Do dividends signal more earnings?

A theoretical analysis

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Abstract

The signaling models have contributed to the literature of corporate finance by the formalization of “the informational content of dividends hypothesis”. However, these models are under criticism of empirical works, as weak evidences were found supporting one of the main predictions: the positive relation between changes in dividends and changes in earnings. We claim that the failure to verify this prediction does not invalidate the signaling approach. The models developed up to now assume or derive utility functions with the single-crossing property. We show that signaling is possible in the absence of this property and, in this case, changes in dividend and changes in earnings can be positively or negatively related.

Keywords: Dividend policy, non-monotone contracts, signaling, single-crossing property.

JEL Codes: C72, D82, G35.
The information content of dividends is a controversial issue in corporate finance. The research started when Miller and Modigliani (1961) suggested that managers use dividend policy to convey their expectations of future prospects of the firm. With this hypothesis they proposed to explain the effect of dividend changes on the prices of shares. Since then, theoretical and empirical research advanced. Signaling models were the main tool that formalized the original intuition. Bhattacharya (1979), Miller and Rock (1985), and John and Williams (1985) were the initiators of a long list of signaling models\(^1\). The basic idea is that firm managers possess private information about future earnings and they like to convey it to the market. However, they cannot simply announce their expectations of future earnings publicly because every firm could imitate them. The information is conveyed by a costly signal. In the cited models, the respective costs are: financing of a committed level of dividend, suboptimal investment and tax on dividends.

On the empirical side, researchers tries to verify the testable implications derived from the models. In all these models the single-crossing property holds. As a consequence, the models predict that dividends, market price and future or current earnings are positively related. The correlation between dividend and returns was a strongly established result even before the signaling models have appeared. Aharony and Swary (1980) show that announcements of dividend increases or decreases result in, respectively, positive or negative abnormal returns.

The controversy lays on the relationship between dividends and subsequent earnings. Watts (1973) analysis found the positive relation, however, the effect was very small and not conclusive. Healy and Palepu (1988) found a significant relation, but they focused in the particular situation of initiation and omission of dividend payment. Exploring a larger data set Benartzi, Michaely and Thaler (1997) found no significant relation between dividends and future earnings and concluded that dividends are more related to past and present

\(^1\)See Allen and Michaely (1995) for a survey on theoretical and empirical issues on dividend policy.
earnings. More recently, Nissim and Ziv (2001) using an improved measure of future earnings concluded that dividends matters for earnings prediction. Many other works contributed to this debate and a definitive conclusion seems far to be reached.

We claim that the lack of a clear relation between dividends and earnings is not incompatible with information content of dividends. Common to all the previous models of signaling is the existence of single-crossing in the objective function of manager, i.e., the marginal cost of signaling is monotonic in the type of firms. This property generates the monotonic relationship between dividends and earnings. In this work, we drop this assumption and develop a model employing the techniques presented in Araujo and Moreira (2001a). In non-single-crossing signaling, a new kind of equilibrium may exist. In this equilibrium the relation between firms earnings and dividends is U-shaped, so that two types of firms signal with the same level of dividend. The two types are indistinguishable by the market and this situation can be interpreted as low types pretending to be high types. The market value of shares is an average of the values of two types and increases with dividend. So, signaling models may provide a positive relation between dividends and market price and, at same time, an ambiguous relation between dividends and earnings.

The model is presented in section 1 and it is similar to the model of Miller and Rock (1985). For concreteness, we assume a quadratic production function, in section 2, and discuss an example. Computations are performed for a range of parameters values and results are presented in section 3. The conclusions are presented in section 4.

1 The model

This model builds on Miller and Rock (1985). There is a firm with production function $F(\cdot)$. The usual properties, $F'(\cdot) > 0$ and $F''(\cdot) < 0$, are assumed. Let $X$ and $Y$ be the earnings, respectively, at period 1 and 2. At the beginning of period 1, managers know $X$ and announce dividends, $D$. Then shareholders
may sell their shares, dividends are distributed and $X - D$ are invested. At the end of period 1, the firm production is subject to a multiplicative shock, $\delta > 0$, so that

$$Y = \delta F(X - D),$$

where $0 \leq D \leq X$. At period 2 the earnings are distributed and the firm is disassembled. The information asymmetry is on the knowledge of $X$, which we assume randomly distributed on $[X_1, X_2]$, with density $p(X)$. Consistent with the terminology used in theory of contracts, we refer to $X$ as the type of the firm. At period 1, managers know $X$ before the dividend announcement, but the market does not, and managers cannot credibly convey their private information to the market. The shock $\delta$ is unknown to both manager and market, but may be correlated with $X$.

Both $X$ and $\delta$ affect the value of firm. Now suppose manager has an estimate of $\delta$ based on a private information in period 1. If managers are interested on market value of firm, they would like to sign $X$ and $\delta$ when the implied value is high. The signaling of $X$ is clear. If a firm pays more dividends, it incurs in increasing costs due to underinvestment. Since higher type firms have higher earnings, their sacrifice in output is lower for the same level of dividend. The decreasing cost of dividend allows signaling in a way that higher types signal with higher dividends. Also, they would like to reveal $\delta$ to the market but, in this case, the signaling is not possible. They cannot signal high productivity distributing more dividends, as high $\delta$ firms has higher marginal cost of dividend because optimal investment level is higher and sacrifice in output is higher. On the other side, a high $\delta$ firm cannot signal higher investment opportunity paying less dividends because low $\delta$ firms could imitate paying low dividends. However, we will show that if $X$ and $\delta$ are correlated, the shock in productivity may produce a signaling in which, for a subset of firms, dividend choice is decreasing with respect of type.
Assumption 1 The shocks are correlated,

\[ E[\delta |X] = \varepsilon(X) > 0. \]

1.1 The value of the firm

At period 1, managers estimate the fundamental cum-dividend value of the firm as the present value of dividend flow:

\[ V(X, D) = D + \frac{1}{1+i} E[\delta F(X - D)|X] \]

\[ = D + \frac{1}{1+i} \varepsilon(X) F(X - D). \] (1)

Under symmetric information, this would be the value of shares and the manager would choose the investment in order to maximize \( V \). The first-best dividend level, \( D^* \), would be given by the Kuhn-Tucker conditions:

\[ F'(X - D^*) - \frac{1 + i}{\varepsilon(X)} \begin{cases} = 0, & \text{if } D^* > 0, \\ \geq 0, & \text{if } D^* = 0. \end{cases} \] (2)

But under asymmetric information, the market value may not coincide with \( V \). Let \( V^m \) denote the market value. We will assume that \( V^m \) is determined as a signaling equilibrium, that is, firms signal to the market by the choice of dividend level and market estimates the value observing the dividend choice. The firms will choose dividend above the optimal level, paying an underinvestment cost for signaling.

Shareholders want to maximize \( V \) if they keep the share with them until period 2. The ones who intend to sell at period 1 prefer the maximization of the market value, \( V^m \). As in Miller and Rock (1985), we assume the firm’s managers are maximizing a welfare function that aggregates the interests of shareholders that desire to sell the shares and the ones who do not. Let \( k \in (0,1] \) be the fraction of shareholders that sell at period 1. This fraction is exogenous and can be motivated by necessity of liquidity by shareholders. The welfare function is

\[ W(X, D, V^m) = k V^m + (1 - k) V(X, D). \]
For the purpose of signaling analysis, we are interested on marginal rate of substitution between $V^m$ and $D$. Since $W$ is quasi-linear with respect to $V^m$, all the properties are found in marginal welfare of $D$,

$$W_D(X, D) = (1 - k) \left( 1 - \frac{1}{1 + \frac{1}{i}} \epsilon(X) F'(X - D) \right).$$

The dependence of productivity shock on type may make $W_D$ non-monotone on type. More precisely, higher $X$ increases $\epsilon$, but reduces $F'$, for the same level of dividend.

In terms of the cross-derivative of $W$,

$$W_{XD}(X, D) = \frac{1 - k}{1 + \frac{1}{i}} \left[ -\epsilon(X) F''(X - D) - \epsilon'(X) F'(X - D) \right], \tag{3}$$

may change its sign. We can define two regions a in $X \times D$ plan, according to this sign.

**Definition 1** The $CS^+$ region (resp. $CS^-$ region) is the set of points in $X \times D$ plan such that $W_{XD} > 0$ (resp. $W_{XD} < 0$).

In equation (3), the first term in the brackets is the investment effect. Firms with higher earnings invest more and have lower marginal product. Consequently, the marginal cost of dividend is lower for higher types. The second term is the productivity effect. Firm earnings provide information about the future productivity, which affect the expected marginal cost of the signal.

**Negative correlation between earnings and productivity**

When $\epsilon'(X) \leq 0$, higher earnings reduce the expected productivity and the cost of signaling is lower. Welfare function has the single-crossing property since both productivity and investment effects collaborate on $W_{XD} > 0$. The results are, therefore, similar to the ones found by Miller and Rock (1985).

**Positive correlation between earnings and productivity**

If $\epsilon'(X) > 0$, higher earnings correspond to a higher optimal investment level and dividend becomes costlier since more earnings should be retained for investment. If, for some types and dividend level, productivity effect dominates, then
$W_{XD} < 0$ and higher types will be more reluctant to pay dividends because of
lost of investment opportunities. Conversely, $W_{XD} > 0$ holds when investment
effect dominates. In this case, lower types are more reluctant to pay dividends
because they have lesser investment resources. Note that from (2) the first-best
dividend as a function of types, $D^*(X)$, is increasing for $W_{XD} > 0$ and decreasing
for $W_{XD} < 0$. If investment effect dominates, firms with higher earnings
may pay more dividends and, if productivity effect dominates, they should in-
vest more paying less dividends. When $\varepsilon'(X) > 0$ is such that the signal of
$W_{XD}$ is ambiguous, the single-crossing propriety does not hold. The assump-
tion on the constancy of sign (for instance in Riley (1979)) for cross-derivative
of objective function is then violated and we need another approach developed
in Araujo and Moreira (2001a,b).

1.2 The signaling equilibrium

As usual, the signaling equilibrium is a perfect Bayesian one (the formal defi-
nition is provided in appendix A.1). The basic description remains. The market
generates a value function, $V^m(\cdot)$, and each type of firm, $X$, chooses a dividend
level, $D$, that maximizes $W$. We have an equilibrium if zero expected profit
condition holds, that is,

$$V^m(D) = E_D[V(X, D)], \tag{4}$$

where $E_D$ denotes the expectation taken on the Bayesian updated distribution
on $X$. The market value $V^m$ should be the expected value of the firm with
respect to the probability distribution of $X$, resulting from the Bayesian update
given the choice of $D$ by the firm.

Formally, the signaling problem consists in finding functions $\mathcal{V}^m(X)$ and
$\mathcal{D}(X)$ such that the type $X$ firm chooses a dividend level $\mathcal{D}(X)$ and is evaluated
as $\mathcal{V}^m(X)$ by the market. Since dividends and market value are linked by $V^m(\cdot)$,
these functions are related by $\mathcal{V}^m(X) = V^m(\mathcal{D}(X))$. 
Define the welfare of type $X$ firm that declares to be type $\hat{X}$ as

$$\mathcal{W}(\hat{X}, X) = W(X, D(\hat{X}), V^m(D(\hat{X})))$$

$$= kV^m(D(\hat{X})) + (1 - k)V(X, D(\hat{X})).$$

In order to be incentive compatible, each firm should prefer to tell the truth, that is

$$\mathcal{W}(X, X) \geq \mathcal{W}(\hat{X}, X),$$

for all $X, \hat{X} \in [X_1, X_2]$. A differential equation for $D$ is derived from the first order condition

$$\frac{\partial \mathcal{W}}{\partial X}(X, X) = 0.$$ (6)

It should be noted that the first order condition is not sufficient condition for implementability when single-crossing property does not hold. Incentive compatibility should be checked globally after a candidate for equilibrium is obtained.

Additionally, the second order condition constrains $D'(X)$:

**Proposition 1** In signaling equilibrium, $D(X)$ is non-decreasing in $CS+$ region and non-increasing in $CS-$ region.

**Proof:** See the appendix.

When single-crossing property is present, $CS+$ and $CS-$ do not show up simultaneously and contracts should be monotone. As a consequence, types are separated when $D'(X) \neq 0$, or an interval of types is bunched when $D'(X) = 0$. When single-crossing property does not hold, monotonicity is not assured and the relationship between type and signal may be, for example, U-shaped and a disconnected set of types may signal with the same dividend level.

### 1.3 Equilibria diversity

In a equilibrium, the same signal, $D$, may be chosen by many types. We are interested in classifying the equilibrium according to its degree of separability. The following definition will be useful:
Definition 2 The pooling set, $\Theta(D)$, is the set of types whose signal is $D$, that is, $\Theta = \{X \in [X_1, X_2] | D(X) = D\}$.

In particular, in a separating equilibrium, $\Theta(D)$ is singleton for every $D$ that is chosen by a firm.

Definition 3 The type $X$ is separated if $\Theta(D(X)) = \{X\}$. A separating equilibrium is a signaling equilibrium such that every $X$ is separated.

When $X$ is separated, market correctly infers the type by the observation of $D$. So $V^m(D) = V(X, D)$, where $X$ is the type that choose dividend level $D$.

Proposition 2 In a interval of separated types, $\mathcal{D}(X)$ follows the differential equation

$$\mathcal{D}'(X) = \frac{-kV_X(X, \mathcal{D}(X))}{V_D(X, \mathcal{D}(X))}.$$  \hspace{1cm} (7)

Proof: See the appendix.

As in the single-crossing case, a pooling equilibrium may be characterized by a continuum of types that chooses the same signal level.

Definition 4 The type $X$ is continuously pooled, if $\Theta(D(X))$ is a continuous set. A continuous pooling equilibrium is a signaling equilibrium such that for every $X$, $\Theta(D(X)) = [X_1, X_2]$.

In signaling games without single-crossing condition, a new kind of pooling arises. As in the continuous pooling, some values of $D$ will be chosen by more than one type of firm. However, the number of pooling types may be finite.

Definition 5 The type $X$ is discretely pooled, if $\Theta(D(X))$ is a discrete and finite set.

The property aggregating the discretely pooled types is that they must have the same marginal welfare $W_D$.

Proposition 3 If $X_a$ and $X_b$ are discretely pooled and $D = D(X_a) = D(X_b) \neq 0$, then

$$W_D(X_a, D) = W_D(X_b, D),$$  \hspace{1cm} (8)
Proof: See the appendix.

Equation (8) gives \( \varepsilon(X_a)F'(X_a - D) = \varepsilon(X_b)F'(X_b - D) \). So different types can choose the same level of dividend when, for higher types, the higher productivity shock compensate the reduction in marginal productivity resulted from higher investment. In the discrete pooling, dividend choice does not fully reveal the type of the firm. The market knows the set of possible types but it cannot distinguish one type from the other. This fact is taken into account when the market estimates the value, so \( E_D[V(X, D)] \) is the average value of types in the pool.

**Assumption 2** The type \( X \) is uniformly distributed on interval \([X_1, X_2]\).

With assumption 2, each type has the same probability. In particular, when there are only two types in the pool, the expected value of firms is

\[
E_D[V(X, D)] = \frac{1}{2}V(X_a, D) + \frac{1}{2}V(X_b, D),
\]

where \( X_a \) and \( X_b \) are the types that choose \( D \).

**Proposition 4** Under assumption 2, in a interval with discretely pooled types, if exactly two types chooses the same dividend, \( D(X) \) follows the differential equation

\[
D' = \frac{-k \left[ V_X(X, D) + V_X(\overline{X}(X, D), D)\overline{X}_X(X, D) \right]}{kV_X(\overline{X}(X, D), D)\overline{X}_D(X, D) + 2V_D(X, D)},
\]

(9)

where \( \overline{X}(X, D) \), derived from (8), is the type pooled together with type \( X \), when dividend \( D \) is chosen.

Proof: See the appendix.

1.4 Equilibrium refinement

The disturbing fact in any signaling model is the existence of many equilibria. For the same parameters, different kinds of equilibrium may exist, and the choice of initial conditions may generate a continuum of equilibria. At this point a selection criterion is needed. The pro-separation equilibrium, defined below,
choose, among different kinds of equilibria, the one that minimizes pooling and maximizes efficiency.

**Assumption 3**  Separability degree of a continuous pooled type, a discretely pooled type, and a separated type are, respectively, 1, 2, and 3.

**Definition 6**  Let \( \Pi(d) = \{ X \in [X_1, X_2] | X \text{ has separability degree } d \} \).

Therefore \( \Pi(1) \) is the set of continuously pooled types, \( \Pi(2) \) is the set of discretely pooled types and \( \Pi(3) \) is the set of separated types.

**Definition 7**  The separation floor of a signaling equilibrium is the lowest separability degree associated to a type in \([X_1, X_2]\).

**Definition 8**  A pro-separation equilibrium is a signaling equilibrium with separating floor \( \varphi \), such that (a) there is no other equilibrium with higher separation floor; (b) among equilibria with same separation floor, there is no other with lower probability of \( \Pi(\varphi) \), according to density \( p(\cdot) \); and (c) among equilibria with same separation floor and same probability of \( \Pi(\varphi) \), there is no other equilibria with higher expected value, according to density \( p(\cdot) \).

Therefore, pro-separation equilibrium criterion chooses an equilibrium eliminating poorly separated equilibria and taking the most efficient among the surviving equilibria.

## 2 The quadratic case

For the computations we consider a quadratic production function

\[
F(I) = aI(b - I),
\]  
(10)

where \( 0 \leq I \leq b/2, a > 0, \) and \( b > 0 \). We assume a linear expected productivity shock

\[
\varepsilon(X) = g + hX,
\]  
(11)

where \( h > 0 \).
The cross-derivative of welfare function of the firms is

\[ W_{XD} = \frac{4ah(1-k)}{1+i} \left( X - \frac{D}{2} - \frac{b}{4} + \frac{g}{2h} \right). \]

The \( CS^+ / CS^- \) frontier is a straight line and is described by

\[ X = \frac{D}{2} + \frac{b}{4} - \frac{g}{2h}. \]  

and \( CS^+ \) region is observed for \( X \) above or \( D \) below the frontier. Only two types can be present in the discrete pooling equilibrium, and types are associated by the function \( X \), derived from marginal condition (8), i.e.,

\[ X(X, D) = -\frac{g}{h} + \frac{b}{2} + D - X. \]

For a given level of dividend, higher type in \( CS^+ \) region pools with lower type in \( CS^- \) region. The marginal cost of signaling is decreasing for high types, due to high investment, and is increasing for low types, due to productivity shock.

The differential equation for separating equilibrium is derived from (7),

\[ D' = \frac{-ka \left[ -3hX^2 + 2(hD + hb - g)X - hD^2 + (2g - hb)D + bg \right]}{-2a(hX + g)D + a(2hX^2 + (2g - hb)X - bg) + 1 + i}, \]  

and, from (9), the differential equation for discrete pooling equilibrium is

\[ D' = \frac{-k \left( D + \frac{b}{2} + \frac{g}{h} \right) \left( 2X - (D + \frac{b}{2} - \frac{g}{h}) \right)}{R(X, D)}, \]

where

\[ R(X, D) = (4 - 3k)X^2 - \left[ (2 - k)(b + 2D) - 4(1 - k)\frac{g}{h} \right] X \\
- (2 - k)(b + 2D)\frac{g}{h} + k \left( \frac{b^2}{4} - \frac{g^2}{h^2} \right) + \frac{2(1 + i)}{ah}. \]

### 2.1 An example

In this example, we assume the following parameter values: \( a = 1, b = 4, i = 0, k = \frac{1}{2}, g = 0 \) and \( h = 1 \), that is,

\[ F(I) = I(4 - I), \]

\[ \varepsilon(X) = X. \]
where $0 \leq I \leq 2$, and $X$ uniformly distributed over $[X_1, X_2] = [0.8, 2]$.

In this context we have

$$W_{XD} = 2X - D - 2$$

and the $CS+$ region is defined by $X > \frac{D}{3} + 1$. The thick line in figure 1 divides the $X \times D$ plan in $CS+$ and $CS-$ regions. Note that we cannot have equilibrium in region above $D = X$, since dividend must be less than earnings in period 1.

The distribution of probabilities of types imposes more restrictions on the possible contracts. The grayed area in figure 1 is the region where the discrete pooling is possible for our choice of support. For each level of dividend, every type $X$, in grayed area has a correspondent type, $\Xi(X, D) = D + 2 - X$, inside the support of the distribution function. Note that for low dividend, high types cannot discretely pool because the correspondent type is out of the interval of possible types.

From (13), the differential equation for separating equilibria is

$$D' = \frac{3X^2 - 4(D + 2)X + D(D + 4)}{2 - 4X(2 - X + D)},$$

and, from (14), the differential equation for discrete pooling is

$$D' = \frac{-2(D + 2)X + (D + 2)^2}{5X^2 - 6(D + 2)X + 8}.$$  

Figure 2 shows some possible paths derived from equation (16), which has a singularity point at $D = -2 + \frac{4}{\sqrt{14}} \approx 0.1381$. Only paths on or above the saddle point are implementable, as they satisfy second order condition. The dashed curve is the first-best optimal dividend level given by equation (2). That is, for $X \in [1 - \frac{\sqrt{2}}{2}, 1 + \frac{\sqrt{2}}{2}]$ the firm should not pay dividend and invest all the earnings. For all other types $D^* = X - 2 + \frac{1}{2X}$.

An initial condition must be provided for differential equations (15) and (16). For each initial condition we have a different solution. We will choose the signaling schedule with the lowest level of dividend. This is an efficiency criterion, since signaling is costly and dividend implies low investments. The
efficient signaling is the path crossing the singularity point because this is the path closer to the optimal dividend level, if the market knows types.

The efficient discrete pooling path does not cover all types. Denote as $X_s$ the type on which the saddle path crosses the boundary of grayed area. For types higher than $X_s$ the matching types are out of the support of distribution. These types must be covered by a separating equilibrium that is a solution of the differential equation (15). The initial dividend level for the separating schedule should not break incentive compatibility. This dividend level should be on the indifference curve of $X_s$ type that passes through the last point of discrete pooling schedule and satisfy zero-profit condition for separating equilibrium.

The optimal contract is plotted on $D \times V^m$ plan of figure 3. Observe that there is a jump in dividend and in value at type $X_s$ assigned contract. The intermediate values can be filled by any contract that is not preferred for any type. In particular, the dotted curve is the lower envelope of the indifference curves that crosses the pair $(D,V^m)$ assigned to each type by the signaling schedule.

On the $X \times D$ plan there is a jump when equilibrium changes from pooling to separating. This jump does not exist if the distribution were continuous and equal to zero at $X_1$. Separating equilibrium is full informative and market knows that only higher types are selected. Then, dividends must be higher in separating equilibrium than in discrete pooling to satisfy zero profit.

In discrete pooling equilibrium, lower types pools with higher types because, in $CS-$, benefits from being treated equally as higher types compensates the cost of higher dividend distribution. The market knows the disguising behavior and adjust the evaluation, but signaling is preserved since lower types do not reduce the expect valuation to the point that incentives are broken.

Whenever the single-crossing property holds, the first and second order conditions assure that incentive compatibility is valid globally. However, without single-crossing, additional checking is needed. In general, every type should correctly choose the signal level assigned to him. In the quadratic case, sometimes a type in the saddle path prefers a contract in the separating region (this
situation does not occur in the example illustrated in figure 3). If incentive compatibility does not hold for the saddle path, there exist paths above it that satisfy global incentive compatibility. The lowest of these paths is chosen as the equilibrium of our model.

### 2.2 Other cases

The combination of discrete pooling with separating equilibrium is not the only equilibrium case for this model. Depending on the support of distribution and parameter values, other kinds of equilibrium may arise. Figure 4 shows other three cases of equilibrium. In graph (a) the $CS-$ set is small and a separating equilibrium schedule exists is $CS+$ region. In graphs (b) and (c) the saddle path cross the upper limit of discrete pooling region and the contract is not implementable because dividends exceed earnings or because correspondent type is above support. In this case a combination of discrete pooling with continuous pooling may be a bunching equilibrium. Graph (b) shows a discrete pooling with bunching. A continuum of low types pools with a continuum of higher types so that the value of the pool coincides with the value of discrete pool, for the same level of dividend. In graph (c), the saddle path requires a higher level of dividend, such that discrete pooling is not possible even with bunching. The equilibrium is a continuous pooling of all types at efficient dividend level. Graph (d) shows the three cases on $D \times V^m$ plan.

### 3 Results

Figure 5 shows equilibria for some values of the parameter $k$ and the remaining parameter values equal to the example above. The parameter $k$ denote the fraction of shareholder who wants to sell the shares at period 1 at price $V^m$. Higher $k$ means higher benefit of signaling and the manager would pay more dividend for getting higher market value. So, for higher $k$ we observe a flatter contract curve at $D \times V^m$ plan. For lower $k$, the relatively high level of $V^m$
increases the probability of violation of global incentive compatibility condition. Consequently, the equilibrium path is higher and the set of separated types is shorter. On the other side, if \( k \) is high enough, the lowest types cannot pay the dividends required for eliminating continuous pooling and the equilibrium with bunching occurs.

The influence of parameters \( g, h \) and \( b \) on equilibrium is mainly by the placement of \( CS^+/CS^- \) frontier. By equation (12), higher \( b \), lower \( g \), higher \( h \) for positive \( g \) or lower \( h \) for negative \( g \) moves the frontier to right and \( CS^- \) region enlarges. Consequently, separating equilibria is less frequent and continuous pooling more common. On the other side, with changes of parameters in opposite direction, \( CS^+ \) region enlarges and separating equilibrium is more frequent. Discrete pooling is an intermediate case and will be present when \( CS^+ \) and \( CS^- \) coexist in a favorable way. We verify this effect in figures 6 and 7. They repeat equilibria of figure 5 for \( g = -0.2 \) and \( g = 0.2 \). For \( g = -0.2 \) we observe more instances of bunching. Continuum set of high types pools with types lower than the types on the decreasing side of discrete pooling.

4 Conclusion

Traditional models of signaling satisfy the single-crossing property for the objective function of the firm. This property leads to a monotone relationship between types and signals. We believe that this assumption is not essential and the results derived from non-single-crossing signaling are also plausible. For instance, in our dividend signaling model, low quality firms may pay high dividends in order to be considered as high quality firms. The general result is that the monotone relationship between type (earnings) and signal (dividends) is not assured.

This result has impact on empirical works that test for the presence of asymmetric information assuming single-crossing\(^2\). Critics on the informational contents of dividends claim that statistical evidences of positive relation between

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\(^2\)For instance, see Araujo and Moreira (2001b) for the insurance case.
dividends and earnings are weak. But, as we have shown, this is an expected result when discrete pooling equilibrium occurs.

We believe that the absence of the single-crossing property is not an uncommon situation and empirical research must take this possibility into account. The presence of discrete pooling enriches the interrelation among variables in signaling models and tests based on monotonicity should not be used to reject asymmetric information.

A Appendix

A.1 The perfect Bayesian equilibrium

A perfect Bayesian equilibrium (PBE) for dividend signaling model is a profile of strategies \( \{c(X) = (D(X), \mathcal{V}^m(X))\}_{X \in [X_1, X_2]} \) and ex-post beliefs \( \mu(\cdot | c) \) such that the following conditions are satisfied:

1. Zero expected profit constraint:

\[
\mathcal{V}^m(X) = \int V(\hat{X}, D(X))d\mu(\hat{X}|c(X))
\]

2. Maximization of firm welfare:

\[
X \in \arg\max_{\hat{X} \in [X_1, X_2]} k\mathcal{V}^m(\hat{X}) + (1 - k)V(X, D(\hat{X}))
\]

3. Consistence of beliefs: \( \mu(X|c) \) is the Bayesian updating given 1 and 2, i.e., it is the probability a posteriori of \( X \) given \( c \).

A.2 Proof of propositions

A.2.1 Proof of proposition 1

The first order condition, (6) can be written as

\[
W_D(X, D(X))D'(X) + kV^m(D(X))D'(X) = 0,
\]
whose derivative is

\[
W_{XD}(X, D(X))D'(X) + \left[ W_{DD}(X, D(\hat{X})) + k V''m''(D(\hat{X})) \right] (D'(\hat{X}))^2 \\
+ \left[ W_D(X, D(\hat{X})) + k V''m'(D(\hat{X})) \right] D''(\hat{X}) = 0. 
\] (17)

From definition of \( \mathcal{W} \),

\[
\frac{\partial^2 \mathcal{W}}{\partial X^2}(\hat{X}, X) = \left[ W_{DD}(X, D(\hat{X})) + k V''m''(D(\hat{X})) \right] (D'(\hat{X}))^2 \\
+ \left[ W_D(X, D(\hat{X})) + k V''m'(D(\hat{X})) \right] D''(\hat{X}). 
\] (18)

Equations (17) and (18) simplifies second order condition to

\[
\frac{\partial^2 \mathcal{W}}{\partial X^2}(X, X) = -W_{XD}(X, D(X))D'(X) \leq 0, 
\]

which proves the proposition.

**A.2.2 Proof of proposition 2**

Since type is perfectly revealed by observation of \( D \), the market correctly evaluate the firm, that is, \( V^m(D) = V(X, D) \), where \( X = D^{-1}(D) \). The welfare is

\[
\mathcal{W}(\hat{X}, X) = k V(\hat{X}, D(\hat{X})) + (1 - k) V(X, D(\hat{X})),
\]

and the first order condition (6), results (7).

**A.2.3 Proof of proposition 3**

The first order condition is

\[
W_D(X, D(X))D'(X) + k V''m'(D(X))D'(X) = 0,
\]

and, since \( D(X) \neq 0 \),

\[
-k V''m'(D(X)) = W_D(X, D(X)).
\]

The left hand side is independent of \( X \), if types are drawn from the same pool, so \( W_D(X, D(X)) \) is the same for pooled types.
A.2.4 Proof of proposition 4

Let $X$ be the lowest type that chooses $D$ and $\bar{X}(X, D)$ the highest. The market value is an average of the value of the types pooled in the same dividend level,

$$V^m(D(X)) = \frac{1}{2} V(X, D(X)) + \frac{1}{2} V(\bar{X}(X, D(X)), D(X)),$$

and the welfare as a function of declaration is

$$\mathcal{W}(\hat{X}, X) = \frac{k}{2} V(\hat{X}, D(\hat{X})) + \frac{k}{2} V(\bar{X}(\hat{X}, D(\hat{X})), D(\hat{X}))) + (1 - k)V(X, D(\hat{X})).$$

Derivating in $\hat{X}$ and taking into account that (8) implies $V_D(X, D(X)) = V_D(\bar{X}(X, D(X)), D(X))$, condition (6) results (9).

References


Bhattacharya, S., 1979, “Imperfect information, dividend policy, and ‘the bird in the hand’ fallacy”, Bell Journal of Economics, 10, 259–270.


Figure 1: CS split and discrete pooling region.

Figure 2: Discrete pooling paths.
Figure 3: Signaling Equilibrium.
Figure 4: Signaling Equilibria.

Figure 5: Equilibria for different values for $k$. 
Figure 6: Equilibria for $g = -0.2$.

Figure 7: Equilibria for $g = 0.2$. 

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