Common Agency with Informed Principals[1]

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Comments most welcome.

Abstract

We analyze a common agency game under asymmetric information on the preferences of the non-cooperating principals. Asymmetric information introduces incentive compatibility constraints which rationalize the requirement of *truthfulness* made in the earlier literature on common agency games under complete information. There exists a large class of differentiable equilibria which are ex post inefficient and exhibit free-riding. We then characterize some interim efficient equilibria. Finally, there exists also a unique equilibrium allocation which is robust to random perturbations. This focal equilibrium is characterized for any distribution of types.

Keywords: Common agency, public goods, incentive mechanisms.

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1 Introduction

Over the past twenty years and following the seminal contributions of Pauly (1974), Wilson (1979) and Bernheim and Whinston (1986a and b), the common agency literature has developed an analytical framework which enabled modelers to tackle a variety of important problems such as menu auctions,[1] public good provisions through voluntary contributions,[2] or policy formation under the influence of lobbying groups.[3] Given this broad range of applications, it is fair to say that the common agency model is by now viewed as a major piece of the toolkit of many economists, most noticeably within the field of political economy.

In our view, the tremendous success of the model relies both on the clear and simple underlying assumptions on which it is based but also in the very precise predictions it conveys. In a nutshell, all common agency models are based on the fact that several principals design non-cooperatively contribution schedules $t_i(q)$ for a common agent. This common agent in turn decides of the quantity q of public good that should be provided for the principals. There is complete information on the preferences of the players who are all risk-neutral. A priori, many equilibria of this two-stage game can be obtained thanks to the freedom in specifying how contributions are designed for out of equilibrium levels of the public good. The out of equilibrium behavior specified by each principal in the schedule he offers to the common agent acts as a threat to prevent deviations away from a particular equilibrium outcome. By imposing that contributions are *truthful*, i.e., reflect the relative preferences of the principals among alternatives, Bernheim and Whinston (1986a) were able to reduce this indeterminacy and to select equilibria which are essentially unique in terms of the level of public good provided.[4] Truthfulness of the contribution schedules has an important consequence in terms of the efficiency of the equilibrium allocation. Since a principal's marginal preferences between alternatives is fully reflected by his contribution, what this principal pays at the margin for inducing a change in the level of public good is exactly what it is worth for him. Modulo the truthfulness requirement, common agency games provide an efficient way of aggregating preferences in a world of complete information and an efficient Coasian bargaining is obtained. The level of public good necessarily maximizes the aggregate payoff of the grand-coalition made of the contributing principals and this common agent.

The goal of this paper is to extend the common agency literature to the case where

[1]Wilson (1979), Bernheim and Whinston (1986a), Anton and Yao (1989 and 1992).

[2]See Laussel and Lebreton (1998) and (2001).

[3]Grossman and Helpman (1994) and Dixit, Grossman and Helpman (1997), among many others.

[4]Multiplicity may still exists but it comes only from the possible flexibility in sharing the aggregate surplus among the contributing principals and their common agent. Nevertheless the various feasible redistributions of the aggregate surplus can be fully described by means of a set of inequalities. See Bernheim and Whinston (1986a) and Laussel and Lebreton (2001).

principals have private information on their preferences for the public good. Clearly, this extension is necessary in a variety of circumstances. Voluntary contributions to a public good are designed by donors with an eye on the information they convey on their willingness to pay for the public good. In the political arena, lobbying groups have private information on the benefits they withdraw from a given policy and much of their activity consists in conveying information to a less-well informed policy-maker.[5] Introducing asymmetric information on the principals' preferences imposes which replace and give firmer foundations to the *truthfulness* requirement imposed so far in the complete information literature.

One could a priori conjecture that this seemingly minor perturbation of the standard model would not modify its main insights. This is however not true. Far from ensuring uniqueness of the equilibrium allocation and by contrast with truthfulness, incentive compatibility introduces a new reason for the multiplicity of equilibria. In a Bayesian setting, the strategy of each principal depends on its type and, at a best response, a given principal forms a conjecture on how the marginal contributions of others evolve with their types. Different conjectures may lead to different equilibria. We show that the size of the set of symmetric equilibria is really *huge* in the following sense: Starting from *any* monotonically increasing equilibrium level of public good which is below the first-best along the 45 degree line where principals have the same willingness to pay, we can reconstruct the whole marginal contributions and equilibrium output off this ray to complete the description of a symmetric differentiable equilibrium of the common agency game.

To better understand this new source of multiplicity, it is necessary to describe the behavior of each principal at an equilibrium of the contribution game. When choosing how much to contribute at the margin for q units of the public good, each principal behaves as a monopsonist in front of a residual supply curve. This residual supply curve is obtained by substracting the *expected* marginal contributions of other principals from the common agent's marginal cost function of supplying the public good. This principal chooses thus to increase his marginal contribution for q units up to the point where the marginal benefit he withdraws from all the inframarginal units produced at that price is just equal to the added supply that such an increase in contribution induces. When other principals contribute on average less at the margin, the residual supply is shifted downwards and, at a best response, a given principal chooses also to contribute less at the

[5] In this respect, it is striking to see that Grossman and Helpman themselves have stressed this point in their recent book (Grossman and Helpman (2001, Chapter 4)). The difficulty in extending their basic model of common agency with monetary contributions to a framework with asymmetric information led them to give up any monetary exchanges between principals (lobbying groups) and their common agent (the policy-maker) and to develop alternative models of cheap-talk à la Crawford and Sobel (1982). Our paper can be viewed as giving a first step towards a more synthetic treatment with both monetary transfers and asymmetric information on the preferences of the lobbyists.

margin. This creates some form of complementarity among the principals and generates multiple equilibria.

Given that particularly severe multiplicity problem, we first follow the tradition of the common agency literature under complete information in looking for equilibrium allocations which also satisfy some efficiency property. Ex post efficiency is by far too demanding a concept. All equilibria are ex post inefficient. Downward distortions below the first-best result from some *free-riding* between principals who contribute less at the margin than what the good is worth to them.

We then relax the efficiency criterion and turn to the weaker concept of *interim efficiency* due to Holmström and Myerson (1983). We show that an equilibrium allocation may be interim efficient under specific assumptions on the principals' distributions of types. Not all equilibria of the common agency game are interim efficient since common agency games are in fact less efficient ways of communicating information than the centralized mechanisms used in the Holmström and Myerson (1983)'s framework. Indeed, equilibrium allocations are not obtained through a centralized mechanism organized by an uninformed mediator[6] but instead result from the interaction between decentralized mechanisms offered non-cooperatively by principals.

Interim efficient equilibria may nevertheless sometimes exist, in which case they can be characterized in terms of the social weights given to the different principals types in the welfare function that would be maximized by the mediator. For instance, when the principals' types are independently and identically distributed according to a uniform distribution, all types of principals may receive the same weight of one half in this social welfare function in one particular interim efficient equilibrium of the common agency game.

We then turn to another selection device and introduce a non-observable shock on the common agent's preferences. An equilibrium nonlinear contribution should go through all quantities corresponding to various realizations of that shock. This robustness requirement severely constrains the nonlinear contributions that might be offered at an equilibrium. In a quite spectacular way, this criterion pins down a one-dimensional family of equilibria, reducing thereby the multiplicity problem by a tall order. Moreover, these possible solutions are Pareto ranked and, one may further select the Pareto-dominating one. This robust equilibrium is solution to a first order partial derivative equation which links the derivatives of the equilibrium output with respect to types and to the random shock. That PDE can be solved explicitly *whatever* the type distribution and the agent's cost function. Looking at the trace of the characteristic surface on the hyperplane corresponding to a null shock yields a unique equilibrium output for a model without shock.

[6]Note that this mediator could be the uninformed agent himself.

Finally, a last contribution of our paper is to propose a mechanism design approach which is useful to find the subclass of so-called *pointwise optimal* equilibria of our common agency game. This approach helps understanding how each principal designs his contribution not only to convey information to the common agent on his own preferences but also to extract the information that this agent may learn on other principals when observing their mere offers. This double-sided role of a contribution in a common agency environment points at the fact that *"market information"*[7] has to be learned in equilibrium. Of course, the difficulty is that "market information" is by large endogenous: this is what other principals are revealing to the common agent in an equilibrium. Standard mechanism design techniques can still be used to compute those common agency equilibria.

Review of the Literature: The results of the common agency literature developed in complete information environments have been viewed as so attractive that they were extended in many different directions. Dixit, Grossman and Helpman (1997) have relaxed the assumption of quasi-linear preferences over output and monetary transfer made in Berheim and Whinston (1986a) and introduced redistributive concerns which may be quite relevant for political economy applications. Laussel and Lebreton (2001) have introduced uncertainty on the preferences of the common agent.[8] Given that contribution schedules are offered at the ex ante stage, i.e., before principals and the agent learn the realization of the preference parameter, efficiency is unsurprisingly still obtained. Prat and Rustichini (2003) have extended the basic framework to allow competition among principals trying to influence multiple agents. Finally, Bergemann and Välimäki (2003) consider the dynamics of common agency games. The main features of the game remain.

Paralleling that trend of the literature which does not stress at all any incentive problems, other authors have looked at the oligopolistic screening environments where different principals try to elicit a piece of information which is privately known by the common agent at the contracting stage. Stole (1991), Martimort (1992, 1996), Biais, Martimort and Rochet (2000) and Martimort and Stole (2002, 2003) among others have analyzed such models. A basic assumption made there is that common agency is *intrinsic*, i.e., the agent has no other choice than accepting or refusing all contributions at once. The focus of these papers is on the inefficiency introduced by the lack of coordination under oligopolistic screening and its impact on the distribution of information rents. Contrary to this literature, our focus is on asymmetric information on the principals' side. The contract offered by a given principal has not only to signal his type to the common agent but also to screen the endogenous information that the latter learns, in equilibrium, on other principals' preferences.

[7] Borrowing an expression due to Epstein and Peters (1999) and Peters (2001).

[8] See also Lebreton and Salanié (2003) and Epstein and O'Halloran (1996) on the case of asymmetric information on the preferences of the common agent in lobbying models.

Our paper is also linked to the literature on voluntary contributions, most noticeably those papers which assume private information on the contributors' side. Menezes, Monteiro and Temini (2001) and Laussel and Palfrey (2003) have both analyzed such games when the public good is a 0-1 decision and contributors have private information on their willingness to pay. Even though both papers stress the multiplicity of equilibria that arises in those environments, they do not provide a complete account of the dimensionality of the equilibrium set because of the restriction to a 0-1 decision. Menezes, Monteiro and Temini (2001) highlight the strong ex post inefficiency of equilibria of the contribution game. Laussel and Palfrey (2003) are instead interested in the interim efficiency of the equilibrium allocations. We also derive simple Lindahl-Samuelson conditions characterizing the provision of a continuous public good and show that downward distortions below the first-best exist in any equilibrium satisfying a simple monotonicity condition. Working with a continuous level of public good yields also a more tractable way of getting interim efficient equilibrium outcomes than in Laussel and Palfrey (2003).

Our techniques of selecting among Bayesian-Nash equilibrium allocations as being robust to perturbations of the game is clearly reminiscent of Klemperer and Meyer (1989)'s device for selecting among Nash equilibria in supply functions equilibria. We extend this approach to a Bayesian game and provide a closed form expression of the equilibrium allocation for any type distribution and cost function.[9]

Section 2 presents the model. Section 3 characterizes the equilibria of our common agency game under asymmetric information. We give a special attention to what we call *pointwise* optimal equilibria. We derive the Lindahl-Samuelson conditions in our framework and give a few properties of the equilibrium nonlinear contributions. Section 4 is devoted to the multiplicity problem and the ex post inefficiency of these equilibria. Section 5 discusses interim efficiency. Section 6 introduces perturbations to select a unique equilibrium allocation that we characterize. Section 7 reinterprets the pointwise optimal equilibria using a mechanism design approach. Section 8 briefly concludes. Proofs are relegated to an Appendix. For completeness, we show there also how to construct non-differentiable equilibria which are inefficient.

[9]Our paper is close in this respect to Wang and Zender (2002) who solve for one particular equilibrium in supply functions in the case of asymmetric information on the bidders's types and for specific assumptions on those types.

2 The Model

There are two risk-neutral principals P_i $(i = 1, 2)$ who derive utility from consuming a public good which is produced in non-negative quantity q.[10] This public good may be an infrastructure of variable size, a charitable activity, or it may also have a more abstract interpretation as a policy variable as it would be the case in a lobbying game à la Grossman and Helpman (1994).

Principal i gets a utility $V_i = \theta_i v(q) - t_i$ from consuming q units of the good where $v(\cdot)$ is twice differentiable, increasing and strictly concave and t_i is the corresponding payment. It should be already clear that with a convenient renormalization of utils we may as well set $v(q) \equiv q$. We will adopt that formulation for simplicity.[11]

Contributions are collected by a common agent A who produces at cost $C(q)$ the public good. The function $C(\cdot)$ is twice differentiable, increasing and concave. To avoid unnecessary technicalities due to corner conditions, we will assume that the Inada condition $C'(0) = 0$ holds except when stated explicitly.

Principals are privately informed on their respective valuations θ_i which are independently drawn from the same common knowledge and atomless distribution on $\Theta = [\underline{\theta}, \bar{\theta}]$ with c.d.f. $F(\cdot)$ and everywhere positive density $f = F'$ except, when needed, at $\bar{\theta}$ but then this extra assumption is made explicit.

The contribution game between the principals unfolds as follows. First, principals learn their preferences and offer non-cooperatively contribution schedules $\{t_i(q, \hat{\theta}_i)\}_{\hat{\theta}_i \in \Theta}$ to the agent. The agent accepts or refuses all those contracts at once. If he refuses, the game ends. Upon acceptance, the agent chooses to produce an amount of public good. Corresponding payments are then made.

We will be interested in characterizing various classes of Perfect Bayesian Equilibria (PBE) -or equilibria in short - of this game.

Even though the contexts are quite different, we follow the same strategy as when computing the equilibrium of a first-price auction to characterize equilibria of the game. Facing the menu of bidding contributions $\{t_i(q, \hat{\theta}_i)\}_{\hat{\theta}_i \in \Theta}$, principal P_i with type θ_i picks whatever schedule he prefers. By the Revelation Principle applied to that Bayesian game, there is no loss of generality in restricting the menus to be incentive compatible.[12] Each principal P_i picks then the contribution corresponding to his own type. This leads us to

[10]It is possible to extend the analysis to the case of $n > 2$ principals at the cost of an increased complexity.

[11]This formulation is also convenient to interpret q in $[0, 1]$ as the probability of producing the public good in the case of a discrete 0-1 project.

[12]Or *truthful* in the sense of incentive theory not in the sense of the common agency literature.

state the following definition.

Definition 1 *: A family of nonlinear schedules* $\{t_i(q, \hat{\theta}_i)\}_{\hat{\theta}_i \in \Theta}$ *is incentive compatible if and only if principal* P_i *finds it optimal to truthfully reveal his type at a Bayesian equilibrium of the contribution game.*

It is important to stress the difference between incentive compatibility and the notion of truthfulness developed in Bernheim and Whinston (1986a). In that latter piece, the principals' preferences are common knowledge and that truthfulness requirement simply means that the marginal contribution of each principal is equal to his marginal valuation for the good. In other words, and when q takes values in a continuum, "truthfulness" means that $\frac{\partial t_i}{\partial q}(q, \hat{\theta}_i) = \hat{\theta}_i$ for all $\hat{\theta}_i$ and all q. Under incomplete information, instead, incentive compatibility requires only that each principal finds optimal to report truthfully his type by choosing the right contribution knowing that at the second stage, the agent chooses how much to produce.

Sometimes, we will refer to *differentiable* menus as menus of incentive compatible nonlinear prices which are three times piece-wise differentiable with respect to q and $\hat{\theta}_i$.[13]

Remark 1: Our specification of the bidding game and the contribution schedules available to each principal seem to preclude that any principal deviates by offering a more complex mechanism if needed. Section 7 precisely shows that the equilibria we obtain are robust and are also equilibria when principals can deviate by offering mechanisms belonging to a larger strategy space. ■

Remark 2: Readers acquainted with the common agency literature will have noticed that we model a game of *intrinsic common agency*[14] in which all contributions are accepted or refused at once. Following Bernheim and Whinston (1986a), this literature has mostly studied models of *delegated common agency* where the agent may choose to turn down one of the offers. Of course, this extra option somewhat restricts the distribution of the aggregate surplus between the contributing principals and the agent but, because of complete information, those redistributive issues have no impact on the actual decision that the agent takes. Instead, we put here at the core of the analysis asymmetric information which already links redistributive concerns and efficiency. Giving up the more complex model of delegated common agency is, in a sense, less of an issue in this context. ■

Benchmark: To conclude, and for further references, we denote by $q^*(\theta_1, \theta_2)$ the first-best level of public good which solves the well-known Lindahl-Samuelson conditions

[13] In Section 8, we discuss non-differentiable equilibria.

[14] See Bernheim and Whinston (1986b) who coined this term.

under complete information:

$$C'(q^*(\theta_1,\theta_2)) = \sum_{i=1}^{2} \theta_i. \quad (1)$$

Note that $q^*(\cdot)$ is (strictly) monotonically increasing in each of its arguments.

3 Preliminary Results on Equilibrium Contributions

In this section, we are interested in characterizing the truthful menus $\{t(q,\hat{\theta})\}_{\hat{\theta}\in\Theta}$ offered at a *symmetric* equilibrium of the contribution game and we thus omit the index i.

For ease of notation, let us denote $p(q,\theta_i) = \frac{\partial}{\partial q} t(q,\theta_i)$ the marginal contribution of a principal with type θ_i when q units of public good are produced.

At the last stage of the game, the agent's problem is:

$$(A): \qquad \max_q \sum_{i=1}^{2} t(q,\theta_i) - C(q).$$

The level of public good is thus given by the first-order condition

$$\sum_{i=1}^{2} p(q(\theta_1,\theta_2),\theta_i) = C'(q(\theta_1,\theta_2)), \quad (2)$$

provided the local second-order condition for the agent's problem holds:

$$\sum_{i=1}^{2} \frac{\partial p}{\partial q}(q(\theta_1,\theta_2),\theta_i) - C''(q(\theta_1,\theta_2)) \leq 0. \quad (3)$$

We will first omit this last constraint in our analysis and will check ex post that it is satisfied at equilibrium.

Of particular relevance are contribution schedules such that an upwards shift in the principal's valuation (weakly) increases the equilibrium quantity. Using (**??**) and a revealed preference argument, this is of course obtained when $\frac{\partial p}{\partial \theta_i}(q,\theta_i) \geq 0$ for all (q,θ_i). Equilibrium schedules exhibit thus the same Spence-Mirrlees property (SMP) than the principals' preferences.

Definition 2 *: A menu of differentiable contribution schedules satisfies the Spence-Mirrlees Property (SMP) when*

$$\frac{\partial p}{\partial \theta_i}(q,\theta_i) \geq 0 \quad \textit{for all} \quad (q,\theta_i).$$

For such a schedule, a standard revealed preference argument yields:

Lemma 1 *: In any PBE of the contribution game with contribution schedules satisfying SMP:*

- $q(\theta_i, \theta_{-i})$ *is almost everywhere differentiable,*
- $\frac{\partial q}{\partial \theta_i}(\theta_i, \theta_{-i}) \geq 0$, *for all* (θ_i, θ_{-i}) *in* Θ^2.

Given the outcome of that last stage of the game, the Revelation Principle applies at the first stage, i.e., when principals make their choices. It tells us that:

$$\theta_i = \arg\max_{\hat{\theta}_i} \Phi(\hat{\theta}_i, \theta_i) \tag{4}$$

where $\Phi(\hat{\theta}_i, \theta_i) = E\left[\theta_i q(\hat{\theta}_i, \cdot) - t(q(\hat{\theta}_i, \cdot), \hat{\theta}_i)\right]$ and $E[\cdot]$ is the expectation operator with respect to the distribution F.

Integrating by parts yields:

$$E\left[t(q(\hat{\theta}_i, \cdot), \hat{\theta}_i)\right] = (F(\cdot) - 1)t(q(\hat{\theta}_i, \cdot), \hat{\theta}_i)\Big|_{\underline{\theta}}^{\bar{\theta}} + E\left[\frac{1 - F(\cdot)}{f(\cdot)} p(q(\hat{\theta}_i, \cdot), \hat{\theta}_i) \frac{\partial q}{\partial \theta_{-i}}(\hat{\theta}_i, \cdot)\right]$$
$$= t(q(\hat{\theta}_i, \underline{\theta}), \hat{\theta}_i) + E\left[\frac{1 - F(\cdot)}{f(\cdot)} \left(C'(q(\hat{\theta}_i, \cdot)) - p(q(\hat{\theta}_i, \cdot), \hat{\theta}_i)\right) \frac{\partial q}{\partial \theta_{-i}}(\hat{\theta}_i, \cdot)\right]$$

Inserting into the maximand of (**??**) yields finally:

$$\Phi(\hat{\theta}_i, \theta_i) = E\left[\left(\theta_i q(\hat{\theta}_i, \cdot) - \frac{1 - F(\cdot)}{f(\cdot)} \left(C'(q(\hat{\theta}_i, \cdot)) - p(q(\hat{\theta}_i, \cdot), \cdot)\right)\right) \frac{\partial q}{\partial \theta_{-i}}(\hat{\theta}_i, \cdot)\right] - t(q(\hat{\theta}_i, \underline{\theta}), \hat{\theta}_i). \tag{5}$$

Proposition 1 *: The first- and second-order conditions for optimality of the principal's problem (**??**) are respectively given by:*

$$(FOC): E\left[\frac{\partial q}{\partial \theta_i}(\theta_i, \cdot)\left[\theta_i + p_{-i}(q(\theta_i, \cdot), \cdot) - C'(q(\theta_i, \cdot)) - \frac{1 - F(\cdot)}{f(\cdot)} \frac{\partial p}{\partial \theta_{-i}}(q(\theta_i, \cdot), \cdot)\right]\right] = 0, \tag{6}$$

$$(SOC): E\left[\frac{\partial q}{\partial \theta_i}(\theta_i, \cdot)\right] \geq 0 \tag{7}$$

for all θ_i *in* Θ *and* $i = 1, 2$.

Among the equilibrium schedules satisfying conditions (**??**) and (**??**), we focus on those which are *pointwise optimal* and monotonic in the sense that:

$$\theta_i + p(q(\theta_i, \theta_{-i}), \theta_i) - C'(q(\theta_i, \theta_{-i})) = \frac{1 - F(\theta_{-i})}{f(\theta_{-i})} \frac{\partial p}{\partial \theta_i}(q(\theta_i, \theta_{-i}), \theta_{-i}). \tag{8}$$

and

$$\frac{\partial q}{\partial \theta_i}(\theta_i, \theta_{-i}) \geq 0 \tag{9}$$

for all (θ_i, θ_{-i}) in Θ^2 , $i = 1, 2$.

This focus on *pointwise optimality* will be motivated later on in Section 7 when we turn to one particular implementation of the PBE just described. From now on, the definition of an equilibrium will implicitly include that pointwise optimality requirement.

The following lemma guarantees that every profile of public good and marginal contribution which satisfy (**??**) and (**??**) satisfies also the conditions for global optimality.

Lemma 2 *: Let $\{q(\theta_1, \theta_2), p(q, \theta)\}$ be a pair of public good level and marginal contribution schedule satisfying (**??**) and (**??**). Thus, this profile, if it exists, constitutes a PBE of the contribution game.*

Condition (**??**) has an intuitive meaning. It looks like the traditional optimality condition for a simple problem involving principal P_i and an agent with preferences $t + t_{-i}(q, \theta_{-i}) - C(q)$ who has private information on θ_{-i}. The right-hand side of (**??**) represents then the standard distortion due to the fact that, under asymmetric information, this is the *virtual demand* of the agent which should be taken into account at the time of finding P_i's best-response. We will come back on that later in Section 7 where this analogy becomes enlighting.

Condition (**??**) is also helpful in already deriving a few properties of the equilibrium schedules. To do so, let us focus on strictly increasing output schedules such that (**??**) is strict over the range of $q(\cdot)$. We can thus uniquely define the inverse function $\psi(q, \theta_i)$ as $q(\theta_i, \psi(q, \theta_i)) = q$ for all θ_i and q in the range of $q(\theta_i, \cdot)$. Note that because q is in the range of $q(\theta_i, \cdot)$, $\psi(q, \theta_i)$ belongs to $[\underline{\theta}, \bar{\theta}]$. Condition (**??**) becomes thus:

$$\psi(q, \theta_i) + p(q, \theta_i) - C'(q) = \frac{1 - F(\theta_i)}{f(\theta_i)} \frac{\partial p}{\partial \theta_i}(q, \theta_i), \tag{10}$$

for all q in the range of $q(\theta_i, \cdot)$. This can be rewritten as:

$$\frac{\partial}{\partial \theta_i}[p(q, \theta_i)(1 - F(\theta_i))] = (\psi(q, \theta_i) - C'(q))f(\theta_i)$$

This is a differential equation in θ_i which can be integrated to get $p(q, \theta_i)$ as

$$p(q, \theta_i) = \frac{\varphi(q)}{1 - F(\theta_i)} + C'(q) - \frac{1}{1 - F(\theta_i)} \int_{\theta_i}^{\bar{\theta}} \psi(q, x) f(x) dx, \tag{11}$$

where $\varphi(q)$ is an integration constant.

If we impose that $\frac{\partial p}{\partial \theta_i}(q, \theta_i)$ is bounded around $\theta_i = \bar{\theta}$, we must have $\varphi(q) = 0$. Finally, the equilibrium schedule writes as:

$$p(q, \theta_i) = C'(q) - \frac{1}{1 - F(\theta_i)} \int_{\theta_i}^{\bar{\theta}} \psi(q, x) f(x) dx. \tag{12}$$

Taking into account that

$$p(q, \theta_i) + p(q, \psi(q, \theta_i)) = C'(q) \tag{13}$$

for any q in the range of $q(\cdot)$ yields

$$p(q, \psi(q, \theta_i)) = \frac{1}{1 - F(\theta_i)} \int_{\theta_i}^{\bar{\theta}} \psi(q, x) f(x) dx,$$

or using that $\psi(q, \psi(q, \theta_i)) = \theta_i$ for all θ_i

$$p(q, \theta_i) = \frac{1}{1 - F(\psi(q, \theta_i))} \int_{\psi(q, \theta_i)}^{\bar{\theta}} \psi(q, x) f(x) dx. \tag{14}$$

Finally, integrating by parts, we obtain:

$$p(q, \theta_i) = \theta_i + \frac{1}{1 - F(\psi(q, \theta_i))} \int_{\psi(q, \theta_i)}^{\bar{\theta}} \frac{\partial \psi}{\partial x}(q, x)(1 - F(x)) dx. \tag{15}$$

From the definition of $\psi(\cdot)$,

$$\frac{\partial \psi}{\partial x}(q, x) = -\frac{\frac{\partial q}{\partial \theta_1}}{\frac{\partial q}{\partial \theta_2}}(x, \psi(q, x)) < 0$$

when output is monotonically increasing. Therefore, in any symmetric PBE of the contribution game satisfying SMP (if such an equilibrium exists) and corresponding to a strictly monotonic output and nonlinear schedules having bounded derivative $\frac{\partial p}{\partial \theta}(\cdot)$ around $\bar{\theta}$, the equilibrium schedule $t(q, \theta_i)$ *does not* reflect the preferences of the principal with types θ_i:

$$p(q, \theta_i) \leq \theta_i.$$

This contrasts sharply with the findings of Bernheim and Whinston (1986a) who show that, under complete information, equilibrium schedules can be chosen so that they reflect the principal's preferences. Under asymmetric information instead, those preferences are not reflected in the equilibrium schedule.

By summing the expressions of the marginal contributions obtained from (**??**), we get also an expression of the modified Lindahl-Samuelson conditions in our context as:

$$C'(q(\theta_1,\theta_2)) = \sum_{i=1}^{2} \frac{1}{1-F(\theta_i)} \int_{\theta_i}^{\bar{\theta}} \psi(q(\theta_1,\theta_2),x)f(x)dx. \tag{16}$$

To understand (**??**), it is useful to come back on the definition of the equilibrium schedule given in (**??**). Given the equilibrium conjecture $p(\cdot)$, one may define for any type θ_i and output q, the *conjugate type* $\psi(q,\theta_i)$ which is such that the quantity q is produced when both types follow the equilibrium strategy. All types corresponding to a valuation x greater than $\psi(q,\theta_i)$ are thus ready to contribute at the margin at least $p(q,\psi(q,\theta_i))$ for q units of public good in any SMP equilibrium. This is in front of those types that principal P_i with type θ_i can in fact underestimate his valuation and contribute less than his true willingness to pay for q units of the good. How much can he underestimate his valuation? Indeed, facing such a type x, the marginal contribution of principal P_i with conjugate type $\psi(q,x)$ is $p(q,\psi(q,x))$. Once q units of the good are produced with type x for principal P_{-i}, one can infer that the marginal valuation of principal P_i is at least $\psi(q,x)$. What (**??**) shows is that the marginal contribution of type θ_i is an average of all such inframarginal valuations. Since x is greater than θ_i, and $\psi(q,\cdot)$ is decreasing in its second argument, that average is lower than θ_i. This already shows the extent of the principals'*bid-shading* in that game.

4 Equilibrium Existence and Multiplicity

The qualitative properties of the equilibria of our game derived above do not give us much information on whether an equilibrium exists or not and whether it is unique when it exists. After all, the Lindahl-Samuelson rule (**??**) is rather complex and only defines $q(\cdot)$ implicitly in terms of its inverse functions $\psi(q,\cdot)$ which is a quite unusual feature.

Constructing Equilibria: To get further insights on the existence and multiplicity of SMP equilibria, it is useful to come back on the two conditions which define the marginal contribution $p(q,\cdot)$ and the conjugate type $\psi(q,\cdot)$ and to reconstruct from there an equilibrium:

$$p(q,\theta) + p(q,\psi(q,\theta)) = C'(q), \tag{17}$$

$$\theta - p(q,\theta) = \frac{1-F(\psi(q,\theta))}{f(\psi(q,\theta))}\frac{\partial p}{\partial \theta}(q,\psi(q,\theta)), \tag{18}$$

for all (q,θ), where q is in the range of $q(\cdot)$ the equilibrium schedule of output.

In fact, those two equations do not yet uniquely define an equilibrium marginal schedule. We need first to define which type $\tilde{\theta}$ is such that $q(\tilde{\theta},\tilde{\theta}) = q$ in the equilibrium under scrutiny. When both principals have that type, their marginal contributions are the same. For such a $\tilde{\theta}$, we must have $\psi(q, \tilde{\theta}) = \tilde{\theta}$ and

$$p(q, \tilde{\theta}) = \frac{C'(q)}{2}. \tag{19}$$

Moreover, by definition of a conjugate type, it must be that:

$$\psi(q, \psi(q, \theta)) = \theta, \tag{20}$$

for all θ in $[\underline{\theta}, \bar{\theta}]$ and q in the range of $q(\cdot)$.

The particular role played by the 45 degree line $\theta_1 = \theta_2$ in defining the "initial conditions" of the system (**??**)-(**??**) shows already one first degree of freedom that we have in defining an equilibrium. The same quantity q can a priori be given to two different types $\tilde{\theta}$ in two different equilibria.

A second degree of freedom comes from the flexibility in defining the function $\psi(q, \theta)$. Any function $\psi(q, \theta)$ is its own conjugate as soon as its graph is symmetric with respect to the 45 degree line.

Instead of defining the equilibrium output $q(\cdot)$, we may describe as well an equilibrium in terms of its isoquant lines $\psi(q, \theta)$. The (strict) monotonicity properties $\frac{\partial q}{\partial \theta_1}(\theta_1, \theta_2) > 0$ and $\frac{\partial q}{\partial \theta_2}(\theta_1, \theta_2) > 0$ are then satisfied whenever

$$\frac{\partial \psi}{\partial \theta}(q, \theta) < 0 \quad \text{and} \quad \frac{\partial \psi}{\partial q}(q, \theta) > 0 \tag{21}$$

over the whole domain of definition of $\psi(\cdot)$.

This approach in terms of isoquants is used thereafter to characterize the equilibrium schedules because it illuminates the two degrees of freedom left in specifying both the equilibrium output $Q(\theta) = q(\theta, \theta)$ along the 45 degree line and the conjugate type.

Once we define a function $\psi(q, \theta)$ satisfying (**??**) over $[\underline{\theta}, \tilde{\theta}]$ and the monotonically increasing output along the 45 degree line $Q(\theta)$, we can reconstruct the marginal contribution $p(q, \theta)$ on $[\underline{\theta}, \tilde{\theta}]$ using (**??**) and thus on the whole interval $[\underline{\theta}, \bar{\theta}]$ using (**??**). This procedure is made explicit in the next proposition.

Proposition 2 *: Fix any monotonically increasing output schedule $Q(\theta) = q(\theta, \theta)$ such that $Q(\theta) \leq q^*(\theta, \theta)$ (with an equality only at $\bar{\theta}$) and fix a function $\psi(q, \theta)$ such that:*

- *(**??**) and (**??**) hold,*
- *$\psi(q, \tilde{\theta}) = \tilde{\theta}$ for some $\tilde{\theta}$ such that $Q(\tilde{\theta}) = q$.*

*Provided that the second-order condition for the agent's problem (**??**) is satisfied, there exists a unique marginal contribution* $p(q,\theta)$ *that generates isoquants having equation* $\theta_2 = \psi(q,\theta_1)$ *in the corresponding SMP equilibrium. This marginal contribution solves the first-order differential equation*

$$\frac{\partial p}{\partial \theta}(q,\theta) = \frac{\psi(q,\theta) + p(q,\theta) - C'(q)}{\frac{1-F(\theta)}{f(\theta)}}, \tag{22}$$

over $[\underline{\theta}, \tilde{\theta}]$ *with the boundary condition (**??**). The marginal contribution over the interval* $[\tilde{\theta}, \bar{\theta}]$ *is defined by*

$$p(q,\theta) = C'(q) - p(q,\psi(q,\theta)) \tag{23}$$

where $\psi(q,\theta)$ *belongs to* $[\underline{\theta}, \tilde{\theta}]$*.*

It is by now in the Folklore of the profession to justify focusing on nonlinear contributions in models of complete information because those nonlinearities are needed to convey information or screen preferences in a world where those preferences are not common knowledge. When the concept of "truthfulness" is given a more precise meaning by explicitly introducing an information problem, we obtain a rather disappointing result: still a large set of equilibria survives.

To better understand this multiplicity, it is necessary to describe the behavior of each principal at an equilibrium of the contribution game. When choosing how much to contribute at the margin for q units of the public good, each principal behaves as a monopsonist in front of a residual supply curve. This residual supply curve is obtained by substracting the *expected* marginal contributions of the other principal from the common agent's marginal cost function. This principal chooses thus to increase his marginal contribution for q units up to the point where the marginal benefit he withdraws from all the inframarginal units produced at that price is just equal to the increase in the supply of the good that such an increased contribution induces. When the other principal contributes at the margin on average less, the residual supply is shifted downwards and, at a best response, a given principal chooses also to contribute less at the margin. This creates some form of complementarity between principals and generates multiple equilibria.

Remark 1: When an isoquant $\psi(q,\theta)$ is defined over $[\underline{\theta}, \tilde{\theta}]$ it must be, *by definition*, that $\psi(q,\underline{\theta}) \leq \bar{\theta}$. Reciprocally, when $\psi(q,\theta)$ is only defined over an interval $[\theta_1, \tilde{\theta}]$ with $\theta_1 > \underline{\theta}$, we have $\psi(q,\theta_1) = \bar{\theta}$. In other words, q does not belong to the range of the equilibrium schedule $q(\theta,\cdot)$ for $\theta < \theta_1$. ■

Remark 2: The resolution techniques and the multiplicity of equilibria found above are reminiscent of the analysis of equilibria in double auctions made in Leininger, Linhart and Radner (1989). Those authors have developed a procedure that consists in fixing the equilibrium strategies for the buyer and the seller when their valuations coincide and reconstruct numerically the bidding strategies as solutions of differential equations with lags on both sides of these critical values. Menezes, Monteiro and Temini (2001) and Laussel and Palfrey (2003, p. 460) use also a similar technique in their public good model with a 0-1 decision. The flexibility in choosing the equilibrium quantities in a continuum allows us to solve *explicitly* a similar differential equation for the marginal contributions as a function of a $\psi(q,\theta)$ function which represents a degree of freedom. Menezes, Monteiro and Temini (2001) argue that one should be careful in checking for the monotonicity conditions of the equilibrium schedule. Our approach avoids this problem. We start from specifying a $\psi(q,\theta)$ which satisfies those monotonicity conditions and then reconstruct equilibrium strategies. ■

Ex Post Inefficiency: Surprisingly, the Lindahl-Samuelson conditions (**??**) put very little restrictions on the possible equilibrium outputs, except the fact that equilibrium outputs are downward distorted below the first-best. Indeed,we have:

Proposition 3 *: For all θ in $[\underline{\theta},\bar{\theta}]$, we have:*

$$p(q,\theta) \leq \theta, \tag{24}$$

with a strict inequality everywhere except at $\theta = \bar{\theta}$ and for $q = q^*(\bar{\theta},\bar{\theta})$.

Marginal contributions never reflect the true valuations of the principals and "bid-shading" occurs in any equilibrium. Intuitively, this phenomenon is nothing else than the usual "free-rider" problem for public good. Instead of being cast in a centralized Bayesian mechanism as in the framework of Mailath and Postlewaite (1990), free-riding appears now at the symmetric equilibrium of a game with voluntary contributions. Principals shade their valuations and their marginal contributions to the public good are less than what it is worth to each of them. As a result of this phenomenon, there is underprovision of the public good.

As a trivial consequence of bid-shading, we have:

Corollary 1 *: An ex post efficient outcome can never be implemented at a SMP equilibrium of the common agency game. Downward distortions below the first-best always occur.*

Remark 3: It is worth noticing that, under complete information, free-riding does not occur in the Bernheim and Whinston (1986a)'s framework since the public good level is efficient when schedules are truthful. If one is interested in the normative properties of the equilibria, one needs to relax the efficiency concept. This is what will be done in the next section. ■

Multiplicity Revisited: To sharpen intuition for why multiple equilibria are possible, it is useful to give an alternative expression of the Lindahl-Samuleson conditions. This expression is obtained by summing up the conditions of pointwise optimality for both principals to get first:

$$C'(q(\theta_1,\theta_2)) = \sum_{i=1}^{2}\left(\theta_i - \frac{1-F(\theta_i)}{f(\theta_i)}\frac{\partial p}{\partial \theta_i}(q(\theta_1,\theta_2),\theta_i)\right). \tag{25}$$

Using (**??**) and differentiating with respect to θ_i yields

$$\frac{\partial q}{\partial \theta_i}\left(\sum_{i=1}^{2}\frac{\partial p}{\partial q}(q,\theta_i) - C''(q)\right) = -\frac{\partial p}{\partial \theta_i}(q,\theta_i).$$

Inserting into (**??**) gives a new expression for the Lindahl-Samuelson conditions:

$$C'(q(\theta_1,\theta_2)) = \sum_{i=1}^{2}\theta_i + \left(\sum_{i=1}^{2}\frac{\partial p}{\partial q}(q(\theta_1,\theta_2),\theta_i) - C''(q(\theta_1,\theta_2))\right)\left(\sum_{i=1}^{2}\frac{1-F(\theta_i)}{f(\theta_i)}\frac{\partial q}{\partial \theta_i}(\theta_1,\theta_2)\right). \tag{26}$$

This expression illuminates also the degree of freedom available to describe equilibria. All this freedom can be captured by the term $\sum_{i=1}^{2}\frac{\partial p}{\partial q}(q(\theta_1,\theta_2),\theta_i) - C''(q(\theta_1,\theta_2)$ which is the second derivative of the agent's objective function evaluated at the equilibrium quantities. That term plays the role of a *conjecture* that, when it varies, allows to trace out the different equilibrium quantities. The purpose of the next two sections is precisely to pin down this conjecture by imposing various requirements (interim efficiency or robustness to perturbations).

For the time being, this expression allows us to get:

Corollary 2 *: At a SMP equilibrium charcaterized in Proposition 4, the local second-order condition of the agent's problem holds:*

$$\sum_{i=1}^{2}\frac{\partial p}{\partial q}(q(\theta_1,\theta_2),\theta_i) - C''(q(\theta_1,\theta_2)) \leq 0 \tag{27}$$

for all (θ_1,θ_2) *in* Θ^2.

5 Interim Efficiency

Under complete information, Bernheim and Whinston (1986a) have observed that the "truthful" equilibrium of common agency game lies on the Pareto-frontier of what the contributing principals could achieve by binding themselves through a contract. Under asymmetric information, one can still be interested by the normative properties of the equilibria provided Pareto efficiency is replaced by *interim efficiency* to take into account asymmetric information.[15] Following Laussel and Palfrey (2003), we will thus investigate under which circumstances, an equilibrium of our common agency game under asymmetric information is *interim efficient.*

We first describe the interim efficient allocations. Those allocations are obtained as the solution of a centralized mechanism design problem where an uninformed mediator (possibly the agent) offers a single mechanism to both principals, who then report their types to this mediator. This mediator maximizes a weighted sum of both the principals and the agent's utilities with the weights given to different types of the principals being possibly different. For simplicity, we restrict to symmetric allocations so that the weights do not depend on the principal's identity.

Proposition 4 *:(Ledyard and Palfrey (1999)) A level of public good $q(\theta_1, \theta_2)$ is interim efficient if and only if there exists positive social weights $\alpha(\theta_i)$ such that:*

$$\int_{\underline{\theta}}^{\bar{\theta}} \alpha(\theta_i) f(\theta_i) d\theta_i \leq 1, \tag{28}$$

$$C'(q(\theta_1, \theta_2)) = \sum_{i=1}^{2} b(\theta_i) \tag{29}$$

where

$$b(\theta_i) = \theta_i - \frac{1 - F(\theta_i)}{f(\theta_i)}(1 - \tilde{\alpha}(\theta_i))$$

is increasing with well-specified weights $\alpha_i(\cdot)$ and where

$$\tilde{\alpha}(\theta_i) = \frac{1}{1 - F(\theta_i)} \int_{\theta_i}^{\bar{\theta}} \alpha(x) f(x) dx.$$

*This formula is valid as long as the right-hand side of (**??**) is monotonically increasing in each θ_i for $i = 1, 2$.*

The fact that $\int_{\underline{\theta}}^{\bar{\theta}} \alpha(x) f(x) dx < 1$ captures the possibility that a positive social weight is given to the common agent in the social welfare function.

As a preliminary remark, notice that an interim efficient allocation is necessarily such that $C'(q(\theta_1, \theta_2))$ is separable in θ_1 and θ_2; an extremely restrictive condition which may kill much equilibria of our common agency game.

This separability is not pure luck. Indeed, to derive interim efficient allocations, Holmström and Myerson (1983) and later on Ledyard and Palfrey (1999) and Laussel and

[15] See Holmström and Myerson (1983).

Palfrey (2003) use a centralized mechanism: all information needed to decide upon the level of public good is reported to an uninformed mediator who commits to a mechanism which specifies public good levels and compensations as functions of those reports. Under common agency instead, allocations result from an equilibrium between a pair of decentralized mechanisms. As Section 7 will make clear, each principal signals his own type through the mere offer he makes to the agent whereas, at the same time, he designs his mechanism to screen the preferences of the other. For each piece of information, there is in a sense too much communication and an unnecessary duplication of reports in the common agency game. That lack of coordinated communication makes it unlikely that interim efficiency is achieved in all equilibria of our game. We will be interested in a weaker statement which is to assess whether there nevertheless exist equilibria which are indeed interim efficient.

If $q(\cdot)$ interim efficient is implemented through a common agency equilibrium, we must have

$$\psi(q,\theta) = b^{-1}(C'(q) - b(\theta))$$

and using (**??**), we get that $b(\cdot)$ must satisfy the following functional equation:

$$\sum_{i=1}^{2} b(\theta_i) = \sum_{i=1}^{2} \frac{1}{1-F(\theta_i)} \int_{\theta_i}^{\bar{\theta}} b^{-1}(b(\theta_1)+b(\theta_2)-b(x))f(x)dx \quad \text{for all} \quad (\theta_1,\theta_2) \in \Theta^2. \tag{30}$$

Finding directly the solutions (if any) to (**??**) is difficult in general, let us first content ourselves with looking at linear solutions of the form $b(\theta) = \bar{\theta} + \lambda(\theta - \bar{\theta})$ and imposing conditions on the the type distribution which ensure that such a linear solution exists.

Proposition 5 *: Assume that* $1 - F(\theta) = \left(\frac{\bar{\theta}-\theta}{\bar{\theta}-\underline{\theta}}\right)^{\beta}$ *where* $\beta \geq 0$*, then there exists at least one equilibrium of the common agency game under asymmetric information which is interim efficient with:*

$$C'(q^{IE}(\theta_1,\theta_2)) = \left(\frac{\beta+2}{\beta+1}\right)(\theta_1+\theta_2) - \frac{2\bar{\theta}}{\beta+1}. \tag{31}$$

It corresponds to social weights $\alpha(\theta) = \frac{1}{\beta+1}$ *for all* $\theta \in [\underline{\theta}, \bar{\theta}[$*, and to marginal contributions*

$$p^{IE}(q,\theta) = \frac{C'(q)+\theta}{\beta+1} + \frac{\bar{\theta}(1-2\beta)}{\beta(\beta+1)} \tag{32}$$

and linear isoquants

$$\psi^{IE}(q,\theta) = \frac{(\beta+1)C'(q)+2\bar{\theta}}{\beta+2} - \theta. \tag{33}$$

The case for interim efficiency exists under those strong assumptions on the type distribution. We see on (??) that the common agency equilibrium selected by the efficiency criterion satisfies the separability property already stressed. With the generalized β-distributions proposed, one equilibrium output depends linearly on the sum of the principal's marginal preferences for the public good whatever the cost function. This linearity ensures separability.[16]

The social weights on the principal's different types are constant and do not sum to one, reflecting the fact that the mediator proposing the centralized mechanism implementing that interim efficient allocation puts a positive weight on the agent. An alternative interpretation of this outcome is worth stressing. Everything happens as if, with this centralized mechanism, the agent had now all the bargaining power in proposing a mechanism to the principals but would give them a weight $\frac{1}{\beta+1} < 1$ in his objective function as in the seminal paper of Baron and Myerson (1982) on regulation.

Instead of working with (??), we may as well try to identify the Lindahl-Samuelson conditions (??) with those conditions (??) obtained at interim efficient allocations. Proceeding that way, we obtain:

Proposition 6 *: Assume that* $lim_{\theta\to\bar{\theta}} f(\theta)\int_{\underline{\theta}}^{\theta}\frac{dx}{1-F(x)} \leq 1$[17] *and that the monotone hazard rate property* $\frac{d}{d\theta}\left(\frac{1-F(\theta)}{f(\theta)}\right) < 0$ *holds, then there exists at least one equilibrium of the common agency game under asymmetric information which is interim efficient with:*

$$C'(q^{IE}(\theta_1,\theta_2)) = \sum_{i=1}^{2} b^{IE}(\theta_i). \tag{34}$$

where

$$b^{IE}(\theta_i) = \theta_i - (1-F(\theta_i))\int_{\underline{\theta}}^{\theta_i}\frac{dx}{1-F(x)} \leq \theta_i \tag{35}$$

with equalities at both $\underline{\theta}$ *and* $\bar{\theta}$*. It corresponds to an output* $Q_0(\theta)$ *along the 45 degree line such that:*

$$C'(Q_0(\theta)) = 2\left(\theta - (1-F(\theta))\int_{\underline{\theta}}^{\theta}\frac{dx}{1-F(x)}\right). \tag{36}$$

Propositions **??** and **??** altogether show that the common agency game may generate one equilibrium which implements what a planner having a particular specification of the social weights of the different principals in his objective function would do by himself.

[16]Lausel and Palfrey (2003) derive also such a linear interim efficient outcome in the 0-1 case.

[17]Note that this implies that $f(\bar{\theta}) = 0$.

This can be viewed as a Decentralization Theorem along the lines of the Second Welfare Theorem but, of course, in a rather different context.[18]

Even though it is quite attractive this result relies heavily on the distribution of types belonging to a rather special class. When those assumptions are not satisfied, one needs something else to select among equilibria. This is the purpose of next section.

6 Robustness to Perturbations

We follow now the so-called *"Wilson doctrine"* and look for an equilibrium allocation that is unique and robust to the details of the environments. More specifically, we assume that the cost function of the agent writes as $C(q) + \varepsilon q$ for some shock ε drawn on an interval centered around zero $]-\bar{\varepsilon}, \bar{\varepsilon}[$ according to a cumulative distribution $H(\cdot)$ that we leave unspecified. We do not make any assumption on the support of this distribution.[19] Principals commit to a nonlinear schedule before the realization of ε. This shock is observable ex post only by the agent after he has accepted the principals' offers. The same nonlinear contribution $t(q, \theta_i)$ must thus be used for all realizations of ε since, with nonlinear contributions, no possibility to screen ε is left to the principals.

For a triplet $(\theta_1, \theta_2, \varepsilon)$, the quantity of public good $q(\theta_1, \theta_2, \varepsilon)$ chosen by the agent satisfies:

$$\sum_{i=1}^{2} p(q(\theta_1, \theta_2, \varepsilon), \theta_i) = C'(q(\theta_1, \theta_2, \varepsilon)) + \varepsilon. \tag{37}$$

From which we deduce by differentiating (**??**) with respect to θ_i and ε respectively:

$$\frac{\partial p}{\partial \theta_i}(q, \theta_i) = \frac{\partial q}{\partial \theta_i}\left(C''(q) - \sum_{i=1}^{2} \frac{\partial p}{\partial q}(q, \theta_i)\right), \tag{38}$$

$$-1 = \frac{\partial q}{\partial \varepsilon}\left(C''(q) - \sum_{i=1}^{2} \frac{\partial p}{\partial q}(q, \theta_i)\right). \tag{39}$$

Proceeding as in Section 3, and focusing again on equilibria which are *pointwise optimal*,[20] we find:

$$\theta_i + p(q, \theta_{-i}) - \varepsilon - C'(q) = \frac{1 - F(\theta_i)}{f(\theta_i)} \frac{\partial p}{\partial \theta_i}(q, \theta_i), \tag{40}$$

[18] In our view, that result is of the same nature as the standard result in bargaining theory that the optimal trading mechanism between privately informed buyer and seller can be implemented as the equilibrium of a double auction when types are uniformly distributed on the same support. See Myerson and Sattherwaite (1983).

[19] Principals may not even share the same beliefs on ε.

[20] Pointwise optimality should now be also meant with respect to the realization of ε also.

with the monotonicity condition $\frac{\partial q}{\partial \theta_i} \geq 0$ for $i = 1, 2$.

Summing (**??**) when $i = 1$ and $i = 2$ and using (**??**) and (**??**) yields:

$$\sum_{i=1}^{2} \theta_i - \varepsilon - C'(q) = \left(C''(q) - \sum_{i=1}^{2} \frac{\partial p}{\partial q}(q, \theta_i) \right) \left(\sum_{i=1}^{2} \frac{1 - F(\theta_i)}{f(\theta_i)} \frac{\partial q}{\partial \theta_i} \right). \tag{41}$$

Had ε being identically fixed at zero, the multiplicity of equilibria would come from the flexibility in specifying the second derivative of the agent's objective function $-C''(q) + \sum_{i=1}^{2} \frac{\partial p}{\partial q}(q, \theta_i)$. When the *same* marginal contribution is used for all values of the shock ε, this flexibility is pinned down by (**??**). The equilibrium condition (**??**) becomes a first-order partial derivative equation:

$$\left(\sum_{i=1}^{2} \theta_i - \varepsilon - C'(q) \right) \frac{\partial q}{\partial \varepsilon} + \sum_{i=1}^{2} \frac{1 - F(\theta_i)}{f(\theta_i)} \frac{\partial q}{\partial \theta_i} = 0. \tag{42}$$

This system can be solved explicitly to find the surface $q = q(\theta_1, \theta_2, \varepsilon)$ in $\mathbb{R}^4$. Following for instance John (1982), it is well known that every such integral surface is the union of characteristic curves obtained as solutions to the following of ordinary differential equations:

$$\frac{d\theta_i}{dt} = \frac{1 - F(\theta_i)}{f(\theta_i)}, \quad \text{for } i = 1, 2; \tag{43}$$

$$\frac{d\varepsilon}{dt} = \sum_{i=1}^{2} \theta_i - \varepsilon - C'(q), \tag{44}$$

$$\frac{dq}{dt} = 0, \tag{45}$$

where t in $\mathbb{R}_+$ is an arbitrary parametrization of these characteristic curves.

(**??**) can be integrated directly as

$$1 - F(\theta_i(t)) = k_i e^{-t} \quad \text{or} \quad \theta_i(t) = \Psi(1 - k_i e^{-t}) \tag{46}$$

where $\Psi = F^{-1}$ and k_i's are arbitrary positive constants. Note that one can arbitrarily choose $k_1 = 1$ which amounts to a rescaling of the parameter t.

Inserting into (**??**) yields

$$\varepsilon(t) = K e^{-t} + e^{-t} \int_{t_0}^{t} e^{x} \left(\sum_{i=1}^{2} \Psi(1 - k_i e^{-x}) \right) dx - C'(q), \tag{47}$$

where K is an arbitrary constant and $t_0 = \max\{\ln k_1, \ln k_2, 0\}$ since $\Psi(\cdot)$ is defined over $[0, 1]$.[21]

[21] Note that for $t \to +\infty$, all solutions to (**??**)-(**??**) converge towards $(\bar{\theta}, \bar{\theta}, 2\bar{\theta} - C'(q))$.

Since we are interested in defining the trace of integral surfaces on the hyperplane $\varepsilon = 0$ to select robust equilibria, it must be that $q(\theta_1, \theta_2, 0)$ (denoted thereafter $q(\theta_1, \theta_2)$ to simplify notation) satisfies:

$$C'(q(\theta_1, \theta_2)) = Ke^{-t} + e^{-t} \int_{t_0}^{t} e^x \left(\sum_{i=1}^{2} \Psi(1 - k_i e^{-x}) \right) dx$$

with t satisfying (**??**).

After simplifications, we get the following family of solutions parametrized by the one-dimensional parameter K:

$$C'(q(\theta_1, \theta_2)) = (1 - F(\theta_1)) \left\{ K + \int_0^{-\ln(1-F(\theta_1))} e^x \left(\Psi(1 - e^{-x}) + \Psi\left(1 - \frac{F(\theta_2)}{1 - F(\theta_1)} e^{-x}\right)\right) dx \right\}, \tag{48}$$

for $\theta_2 \geq \theta_1$ (i.e. above the 45 degree line) so that $k_2 \leq 1$.

To better understand the structure of those solutions, let us compute their values $q(\theta, \theta) = Q(\theta)$ along the ray $\theta_1 = \theta_2 = \theta$:

$$C'(Q(\theta)) = (1 - F(\theta)) \left(K + 2 \int_0^{-\ln(1-F(\theta))} e^x \Psi(1 - e^{-x}) dx \right).$$

Integrating by parts and changing variables, we finally obtain:

$$C'(Q(\theta)) = K'(1 - F(\theta)) + 2 \left(\theta - (1 - F(\theta)) \int_{\underline{\theta}}^{\theta} \frac{dx}{1 - F(x)} \right)$$

for some K'.

Such a solution is always monotonically increasing in θ on $[\underline{\theta}, \bar{\theta}]$ when $K' \leq 0$.

Proposition 7 *: There exists a one-dimensional set of equilibrium outputs which are robust to perturbations of the agent's cost function.*

*Within that set, the equilibrium outputs on the ray $\theta_1 = \theta_2 = \theta$ can be ranked with a higher output $Q_0(\theta)$ being defined by (**??**).*

Of course $Q_0(\theta) \leq q^*(\theta, \theta)$ because of free-riding between the agents. However, there are no distortions at both sides of the type interval, $Q_0(\bar{\theta}) = q^*(\bar{\theta}, \bar{\theta})$ and $Q_0(\underline{\theta}) = q^*(\underline{\theta}, \underline{\theta})$.[22]

[22]This can be seen by using Lhôspital's rule since

$$\lim_{\theta \to \bar{\theta}} (1 - F(\theta)) \int_{\underline{\theta}}^{\theta} \frac{dx}{1 - F(x)} = \lim_{\theta \to \bar{\theta}} \frac{1 - F(\theta)}{f(\theta)} = 0.$$

Interestingly, the equilibria selected through interim efficiency (Proposition **??**) and that robust to perturbations are identical on the 45 degree line. Tedious computations show nevertheless that the two equilibria differ off the diagonal. In that case, interim efficiency and robustness to perturbations may somewhat be conflicting.

Remark 1: Importantly, our selection device yields an allocation $q_0(\theta_1, \theta_2)$ which can be computed for *any* distributions $F(\cdot)$ whereas interim efficiency is explicitly obtained only with restrictive assumptions on those distributions. Our selection device is thus applicable in a broader range of environments. ■

Remark 2: It is worth mentioning that perturbations could come from the presence of an extra principal whose preferences for the public good ϵq were observable only by the agent. ■

Remark 3: Think about an econometrician having observed data (marginal contributions and outputs), aware of the structure of the game and willing to estimate which equilibrium is played among those presented in Section 4. This econometrician would like to introduce some shock exactly as we did above and would estimate the robust equilibrium allocation. ■

The special case of a 0-1 project: As an application, consider the special case of a 0-1 project whose unit cost is c as in Laussel and Palfrey (2003). We assume that the principals' valuations are drawn in $[0, 1]$ and that $c < 2$ so that, under complete information, it would be optimal to make the project if both principals have a sufficiently high valuation, namely $\theta_1 + \theta_2 \geq c$. The quantity q is now interpreted as the probability of building the project.

Using (**??**), the boundary of the area where the project is done is defined implicitly above the bissectrice $\theta_1 = \theta_2$ by the curve:

$$c = (1 - F(\theta_1)) \int_0^{-\ln(1-F(\theta_1))} e^x \left(\Psi(1 - e^{-x}) + \Psi\left(1 - \frac{1 - F(\theta_2)}{1 - F(\theta_1)} e^{-x}\right)\right) dx,$$

for $\theta_2 \geq \theta_1$ and a symmetric expression for $\theta_2 \leq \theta_1$.

It is easy to check that the line which separates the areas where the project is either done or not crosses the horizontal axes $\theta_2 = 1$ for θ_1^* which solves:

$$c = (1 - F(\theta_1^*)) \int_0^{-\ln(1-F(\theta_1^*))} e^x [\Phi(1 - e^{-x}) + 1] dx.$$

For a uniform distribution, we find

$$c = 2\theta^* + (1 - \theta^*) \ln(1 - \theta^*)$$

and thus $1 > \theta^* > \frac{c}{2}$ necessarily.

[22] Consistently with Proposition 7, we select $K' = 0$.

7 Mechanism Design Approach

In this section, we propose an alternative approach to characterize the *pointwise optimal* equilibria of the common agency game using mechanism design techniques. This new approach illustrates the role of the nonlinear contribution of any given principal in simultaneously screening the other principal's type and signaling his own to the agent.

Viewing the strategy of each principal as a choice within a menu $\{t(q,\hat{\theta}_i)\}_{\hat{\theta}_i\in\Theta}$ and writing down the condition for incentive compatibility, may *a priory* entails a loss of generality since a given principal might like to deviate to a more complex mechanism. In this section, we show that this is indeed not the case. We will describe explicitly P_i's best-response to P_{-i}'s own offer within the largest class of mechanisms available and show that it can be implemented as a contribution schedule.

Definition 3 *: A symmetric menu of contributions $\{t(q,\hat{\theta}_i)\}_{\hat{\theta}_i\in\Theta}$ is separating if two different types of the same principal P_i offer two different contributions so that these offers reveal all information on this principal's preferences to the agent.*

For a fixed contribution made by principal P_i, the design of P_{-i}'s contribution is an informed principal problem under private values. We know from Maskin and Tirole (1990) that, under risk-neutrality, there is no loss of generality in having principal P_{-i} offering a contract to the agent exactly as if the latter was informed on his type. Intuitively, the mechanism consisting in pilling up the various contracts that would be signed by those different types if the agent was informed on P_{-i}'s preferences is incentive compatible from the principal's point of view and achieves a lower bound on the principal's payoffs. The key insight due to Maskin and Tirole (1990) is that, higher payoffs can only be achieved if the principal is risk-averse by pooling those contracts at the time of offering contracts and revealing the principal's type at a later stage. This relaxes the agent's incentive and participation constraints and improves risk-sharing among the different types of principal. With risk-neutrality, this insurance motive is absent and the lowest bound on the principal's payoff is also an upper bound. In that case, instead of offering a mechanism to the agent where both he and the agent report their types after contract's acceptance, the principal is as well off revealing his type right away by offering only one contract. For each contribution offered by P_i, P_{-i} has thus always in his best-response correspondence a separating menu of contributions.[23]

[23] The reader will recognize in our approach a feature of Bernheim and Whinston (1986a)'s original paper. To select among all equilibria of their contribution game, they indeed first noticed that each principal has a best response which is truthful and thus justified that looking at equilibria in truthful schedules is meaningful.

In the single principal environment of Maskin and Tirole (1990), this equivalence between two contracting modes has no consequence. In our common agency environment instead, that seemingly innocuous difference in the timing of information revelation has a strategic value since it affects the way principal P_i will himself contract with the agent. Provided that P_{-i}'s offer reveals his type to the agent, P_i knows that he should design his contribution not only to signal his own type to the agent but also to learn also P_{-i}'s type which is "endogenously" learned in equilibrium by the agent.

This points at the major role that nonlinear contributions play in a common agency environment: learning over what Epstein and Peters (1999) and Peters (2001) would call *market information*; i.e., everything which is not known to a given principal and, most specifically in our context, the preferences of others.

Following Martimort and Stole (2002 and 2003), we focus on pure strategy PBE with separating menus. To compute P_i's best response to P_{-i}'s nonlinear contribution $t_{-i}(q, \theta_{-i})$, we may thus as well restrict the analysis to direct truthful revelation mechanisms $\{t_i^D(\hat{\theta}_{-i}|\theta_i), q(\hat{\theta}_{-i}|\theta_i)\}$ where $\hat{\theta}_{-i}$ is the agent's report on θ_{-i} (that he has learned through the revelation induced by P_{-i}'s offer).

We will compute these direct revelation mechanisms *as if* θ_i was known by the agent. Again, this is justified by our discussion above and the result of Maskin and Tirole (1990). The agent's utility can then be written as:

$$\hat{U}(\hat{\theta}_{-i}, \theta_{-i}|\theta_i) = t_i^D(\hat{\theta}_{-i}|\theta_i) + t_{-i}(q(\hat{\theta}_{-i}|\theta_i), \theta_{-i}) - C(q(\hat{\theta}_{-i}|\theta_i)). \tag{49}$$

From incentive compatibility we get:

$$U(\theta_{-i}|\theta_i) = \hat{U}(\theta_{-i}, \theta_{-i}|\theta_i) = \max_{\hat{\theta}_{-i}} \hat{U}(\hat{\theta}_{-i}, \theta_{-i}|\theta_i).$$

We assume that $t_{-i}(q, \theta_{-i})$ is twice differentiable and satisfies the SMP. Using standard techniques, we get:

- $q(\theta_{-i}|\theta_i)$ is monotonically increasing and thus almost everywhere differentiable with respect to θ_{-i} with,
$$\frac{\partial q}{\partial \theta_{-i}}(\theta_{-i}|\theta_i) \geq 0 \qquad \text{a.e..} \tag{50}$$

- $U(\theta_{-i}|\theta_i)$ is almost everywhere differentiable in θ_{-i} with
$$\frac{\partial U}{\partial \theta_{-i}}(\theta_{-i}|\theta_i) = \frac{\partial t_{-i}}{\partial \theta_{-i}}(q(\theta_{-i}|\theta_i), \theta_{-i}). \tag{51}$$

At a best-response to $t_{-i}(q, \theta_{-i})$, P_i with type θ_i must solve the following problem:

$$(P_i(\theta_i)) : \quad \max_{\{U(\cdot);q(\cdot)\}} E\left[\theta_i q(\cdot|\theta_i) + t_{-i}(q(\cdot|\theta_i), \cdot) - C(q(\cdot|\theta_i)) - U(\cdot|\theta_i)\right], \tag{52}$$

subject to (**??**)-(**??**) and

$$U(\theta_{-i}|\theta_i) \geq 0, \qquad \text{for all } \theta_{-i} \in \Theta. \tag{53}$$

where (**??**) is the agent's ex post participation which guarantees that he makes a positive profit for all preference profiles (θ_i, θ_{-i}).

A solution to (P_i) is an allocation $\{U(\theta_{-i}|\theta_i); q(\theta_{-i}|\theta_i\}$ or equivalently a direct revelation mechanism $\{t_i^D(\theta_{-i}|\theta_i), q(\theta_{-i}|\theta_i)\}$ (we omit the dependence on $t_{-i}(q, \theta_{-i})$) from which we can reconstruct a nonlinear contribution $t_i(q, \theta_i)$ when $q(\theta_{-i}|\theta_i)$ is invertible. Of course, since all problems $(P_i(\theta_i))$ have the same constrained set, the menu $\{t_i(q|\hat{\theta}_i)\}_{\hat{\theta}_i \in \Theta}$ obtained is incentive compatible from principal P_i's point of view.

Proposition 8 *: Provided that (**??**) holds, an equilibrium with separating contributions satisfies (**??**) and (**??**) and is thus pointwise optimal.*

Comparing with the more direct approach taken in Proposition 1, we observe that the equilibria with separating contributions describe indeed the *pointwise optimal* allocations that we selected in Section 3. By the same token, it is easy to see that the equilibria satisfying the weaker condition (**??**) correspond in fact to cases where a subset S of the principals P_{-i} pool and offer the whole set of contributions $\{t_{-i}(q, \theta_{-i})\}_{\theta_{-i} \in S}$.

Proposition **??** shows that the focus on *pointwise* optimal allocations is a rather natural requirement. It comes immediately from the fact that each principal may as well reveal truthfully his type to the agent through his mere offer of a contract at a best response.

The mechanism design approach is also useful to better understand why interim efficiency is in general so difficult to achieve. Indeed, at an equilibrium of the common agency game, each principal offers a contribution to influence the quantity of public good that will be produced and the contribution effectively made by the other principal. The decision rule about quantity of public good chosen is not committed ex ante as under centralized contracting but decided ex post, i.e., after that contributions are chosen. This imperfect commitment on the agent's side and the duplication of information channel it implies may introduce some inefficiency.

8 Conclusion

In this paper, we analyzed a common agency game under asymmetric information with, in mind, the objectives of checking whether the earlier lessons of common agency games under complete information are in fact robust and of extending their insights. In that respect, our results leave us with contrasted feelings. Under asymmetric information, incentive compatibility conditions replace the “truthfulness” requirement used in the earlier literature but far from helping in selecting an equilibrium allocation, it still generates a multiplicity of outcomes. “Almost anything” is an equilibrium allocation provided that this allocation leads to under-provision of the public good below the first-best. Free-riding and ex post inefficiency are thus pervasive in common agency games under asymmetric information.

Nevertheless, a few more positive insights emerge from our analysis. First, we have been able to specify conditions under which equilibria of the common agency game yield interim efficient allocations. This suggests that modelers could as well forget about the complexity of the decentralized approach and instead look at a centralized mechanism design approach provided that the social weights on the different types of principals are conveniently specified. Extending the conditions under which that decentralization result obtains seems a fruitful alley for research. Second, if one is not particularly comfortable with an efficiency criterion, one may prefer to select among all equilibrium allocations those which are robust to random perturbations of the game. Finally, the class of pointwise optimal equilibria of common agency games under asymmetric information can be easily analyzed with standard mechanism design techniques and is robust in the sense that a principal would not like to deviate to a larger space of mechanisms to improve his payoff.

Our model should certainly be extended along several directions. First, we should analyze also delegated common agency games. Taking a mechanism design approach in computing best-responses, this possibility introduces a type-dependent participation constraint which affects the distribution of surplus between principal. An open question is whether it also affects the equilibrium allocations. Second, other information structures could possibly be analyzed. One may think of the case of correlation between the principals' types and of the case where the agent has also some private information on his own. Third, following Maskin and Tirole (1990), we know that risk-aversion on the principals' side forces pooling in informed principal games. This may significantly change equilibrium patterns in common agency environments. Fourth, the robust equilibrium we have selected may be amenable to econometric analysis.[24] Lastly, in other institutional contexts, allocations do not result from a well-centralized mechanisms but come out of the equilibria among various decentralized mechanisms. One may think of multi-unit auctions on financial or electricity markets for instance. It would be nice to extend the approach taken in this paper to these environments. We hope to investigate some of these issues in future research.

[24] This can be useful in view of the recent vintage of empirical works having taken the Grossman and Helpman (1994) political economy model to estimate policy distortions. See Gawande and Bandyopadhyay (2000) for instance.

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Appendix

Proof of Lemma ??: Let us fix (θ_i, θ_{-i}) and consider $\theta_i > \theta_i'$.

By definition, we have

$$t(q(\theta_i,\theta_{-i}),\theta_i) + t(q(\theta_i,\theta_{-i}),\theta_{-i}) - c(q(\theta_i,\theta_{-i})) \geq t(\tilde{q},\theta_i) + t(\tilde{q},\theta_{-i}) - c(\tilde{q}), \qquad \forall \tilde{q}.$$

Thus,

$$t(q(\theta_i,\theta_{-i}),\theta_i) - t(\tilde{q},\theta_i) \geq t(\tilde{q},\theta_{-i}) - c(\tilde{q}) - [t(q(\theta_i,\theta_{-i}),\theta_{-i}) - c(q(\theta_i,\theta_{-i}))]$$

$$\text{for all } \tilde{q} \leq q(\theta_i,\theta_{-i}).$$

Using (SMP), the l.h.s. above is lower than

$$t(q(\theta_i,\theta_{-i}),\theta_i') - t(\tilde{q},\theta_i') \quad \text{for all } \tilde{q} \leq q(\theta_i,\theta_{-i}).$$

Then $q(\theta_i,\theta_{-i}) \geq q(\theta_i',\theta_{-i})$ and $q(\cdot)$ is almost everywhere differentiable in each of its arguments. ■

Proof of Proposition ??: Using (??), we get the following first-order derivative of Φ with respect to $\hat{\theta}_i$:

$$\frac{\partial \Phi}{\partial \hat{\theta}_i}(\hat{\theta}_i,\theta_i) = E\left[\left(\theta_i - \frac{1-F(\cdot)}{f(\cdot)}\left(C''(q(\hat{\theta}_i,\cdot)) - \frac{\partial p}{\partial q}(q(\hat{\theta}_i,\cdot),\cdot)\right)\right)\frac{\partial q}{\partial \theta_i}(\hat{\theta}_i,\cdot)\frac{\partial q}{\partial \theta_{-i}}(\hat{\theta}_i,\cdot)\right]$$
$$-E\left[\frac{1-F(\cdot)}{f(\cdot)}(C'(q(\hat{\theta}_i,\cdot)) - p(q(\hat{\theta}_i,\cdot),\cdot))\frac{\partial^2 q}{\partial\theta_i\partial\theta_{-i}}(\hat{\theta}_i,\cdot)\right]$$
$$-p(q(\hat{\theta}_i,\underline{\theta}),\hat{\theta}_i)\frac{\partial q}{\partial \theta_i}(\hat{\theta}_i,\underline{\theta}) - \frac{\partial t}{\partial \theta_i}(q(\hat{\theta}_i,\underline{\theta}),\hat{\theta}_i). \qquad \text{(A1)}$$

Integrating by parts the second term yields

$$E\left[\frac{1-F(\cdot)}{f(\cdot)}(C'(q(\hat{\theta}_i,\cdot))-p(q(\hat{\theta}_i,\cdot),\cdot))\frac{\partial^2 q}{\partial\theta_i\partial\theta_{-i}}(\hat{\theta}_i,\cdot)\right]=$$

$$(1-F(\cdot))(C'(q(\hat{\theta}_i,\cdot))-p(q(\hat{\theta}_i,\cdot),\cdot))\frac{\partial q}{\partial\theta_i}(\hat{\theta}_i,\cdot)\Big|_{\underline{\theta}}^{\bar{\theta}}$$

$$-E\left[\frac{1-F(\cdot)}{f(\cdot)}\frac{\partial q}{\partial\theta_i}(\hat{\theta}_i,\cdot)\left(\frac{\partial q}{\partial\theta_{-i}}(\hat{\theta}_i,\cdot)\left(C''(q(\hat{\theta}_i,\cdot))-\frac{\partial p}{\partial q}(q(\hat{\theta}_i,\cdot),\cdot)\right)+\frac{\partial p}{\partial\theta_i}(q(\hat{\theta}_i,\cdot),\cdot)\right)\right]$$

$$+E\left[C'(q(\hat{\theta}_i,\cdot))-p(q(\hat{\theta}_i,\cdot),\cdot)\right]$$

$$=-p(q(\hat{\theta}_i,\underline{\theta}),\hat{\theta}_i)\frac{\partial q}{\partial\theta_i}(\hat{\theta}_i,\underline{\theta})$$

$$-E\left[\frac{1-F(\cdot)}{f(\cdot)}\left\{\left(C''(q(\hat{\theta}_i,\cdot))-\frac{\partial p}{\partial q}(q(\hat{\theta}_i,\cdot),\cdot)\right)\frac{\partial q}{\partial\theta_i}(\hat{\theta}_i,\cdot)\frac{\partial q}{\partial\theta_{-i}}(\hat{\theta}_i,\cdot)+\frac{\partial q}{\partial\theta_i}(\hat{\theta}_i,\cdot)\frac{\partial p}{\partial\theta_i}(q(\hat{\theta}_i,\cdot),\cdot)\right\}\right]$$

$$+E\left[C'(q(\hat{\theta}_i,\cdot))-p(q(\hat{\theta}_i,\cdot),\cdot)\right]. \tag{A2}$$

where the last equality comes from (**??**) for $\theta_1=\hat{\theta}_i$ and $\theta_2=\underline{\theta}$.

Moreover, it must be that the agent's payoff with type $\hat{\theta}_i$ when $\theta_{-i}=\underline{\theta}$ is zero

$$t(q(\hat{\theta}_i,\underline{\theta}),\hat{\theta}_i)+t(q(\hat{\theta}_i,\underline{\theta}),\underline{\theta})=C(q(\hat{\theta}_i,\underline{\theta})),\quad \text{for all } \hat{\theta}_i. \tag{A3}$$

Differentiating w.r.t. $\hat{\theta}_i$ yields:

$$\left(p(q(\hat{\theta}_i,\underline{\theta}),\hat{\theta}_i)+p(q(\hat{\theta}_i,\underline{\theta}),\underline{\theta})-C'(q(\hat{\theta}_i,\underline{\theta}))\right)\frac{\partial q}{\partial\theta_i}(\hat{\theta}_i,\underline{\theta})+\frac{\partial t}{\partial\theta_i}(q(\hat{\theta}_i,\underline{\theta}),\hat{\theta}_i)=0$$

and thus using (**??**),

$$\frac{\partial t}{\partial\theta_i}(q(\hat{\theta}_i,\underline{\theta}),\hat{\theta}_i)=0 \tag{A4}$$

for all $\hat{\theta}_i$.

Inserting into (**??**) yields:

$$\frac{\partial\Phi}{\partial\hat{\theta}_i}(\hat{\theta}_i,\theta_i)=E\left[\frac{\partial q}{\partial\theta_i}(\hat{\theta}_i,\cdot)\left(\theta_i+p(q(\hat{\theta}_i,\cdot),\cdot)-C'(q(\hat{\theta}_i,\cdot))-\frac{1-F(\cdot)}{f(\cdot)}\frac{\partial p}{\partial\theta_{-i}}(q(\hat{\theta}_i,\cdot),\cdot))\right)\right]. \tag{A5}$$

For $\hat{\theta}_i=\theta_i$ being the optimal report, i.e., $\frac{\partial\Phi}{\partial\hat{\theta}_i}(\theta_i,\theta_i)=0$, we obtain the first-order condition (**??**).

The second-order condition for the principal's problem is

$$\frac{\partial^2\Phi}{\partial\hat{\theta}_i^2}(\hat{\theta}_i,\theta_i)\Big|_{\hat{\theta}_i=\theta_i}\leq 0.$$

But using (**??**) and the envelope theorem and taking the total derivative of (**??**) with respect to $\hat{\theta}_i$, we get

$$\frac{\partial^2 \Phi}{\partial \hat{\theta}_i^2}(\theta_i, \theta_i) = -E\left[\frac{\partial q}{\partial \theta_i}(\theta_i, \cdot)\right],$$

hence (**??**). ■

Proof of Lemma ??: The proof of this lemma reduces to show that the schedule satisfying (**??**) and (**??**) is not only locally incentive compatible but also globally. From (**??**) and (**??**) we have that

$$\Phi(\theta_i, \theta_i) - \Phi(\hat{\theta}_i, \theta_i) = \int_{\hat{\theta}_i}^{\theta_i} \frac{\partial \Phi}{\partial \hat{\theta}_i}(x, \theta_i)dx = \int_{\hat{\theta}_i}^{\theta_i} \left[\frac{\partial \Phi}{\partial \hat{\theta}_i}(x, \theta_i) - \frac{\partial \Phi}{\partial \hat{\theta}_i}(x, x)\right] dx$$
$$= \int_{\hat{\theta}_i}^{\theta_i} E\left[\frac{\partial q}{\partial \theta_i}(x, \cdot)\right](\theta_i - x)dx.$$

By (**??**) this last expression is always non-negative. ■

Proofs of Propositions ?? and ??: Fix $\psi(q, \theta)$ and $\tilde{\theta}$ in $[\underline{\theta}, \bar{\theta}]$ such that $\psi(q, \tilde{\theta}) = \tilde{\theta}$ and $q(\tilde{\theta}, \tilde{\theta}) = q$.

From (**??**) taken for $\psi(q, \theta)$, we have:

$$\frac{1 - F(\theta)}{f(\theta)} \frac{\partial p}{\partial \theta}(q, \theta) = \psi(q, \theta) - p(\psi(q, \theta)). \tag{A6}$$

Using (**??**), we get (**??**). Still using (**??**) at $\theta = \tilde{\theta}$, we obtain the initial condition (**??**).

Consider (**??**) and note that it can be rewritten as:

$$\frac{\partial}{\partial \theta}\left[(1 - F(\theta))p(q, \theta)\right] = (\psi(q, \theta) - C'(q))f(\theta).$$

Integrating yields

$$(1 - F(\theta))p(q, \theta) = k + \int_{\tilde{\theta}}^{\theta} \psi(q, x)f(x)dx + C'(q)(1 - F(\theta)).$$

But using (**??**), we get $k = -\frac{C'(q)}{2}(1 - F(\tilde{\theta}))$ and finally:

$$p(q, \theta) = C'(q)\left(1 - \frac{1 - F(\tilde{\theta})}{2(1 - F(\theta))}\right) + \frac{1}{1 - F(\theta)} \int_{\tilde{\theta}}^{\theta} \psi(q, x)f(x)dx. \tag{A7}$$

We must check that $\frac{\partial p}{\partial \theta}(q, \theta) > 0$ over $[\underline{\theta}, \tilde{\theta}]$ to have a SMP equilibrium. Then, because $\frac{\partial \psi}{\partial \theta}(q, \theta) < 0$ and from (**??**), we have

$$\frac{\partial p}{\partial \theta}(q, \theta) = -\frac{\partial p}{\partial \theta}(q, \psi(q, \theta))\frac{\partial \psi}{\partial \theta}(q, \theta) > 0$$

also on $[\tilde{\theta}, \bar{\theta}]$, by (**??**).

To guarantee $\frac{\partial p}{\partial \theta}(q, \theta) > 0$ on $[\underline{\theta}, \tilde{\theta}]$, we must have

$$\psi(q, \theta) + p(q, \theta) - C'(q) > 0$$

on $[\underline{\theta}, \tilde{\theta}]$.

Using (**??**), this amounts to proving that

$$\Phi(\theta) = 2(1 - F(\theta))\psi(q, \theta) + 2\int_{\tilde{\theta}}^{\theta} \psi(q, x)f(x)dx - (1 - F(\tilde{\theta}))C'(q)$$

is positive over $[\underline{\theta}, \tilde{\theta}]$.

Note that

$$\dot{\Phi}(\theta) = 2\frac{\partial \psi}{\partial \theta}(q, \theta)(1 - F(\theta)) \leq 0$$

so that $\Phi(\theta)$ is decreasing over $[\underline{\theta}, \tilde{\theta}]$ and thus minimized at $\tilde{\theta}$ for $\Phi(\tilde{\theta}) = (1 - F(\tilde{\theta}))(2\tilde{\theta} - C'(q)) > 0$. Hence, $\frac{\partial p}{\partial \theta}(q, \theta) > 0$ over $[\underline{\theta}, \tilde{\theta}]$.

From that, we immediately obtain that $\psi(q, \theta) \geq p(q, \psi(q, \theta))$ on $[\underline{\theta}, \tilde{\theta}]$ and by differentiating (**??**) that $\frac{\partial p}{\partial \theta}(q, \theta) > 0$ also on $[\tilde{\theta}, \bar{\theta}]$ which implies that $p(\theta) \leq \theta$ on $[\underline{\theta}, \tilde{\theta}]$ also. ■

Proof of Corollary ??: From Corollary **??**, we know that $C'(q(\theta_1, \theta_2) \leq \theta_1 + \theta_2$, moreover, from construction in Proposition **??**, we have $\frac{\partial q}{\partial \theta_i} > 0$. Hence the result follows. ■

Proof of Proposition ??: (**??**) amounts to:

$$\sum_{i=1}^{2} b(\theta_i) = \sum_{i=1}^{2} 2\theta_i - \frac{1}{1 - F(\theta_i)} \int_{\theta_i}^{\bar{\theta}} xf(x)dx,$$

which admits the (unique) solution $b(\cdot)$ defined by

$$b(\theta) = 2\theta - \frac{1}{1 - F(\theta)} \int_{\theta}^{\bar{\theta}} xf(x)dx = \theta - \frac{1}{1 - F(\theta)} \int_{\theta}^{\bar{\theta}} (1 - F(x))dx.$$

When $1 - F(\theta) = \left(\frac{\bar{\theta} - \theta}{\bar{\theta} - \underline{\theta}}\right)^{\beta}$, we get

$$b(\theta) = \frac{\beta + 2}{\beta + 1}\theta - \frac{\bar{\theta}}{\beta + 1}$$

which finally yields (**??**).

From this, we also get

$$1-\tilde{\alpha}(\theta)=\frac{f(\theta)}{(1-F(\theta))^2}\int_\theta^{\bar{\theta}}(1-F(x))dx$$

and

$$\int_\theta^{\bar{\theta}}\alpha(x)f(x)dx=1-F(\theta)-\frac{f(\theta)}{1-F(\theta)}\int_\theta^{\bar{\theta}}(1-F(x))dx.$$

Note that

$$\int_{\underline{\theta}}^{\bar{\theta}}\alpha(x)f(x)dx<1$$

so that a positive social weight is necessarily given to the agent.

Moreover, we get:

$$\alpha(\theta)f(\theta)=\frac{d}{d\theta}\left(\frac{f(\theta)}{1-F(\theta)}\right)\int_\theta^{\bar{\theta}}(1-F(x))dx$$

and finally $\alpha(\theta)=\frac{1}{\beta+1}$ for all θ in $[\underline{\theta},\bar{\theta}]$. Finally, $\psi(q,\theta)$ is directly obtained from (**??**) and $p(q,\theta)$ is derived from (**??**). ■

Proof of Proposition ??: We can identity conditions (**??**) and (**??**) by setting

$$1-\tilde{\alpha}(\theta_i)=\frac{\partial q}{\partial\theta_i}\left(\sum_{i=1}^{2}\frac{\partial p}{\partial q}(q,\theta_i)-C''(q)\right).$$

By this, we get along an isoquant q:

$$-\frac{\partial\psi}{\partial\theta_1}=\frac{\frac{\partial q}{\partial\theta_1}}{\frac{\partial q}{\partial\theta_2}}=\frac{1-\tilde{\alpha}(\theta_1)}{1-\tilde{\alpha}(\psi(q,\theta_1))}. \quad \text{(A8)}$$

For an equilibrium to be interim efficient, we must have

$$\psi(q,\theta_1)=b^{-1}(C'(q)-b(\theta_1))$$

and thus

$$-\frac{\partial\psi}{\partial\theta_1}=\frac{\dot{b}(\theta_1)}{\dot{b}(\psi(q,\theta_1))}. \quad \text{(A9)}$$

Identifying (**??**) and (**??**), one possibility is to set

$$\dot{b}(\theta)=1-\tilde{\alpha}(\theta).$$

Inserting this expression in the definition of $b(\cdot)$ yields the differential equation:

$$b(\theta)=\theta-\frac{1-F(\theta)}{f(\theta)}\dot{b}(\theta), \quad \text{(A10)}$$

with the boundary condition $b(\bar{\theta}) = \bar{\theta}$.

Solving (**??**) gives

$$b^{IE}(\theta) = k(1 - F(\theta)) + \theta - (1 - F(\theta)) \int_{\underline{\theta}}^{\theta} \frac{dx}{1 - F(x)}, \tag{A11}$$

with $k \leq 0$ to insure that $\dot{b}^{IE}(\theta) > 0$ everywhere on $[\underline{\theta}, \bar{\theta}]$. For $k = 0$, we get the less distorted outcome in the ex post sense.

We have also

$$1 - \tilde{\alpha}(\theta) = \dot{b}^{IE}(\theta) = f(\theta) \int_{\underline{\theta}}^{\theta} \frac{dx}{1 - F(x)}.$$

$\tilde{\alpha}(\theta)$ remains positive as long as $1 \geq \Phi(\theta) = f(\theta) \int_{\underline{\theta}}^{\theta} \frac{dx}{1-F(x)}$.

One can show that $\Phi(\theta)$ remains increasing as long as $\Phi(\theta) < 1$. Hence, under the assumption of the proposition, $\tilde{\alpha}(\theta)$ remains positive.

Moreover, by differentiating in θ, we have:

$$\alpha(\theta) = 2 - f(\theta) \int_{\underline{\theta}}^{\theta} \frac{dx}{1 - F(x)} + \frac{\dot{f}(\theta)(1 - F(\theta))}{f(\theta)} \int_{\underline{\theta}}^{\theta} \frac{dx}{1 - F(x)}.$$

When the monotone hazard rate property holds, we have:

$$\frac{\dot{f}(\theta)(1 - F(\theta))}{f(\theta)} > -f(\theta)$$

and thus

$$\alpha(\theta) \geq 2(1 - \Phi(\theta)) > 0$$

ensuring that all social weights are positive. ■

Derivation of (??): Observe that now:

$$\Phi(\hat{\theta}_i, \theta_i) = E[\theta_i q(\hat{\theta}_i, \cdot) - t(q(\hat{\theta}_i, \cdot), \hat{\theta}_i)]$$

where $q(\hat{\theta}_i, \cdot)$ is meant for $q(\hat{\theta}_i, \theta_i, \varepsilon)$ and $E[\cdot]$ denotes now the expectation operator with respect to the joint distribution of θ_{-i} and ε which has density $f(\theta_{-i})h(\varepsilon)$.

Proceeding as in the main text, we get again:

$$\Phi(\hat{\theta}_i, \theta_i) = E\left[\left(\theta_i q(\hat{\theta}_i, \cdot) - \frac{1 - F(\cdot)}{f(\cdot)} \left(C'(q(\hat{\theta}_i, \cdot)) - p(q(\hat{\theta}_i, \cdot), \cdot)\right)\right) \frac{\partial q}{\partial \theta_{-i}}(\hat{\theta}_i, \cdot)\right]$$
$$- E_\varepsilon[t(q(\hat{\theta}_i, \underline{\theta}, \varepsilon), \hat{\theta}_i)], \tag{A12}$$

where $E_\varepsilon[\cdot]$ denotes now the expectation operator with respect to the distribution of ε. The proof then is similar to that of Proposition **??** once we replace (**??**) by the participation constraint in expectation over ε since the agent accepts the contracts before the realization of ε:

$$E_\varepsilon[t(q(\hat{\theta}_i,\underline{\theta},\varepsilon),\hat{\theta}_i)+t(q(\hat{\theta}_i,\underline{\theta},\varepsilon),\underline{\theta})-C(q(\hat{\theta}_i,\underline{\theta},\varepsilon))]=0,\quad \text{for all } \hat{\theta}_i. \tag{A13}$$

The optimality condition $\frac{\partial\Phi}{\partial\hat{\theta}_i}(\theta_i,\theta_i)=0$ and the corresponding second-order condition $\frac{\partial^2\Phi}{\partial\hat{\theta}_i^2}(\theta_i,\theta_i)=0$ can be handled as in the proof of Proposition **??**.

Proof of Proposition ??: For equilibria where $\frac{\partial t_{-i}}{\partial\theta_{-i}}(q,\theta_{-i})\geq 0$, $U(\theta_{-i}|\theta_i)$ is increasing and (**??**) is binding at $\theta_{-i}=\underline{\theta}$. Integrating by parts, we obtain:

$$E\left[U(\cdot|\theta_i)\right]=E\left[\frac{1-F(\cdot)}{f(\cdot)}\frac{\partial t_i}{\partial\theta_{-i}}(q(\cdot|\theta_i),\cdot)\right].$$

Inserting into (**??**) and optimizing with respect to $q(\theta_{-i}|\theta_i)$ yields (**??**).

Provided that $\frac{\partial q}{\partial\theta_{-i}}(\theta_{-i}|\theta_i)\geq 0$, this gives the pointwise optimal output. ■

Non-Differentiable Equilibria: For the sake of completeness, we present in this section a class of non-differentiable equilibria. To analyze those equilibria, it turns out that the most useful procedure is based on the *supply profile* due to Wilson (1993).[25]

Consider a principal P_i with type θ_i willing to pay a marginal contribution p_i for q units of the public good. This principal has to evaluate the measure of types of principal P_{-i} who are also willing to contribute enough so that this amount is produced.

Formally, if P_{-i} follows the strategy $p_{-i}(q,\cdot)$, the likelihood that q units of public good are produced is

$$\text{proba }\left\{p_{-i}(q,\tilde{\theta}_{-i})+p_i\geq C'(q)\right\}=1-G_{-i}(C'(q)-p_i|q)$$

where $G_{-i}(\cdot|q)$ is the cumulative distribution of the marginal contribution of principal P_{-i} for q units of the public good. Given that residual supply schedule, P_i acts in fact as a monopsonist and offers a marginal contribution for q units of the public good which solves:

$$p_i(q,\theta_i)=\arg\max_{p_i}(\theta_i-p_i)(1-G_{-i}(C'(q)-p_i|q)).$$

These best responses for each θ_i induce a distribution of marginal contributions $G_i(\cdot|q)$ for principal P_i. A symmetric equilibrium of the common agency game is thus a family of distributions (one for each value of q) $G(\cdot|q)$ which are fixed-points of these processes.

To find an interesting class of non-differentiable equilibria, it is in fact enough to specify marginal contributions having two steps and a threshold $\theta^*(q)$ such that:

[25] Wilson (1993) is interested in nonlinear pricing and thus consider in fact *demande profiles*.

- for $\theta \geq \theta^*(q)$, $p(q,\theta) = \bar{p}(q)$;
- for $\theta \leq \theta^*(q)$, $p(q,\theta) = \underline{p}(q)$ $(< \bar{p}(q))$.

As we will see below, the three functions $\bar{p}(\cdot)$, $\underline{p}(\cdot)$ and $\theta^*(\cdot)$ are linked altogether by some equilibrium conditions. Given a function $\theta^*(q)$ (satisfying some properties to be made precise below), one can certainly find a two-step equilibrium (or the marginal contribution associated to it) using those conditions.

For a two-step symmetric equilibrium, let us describe the probability that q units of the public good are produced given a marginal contribution p_i:

$$
\begin{array}{rcll}
G(C'(q) - p_i|q) & = & 0 & \text{if} \quad p_i > C'(q) - \underline{p}(q) \\
G(C'(q) - p_i|q) & = & F(\theta^*(q)) & \text{if} \quad C'(q) - \bar{p}(q) \leq p_i \leq C'(q) - \underline{p}(q) \\
G(C'(q) - p_i|q) & = & 1 & \text{if} \quad p_i \leq C'(q) - \bar{p}(q)
\end{array}
$$

where in first (last) case q units of the public good are (never) produced and the second case there is a probability $(1 - F(\theta^*(q)))$ to be produced.

For each quantity q and θ_i a type for P_i, P_i's best response is to offer a marginal contribution $\bar{p}(q) = C'(q) - \underline{p}(q)$ whenever

$$
\begin{aligned}
\theta_i - C'(q) + \underline{p}(q) \geq & \max_{C'(q)-\bar{p}(q) \leq p_i \leq C'(q) - \underline{p}(q)} (\theta_i - p_i)(1 - F(\theta^*(q))) \qquad \text{(A14)} \\
& = (\theta_i - C'(q) + \bar{p}(q))(1 - F(\theta^*(q))).
\end{aligned}
$$

The set of such types θ_i is thus of the form $[\theta^*(q), \bar{\theta}]$ as requested by the structure postulate for the equilibrium.

At a symmetric two-step equilibrium, it must thus be that the two following conditions hold:

$$\bar{p}(q) + \underline{p}(q) = C'(q), \tag{A15}$$

and

$$\theta_i = \theta^*(q) \text{ solves } (\textbf{??}) \text{ as an equality, i.e.,} \tag{A16}$$

$$\theta^*(q)F(\theta^*(q)) = \bar{p}(q)(2 - F(\theta^*(q)) - C'(q)(1 - F(\theta^*(q))). \tag{A17}$$

For q such that $\frac{C'(q)}{2} \leq \bar{p}(q) \leq \bar{\theta}$, (**??**) defines uniquely $\theta^*(q)$ in $[\underline{\theta}, \bar{\theta}]$. Alternatively, given an increasing schedule $Q^*(\theta)$ which admits an inverse $\theta^*(q)$ which is almost everywhere differentiable, one can reconstruct $\bar{p}(q)$ from (**??**) and $\underline{p}(q)$ from (**??**). Note that $\bar{p}(q)$ is such that $\bar{p}(q) \geq \frac{C'(q)}{2}$.

Proposition 9 *: There exists a multiplicity of equilibria with two-steps marginal contributions. For each $Q^*(\theta)$ monotonically increasing with $Q^*(\bar{\theta}) = q^*(\bar{\theta}, \bar{\theta})$ and $Q^*(\theta) \leq q^*(\theta, \theta)$, there exists an equilibrium described by (**??**) and (**??**).*

Proof: The only thing to note is that for $\theta_1 < \theta^*(q) \leq \theta_2$, P_2 offers a marginal contribution $\bar{p}(q)$ whereas P_1 offers $\underline{p}(q)$, leading to the choice of q units. Idem for $\theta_2 \leq \theta^*(q) < \theta_1$ with the identity of the principals being reversed. When $\theta_1 = \theta_2 = \theta^*(q)$, note that both principals are indifferent between paying $\bar{p}(q)$ or $\underline{p}(q)$ at the margin. Break this indifference with a lexicographic order in favor of principal P_1 who pays indeed $\underline{p}(q)$ when both contributions are the same. Then the isoquant for q units cuts the diagonal at $\theta_1 = \theta_2 = \theta^*(q)$. Note that (**??**) and $\bar{p}(q) \geq \frac{C'(q)}{2}$ imply that $2\theta^*(q) \geq C'(q)$. ■

It is worth describing the isoquants corresponding to those non-differentiable equilibria. In fact, those curves are the reunion made of the horizontal segment $\{\theta_1 \geq \theta^*(q)\}$ with the vertical segment $\{\theta_2 \geq \theta^*(q)\}$. Those non-differentiable equilibria allows us to describe settings where isoquants are not strictly decreasing ($\psi(q, \cdot)$ being not invertible).

Remark 1: Equilibria with more than two steps can also be constructed following the same procedure. ■

Remark 2: Those non-differentiable equilibria are clearly not robust to the introduction of perturbations. ■