

Environmental Policy and Growth when Inputs are Differentiated in Pollution Intensity

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Abstract

Environmental policy affects the distribution of market shares if intermediate goods are differentiated in their pollution intensity. When innovations are environment-friendly, a tax on emissions skews demand towards new goods which are the most productive. In this case, the tax has to increase along a balanced growth path to keep the market shares of goods of different vintages constant. Comparing balanced growth paths, we find that an increase in the burden of environmental taxation spurs innovation because it increases the market share of recent vintages. As a result the cost of environmental policy in terms of slower growth is weaker and may even be absent.

Keywords: Endogenous growth; Environmental policy; Induced technological change

JEL Classification Codes: O41; Q28; H32; O30

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1 Introduction

Production activities have grown so much that they now play a primary role in the functioning of the ecosystem. The conservation of the ecosystem may significantly constrain further expansion of economic activity. The case of global warming is particularly important in the current international agenda of environmental policy. There is substantial agreement among the scientific community that climate change is exacerbated by emissions of green-house gases, such as methane and carbon dioxide (CO₂). Carbon emissions result from burning fossil fuel, the main source of primary energy. With current technology and equipment, a reduction in emissions requires a fall in energy inputs which is likely to lower output below its potential level. Emissions can be controlled by imposing a tax or a quota on emissions (supported for instance by a market for pollution permits). We define this policy a restrictive environmental policy.

For a positive analysis, a question arises:¹ how does a restrictive environmental policy influence the prospects of economic growth? The answer is not so obvious as the one discussed above concerning the impact on current level of production. This is because technology and equipment change in the long run. In recent years a body of theoretical papers has addressed this question using endogenous growth models. In these analyses emissions are formalized as an input or a by-product. In any case environmental policy operates through a *direct channel* of transmission. It increases the cost of emission inputs or forces firms to engage in abatement expenditure and therefore reduces the return on capital. As a result, the rate of investment falls and this slows output growth rate. In short, there is a trade-off between economic growth and environmental quality protection.²

A number of analyses have explored the possibility that some other channels of transmission of environmental policy on growth can relax this trade-off. Within the class of one-sector growth models two main channels have been proposed. A first strand of papers examines how environmental policy affects growth through its influence on households' saving behavior (e.g. Fisher and Marrewijk 1998). In particular, if consumption and environmental quality are complements, expected improvements in environmental quality induce households to save more and postpone consumption (Mohtadi 1996, Michel and Rotillon 1995). It is questionable though that environmental policy may play a primary role in the determination of households' savings. A second approach assumes increasing

¹The question may not be relevant for the design of environmental policy, given that the latter should target social welfare. Yet the answer is interesting on its own, at least in view of governments' reluctance to engage in stringent targets for the reduction of CO₂ emissions.

²See, e.g. van Marrewijk et al. (1993), Stokey (1998), Grimaud (1999), van Zon and Yekiner (2003).

returns in pollution abatement, and finds that growth makes it possible to increase the efficiency of abatement activities (Michel 1993, Xepapadeas 1994, Andreoni and Levinson 2001). Within the class of multi-sector growth models instead two arguments have been put forward to justify positive channels of transmission of environmental policy on growth. On the one hand, improvements in environmental quality may enhance either total factor productivity or productivity in the core sector for growth, such as human capital accumulation.³ These assumptions seem plausible for economies that rely heavily on the exploitation of natural resources or economies where pollution is so serious as to weaken pupils' cognitive ability. In the case of most industrialized economies it is debatable whether improvements in the quality of the environment can have first order effects on factors' productivity. On the other hand, some authors argue that environmental policy entails a reallocation of resources out of the production sector and into the dynamic sector of the economy, which may result in faster growth. Environmental policy may then foster human capital accumulation, as in Hettich (1998), or R&D activity as in Elbasha and Roe (1996) or Bretschger (1998). Our contribution adds to the latter strand of literature. For a survey of these approaches see Ricci (2004).

This paper provides the rationale for an alternative channel of transmission of environmental policy on growth which tends to relax the trade-off between environmental quality protection and economic growth. The theory presented in the following pages gives structure to the statement "environmental policy fosters innovations and can therefore promote growth, regardless of any productive or amenity role of environmental quality". In order to sharpen the argument we deliberately abstract from the functioning of the ecosystem, and from the influence of environmental quality or pollution on welfare and on factors' productivity. As a result, our analysis does not include any normative justification for environmental policy. This feature is meant to strengthen our argument and to single out its originality: the independence of the transmission channel from any externality resulting from improvements in environmental quality. The analysis can easily be generalized to include welfare considerations for normative enquiry.⁴

This paper builds on the Schumpeterian growth theory, which explicitly models incentives to engage in productivity enhancing activities (hereafter R&D) (Grossman and

³See e.g. Bovenberg and Smulders 1995, Smulders and Gradus 1996, Gradus and Smulders 1993.

⁴The welfare analysis is presented in Ricci (2000). The results are very close to those of Aghion and Howitt (1998, ch.5). The only difference is that in our model, as compared to theirs, the optimal output growth rate is larger, while the optimal emissions growth rate is lower because both pollution control and output growth require R&D.

Helpman 1991, Aghion and Howitt 1992).⁵ It is assumed that innovations improve the quality of capital goods in two dimensions: their productivity and their pollution intensity. New goods are characterized by higher productivity and, possibly, lower pollution intensity than existing goods.⁶ If innovations embody the same pollution intensity as the goods they replace, emissions grow at the same rate as output. If instead innovations have a cleaner technology, emissions grow at a slower rate than output. The flow of services that capital goods provide are called intermediate goods. Their operating cost is composed of the rental rate of capital and the tax on emissions associated with their use. The relative weight of these components can be managed by the environmental policy-maker and affects incentives to invest and the pace of growth if it varies across vintages. The next section presents the model, while section 3 characterizes balanced growth paths.

In section 4, we analyze the case where the extent to which innovations are cleaner is exogenous, and in particular independent of environmental policy. This assumption allows us to focus on the description of an original channel of transmission through which environmental policy affects the pace of technological change and the rate of growth. We show that a tax on emissions has a *distortionary impact* on competition across sectors, when goods are differentiated in their pollution intensity. To the extent that there is a (negative) correlation between the productivity of goods and their pollution intensity, tax on emissions acts as a (relative) reward to innovators. In fact, tax on emissions increase the market share of the newer and more productive goods and increase the relative pay-off to R&D investment. We find that when goods are differentiated in emission intensity, an increase in the burden of emissions taxes raises the long-run rate of growth by fostering R&D activity, even though - on impact - it may reduce the level of aggregate output.

Our argument is close to Xepapadeas and de Zeeuw (1999) and Feichtinger et al. (2004), who analyze a single firm's choice of the vintage composition of its capital stock. They show that an increase in emissions taxes induces firms to reduce the average age of their capital stock, and identify the conditions under which this response implies higher productivity of capital. The theory presented in the following sections incorporates this mechanism in a dynamic general equilibrium model, where the supply-of-capital side of the economy is endogenous in both its quality (designs of innovations) and quantity (saving-

⁵Our model is an extension of Aghion and Howitt's (1998, p.85-92). As explained in Ricci (2002), the Aghion-Howitt framework is the only dynamic general equilibrium model where the degree of heterogeneity of profits across sectors is endogenously determined. This feature is crucial for our results.

⁶We do not allow for increased pollution intensity of capital goods. While this assumption seems uncontroversial for industrialized economies, there is evidence that even in developing countries "the intensity of industrial pollution at the end-of-pipe declines strongly with income" (Hettige et al. 1997).

investment decision) dimensions. Our model nevertheless is simpler in some respects, namely in assuming that capital is putty-putty while only technologies are putty-clay.

Section 5 considers the case where R&D laboratories choose endogenously the pollution intensity of innovations. Here environmental policy affects the extent to which innovations embody cleaner technologies, i.e. the *direction of R&D*.⁷ In this way we reintroduce the possibility that environmental policy has a negative impact on growth. In fact, emissions are an implicit input of production. Cleaner innovations are relatively less productive, because they use a smaller amount of the complementary emissions input. Hence, as environmental policy induces R&D laboratories to design cleaner goods, the marginal effect of R&D on productivity growth decreases. This *direct input effect* runs in a direction opposite to the one resulting from the distortionary impact of emissions taxes (which tends to foster R&D). Solving the model numerically, we find that the direct effect dominates. However, the distortionary impact of taxation is active and relaxes the growth-environment trade-off.

The analysis by Hart (2004 a) is very close to ours. Similarly, it builds on the Aghion-Howitt framework where environmental policy heterogeneously affects profits across vintages, if these embody different pollution intensity technologies. Unlike in our case, attention is restricted to state contingent technology standards, implying an exogenously determined maximum age of equipment in use.⁸ Also analytical results are obtained under a simplifying assumption: R&D labs are constrained to choose their technology out of a discrete two-point set. Because of this rigidity, the direct input effect may be weak enough for the distortionary effect to dominate, giving rise to a win-win outcome with environmental policy improving both growth rate and environmental quality. We find that this result is not robust to an increase in flexibility in R&D labs' technological choice.

We first present the model and its balanced growth properties. Next we analyze the consequences of tightening environmental policy on innovation, direction of technological change and growth rate.

2 The model economy

We extend the Schumpeterian model of endogenous growth (like that of Aghion and Howitt 1998, p.85-92) to consider that production emits pollutants. Production takes

⁷See Popp (2002) and Jaffe et al. (2002) for relevant empirical evidence.

⁸Hart (2004 a) considers environmental sales taxes that are individually tailored on vintages. The optimal policy consists in raising the vintage-specific tax rate upon arrival of innovations, ultimately banning sales of older vintages. Hart (2004 b) explicitly considers technology standards.

place in three stages. First, labor is competitively engaged in research and development (R&D) activities aimed at designing higher-quality intermediate goods. Successful innovations are characterized by higher productivity and, possibly, lower pollution intensity. Pollution intensity is a technological variable of the intermediate good, which is chosen by the R&D laboratory when it introduces the good on the market. This choice is irreversible and a lower pollution intensity implies a lower productivity of the good. Second, designs are protected by patents, so that intermediate goods are supplied under local monopoly power. These goods are produced employing capital. Their production also implies emissions of pollution. We assume a continuum of intermediate goods. Producers rent capital from households and pay a tax per unit of emissions resulting from their activities. Third, intermediate goods are combined with labor in the final sector to produce a homogeneous good which can be consumed or invested.

In this section we first present the production functions of the final and intermediate sectors, and emissions. Next we study the behavior of the agents: the final sector, a representative intermediate good monopolist, the R&D sector, consumers and the government.

2.1 Production and the environment

Final output is produced employing labor and a continuum of intermediate inputs according to the production function:

$$Y_\tau = (1 - n_\tau)^{1-\alpha} \int_0^1 Z_{j\tau} A_{j\tau} x_{j\tau}^\alpha dj \quad (1)$$

where $\alpha \in (0, 1)$; labor supply is fixed and normalized to unit mass; a share $(1 - n)$ of labor is employed in production and n in R&D activities (if the labor market clears); $x_{j\tau}$ is the quantity of intermediate good $j \in [0, 1]$ used at date τ . The technology embodied in intermediate goods is described by a two-dimensional vector characterized by parameters A and Z . $A_{j\tau}$ is the implicit labor productivity index and $Z_{j\tau}$ the pollution intensity index of intermediate good j at date τ . As shown in the production function (1), the productivity of good j at date τ depends on the product $Z_{j\tau} A_{j\tau}$. These definitions and the link between parameter Z and productivity are explained at the end of this subsection.

Intermediate goods are produced employing capital, according to:

$$x_{j\tau} = \frac{K_{j\tau}}{A_{j\tau}} \quad (2)$$

Thus, intermediate goods are services from capital goods, and the more productive the good the higher its capital intensity.⁹

The flow of emissions, $P_{j\tau}$, associated with the use of a capital good depends on its pollution intensity index at date τ , $Z_{j\tau}$, and is given by, $\forall j \in [0, 1]$:

$$P_{j\tau} = Z_{j\tau}^{1/\alpha\beta} K_{j\tau} \quad (3)$$

with $\beta \in (0, 1)$. Thus aggregate emissions, P , can be defined as follows:

$$P_\tau = \int_0^1 P_{j\tau} dj = \int_0^1 Z_{j\tau}^{1/\alpha\beta} K_{j\tau} dj \quad (4)$$

Substituting $K_{j\tau}$ in (2) using (3), we see that good j is produced out of emissions:

$$x_{j\tau} = \frac{P_{j\tau}}{Z_{j\tau}^{1/\alpha\beta} A_{j\tau}}.$$

(3) can be written as:

$$Z_{j\tau} = \left(\frac{P_{j\tau}}{K_{j\tau}} \right)^{\alpha\beta}$$

Thus $Z_{j\tau}$ is a measure of the emissions-capital ratio characteristic of good j at date τ . For a given technology Z_j , the substitution of capital for emissions cannot take place, e.g. in response to a shift in their relative price. Substitution can take place only with the introduction of a new technology in sector j , say Z'_j . Pollution intensity is reduced if the new technology satisfies $Z'_j < Z_j$. A new technology can be introduced only through R&D. At the industry level, substitution is therefore costly and discontinuous over time.

Emissions are implicit inputs that are combined with intermediate inputs according to their pollution intensity. Substituting $Z_{j\tau}$ above and $x_{j\tau}$ in (1) using (2), we get:

$$Y_\tau = \int_0^1 [(1 - n_\tau) A_{j\tau}]^{1-\alpha} \left[P_{j\tau}^\beta K_{j\tau}^{1-\beta} \right]^\alpha dj$$

We can think of machinery that is employed in the production process. Labor is required to operate it, and its use implies some pollution. Labor productivity and the dirtiness of the production process depend on the design of the machinery, according to its technological parameters A and Z . The productivity of intermediate goods is increasing in the

⁹This is a standard assumption necessary for balanced growth paths to exist (see e.g. Aghion and Howitt 1998, ch.3).

pollution intensity index because emissions represent an input complementary to capital.

2.2 Prices and the green tax

The price of final output is normalized to unity. We denote by w the wage, by p_j the price of intermediate input $j \in [0, 1]$, by r the rate of return on savings, by V_τ the value of an innovation introduced at date τ . Moreover, the government levies a tax per unit of emissions, h , on intermediate goods producers for emissions associated with their sales.

2.3 The final sector

The instantaneous profits of the fictitious competitive final firm are:

$$\psi_\tau = (1 - n_\tau)^{1-\alpha} \int_0^1 Z_{j\tau} A_{j\tau} x_{j\tau}^\alpha dj - w_\tau(1 - n_\tau) - \int_0^1 p_{j\tau} x_{j\tau} dj$$

Therefore the (inverse) demand for labor from the final sector is given by:

$$w_\tau = (1 - \alpha)(1 - n_\tau)^{-\alpha} \int_0^1 Z_{j\tau} A_{j\tau} x_{j\tau}^\alpha dj \quad (5)$$

and the (inverse) demand for intermediate inputs is given by, $\forall j \in [0, 1]$:

$$p_{j\tau} = Z_{j\tau} A_{j\tau} \alpha (1 - n_\tau)^{1-\alpha} x_{j\tau}^{\alpha-1} \quad (6)$$

2.4 The intermediate goods monopolists

Consider the problem of the monopolist in sector j characterized by technology $\{A_j, Z_j\}$. It rents A_j units of capital from households and is subject to a green tax burden $h_\tau P_{j\tau}/x_{j\tau} = h_\tau A_j Z_j^{1/\alpha\beta}$ per unit produced, from (2) and (3). Hence, the monopolist maximizes instantaneous profits $\Pi_{j\tau} = [p_{j\tau} - A_j(r_\tau + h_\tau Z_j^{1/\alpha\beta})]x_{j\tau}$. Substituting the demand from the final sector, (6), and proceeding for maximization, we obtain partial equilibrium sales, the pricing rule and profits of the monopolist in sector j :¹⁰

$$\hat{x}_{j\tau} = (1 - n_\tau) \left(\frac{\alpha^2 Z_j}{r_\tau + h_\tau Z_j^{1/\alpha\beta}} \right)^{\frac{1}{1-\alpha}} \quad (7)$$

¹⁰Results do not change if the green tax were levied on the final sector. The demand for good j is in this case $p_{j\tau} = A_j[\alpha(1 - n_\tau)^{1-\alpha} Z_j x_{j\tau}^{\alpha-1} - h_\tau Z_j^{1/\alpha\beta}]$. The monopolist maximizes $\pi = (p_{j\tau} - A_j r_\tau)x_{j\tau}$. Sales and profits are given by (7) and (8), the price is lower, i.e. $p_{j\tau} = A_j[r_\tau/\alpha + (1 - \alpha)h_\tau Z_j^{1/\alpha\beta}/\alpha]$.

$$\hat{p}_{j\tau} = A_j \frac{r_\tau + h_\tau Z_j^{1/\alpha\beta}}{\alpha}$$

$$\hat{\Pi}_{j\tau} = A_j \frac{1-\alpha}{\alpha} [r_\tau + h_\tau Z_j^{1/\alpha\beta}] \hat{x}_{j\tau} \quad (8)$$

Notice that profits are:

- increasing in the *total productivity* index $A_j Z_j^{\frac{1}{1-\alpha}}$ of good j ;
- decreasing in the *marginal cost* of firm j : $m_{j\tau} = [r_\tau + h_\tau Z_j^{1/\alpha\beta}]$

The green tax depresses sales and profits and more so the dirtier is the good (the higher Z_j). This feature of the model is crucial to our results: the green tax has a heterogeneous impact on profits across goods, when they are differentiated in pollution intensity, i.e.

$$Z_{j\tau} \neq Z_{i\tau} \text{ for } j \neq i, h_\tau > 0 \Rightarrow \hat{\Pi}_{j\tau} \neq \hat{\Pi}_{i\tau}$$

2.5 R&D

In the competitive R&D sector, every firm targets improvements of one particular intermediate good. R&D activity is modelled as a Poisson process with instantaneous arrival rate λn_j , where n_j is the mass of labor employed in R&D in sector j and $\lambda > 0$ is a productivity parameter. Each innovation improves the quality of the intermediate good in both dimensions, A and Z . Specifically, an innovation allows the patent holder to produce the intermediate good characterized by the leading-edge technology, that is, the highest of all A 's, denoted by \bar{A} , and the lowest of all Z 's, denoted by \underline{Z} , at the date of arrival of the innovation (an intersectorial spillover).¹¹ Each innovation marginally contributes to an improvement of the leading-edge technology in the two dimensions. This intertemporal spillover - a pure production externality - is modelled by assuming that the rate of growth of the leading-edge technology is proportional to the aggregate flow of innovations $\lambda n = \int_0^1 \lambda n_j dj$:

$$\frac{\dot{\bar{A}}_\tau}{\bar{A}_\tau} = \gamma \lambda n_\tau \quad \gamma > 0 \quad (9)$$

$$\frac{\dot{\underline{Z}}_\tau}{\underline{Z}_\tau} = \zeta \lambda n_\tau \quad \zeta \leq 0 \quad (10)$$

ζ is the aggregate index of the *direction of R&D*, because it measures to what extent innovations are environment-friendly. If $\zeta = 0$ innovations have the same pollution intensity as the goods they replace, and emissions associated with their use are larger because

¹¹The leading-edge technology, i.e. the state of knowledge, is a public good as an input for R&D.

innovations are more capital intensive. Instead, innovations are cleaner if $\zeta < 0$, that is, if their pollution intensity is lower. Whether the emissions associated with the use of new goods are lower than those generated by old goods depends on the size of ζ . Note that in any case, as soon as $\zeta < 0$ emission intensity is correlated to the productivity of goods. If $\zeta < 0$ a tax on emissions is a policy tool that makes it possible to indirectly discriminate goods according to their productivity.

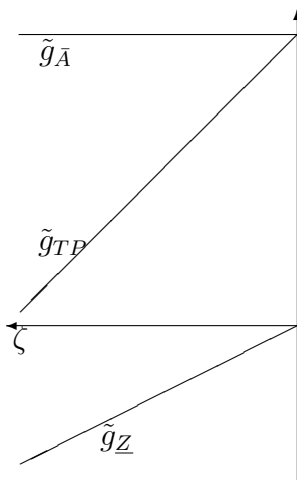


Figure 1: The direction of R&D

Figure 1 plots the marginal impact of R&D employment on the growth rate of three technological parameters as function of ζ . Line $dg_{\bar{A}}/dn = \gamma\lambda$ represents the growth rate of the leading-edge implicit labor productivity index, \bar{A} , which is independent of ζ . Line $dg_{\underline{Z}}/dn = \zeta\lambda$ depicts the growth rate of the leading-edge pollution intensity, \underline{Z} . Line $dg_{TP}/dn = \gamma\lambda + \zeta\lambda/(1 - \alpha)$ is the growth rate of the total productivity index of the leading-edge good, $\bar{A}\underline{Z}^{\frac{1}{1-\alpha}}$. It is clear that ζ measures the direction of technological change: it determines whether and by how much R&D improves the total productivity and the cleanliness of goods.¹²

Free entry in R&D ensures that at equilibrium the following arbitrage condition holds:

$$n_{\tau} \in (0, 1) \quad \Rightarrow \quad w_{\tau} = \lambda V_{\tau} \quad (11)$$

where V_{τ} is the value of an innovation arrived at date τ . If R&D activity takes place at all, then its marginal cost (the wage) equals its expected marginal return.

¹²Targeting cleaner innovations does not affect the cost of R&D, nor the difficulty of R&D, as assumed by Verdier (1995). Yet increasing the degree of targeted cleanliness reduces the total productivity of innovations. This trade-off is equivalent to the case of explicit R&D cost, as explained in section 5.

An innovation is worth the present value of the expected stream of profits:

$$V_\tau = \int_\tau^\infty e^{-\int_\tau^t r_s ds} e^{-\lambda \int_\tau^t n_s ds} \hat{\Pi}_t(\bar{A}_\tau, \underline{Z}_\tau) dt$$

where $\hat{\Pi}_t(\bar{A}_\tau, \underline{Z}_\tau)$ denotes profits at date t of a monopoly characterized by technology $\{\bar{A}_\tau, \underline{Z}_\tau\}$. The first discount factor takes into account the opportunity cost, i.e. the return on savings. The second discount factor is the monopoly's survival probability, because the next innovation in the sector will make its patent obsolete.

2.6 Consumers and the government

The representative consumer chooses the path of consumption to maximize the present value stream of instantaneous isoelastic utilities, subject to a dynamic budget constraint:¹³

$$\max_{\{c\}_0^\infty} \int_0^\infty e^{-\rho\tau} \frac{c_\tau^{1-\varepsilon}}{1-\varepsilon} d\tau$$

$$\dot{W} = w_\tau + r_\tau W_\tau - c_\tau + T_\tau$$

where W is financial wealth and T are transfers from the government. The solution links the rate of growth of consumption to the rate of return on savings and preference parameters, according to the Ramsey rule:

$$g_c = \frac{r_\tau - \rho}{\varepsilon} \quad (12)$$

where g_i denotes the growth rate of variable i . To rule out trivial paths of savings, the solution must satisfy the no-Ponzi game condition:

$$\lim_{\tau \rightarrow \infty} e^{-\int_0^\tau r_s ds} W_\tau = 0$$

Finally, we turn to the government's budget constraint. To be simple but without loss of generality, we assume that the budget is held to be balanced at any given date:

$$h_\tau P_\tau = T_\tau$$

¹³With this formalization of the utility function we deliberately abstract from the direct impact that environmental policy may have on saving behavior, via amenities and externalities. Of course, savings react to changes in their rate of return.

3 Balanced growth path analysis

Along a balanced growth path, n and ζ must be constant for \bar{A} and \underline{Z} to grow at constant rates from (9) and (10). Furthermore, the law of motion of capital, $\dot{K}_\tau = Y_\tau - c_\tau$, implies that capital, output and consumption grow at a common rate $g = g_c$. Finally, according to the Ramsey rule (12) g is constant only if r is constant. Hence, we obtain the following:

Proposition 1 *A balanced growth path exists if the green tax increases according to the following policy rule:*

$$g_h = -\frac{g\underline{Z}}{\alpha\beta} = \frac{-\zeta}{\alpha\beta}\lambda n \quad (13)$$

Along this path output growth is function of n and ζ according to:

$$g = \left(\gamma + \frac{\zeta}{1-\alpha} \right) \lambda n \quad (14)$$

Therefore, growth is positive only if:

$$\zeta \in ((\alpha - 1)\gamma, 0] \quad (15)$$

Proof. We compute the value of an innovation using (7) and (8) for $Z_j = \underline{Z}_\tau$, $A_j = \bar{A}_\tau$:

$$\begin{aligned} V_\tau &= \frac{1-\alpha}{\alpha} \bar{A}_\tau (\alpha^2 \underline{Z}_\tau)^{\frac{1}{1-\alpha}} (1-n) \int_\tau^\infty e^{-(r+\lambda n)(t-\tau)} \left[r + h_t \underline{Z}_\tau^{1/\alpha\beta} \right]^{\frac{-\alpha}{1-\alpha}} dt \\ &= \bar{\Pi}_\tau \int_\tau^\infty e^{-(r+\lambda n)(t-\tau)} \left(\frac{\bar{m}_\tau}{m_t} \right)^{\frac{\alpha}{1-\alpha}} dt \end{aligned} \quad (16)$$

Where $\bar{\Pi}_\tau$ and \bar{m}_τ denote initial profits and the marginal cost of an innovator at date τ , and $m_t = r + h_t \underline{Z}_\tau^{1/\alpha\beta}$ is the marginal cost at future dates $t > \tau$ of the firm innovating at τ . The latter increases over time, and profits are crowded out, if and only if the green tax increases. The integral in the first expression is constant over time if the marginal cost of the leading-edge monopolist, \bar{m}_τ , is constant, i.e. if $h_\tau \underline{Z}_\tau^{1/\alpha\beta}$ is independent of τ . This is ensured by (13).

For n to be constant the arbitrage condition (11) must hold at all times for the equilibrium level of n . Under policy rule (13) the value of patents (proportional to the right-hand-side of (11)) grows at the same rate as $\bar{\Pi}_\tau$, that is $g_V = g_{\bar{A}} + \frac{1}{1-\alpha} g_{\underline{Z}}$. The left-hand-side of (11) increases with the wage. The latter is the productivity of labor in the final sector, i.e. $w_\tau = (1-\alpha) Y_\tau / (1-n)$. Hence $g_w = g$ for n constant. Taking logarithms and differentiating (11), we get:

$$g = g_w = g_V = g_{\bar{A}} + \frac{1}{1-\alpha} g_{\underline{Z}}$$

(14) is derived using (9) and (10). ■

Along a balanced growth path, the green tax increases at a constant rate if innovations are environment-friendly. This is the case when the emissions-capital ratio of innovations

is lower than that of the goods they replace, i.e. when $\zeta < 0$.¹⁴ To understand this result in terms of time-invariancy of the distribution of market shares across vintages, let us suppose that the green tax is held constant although innovations are environment-friendly. In this case the weight of the green tax burden over marginal cost for innovations would fall over time. Then innovations would become increasingly competitive relative to existing intermediate goods. As a result the market share of innovations would progressively increase. This is incompatible with the concept of balanced growth.¹⁵

The crucial feature of environmental policy in this economy is that it affects the relative costs across goods of different vintages. Let $H = h_\tau Z_\tau^{1/\alpha\beta}$ define the burden of green tax on innovations. Under policy rule (13) H and the marginal cost of the leading-edge good, $\bar{m} = r + H$, are constant. Thereafter the marginal cost increases with the age of the technology, so that at some later date $t > \tau$ it is equal to $m_t = r + e^{g_h(t-\tau)}H$. Hence the distribution of intermediate goods according to their technological age is characterized by the marginal cost of the leading-edge sector relative to that of firms of age s :

$$\frac{\bar{m}}{m_s} = \frac{r + H}{r + e^{g_h s} H}$$

Older technologies are less competitive than new ones because of their relative dirtiness, implying a larger green tax burden. The ratio would indeed be constant in the absence of environment-friendly technological progress (i.e. $\zeta = 0$ and $g_h = 0$) or in the absence of taxation ($h = 0$). This effect of environmental policy is called the “green crowding-out” effect, because the policy reduces the competitiveness of aging technologies and crowds out their profit generating capacity.

The loss of competitiveness follows the path illustrated in figure 2. Under policy rule (13), the distribution of market shares across goods of different age is time invariant. Therefore the two competitiveness-loss functions (one backward looking, \bar{m}/m_s , the other forward looking, \bar{m}_τ/m_t , in (16)) are independent of τ and coincide for $s \equiv (t - \tau)$.

¹⁴That the tax per unit of emissions increases along balanced growth paths with declining pollution intensity of output is a result common to all models with emissions inputs. In fact, as P/Y declines, the marginal product of emissions increases, and this is reflected in their implicit price.

¹⁵As the market share of innovations increases, so does the value of innovations (which is forward looking) relative to the cost of innovation (which reflects the current cross-sectoral distribution of market shares). Then the incentive to engage in R&D grows faster than its cost, and R&D activity intensifies over time (n grows).

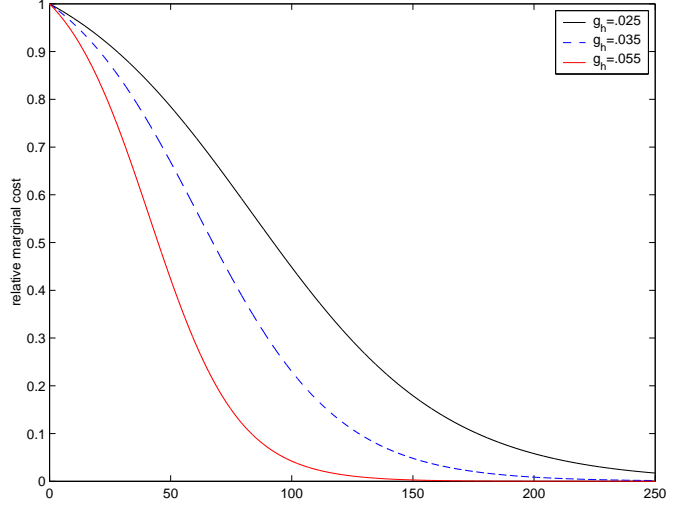


Figure 2: Competitiveness-loss or “green crowding-out” effect

3.1 The aggregate economy

We use three aggregation factors, Γ , Λ and Δ , to express aggregate variables as proportional to the corresponding variables in the leading-edge sector. In appendix A.1 we define these factors and present their properties.¹⁶ We can then compute aggregate demand for capital from the intermediate goods sector by integrating over the space of goods the rearranged production function (2), to obtain:

$$K_\tau = \bar{A}_\tau \bar{x}_\tau \Gamma \quad (17)$$

Similarly, using (1), (2) and (4) the flow of aggregate emissions and output is:

$$P_\tau = \underline{Z}_\tau^{\frac{1}{\alpha\beta}} \bar{A}_\tau \bar{x}_\tau \Lambda \quad (18)$$

$$Y_\tau = (1-n)^{1-\alpha} \underline{Z}_\tau \bar{A}_\tau \bar{x}_\tau^\alpha \Delta \quad (19)$$

It is worth spending a few words to explain the meaning of these aggregation factors. Consider Δ that measures the relative contribution of older goods to production:

$$\Delta = \int_0^1 \frac{Z_{j\tau} A_{j\tau} x_{j\tau}^\alpha}{\underline{Z}_\tau \bar{A}_\tau \bar{x}_\tau^\alpha} dj = \lambda n \int_0^\infty e^{-(\lambda n + g)s} \left(\frac{\bar{m}}{m_s} \right)^{\frac{\alpha}{1-\alpha}} ds \quad (20)$$

¹⁶The aggregation factors are constant along a balanced growth path.

It is computed by taking into account the mass of existing goods of age s , their total productivity gap, and their relative use. Initially, each technology is embodied in a mass λn of goods, out of which only a proportion $e^{-\lambda ns}$ of goods of age s survives at date τ . Older goods are less productive than the leading-edge good, and their productivity gap is ruled by the growth rate of total productivity, $AZ^{1/(1-\alpha)}$, which equals g by (14). Finally, sales of older goods are affected by environmental policy according to the competitiveness-loss function \bar{m}/m_s .

Substituting \bar{x}_τ in (18) using (17), aggregate emissions are:

$$P_\tau = \underline{Z}_\tau^{\frac{1}{\alpha\beta}} K_\tau \frac{\Lambda}{\Gamma} \quad (21)$$

which can be rearranged as follows :

$$\underline{Z}_\tau = \left(\frac{P_\tau \Gamma}{K_\tau \Lambda} \right)^{\alpha\beta} \quad (22)$$

Thus leading-edge pollution intensity also measures the pollution intensity of aggregate output. If $\zeta < 0$ pollution intensity declines continuously for the economy as a whole, although the process is discontinuous at the firm level.

Next, substituting \bar{x}_τ in (19) using (17), we obtain aggregate output as follows:¹⁷

$$Y_\tau = \underline{Z}_\tau [(1-n)\bar{A}_\tau]^{1-\alpha} K_\tau^\alpha \frac{\Delta}{\Gamma^\alpha} \quad (23)$$

Finally, substituting \underline{Z} in (23) using (22), we can write emissions explicitly as inputs in the production function:

$$Y_\tau = \Delta [(1-n)\bar{A}_\tau]^{1-\alpha} \left[\left(\frac{P_\tau}{\Lambda} \right)^\beta \left(\frac{K_\tau}{\Gamma} \right)^{1-\beta} \right]^\alpha \quad (24)$$

It is clear then that the lower are emissions inputs the lower is output. Moreover, equation (24) shows that emissions are combined with services from capital goods, with unitary elasticity of substitution, and then this composite good is combined with labor.

To conclude on the aggregate picture of the economy, we find that green tax revenue

¹⁷This functional form is similar to the one used by Stokey (1998), Aghion and Howitt (1998, ch.5), Grimaud (1999) and Grimaud and Ricci (1999). In those models emissions are an input in the final sector of the economy, independent of the mix of capital goods employed in production. Final sector firms control pollution intensity in response to the relative price of pollutants. In the present paper instead pollution intensity is a technological state variable and its improvement is gradual, it requires R&D investment and a change in the composition of capital stock. This feature gives rise to new results.

grows at the same rate as output. In fact, green tax revenue grows at rate $g_h + g_P$ which equals g from (21) under policy rule (13). Hence, transfers to households grow at this rate to keep the budget balanced. This property provides a simple implementation rule for policy (13): the tax level must be set so as to maintain constant the weight of green tax revenue over output.

3.2 General equilibrium

The dynamic general equilibrium is determined when the labor market clears and workers are indifferent as to whether they work in final sector firms or in R&D firms. The equilibrium level of R&D employment, n , equates labor productivity in the final sector (5) to the expected marginal return on R&D from the arbitrage condition (11), using (19):

$$(1 - \alpha)(1 - n)^{-\alpha} \underline{Z}_\tau \bar{A}_\tau \bar{x}_\tau^\alpha \Delta = \lambda V_\tau$$

Substituting V_τ using (16) and simplifying, the condition is:

$$(1 - n)^{-\alpha} \underline{Z}_\tau \bar{x}_\tau^\alpha \Delta = \frac{\lambda}{\alpha} (r + H) \bar{x}_\tau \int_\tau^\infty e^{-(r+\lambda n)(t-\tau)} \left(\frac{\bar{m}_\tau}{m_{t-\tau}} \right)^{\frac{\alpha}{1-\alpha}} dt \quad (\text{E})$$

Figure 3 depicts the left-hand-side of equation (E) at a given date τ as an upward sloping, and the right-hand-side as a downward sloping schedule in the (n, value) space.¹⁸ The equilibrium level of R&D, n^e , is determined at the intersection of the two schedules.

The left-hand-side is proportional to the marginal product of labor in the final sector. Due to diminishing returns, the latter decreases monotonically and tends to infinity as all labor is employed in R&D. The right-hand-side is proportional to the value of an innovation. The first factor before the integral, is proportional to the initial instantaneous profit of the innovator which is decreasing in n (see 16).¹⁹ Also the integral is strictly decreasing in n . This is straightforward for the discount factor and the survival probability. Furthermore, competitiveness loss proceeds at a faster pace, because the greater is the rate of innovation, the faster the green tax will be increasing, according to policy rule (13). The expected flow of profits is therefore crowded out at a faster rate.

It follows that, if at $n = 0$, the expected return to R&D is greater than the cost, there

¹⁸Both the LHS and the RHS in figure 3 shift downwards over time. However they cross at a constant level of n (see 25).

¹⁹An increase in n has two negative effects on this profit rate (see 8). First, lower labor inputs in the final sector reduce the marginal product of intermediate inputs, and thus depress demand (from 6). Second, the faster the growth rate, the higher the interest rate (from 12) and thus the marginal cost.

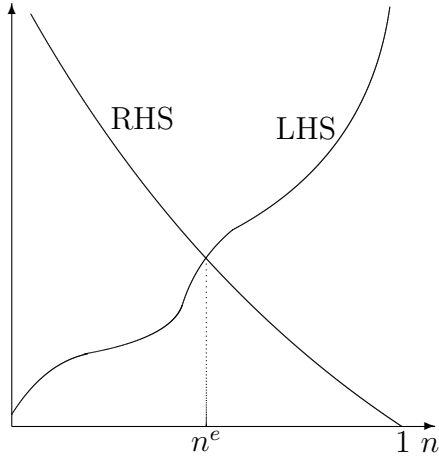


Figure 3: The equilibrium condition (E)

exists a unique equilibrium level of R&D activity, $n^e > 0$. Substituting \bar{x}_τ and Δ in (E) using (7) and (20), and simplifying, n^e is defined as the implicit solution of:

$$\frac{n}{\alpha(1-n)} \int_0^\infty e^{-(g+\lambda n)s} \left(\frac{\bar{m}}{m_s}\right)^{\frac{\alpha}{1-\alpha}} ds = \int_0^\infty e^{-(r+\lambda n)t} \left(\frac{\bar{m}}{m_t}\right)^{\frac{\alpha}{1-\alpha}} dt \quad (25)$$

4 The impact of environmental policy when the direction of technological change is exogenous

Let us assume that the aggregate index of the direction of R&D, ζ , is exogenous, and in particular independent of environmental policy. Under policy rule (13) $g_h = -\lambda n \zeta / \alpha \beta$ always, and therefore the policy tool of interest is the level of the green tax burden levied on the leading-edge producer: H . In fact, at any given date \underline{Z} is known, so that the policy-maker is able to set $H = h \underline{Z}^{1/\alpha \beta}$ by controlling the tax h . In doing so it can actually set the burden of green tax levied on each vintage and can effectively manage the shape of the competitiveness-loss schedule. We find the following result.

Proposition 2 *A marginal increase in the burden of the green tax levied on innovations, H , increases the level of R&D employment on the balanced growth path if and only if innovations are environment-friendly. That is:*

$$\frac{\partial n^e}{\partial H} > 0 \quad \text{iff } \zeta < 0$$

Therefore the rate of growth of the economy, g^e , is increasing in H (see 14).

Proof. See appendix A.2. The proof shows that, at the original equilibrium level of n , the left-hand-side of the equilibrium condition (E) falls more than its right-hand-side, if $\zeta < 0$. These shifts of the schedules result in a new equilibrium with a higher n . ■

A larger green tax burden directly reduces the innovator's prospective profits and the value of innovations. The larger burden of taxation, however, also translates into a lower demand for labor from the final sector, which reduces the cost of R&D. The proposition establishes that the fall in wages outweighs the fall in the value of innovations, when these are environment-friendly. As a result R&D activity increases at equilibrium.

The asymmetric impact of the tax on the cost of and reward to R&D is due to the fact that an increase in the green tax weighs more heavily on older technologies than on more recent ones. In other words, market shares are skewed in favor of modern intermediate goods, because they are cleaner. It must be emphasized that in our framework environmental policy can foster R&D activity only if intermediate goods are differentiated in their pollution intensity and only to the extent that the tax affects the distribution of market shares across vintages.²⁰ From this point of view, our result differs from those obtained in other multi-sector growth models, where environmental policy is favorable to the dynamic sector because it curbs demand for factors from the production sector (e.g. Verdier 1995, Elbasha and Roe 1996, Bretschger 1998, Hettich 1998).

Consider the reduced form of the equilibrium condition, (25). The downward shift of the competitiveness-loss schedule affects both sides of the equation. The left-hand-side represents the demand for labor from the final sector, which depends upon the distribution of sales across vintages. The reduction in the market share of relatively old goods is discounted according to their productivity gap, that is at rate g . The right-hand-side represents the demand for labor from the R&D sector, proportional to the value of an innovation. The latter depends upon the expected evolution of the market share of the patent holder. An expected fall in the future market share reduces the value of the patent according to discount rate r . Recall that along a balanced growth path $r > g$ (the no-Ponzi games condition). Since the negative impact of an increase in the green tax burden falls more heavily on older sectors and on future profits, $r > g$ implies that the loss has a heavier repercussion on labor demand from the final sector than from the R&D sector.

²⁰That $\partial n/\partial H = 0$ if $\zeta = 0$ is established in the proposition. Furthermore, H affects the dynamics of the system only to the extent that it modifies the shape of the competitiveness-loss schedule. Suppose for instance that the marginal cost consists exclusively of the green tax burden, so that $\bar{m}/m_t = e^{-g_h t}$, with g_h exogenous in this setting. In this case the equilibrium R&D employment is independent of H . (For proof of this statement see the FEEM working paper, n. 16.02, version of this article, or ask the author by email.)

In other words, the current situation of the labor market depends less on expected future events than on past events, as summarized by the distribution of technologies embodied in the capital stock.

The main lesson we want to retain from this result is that environmental policies are in general not neutral with respect to the cross-sectorial distribution of demand. Green taxes skew sales towards cleaner intermediate goods. When there is a negative correlation between pollution intensity and technological age, the tax on emissions acts as a (relative) reward for most recent vintages. Therefore, if innovations are environment-friendly, environmental policy fosters innovative activities and leads to faster productivity growth (given ζ). The result can be interpreted along the lines proposed by Xepapadeas and de Zeeuw (1999).²¹ An increase in the green tax reduces the average age of capital goods in use, increasing the average productivity of capital. As in any endogenous growth model, higher return on capital fosters investment and increases the growth rate.

A complete study of the dynamics of the system is beyond the scope of this paper. Such an analysis requires solving functional differential equations of mixed type, since investment in R&D at any given point in time involves delays (productivity gaps and market shares of existing goods) and advances (expectations on competitiveness loss for innovations). As far as we know, recent mathematical results have been applied only to a few special cases in economics (Benhabib and Rustichini 1991). Among these studies, those that run numerical simulations find that on impact an unexpected permanent shock makes the control variable (investment) overshoot its new long term level (e.g. Boucekine et al. 2004, Collard et al. 2004). These findings suggest that we can expect R&D employment to increase on impact as a result of an unexpected increase in H . This is a sufficient condition to establish that along a balanced growth path a marginal increase in H , reduces on impact the level of aggregate output:²²

$$\frac{\partial Y_\tau}{\partial H_\tau} < 0$$

This is in fact true if R&D employment, n_τ , does not decrease on impact. Moreover, the recession is deeper the more environment-friendly are innovations, i.e. the lower ζ .

Such a response from the economic system would not be surprising since we know from (24) that emissions are inputs in the aggregate production function. Thus the higher their relative price, the lower will be their employment and the lower the aggregate output for a

²¹See Feichtinger et al. (2004) for a rigorous and thorough exploration of this argument.

²²See the remark and (29) at the beginning of appendix A.2.

given distribution of technologies embodied in the capital stock. This input effect is active even in the absence of differentiation in pollution intensity. However, with differentiation the impact is magnified because the policy change affects older (i.e. dirtier) goods more heavily. That is, for a given dH the average burden of green taxes is increasing in $|\zeta|$.

5 The impact of environmental policy when the direction of technological change is endogenous

In this section the model is extended to allow R&D laboratories to respond to tighter environmental policy by adopting cleaner technologies. Embodying cleaner technologies in blueprints, however, is costly for R&D firms. Pollution is a complementary input to capital for a given design, hence the use of lower emissions is, *ceteris paribus*, detrimental to a patent's value. Consequently, R&D labs adopt cleaner technologies only if the incentives arising from environmental policy are strong enough. If this is the case then we can establish a correlation between environmental policy and the direction of technological change at the aggregate level: the tighter is environmental policy, the more environment-friendly are technologies. We explain this in the first and second parts of this section.

As is evident from (14), cleaner innovations (a larger $|\zeta|$) reduce the expected contribution of any given amount of R&D to total productivity growth. This is nothing but the direct channel of transmission of environmental policy on growth, i.e. the dynamic version of the statement: "Given that pollution is an input, polluting less means producing less". This effect runs counter to the distortionary impact of environmental policy on market shares across vintages and thereby on R&D activity, described in the previous section. It follows that the comprehensive impact of tightening environmental policy on total productivity growth is ambiguous. By running simulations, however, we are able to show that, under the current specification of the model, growth is reduced by tighter environmental policy. The second and third parts of the section explore this argument.

5.1 Microeconomic background: the R&D lab's choice of pollution intensity

Consider the problem of R&D firm i that has obtained an innovation in sector j , at date τ , with attached implicit labor productivity \bar{A}_τ . Suppose that it can choose to adopt any pollution intensity for its innovation out of the technological menu $(0, \bar{Z}_\tau]$,

with $\bar{Z}_\tau = \int_0^1 Z_{j\tau} dj$ denoting the average pollution intensity in the economy at date τ .²³ Note that the cross-sectorial spillovers concerning the two technological parameters are assumed to be similar in some respects. First and foremost, the set from which a successful R&D lab can choose the technology to adopt is determined entirely by spillovers from other sectors and is not sector specific.²⁴ Second, while we assume that the leading-edge implicit labor productivity index characterizes innovations, we leave the choice of pollution intensity index up to the R&D lab. This is because this choice is not trivial and entails a trade-off, as illustrated below.

Let us denote by $\hat{Z}_{ij\tau} \leq \bar{Z}_\tau$ the choice of the innovating R&D firm i in sector j . If it employs $\hat{n}_{ij\tau}$ researchers it produces an expected instantaneous improvement in total productivity equal to $\lambda \hat{n}_{ij\tau} \bar{A}_\tau \hat{Z}_{ij\tau}^{\frac{1}{1-\alpha}}$. This is the correct measure of the (private) output of an R&D firm. The lab could obtain the same expected instantaneous improvement in total productivity by employing less researchers, say $\bar{n}_{ij\tau}$, if it were to adopt the most polluting technology \bar{Z}_τ instead of $\hat{Z}_{ij\tau}$. The mass of researchers $\bar{n}_{ij\tau}$ is given by $\lambda \bar{n}_{ij\tau} \bar{A}_\tau \bar{Z}_\tau^{\frac{1}{1-\alpha}} = \lambda \hat{n}_{ij\tau} \bar{A}_\tau \hat{Z}_{ij\tau}^{\frac{1}{1-\alpha}} = \lambda \bar{n}_{ij\tau} \bar{A}_\tau \bar{Z}_\tau^{\frac{1}{1-\alpha}}$. We can thus define the opportunity cost of targeting cleaner innovations in terms of the percentage increase in research employment:

$$\frac{\hat{n}_{ij\tau}}{\bar{n}_{ij\tau}} = \left(\frac{\bar{Z}_\tau}{\hat{Z}_{ij\tau}} \right)^{\frac{1}{1-\alpha}} \equiv \nu \left(\bar{Z}_\tau / \hat{Z}_{ij\tau} \right)$$

We are now ready to formulate the problem of the R&D lab. It needs to decide the amount of “regular” researchers to be employed, $\bar{n}_{ij\tau}$, and the degree of cleanliness of the innovation it targets, $\bar{Z}_\tau / \hat{Z}_{ij\tau}$, taking as given all expected aggregate variables: $\{r, w, n\}_\tau^\infty$,

²³It is fair to admit that imitation of the average technology is feasible. There is no concern on whether non-innovating firms adopt average pollution intensity. In fact, if the incumbent patent holder in sector j were to adopt the cleaner technology \bar{Z} , it would reduce the total productivity of its intermediate good. As a result it would face competition from previous, displaced producers in the sector. Only an innovator can adopt the clean technology, while keeping a safe margin over previous producers in terms of total productivity, thanks to its monopoly of technology \bar{A} (if (15) holds).

²⁴This crucial feature is necessary to keep the model tractable and for a balanced growth path to be feasible. In fact, only in this case the reward to R&D is independent of the sector, so that R&D activity is uniform across sectors, and the distribution of technological gaps across vintages is stationary on a balanced growth path.

It should now be clear why it is unwise to assume two different types of R&D, one targeting improvements on A , the other on Z . In such a case the value of innovation would depend upon the incumbent technology for the non-improved parameter. This would be true whatever rent-sharing arrangement is assumed between the incumbent and the innovator. Hence the reward to R&D would be sector specific. Furthermore, a successful R&D lab that has invested only in improving cleanliness would offer a patent to produce an intermediate good with lower total productivity than the one supplied by the incumbent. Hence its market share would be positive only with a sufficiently high green tax and our assumption of local monopoly for the innovator would no longer be justified.

$h_\tau, g_h, \bar{A}_\tau, \bar{Z}_\tau$. The choice is made in order to maximize the expected instantaneous return on R&D employment, i.e.

$$\begin{aligned} & \max_{\bar{n}_{ij\tau}, \hat{Z}_{ij\tau}} \lambda \nu \left(\bar{Z}_\tau / \hat{Z}_{ij\tau} \right) \bar{n}_{ij\tau} V_\tau - \nu \left(\bar{Z}_\tau / \hat{Z}_{ij\tau} \right) \bar{n}_{ij\tau} w_\tau \\ & \text{with } V_\tau = \int_\tau^\infty e^{-\int_\tau^t r_s ds} e^{-\lambda \int_\tau^t n_s ds} \hat{\Pi}_t(\bar{A}_\tau, \hat{Z}_{ij\tau}) dt \end{aligned}$$

The first order conditions of this problem give:

$$\lambda V_\tau = w_\tau$$

which is a restatement of (11), and:²⁵

$$\frac{\partial V_\tau}{\partial \hat{Z}_{ij\tau}} = \int_\tau^\infty e^{-\int_\tau^t r_s ds} e^{-\lambda \int_\tau^t n_s ds} \left(\frac{r_t + h_\tau \hat{Z}_{ij\tau}^{1/\alpha\beta}}{r_t + e^{g_h(t-\tau)} h_\tau \hat{Z}_{ij\tau}^{1/\alpha\beta}} \right)^{\frac{\alpha}{1-\alpha}} \left[\frac{e^{g_h(t-\tau)} h_\tau \hat{Z}_{ij\tau}^{1/\alpha\beta}}{r_t + e^{g_h(t-\tau)} h_\tau \hat{Z}_{ij\tau}^{1/\alpha\beta}} - \beta \right] dt = 0 \quad (26)$$

Improving the cleanliness of the innovation affects its value in two opposite directions. On the one hand, profits accruing to the patent holder are reduced because the innovation's total productivity, $\bar{A}_\tau \hat{Z}_{ij\tau}^{\frac{1}{1-\alpha}}$, falls. On the other hand, if emissions are taxed, profits increase because the burden of the green tax on marginal cost, $h_\tau \hat{Z}_{ij\tau}^{\frac{1}{\alpha\beta}}$, is reduced. This gain is in fact increasing in the expected growth rate of the green tax, g_h .

Condition (26) balances out the cost and benefit of lower pollution intensity in expected present value terms. Its solution is such that the net present value of marginal investment in cleanliness equals zero. The solution is:

- $\hat{Z}_{ij\tau} = \bar{Z}_\tau$ if $g_h = 0$ and $h_\tau \leq \frac{\beta}{1-\beta} r_\tau \bar{Z}_\tau^{-1/\alpha\beta}$;
- $\hat{Z}_{ij\tau} < \bar{Z}_\tau$ if $g_h > 0$ or $h_\tau > \frac{\beta}{1-\beta} r_\tau \bar{Z}_\tau^{-1/\alpha\beta}$.²⁶

5.2 Macroeconomic implications

We now examine the relationship between the behavior of the representative R&D laboratory and the evolution of key macroeconomic variables and policy instruments. The

²⁵In fact, the condition is $\lambda \nu(\cdot) \bar{n}_{ij\tau} \partial V_\tau / \partial \hat{Z}_{ij\tau} - \nu(\cdot) \bar{n}_{ij\tau} [\lambda V_\tau - w_\tau] \frac{1}{1-\alpha} / \hat{Z}_{ij\tau} = 0$, where the second term equals zero according to the other first order condition.

²⁶Notice that for \bar{Z} constant (i.e. when initially $\hat{Z} = \bar{Z}$), $g_h > 0$ implies that $h_\tau > \frac{\beta}{1-\beta} r_\tau \bar{Z}_\tau^{-1/\alpha\beta}$ at some finite date τ .

analysis is restricted to balanced growth paths. First consider the macroeconomic variable determining the direction of technological change, i.e. parameter ζ . Clearly when the representative firm chooses to adopt average pollution intensity - i.e. the most polluting technology in the menu - the distribution of pollution intensity indexes across vintages will converge to a uniform distribution, with all sectors sharing the same pollution intensity. This is the case when $\zeta = 0$.

Let us define the leading edge pollution intensity as the one targeted by R&D laboratories, i.e. $\underline{Z}_\tau \equiv \hat{Z}_\tau$. Then we can compute the average pollution intensity along a balanced growth path, as:

$$\bar{Z}_\tau = \int_0^1 Z_{j\tau} dj = \lambda n \int_0^\infty e^{-\lambda ns} \underline{Z}_{\tau-s} ds = \underline{Z}_\tau \frac{1}{1 + \zeta}$$

where we switch the space of integration as in appendix A.1. It follows that

$$\zeta = 1 - \frac{\hat{Z}_\tau}{\bar{Z}_\tau} \quad (27)$$

leading to:

- $\zeta = 0$ if $g_h = 0$ and $h_\tau \leq \frac{\beta}{1-\beta} r_\tau \bar{Z}_\tau^{-1/\alpha\beta}$;
- $\zeta < 0$ if $g_h > 0$ or $h_\tau > \frac{\beta}{1-\beta} r_\tau \bar{Z}_\tau^{-1/\alpha\beta}$;
- $\partial|\zeta|/\partial g_h > 0$.²⁷

It is also to be noticed that the instruments of environmental policy are now different from those of the previous section. In that case the direction of technological change was fixed, so that the growth rate of the green tax, g_h , was set according to rule (13). As a consequence the policy-maker had some degree of freedom only in setting the level of tax, h_τ , and thus the burden of the green tax levied on innovations, H . Instead when innovators choose the pollution intensity of their blueprints, the environmental policy-maker loses control over the burden of the green tax levied on these agents, H . In fact when $\hat{Z}_\tau < \bar{Z}_\tau$, a once and for all upward jump in the green tax, h_τ , entails a symmetric

²⁷From the term in brackets of the integrand of (26), it can be seen that a higher g_h implies a lower H/r . Since h_τ is a policy tool, it follows that \hat{Z}_τ is a decreasing function of g_h .

adjustment of the pollution intensity target, according to²⁸

$$\frac{d\hat{Z}_\tau}{dh_\tau} \frac{h_\tau}{\hat{Z}_\tau} = -\alpha\beta$$

Given $\hat{Z}_\tau \equiv \underline{Z}_\tau$, this response is such that $H_\tau = h_\tau \underline{Z}_\tau^{1/\alpha\beta}$ is independent of h_τ . In this case, therefore, the policy tool upon which the policy-maker has some leverage is g_h . We assume that a credible commitment on the growth rate of the green tax is possible. This seems plausible because it only requires that the government keep constant the share of green tax revenue over GDP. As we have argued, expectations over g_h do determine the direction of technological change. Hence, by allowing g_h to be set freely policy rule (13) still holds, although causality is now reversed.

We can now ask how the balanced growth path of the economic system is affected by a stricter environmental policy. A commitment to a higher g_h augments the incentive to adopt cleaner technologies, a choice that lowers the total productivity of innovations. This translates directly into a slower productivity growth. It can be seen from (14) that, holding n constant, g falls as $|\zeta|$ increases. This is the direct channel of transmission of environmental policy on growth. Nevertheless, when $g_h > 0$ innovations are environment-friendly and pollution intensity is heterogeneous across goods. As a result, tightening environmental policy permanently modifies the distribution of market shares across vintages and, through this indirect effect, affects the equilibrium level of R&D employment, n . The rise in g_h affects this distribution in two ways: the relative weight of the initial tax burden (H/r) changes, and the tax burden increases at a faster pace with age. The competitiveness loss schedule shifts downwards and favors innovations. The expected implication of this structural change is to foster R&D activity, i.e. to increase n . Unfortunately, however, this last indirect effect cannot be determined exactly. To establish how n responds to g_h it is necessary to solve the dynamic equilibrium, since R&D is a form of investment that reacts to technological opportunities according to preference parameters, such as the willingness to smooth consumption. Moreover, even when a higher g_h does foster R&D activity, the global impact of stricter environmental policy on growth is ambiguous. In fact, even if more researchers were employed, the expected contribution of each of them to productivity growth is reduced by the direct channel of transmission.

²⁸The proof of this result is omitted because it would make the paper excessively long. It is provided by the author upon request.

5.3 Numerical simulations

In the case of an endogenous direction of R&D we solve numerically the system given by the equilibrium condition (25), the first order condition (26) and the policy rule (13). Solutions in terms of $\{n, \underline{Z}_\tau, \zeta\}$ are derived for balanced growth paths prevailing for different policies g_h . Figure 4 plots the results obtained for the baseline case.²⁹ The simulations confirm that the direct channel of transmission of environmental policy on growth dominates the indirect effect. The growth rate declines monotonically with g_h . However, a faster growth of green taxes stimulates R&D activity, a secondary effect that eases the slowdown in productivity growth. The cost of environmental policy in terms of slower growth is reduced by its positive impact on incentives to engage in R&D activity. Moreover, the adoption of cleaner technologies improves the efficiency of environmental policy in targeting any given goal in terms of rate of growth of polluting emissions.

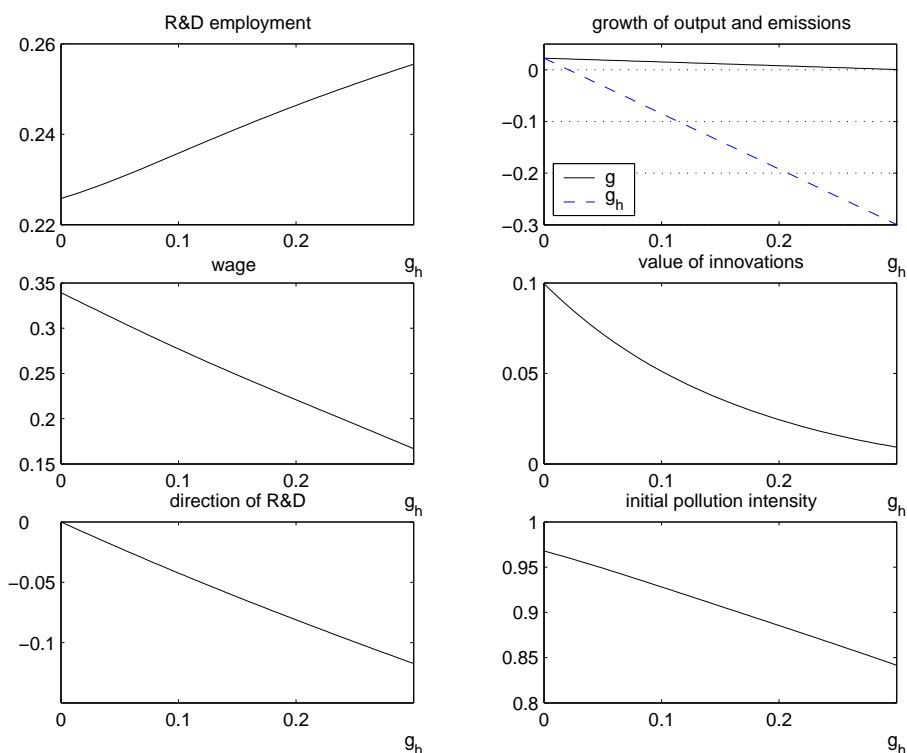


Figure 4: Endogenous direction of R&D: equilibria as function of g_h .

²⁹The values of parameters are set at $\alpha = .4$, $\beta = .125$, $\lambda = .5$, $\gamma = .2$, $A_0 = 1$, $h_\tau = .02$, $\rho = .03$, and $\varepsilon = 1.5$. We concentrate on the presentation of the baseline case because changing the parameters (but ε) affects only the scale of the results, but not their qualitative features.

In order to assess the importance of the distortionary impact of environmental policy in easing the growth-environment trade-off, we compare the results with those obtained from a benchmark model based on Grimaud and Ricci (1999). In the benchmark model intermediate goods are not differentiated in pollution intensity, so that intermediate-goods producers choose Z at each date to maximize their instantaneous profits. Pollution intensity is in effect a control variable, rather than a state variable, in the benchmark model. Figure 5 compares the results concerning R&D employment and growth rate, for the case with $\varepsilon = .9$. This case is particularly interesting because the substitution effect dominates the income effect, so that households invest less when the rate of return on savings falls. Since R&D is a form of investment, the standard result is that stricter environmental policy is detrimental to R&D activity. In our case instead the distortionary impact of green taxes is strong enough to reverse the standard outcome.

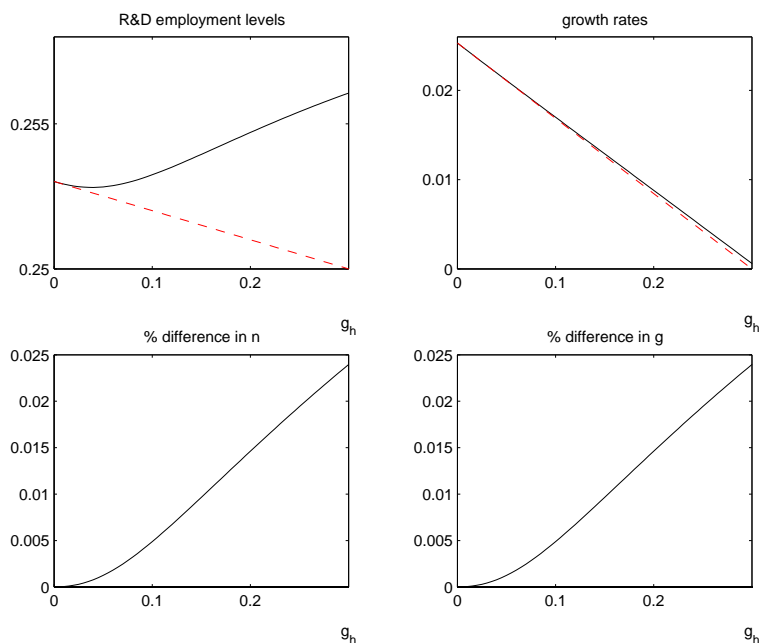


Figure 5: The distortionary impact of green taxes: comparison of models with (-) and without (- -) differentiation in pollution intensity.

In conclusion we have shown that the original transmission channel put forward in this paper, and singled out in section 4, is active also in a more general framework. This transmission channel is potentially important in appraising the costs and benefits of a strict environmental policy when using a dynamic framework with induced technological change. Moreover, this mechanism could prevail and make a win-win outcome feasible if improvements in the cleanliness of innovations were constrained. In fact, the main reason

for our unambiguous numerical result is the strong response of R&D labs in adopting much cleaner technologies when environmental policy is tightened. In turn, this is the consequence of two assumptions. First, differently from Hart (2004 a), R&D labs face an almost unconstrained technology menu $\hat{Z} \in (0, \bar{Z}]$. Second, production technology is characterized by unitary elasticity of substitution for emissions with capital and effective labor. However, recent findings suggest that dynamic multi-sector models depend crucially on the assumed degree of elasticity of substitution (e.g. Bretschger and Smulders 2004). Thus an applied version of our theory should go beyond these assumptions.

6 Conclusion

In the theory presented in this paper, economic growth results from the design of new, more productive capital goods by profit seeking agents, the R&D firms. Emissions represent implicit inputs complementary to capital. Hence, if new goods are designed as cleaner, their productivity is below their potential level. If the tax on emissions is large enough, innovations are relatively clean. This has two implications. First, the contribution of innovation to growth in productivity is weakened. Second, capital goods are differentiated in pollution intensity. The latter effect gives scope for taxation to distort the distribution of market shares across goods of different vintages. In particular, the green tax increases the market share of relatively modern goods (which are the most productive and the least polluting), and this effect improves incentives to engage in R&D activities. The crucial assumption underlying this result is that R&D is labor intensive.³⁰ The identification and exploration of this second channel of transmission of environmental policy constitutes the main original contribution of the paper.

To summarize, a restrictive environmental policy affects economic growth through two channels of transmission that operate in two opposite directions: the first channel lowers the marginal impact of innovation on productivity growth, while the second channel spurs innovation. The second channel is unlikely to dominate unless R&D labs have little scope for reducing pollution intensity. In any case, when carrying out cost-benefit analyses of environmental policy, its cost in terms of slower growth is reduced once this distortionary impact of policy on competition across vintages is taken into account.

Finally, we would like to point out that the distortionary impact of green taxes highlights a more general property of the Schumpeterian model of growth. In fact, in the

³⁰In fact, in this case the tax reduces the cost of R&D along with its pay-off.

multi-sector model of vertical innovation where labor is the sole input in R&D, any policy that affects heterogeneously the producers of intermediate goods has an impact on the equilibrium growth of output (Ricci 2002). Various policies influence the distribution of market shares across goods of different vintages, for instance tax breaks for new firms or fiscal incentives to adopt new equipment. This additional property confirms the fact that the Schumpeterian theory of growth is a fruitful tool for policy analysis.

References

- Aghion, P. and P. Howitt (1992), 'A Model of Growth Through Creative Destruction'. *Econometrica* **60**(2), 323-351.
- Aghion, P. and P. Howitt (1998) *Endogenous Growth Theory*. First ed. Cambridge: The MIT Press.
- Andreoni, J and A. Levinson (2001), 'The Simple Analytics of the Environmental Kuznets Curve'. *Journal of Public Economics* **80**(2), 269-286.
- Benhabib, J. and A. Rustichini (1991), 'Vintage Capital, Investment and Growth'. *Journal of Economic Theory* **55**: 323-339.
- Bretschger, L. (1998), 'How to Substitute in order to Sustain: Knowledge Driven Growth under Environmental Restrictions'. *Environmental and Development Economics* **3**: 425-442.
- Bretschger, L. and S. Smulders (2004), 'Sustainability and Substitution of Exhaustible Natural Resources. How Resource Prices Affect Long-Term R&D Investment'. WIF-Institute of Economic Research, Economics w.p. 03/26, ETH Zurich.
- Bovenberg, A.L. and S. Smulders (1995), 'Environmental Quality and Pollution-Augmenting Technological Change in a Two-Sector Endogenous Growth Model'. *Journal of Public Economics* **57**(3): 369-391.
- Boucekkine, R., O. Licandro, L.A. Puch and F. del Rio (2004), 'Vintage Capital and the Dynamics of the AK Model'. *Journal of Economic Theory*, forthcoming.
- Collard, F., O. Licandro and L.A. Puch (2004), 'The Short-run Dynamics of Optimal Growth Models with Delays'. Mimeo, European University Institute, Florence.
- Elbasha, E. and T. Roe (1996), 'On Endogenous Growth: The Implications of Environmental Externalities'. *Journal of Environmental Economics and Management* **31**: 240-268.
- Feichtinger, G., R. Hartl, P. Kort and V. Veliov (2004), 'Environmental Policy, the Porter Hypothesis and Composition of Capital: Effects of Learning and Technological Progress'. *Journal of Environmental Economics and Management*, forthcoming.
- Fisher, E. and C. van Marrewijk (1998), 'Pollution and Economic Growth'. *Journal of International Trade & Economic Development* **7**(1): 55-69.

- Gradus, R. and S. Smulders (1993), ‘The Trade-Off Between Environmental Care and Long-Term Growth–Pollution in Three Prototype Growth Models’. *Journal of Economics* **58**(1): 25-51.
- Grimaud, A. (1999), ‘Pollution Permits and Sustainable Growth in a Schumpeterian Model’. *Journal of Environmental Economics and Management* **38**(3): 249-266.
- Grimaud, A. and F. Ricci (1999), ‘The Growth-Environment Trade-off: Horizontal vs Vertical Innovations’. Fondazione ENI E. Mattei w.p. 99.34, Milan.
- Grossman, G. and E. Helpman (1991), ‘Quality Ladders in the Theory of Growth’. *Review of Economic Studies* **58**(1): 43-61.
- Hettich, F. (1998), ‘Growth Effects of a Revenue-Neutral Environmental Tax Reform’. *Journal of Economics* **67**(3): 287-316.
- Hettige, H., M. Mani and D. Wheeler (1998), ‘Industrial Pollution in Economic Development (Kuznets Revisited)’. World Bank, Policy Research w.p. 1876, Washington.
- Hart, R. (2004 a), ‘Growth, Environment and Innovation: A Model with Production Vintages and Environmentally Oriented Research’. *Journal of Environmental Economics and Management* **48**(3): 1078-1098.
- Hart, R. (2004 b), ‘Can Environmental Policy Boost Growth?’. Paper presented at the Monte Verità Conference on Sustainable Resource Use and Economic Dynamics, Ascona (Switzerland), June 7-10. Organized by the Institute of Economic Research of ETH Zurich.
- Jaffe, A., R. Newell and R. Stavins (2002), ‘Environmental Policy and Technological Change’. *Environmental and Resource Economics* **22**(1-2): 41-69.
- Michel, P. (1993), ‘Pollution and Growth Towards the Ecological Paradise’. Fondazione ENI E. Mattei w.p. 80.93, Milan.
- Michel, P. and G. Rotillon (1995), ‘Disutility of Pollution and Endogenous Growth’. *Environmental and Resource Economics* **6**(3): 279-300.
- Mohtadi, H. (1996), ‘Environment, Growth, and Optimal Policy Design’. *Journal of Public Economics* **63**(1): 119-140.
- Popp, D. (2002), ‘Induced Innovation and Energy Prices’. *American Economic Review* **92**: 160-180.
- Ricci, F. (2000), ‘Essays on the Theory of Sustainable Development’. Ph.D. dissertation, Université de Toulouse 1.
- Ricci, F. (2002), ‘The Growth Effect of Cross-Sectoral Distortions’. Temi di Ricerca dell’Ente “Luigi Einaudi” w.p. no.26, Rome.
- Ricci, F. (2004), ‘Channels of Transmission of Environmental Policy to Economic Growth: A Survey of the Theory’. Fondazione ENI E. Mattei w.p. 52-04.
- Smulders, S. and R. Gradus (1996), ‘Pollution Abatement and Long-Term Growth’. *European Journal of Political Economy* **12**(3): 505-532.

Stokey, N. (1998), ‘Are there Limits to Growth ?’. *International Economic Review* **39**(1): 1-31.

van Marrewijk, C., F. van der Ploeg and J. Verbeek (1993), ‘Is Growth Bad for the Environment? Pollution, Abatement and Endogenous Growth’. World Bank, Policy Research (Environment) w.p. 1151, Washington.

Van Zon, A. and H. Yetkiner (2003), ‘An Endogenous Growth Model with Embodied Energy-Saving Technical Change’. *Resource and Energy Economics* **25**: 81-103.

Verdier, T. (1995), ‘Environmental Pollution and Endogenous Growth: a Comparison between Emission taxes and Technological Standards’, in C. Carraro and J.A. Filar, ed.s, *Control and Game-Theoretic Models of the Environment (Annals of the International Society of Dynamic Games)*. Boston, Basel and Berlin: Birkhauser. Vol.2: 175-200.

Xepapadeas, A. (1994), ‘Long-Run Growth, Environmental Pollution, and Increasing Returns’. Fondazione ENI E. Mattei w.p. 67.94, Milan.

Xepapadeas, A. and A. de Zeeuw (1999), ‘Environmental Policy and Competitiveness: The Porter Hypothesis and the Composition of Capital’. *Journal of Environmental Economics and Management* **37**(2): 165-182.

A Appendix

A.1 The aggregation factors Γ , Λ and Δ

We compute sales of good j relative to the leading-edge sector, using (7), as:

$$\frac{\hat{x}_{j\tau}}{\bar{x}_\tau} = \left(\frac{Z_{j\tau}}{\underline{Z}_\tau} \right)^{\frac{1}{1-\alpha}} \left(\frac{\bar{m}}{m_j} \right)^{\frac{1}{1-\alpha}}$$

By definition (see (2) and (17)):

$$\Gamma = \int_0^1 \frac{A_{j\tau}}{\bar{A}_\tau} \frac{\hat{x}_{j\tau}}{\bar{x}_\tau} dj$$

The integral has no mathematical sense because $\frac{A_{j\tau}}{\bar{A}_\tau}$ and $\frac{Z_{j\tau}}{\underline{Z}_\tau}$ are distributed stochastically (and *a priori* discontinuously) over the space of goods $[0, 1]$. However at any date τ we can reshuffle goods according to their technological gap, i.e. their age s . Along a balanced growth path each technology is initially adopted by λn of firms, out of which only a proportion $e^{-\lambda n s}$ of those aged s survives at date τ . The productivity gap for firms of age s is: $\frac{A_{\tau-s}}{\bar{A}_\tau} = e^{-g\bar{A}s} = e^{-\lambda n \gamma s}$; and the pollution intensity gap is: $\frac{\underline{Z}_\tau}{\underline{Z}_{\tau-s}} = e^{g\underline{Z}s} = e^{\lambda n \zeta s}$. Under policy rule (13) older goods sell

less, according to the competitiveness-loss function, \bar{m}/m_s . Using (14) we have:

$$\begin{aligned}\Gamma &= \int_0^\infty \lambda n e^{-\lambda n s} \frac{\bar{A}_{\tau-s}}{\bar{A}_\tau} \left(\frac{\underline{Z}_{\tau-s}}{\underline{Z}_\tau} \right)^{\frac{1}{1-\alpha}} \left(\frac{\bar{m}}{m_s} \right)^{\frac{1}{1-\alpha}} ds \\ &= \lambda n \int_0^\infty e^{-(\lambda n + g)s} \left(\frac{\bar{m}}{m_s} \right)^{\frac{1}{1-\alpha}} ds\end{aligned}$$

By definition (see (4), (17) and (18)) and proceeding as above:

$$\begin{aligned}\Lambda &= \int_0^1 \left(\frac{Z_{j\tau}}{\underline{Z}_\tau} \right)^{\frac{1}{\alpha\beta}} \frac{A_{j\tau}}{\bar{A}_\tau} \frac{\hat{x}_{j\tau}}{\bar{x}_\tau} dj \\ &= \lambda n \int_0^\infty e^{-(\lambda n + g + \frac{\zeta}{\alpha\beta} \lambda n)s} \left(\frac{\bar{m}}{m_s} \right)^{\frac{1}{1-\alpha}} ds\end{aligned}\tag{28}$$

By definition (see (1) and (19)) and using (14):

$$\begin{aligned}\Delta &= \int_0^1 \frac{Z_{j\tau}}{\underline{Z}_\tau} \frac{A_{j\tau}}{\bar{A}_\tau} \left(\frac{\hat{x}_{j\tau}}{\bar{x}_\tau} \right)^\alpha dj \\ &= \lambda n \int_0^\infty e^{-(\lambda n + \lambda n \gamma + \lambda n \zeta)s} \left(\frac{\underline{Z}_{\tau-s}}{\underline{Z}_\tau} \right)^{\frac{\alpha}{1-\alpha}} \left(\frac{\bar{m}}{m_s} \right)^{\frac{\alpha}{1-\alpha}} ds \\ &= \lambda n \int_0^\infty e^{-(\lambda n + g)s} \left(\frac{\bar{m}}{m_s} \right)^{\frac{\alpha}{1-\alpha}} ds\end{aligned}$$

We have the following properties :

Property 1 : $\Gamma < 1$, if $\zeta < 0$, from (13), (14) and (15).

Property 2 : $\Lambda > \Gamma$ if $\zeta < 0$ since $z_j^{-1/\alpha\beta} \in [1, \infty)$.

Property 3 : $\Lambda > \Delta > \Gamma$, if $\zeta < 0$.

Indeed using (13) :

$$\begin{aligned}\Delta &= \int_0^1 \frac{A_{j\tau}}{\bar{A}_\tau} \left(\frac{Z_{j\tau}}{\underline{Z}_\tau} \right)^{\frac{1}{1-\alpha}} \left(\frac{\bar{m}}{m_j} \right)^{\frac{\alpha}{1-\alpha}} dj \\ &= \int_0^1 \frac{A_{j\tau}}{\bar{A}_\tau} \left(\frac{Z_{j\tau}}{\underline{Z}_\tau} \right)^{\frac{1}{1-\alpha}} \left(\frac{\bar{m}}{m_j} \right)^{\frac{1}{1-\alpha}} \frac{r + H \left(\frac{Z_{j\tau}}{\underline{Z}_\tau} \right)^{\frac{1}{\alpha\beta}}}{r + H} dj \\ &= \frac{r}{r + H} \Gamma + \frac{H}{r + H} \Lambda\end{aligned}$$

Thus : $\Delta - \Lambda = \frac{r}{r+H}(\Gamma - \Lambda) < 0$, by property 2.

Property 4 : $\Delta \Gamma^{-\alpha} \leq (1 + \gamma + \zeta / (1 - \alpha))^{\alpha-1} < 1$.

Define $G = \gamma + \zeta / (1 - \alpha)$. By Jensen's inequality:

$$\begin{aligned} [\Delta(1+G)]^{\frac{1}{\alpha}} &= \left\{ \lambda n \int_0^\infty (1+G)e^{-(1+G)\lambda ns} \left[\left(\frac{\bar{m}}{m_s} \right)^{\frac{1}{1-\alpha}} \right]^\alpha ds \right\}^{\frac{1}{\alpha}} \leq \\ &\leq \lambda n \int_0^\infty (1+G)e^{-(1+G)\lambda ns} \left(\frac{\bar{m}}{m_s} \right)^{\frac{1}{1-\alpha}} ds = \Gamma(1+G) \end{aligned}$$

Property 5 : $\Gamma = \Delta = \Lambda = \int_0^1 \frac{A_{j\tau}}{\bar{A}_\tau} dj = (1 + \gamma)^{-1}$ if $\zeta = 0$

If there is no differentiation in pollution intensity, that is if $\zeta = 0$, then $Z_{j\tau}/\underline{Z}_\tau = 1 \forall j \in [0, 1]$, and $g_h = 0$ by policy rule (13), so that $\hat{x}_{j\tau}/\bar{x}_\tau = 1 \forall j \in [0, 1]$, which proves the result.

A.2 Proof of proposition 2

To prove the result we show that, at the original equilibrium level of n , the left-hand-side (LHS) of the equilibrium condition (E) falls more than the right-hand-side (RHS). These shifts of the schedules depicted in figure 4.b result in a higher equilibrium level of R&D employment, n .

Preliminary: The LHS of (E) can also be written as $Y_\tau / [(1-n)\bar{A}_\tau]$ and therefore falls along with output. Let us first differentiate output given by (19) with respect to $H \equiv h\underline{Z}^{1/\alpha\beta}$, holding n constant. Using (7), (20), (28), definitions in appendix A.1 and property 3, we have:

$$\begin{aligned} \left. \frac{\partial Y}{\partial H} \right|_{dn=0} &= (1-n)^{1-\alpha} \underline{Z}_\tau \bar{A}_\tau \bar{x}_\tau^\alpha \left[\frac{\partial \Delta}{\partial H} + \frac{\alpha \Delta}{\bar{x}_\tau} \frac{\partial \bar{x}_\tau}{\partial H} \right] \\ &= \frac{\alpha(1-n)^{1-\alpha} \underline{Z}_\tau \bar{A}_\tau \bar{x}_\tau^\alpha}{(1-\alpha)[r+H]} \left[\int_0^1 \frac{Z_{j\tau}}{\underline{Z}_\tau} \frac{A_{j\tau}}{\bar{A}_\tau} \left(\frac{\hat{x}_{j\tau}}{\bar{x}_\tau} \right)^\alpha \left[1 - \left(\frac{Z_{j\tau}}{\underline{Z}_\tau} \right)^{\frac{1}{\alpha\beta}} \right] \frac{r}{m_j} dj - \Delta \right] \\ &= \frac{\bar{A}_\tau \bar{x}_\tau}{\alpha(1-\alpha)} \left[\frac{r}{\bar{m}} \int_0^1 \frac{A_{j\tau}}{\bar{A}_\tau} \frac{\hat{x}_{j\tau}}{\bar{x}_\tau} dj - \frac{r}{\bar{m}} \int_0^1 \frac{A_{j\tau}}{\bar{A}_\tau} \frac{\hat{x}_{j\tau}}{\bar{x}_\tau} \left(\frac{Z_{j\tau}}{\underline{Z}_\tau} \right)^{\frac{1}{\alpha\beta}} dj - \Delta \right] \\ &= \frac{\bar{A}_\tau \bar{x}_\tau}{\alpha(1-\alpha)} \left[\frac{r}{\bar{m}} \Gamma - \frac{r}{\bar{m}} \Lambda - \Delta \right] \\ &= \frac{\bar{A}_\tau \bar{x}_\tau}{\alpha(1-\alpha)} \left[\Delta - \frac{H}{\bar{m}} \Lambda - \frac{r}{\bar{m}} \Lambda - \Delta \right] = \frac{-\bar{A}_\tau \bar{x}_\tau}{\alpha(1-\alpha)} \Lambda < 0 \end{aligned} \quad (29)$$

Remark: $\left| \frac{\partial Y}{\partial H} \right|_{dn=0}$ is increasing in $|\zeta|$. In fact $\zeta = 0 \Rightarrow \partial \Delta / \partial H = 0$ by property 5 in appendix A.1, $\Rightarrow \int_0^1 \frac{Z_{j\tau}}{\underline{Z}_\tau} \frac{A_{j\tau}}{\bar{A}_\tau} \left(\frac{\hat{x}_{j\tau}}{\bar{x}_\tau} \right)^\alpha \left[1 - \left(\frac{Z_{j\tau}}{\underline{Z}_\tau} \right)^{\frac{1}{\alpha\beta}} \right] \frac{r}{m_j} dj = 0$. Instead $\zeta < 0 \Rightarrow \left(\frac{Z_{j\tau}}{\underline{Z}_\tau} \right)^{\frac{1}{\alpha\beta}} > 1 \forall j$ but one, $\Rightarrow \int_0^1 \frac{Z_{j\tau}}{\underline{Z}_\tau} \frac{A_{j\tau}}{\bar{A}_\tau} \left(\frac{\hat{x}_{j\tau}}{\bar{x}_\tau} \right)^\alpha \left[1 - \left(\frac{Z_{j\tau}}{\underline{Z}_\tau} \right)^{\frac{1}{\alpha\beta}} \right] \frac{r}{m_j} dj < 0$.

Using (29) in $LHS \equiv Y_\tau / [(1-n)\bar{A}_\tau]$ we obtain:

$$\frac{\partial LHS}{\partial H} = \frac{-\bar{x}_\tau \Lambda}{\alpha(1-\alpha)(1-n)} \quad (L)$$

The RHS of (E) is equal to $\lambda V_\tau / [\bar{A}_\tau (1 - \alpha)]$. First we compute the impact of a marginal change in H on the value of innovations V_τ given by (16). This is:

$$\begin{aligned} \left. \frac{\partial V_\tau}{\partial H} \right|_{dn=0} &= \frac{1 - \alpha}{\alpha} \bar{A}_\tau (\alpha^2 \underline{Z}_\tau)^{\frac{1}{1-\alpha}} (1 - n) \int_0^\infty e^{-(r+\lambda n)t} \left(\frac{-\alpha}{1 - \alpha} \right) e^{g_h t} [r + e^{g_h t} H]^{\frac{-\alpha}{1-\alpha} - 1} dt \\ &= -\bar{x}_\tau \bar{A}_\tau \int_0^\infty e^{-(r+\lambda n)t} e^{g_h t} \left(\frac{\bar{m}}{m_t} \right)^{\frac{1}{1-\alpha}} dt < 0 \end{aligned}$$

where (7) is used. Hence $RHS \equiv \lambda V_\tau / [\bar{A}_\tau (1 - \alpha)]$ equals:

$$\frac{\partial RHS}{\partial H} = -\frac{\lambda \bar{x}_\tau}{(1 - \alpha)} \int_0^\infty e^{-(r+\lambda n)t} e^{g_h t} \left(\frac{\bar{m}}{m_t} \right)^{\frac{1}{1-\alpha}} dt \quad (\text{R})$$

Core : The increase in H reduces more the LHS than the RHS of (E) if (L) is smaller than (R). Rearranging using (28) and (14), this condition requires that :

$$\frac{n}{\alpha(1 - n)} > \frac{\int_0^\infty e^{-(r+\lambda n)t} e^{g_h t} \left(\frac{\bar{m}}{m_t} \right)^{\frac{1}{1-\alpha}} dt}{\int_0^\infty e^{-(g+\lambda n)s} e^{g_h s} \left(\frac{\bar{m}}{m_s} \right)^{\frac{1}{1-\alpha}} ds}$$

We can substitute the left-hand-side of this last inequality by its value prevailing at the original equilibrium, given by the equilibrium condition (E), which rearranged gives:

$$\frac{n}{\alpha(1 - n)} = \frac{\int_0^\infty e^{-(r+\lambda n)t} \left(\frac{\bar{m}}{m_t} \right)^{\frac{\alpha}{1-\alpha}} dt}{\Delta / \lambda n} = \frac{\int_0^\infty e^{-(r+\lambda n)t} \left(\frac{\bar{m}}{m_t} \right)^{\frac{\alpha}{1-\alpha}} dt}{\int_0^\infty e^{-(g+\lambda n)s} \left(\frac{\bar{m}}{m_s} \right)^{\frac{\alpha}{1-\alpha}} ds}$$

That the LHS of (E) falls more than the RHS around the original equilibrium if (L) is smaller than (R) implies:

$$\frac{\int_0^\infty e^{-(g+\lambda n)s} e^{g_h s} \left(\frac{\bar{m}}{m_s} \right)^{\frac{1}{1-\alpha}} ds}{\int_0^\infty e^{-(g+\lambda n)s} \left(\frac{\bar{m}}{m_s} \right)^{\frac{\alpha}{1-\alpha}} ds} > \frac{\int_0^\infty e^{-(r+\lambda n)t} e^{g_h t} \left(\frac{\bar{m}}{m_t} \right)^{\frac{1}{1-\alpha}} dt}{\int_0^\infty e^{-(r+\lambda n)t} \left(\frac{\bar{m}}{m_t} \right)^{\frac{\alpha}{1-\alpha}} dt} \quad (\text{I})$$

The two sides of (I) are equal if $\zeta = 0$ since this implies $g_h = 0$ and $\bar{m}/m_t = \bar{m}/m_s = 1 \forall t, s$. The difference between the two sides of the inequality lies in the discount rate of the integrands.

Lemma 1: The ratio of the two integrals is decreasing in the discount rate:

$$f(\delta) = \frac{\int_0^\infty e^{-\delta t} e^{g_h t} \left(\frac{\bar{m}}{m_t} \right)^{\frac{1}{1-\alpha}} dt}{\int_0^\infty e^{-\delta t} \left(\frac{\bar{m}}{m_t} \right)^{\frac{\alpha}{1-\alpha}} dt}, \text{ with } \delta > 0 \Rightarrow \frac{\partial f(\delta)}{\partial \delta} < 0$$

Proof of the lemma 1: Define

$$\Psi(t) = \frac{e^{g_h t} \left(\frac{\bar{m}}{m_t}\right)^{\frac{1}{1-\alpha}}}{\left(\frac{\bar{m}}{m_t}\right)^{\frac{\alpha}{1-\alpha}}} = e^{g_h t} \frac{r + H}{r + e^{g_h t} H}$$

to write the ratio of the integrals in (I) as:

$$f(\delta) = \int_0^\infty \Omega(t, \delta) \Psi(t) dt$$

where:

$$\Omega(t, \delta) = \frac{e^{-\delta t} \left(\frac{\bar{m}}{m_t}\right)^{\frac{\alpha}{1-\alpha}}}{\int_0^\infty e^{-\delta u} \left(\frac{\bar{m}}{m_u}\right)^{\frac{\alpha}{1-\alpha}} du}$$

is a normalized weight function, characterized by:

$$\begin{aligned} \frac{\partial \Omega}{\partial \delta} &\propto \int_0^\infty u e^{-\delta u} \left(\frac{\bar{m}}{m_u}\right)^{\frac{\alpha}{1-\alpha}} du - t \int_0^\infty e^{-\delta u} \left(\frac{\bar{m}}{m_u}\right)^{\frac{\alpha}{1-\alpha}} du \\ \Rightarrow \exists \tilde{t} &= \frac{\int_0^\infty u e^{-\delta u} \left(\frac{\bar{m}}{m_u}\right)^{\frac{\alpha}{1-\alpha}} du}{\int_0^\infty e^{-\delta u} \left(\frac{\bar{m}}{m_u}\right)^{\frac{\alpha}{1-\alpha}} du} \\ \text{such that: } &\frac{\partial \Omega}{\partial \delta} > 0 \quad \forall t < \tilde{t} \quad \text{and} \quad \frac{\partial \Omega}{\partial \delta} < 0 \quad \forall t > \tilde{t} \end{aligned}$$

That is, an increase in the discount rate δ shifts weight from high values of t to low values of t . Hence $f(\delta)$ falls with δ because $\Psi(t)$ is increasing for $r > 0$ and $\zeta < 0$:

$$\frac{\partial \Psi(t)}{\partial t} \propto 1 - \frac{e^{g_h t} H}{r + e^{g_h t} H} > 0$$

Corollary: Inequality (I) holds as long as the discount rate on its left-hand-side, $g + \lambda n$, is lower than the one on its right-hand-side, $r + \lambda n$, that is whenever $r > g$.

Lemma 2: At steady state equilibrium $r > g$.

Proof of lemma 2: This is the well known no-Ponzi game condition: the present value of households' wealth, W , is nil asymptotically. It is nothing but the transversality condition of households' intertemporal optimization problem. Since W grows with income:

$$\lim_{t \rightarrow \infty} e^{-rt} W_t = \lim_{t \rightarrow \infty} e^{-(r-g)t} W_0 = 0 \quad \Leftrightarrow \quad r > g$$

Conclusion: We have established that in the neighborhood of the equilibrium defined by (E):

$$r > g \quad \Rightarrow \quad \text{(I) holds} \quad \Rightarrow \quad \frac{\partial LHS}{\partial H} < \frac{\partial RHS}{\partial H} < 0 \quad \Rightarrow \quad \frac{\partial n^e}{\partial H} > 0 .$$