

$$\frac{\partial SWE}{\partial \gamma} = \frac{1 - \beta(1-\gamma)^2}{(1-\gamma)^2} > 0 \quad (58)$$

since $0 < \beta < 1$ and $0 < \gamma < 1$.

Assuming positive real wealth ($Q + D$) in the steady state:

$$\frac{\partial \psi}{\partial \gamma} = \beta^2 \left(\frac{Q + D}{Y} \right) \frac{dSWE}{d\gamma} > 0 \quad (59)$$

$$\frac{\partial \tilde{\beta}}{\partial \gamma} = -\frac{\beta}{(1 + \psi)^2} \frac{d\psi}{d\gamma} < 0 \quad (60)$$

Finally,

$$\frac{d\alpha_3}{d\gamma} = -(1 + \chi)(1 - \phi) \frac{\partial \tilde{\beta}}{\partial \gamma} > 0 \quad (61)$$

$$\frac{d\alpha_4}{d\gamma} = -(1 - \phi) \frac{\partial \tilde{\beta}}{\partial \gamma} > 0 \quad (62)$$

I am now in the position to state the following result:

Result 8: The monetary policy design problem in the presence of wealth effects is isomorphic to the same problem in a simple representative agent New Keynesian closed economy with an aggregate demand equation implied by logarithmic preferences, more volatile cost shocks and a steeper output gap-inflation trade-off⁵.

This last result depends crucially on the assumption that all shocks are white noise. With persistent shocks, I cannot shut down all expectations and obtain $q_t = -r_t$. The impact of monetary policy on equity prices will depend upon additional effects working through expectations; therefore, there is no way to disregard equity-price dynamics and solve the robust policy problem considering only equations (54) and (55).

3.2 Further Analytical Results

3.2.1 The Effects of Robustness

Expressions for output gap, interest rate and equity price are the same in the worst-case model and in the approximating model. They take the form $x_t = -A_2 \varepsilon_t^u$, $r_t = A_2 \varepsilon_t^u - \sigma_n \varepsilon_t^n$ and $q_t = -A_2 \varepsilon_t^u + \sigma_n \varepsilon_t^n$, where $A_2 = \frac{\sigma_u(1+\chi)\alpha_4^2}{\lambda + \alpha_4^2 A_1}$ and $A_1 = (1 + \chi)^2 - \frac{\lambda}{\theta}$.

Since α_4 does not depend on θ , I have

$$\frac{\partial A_2}{\partial \theta} = \frac{\partial A_2}{\partial A_1} \frac{\partial A_1}{\partial \theta} = -\frac{\sigma_u(1+\chi)\alpha_4^2}{(\lambda + \alpha_4^2 A_1)^2} (\alpha_4^2) \left(\frac{\lambda}{\theta^2} \right) < 0$$

Remember that the analysis focus on small amounts of misspecification (large values for θ , very close to ∞) and a decreasing θ means an increasing preference for robustness. Thus, an increasing preference for robustness implies an increasing coefficient A_2 .

⁵This result resembles the findings in Clarida, Gali and Gertler (2001), showing that the optimal monetary policy for a small open economy is, under certain conditions, isomorphic to the same problem for a closed economy.

I am now in the position to present the next result:

Result 9: An increased preference for robustness makes output gap, interest rate and equity price more sensitive to cost shocks; therefore, more volatile. The increased volatility due to cost shocks is the same in the worst-case solution and in the approximating solution.

Inflation, in the worst case model, is given by $\pi_t = A_3 \varepsilon_t^u$, where $A_3 = \frac{\lambda \sigma_u \alpha_4}{\lambda + \alpha_4^2 A_1}$.

Again, since α_4 does not depend on θ , I have

$$\frac{\partial A_3}{\partial \theta} = \frac{\partial A_3}{\partial A_1} \frac{\partial A_1}{\partial \theta} = -\frac{\lambda \sigma_u \alpha_4}{(\lambda + \alpha_4^2 A_1)^2} (\alpha_4^2) \left(\frac{\lambda}{\theta^2}\right) < 0$$

In summary,

Result 10: In the worst-case model, an increased preference for robustness makes inflation more sensitive to cost shocks; therefore, more volatile.

Inflation, in the approximating model, takes the form $\pi_t = A_4 \varepsilon_t^u$, where $A_4 = \sigma_u \alpha_4 - \frac{\sigma_u (1+\chi)^2 \alpha_4^2}{\lambda + \alpha_4^2 A_1}$.

The derivative for the coefficients A_4 is

$$\frac{\partial A_4}{\partial \theta} = \frac{\partial A_4}{\partial A_1} \frac{\partial A_1}{\partial \theta} = \frac{\sigma_u (1+\chi) \alpha_4^2}{(\lambda + \alpha_4^2 A_1)^2} (\alpha_4^2) \left(\frac{\lambda}{\theta^2}\right) > 0$$

To summarize, I have

Result 11: In the approximating model, an increased preference for robustness makes inflation less sensitive to cost shocks; therefore, less volatile.

These results are all in line with the findings in Leitemo & Söderstorm (2004). Concerning the effects of robustness, the presence of wealth effects does not change the qualitative results obtained by analyzing the basic New Keynesian framework.

The difference between results 10 and 11 are due to the action of the evil agent. The fear that cost shocks are more volatile leads the monetary authority to increase the interest rate and to contract the economy. Accordingly, inflation is curbed in the absence of the evil agent. In the worst-case model, the evil agent is, in reality, adding more noise. Thus, inflation tends to be more volatile in this situation.

3.2.2 The Role of Wealth Effects

Inspecting equations (37), (38) and (39), it is easy to see that the parameter ψ is governing the impact of equity prices on the economy. The parameter ψ measures the total size of the wealth effect in the linear rational expectations system. According to equation (57), ψ combines the direct strength of the wealth effect on consumption (*SWE*) with the financial wealth to GDP ratio in steady state, which can be interpreted as a measure of the size of the stock market wealth. So, market-based financial systems tend to have high values for ψ , as suggested by some empirical studies discussed in the introduction.

In addition, equation (56) establishes the connection between SWE and the probability of death γ . In the closed related model due to Weil (1991), the direct size of the wealth effect on consumption is a function of population growth, since in his model all consumers in each cohort have infinite lives. The point is that γ can have a broader meaning, capturing the turnover of consumers with different levels of wealth. In a more liberal interpretation, γ is a measure of demographic changes that impact the total amount of wealth in the economy.

Equations (59) and (62) show that α_4 is an increasing function in the total size of the wealth effect. In addition, the coefficient A_1 does not depend on α_4 . For this reason, to assess the effect of an increased strength of the wealth effect, it is sufficient to study how A_2 , A_3 and A_4 change when α_4 increases or decreases.

Computing the derivative of A_2 with respect to α_4 :

$$\frac{\partial A_2}{\partial \alpha_4} = \frac{\sigma_u(1+\chi)2\alpha_4\lambda}{(\lambda + \alpha_4^2 A_1)^2} > 0$$

I summarize this finding in result 12:

Result 12: A stronger wealth effect makes output gap, interest rate and equity price more sensitive to cost shocks; therefore, more volatile. The increased volatility due to cost shocks is the same in the worst-case solution and in the approximating solution.

The derivatives for the coefficients A_3 and A_4 are

$$\frac{\partial A_3}{\partial \alpha_4} = \frac{\lambda\sigma_u(\lambda - \alpha_4^2 A_1)}{(\lambda + \alpha_4^2 A_1)^2}$$

$$\frac{\partial A_4}{\partial \alpha_4} = \frac{\lambda^2 + (\alpha_4^2 A_1)^2 + 2\lambda\alpha_4(\alpha_4 A_1 - (1+\chi)^2)}{(\lambda + \alpha_4^2 A_1)^2}$$

Both derivatives are ambiguous. The first one, for a given amount of preference for robustness, depends on the relative importance of output stabilization vis-a-vis the variability of cost shocks. The second one depends on the variability of cost shocks and the slope of the Phillips Curve, since $(1+\chi)^2 = \left(\frac{\alpha_3^2}{\alpha_4^2}\right)$.

I can state the following result:

Result 13: The impact of a stronger wealth effect on inflation depends on particular parameter values in the worst-case solution and in the approximating solution.

In fact, the wealth effect in the Blanchard-Yaari framework does affect both demand and supply blocks. Indeed, equity prices play a role in aggregate demand. Moreover, the Phillips Curve is also affected by the wealth effect since firms use the aggregate stochastic discount factor, $\Lambda_{t,t+k}$, in order to discount future profits. As a result, since $\Lambda_{t,t+k}$ is an average over agents which gives more importance to newborns with no wealth and high marginal utility of consumption, shocks to marginal costs today have a greater impact compared to the representative agent case. This discounting behavior affects the slope of the Phillips Curve as well as the variability of cost shocks.

With a more favorable inflation-output gap trade-off, the central bank tends to act more aggressively, especially because it is known that cost shocks effects are short-lived. Thus, output gap and equity prices become more sensitive to cost shocks. The behavior of inflation, on the other hand, depends on the relative strength of demand and supply repercussions caused by changes in the size of the wealth effect.

3.3 Extension to the Case of Persistent Shocks

Now, I extend the analysis to the case of persistent shocks using numerical methods. I present results for the approximating model. The findings for the worst-case model under the chosen set for the parameter values are qualitatively the same. In what follows, I study a situation with low persistence ($\rho = 0.2$) and another with high persistence ($\rho = 0.9$). I use the same degree of persistence for both disturbances. I also set their variances to 1 ($\sigma_u = \sigma_n = 1$). Results do change quantitatively as a function of persistence, but their qualitative features remain the same.

I calibrate the artificial economy using standard parameters for the US economy: $\beta = 0.99$, $\phi = 0.75$, $\mu = 1.2$, $\chi = 0.5$ and $\lambda = 0.5$.

The solution to the monetary policy problem can be represented as follows. The shocks are governed by the laws of motion: $u_t = \rho u_{t-1} + \sigma_u \varepsilon_t^u$ and $y_t^n = \rho y_{t-1}^n + \sigma_n \varepsilon_t^n$. Each forward-looking variable can be written as a linear function of the state variables u_t and y_t^n according to the expressions: $q_t = a_q u_t + b_q y_t^n$, $x_t = a_x u_t + b_x y_t^n$, $\pi_t = a_\pi u_t + b_\pi y_t^n$ and $r_t = a_r u_t + b_r y_t^n$. I solve the model numerically, following the procedures described in Giordani & Söderlind (2004).

The numerical experiments are designed to study how the responses of the key macroeconomic variables to the shocks vary as a function of the preference for robustness ($\frac{1}{\theta}$) and the size of the wealth effect (ψ).

In what follows, I present, for a given parameterization, the coefficients in the linear rules connecting the forward-looking variables to the disturbances. In fact, variances and impulse responses are functions of these coefficients; therefore they fully characterize the dynamic behavior of the artificial economy studied.

3.3.1 Numerical Results on the Effects of Robustness

To isolate the effects of increased preference for robustness from wealth effects, I set $\psi = 0$. Figures 1 and 2 show the coefficients a_q , a_x , a_π , a_r , b_q and b_r as a function of $\frac{1}{\theta}$. As in the case of white noise disturbances, only cost shocks matter for inflation and the output gap; therefore $b_x = b_\pi = 0$.

In figures 1 and 2, the sensitivity of all variables to cost shocks are much stronger if cost shocks are very persistent. The coefficient b_r does not depend on $\frac{1}{\theta}$, since the evil agent sets $v^n = 0$, introducing noise into the system only via cost shocks ($v^u \neq 0$).

All key macroeconomic variables respond with more intensity to cost shocks when the preference for robustness increases. In particular, monetary policy

becomes more aggressive, since a_r is increasing in $\frac{1}{\theta}$. In addition, the degree of aggressiveness is an increasing function of persistence.

Results are qualitatively the same for alternative values concerning ρ and λ . Therefore, the aggressiveness result holds independently of the nature of the shocks.

Since $\psi = 0$, I am back to the basic New Keynesian framework analyzed by Leitimo & Söderstorm (2004). In figures 1 and 2, it is shown that the findings in their paper, concerning the aggressiveness result, carry over to the case of persistent shocks.

Summing up, numerical results show that the analytical results 4 and 9 are also valid when shocks are persistent. Furthermore, result 10 seems to apply also to the approximating model, at least for the parameter configurations analyzed.

3.3.2 Numerical Results on the Role of Wealth Effects

I present in figures 3 and 4, numerical results concerning how the coefficients a_q , a_x , a_π , a_r , b_q and b_r vary as a function of the wealth effect (ψ). For the low persistence case, equity prices, output gap and inflation become more sensitive to cost shocks. This seems to corroborate, for the case of persistent shocks, some statements summarized in result 12. However, the behavior of interest rates does not conform with result 12. In fact, interest rates are decreasing in ψ . In some cases, after ψ hits a threshold, the central bank reduces interest rates responding to cost shocks. This result seems a priori odd. But it is a plausible outcome, which depends on the importance of output gap stabilization for the monetary authority.

With persistent shocks, the effect of the shock itself and monetary policy actions taken today may have repercussions in the future. An adverse cost shock today, due to persistence, may ignite a string of adverse cost shocks, which, in turn, will have a negative impact on marginal costs and dividends, affecting, finally, equity prices. An aggressive response to cost shocks today may signal more interest rate hikes tomorrow and lower values for output gap. As a result expected dividends will be lower, leading to a plunge in equity prices.

Persistent cost shocks and aggressive monetary policy may depress the economy, through the effects of equity prices on aggregate demand, beyond the level compatible with the preference for output stabilization characterizing the central banker. Therefore, to avoid a big economic slowdown, catalyzed by the consumption-wealth effect, the central bank acts softly. In some cases, a reduction in interest rates is needed in order to counteract the negative effects of a plunge in asset prices in aggregate demand.

For the high persistence case, figure 4 shows that the reduction in interest rates is so strong that its impact on asset prices leads to less sensitivity to cost shocks for high values of ψ . In addition, the threshold for ψ triggering a reduction in interest rates after a cost shock is smaller if persistence is higher.

Figure 5 and 6 show the importance of persistence and preference for output stabilization in explaining a reduction of interest rate after a positive cost shock. In figure 5, shocks are white noise and the plot illustrate results 12 and 13. With

short-lived shocks, the propagation of its effects via expectations is absent, so monetary authority can take advantage of a more favorable output gap-inflation trade-off, induced by the wealth effect, in the Phillips Curve. In figure 6, the preference for output gap stabilization is low; therefore the central bank never reduces the interest rate and always fights cost shocks. However, monetary policy still becomes soft as ψ increases.

4 Conclusion

In a simple overlapping generations model with sticky prices, I study how optimal monetary policy is affected by the central bank's concerns about model uncertainty. The artificial economy considered allows monetary policy to be transmitted through the consumption-wealth channel. Consequently, equity prices become relevant for the behavior of the economy responding to monetary policy actions.

This essay analyzes how central banks's desire to be robust against model misspecification impact the behavior of the economy according to the importance of the size of the wealth effect. To be able to find closed-form solutions to the robust monetary policy problem, paralleling Leitemo & Söderstorm (2004), I assume initially that all shocks are white noise.

I find that the robust policy always respond aggressively to cost shocks than the non-robust policy. Consequently, inflation tend to be less volatile in the approximating model while output gap is more volatile. Preference for robustness does not affect the response to shocks to the natural level of output. In addition, equity prices become more sensitive to cost shocks and, as a consequence, more volatile under the robust policy.

Concerning the interaction between robustness and the size of wealth effects, I find that monetary policy responses to cost shocks are even more aggressive if the consumption-wealth channel has an important role in the transmission mechanism. Output gap and equity prices become more volatile but the effect on inflation is ambiguous.

I also show that the monetary policy design problem in the overlapping generations model is isomorphic to the same problem in a simple representative agent New Keynesian closed economy with the IS implied by logarithmic preferences, more volatile cost shocks and a steeper output gap-inflation trade-off.

All these findings depend on the assumption that all disturbances are white noise. The next logical step is to relax that assumption. Since closed-form solutions for the case of persistent shocks are hard to get, I use numerical methods to solve the model. Under persistent shocks, an increased preference for robustness also leads to more aggressive responses to cost shocks. In contrast, stronger wealth effects are associated with less aggressive responses.

If shocks are short-lived, the central bank can act more aggressively since the impact of monetary policy on equity prices and the output gap will not depend upon additional effects working through expectations. With persistent shocks, the effect of the shock itself and monetary policy actions taken today may have

repercussions in the future. An aggressive response to cost shocks today may signal even lower values for output gap and dividends tomorrow, leading to a plunge in equity prices, which, in turn, may depress the economy beyond the level compatible with the preference for output stabilization characterizing the central banker.

This paper can be extended in some interesting directions. For instance, it is possible to consider how endogenous persistence in inflation (hybrid Phillips Curve) and aggregate demand (habit persistence) would change the results presented here. In addition, issues concerning the importance of the gains from commitment when stock-price dynamics play a role in the transmission mechanism could be addressed using the framework discussed in the paper.

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Figure 1: $\psi = 0$ and $\rho = 0.2$

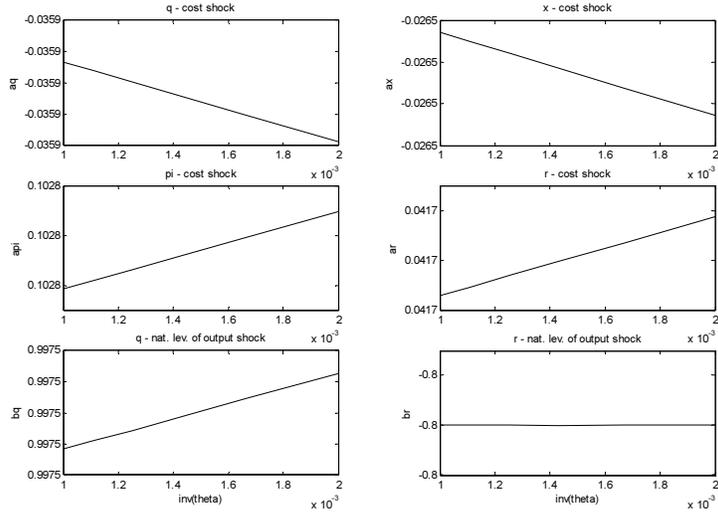


Figure 2: $\psi = 0$ and $\rho = 0.9$

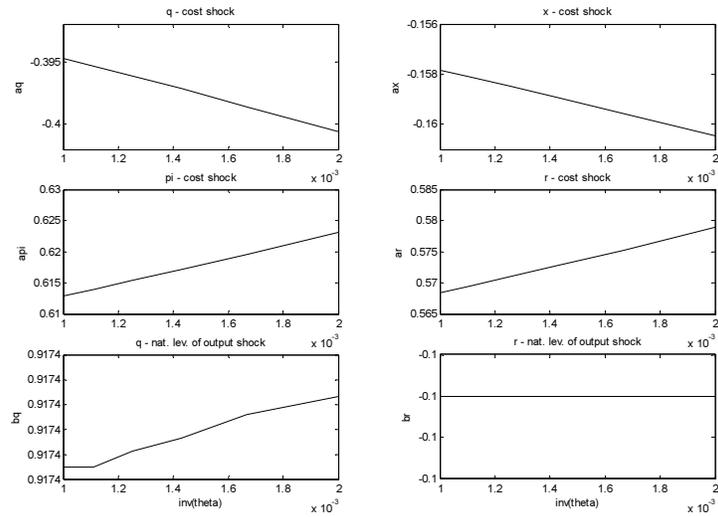


Figure 3: $\theta = 1000$ and $\rho = 0.2$

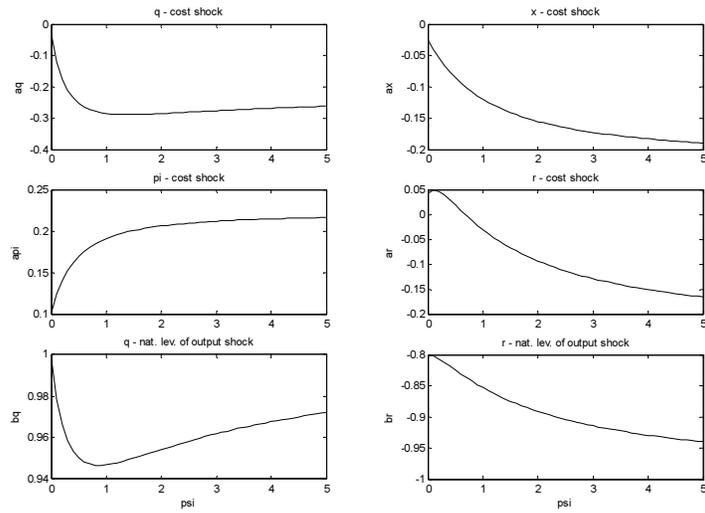


Figure 4: $\theta = 1000$ and $\rho = 0.9$

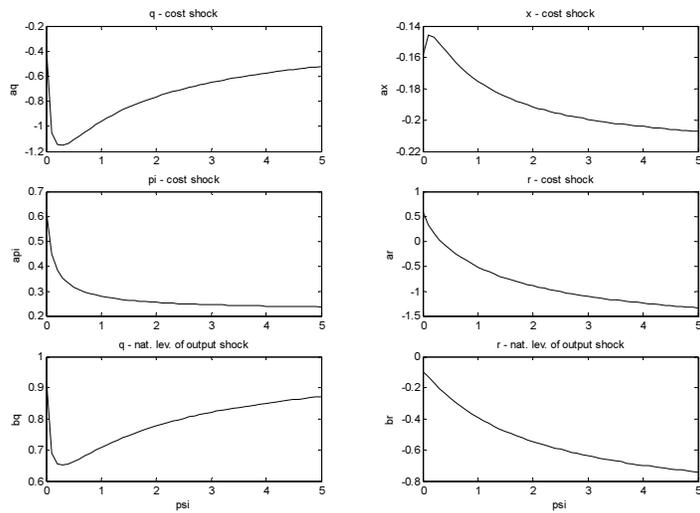


Figure 5: $\theta = 1000$ and $\rho = 0$

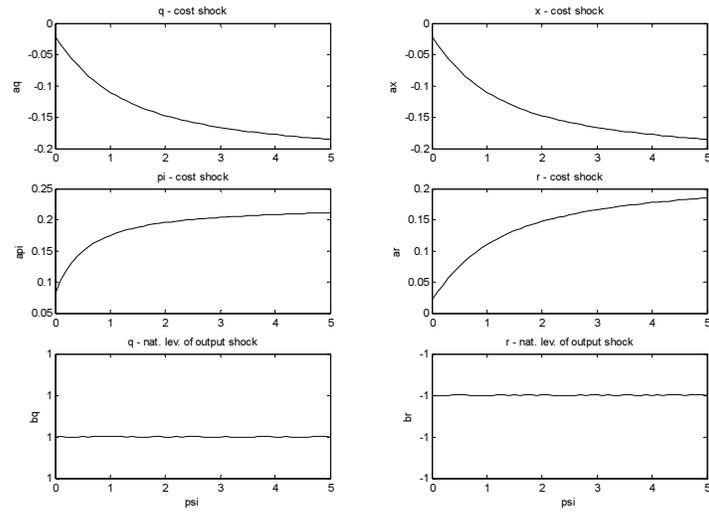


Figure 6:

Figure 7: $\theta = 1000$, $\rho = 0.9$ and $\lambda = 0.07$

