

Three Applications of the Marginal Cost of Public Funds Concept

Bev Dahlby
Department of Economics
University of Alberta
Edmonton Alberta
Canada, T6G 2H4

bev.dahlby@ualberta.ca

(April 2005)

Abstract

This paper illustrates the use of the marginal cost of public funds concept in three contexts. First, we extend Parry's (2003) analysis of the efficiency effects excise taxes in the U.K., primarily by incorporating the distortion caused by imperfect competition in the cigarette market and distinguishing between the MCFs for per unit and ad valorem taxes on cigarettes. Our computations show, contrary to the standard result in the literature, that the per unit tax on cigarettes has a slightly lower MCF than the ad valorem tax on cigarettes. Second, we calculate the MCF for a payroll tax in a labour market with involuntary unemployment, using the Shapiro and Stiglitz (1984) efficiency wage model as our framework. Our computations, based on Canadian labour market data, indicate that incorporating the distortion caused by involuntary unemployment raises the MCF by 25 to 50 percent. Third, we derive expressions for the distributionally-weighted MCFs for the exemption level and the marginal tax rate for a "flat tax", such as the one that has been adopted by the province of Alberta. This allows us to develop a restricted, but tractable, version of the optimal income tax problem. Computations indicate that the optimal marginal tax rate may be quite high, even with relatively modest pro-poor distributional preferences.

This paper illustrates the use of the marginal cost of public funds (MCF) concept in three contexts:

- the computation of the MCFs for excise taxes in the United Kingdom;
- the computation of the MCF for payroll taxes in a labour market with involuntary unemployment
- the derivation and computation of an optimal flat tax.

In the first application, we extend the recent paper by Parry (2003) by (a) distinguishing between consumption externalities (e.g. second-hand smoke from cigarettes) and public expenditure externalities (e.g. the increase in public health care costs caused by smoking), (b) incorporating the distortions caused by imperfect competition in the cigarette market, and (c) distinguishing between the MCFs for per unit and ad valorem taxes on cigarettes. Our base case calculations indicate that the MCFs for taxes on petrol and alcohol are 3.00 and 1.42 respectively. Incorporating the distortion caused by market power increases the MCF for the excise taxes on cigarettes, but these MCFs are still less than one and lower than the MCFs for the other excise taxes. The per unit excise tax on cigarettes has an MCF of 0.978, while the ad valorem tax on cigarettes is slightly higher at 0.981. These results contradict the standard result in the literature—ad valorem taxes are superior to per unit excise taxes in an imperfectly competitive market. The standard result is not necessarily valid when there are other distortions in the market because under these conditions social welfare may be improved by raising the price of the product, and a per unit tax is more effective in raising the price of the product than an ad valorem tax.¹

¹ A more detailed discussion of the conditions under which a per unit tax is superior to an ad valorem tax under imperfect competition is contained in Dahlby (2005, Chapter 3).

In the second application, we incorporate one of the most important labour market distortions, involuntary unemployment, in the measurement of the MCF. The MCF from taxing labour has been extensively studied.² However, none of the studies incorporates the distortion created by the involuntary unemployment in the calculation of the MCF. We use the Shapiro and Stiglitz (1982) efficiency wage model as a framework for modeling the effect of taxes on level of unemployment. Our analysis shows, using numerical values based on the Canadian labour market, that incorporating involuntary unemployment significantly increases the MCF for an employer payroll tax by 25 to 50 percent.

In the third application, we analyze the optimal “flat tax”, which is an income tax where all earnings above a given exemption level, X , are taxed a constant marginal tax rate, m . The flat tax is a progressive tax because the average tax rate increases with the taxpayer’s income if they earn more than the basic exemption level. The flat tax can be made more progressive, for a given tax yield, by increasing X and m . One reason for focusing on the flat tax is that it has been advocated by tax reformers such as Hall and Rabushka (1995) in the United States, and the flat tax has been adopted by the province of Alberta. However, the basic analytics of an optimal flat tax have not been fully articulated in the literature. In this section, we derive expressions for the distributionally-weighted marginal cost of funds from increasing the basic exemption level, $SMCF_X$, and for an increase in the marginal tax rate, $SMCF_m$. The optimal flat tax is characterized by the condition that $SMCF_X = SMCF_m$, and we use this condition to provide some insights into the structure of an optimal flat tax. However, like most optimal income tax

² See, for example, Stuart (1984), Mayshar (1991), and Kleven and Kreiner (2003).

problems, the optimality conditions are sufficiently complex that we need to simulate the model in order to fully appreciate their implications. We use the model to compute the optimal flat tax for a government that needs to raise the same revenues as a 20 percent proportional tax on earnings. Our computations indicate that the optimal exemption level would be relatively high (43 percent of earners would not pay the tax) and the optimal marginal tax rate would be over 40 percent even with relatively modest distributional objectives. It is hoped that future research will show how this approach can be generalized and used to evaluate multi-bracket optimal tax systems.

1.0 The MCFs for Excise Taxes in the U.K.

In a recent paper, Parry (2003) calculated the marginal excess burdens for excise taxes on petrol, alcoholic beverages, and cigarettes in the United Kingdom. In this section, we use Parry's benchmark parameter values and extend his analysis of the efficiency effects of excise taxation by:

- including commodity taxes on all other goods,
- distinguishing between consumption and public expenditure externalities,
- incorporating the distortions caused by imperfect competition in the cigarette market
- distinguishing between the MCFs for per unit and ad valorem taxes on cigarettes.

Our calculations are primarily meant to illustrate how these various factors can be incorporated in the calculation of the MCFs for excise taxes. Perhaps the most significant result is that they reveal that the importance of including the distortion caused

by imperfect competition in the cigarette industry in calculating the MCF from cigarette taxes.

The MCFs for the excise taxes are calculated in the context of a model where a representative individual allocates his expenditures among four commodities, the x_j s, and his time between work and leisure. We will define commodity 1 as petrol, commodity 2 as alcoholic beverages, commodity 3 as tobacco products, and commodity 4 as all other goods. His budget constraint is:

$$\sum_{j=1}^4 q_j x_j = (1 - \tau_w) w L \quad (1)$$

where q_j is the consumer price of commodity j , the after-tax wage rate is $(1 - \tau_w)w$, and L is amount of labour supplied. Since $L = T - x_0$, where x_0 is the individual's leisure time and T is total amount of time available for work or leisure, the budget constraint can also be written in the following form:

$$\sum_{j=0}^4 q_j x_j = q_0 T \quad (2)$$

where $q_0 = (1 - \tau_w)w$ is the "price" of leisure, and $q_0 T$ is the individual's total potential net earnings. Total tax revenues are equal to:

$$R = \sum_{j=1}^4 \tau_j q_j x_j + \tau_w w L + \sum_{j=1}^4 \tau_{\pi_j} \Pi_j = \sum_{j=0}^4 \tau_j q_j x_j + \tau_w w T + \sum_{j=1}^4 \tau_{\pi_j} \Pi_j \quad (3)$$

where τ_{π_j} is the tax rate on profit earned in the j th market, Π_j . The effective tax rate on leisure, $\tau_0 = -\tau_w/(1 - \tau_w)$, is negative, reflecting the fact that wage taxes reduce the opportunity cost of leisure.

The MCFs depend on the price sensitivity of the tax bases; the distortions in each market caused by taxes, externalities, and imperfect competition; and the interactions

between tax bases based on their substitutability or complementarity. All of these elements are incorporated in the following formula for the MCF for a tax on commodity i:

$$\text{MCF}_{t_i} = \frac{b_i(1 - \tau_{\pi_i}) + b_i \tau_{\pi_i} \frac{dq_i}{dt_i} - \sum_{j=0}^4 b_j (\delta_{E_j} + (1 - \tau_{\pi_j}) \delta_{M_j}) \epsilon_{ji} \frac{dq_i}{dt_i}}{b_i(1 - \tau_{\pi_j}) + b_i \tau_{\pi_i} \frac{dq_i}{dt_i} + \sum_{j=0}^4 b_j (\tau_j + \tau_{\pi_j} \delta_{M_j} - \delta_{G_j}) \epsilon_{ji} \frac{dq_i}{dt_i}} \quad (4)$$

where:

b_i is the budget share of commodity i, $q_i x_i / q_0 T$;

δ_{E_j} is the distortion in the market for commodity j caused by environmental externalities;

δ_{M_j} is the distortion in the market for commodity j caused by imperfect competition;

δ_{G_j} is the distortion in the market for commodity j caused by government expenditure externalities;

ϵ_{ji} is the (uncompensated) elasticity of demand for commodity j with respect to the price of commodity i;

τ_j is the ad valorem (equivalent) tax rate on commodity j;

dq_i/dt_i is the degree to which the per unit excise taxes, t_i , are shifted to consumers.

In order to calculate the MCFs for the excise taxes on petrol, alcoholic beverages, and cigarettes, we need values for the excise and profit tax rates, the budget shares of the commodities, the own and cross-price elasticities of demand for each of the commodities, the distortions caused by externalities and imperfect competition, and the degree of tax shifting. Each of these components is discussed below.

1.1 Tax Rates

$$\tau_w = 0.318$$

$$\tau = (-0.466, 0.863, 0.45, 0.823, 0.109)$$

$$\tau_{av_3} = 0.375 \text{ and } \tau_{pu_3} = 0.448 .$$

$$\tau_\pi = (0, 0.30, 0.30, 0.30, 0.30)$$

Parry used a value of 0.42 for τ_w in his calculation, based on a study of effective tax rates on labour by Mendoza, Razin and Tesar (1994). His measure of the effective tax rate on labour income included the value-added taxes on commodities in addition to income and payroll taxes. In our model, those taxes are reflected in the commodity tax rates, including the tax rate on commodity 4, and therefore they should not be included in our measure of τ_w . In our calculations, the average commodity tax rate is about 17.6 percent and therefore the wage tax rate that is equivalent to Parry's tax wedge is a tax rate of 31.8 percent. The effective tax rate on leisure is $\tau_0 = -0.318/(1 - 0.318) = -0.466$. It was assumed that profits are taxed at a standard corporate income tax rate of 30 percent.

The excise tax rates—86.3 percent on petrol, 45.0 percent on alcoholic beverages, and 82.3 percent on cigarettes—are based on the tax rate data for 1999 in the Institute for Fiscal Studies publication by Chennells, Dilnot, and Roback (1999).³ Note that these tax rates are expressed as a percentage of the consumer price and include the VAT rate of 17.5 percent, whereas Parry expressed tax rates as a percentage of the producer price and excluded the VAT from the tax rates on these commodities. We believe that the VAT on petrol, alcoholic beverages, and cigarettes should be included in the effective tax rates because they contribute to the tax wedge between the consumer price and the producer

³ The tax rate for alcohol is a weighted average of the ad valorem equivalent tax rates for beer (31.8 percent), wine (52.9 percent), and spirits (63.5 percent) where beer has a 50 percent weighting and wine and spirits have 25 percent weights.

price. The excise tax on petrol generated £23.1 billion in 1999-2000. The excise taxes on alcoholic beverages generated £6.1 billion, and the cigarette taxes yielded £7.0 billion. Using the commodity tax rates, with an adjustment for the VAT component in the tax rates, we calculated $q_1x_1 = £32.25$ billion, $q_2x_2 = £20.26$ billion, and $q_3x_3 = £10.38$ billion. Given that total consumer expenditure in the U.K. was £570.44 billion in 1999, this implies that $q_4x_4 = £507.44$ billion. In 1999-2000, VAT revenues were £54 billion and other excise taxes, such as vehicle excise duties and betting and gambling duties, were £10.5 billion. After adjusting for the VAT collected from sales of petrol, alcohol beverages and tobacco products, the average tax rate on all other goods was 10.9 percent.⁴ Petrol and alcohol taxes are levied as per unit taxes in the U.K. while the tobacco taxes in the U.K. are a mix of per unit and ad valorem taxes, with per unit taxes representing 54.41 percent of the total tax rate. We have modeled the tobacco taxes as a combination of an ad valorem excise tax with $\tau_{av_3} = 0.375$ and as a per unit excise tax at the ad valorem equivalent rate of $\tau_{pu_3} = 0.448$. Distinguishing between ad valorem and per unit excise taxes is important because the cigarette industry is highly concentrated in most countries, and there is considerable evidence of non-competitive behaviour in cigarette pricing. It is well-known that ad valorem and per unit taxes will have different effects on consumer prices under imperfect competition and that ad valorem taxes are a more efficient way of raising a given amount of tax revenue than per unit taxes under imperfect competition, whereas they are equivalent under perfect competition.⁵

⁴ The average tax rate on all other goods is below the VAT rate because many goods are zero-rated or exempt from VAT. Chennells, Dilnot, and Roback (1999, Table 6, p.8) indicates that the reduction in tax revenue due to zero-rating, exemption, and reduced VAT rates for domestic fuels was £26.9 billion in 1998-99.

⁵ See Delipalla and Keen (1992, Proposition 6).

1.2 Budget Shares

$$b = (0.5, 0.0284, 0.0177, 0.00910, 0.445)$$

The budget share calculations were based on the assumption that half of the representative individual's available time is devoted to leisure (non-market activities), and therefore $b_0 = 0.5$. Since $(1 - b_0)q_0T$ is equal to £570.44 billion, total potential net earnings, q_0T , were estimated to be £1.114 trillion. This figure was used to calculate the budget shares of the other commodities given the values for the q_jx_j s calculated in the previous section. Consequently, other commodities represent 44.5 percent of total net potential earnings, and total expenditures on petrol, alcoholic beverages and cigarettes collectively represent only 5.5 percent of total potential earnings.

1.3 Demand Elasticities

$$\mu = \begin{pmatrix} 0.3 \\ 0.7 \\ 0 \\ 0 \\ 1.867 \end{pmatrix} \quad \varepsilon = \begin{pmatrix} -0.2 & 0.008 & 0.001 & -0.001 & 0.191 \\ -0.06 & -0.7 & -0.012 & -0.006 & 0.779 \\ 0.19 & 0 & -0.6 & 0 & 0.41 \\ 0.11 & 0 & 0 & -0.4 & 0.29 \\ 1.343 & -0.028 & -0.017 & -0.011 & -1.287 \end{pmatrix}$$

The income elasticities, the μ_{iS} , and price elasticities, the ε_{ijS} , used in computations are given above. Although, the income elasticities do not appear in the formula for the MCF in (4), they were used to derive the complete set of price elasticities used in the model, and together with the price elasticities, they determine the magnitudes of the substitution effects that underlie the efficiency losses from taxation. The normal

practice for economists is to state the income effects of a change in the wage rate in terms of changes in the supply of labour, and not leisure. Let $\theta = (1 - \tau_w)w dL/dM$ be the income effect on labour supply. Parry adopted a widely-used value of -0.15 in his calculations. It can be shown that income elasticity of demand for leisure is equal to $\mu_0 = -\theta/b_0$. Given our assumption that b_0 is 0.50, this implies that the equivalent income elasticity of demand for leisure is 0.30. We have also adopted Parry's assumptions regarding the income elasticities of demand for petrol, alcoholic beverages, and cigarettes, 0.7, 0.0, and 0.0 respectively. Since $\sum_{j=0}^4 b_j \mu_j = 1$, our budget share estimates imply that the income elasticity of demand for all other good is 1.867.

We have also tried to use values for the own-price elasticities of demand that are equivalent to those used by Parry. He assumed in his benchmark estimates that the labour supply elasticity, η , is 0.20. The equivalent value for the own-price elasticity of demand for leisure is $\varepsilon_{00} = -\eta(1 - b_0)/b_0 = -0.20$. In addition, we have used his values of $\varepsilon_{11} = -0.70$ for the price elasticity of demand for gasoline, $\varepsilon_{22} = -0.60$ for the price elasticity of demand for alcoholic beverages, and $\varepsilon_{33} = -0.40$ for the price elasticity of demand for cigarettes.

Parry assumed that petrol and alcoholic beverages are substitutes for leisure and that cigarettes are a complement, i.e. $\varepsilon_{01} = 0.008$, $\varepsilon_{02} = 0.001$, and $\varepsilon_{03} = -0.001$. Note that the values of these cross-price elasticities are close to zero, implying that the excise tax changes will have little direct effect on labour supply. Parry also assumed that changes in the net wage rate have no effect on the demand for petrol, alcohol beverages, and cigarettes, i.e. $\varepsilon_{10} = \varepsilon_{20} = \varepsilon_{30} = 0$. However, this assumption is not consistent with his

assumptions regarding the ε_{0j} s if consumer demand satisfies the symmetry condition that

$b_j \varepsilon_{ji}^c = b_i \varepsilon_{ij}^c$ or using the Slutsky equation:

$$\varepsilon_{ji} = \frac{b_i}{b_j} \varepsilon_{ij} + b_i (\mu_i - \mu_j) \quad (5)$$

We have used (5) to calculate the ε_{j0} s given the ε_{0j} s and the income elasticities. We also used this condition to compute ε_{12} , ε_{13} , and ε_{23} given Parry's assumption that $\varepsilon_{21} = \varepsilon_{31} = \varepsilon_{32} = 0$. Finally, given the values for the cross-price elasticities, we used the homogeneity

condition, $\sum_{i=0}^4 \varepsilon_{ji} = 0$, $j = 0, 1, \dots, 4$, and the Cournot aggregation

condition, $\sum_{j=0}^4 b_j \varepsilon_{ji} + b_i = 0$, $i = 1, 2, \dots, 4$, to compute the cross-price elasticities for all other

goods and its own price elasticity. In other words, the price elasticities in each column of the ε matrix should sum to zero because an equi-proportional increase in all prices, including q_0 , does not change the budget constraint of the individual. The Cournot aggregation condition requires that the weighted sum of the price elasticities in rows 1 to 4 of the above matrix equals (minus) the budget share of the commodity in that row.

These nine equations were used to calculate the nine price elasticities in the last column and the last row of the ε matrix.

1.3 Distortions

$$\delta_E = \begin{pmatrix} 0 \\ -0.16 \\ -0.083 \\ -0.071 \\ 0 \end{pmatrix} \quad \delta_G = \begin{pmatrix} 0 \\ 0.018 \\ 0.028 \\ 0.212 \\ 0 \end{pmatrix} \quad \delta_M = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0.219 \\ 0 \end{pmatrix}$$

Incorporating the relevant non-tax distortions is one of the most important elements in calculating the MCFs. We have defined δ_E as the proportional marginal external benefit generated by the consumption of a commodity. It is equal to the difference between the marginal social benefit and the marginal social cost of producing commodity, expressed as a proportion of the consumer price. For harmful externalities, $\delta_E < 0$. We will refer to the δ_{ES} as the distortions caused by direct consumption externalities.

There is another type of externality that has an indirect effect on individuals' well-being through the government's budget constraint. For example, an increase in cigarette consumption may increase public expenditures on health care. Even in the absence of a "second-hand" smoke externality, non-smokers as well as smokers will be adversely affected by the higher taxes that they have to pay to finance the higher public health care expenditures. The health care costs associated with smoking are often used to justify high taxes on tobacco products. Suppose that the government provides a service, G , and let the cost of providing this service be $C(G, x)$ where $\partial C/\partial G > 0$ is the marginal cost of providing the service and $\partial C/\partial x$ is the increase in the cost of providing a given level of service (say health care) as a result of an increase in the consumption of a private good x . We will define the government expenditure externality as $\delta_G = (\partial C/\partial x)$,

i.e. the change in the cost of public services when individuals spend another dollar on x . If $\delta_G > 0$, taxing x will reduce the cost of producing public services, and this will reduce the MCF from taxing x .

Parry (2003) provided an extensive review of the empirical literature on the externalities generated by the consumption of gasoline, alcohol and cigarettes. Obviously, there is still a great deal of uncertainty concerning the magnitudes of these parameters, but Parry's choices for his base case estimates seem reasonable. Based on his review of the literature, Parry concluded tobacco products impose the largest harmful externalities, representing 28.3 percent of the consumer price of the product, and alcohol consumption imposes the smallest harmful externality, at just 11 percent of the product price. However, Parry treated all externalities as direct consumption externalities even though his discussion and the literature indicate that these externalities, especially for smoking and alcohol consumption, take the form of higher public expenditures on health care, and in our framework would be included in the δ_G parameters. In order to distinguish between direct consumption externalities and public expenditure externalities, we somewhat arbitrarily assumed that 10 percent of the petrol externalities, 25 percent of the alcohol-related externalities, and 75 percent of the tobacco-related externalities were public expenditure externalities.

A third type of distortion that drives a wedge between the market price of a good and its cost of production is imperfect competition. Let the market power distortion be defined as $d_M = q - (MC + t)$ where q is the consumer price, MC is the marginal cost of production and t is the excise tax imposed on the commodity. The market power distortion expressed as a percentage of the consumer price is $\delta_M = d_M/q$. Under

monopoly, the proportional mark-up over the marginal cost is inversely related to the elasticity of demand for the monopolist's product or $\delta_M = -1/\epsilon$. In the case of oligopoly, the market power distortion will be equal to $-\gamma/\epsilon$ where γ is the firms' conjectural variation parameter reflecting each firm's beliefs concerning how other firms will respond to a change in its output. (We assume that all firms have identical costs and the same conjectural variations.) For a perfectly competitive industry, $\gamma = 0$ because each firm expects the price of the product (and hence total output) to remain constant when it increases its output. With a symmetric Cournot duopoly, $\gamma = 0.5$. If the firms in the industry collude to maximize their joint profits, then $\gamma = 1$. This is equivalent to the monopoly case.⁶

Parry did not consider the implications of imperfect competition in his measures of the efficiency cost of taxes. However, the cigarette industry is highly concentrated in most countries, and there is considerable evidence of non-competitive behaviour in cigarette pricing. Delipalla and O'Donnell (2001) used a conjectural variations framework to estimate the responsiveness of cigarette prices to tax changes in European countries. Their estimates of the tax shifting parameters were consistent with the theoretical prediction that ad valorem taxes produce smaller price increases than per unit taxes in an imperfectly competitive market. The ratio of the tax shifting effects for per unit and ad valorem taxes yields an estimate of the market power distortion. Their parameter estimates, which are given in the next section, imply that the distortion in the market price caused imperfect competition is 0.219 if the price elasticity of demand for cigarettes is -0.40.

⁶ See Dahlby (1992) for a conjectural variations model applied to cigarette excise taxes.

1.4 Tax Shifting

$$dq_0/dt_w = -1$$

$$dq_i/dt_i = 1 \quad i = 1, 2, \text{ and } 4$$

$$(1 - \tau_{av_3}) \frac{dq_3}{dt_3} = 0.92 \quad \text{and} \quad (1 - \tau_{av_3}) \frac{dq_3}{q_3 d\tau_{av_3}} = 0.72$$

We have assumed that producer prices are constants and that all markets, except the tobacco market, are competitive. These assumptions imply that excise taxes on petrol, alcohol beverages and all other goods are fully shifted to consumers. It also implies that $dq_j/dt_i = 0$ for $j \neq i$ and that a tax on labour is borne by workers.

Consequently, a per unit labour tax reduces the price of leisure by the amount of the tax.

As noted above, the cigarette industry is highly concentrated in most countries, and there is considerable evidence of non-competitive behaviour in cigarette pricing. Delipalla and O'Donnell's estimates of tax shifting in a subset of European countries are given above, and they are consistent with the prediction that an ad valorem tax induces a smaller price increase than the equivalent per unit excise tax in an imperfectly competitive market. Their estimates also imply that excise taxes are not fully shifted to consumers because net price to the producer does not increase by the full amount of the tax.

1.5 Computations of the MCFs

Given these parameter values, the calculated values of the MCFs are given below:⁷

$$\text{MCF}_{t_w} = 1.290 \quad \text{MCF}_{t_1} = 3.008 \quad \text{MCF}_{t_2} = 1.424 \quad \text{MCF}_{t_3} = 0.978 \quad \text{MCF}_{\text{rav}_3} = 0.981 \quad \text{MCF}_{t_4} = 1.242$$

In particular, MCFs for petrol and alcohol are 3.00 and 1.42 respectively. The per unit excise tax on cigarettes has an MCF of 0.978, while the ad valorem tax on cigarettes is slightly higher at 0.981. For purposes of comparison, Parry's MEBs for his benchmark case are also shown

$$\text{MEB}_{t_w} = 0.30 \quad \text{MEB}_{t_1} = 1.00 \quad \text{MEB}_{t_2} = 0.24 \quad \text{MEB}_{t_3} = 0.11$$

A direct comparison of the magnitudes of Parry's MEBs and our MCFs is not possible, and in particular the MCFs will generally not be equal to $1 + \text{MEB}$.⁸ However, a comparison of the ordinal ranking is valid and important because an efficiency-enhancing tax reform would (in general) increase the tax with the lowest MEB or MCF and reduce the tax rate with the highest MEB or MCF. Both sets of calculations indicate that petrol taxes have the highest efficiency cost while cigarette taxes have the lowest efficiency cost. Parry's MEB calculations indicate that alcohol taxes have a lower marginal efficiency cost than a tax on labour, whereas the MCF calculations indicate that the labour taxes have lower efficiency costs than alcohol taxes. Our MCF calculations also indicate that a tax on labour has a slightly higher marginal efficiency cost than a tax on all other goods.⁹ In summary, our calculations of the MCFs produce a very similar

⁷ A pdf file describing the calculations is available from the author upon request.

⁸ On the relationship between the MEB and the MCF, see Triest (1990) Håkonsen (1997) and Browning, Gronberg, and Liu (2000).

⁹ Recall that Parry did not model or compute the MEBs for taxes on all other goods.

ranking of the marginal efficiency costs of the various taxes, but our MCF calculations emphasize the substantial gains from raising cigarette taxes and lowering petrol taxes because the differences in the MCFs is 2.03 whereas the difference in Parry's MEBs for the two taxes is only 0.89.

Our most significant departure from Parry's analysis is that we have included the market power distortion in tobacco products market in the calculation of the MCFs. Incorporating the market power distortion increases the MCF for the excise taxes on cigarettes, but these MCFs are still less than one, and they remain the lowest MCFs for all of the excise taxes. (Incorporating δ_M greatly reduces the magnitude of the total distortion since $\delta_G - \delta_E - \delta_M = 0.064$, such that the marginal social cost of smoking is only 6.4 percent above the market price.) In spite of this reduction in the total distortion, the MCF for the excise taxes on cigarettes is still less than one, in part because Delipalla and O'Donnell's estimates imply that producer prices do not increase by the amount of the tax, and therefore consumers do not bear the full amount of the tax burden. Finally, note that the MCF for the per unit excise tax is slightly lower than the MCF for the ad valorem taxes, and therefore these calculations indicate that the standard result—ad valorem taxes are superior to per unit excise taxes in an imperfectly competitive market—is not valid when there are other distortions in the market.¹⁰ If the marginal environmental damage from the product exceeds the price distortion caused by monopoly, implying that the monopoly price of the product is “too” low, and if the marginal expenditure distortion exceeds the total tax rate on the commodity, then a per unit tax has a lower MCF than an ad valorem tax. Intuitively, the per unit tax has a lower MCF under these conditions

¹⁰ A more detailed discussion of the conditions under which a per unit tax is superior to an ad valorem tax under imperfect competition is contained in Dahlby (2005, Chapter 3).

because social welfare is improved by raising the price of the product, and a per unit tax is more effective in raising the price of the product than an ad valorem tax.

2.0 The MCF for a Payroll Tax with Involuntary Unemployment in the Labour Market

In the previous section, we argued that it is important to incorporate non-tax distortions in measuring the MCFs for commodity taxes. Similarly, it may be important to incorporate one of the most important labour market distortions, involuntary unemployment, in measuring the MCF for taxing labour income. In this section, we use a simple efficiency wage model, based on Shapiro and Stiglitz (1984), to incorporate the distortion caused by unemployment in the MCF. We use their efficiency wage model as a framework because it provides a relatively simple explanation for the existence of unemployment. Other explanations for unemployment—wage rigidities caused by nominal wage contracts or minimum wage laws—would likely produce expressions for the MCF that are similar to the ones derived in below.

2.1 The Shapiro and Stiglitz Efficiency Wage Model

Suppose there are N identical workers in the labour market, and each worker, if employed, supplies one unit of time. The work effort that is expended by an employed worker can take on two values—0 and $e > 0$. The production function is $x = F(L)$ if the L employed workers supply e units of effort. If workers' effort level is zero, i.e. if workers “shirk”, nothing can be produced. An employed worker, who does not shirk, has a utility level of $(w - e)$, where w is the worker's wage rate and e is the money measure of the

disutility of effort. An employed worker who “shirks” and supplies no effort has a utility level of w .

Employers observe only imperfectly the effort that a worker applies on the job. Let $0 < \chi < 1$ be the exogenously determined probability that a shirking worker is detected and fired from his job. A fired worker joins a pool of unemployed workers and receives unemployment benefits equal to B . Let $U = N - L$ be the number of unemployed workers, and $ur = U/N$ be the unemployment rate in the economy. Each period, an exogenously determined fraction of the employed workers, l , is laid off because of firm turnover. Also each period, a fraction of the pool of unemployed workers, h , are hired by firms. An equilibrium unemployment rate implies that $hU = lL$ or $h = l(1 - ur)/ur$. The utility level of an individual who is employed and not shirking is given by:

$$\rho V_E^N = w - e + l(V_U - V_E^N) \quad (6)$$

where ρ is the individual’s discount rate, and V_U is the present value of the utility of an unemployed worker. The above equation expresses the notion that the value of an asset (in this case a job) times the market interest rate equal the flow of income plus the expected capital gain or loss on the asset. The utility level of a “non-shirking” worker can also be expressed as:

$$V_E^N = \left(\frac{\rho}{\rho + l} \right) \left[\frac{w - e}{\rho} \right] + \left(\frac{l}{\rho + l} \right) V_U \quad (7)$$

The first term in this expression can be thought of as the present value of net income from employment multiplied by the fraction of time that the worker can expect to be employed, and the second term is the fraction of time that the worker will be laid off

multiplied by the utility level of an unemployed worker. The utility level of an employed worker who shirks will be equal to $\rho V_E^S = w + (l + \chi)(V_U - V_E^S)$ or:

$$V_E^S = \left(\frac{\rho}{\rho + l + \chi} \right) \frac{w}{\rho} + \left(\frac{l + \chi}{\rho + l + \chi} \right) V_U \quad (8)$$

Finally the utility level of an unemployed worker will be $\rho V_U = B + h(V_E - V_U)$ or,

$$V_U = \frac{\rho}{\rho + h} \left(\frac{B}{\rho} \right) + \frac{h}{\rho + h} V_E \quad (9)$$

Since nothing is produced when workers shirk, it must be the case that $V_E^N \geq V_E^S$ and that employed workers do not have the incentive to shirk. Using (7) to (9), this non-shirk constraint (NSC) imposes the following condition on the equilibrium wage rate and unemployment rate:

$$w \geq B + e + \frac{e}{\chi} \left(\frac{l}{ur} + \rho \right) \quad (10)$$

The model predicts that the market wage rate will be higher when unemployment benefits, the cost of effort, the layoff rate, or the discount rate are higher. A higher detection rate or a higher unemployment rate will reduce the wage rate that employers need to offer in order to prevent shirking. In equilibrium, the wage rate will satisfy the NSC and equal the value of the marginal product of labour F_L .

Figure 1 show the equilibrium wage and unemployment rate if the total labour force is normalized to equal one and therefore $L = 1 - ur$. The NSC has a positive slope because as employment increases and the unemployment rate declines, the wage rate that employers have to pay in order to prevent shirking increases. The equilibrium wage rate w_0 occurs where the F_L curve intersects the NSC curve. The equilibrium unemployment

rate is ur_0 . In contrast with the conventional competitive labour market model, unemployment is not eliminated through a reduction in the wage rate that would increase the number of workers employed because at a lower wage rate, the opportunity cost of shirking would be too low, and all workers would shirk. Thus the model provides an explanation for the existence of involuntary unemployment that does not rely on government-imposed minimum wages or union wage determination that prevents wage rates from adjusting to eliminate unemployment.

2.2 The MCF in an Efficiency Wage Model

We want to use this framework to investigate how unemployment could affect the MCF from taxing labour income. It will be assumed that a per worker payroll tax is imposed on employers, and therefore the profit-maximizing employment level is $w + t = F_L$. As shown in Figure 1, the net marginal product of labour declines by the tax per worker, and a new equilibrium will be established with a lower wage rate and a higher unemployment rate. Part of the burden of the employer payroll tax is shifted to employed workers through a lower wage rate. Their expected utility is also reduced because of the increase in the unemployment rate, given the positive layoff rate. Taking the total differentials of the NSC and the profit-maximizing employment condition, the following comparative static results can be obtained:

$$\frac{dw}{dt} = \frac{(1 - ur)z\varepsilon_{Lw}}{(w + t) - (1 - ur)z\varepsilon_{Lw}} \quad (11)$$

$$\frac{dur}{dt} = \frac{(1 - ur)\varepsilon_{Lw}}{(1 - ur)z\varepsilon_{Lw} - (w + t)} \quad (12)$$

where $\varepsilon_{Lw} = (w + t)(L \cdot F_{LL})^{-1} < 0$ is the elasticity of demand for labour and $z = (e \cdot l) / (\chi \cdot ur^2)$ is the slope of the NSC. When the unemployment rate is low and the NSC is steeper,

more of the tax burden is shifted to workers through lower wages. Note however, that $-1 < dw/dt < 0$. Therefore some of the burden of the employer payroll tax is shifted to the owners of the other inputs used in production even though the total potential supply of labour is fixed.

An increase in an employer payroll tax adversely affects three groups in this economy. First, the unemployed are worse off because the unemployment rate increases, and they can expect to spend a longer time unemployed before being re-hired. Second, employed workers bear part of the burden through a lower wage rate. Third, firms receive lower after-tax profits because the wage rate does not fall by the full amount of the tax. Using the envelope theorem, it can be shown that the firms' burden is:

$$\frac{d\Pi}{dt} = -L \left(1 + \frac{dw}{dt} \right) \quad (13)$$

where $\Pi = x - (w + t)L$ is the after-profit earned by firms. It will be assumed that the social welfare function that is used to evaluate the losses imposed by taxation has the following modified utilitarian form:

$$S = ur \cdot \rho \cdot V_U + (1 - ur) \cdot \rho \cdot V_E^N + \beta \cdot \Pi \quad (14)$$

where β is a distributional weight that is applied to profit-income. It will be assumed that $0 \leq \beta \leq 1$, perhaps because profits accrue to a small group of individuals in the economy who are relatively well off. From the equilibrium values for V_U and V_E^N in (7) and (9), this social welfare function has the following simple form:

$$S = ur \cdot B + (1 - ur) \cdot (w - e) + \beta \cdot \Pi \quad (15)$$

That is, social welfare is equal to (a) the unemployment rate multiplied by the "surplus" that is received by the unemployed, i.e. their unemployment benefits, plus (b) the

employment rate $(1 - ur)$ multiplied by the surplus that the employed receive, which is the difference between their wage rate and the cost of effort, plus (c) the distributionally-weighted profit that accrues to the owners of firms.

Taking the total differential of the social welfare function in (15) gives the following expression for the marginal social impact of an increase in the employer payroll tax:

$$\frac{-dS}{dt} = \beta \cdot (1 - ur) + (w - (e + B)) \cdot \frac{dur}{dt} + (\beta - 1)(1 - ur) \cdot \frac{dw}{dt} \quad (16)$$

This expression shows that there are three components to the social cost of an employer payroll tax increase. The first term is the distributionally-weighted direct impact of the tax increase on employers, which is proportional to the level of employment. The second component is the net loss $(w - (e + B))$ that is sustained by a worker if unemployment increases due to a tax rate increase. The third component is the net social impact of the reduction in the wage rate caused by the tax increase. Note that this term vanishes, if $\beta = 1$ and profit income has the same social marginal utility as employment income, because the loss sustained by employees from a decline in the wage rate just offsets the increase in profits that arises because of the wage rate reduction.

To compute the MCF for the payroll tax increase, we need to divide (16) by the rate at which the government's net revenues increase when it increases its tax rate. It will be assumed that unemployment benefits are financed by the public sector and therefore:

$$\frac{dR}{dt} = (1 - ur) - (t + B) \frac{dur}{dt} \quad (17)$$

The first term reflects the size of the tax base, while the second term reflects the reduction in net tax revenues because few workers are employed and more workers are

drawing unemployment benefits. Combining (16) and (17), we obtain the following general expression for the MCF for an employer payroll tax in this efficiency wage model:

$$\text{MCF}_t = \frac{\left[\beta \left(1 + \frac{dw}{dt} \right) - \frac{dw}{dt} \right] + \delta_U \frac{w}{(1-ur)} \frac{dur}{dt}}{1 - \left(\frac{t+B}{w} \right) \frac{w}{(1-ur)} \frac{dur}{dt}} \quad (18)$$

where $\delta_U = (w - e - B)/w$ is the labour market distortion caused by involuntary unemployment expressed as a percentage of the market wage rate. Note that this measure of the labour market distortion has the same general form as the environmental and market power distortions, δ_E and δ_M , that were discussed in the previous section. This distortion, which arises because of an asymmetric information problem, is multiplied by the loss of labour income per dollar of tax imposed due to the increase in unemployment caused by the tax increase to obtain the total social loss due to the tax rate increase. The term in square brackets in the numerator of (18) measures the distributionally weighted effect of the tax increase on employers and employees. In the denominator of (18), the rate of change in the payroll tax base per dollar is multiplied by the payroll tax rate and the unemployment benefit rate, expressed as a percentage of the market wage rate. In this model, B/w reflects the public expenditure externality, δ_G , that was discussed in the previous section.

2.3 Computing the MCF in an Efficiency Wage Model

In order to calculate dw/dt and dur/dt using (11) and (12), we need to specify values for χ and e , and these parameters are not directly observable. Our strategy in selecting values for these parameters is to choose values for the other parameters of the

model and then use to the model to calculate the values of χ and e that are consistent with that equilibrium. It is assumed that the production function is Cobb-Douglas with $x = (1 - ur)^\alpha$ where $0 < \alpha < 1$. The marginal product of labour is equal to $\alpha(1 - ur)^{\alpha-1}$. The wage rate, in the absence of unemployment and payroll taxes, would be α . In our calculations, it will be assumed that $\alpha = 2/3$. This implies that the elasticity of demand for labour is $\epsilon_{Lw} = -(1 - \alpha)^{-1} = -3$. The other labour market parameters were chosen to roughly approximate the Canadian labour market in the 1976-91 period based on the labour market flows analyzed by Jones (1993). The average unemployment rate over this period was 8.9 percent, and the average layoff rate was 1.9 percent. It was assumed that the average tax rate was 30 percent of α and the average unemployment insurance benefit was 25 percent of α . It was also assumed that the discount rate is 0.05. Using these parameter values, values of χ from 0.10 to 0.90 were specified, and the model was solved for the value of the effort parameter that would be consistent with the specified labour market equilibrium.

Table 1 shows that if the detection rate was 0.90, the corresponding cost of effort would have to be 57.9 percent of the wage rate to generate the observed labour market equilibrium. With these values, about three-quarters of the wage tax would be borne by employees through a lower wage rate. The MCF would be 1.56 if profit income has the same distributional weight as labour income. Since the MCF in a competitive labour market with full employment and a fixed labour supply is 1.00, we can see that the distortion caused by unemployment adds 0.56 to the MCF if the detection rate is high. However, the table shows that with lower values for χ , and correspondingly lower values for the cost of effort, the MCF would be lower. With $\chi = 0.10$, unemployment adds

“only” 0.25 to the MCF. So while the effect of incorporating unemployment in the calculation of the MCF is quite sensitive to the chosen values of the χ and e , it increases the measured MCF by a significant amount even when the χ is low. The reason for the decline in the MCF as the χ and e values decline is that although the distortion from unemployment, δ_U , increases, more of the tax burden is shifted to workers and therefore the tax increase has a smaller effect on the overall level of unemployment. In other words, the unemployment distortion per dollar of tax revenue is relatively constant, at around 0.10, when χ varies from 0.9 to 0.1 and the MCF declines because the loss of labour income per dollar of additional tax revenue declines.

Table 1 also shows the calculated values of the MCF when the distributional weight on profit is zero. These MCFs are, not surprisingly, lower because the costs of tax increases that are borne by firms are “ignored” in these calculations. These values for the MCF also declined as χ and e decline, but within a narrower range from 1.15 to 1.22.

The model indicates that it may be important to incorporate the distortion caused by unemployment in the calculation of the MCF. While we have used the framework of the Shapiro-Stiglitz model to calculate δ_U , dur/dt , and dw/dt terms, the results from econometric studies on the impact of payroll taxes on labour markets could be used instead, and this might narrow the range of values for the estimated MCF.

3.0 The Optimal Flat Tax

Einstein is reputed to have said “Nothing is as complicated as income tax.” From this remark, we may speculate that Einstein tried to solve the optimal income tax problem, found it too difficult, and went back to his research on relativity. The solution

to the optimal income tax developed by Mirrlees (1971) is very complicated because Mirrlees imposed very few restrictions on the structure of the income tax. Few predictions come out of the general optimal income tax model, other than that the marginal tax rate should be zero for the taxpayer with the highest wage rate and positive for other taxpayers. Slemrod et al. (1994) tackled a more restrictive version of the optimal income tax problem. They restricted the government's choices to the tax rates in two tax brackets and a lump-sum demogrant. Using a numerical simulation model, the authors found, for a wide variety of parameter values, that the optimal two bracket income tax has a lower marginal tax rate in the higher tax bracket. However, this result might be specific to the type of utility function that they used to model individuals' labour leisure decisions. Furthermore, they did not derive any analytical expressions describing the properties of the optimal two bracket income tax. Hence the relationship between the optimal income tax and the concept of the SMCF is not well articulated in the literature, except in the work of Saez (2001).

In this section, we will impose even more structure on the optimal income tax problem than Slemrod et al. (1994) by focusing on the optimal "flat tax", i.e. an income tax in which the marginal income tax rate in the first tax bracket is zero and the marginal income tax rate in the second bracket is positive. The government's only policy variables are X_1 , which determines the size of the first bracket, and m_2 , the marginal tax rate on income in excess of X_1 . We take the government's expenditures (including income transfer programs) as given. In other words, we do not include a lump-sum transfer to all individuals as policy variable of the government. Imposing this structure on the optimal income tax problem has two advantages. First, it allows us to develop tractable

expressions for the social marginal cost of funds (SMCF) for the two policy instruments, X_1 , and m_2 . Hopefully, this will give some insights into the nature of the solution to the more general income tax design problem. Second, the optimal design of a flat tax is an interesting policy question in its own right because some economists, such as Hall and Rabuska (1995) have advocated the adoption of a flat tax for the United States. Indeed, Alberta's provincial income tax is a flat tax, with an exemption level of \$14,337 and a marginal tax rate of 10 percent in 2004.

We derive the optimal flat tax by developing expressions for the SMCF using the framework described in Dahlby (1998), except that it now assumed that there is a continuous distribution of wage rates among the taxpayers. Suppose that the wage rate varies between 0 and w_{top} . Let $F(w)$ be the cumulative distribution function for wage rates, with $F(0) = 0$ and $F(w_{top}) = 1$. The density function will be denoted by $f(w)$. The total population of individuals will be normalized to equal one.

Individuals have an identical utility function $U(C, L)$, where C is consumption and L is total labour supplied with $U_C > 0$ and $U_L < 0$. Let $Y = wL$ be the individual's income. (Non-labour market income is assumed to be zero.) The individual's consumption opportunities are given by:

$$\begin{aligned} C &= Y && \text{for } Y \leq X_1 \\ C &= Y - m_2(Y - X_1) && \text{for } Y > X_1 \end{aligned} \tag{19}$$

The individual's preferences over consumption and income are given by $U(C, Y/w)$, and the slope of an individual's indifference curves in (C, Y) space is $-(U_L/U_C)(1/w)$. This implies that individuals with higher wage rates have less steeply sloped indifference curves. Intuitively, for any (C, Y) combination, a high wage person needs to work fewer hours to earn an additional dollar of income, and therefore he needs less additional

consumption to compensate for the additional effort required to earn that additional dollar of income. We can also represent individuals' preferences using the indirect utility function $V((1 - m)w, Z)$ where m is zero if $Y < X_1$ and $m = m_2$ if $Y > X_1$. The individual's virtual income is $Z = mX_1$, with $Z = 0$ if $Y \leq X_1$ and $Z = m_2X_1$ if $Y > X_1$.

With a continuous distribution of wage rates across individuals, some taxpayers will find it optimal to earn X_1 where there is a kink in their income–consumption opportunity locus. This possibility is illustrated in Figure 2 where individuals with wage rates ranging from w_1 to w_2 have indifference curves that are tangent at kink in their consumption-income opportunity curve at X_1 . These wage rates are defined by the following equations:

$$w_1 = - \frac{U_L \left(X_1, \frac{X_1}{w_1} \right)}{U_C \left(X_1, \frac{X_1}{w_1} \right)} \quad (20)$$

$$w_2 = - \frac{U_L \left(X_1, \frac{X_1}{w_2} \right)}{U_C \left(X_1, \frac{X_1}{w_2} \right) (1 - m_2)} \quad (21)$$

The total number of individuals who earn exactly X_1 is $F(w_2) - F(w_1)$, and the total number of taxpaying individuals is $1 - F(w_2)$. An increase in X_1 (holding m_2 constant) will increase w_1 and w_2 because individuals with higher wage rates have less steeply sloped indifference curves, and similarly, an increase in m_2 will increase w_2 .

The government's total tax revenue is equal to:

$$R = \int_{w_2(m_2, X_1)}^{w_{top}} m_2 (wL[(1 - m_2)w, Z] - X_1) f(w) dw \quad (22)$$

where $L[(1 - m_2)w, Z]$ is the labour supply function of a taxpayer who receives a wage rate of w . The government can increase revenue by raising m_2 or by decreasing X_1 . We begin by developing a measure of the marginal social cost of raising revenue by decreasing X_1 . We then develop an expression for the SMCF of increasing m_2

Applying Leibnitz's rule, the derivative of (22) with respect to X_1 is:

$$\frac{dR}{dX_1} = m_2 \int_{w_2}^{w_{top}} w \frac{dL}{dZ} \frac{dZ}{dX_1} f(w) dw - m_2 \int_{w_2}^{w_{top}} f(w) dw - \frac{dw_2}{dX_1} m_2 (w_2 L[(1 - m_2)w_2, Z] - X_1) f(w_2) \quad (23)$$

For taxpayers, i.e. individuals with $w > w_2$, an increase in X_1 is equivalent to a lump-sum tax cut of $m_2 dX_1$, and the first term reflects the income effect of this tax cut on the individuals' supply of labour. The second term is the decline in revenues from an increase in X_1 , given the number of individuals in tax bracket two. The last term is the effect of an increase in X_1 on w_2 , but this effect is zero because an individual with a wage rate of w_2 earns X_1 . We will denote the income effect for an individual earning a wage rate w by $\theta(w) = (1 - m_2)w dL/dZ$, and the effect of an increase in X_1 on revenues can be written as:

$$\frac{dR}{dX_1} = -m_2 [1 - F(w_2)] + m_2 \int_{w_2}^{w_{top}} \frac{m_2}{1 - m_2} \theta(w) f(w) dw \quad (24)$$

The average income effect for taxpayers will be defined as:

$$\bar{\theta}(w_2) = \frac{1}{1 - F(w_2)} \int_{w_2}^{w_{top}} \theta(w) f(w) dw \quad (25)$$

Consequently, the effect on tax revenues of an increase in X_1 can be expressed in terms of the average income effect among individuals earning more than w_2 as:

$$\frac{dR}{dX_1} = -m_2 [1 - F(w_2)] \left(1 - \frac{m_2}{1 - m_2} \bar{\theta}(w_2) \right) \quad (26)$$

We will use the general social welfare function $S(V(w, Z))$ to reflect a government's distributional preferences. It will be assumed that the distributional weights that are implicit in the social welfare function, $\beta(w) = S_V V_Z(w, Z)$, reflect pro-poor preferences. Otherwise, the government would want to set $X_1 \leq 0$. It will be convenient to decompose social welfare as:

$$S = \int_0^{w_1} S(V(w))f(w)dw + \int_{w_1}^{w_2} S\left(U\left(X_1, \frac{X_1}{w}\right)\right)f(w)dw + \int_{w_2}^{w_{top}} S(V((1 - m_2)w, Z))f(w)dw \quad (27)$$

where the first term reflects the well-being of individuals who do not pay the tax, the second term reflects the well-being of individuals who are the kink in the income-consumption curve, and the third term reflects the well-being of taxpayers. Applying Leibnitz's rule, the effect of an increase in X_1 can be written as:

$$\begin{aligned} \frac{dS}{dX_1} = & \frac{dw_1}{dX_1} S_V \left[V(w_1) - U\left(X_1, \frac{X_1}{w_1}\right) \right] f(w_1) + \frac{dw_2}{dX_1} S_V \left[U\left(X_1, \frac{X_1}{w_1}\right) - V((1 - m_2)w_2, m_2 X_1) \right] f(w_2) \\ & + \int_{w_1}^{w_2} S_V \frac{dU\left(X_1, \frac{X_1}{w}\right)}{dX_1} f(w)dw + \int_{w_2}^{w_{top}} S_V \frac{dV((1 - m_2)w, Z)}{dX_1} f(w) \end{aligned} \quad (28)$$

The first two terms in this expression are equal to zero and therefore the effect of an increase in X_1 on social welfare is measured by the third term—the effect of an increase in X_1 on the well-being of individuals at the kink—and fourth term—the effect on the tax paying individuals. For the taxpaying individuals, the marginal benefit from an increase in X_1 is simply m_2 . For the individuals who are at the kink, the marginal benefit from an increase in X_1 , mb_x , can be expressed as:

$$\frac{dU}{dX_1} = U_C \left[1 - \frac{\left(\frac{-U_L}{U_C} \right)}{w} \right] = U_C mb_X(w) \quad (29)$$

Note that the mb_X will vary from zero, for individuals earning w_1 , to m_2 for individuals earning w_2 . The mb_X for individuals who earn between w_1 and w_2 is less than m_2 because these individuals are “off their labour supply curves” and consuming too much leisure.

We will define the $smb_X(w_1, w_2)$ as the distributionally-weight average mb_X for individuals at the kink where:

$$smb_X(w_1, w_2) = \frac{1}{F(w_2) - F(w_1)} \int_{w_1}^{w_2} \beta(w) mb_X(w) f(w) dw \quad (30)$$

Therefore the effect of an increase in X_1 on social welfare can be written as:

$$\frac{dS}{dX_1} = smb_{X_1} [F(w_2) - F(w_1)] + \bar{\beta}(w_2) m_2 [1 - F(w_2)] \quad (31)$$

where $\bar{\beta}(w_2)$ is the average distributional weight among individuals earning more than w_2 :

$$\bar{\beta}(w_2) = \frac{1}{1 - F(w_2)} \int_{w_2}^{w_{top}} \beta(w) f(w) dw \quad (32)$$

Combining (26) with (31), we obtain the following expression for the social marginal cost of raising revenue through a reduction in X_1 :

$$SMCF_{X_1} = - \frac{\frac{dS}{dX_1}}{\frac{dR}{dX_1}} = \frac{smb_{X_1} [F(w_2) - F(w_1)] + \bar{\beta}(w_2) m_2 [1 - F(w_2)]}{m_2 [1 - F(w_2)] \left(1 - \frac{m_2}{1 - m_2} \bar{\theta}(w_2) \right)} \quad (33)$$

Note that the first term in the numerator of this expression is the distributionally-weighted measure of the harm done to individuals who are at the kink in the income-

consumption curve. The smb_{X_1} may be greater than $\bar{\beta}(w_2)m_2$ if the distributional preferences implicit in the social welfare function are sufficiently pro-poor. Therefore, even though the number of individuals who are at the kink will likely be much smaller than the number of taxpayers, this first term may be important in the measuring the SMCF_{X_1} if preferences are sufficiently pro-poor. Note that in the absence of this term, the expression for the SMCF_{X_1} would be very similar to the expression for the MCF for a lump-sum tax increase in the presence of a proportional wage tax. Equation (33) indicates, not surprisingly, that the social marginal cost of raising revenue through lowering X_1 and imposing taxes on a larger percentage of the population will be lower the stronger the average income effect.

The derivation of the SMCF_{m_2} proceeds in a similar fashion. First, taking the derivative of tax revenue with respect to m_2 , we can obtain:

$$\begin{aligned} \frac{dR}{dm_2} &= \int_{w_2}^{w_{\text{top}}} (wL - X_1)f(w)dw + \int_{w_2}^{w_{\text{top}}} m_2 w \left[\frac{dL}{d((1-m_2)w)}(-w) + \frac{dL}{dZ}X_1 \right] f(w)dw \\ &= \bar{Y}(w_2)[1 - F(w_2)] \left(1 - \zeta - \frac{m_2}{1 - m_2} [\hat{\eta}(w_2) - \zeta \bar{\theta}(w_2)] \right) \end{aligned} \quad (34)$$

where $\bar{Y}(w_2)$ is the average income of the taxpayers:

$$\bar{Y}(w_2) = \frac{1}{1 - F(w_2)} \int_{w_2}^{w_{\text{top}}} wL(w, Z)f(w)dw \quad (35)$$

ζ is the ratio of X_1 to $\bar{Y}(w_2)$, and $\hat{\eta}(w_2)$ is the (uncompensated) income-weighted average labour supply elasticity for taxpayers:

$$\hat{\eta}(w_2) = \frac{1}{1 - F(w_2)} \int_{w_2}^{w_{\text{top}}} \frac{wL}{\bar{Y}(w_2)} \eta(w)f(w)dw \quad (36)$$

Taking the derivative of the social welfare function, we obtain:

$$-\frac{dS}{dm_2} = \int_{w_2}^{w_{top}} S_v \lambda(w) [wL(w, Z) - X_1] f(w) dw = \bar{Y}(w_2) [1 - F(w_2)] (\hat{\beta}(w_2) - \bar{\beta}(w_2) \zeta) \quad (37)$$

where $\hat{\beta}(w_2)$ is the income-weighted average distributional weight for the taxpayers:

$$\hat{\beta}(w_2) = \frac{1}{1 - F(w_2)} \int_{w_2}^{w_{top}} \frac{wL}{\bar{Y}(w_2)} \beta(w) f(w) dw \quad (38)$$

Combining (34) with (37), we obtain the following expression for the $SMCF_{m_2}$:

$$SMCF_{m_2} = \frac{\hat{\beta}(w_2) - \bar{\beta}(w_2) \zeta}{1 - \zeta - \frac{m_2}{1 - m_2} [\hat{\eta}(w_2) - \zeta \bar{\theta}(w_2)]} \quad (39)$$

The roles played by each component of the SMCF can be easily identified. In the numerator, the two β s reflect the social cost of the tax rate increase. The first β is an income-weighted average value because individuals with higher incomes will have higher tax increases. The second β enters the formula with a negative sign because it reflects the off-setting increase in virtual income that occurs when m_2 increases. Since the increase in virtual income is the same for each taxpayer, this β is an unweighted average. It is multiplied by ζ because the increase in virtual income is larger when X_1 is higher. In the denominator, the $(1 - \zeta)$ component reflects the proportion of income, on average, that is subject to the tax rate increase. The term in square brackets indicates the labour supply responses, where $\hat{\eta}(w_2)$ reflects the income-weighted average response to the decline in taxpayers' net wage rates. It is an income-weighted average because the labour supply changes by high income taxpayers have a proportionately larger effect on tax revenues. The $\zeta \bar{\theta}(w_2)$ term in square brackets reflects the reduction in labour supplies because of the increase in virtual income. The reduction in labour supply will be larger when X_1 is

relatively high and when the average income effect among taxpayers is relatively strong.

The optimal flat tax can be defined as the (m_2, X_1) combination that, for a given level of tax revenue, maximizes the social welfare function. The optimal (m_2, X_1) combination will be found by equating the $SMCF_{m_2}$ in (39) with the $SMCF_{X_1}$ in (33).

Note that this solution involves various parameters, such as $\hat{\eta}(w_2)$, $\bar{\theta}(w_2)$, $\hat{\beta}(w_2)$, and $\bar{\beta}(w_2)$ which are functions of w_2 and therefore functions of m_2 and X_1 . Therefore, it is not possible to derive simple reduced-form equations for m_2 and X_1 for the most general case where the labour supply responses and distributional weights vary with the individuals' wage rates. However, some insights can be gleaned by equating (33) with (39), and solving for the $m_2/(1 - m_2)$ ratio:

$$\frac{m_2}{1 - m_2} = \frac{(1 - \zeta)[F(w_2) - F(w_1)]\beta_{\text{kink}} + (\bar{\beta} - \hat{\beta})(1 - F(w_2))}{[F(w_2) - F(w_1)]\beta_{\text{kink}}(\hat{\eta} - \zeta\bar{\theta}) + (1 - F(w_2))(\bar{\beta}\hat{\eta} - \hat{\beta}\bar{\theta})} \quad (40)$$

where $\beta_{\text{kink}} = \text{smb}_x/m_2$, can be interpreted as the average effective distributional weight that is applied to the individuals who are at the kink in the income-consumption curve.

The numerator in (40) can be interpreted as the marginal social gain from a lump-sum transfer financed by a marginal tax rate increase, β_{MTT} . To provide an intuitive interpretation of the denominator, we make the simplifying assumption that η and θ are constant, and using the Slutsky decomposition $\eta = \eta^c + \theta$ to eliminate η , we can obtain the following:

$$\frac{m_2}{1 - m_2} = \frac{\beta_{\text{MTT}}}{\beta_{\text{LST}}\eta^c + \beta_{\text{MTT}}\theta} \quad (41)$$

β_{LST} is the marginal social gain from a pure lump-sum transfer, or:

$$\beta_{\text{LST}} = [F(w_2) - F(w_1)]\beta_{\text{kink}} + (1 - F(w_2))\bar{\beta} \quad (42)$$

Since the left-hand side of (41) is increasing in m_2 , we can interpret this relation as indicating that the optimal marginal tax rate will be higher when (a) the marginal social gain from a marginal tax rate financed transfer is higher relative to a pure lump-sum transfer, (b) the income effect on earnings of a lump-sum transfer is large (in absolute value) and (c) if the substitution effect of a wage rate increase is low. All of these factors accord well with our intuition, but unfortunately (41) does not allow us to solve for m_2 because β_{MTT} and β_{LST} both depend on m_2 and X_1 .

3.1 Computing the Optimal Flat Tax

To gain more insight into the nature of the optimal flat tax, we have used the model developed above to compute the optimal m_2 and X_1 . Suppose individuals have the following CES utility function:

$$U = \left[(1 - \alpha) \cdot C^{\frac{\sigma-1}{\sigma}} + \alpha \cdot (T - L)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \quad (43)$$

where $T - L$ is the total amount of time available for leisure, σ is the elasticity of substitution between consumption of goods, C , and leisure, and α is a positive parameter determining strength of the preference for leisure. Following Stern (1976), the individual's labour supply function will be equal to:

$$L = \frac{T - Z \cdot \left[\frac{\alpha}{(1 - \alpha) \cdot [(1 - m) \cdot w]} \right]^{\sigma}}{1 + (1 - m) \cdot w \cdot \left[\frac{\alpha}{(1 - \alpha) \cdot [(1 - m) \cdot w]} \right]^{\sigma}} \quad (44)$$

This implies that the income effect will depend on the after-tax wage rate:

$$\theta = \frac{-[(1-m) \cdot w] \cdot \left[\frac{\alpha}{(1-\alpha) \cdot [(1-m) \cdot w]} \right]^\sigma}{1 + (1-m) \cdot w \cdot \left[\frac{\alpha}{(1-\alpha) \cdot [(1-m) \cdot w]} \right]^\sigma} \quad (45)$$

It also implies that the individuals at the kink in the income-consumption locus will have wage rates between:

$$w_1 = \left(\frac{\alpha}{1-\alpha} \right) \left(\frac{X_1}{T - \frac{X_1}{w_1}} \right)^{\frac{1}{\sigma}} \quad (46)$$

$$w_2 = \left(\frac{\alpha}{1-\alpha} \right) \left(\frac{X_1}{T - \frac{X_1}{(1-m_2)w_2}} \right)^{\frac{1}{\sigma}} \left(\frac{1}{1-m_2} \right) \quad (47)$$

It was assumed that hourly wage rates follow a log normal distribution, where the mean wage rate is $\bar{w} = 20$. Therefore $\mu_w = \ln(\bar{w}) = 2.99573$. The standard deviation of the $\ln w$ was assumed to be $\sigma_w = 0.75$. The highest hourly wage rate was assumed to be 400. The corresponding distribution of wage rates across the population is shown in Figure 3.

The preference parameters were selected in the following manner. It was assumed that an individual earning the average hourly wage rate would choose to earn \$36,000 per year in the absence of taxation and that the income effect at the average wage rate would be equal to -0.15. It was also assumed that $\alpha = 0.5$. This implies that $T = 2117.65$ and $\sigma = 1.579$. This elasticity of substitution is considerably higher than the ones that are conventionally used in the literature, and it might be expected that this

would lead to relatively low values for the optimal marginal tax rate in our calculations. As we will see, this is not the case. The social welfare function used to calculate the distributional weights was $S = \int (1 - \xi)^{-1} (V(w))^{1-\xi} f(w) dw$, where $\xi > 0$ reflects the strength of social preference for equality. The distributional weights were normalized so that $\beta(\bar{w}) = 1$. The optimal flat tax was calculated such that it would yield the same revenue as a 20 percent proportional tax on earnings.

Table 2 shows our results for $\xi = 0.5$ and $\xi = 1.5$. With $\xi = 0.5$, the optimal marginal tax rate is close to 41 percent, with an exemption level of \$25,661 such that only 57 percent of the population would pay the tax. However, 8.8 percent of the population would be at the kink in the income consumption curve. The average income of a taxpayer would be \$67,089. The uncompensated labour supply elasticities would be positive but decline as the wage rate rose. The income-weighted average labour supply elasticity would be 0.14169, which is within the usual range of values used in simulating the labour supply effects of tax policies. Similarly the income effect of a lump-sum transfer on earnings would decline (in absolute value) at higher income levels with the average value for the income effect equal to -0.15425. The distributional weights that underlie these computations ranged from 1.0628 at the bottom of the tax bracket to 0.15953 at the highest income level. The model implies that even with ‘relatively moderate’ distributional preferences and with labour supply elasticities that are in the normal range of values used in applied tax policies studies, the marginal tax rate under the optimal flat tax would be significantly higher than the flat tax rates proposed for the U.S. economy. In other words, the optimal flat tax may be considerably more progressive than either its critics or its proponents have supposed. Table 2 also shows

that with $\xi = 1.5$ and distributional weights that range from 0.929 at the bottom of the tax bracket to 0.1057 at the top of the tax bracket, the optimal flat tax rate would be over 50 percent.

Of course these computations are based on a simple model that used a rather arbitrary distribution function for wage rates and ignored other features of taxpayer behaviour, such as the ability of taxpayers, especially at high income levels, to receive income in non-taxable, or low tax, forms such as fringe benefits. It has also ignored the potential for under-reporting of earnings or tax evasion by working in the underground economy. All of these factors are relevant in designing the optimal income tax and should be incorporated in future models of the optimal flat tax.

References

- Browning, E.K., T. Gronberg, and L. Liu (2000) "Alternative Measures of the Marginal Cost of Funds". *Economic Inquiry* 38:591-599.
- Chennells, L., A. Dilnot, and N. Roback (1999) "A Survey of the UK Tax System" Institute for Fiscal Studies, London.
- Dahlby, B (2005) *The Marginal Cost of Public Funds: Theory and Applications*, draft.
- Dahlby, B. (1992) "The Efficiency Effects of an Excise Tax in a Generalized Cournot Oligopoly Model With an Application to the U.S. Cigarette Industry" *Public Finance* 47: 378-389.
- Dahlby, B. (1998). "Progressive Taxation and the Social Marginal Cost of Public Funds" *Journal of Public Economics* 67: 105–122
- Delipalla, S. and M. Keen (1992) "The Comparison Between Ad Valorem and Specific Taxes in Noncompetitive Environments", *Journal of Public Economics*, 53, 53-71.
- Delipalla, S. and O. O'Donnell (2001) "Estimating Tax Incidence, Market Power and Market Conduct: The European Cigarette Industry. *International Journal of Industrial Organization* 19: 885-908.
- Håkonsen, L. (1998) "An Investigation into Alternative Representations on the Marginal Cost of Public Funds" *International Tax and Public Finance* 5: 329-43.
- Hall, R.E. and A. Rabushka (1995), *The Flat Tax*; Second Edition. Stanford: Hoover Institution.
- Jones, S. (1993) "Cyclical and Seasonal Properties of Canadian Gross Flows of Labour" *Canadian Public Policy* 19(1) (March):1-17.
- Kleven, H.J. and C.T. Kreiner (2003) "The Marginal Cost of Public Funds In OECD Countries: Hours of Work Versus Labor Force Participation", CESIFO Working Paper No. 935.
- Mayshar, J. (1991) "On Measuring the Marginal Cost of Funds Analytically". *American Economic Review* 81(5) 1329–1335.
- Mendoza, E. G., A. Razin and L. L. Tesar (1994) "Effective Tax Rates in Macroeconomics: Cross-country Estimates of Tax Rates on Factor Incomes and Consumption." *Journal of Monetary Economics* 34: 297–323.
- Mirrlees, J.A. (1971) "An Exploration in the Theory of Optimal Income Taxation" *Review of Economic Studies* 38: 175-208.
- Parry, I.W.H. (2003) "On the costs of excise taxes and income taxes in the UK" *International Tax and Public Finance* 10: 281-304.
- Saez, E. (2001) "Using Elasticities to Derive Optimal Income Tax Rates" *Review of Economic Studies* 68: 205-229.
- Shapiro, C. and J.E. Stiglitz (1984) "Equilibrium Unemployment as a Worker Discipline Device" *American Economic Review* 74(3) (June):433-444.

- Slemrod, J., S. Yitzhaki, J. Mayshar, and M. Lundholm (1994) "The Optimal Two-Bracket Linear Income Tax" *Journal of Public Economics* 53: 269–290.
- Stern, N.H. (1976) "On the Specification of Models of Optimum Taxation" *Journal of Public Economics* 6: 123-162.
- Stuart, C. (1984) "Welfare Costs per Dollar of Additional Tax Revenue in the United States" *American Economic Review* 74: 352-362.
- Triest, R.K. (1990) "The Relationship Between the Marginal Cost of Public Funds and Marginal Excess Burden" *American Economic Review* 80: 557-566.

Table 1 Computation of the MCF in a Efficiency Wage Model

χ	e	δ_U	$\frac{w}{(1-ur)} \frac{dur}{dt}$	$\delta_U \frac{w}{(1-ur)} \frac{dur}{dt}$	$\frac{dw}{dt}$	$MCF_{\beta=0}$	$MCF_{\beta=1}$
0.9	0.306	0.170	0.543	0.092	-0.765	1.222	1.557
0.8	0.298	0.186	0.506	0.094	-0.781	1.213	1.517
0.7	0.288	0.206	0.468	0.096	-0.797	1.203	1.477
0.6	0.275	0.230	0.428	0.098	-0.814	1.194	1.437
0.5	0.259	0.260	0.387	0.100	-0.832	1.185	1.398
0.4	0.239	0.299	0.344	0.103	-0.851	1.176	1.360
0.3	0.211	0.352	0.299	0.105	-0.871	1.167	1.322
0.2	0.171	0.427	0.251	0.107	-0.891	1.159	1.285
0.1	0.109	0.544	0.202	0.110	-0.912	1.150	1.249

Notes: Computations based on the following parameter $x = (1 - ur)^\alpha$, $\alpha = 0.667$, $\epsilon_{Lw} = -(1 - \alpha)^{-1} = -3$, $ur = 0.089$, $l = 0.019$, $t = 0.3\alpha$, $B = 0.25\alpha$, and $\rho = 0.05$.

Table 2 The Optimal Flat Tax

	$\xi = 0.5$	$\xi = 1.5$
m_2	0.40958	0.5039
X_1	25,661	30,940
w_1	14.68	17.41
w_2	17.44	22.00
$\bar{Y}(w_2)$	67,089	73,799
$\zeta = X_1 / \bar{Y}(w_2)$	0.38249	0.41925
$F(w_2) - F(w_1)$	0.08757	0.12388
$1 - F(w_2)$	0.57252	0.44962
smb_x	0.25563	0.29738
$\eta(w_2)$	0.34468	0.43891
$\eta(\bar{w})$	0.28854	0.50917
$\eta(2\bar{w})$	0.13502	0.19879
$\hat{\eta}(w_2)$	0.14169	0.18453
$\theta(w_2)$	-0.20585	-0.20041
$\theta(\bar{w})$	-0.19318	-0.20937
$\theta(2\bar{w})$	-0.13814	-0.15058
$\bar{\theta}(w_2)$	-0.15425	-0.15579
$\beta(w_2)$	1.0628	0.92908
$\beta(\bar{w})$	1.000	1.000
$\beta(2\bar{w})$	0.72362	0.5392
$\beta(w_{top})$	0.15953	0.01057
$\hat{\beta}(w_2)$	0.69741	0.46532
$\bar{\beta}(w_2)$	0.80432	0.59132

Figure 1 The Effect of an Employment Tax in an Efficiency Wage Model

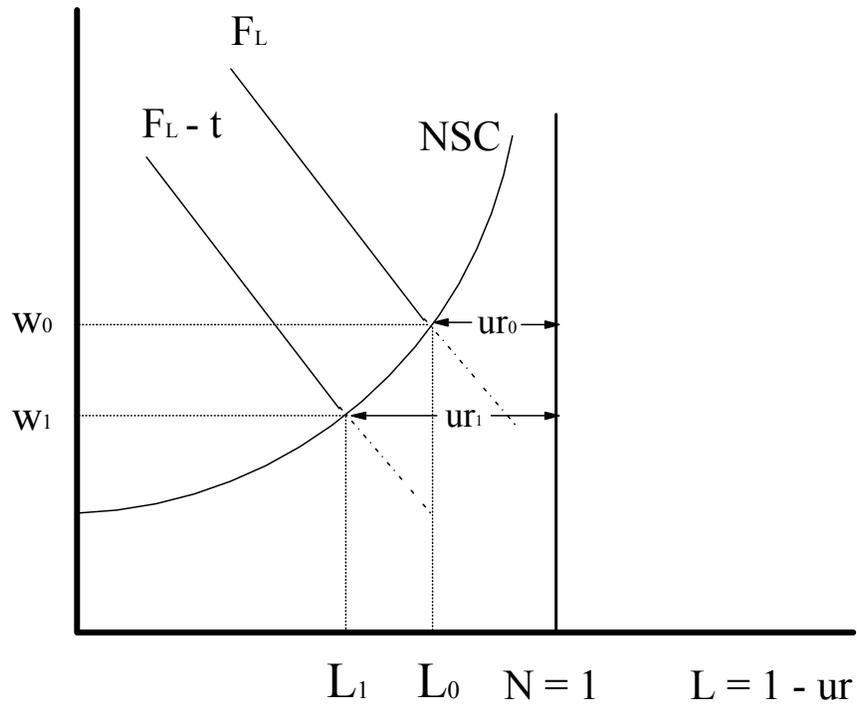


Figure 2 Individuals at the Kink in the Consumption-Earnings Schedule

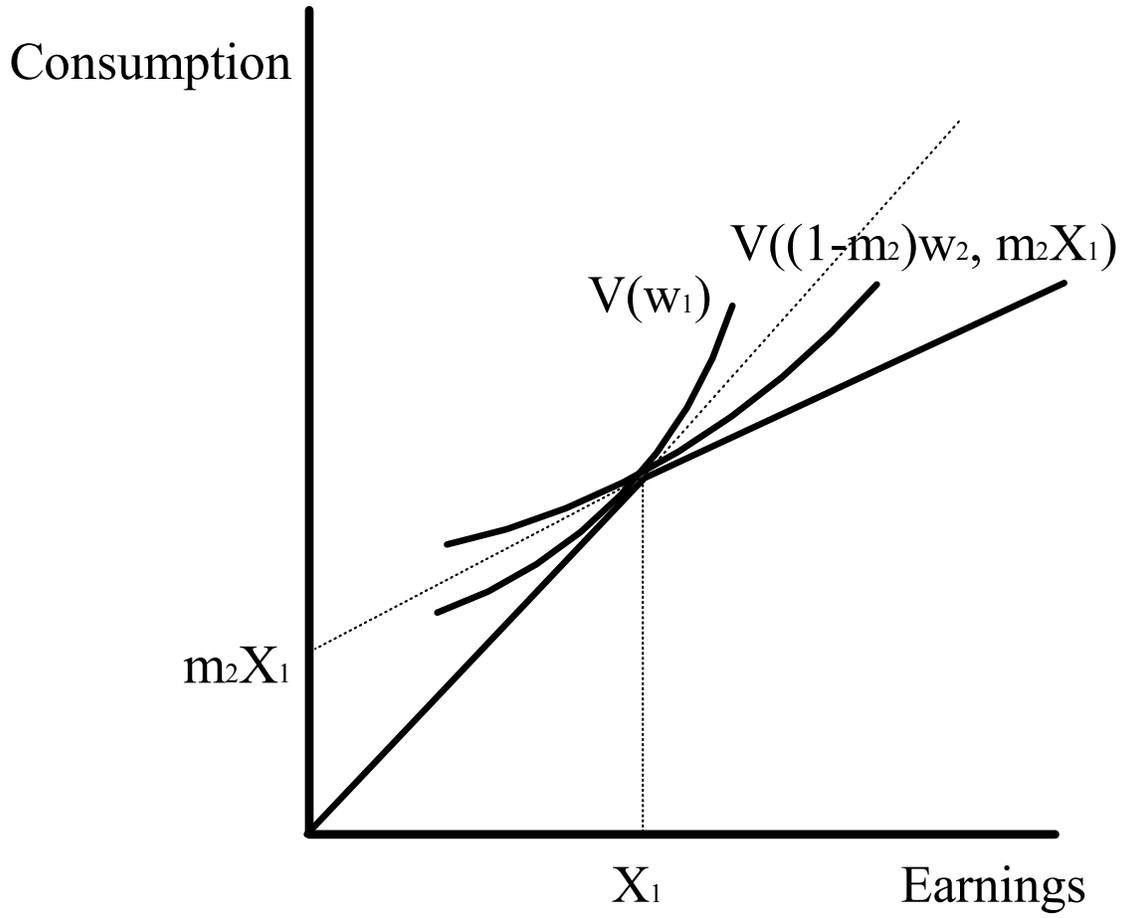


Figure 3 The Distribution of Wage Rates

