

Inflation and Occupational Choices

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Abstract

This paper studies the impact of (high rates) of inflation on occupational choices in a model where the demand for labor is derived from a production technology that uses capital, productive labor, and managerial services done by administrative labor and money; while the supply of both kinds of labor is rigid in the short-run due to irreversible professional choices. The dynamic path of the economy after stabilization plans exhibits the main stylized facts reported in the literature including an initial consumption boon followed by a gradual adjustment. In its open economy version, the initial phase of the transitional dynamics exhibits capital inflight. The model also generates an increase of income inequality during the transitional dynamics.

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1 Introduction

This paper studies the impact of (high rates) of inflation on occupational choices in a model where the demand for labor is derived from a production technology that uses capital, productive labor, and managerial services done by administrative

labor and money; while the supply of both kinds of labor is rigid in the short-run due to irreversible professional choices.

One of the economic costs of high inflationary processes is the distorted allocation of resources within firms and its consequences on the labor market. By a high rate of inflation we mean annual rates above 20%. For three digit annual rates of inflation, these administrative tasks become the vital ones. After all, why should a firm be worried about increasing real productivity by 5% a year in its assembly line, when the gains brought about can be swiftly wiped out by a simple two-days delay of the receipt of a simple bill?

When inflation is high firms are forced to bloat their administrative and finance departments in order to perform activities that would not be necessary if price stability prevailed. We give some examples of administrative costs caused by inflation: price lists must be redrawn periodically; nominal contracts with suppliers have frequently to be renegotiated; wage bargains are not only frequent but also involves real as well as merely nominal corrections, which tend to protract discussions; and, the actual cost of inventories must be recalculated continuously.

The four examples cited above are administrative tasks closely related to the fact that, in the real world, relative prices vary a lot when the rate of inflation is high. These tasks are amplified when the rate of inflation is not only high but also random. These tasks are reflect the fact that when inflation is high money tends to loose its role as a unit of account. They explain why, for a given level of production, a higher rate of inflation requires firms to augment the amount of administrative workers. Modelling these administrative costs would require a complex model which should allow for relative price changes or stochastic inflation. This is not done in the present single good model.

Another group of examples is linked to the need to economize on the use of non interest bearing working capital. The administrative tasks done by the financial departments of firms are made more difficult when less money is used as working capital. Delays in the receipts of nominally denominated bills must desperately be avoided, while the payments of nominally denominated bills should be postponed to the very last day. Cash and cheque payments must be transformed into interest bearing bonds in the shortest possible time. These administrative tasks are present even in an economy with a single good traded at a single price and when inflation is perfectly foresighted. These tasks are reflect the fact that when inflation is high money tends to loose its role as a means of exchange.

For tractability, it is assumed that the only administrative tasks affected by inflation are those performed by financial departments. This is done assuming that the technology of performing managerial services uses administrative labor and money. Since in the model there is one single good and no uncertainty, extra administrative costs due to relative price variability or stochastic inflation are ignored here. Hence, the model underestimates a large gamut of other administrative tasks generated by inflation.

The extra administrative costs associated with the dire need toget rid of cash

balances as soon as possible, however, are reasonably well modelled with the assumption that the availability of non interest bearing working capital facilitates administrative tasks. This is equivalent to assuming that inflation increases administrative costs only indirectly when it increases the opportunity cost of holding liquid assets, thus forcing firms to substitute administrative workers who work in financial departments for non interest bearing assets. This is a mere short-cut which provides a mechanism that endogenously increases the demand for administrative labor whenever inflation increases.

As far as we know, the literature on stabilization policies assume that there is one unique kind of labor. Examples are Van Der Ploeg and Alogoskoufis (1994), Lacker and Schreft (1996), Aiagary, Braun and Ecstein (1998), English (1999), Lucas (2000), and Hence, a household can supply work either as a productive or as an administrative worker.

However, in the real world, specialization of the workforce requires professional choices made by the youth that are not easily reversed later. It takes a few years to form a good financial manager who can not be transformed into an experienced mechanic engineer the day after a successful stabilization program. The lower inflation reduces the need to use as little working capital as possible, thus reducing the demand for financial managers. Likewise, when inflation rises permanently an experienced engineer can not be transformed into a seasoned financial manager.

When the youth make their professional choices, they do so according to the wages paid in the different labor markets. After a stabilization plan that reduces the need for financial managers, potential students of business administration tend to be lured to engineering schools. In the short run the supply of engineers is rigid but it can be increased in the medium run as schools produce new engineers. This creates a rigidity in the labor market that has important consequences whenever the rate of inflation changes significantly and is perceived as permanent.

Modelling the consequences of this phenomenon is the main contribution of this paper. It is shown that the distortions brought about by prolonged inflationary processes tend to generate long lasting rigidities in the labor market that can account for the path followed by some macro variables in countries that have implemented successful stabilization plans. The model also predict reasonably well the path followed by the same macro variables following a permanent rise of the rate of inflation. It is shown that these two phenomena are not symmetric.

The technology represented by the production function requires physical, productive labor and administrative services. The productive workers are those directly engaged in the production of goods. Hence the model adopts the money in the production function approach to monetary theory ¹. Administrative ser-

¹See Fischer (1974) for a list of references. See also Finnerty (1980). Empirical papers include Dennis and Smith (1978), Short (1979), Boyes and Kavanaugh (1979), Sinai and Stokes (1972), (1975), (1977), (1981), Kanh and Kouri (1975), Kahn and Ahmad (1985), Nguyen (1986).

vices are performed both by money used as working capital and administrative workers. For tractability, it is assumed that there is a fixed proportion between the amount of labor directly employed in production and the amount of administrative services that must be performed to keep the productive activities going smoothly.

Firms optimally rent capital, hire both kinds of labor and demand money to be used as working capital. Households work inelastically, and optimally consume and save in order to maximize the present value of their inter temporal utility. Each agent takes as given the prices above when making its choices.

The labor market dynamics is built with the help of the model of perpetual youth of Blanchard (1985). In this model the population is constant for the number of households that die at each moment is equal to those that are born. The labor market dynamics arises from the assumption that the newly born make their professional choices and these choices can not be reversed thereafter. The perpetual youth model is used just to build a tractable endogenous labor market dynamics.

In order to replace a dying group by an equally wealthy newly born group, there is no annuity market nor bequests. So when a household dies its wealth is inherited by the government and immediately transferred to the newly born. This is merely a trick to replace old workers that cannot reverse their professional choices by new workers who can choose whether to work as a productive or as an administrative worker. The average wealth of those that die is identical to that of the newly born. Therefore, unlike in the Blanchard model and others like Marini and van der Ploeg (1988), it is assumed that there is no actuarially fair premium added to the real rate of return on accumulated savings.

Four dynamics are studied. Firstly one studies what happens after a permanent rise of the rate of inflation in a closed economy. Secondly, what occurs after a fall of inflation in a closed economy. The third and fourth dynamics are the open economy versions of the first and second dynamics. It is shown that the response of the economy to a rise of the inflation is not symmetric to its response to a fall.

The results of a permanent rise of the rate of inflation can be described starting from a long run steady state with low inflation. In the closed economy, the sudden unexpected rise of rate of inflation increases the cost of using money as working capital which induces firms to substitute administrative labor for money used as working capital. Since the supply of administrative labor is fixed in the short run, there is no short run reduction in the real demand for money but there is a rise of administrative wages. Since the cost of administrative services increases, the demand for productive workers falls, producing a fall of productive wages. Consumption falls in response to a lower permanent income. As a result there is a positive accumulation of capital in the first phase of the transitional dynamics.

During the transition to the new long run steady state, while the wage paid to administrative workers remain above the wage of productive ones, all the newly

born chose to become administrative workers. The excess supply of productive workers dampens the demand for capital, thus reducing its rate of return, which tends to increase consumption and gradually revert the positive accumulation of capital. As the supply of administrative workers is continuously increased and that of productive workers decreased, the wage differential is gradually reduced down to zero. The average wage falls as capital is depreciated.

As the economy approaches its new steady state with lower capital, the real return on capital tends to rise again to its previous steady state level. In the steady state with high inflation, the economy is poorer for the capital stock was reduced. The new level of wages and of consumption is lower, and a larger share of the work force is doing administrative services.

The model also casts some light on the political economy of prolonged inflationary processes. This issue is not the focus of the paper but arises as a subsidiary consequence of its results. Although in the long run all individuals lose with a higher rate of inflation, in the short run some groups profit from it. Starting at the long run non inflationary equilibrium, if (due to an exogenous reason) the group that gains with the rise of inflation gets the political power, inflation will rise. For some time the members of this group are benefited. As time passes by however, new generations are lured to the activities that are benefited by inflation, thus reducing their attractiveness. As the group that benefits from inflation is enlarged in size their political clout tends to augment. Moreover, as the individual gains absorbed by each of its members is whittled down by competition within the group due to its enlargement, the incentive press for rising the rate of inflation even further increases. This creates an upward inflationary bias in which the longest the inflationary period the smallest is the political willingness to curb it. Monetary stabilization would occur only after a long period of inflation, when the economy is so impoverished that the long run gains from curbing inflation outstrips the short run loss of the group that gains with inflation.

The growth of the banking sector in inflationary economies has already been documented and modeled theoretically. But high rates of inflation generate extra administrative tasks within non banking firms as well.

The paper is organized in 5 sections including this introduction. In the second section one presents the basic framework of the model. In the third one studies the behavior of firms and of households. In the fourth, one describes the determination of prices and the time evolution of state variables and studies the general equilibrium of the model. In the fifth section one studies the dynamic adjustment of the economy in response to changes of the rate of inflation. In the sixth section one concludes the paper.

2 Economic Environment

The economy is modeled in continuous time. There are three kinds of infinitely lived economic agents: the government, households and firms. There is only one good that can be consumed or saved to be used as physical capital. The good is produced by firms that use as inputs physical capital, productive labor, administrative labor and money (working capital). There are two kinds of labor: administrative and productive labor. Each household supplies labor inelastically in one of the two labor markets. Households do not use money.

There are five competitive markets: of money, of the consumption good, of productive labor, of administrative labor, and of capital. In each of the five markets there is one price that equals supply and demand. Normalizing these five prices so that the price of money is one, there remains four prices to be expressed in monetary units: the price of the consumption good $P(t)$, the wage of productive workers $W_1(t) = P(t)w_1(t)$, the wage of administrative workers $W_2(t) = P(t)w_2(t)$, and the nominal rental $R(t) = P(t)r(t)$. These four prices are fully flexible and will later be determined in general equilibrium.

The information structure is very simple. The model has one main exogenous variable, the rate of inflation π , which is defined by the government. At any instant t the prevailing rate of inflation is the best prediction for the future rates of inflation. For simplicity, instead of assuming, as is the wont in the monetary literature, that the nominal monetary supply follows an exogenous path and the rate of inflation is endogenously determined, it was assumed that the reverse holds. In the subsections below one presents the formal characterization of the economic environment faced by agents.

2.1 Firms

The only good of the economy is produced by competitive firms whose technology of production uses physical capital K , productive workers L_1 , administrative workers L_2 and real money $m = M/P$ employed as working capital. For simplicity, it is assumed that there is substitutability between capital and productive labor, but that there is a fixed proportion between productive labor and administrative services S . The flow of goods $Y(t)$ produced at instant t is given by

$$Y(t) = F(K(t), \min\{L_1(t), S(m(t), L_2(t))\})$$

where the function $F : \mathfrak{R}^2 \rightarrow \mathfrak{R}$ describes the amount of goods produced with a given amount of physical capital and productive workers when the amount of administrative services is not binding. The technology of providing administrative services $S : \mathfrak{R}^2 \rightarrow \mathfrak{R}$ uses administrative labor and working capital represented by real money balances which will simply be called money. Both F and S are

assumed to be homogeneous of degree one, twice continuously differentiable and to satisfy the INADA conditions. Capital depreciates at the instantaneous rate δ .

2.2 Households

At each instant of time t a large cohort of size λL is born. Each household of this cohort has a constant instantaneous probability λ of dying. This implies that the probability that a household born at instant s is alive at instant t , $s \leq t$, is given by $e^{-\lambda(t-s)}$. By the Law of Large Numbers, the size of the cohort s as of date t , $s \leq t$, is thus $\lambda L e^{-\lambda(t-s)}$. Hence, the size of the population at instant t is given by $\int_{-\infty}^t \lambda L e^{-\lambda(t-s)} ds = L$. This shows that although the size of each cohort declines continuously at the rate λ , the size of the population is constant.

Household receive two kinds of lump sum transfers from the government during its life time. The first is a once-and-for-all lump sum transfer received when the household is born that is equal to the average wealth of those who die at that instant. The second is a continuous lump sum transfer of money that is equal to the per capita seigneurage.

Each household takes two kinds of decisions. The first decision is its career choice which is chosen at birth. This choice is irreversible thereafter. The second decision is taken continuously. It concerns its consumption and savings pattern that is determined according to the expected present value of its lifetime utility. Each household inelastically supplies one unit of labor while alive.

Letting $\rho > 0$ stand for the instantaneous rate of time preference, the flow of consumption $c(t)$ at instant t , $t \geq s$, yields the flow of utility $U(c(t)) = \ln c(t)$. Hence the expected present value of the household's lifetime utility is

$$E_t \left\{ \int_t^{\infty} e^{-\rho(u-t)} \ln c(u) du \right\} = \int_t^{\infty} e^{-(\rho+\lambda)(u-t)} \ln c(u) du$$

3 Economic Choices

In this section one studies how firms and households take their economic decisions in partial equilibrium, i.e., taking as given the prices of goods $P(t)$, the wage of productive workers $W_1(t) = P(t)w_1(t)$, the wage of administrative workers $W_2(t) = P(t)w_2(t)$, and the nominal rental $R(t) = P(t)r(t)$. The definition and study of the general equilibrium of the model is left for the next section.

3.1 Firms

Firms operate competitively. Defining the auxiliary functions $f(k) \equiv F(k, 1)$ and $s(x) \equiv S(x, 1)$, one can determine the instantaneous flow of nominal profits that can be transferred to firms' owners. It is the difference between nominal

revenues and nominal expenditures with capital and labor net of increases of nominal money:

$$P(t) \{L_1(t)f(K(t)/L_1(t)) - (r(t) + \delta)K(t) - w_1(t)L_1(t) - w_2(t)L_2(t)\} - \dot{M}(t)$$

where $L_2(t)$ is implicitly given by

$$L_1(t) = L_2(t)s(m(t)/L_2(t))$$

and $\dot{M}(t)$ denotes the increase in nominal money demand. From the definition of the rate of inflation $\dot{P}(t)/P(t) = \pi$, one can write $\dot{M}(t)/P(t) = \dot{m}(t) + \pi m(t)$.

Maximization of the present value of transferable real profits at instant $t = T$ is written as:

$$\max_{K, L_1, L_2, m} \int_T^\infty e^{-\int_T^t r(\tau) d\tau} \left\{ L_1 f \left(\frac{K}{L_1} \right) - (r + \delta)K - w_2 L_2 - w_1 L_1 - [\dot{m} + \pi m] \right\} dt$$

where the time variable t has been dropped from $K, L_1, L_2, m, P, w_1, w_2$ and r to avoid cluttering the notation.

Let K^D, L_1^D, L_2^D and m^D stand for the demands for capital, productive labor, administrative labor and real money. Optimization with respect to the capital stock yields:

$$f' \left(K^D / L_1^D \right) = r + \delta \quad (1)$$

where the symbol “ ’ ” represents the first derivative of the underlying function.

Optimization with respect to money implies:

$$\frac{\pi + r}{s'(m^D / L_2^D)} = \frac{w_2}{s(m^D / L_2^D) - m^D / L_2^D s'(m^D / L_2^D)} \quad (2)$$

The left hand side of (2) represents the (opportunity) cost of producing an additional unit of administrative service using more money; whilst the right hand side is that cost when more administrative labor is chosen. The necessary condition for cost minimization is that the cost of an additional unit of administrative service must be the same for both factors of production.

The optimum choice of productive labor gives:

$$f \left(\frac{K^D}{L_1^D} \right) - \frac{K^D}{L_1^D} f' \left(\frac{K^D}{L_1^D} \right) = w_1 + \frac{w_2}{s(m^D / L_2^D) - m^D / L_2^D s'(m^D / L_2^D)} \quad (3)$$

The left hand side of (3) is the amount of goods produced by and additional productive worker, whilst the right hand side is the sum of the wage paid to the additional productive worker with the cost of an additional administrative

service as seen in (2). This equation reflects the assumption that there is a fixed proportion between productive labor and administrative services.

The demand for administrative labor is implicitly given by

$$L_1^D/L_2^D = s (m^D/L_2^D) \quad (4)$$

3.2 Households

The first decision of a household is its career choice. Let $\beta(s)$ stand for the share of the cohort born at instant s that chooses to be productive workers. If the real wage $w_1(s)$ of administrative workers is higher than the real wage $w_2(s)$ of productive ones, all the newly born will prefer to be productive workers, i.e., $\beta(s) = 1$. When $w_1(s) < w_2(s)$ all the newly born will prefer to be administrative workers, i.e., $\beta(s) = 0$. When $w_1(s) = w_2(s)$, the new cohort will be divided between productive and administrative workers in such a way as to keep the two wages equal.

The mathematical expression of the share of workers that keeps both wages equal will be determined later in the section where one studies the general equilibrium. The fraction $\alpha(t)$ of the labor force that work as productive workers at instant t , is the sum of all productive workers of past cohorts that are still alive at instant t :

$$\alpha(t) = \int_{-\infty}^t \lambda e^{-\lambda(t-s)} \beta(s) ds \quad (5)$$

The consumption choice at instant $t \geq s$ of each household of type $i = 1, 2$ of cohort s is subject to the budget constraint

$$\partial v_i(s, t)/\partial t = r(t)v_i(s, t) + w_i(s, t) + \varphi(t) - c_i(s, t), \text{ with } v_i(s, s) = \psi(s) \quad (6)$$

where $v_i(s, t)$ is its wealth or accumulated net savings, $r(t)$ is the real rate of return on accumulated net savings, $w_i(s, t)$ is its flow of labor income, $c_i(s, t)$ is its flow of consumption, $\varphi(t)$ is the inflationary flow of lump sum transfer received from the government, and $\psi(s)$ is the amount of real lump sum transfers received from the government at birth. The amount of these two transfers will later be defined in order to coherently close the general equilibrium of the model.

The optimal consumption choice is described by

$$\dot{c}_i(s, t) = c_i(s, t)[r(t) - (\rho + \lambda)] \quad (7)$$

where the dot over a variable will denote its time derivative.

From (7) and (6), the consumption of a household of cohort s as of instant t is given by

$$c_i(s, t) = (\rho + \lambda)[v_i(s, t) + h_i(t) + x(t)] \quad (8)$$

and the condition

$$\lim_{t_1 \rightarrow \infty} e^{-\int_t^{t_1} r(t_2) dt_2} v_i(s, t_1) = 0 \quad (9)$$

where

$$h_i(t) = h_i(s, t) = \int_t^\infty e^{-\int_t^{t_1} r(t_2) dt_2} w_i(s, t_1) dt_1 = \int_t^\infty e^{-\int_t^{t_1} r(t_2) dt_2} w_i(t_1) dt_1 \quad (10)$$

is the present value of its future labor income, and

$$x(t) = x(s, t) = \int_t^\infty e^{-\int_t^{t_1} r(t_2) dt_2} \varphi(t_1) dt_1 \quad (11)$$

is the present value of its future inflationary transfers. Note that $h_i(s, t)$ is invariant of s since members of different cohorts that have the same occupation get the same wage.

The wealth of the cohort born at instant s as of instant t , $v(s)$, is given by $v(s) = \beta(s)v_1(s, t) + (1 - \beta(s))v_2(s, t)$. The aggregate wealth $V(t)$ at instant t is given by the sum over all cohorts alive at instant t :

$$V(t) = L \int_{-\infty}^t v(s, t) \lambda e^{-\lambda(t-s)} ds = L \int_{-\infty}^t \{\beta(s)v_1(s, t) + (1 - \beta(s))v_2(s, t)\} \lambda e^{-\lambda(t-s)} ds \quad (12)$$

Deriving (12) with respect to time, and recalling that the newly born are equally wealthy, i.e., $v_1(t, t) = v_2(t, t) = \psi(t)$, one gets

$$\begin{aligned} \dot{V}(t) &= \lambda L \psi(t) - \lambda V(t) + \\ &+ L \int_{-\infty}^t \left\{ \beta(s) \frac{\partial v_1(s, t)}{\partial t} + (1 - \beta(s)) \frac{\partial v_2(s, t)}{\partial t} \right\} \lambda e^{-\lambda(t-s)} ds \end{aligned} \quad (13)$$

The first term of (13) represents the lump sum transfer to the newly born and the second the wealth of those who die. The assumption that the wealth of those who die at instant t is equally divided among those who are born at the same instant of time implies that the first two terms of (13) cancel out. Substituting (6) into (13) one gets:

$$\dot{V}(t) = r(t)V(t) + W(t) + X(t) - C(t) \quad (14)$$

where $X(t) = x(t)L$ represents the aggregate inflationary transfer to households at instant t , $W(t)$ stand for the aggregate labor income at instant t

$$W(t) = L \int_{-\infty}^t \lambda e^{-\lambda(t-s)} \{\beta(s)w_1(t) + (1 - \beta(s))w_2(t)\} ds \quad (15)$$

and $C(t)$ for the aggregate consumption at instant t

$$C(t) = \int_{-\infty}^t \lambda e^{-\lambda(t-s)} \{\beta(s)c_1(s, t) + (1 - \beta(s))c_2(s, t)\} ds \quad (16)$$

The present value of the aggregate labor income of the households currently alive $H(t)$ is given by

$$H(t) = L \int_t^\infty e^{-\int_t^{t_1} r(t_2) dt_2} \int_{-\infty}^{t_1} \lambda e^{-\lambda(t_1-s)} \{\beta(s)w_1(t_1) + (1-\beta(s))w_2(t_1)\} ds dt_1 \quad (17)$$

Deriving (17) with respect to time one gets

$$\dot{H}(t) = r(t)H(t) - W(t) + Z(t) \quad (18)$$

where

$$\begin{aligned} Z(t) &= \lambda L \int_t^\infty e^{-\int_t^{t_1} r(t_2) dt_2} \{\beta(t)w_1(t_1) + (1-\beta(t))w_2(t_1)\} dt_1 - \lambda H(t) \quad (19) \\ &= \lambda L [\beta(t) - \int_{-\infty}^t e^{-\lambda(t-s)} \beta(s) ds] \int_t^\infty e^{-\int_t^{t_1} r(t_2) dt_2} [w_1(t_1) - w_2(t_1)] dt_1 \\ &= \lambda L [\beta(t) - \alpha(t)] [h_1(t) - h_2(t)] \end{aligned}$$

It is important to note that $Z(t) \geq 0$. When $h_1(t) > h_2(t)$, all the newly born at instant t chose to be productive workers, i.e. $\beta(t) = 1$. Hence when $h_1(t) > h_2(t)$ one has $\beta(t) - \alpha(t) = 1 - \alpha(t) > 0$. When $h_1(t) < h_2(t)$, all the newly born at instant t chose to be administrative workers, i.e. $\beta(t) = 0$. Hence when $h_1(t) < h_2(t)$, one has $\beta(t) - \alpha(t) = 0 - \alpha(t) < 0$.

The term $Z(t)$ represents the part of instantaneous change of the present value of the aggregate labor income that is due to professional choices. Whenever the two wages differ, this present value is increasing because among those that die there are some that receive the high wage and others that get the low one; but all those that are born get the high wage. This explains why $Z(t) \geq 0$.

From (8), (16), (12) and (17), the aggregate consumption can be written as

$$C(t) = (\lambda + \rho)[V(t) + H(t) + X(t)] \quad (20)$$

where $X(t) = Lx(t)$.

Taking the derivative of (20) with respect to time, and substituting (14), one gets

$$\begin{aligned} \dot{C}(t) &= (\lambda + \rho)\{[r(t)V(t) + W(t) + X(t) - C(t)] + \dot{H}(t) + \dot{X}(t)\} \\ &= (\lambda + \rho)\{[r(t)V(t) - C(t)] + \{W(t) + \dot{H}(t)\} + \{X(t) + \dot{X}(t)\}\} \end{aligned}$$

Substituting (18) and recalling that from (11) $\dot{X}(t) = r(t)X(t) - X(t)$ one gets

$$\dot{C}(t) = (\lambda + \rho)[r(t)\{V(t) + H(t) + X(t)\} + Z(t) - C(t)]$$

Substituting (20) one gets

$$\dot{C}(t) = [r(t) - (\lambda + \rho)]C(t) + (\lambda + \rho)Z(t) \quad (21)$$

Expressions (14) and (21) summarize the aggregate savings and consumption decisions of households. These expressions will play key roles in the study of the general equilibrium ahead.

4 General Equilibrium

There are three state variables in the model: the first is the nominal supply of money $M(t)$, determined by the government; the second is the per capita aggregate supply of capital $k(t) = K(t)/L$, which is the counterpart of the per capita wealth; and the third is the share $\alpha(t) = L_1(t)/L$ of households that supply labor in the productive labor market.

The general equilibrium will be studied in three steps. In the first step, one takes as given the triple of state variables $(k(t), \alpha(t), M(t))$ at instant t and studies the determination of the relative prices $(r(t), w_1(t), w_2(t), P(t))$ that clear the five markets. In the second step, one studies the time derivative of each of the three state variables in response to the prices that clear the markets. In the third step one defines the concept of general equilibrium and show that it is unique.

4.1 Determination of Prices for Given State Variables

Given the state variables $(k(t), \alpha(t), M(t))$, the supply of capital $K^S(t)$ is the counterpart of households' assets: $K^S(t) = V(t)$. The supply of money $M(t)$ is defined by the government. The supply of productive and administrative workers are respectively $L_1^S(t) = \alpha(t)L$ and $L_2^S(t) = (1 - \alpha(t))L$. Hence the three state variables $(M(t), k(t), \alpha(t))$ determine the supply in four markets. In the fifth market the supply of goods is given by the optimal decision of firms $Y^S(t) = L_1^D(t)f(K^D(t)/L_1(t))$.

The demands for capital $K^D(t)$, productive workers $L_1^D(t)$, administrative workers $L_2^D(t)$ and money $M^D(t)$ are determined by the system of equations (1), (2), (3) and (4). The demand for goods is $Y^D(t) = C(t) + \delta K(t) + \dot{K}(t)$.

In order to close the general equilibrium, one assumes that the lump sum transfers to households are equal to the proceeds from monetary creation $X(t) = \dot{M}(t)/P(t) = \dot{m}(t) + \pi m(t)$.

According to Walras's law when supply equals demand in four markets, this will also hold in the fifth one. Hence the characterization of the general equilibrium will be made imposing that supply equals demand in the capital market, in both labor markets and in the money market. Let

$$\mu(t) = m(t)/L = M(t)/P(t)L \quad (22)$$

stand for real per capita money. Since $M(t)$ is a state variable and L is constant, the nominal price of the good $P(t)$ is uniquely determined for a given value of $\mu(t)$. Hence μ can be treated as a relative price.

The state variables $(k(t), \alpha(t), M(t))$ at instant t define the supply of capital, the supply of both kinds of labor and the supply of money. The set of four prices $(r(t), w_1(t), w_2(t), \mu(t))$ must satisfy (1), (2), (3) and (4). Hence at each instant the following four equations must hold

$$f'(k/\alpha) = r + \delta \quad (23)$$

$$\frac{\pi + r}{s'(\mu/(1-\alpha))} = \frac{w_2}{s(\mu/(1-\alpha)) - \mu/(1-\alpha) s'(\mu/(1-\alpha))} \quad (24)$$

$$f(k/\alpha) - k/\alpha f'(k/\alpha) = w_1 + \frac{w_2}{s(\mu/(1-\alpha)) - \mu/(1-\alpha) s'(\mu/(1-\alpha))} \quad (25)$$

$$\alpha/(1-\alpha) = s(\mu/(1-\alpha)) \quad (26)$$

where the time symbol t has been dropped from $k(t)$, $\alpha(t)$, $\mu(t)$, $r(t)$, $w_1(t)$ and $w_2(t)$ to avoid cluttering the notation.

The equilibrium in the goods market is easily checked: the zero profit condition implies

$$L_1 f(K/L_1) = (r + \delta)K + w_2 L_2 + w_1 L_1 + \Phi = \delta K + (rK + W + \Phi) \quad (27)$$

where W is the labor income defined by (15). Substituting $(rK + W + X)$ from (14), and setting $V = K$, one can write

$$L_1 f(K/L_1) = \delta K + \dot{K} + C$$

which states that supply is equal to demand in the goods market in accordance to Walras' law.

4.2 Evolution of State Variables for Given Prices

In this subsection one studies how the state variables $(M(t), k(t), \alpha(t))$ evolve when the relative prices $(r(t), w_1(t), w_2(t), \mu(t))$ are determined by equations (23), (24), (25) and (26).

4.2.1 Dynamics of the Share of Productive Labor

From (25), (24) and (23), the real wage of productive workers is written as

$$w_1 = \left\{ f\left(\frac{k}{\alpha}\right) - \frac{k}{\alpha} f'\left(\frac{k}{\alpha}\right) \right\} - \frac{\pi + f'\left(\frac{k}{\alpha}\right) - \delta}{s'\left(\frac{\mu}{1-\alpha}\right)} \quad (28)$$

From (26) and the concavity of the function s , one concludes that $\mu/(1-\alpha)$ is an increasing function of α . This and the concavity of the functions s and f assure that w_1 can be written as a function of the ratio k/α and α : $w_1 = w_1(k/\alpha, \alpha)$ with

partial derivatives $w_{1_{k/\alpha}} > 0$ and $w_{1_\alpha} < 0$. One concludes that w_1 is increasing in k and decreasing in α .

Likewise, the real wage of administrative workers is written as

$$w_2 = \left\{ \pi + f' \left(\frac{k}{\alpha} \right) - \delta \right\} \frac{s \left(\frac{\mu}{1-\alpha} \right) - \frac{\mu}{1-\alpha} s' \left(\frac{\mu}{1-\alpha} \right)}{s' \left(\frac{\mu}{1-\alpha} \right)} \quad (29)$$

The concavity of the functions s and f assure that w_2 can be written as a function of the ratio k/α and α : $w_2 = w_2(k/\alpha, \alpha)$ with partial derivatives $w_{2_{k/\alpha}} < 0$ and $w_{2_\alpha} > 0$. One concludes that w_1 is decreasing in k and increasing in α .

From (28) and (29), $w_1 > w_2$ if and only if:

$$\frac{f \left(\frac{k}{\alpha} \right) - \frac{k}{\alpha} f' \left(\frac{k}{\alpha} \right)}{\pi + f' \left(\frac{k}{\alpha} \right) - \delta} > \frac{1 + s \left(\frac{\mu}{1-\alpha} \right) - \frac{\mu}{1-\alpha} s' \left(\frac{\mu}{1-\alpha} \right)}{s' \left(\frac{\mu}{1-\alpha} \right)} \quad (30)$$

The right hand side of (30) is an increasing function of $\mu/(1-\alpha)$. Since (26) assures that $\mu/(1-\alpha)$ is an increasing function of α , it follows that the right hand side of this equation is an increasing function of α . Hence one defines the increasing function $g : \mathfrak{R} \rightarrow \mathfrak{R}$ such that $g(\alpha)$ stands for the right hand side of (30).

Likewise, the assumptions about the production function assure that the left hand side of (30) is an increasing function of the ratio k/α and a decreasing function of π . Hence one defines the function $q : \mathfrak{R}^2 \rightarrow \mathfrak{R}$ such that $q(k/\alpha, \pi)$ stands for the left hand side of (30), with partial derivatives $q_{k/\alpha} > 0$ and $q_\pi < 0$.

One concludes that in the plan $(k \times \alpha)$ the set of points where $w_1 = w_2$, for a given π , is represented by $q(k/\alpha, \pi) = g(\alpha)$. It has positive slope. The INADA conditions assure that $\lim_{\alpha \rightarrow +\infty} g(\alpha) = +\infty$, $\lim_{k/\alpha \rightarrow 0} h(k/\alpha, \pi) = 0$, $\lim_{\alpha \rightarrow 0^+} g(\alpha) = 0$. These properties imply that the point $(0, 0)$ belongs to this locus and that this locus approaches asymptotically the vertical line $\alpha = 1$.

Another property of the locus $w_1 = w_2$ is that it crosses the line $f'(k/\alpha) = \rho + \lambda + \delta$ at a single point. In order to see this, one starts at any point (k, α) on the line $f'(k/\alpha) = \rho + \lambda + \delta$ and move upwards along this line. Since the ratio k/α remains fixed, for a given π , $h(k/\alpha, \pi)$ is a positive constant; while $g(\alpha)$ increases monotonically from zero at $\alpha = 0$ to infinite as $\alpha \rightarrow 1$. Continuity of the functions q and g assures the existence of a single crossing point that will be denoted by (α_π^*, k_π^*) .

Since the locus $w_1 = w_2$ moves the left as the rate of inflation increases, one concludes that the higher the rate of inflation the smaller are α_π^* and k_π^* . In order to avoid cluttering the notation, the subscript π will be dropped from (α_π^*, k_π^*) .

Another property of the locus $w_1 = w_2$ is that it remains below the line $f'(k/\alpha) = \rho + \lambda + \delta$ for α in the range $(0, \alpha_\pi^*)$ and above it in the range $(\alpha_\pi^*, 1)$. To see this, one starts at the crossing point (α_π^*, k_π^*) where $f'(\alpha_\pi^*/k_\pi^*) = \rho + \lambda + \delta$ and

$q(\alpha_\pi^*/k_\pi^*, \pi) = g(\alpha_\pi^*)$ and move downwards along the line defined by $f'(k/\alpha) = \rho + \lambda + \delta$. Take any $\alpha' \in (0, \alpha_\pi^*)$ and the corresponding k' such that the point (k', α') belongs to the locus $w_1 = w_2$, i.e., $q(k'/\alpha', \pi) = g(\alpha')$. Since g is increasing $g(\alpha') < g(\alpha_\pi^*)$ and therefore $q(k'/\alpha', \pi) = g(\alpha') < g(\alpha_\pi^*) = q(\alpha_\pi^*/k_\pi^*, \pi)$. Since q is increasing in k/α , one concludes that $k'/\alpha' < \alpha_\pi^*/k_\pi^*$, which shows that the locus $w_1 = w_2$ is located below the line $f'(k/\alpha) = \rho + \lambda + \delta$ for α in the range $(0, \alpha_\pi^*)$.

At any instant t , an amount $\lambda\alpha(t)L$ of the $L_1(t) = \alpha(t)L$ productive workers, die. When $w_1(t) > w_2(t)$ all the λL newly born households decide to be productive workers. Hence the net increase in the supply of these workers is $\dot{L}_1 = \lambda L - \alpha(t)\lambda L = L[1 - \alpha(t)]\lambda$. Hence $\dot{\alpha} = \lambda(1 - \alpha(t)) > 0$. When $w_2(t) > w_1(t)$ the derivative is $\dot{\alpha} = -\lambda\alpha(t) < 0$. When $w_1(t) = w_2(t)$ the share α will evolve in order to keep both wages equal, i.e., $\alpha(t)$ will follow $k(t)$ in a way as to keep the state $(k(t), \alpha(t))$ on the locus $w_1 = w_2$. Points above this locus are those where $\dot{\alpha} = \lambda(1 - \alpha) > 0$, whereas points below this locus are those where $\dot{\alpha} = -\lambda\alpha < 0$. The locus $w_1 = w_2$ is *not* a locus where $\dot{\alpha} = 0$. The dynamics of α is summarized as:

$$\text{If } q(k(t)/\alpha(t), \pi) > g(\alpha(t)), \text{ then } \dot{\alpha}(t) = \lambda(1 - \alpha(t)) > 0 \quad (31)$$

$$\text{If } q(k(t)/\alpha(t), \pi) = g(\alpha(t)), \text{ then } \dot{\alpha}(t) = \dot{k} q_{k/\alpha} \{ \alpha g'(\alpha) + k/\alpha q_{k/\alpha} \}^{-1}$$

$$\text{If } q(k(t)/\alpha(t), \pi) < g(\alpha(t)), \text{ then } \dot{\alpha}(t) = -\lambda\alpha(t) < 0$$

One concludes that $\dot{\alpha}$ is positive (negative) for points (α, k) to the left (right) of the locus $w_1 = w_2$. For points on the locus $w_1 = w_2$, the time derivative $\dot{\alpha}$ is *not* zero, but has the same sign of the time derivative \dot{k} which will be studied ahead.

4.2.2 Dynamics of the Consumption Flow

Substituting (23) into (21) and dividing through by L , the dynamics of the aggregate per capita consumption is

$$\dot{c}(t) = [f'(k(t)/\alpha(t)) - (\lambda + \rho + \delta)]c(t) + Z(t)(\lambda + \rho)/L \quad (32)$$

From (19), one knows that at an instant t , $Z(t)$ is given by future values of the variables $r(t_1)$, $w_1(t_1)$, $w_2(t_1)$ and $\beta(t_1)$, where $t < t_1$. These four variables are functions of the future states $(\alpha(t_1), k(t_1))$. Since $(\alpha(t_1), k(t_1))$ is a function of the current state $(\alpha(t), k(t))$, one concludes that $Z(t)$ is itself a function of the current state $(\alpha(t), k(t))$. Hence one can write $Z(t) = Z(\alpha(t), k(t))$.

From (19), one knows that $Z(k, \alpha) \geq 0$ with equality at the points where the state (α, k) is on the locus $w_1 = w_2$. This implies that $\dot{c} = (\lambda + \rho)Z/L > 0$ for points on the line $f'(k/\alpha) = \rho + \lambda + \delta$. Since $(0, 0)$ belongs to this line and also to the locus $w_1 = w_2$, one concludes that $(0, 0)$ belongs to the locus $\dot{c} = 0$.

One can now locate the locus $\dot{c} = 0$ in the plan $(k \times \alpha)$. Moving along the locus $w_1 = w_2$, where $Z(k, \alpha) = 0$, one has $\dot{c} > 0$ ($\dot{c} < 0$) when the state (k, α) is on the part of the locus $w_1 = w_2$ below (above) the line $f'(k/\alpha) = \rho + \lambda + \delta$. On the line $f'(k/\alpha) = \rho + \lambda + \delta$, one has $\dot{c} = (\lambda + \rho)Z/L > 0$. For (k, α) at the intersection of this line with the locus $w_1 = w_2$, i.e., at (α_π^*, k_π^*) one has $\dot{c} = 0$.

Take any point $A = (k_A, \alpha_A)$ on the line $f'(k/\alpha) = \rho + \lambda + \delta$ and another point $B = (k_B, \alpha_B)$ on the part of the locus $w_1 = w_2$ above (α_π^*, k_π^*) . It has just been shown that $\dot{c}(A) > 0$ and $\dot{c}(B) < 0$. From the continuity of the function \dot{c} there is a point $D = (k_D, \alpha_D)$ in the segment AB where $\dot{c}(D) = 0$.

One concludes that the locus $\dot{c} = 0$ touches the line $f'(k/\alpha) = \rho + \lambda + \delta$ at (α_π^*, k_π^*) and lies to the left of this line elsewhere. At (α_π^*, k_π^*) , the locus $\dot{c} = 0$ crosses the locus $w_1 = w_2$. Points (α, k) above (below) the locus $\dot{c} = 0$ are those where $\dot{c} < 0$ ($\dot{c} > 0$).

4.2.3 Dynamics of the Capital Stock

Setting the aggregate stock of capital $K(t)$ equal to the aggregate wealth $V(t)$, from (14), the dynamics of K is given by

$$\dot{K}(t) = r(t)K(t) + W(t) + \Phi(t) - C(t) \quad (33)$$

Substituting (27) into (33) and dividing through by L the dynamics of the per capita stock of capital is given by

$$\dot{k}(t) = \alpha(t)f(k(t)/\alpha(t)) - \delta k(t) - c(t) \quad (34)$$

One now studies the location of the locus where $\dot{k} = 0$ in the plan $(k \times \alpha)$. Along this locus the second time derivative must also be zero:

$$\ddot{k} = \left\{ f\left(\frac{k}{\alpha}\right) - \frac{k}{\alpha}f'\left(\frac{k}{\alpha}\right) \right\} \dot{\alpha} + \left\{ f'\left(\frac{k}{\alpha}\right) - \delta \right\} \dot{k} - \dot{c} \quad (35)$$

Setting $\dot{k} = 0$ and $\ddot{k} = 0$, one gets:

$$\dot{c} = \left\{ f\left(\frac{k}{\alpha}\right) - \frac{k}{\alpha}f'\left(\frac{k}{\alpha}\right) \right\} \dot{\alpha} \quad (36)$$

Equation (36) implies that the locus $\dot{k} = 0$ is located in the region of the plan $(k \times \alpha)$ where the time derivatives \dot{c} and $\dot{\alpha}$ have the same sign. This motivates the division of the plan $(k \times \alpha)$ into four regions according to the signs of \dot{c} and $\dot{\alpha}$.

The first region is represented by $[\dot{\alpha} > 0, \dot{c} < 0]$. This region is bounded by the k axis, the part of the locus $\dot{c} = 0$ situated between $(0, 0)$ and (α_π^*, k_π^*) , and the part of the locus $w_1 = w_2$ above (α_π^*, k_π^*) . The second region is represented by $[\dot{\alpha} < 0, \dot{c} > 0]$. This region is bounded by the α axis, the part of the locus

$w_1 = w_2$ situated between $(0, 0)$ and (α_π^*, k_π^*) , and the part of the locus $\dot{c} = 0$ above (α_π^*, k_π^*) . The third region is represented by $[\dot{\alpha} < 0, \dot{c} < 0]$. In this region, points (α, k) satisfy $\alpha > \alpha^*$ and $k > k^*$. The left boundary of this region is the part of the locus $w_1 = w_2$ above (α^*, k^*) and its right boundary is the part of the locus $\dot{c} = 0$ above (α_π^*, k_π^*) . The fourth region is represented by $[\dot{\alpha} > 0, \dot{c} > 0]$. In this region, points (α, k) satisfy $\alpha < \alpha^*$ and $k < k^*$. The left boundary of this region is the part of the locus $\dot{c} = 0$ situated between $(0, 0)$ and (α_π^*, k_π^*) , and its right boundary is the part of the locus $w_1 = w_2$ situated between $(0, 0)$ and (α_π^*, k_π^*) . Equation (36) assures that the locus $\dot{k} = 0$ is located in the third and fourth regions.

One now turns to the analysis of the first region $[\dot{\alpha} > 0, \dot{c} < 0]$. When the state (α, k) is in the interior of this region or on its boundary on the locus $\dot{c} = 0$, one has $\dot{k} < 0$, for if one had $\dot{k} > 0$, than (35) would imply that $\ddot{k} > 0$ and the capital stock would increase indefinitely. When the state (α, k) is on the boundary on the part of the locus $w_1 = w_2$ situated between (α_π^*, k_π^*) and $(1, +\infty)$, (31) implies that \dot{k} and $\dot{\alpha}$ have the same sign. Since $\dot{c} < 0$, one has $\dot{k} < 0$, for if one had $\dot{k} > 0$ then (35) would imply that $\ddot{k} > 0$ and the capital stock would increase indefinitely. Hence, on the first region $[\dot{\alpha} > 0, \dot{c} < 0]$ one has $\dot{k} < 0$.

One now turns to the analysis of the second region $[\dot{\alpha} < 0, \dot{c} > 0]$. When the state (α, k) is in the interior of this region or on its boundary on the locus $\dot{c} = 0$, one has $\dot{k} > 0$, for if one had $\dot{k} < 0$, than (35) would imply that $\ddot{k} < 0$ and the capital stock would decrease down to zero. When the state (α, k) is on the boundary on the part of the locus $w_1 = w_2$ situated between $(0, 0)$ and (α_π^*, k_π^*) , then (31) implies that \dot{k} and $\dot{\alpha}$ have the same sign. Since $\dot{c} > 0$, one has $\dot{k} > 0$, for if one had $\dot{k} < 0$ than (35) would imply that $\ddot{k} < 0$ and the capital would decrease down to zero. Hence on the second region $[\dot{\alpha} < 0, \dot{c} > 0]$ one has $\dot{k} > 0$.

The locus $\dot{k} = 0$ which has been shown to be in the third and fourth regions can now be located using the fact that the derivative \dot{k} is a continuous function of the state (α, k) . One starts with the third region $[\dot{\alpha} < 0, \dot{c} < 0]$. On the left boundary of this region where $w_1 = w_2$ one has $\dot{k} < 0$; while on the right boundary where $\dot{c} = 0$ one has $\dot{k} > 0$. Continuity of \dot{k} implies that $\dot{k} = 0$ is somewhere in the interior of this region.

Likewise, on the left boundary of the fourth region $[\dot{\alpha} > 0, \dot{c} > 0]$ where $\dot{c} = 0$ one has $\dot{k} < 0$; while on the right boundary where $w_1 = w_2$ one has $\dot{k} > 0$. Continuity of \dot{k} implies that $\dot{k} = 0$ is located somewhere in the interior of this region. Points above (below) the locus $\dot{k} = 0$ are those where $\dot{k} < 0$ ($\dot{k} > 0$).

But the location of the locus $\dot{k} = 0$ in the fourth region can be established more precisely. One will show that $\dot{k} = 0$ in this region is located to the right of the line $f'(k/\alpha) = \rho + \lambda + \delta$. This is done in two steps. In the first step, one shows that in the vicinity of (α_π^*, k_π^*) in the fourth region the locus $\dot{k} = 0$ is located below the line $f'(k/\alpha) = \rho + \lambda + \delta$. In the second step, one shows that, starting below this line in the vicinity of (α_π^*, k_π^*) , the locus $\dot{k} = 0$ does not cross

the line in any point (α, k) such that $0 < \alpha < \alpha_\pi^*$.

From (32) and (19), for (α, k) on the locus $\dot{k} = 0$ located to the left of the locus $w_1 = w_2$ one has $c = \alpha\{f(\frac{k}{\alpha}) - \delta\frac{k}{\alpha}\}$ and hence

$$\begin{aligned} \dot{c} &= \{f'(\frac{k}{\alpha}) - (\lambda + \rho + \delta)\}\{f(\frac{k}{\alpha}) - \delta\frac{k}{\alpha}\}\alpha + \\ &\quad + (\lambda + \rho)\lambda(1 - \alpha)[h_1(t) - h_2(t)] \end{aligned} \quad (37)$$

From (31), in this region one has $\dot{\alpha} = \lambda(1 - \alpha) > 0$. Hence from (36) and (37), for any (α, k) to the left of the locus $w_1 = w_2$ on the locus $\dot{k} = 0$, one has

$$\begin{aligned} &f(\frac{k}{\alpha}) - \frac{k}{\alpha}f'(\frac{k}{\alpha}) - (\lambda + \rho)[h_1 - h_2] = \\ &= \alpha \frac{[f'(\frac{k}{\alpha}) - (\lambda + \rho + \delta)]\{f(\frac{k}{\alpha}) - \delta\frac{k}{\alpha}\}}{\lambda(1 - \alpha)} \end{aligned} \quad (38)$$

From (24), (25) and (23), when the state is (α, k) one can write:

$$(w_1 - w_2) = \{f(\frac{k}{\alpha}) - \frac{k}{\alpha}f'(\frac{k}{\alpha})\} - \{\pi + f'(\frac{k}{\alpha}) - \delta\}g(\alpha) \quad (39)$$

where $g(\alpha)$ stands for the right hand side of (30) and g is a decreasing function. Hence the difference $w_1 - w_2$ can be written as $w_1 - w_2 = \Psi(k/\alpha, \alpha)$, where the function $\Psi : \mathfrak{R}^2 \rightarrow \mathfrak{R}$ has derivatives $\Psi_1 > 0$ and $\Psi_2 < 0$.

From (38) and (39) one can write:

$$\begin{aligned} (w_1 - w_2) - (\lambda + \rho)(h_1 - h_2) &= \\ &= \alpha \frac{[f'(\frac{k}{\alpha}) - (\lambda + \rho + \delta)]\{f(\frac{k}{\alpha}) - \delta\frac{k}{\alpha}\}}{\lambda(1 - \alpha)} - \{\pi + f'(\frac{k}{\alpha}) - \delta\}g(\alpha) \end{aligned} \quad (40)$$

One now shows that in the vicinity of (α_π^*, k_π^*) in the fourth region the locus $\dot{k} = 0$ is located below the line $f'(k/\alpha) = \rho + \lambda + \delta$. In order to prove it, one assumes otherwise. Since the locus $\dot{k} = 0$ crosses (α_π^*, k_π^*) , this assumption assures that there is segment of the locus $\dot{k} = 0$ in the vicinity (α_π^*, k_π^*) in the fourth region, say between points $(\tilde{\alpha}, \tilde{k})$ and (α_π^*, k_π^*) such that for any point (α, k) on this segment, one has $\tilde{k}/\tilde{\alpha} \geq k/\alpha \geq k_\pi^*/\alpha_\pi^*$. If at instant $t = 0$ the state $(\alpha(0), k(0))$ starts on this segment, then along the convergence path to (α_π^*, k_π^*) at any instant $t_1 > 0$ the state (α_{t_1}, k_{t_1}) will not enter the interior of the region above the locus $\dot{k} = 0$ where $\dot{k} < 0$. Since along the convergence path one would have $\dot{\alpha} > 0$, it follows that for any instant $t_1 > 0$ the state (α_{t_1}, k_{t_1}) satisfies $k(0)/\alpha(0) \geq k(t_1)/\alpha(t_1)$ and $\alpha(0) < \alpha(t_1)$. From (39) one concludes that along the convergence path one would have $(w_1(0) - w_2(0)) \geq w_1(t_1) - w_2(t_1)$.

Letting A stand for the left hand side of (40), from (19) and (10), one can write

$$A = (w_1 - w_2) - (\lambda + \rho) \int_t^\infty e^{-\int_t^{t_1} r(t_2) dt_2} [w_1(t_1) - w_2(t_1)] dt_1$$

Since $r \geq 0$, if at instant $t = 0$ the state $(\alpha(0), k(0))$ starts on the above described segment then

$$\begin{aligned} A &\geq [w_1(0) - w_2(0)] - (\lambda + \rho) \int_0^\infty [w_1(t_1) - w_2(t_1)] dt_1 \\ &= [w_1(0) - w_2(0)] - (\lambda + \rho) \left\{ \int_0^\Delta [w_1(t_1) - w_2(t_1)] dt_1 \right. \end{aligned}$$

where Δ is the time taken by the transition from $(\alpha(0), k(0))$ to (α_π^*, k_π^*) . The dynamics of α assures that Δ is finite. From the inequality $(w_1(0) - w_2(0)) \geq w_1(t_1) - w_2(t_1)$, one can write:

$$\begin{aligned} A &\geq [w_1(0) - w_2(0)] - (\lambda + \rho) \left\{ \int_0^\Delta [w_1(0) - w_2(0)] dt_1 \right. \\ &= [w_1(0) - w_2(0)] [1 - (\lambda + \rho)\Delta] \end{aligned}$$

The initial state $(\alpha(0), k(0))$ on the above described segment of the locus $\dot{k} = 0$ can be arbitrarily chosen sufficiently close to (α_π^*, k_π^*) . In particular it can be sufficiently short for the inequality $\Delta < 1/(\lambda + \rho)$ to hold. Hence, for a suitable choice of $(\alpha(0), k(0))$ one has $A > 0$.

But the assumption that, in the vicinity of (α_π^*, k_π^*) in the fourth region, the locus $\dot{k} = 0$ is located above the line $f'(k/\alpha) = \rho + \lambda + \delta$, implies that for any choice of $(\alpha(0), k(0))$ on that segment of the locus $\dot{k} = 0$, the right hand side of (40) is negative, for $f'' < 0$ implies $f'(k(0)/\alpha(0)) < (\rho + \lambda + \delta)$. This is in contradiction to $A > 0$. This completes the proof that, in the vicinity of (α_π^*, k_π^*) in the fourth region, the locus $\dot{k} = 0$ is located below the line $f'(k/\alpha) = \rho + \lambda + \delta$.

One now shows that the locus $\dot{k} = 0$ in the fourth region, even for (α, k) far from (α_π^*, k_π^*) remains below the line $f'(k/\alpha) = \rho + \lambda + \delta$. Assume that the locus $\dot{k} = 0$ crosses the line $f'(k/\alpha) = \rho + \lambda + \delta$ at some point in the segment where $0 < \alpha < \alpha_\pi^*$. Of the (possibly more than one) crossing points let $(\hat{\alpha}, \hat{k})$ be the one closest to (α_π^*, k_π^*) . Assume the state is initially at the crossing point $(\hat{\alpha}, \hat{k})$ at instant t , i.e., $(\alpha(t), k(t)) = (\hat{\alpha}, \hat{k})$. One will show that this assumption leads to a contradiction.

For any instant $t_1 > t$ the state $(\alpha(t_1), k(t_1))$ can not enter the interior of the region $\dot{k} < 0$ located above the locus $\dot{k} = 0$. Hence the state $(\alpha(t_1), k(t_1))$ converges to (α_π^*, k_π^*) in a path somewhere below or at most on the locus $\dot{k} = 0$. Since this locus has no other intersection with the line $f'(k/\alpha) = \rho + \lambda + \delta$ in the segment where $\hat{\alpha} < \alpha < \alpha^*$, and $\dot{\alpha}(t_1) > 0$, one concludes that along the convergence path, one would have $k(t_1)/\alpha(t_1) < \hat{k}/\hat{\alpha}$ and $\alpha(t_1) > \hat{\alpha}$.

The concavity of f implies that for any $t_2, t < t_2$, one has $r(t_2) = f'(k(t_2)/\alpha(t_2)) - \delta \geq f'(k(t)/\alpha(t)) - \delta = r(t)$. Since $k(t)/\alpha(t) = \hat{k}/\hat{\alpha} = k^*/\alpha^*$ one can write $r(t) = f'(k^*/\alpha^*) - \delta = \rho + \lambda$. Hence $r(t_2) \geq r(t)$ for $t < t_2$, and

$$e^{-\int_t^{t_1} r(t_2) dt_2} < e^{-\int_t^{t_1} (\rho + \lambda) dt_2}$$

From (30) the difference $w_1 - w_2$ can be written as $w_1 - w_2 = \Psi(k/\alpha, \alpha)$, where the function $\Psi : \mathfrak{R}^2 \rightarrow \mathfrak{R}$ has derivatives $\Psi_1 > 0$ and $\Psi_2 < 0$. Since for any instant $t_1 > t$ along the convergence path, one would have $\hat{k}/\hat{\alpha} = k(t)/\alpha(t) \geq k(t_1)/\alpha(t_1)$, and $\hat{\alpha} = \alpha(t) < \alpha(t_1) < \alpha^*$. The properties of Ψ imply that $w_1(t_1) - w_2(t_1) < w_1(t) - w_2(t)$ for $t < t_1$. One can write

$$\int_t^\infty e^{-\int_t^{t_1} r(t_2) dt_2} [w_1(t_1) - w_2(t_1)] dt_1 < \int_t^\infty e^{-\int_t^{t_1} (\rho+\lambda) dt_2} [w_1(t) - w_2(t)] dt_1 \quad (41)$$

The left hand side of (41) is $[h_1 - h_2]$ and the right hand side is $[w_1(t) - w_2(t)]/(\rho + \lambda)$. This implies that at instant t when $(\alpha(t), k(t)) = (\hat{\alpha}, \hat{k})$, (41) assures that

$$[w_1(\hat{\alpha}, \hat{k}) - w_2(\hat{\alpha}, \hat{k})] - (\rho + \lambda)[h_1(\hat{\alpha}, \hat{k}) - h_2(\hat{\alpha}, \hat{k})] > 0 \quad (42)$$

But when the state is $(\hat{\alpha}, \hat{k})$, one has $f'(\hat{k}/\hat{\alpha}) = (\lambda + \rho + \delta)$ and (40) implies

$$\begin{aligned} [w_1(\hat{\alpha}, \hat{k}) - w_2(\hat{\alpha}, \hat{k})] - (\rho + \lambda)[h_1(\hat{\alpha}, \hat{k}) - h_2(\hat{\alpha}, \hat{k})] &= \\ &= -\left\{ \pi + f' \left(\frac{\hat{k}}{\hat{\alpha}} \right) - \delta \right\} g(\alpha) < 0 \end{aligned}$$

which contradicts (42). This completes the proof that the locus \dot{k} does not cross the line $f(k/\alpha) - (\lambda + \rho + \delta) > 0$ in the fourth region.

4.2.4 Dynamics of the Money Supply

For ease of exposition, it was assumed above that the rate of inflation is exogenously chosen by the government and the rate of monetary creation is set accordingly. In this subsection one analyzes the rate of monetary creation that would generate this pattern of inflation.

Let $\nu(t)$ stand for the instantaneous rate of monetary creation at instant t . The money supply follows $\dot{M}(t) = \nu(t)M(t)$. The money demand is given by $\dot{M}^D(t) = P(t)[\dot{m}^D(t) + \pi m^D(t)]$. Recalling that $\mu^D(t) = M^D(t)/P(t)L$, one gets $\dot{\mu}^D(t) = (\nu(t) - \pi)\mu^D(t)$.

From 4 and $L_1 + L_2 = L$ one can write:

$$\frac{\alpha(t)}{1 - \alpha(t)} = s\left(\frac{\mu(t)}{1 - \alpha(t)}\right)$$

Deriving this expression with respect to time one gets

$$\begin{aligned} \dot{\mu}(t) &= (\varphi(t) - \pi)\mu^D(t) = \frac{1 + s\left(\frac{\mu(t)}{1 - \alpha(t)}\right) - \frac{\mu(t)}{1 - \alpha(t)} s' \left(\frac{\mu(t)}{1 - \alpha(t)}\right)}{s' \left(\frac{\mu(t)}{1 - \alpha(t)}\right)} \dot{\alpha}(t) = \\ &= g(\alpha) \dot{\alpha}(t) \end{aligned}$$

where $g(\alpha)$ stands for the right hand side of 30 and g is an increasing function. This expression shows that whenever the share of productive workers is increasing, real money balances are increasing, which is equivalent to stating that inflation is lower than the rate of monetary creation. This is so because the real demand for money is reduced gradually as more administrative workers are hired.

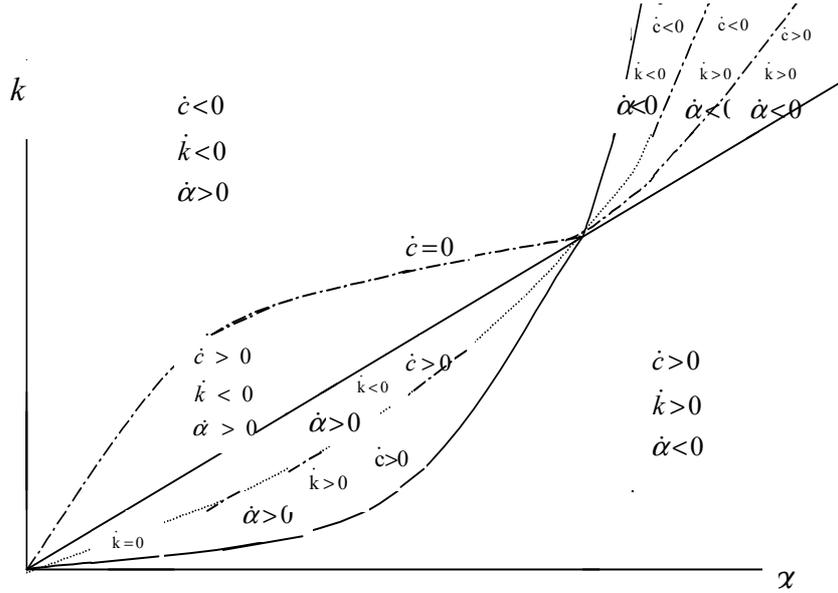
4.3 General Equilibrium: Definition

The previous discussion motivates the following definition:

Definition 1 Starting from a triple of state variables $(\alpha(0), k(0), M(0))$ and a fixed rate of inflation π , a General Equilibrium is characterized by the three following mathematical conditions: (1) a sequence of relative prices $(r(t), w_1(t), w_2(t), P(t))$ satisfying the system of equations (23), (25), (26) and (24); (2) a sequence of state variables $(\alpha(t), k(t), M(t))$ that evolve according to (31), (34), (22) and satisfies (9); (3) a control variable $(c(t))$ that satisfies (32).

From the definition above one concludes that for each triple of state variables $(\alpha(0), k(0), M(0))$ and fixed rate of inflation π , there is a unique general equilibrium. The dynamics of the state variables are presented in figure 1:

Figure 1: Summary of the Time Derivatives of k , α and c in the plan $(k \times \alpha)$



The analysis of the dynamics of the state variables (α, k) has shown that for any starting point $(\alpha(0), k(0), M(0), \pi)$ at instant $t = 0$ there is a unique sequence $(r(t), w_1(t), w_2(t), P(t); \alpha(t), k(t), M(t); c(t))$ that constitute a General Equilibrium. For a chosen rate of inflation π , the state (α, k) eventually converges to (α^*, k^*) . This point will henceforth be denoted by (α_π^*, k_π^*) in order to stress the fact that this point is a function of the rate of inflation.

5 Transitional Dynamics

In this section one studies the time path of the economy in response to an unannounced permanent change of the rate of inflation. These paths are quite different for an open and a closed economy. They are also not symmetric for inflationary increases and decreases.

5.1 The Closed Economy

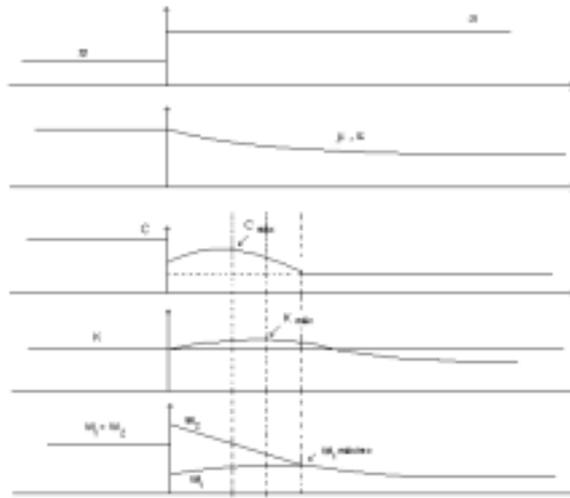
In this subsection one uses the model built in the previous sections without any change. One describes how the the economy reacts after a permanent change of the rate of inflation.

5.1.1 Transition from Low to High Inflation

One assumes that inflation is zero for $t < 0$ and jumps to $\pi > 0$ at $t \geq 0$. The initial steady state with low inflation is represented by (α_0^*, k_0^*) . In the plan $(k \times \alpha)$, the jump of the rate of inflation implies a shift to the left of the locus $w_1 = w_2$. According to 30 the new locus $w_1 = w_2$ crosses the line $f'(k/\alpha) - (\lambda + \rho + \delta)$ at a the new steady state $(\alpha_\pi^*, k_\pi^*) < (\alpha_0^*, k_0^*)$.

Immediately after the rise of the rate of inflation, at $t = 0$, the state (α, k) remains at point (α_0^*, k_0^*) , but at this point the time derivatives $\dot{\alpha}$, \dot{k} and \dot{c} that were zero for $t < 0$ become $\dot{\alpha} < 0$, $\dot{k} > 0$ and $\dot{c} > 0$ for $t \geq 0$.

Figure 2: Transition From Low to High Inflation in Closed Economy



With a higher rate of inflation, the cost of using money as working capital increases and firms wish to economize on its use. But this can only be done if

more administrative workers are hired. However, in the short run the supply of administrative workers is inelastic. As a consequence the real money demand remains unchanged, and the excess demand for administrative labor causes an upward jump of w_1 , as can be ascertained from 28.

Having to pay more for both factors used in the production of administrative services, the demand for productive workers - which determines the need for those services - falls. Since the supply of productive labor is fixed in the short run, as shown by 29, w_2 falls instantaneously in order to keep supply equal to demand in the productive labor market. With unchanged demand for productive workers, the marginal productivity of capital remains unchanged.

Apart from transferring income from productive workers to administrative ones, the higher rate of inflation is a harbinger of lower future income. As a result consumption falls abruptly at $t = 0$. This is shown by the fact that at $t = 0$ the state remains at (α_0^*, k_0^*) and that $\dot{k}(0) > 0$. One concludes that in the short run, the rise of inflation causes a transfer of income from productive to administrative workers, and reduces consumption.

Immediately after $t > 0$, the capital stock follows an increasing path while the supply of productive workers gradually falls as the new cohorts choose to be administrative workers. As a result, the wage of productive workers start to recover its loss while the wage of administrative workers starts to fall due to increased supply. Since the capital/productive labor ratio gradually rises, the rental falls gradually.

Consumption, having fallen abruptly at $t = 0$ starts to increase gradually thereafter. This is due to the fact that, for a given future capital income, the future labor income gradually increases. Since all workers that are born get the higher wage in the administrative labor market, each administrative worker who dies is replaced by another who gets the same wage, but the productive workers that die are replaced by administrative ones that get the higher wage. As a result, the aggregate wage income increases and so does the aggregate consumption.

In finite time consumption reaches a maximum and so does the capital stock and then starts to fall. Wages in both labor markets will also be equal in finite time. After this moment the division of each cohort between the two labor markets will be such that the two wages remain equal. Capital falls in order to increase its return up to the steady state level.

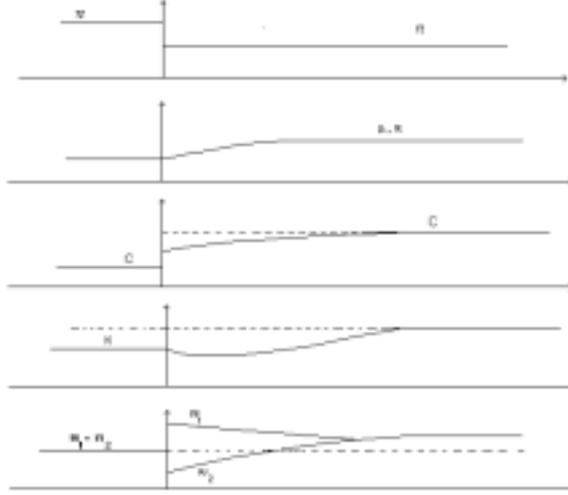
When the new steady state is reached, the economy as a whole has impoverished vis-a-vis its pre inflationary steady state. A larger share of the population is employed in administrative tasks doing services that could be done by money if there were no inflation.

5.1.2 Transition from High to Low Inflation

One assumes that inflation is positive $\pi > 0$ for instants $t < 0$ and falls to zero at $t \geq 0$. The initial steady state with high inflation is represented by (α_π^*, k_π^*) .

In the plan ($k \times \alpha$) the set of points where $w_1 = w_2$, according to 30 shifts to the right and crosses the the line $f'(k/\alpha) - (\lambda + \rho + \delta)$ at $(\alpha_\pi^*, k_\pi^*) < (\alpha_0^*, k_0^*)$.

Figure 3: Transition From High to Low Inflation in Closed Economy



Immediately after the fall of the rate of inflation, at $t = 0$, the state (α, k) remains at point (α_0^*, k_0^*) , but at this point the time derivatives $\dot{\alpha}$, \dot{k} and \dot{c} that were zero for $t < 0$ become $\dot{\alpha} > 0$, $\dot{k} < 0$ and $\dot{c} > 0$ for $t \geq 0$. With a lower rate of inflation, the cost of using money as working capital decreases and firms wish to substitute costless money used as working capital for administrative labor. Since in the short run the supply of administrative workers is inelastic, the real money demand remains unchanged, and the reduced demand for administrative labor produces a sharp drop of w_2 as can be ascertained from 29.

Having to pay less for both factors used in the production of administrative services, the demand for productive workers - which determines the need for those services - at the previous productive wage increases. Since the supply of productive labor is fixed in the short run, w_1 rises instantaneously in order to keep supply equal to demand in the productive labor market. With unchanged demand for productive workers, the marginal productivity of capital remains unchanged.

Apart from transferring income from administrative workers to productive ones, the lower rate of inflation is a harbinger of higher future income. As a result consumption increases abruptly at $t = 0$. This is shown by the fact that at $t = 0$ the state remains at (α_π^*, k_π^*) and that $\dot{k}(0) < 0$. One concludes that in the short run, the fall of inflation causes a transfer of income from administrative to productive workers, and generates a consumption boon.

Immediately after $t > 0$, the capital stock starts to fall due to the higher consumption, while the supply of administrative workers gradually falls as the new

cohorts choose to be productive workers. As a result, the wage of administrative workers start to recover its loss while the wage of productive workers starts to fall due to increased supply. Since the capital/productive labor ratio gradually falls, the rental gradually increases.

Consumption increases continuously up to the point where the high return on capital will curb the consumption boom and the capital stock starts to increase. This is due to the fact that, for a given future capital income, the future labor income gradually increases. Since all workers that are born get the higher wage in the productive labor market, each productive worker who dies is replaced by another who gets the same wage, but the administrative workers that die are replaced by productive ones that get the higher wage. As a result, the aggregate wage income increases and so does the aggregate consumption.

In a finite time the capital stock reaches its minimum and the starts to rise. Wages in both labor markets will equal in finite time. After this moment the division of each cohort between the two labor markets will be such that the two wages remain equal while capital increases up to its steady state level.

When the steady state is reached, the economy as a whole has enriched vis-a-vis its previous inflationary steady state. A larger share of the population is employed in productive tasks and most administrative services are done by money. The economy is richer for it is making greater use of a factor of production whose social cost is zero.

5.2 The Model in Open Economy

The adaptation of the model to study an open economy is based on the assumption that the economy is a small one. The gross return of capital in the international market is assumed to be $\rho + \lambda + \delta$. It is also assumed that there is no rigidity that precludes capital from adjusting discontinuously.

The capital stock k now is not necessarily equal to the aggregate per capita wealth $v = V/L$. The variable k reflects the stock of capital used by firms in the economy, but local households can send their savings abroad as well as firms can borrow in international capital markets. Hence, in the open economy equation (34) is rewritten to reflect the accumulation of wealth by local households, not the accumulation of capital used by firms. This equation is rewritten as

$$\dot{v}(t) = v(t)(\rho + \lambda + \delta) - \delta v(t) - c(t)$$

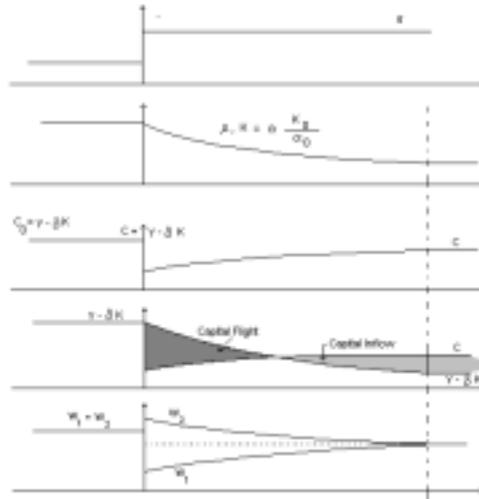
The inexistence of arbitrage requires that the state (α, k) always be on the line $f'(k/\alpha) = \rho + \lambda + \delta$. For a given value of the state variable α , the supply of capital is $k = \alpha f'^{-1}(\rho + \lambda + \delta)$. In the plan $(k \times \alpha)$, for a given level of inflation, the loci $w_1 = w_2$, $\dot{k} = 0$, and $\dot{c} = 0$ are the same of those calculated for the closed economy, but the state (α, k) will always slide on the line $f'(k/\alpha) = \rho + \lambda + \delta$ as k follows α . This means that the capital stock is driven by the supply of productive labor.

5.2.1 Transition from Low to High Inflation in Open Economy

As in the closed economy, one assumes that inflation is zero for $t < 0$ and jumps to $\pi > 0$ at $t \geq 0$. The initial steady state with low inflation is represented by (α_0^*, k_0^*) . In the plan $(k \times \alpha)$ the set of points where $w_1 = w_2$, according to (30) shifts to the left after the rise of the rate of inflation, and crosses the the line $f'(k/\alpha) - (\lambda + \rho + \delta)$ at $(\alpha_\pi^*, k_\pi^*) < (\alpha_0^*, k_0^*)$.

Immediately after the rise of the rate of inflation, at $t = 0$, the state (α, k) remains at point (α_0^*, k_0^*) , but at this point the time derivatives $\dot{\alpha}$, \dot{k} and \dot{c} that were zero for $t < 0$ become $\dot{\alpha}/\alpha = -\lambda$, $\dot{k}/k = \dot{\alpha}/\alpha = -\lambda$ and $\dot{c} > 0$ for $t \geq 0$.

Figure 4: Transition From Low to High Inflation in Open Economy



With a higher rate of inflation, the cost of using money as working capital increases. Firms wish to substitute administrative labor for money. Since in the short run the supply of administrative workers is inelastic, the excess demand for administrative labor produces an upward jump of w_1 . Having to pay more for both factors used in the production of administrative services, the demand for productive workers - which determines the need for those services - at the previous productive wage falls. Since the supply of productive labor is fixed in the short run, w_2 falls instantaneously in order to keep supply equal to demand in the productive labor market. With unchanged demand for productive workers, the marginal productivity of capital remains unchanged. This shows that immediately after the rise of the rate of inflation, the open economy does not differ much from the closed one.

After $t > 0$ the supply of productive workers starts to fall as the new cohorts choose to be administrative workers. As a result, the wage of productive workers start to recover its loss while the wage of administrative workers starts to fall due to increased supply. Again this is similar to what happened in the close economy.

The main difference between the open and closed economies regards the paths of the aggregate consumption and of the capital stock. Having fallen abruptly at $t = 0$ the aggregate consumption increases henceforth and never falls. This is possible because the difference between output net of depreciation and consumption is sent abroad. This capital flight will gradually increase the economy's net foreign assets.

One remarkable result of the open economy version of the model is the fact that the aggregate consumption does *not* adjust instantaneously to its new (lower) long run level, as would happen in a perfect foresight model of an open economy with a representative consumer. The key to understand the partial adjustment of the level of consumption at $t = 0$ is the fact that, during the transition to the new steady state, the (average) wage - and hence the permanent income - of the households that die is lower than that of those that are born. Consumption reaches its steady state when both wages become equal.

Comparing the aggregate consumption in the inflationary steady state in the open and the closed economies, it is higher in the former than in the latter. This is so because the net foreign assets accumulated during the transition of the open economy will finance a higher steady state level of consumption. The capital stock falls at the same rate of the supply of productive workers.

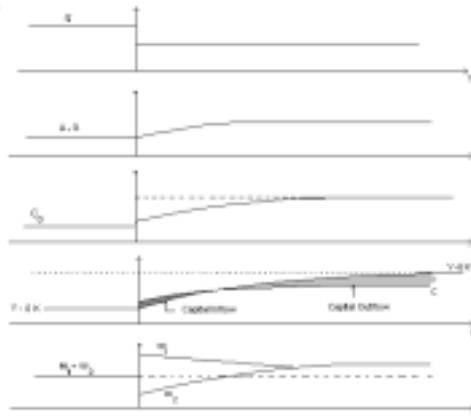
When the inflationary steady state is reached, the economy as a whole has impoverished vis-a-vis its pre inflationary steady state. A larger share of the population is employed in administrative tasks doing services that could be done by money if there were no inflation. The economy will have permanent trade deficits that are counterparts of the surplus on the capital services account.

5.2.2 Transition from High to Low Inflation in Open Economy

As in the closed economy, one assumes that inflation is positive for $t < 0$ and jumps to $\pi = 0$ at $t \geq 0$. The initial steady state with high inflation is represented by (α_π^*, k_π^*) . In the plan $(k \times \alpha)$ the set of points where $w_1 = w_2$, according to (30) shifts to the right and crosses the line $f'(k/\alpha) - (\lambda + \rho + \delta)$ at $(\alpha_\pi^*, k_\pi^*) < (\alpha_0^*, k_0^*)$.

Immediately after the fall of the rate of inflation, at $t = 0$, the state (α, k) remains at point (α_0^*, k_0^*) , but at this point the time derivatives $\dot{\alpha}$, \dot{k} and \dot{c} that were zero for $t < 0$ become $\dot{\alpha} = \lambda(1 - \alpha) > 0$, $\dot{k}/k = \dot{\alpha}/\alpha > 0$ and $\dot{c} > 0$ for $t \geq 0$.

Figure 5: Transition From High to Low Inflation in Open Economy



With a lower rate of inflation, the cost of using money as working capital is reduced and firms wish to substitute money for administrative labor. Since in the short run the supply of administrative workers is inelastic, the excess supply of administrative labor produces a downward jump of w_1 . Having to pay less for both factors used in the production of administrative services, the demand for productive workers - which determines the need for those services - at the previous productive wage rises. Since the supply of productive labor is fixed in the short run, w_2 rises instantaneously in order to keep supply equal to demand in the productive labor market. With unchanged demand for productive workers, the marginal productivity of capital remains unchanged. This shows that immediately after the fall of the rate of inflation, the open economy does differ much from the close one.

After $t > 0$ the supply of productive workers starts to increase as the new cohorts choose to be productive workers. As a result, the wage of productive workers start to recover its loss while the wage of administrative workers starts to fall due to increased supply. Again this is similar to what happens in the close economy.

The main difference between the open and closed economies again is seen in the paths of the aggregate consumption and of the capital stock. Having risen abruptly at $t = 0$ the aggregate consumption increases henceforth up to its new non inflationary steady state level. This is possible because the difference between output net of depreciation and consumption is financed internationally. This capital inflow will gradually increase the economy's net foreign debt.

As when inflation rises, gain the aggregate consumption does *not* adjust instantaneously to its new (higher) long run level, as would happen in a perfect foresight model of an open economy with a representative consumer. The key to understand the partial adjustment of the level of consumption at $t = 0$ is the fact that, during the transition to the new steady state, the (average) wage - and hence the permanent income - of the households that die is lower than that

of those that are born. Consumption reaches its steady state when both wages become equal.

Comparing the aggregate consumption in the inflationary steady state in the open and the closed economies, it is lower in the former than in the latter. This is so because the net foreign liabilities accumulated during the transition of the open economy will generate a capital service. The capital stock falls at the same rate of the supply of productive workers.

When the inflationary steady state is reached, the economy as a whole has enriched vis-a-vis its pre inflationary steady state. A larger share of the population is employed in productive tasks and most of the administrative services are done by money. The economy will have a permanent trade surplus necessary to cover the expenditures with the capital services.

6 Concluding Remarks

Most monetary models that study the impact of inflation on capital accumulation and on the allocation of factors of production are based on the assumption of a representative consumer. In order to study the impact of inflation on income inequality, the model of this paper assumed that there were two kinds of labor and focussed on the impact of inflation on the career choices. The main conclusions were that inflation increases income inequality during the transition from one level of inflation to another; but when it remains constant for a sufficiently long time, its level has no impact on the income inequality.

Another result that is worth mentioning is the theoretical explanation for the path of consumption observed after stabilization plans. The usual representative consumer models of perfect foresight open economies predict that, after a stabilization plan, the consumption level jumps on impact to its new (higher) long run level in response to the higher expected future income. To rule out such instantaneous adjustment, some models rely on installation costs for capital. Here after the initial jump of consumption that follows stabilization, there is a subsequent gradual rise of consumption to its new (higher) steady state level due to the fact that the permanent income of the households that die is lower than that of those that are born. Consumption reaches its steady state when both wages become equal.

The model also casts some insights on the political economy of long inflationary processes. Although this issue cannot be satisfactorily studied within the simple framework presented, some results of the model are worth mentioning. Firstly, if one assumes that societies do not like income inequality, the model provides an explanation for the fact that some societies can be relatively tolerant with inflation, provided that its level remains constant.

Secondly, when, for some unexplained reason, inflation rises, the economic forces attract the new generations of workers to the jobs that profit from high

inflation, thus increasing gradually the number of households that do not want the inflationary process to be curbed. In the short run there are individuals that are willing to support a rise of inflation in order to profit from the fact of being in the jobs that benefit from it. In the long run, however, these individuals end up losing those initial gains as the economy becomes less efficient. When this occurs, these individuals have already increased their share in the voting population and thus potentially form a large pressure group in favor of increasing inflation to a still higher level.

A natural extension of this paper is to go deeper into these political economy issues and address the voting process. In particular intuition suggests that the median voter theorem can be used to define a benchmark beyond which the economy may go into a hyperinflation path.

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