Technology Adoption and E¤ects of Transfers Across Locations

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Abstract

This paper develops a two-period model with heterogeneous agents to analyze the e¤ects of transfers across locations on convergence, growth and welfare. The model has two important features. First, locations are asymmetric as it is assumed that there are more specialized occupations in the more developed one. Second, the returns on the investment to acquire new technology depend positively on the level of each region's knowledge and on the level of the world knowledge assumed to be available to all. In one hand, the poor region has a disadvantage as it has a lower stock of knowledge. On the other hand, it has the advantage of not having yet exploited a greater stock of useable knowledge available in the world. Hence, there are two possible cases. When the returns are greater in the poor region, we obtain the following results: (i) the rich location grows slower; (ii) the transfers to the poor location enhances the country's growth rate; and (iii) there is a positive amount of transfers to the poor region that is welfare improving. When the returns are greater in the rich region, the …rst two results are reversed and transfers to the rich region are welfare improving. In both cases, the optimal amount of transfer increases with the level of income disparity across regions and is not dependent on the level of the country's economic development (measured by its income per capita). Barriers to the adoption of new technology available in the world can constrain the convergence process as it harms in greater length the poor region. The results do not change whether migration is allowed or not.

1   Introduction

An examination of the empirical literature on income disparity across regions leads to three main ...ndings.

First, there is no agreement whether the so-called convergence hypothesis holds when regional data sets are used. Using the traditional cross-country growth regressions, Sala-i-Martin (1996) makes a survey and shows that the sub-sample of OECD countries, the states within the United States, the prefectures of Japan, and regions withing several European countries display strong evidence of ¾-convergence and absolute and conditional ¯-convergence. These ...ndings have been questioned by an opposing literature, which uses alternative econometric methods, and argues that the pattern is not necessarily consistent with convergence. Quah (1996a) and Quah (1996b) are examples of this alternative approach.

Second, even if one follows the classical approach and agrees with the convergence process, the problem is that it is very slow. For example, the south region in the United States had an income per capita that corresponded to 80% of the one in the north region before the Civil War. This percentage dropped to 40% right after the Civil War and it took approximately 100 years to reach the pre-war level again. The empirical ...ndings are that the speed of convergence is close to 2 per cent per year in all cases presented by Sala-i-Martin (1996).

Third, central government in di¤erent countries deal with the income disparity across regions in di¤erent ways. They do make transfers from the richest to the poorest regions in the country. In general, there is a very complex system of transfers of resources from the central to the subnational governments. The degree in which these goals are pursued varies from country to country.[1]

This paper develops a simple theoretical model to examine whether transfers can a¤ect the convergence or divergence process and its speed and how it should be implemented. The goal is to analyse the e¤ects of tranfers across locations on convergence, economic growth, and economic welfare. We ...rst examine the conditions for the existence either of convergence or divergence. We then analyse if transfers to the poor location can be detrimental to growth, that is, if there

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[1] It is worth giving as examples three countries with di¤erent levels of economic development and income disparity across regions that follow di¤erent transfer policies. In the United States, "the link between federal aid and state ...scal capacity - the potential ability to raise revenue relative to the cost of service - is weak". In Canada, the principle of equalization is not only part of the Constitution but also receives broad support. There is a scheme in which equalization payments equalize revenues but the di¤erences in need are not taken into consideration. In Brazil, there are two main vehicles for revenue sharing which are mainly based on redistributive criteria: the Fundo de Participação dos Estados and the Fundo de Participação dos Municípios. However, in a context of growing ...scal stress, the federal government increased the importance of revenues that are not subject to sharing with subnational government in the last years. It reduces the redistributive impact of the system of transfers. For a detailed description of the system of transfers for these and other countries, see Teresa Ter-Minassian (1996). Another example of transfers across locations is the European Union. In 1993, it created the Cohesion Fund to provide money for environmental and transport projects in the member states whose GDP was less than 90% of the EU average. The four bene...ciary countries are Spain, Portugal, Ireland, and Greece.

is a trade-off between the country's economic growth and the pursuit of policies that reduce the income disparity across regions. Moreover, we investigate which type of transfer, either to the poor or the rich locations, is efficient. In other word, which type of policy can be welfare improving. Finally, we examine if the optimal amount of transfers should be dependent on the level of income disparity across regions and on the country's level of economic development.

We develop a two-period model with heterogenous agents in the tradition of the multicommunity models such as Fernandez and Rogerson (1996). There are two locations and the quality of the labor force is greater in the rich one. There are two additional features in the model. First, locations are asymmetric as the more developed one has more specialized occupations. Therefore, the more skilled individuals have no incentive to move to the poor region. Second, individuals in both locations have to invest to acquire new technologies in order to increase their production in the next period. The returns on the investment to acquire new technology depend positively on the level of each region's knowledge. Moreover, as in Parente and Prescott (1994), the technology for adoption is such that it takes fewer resources to move from a lower level to a higher level as greater is the level of world technology, which is assumed to be available to all. In one hand, the poorest region has a disadvantage as it has a lower stock of knowledge. On the other hand, it has the advantage of not having yet exploited a greater stock of useable knowledge available in the world.

Two types of transfer policies are considered. Either individuals who live in the rich location are taxed and the resources are transferred to finance the adoption of new technologies in the poor location or vice-versa. There are two possible explanations for the government intervention. First, one can imagine that investments in education allow individuals in one region to adopt new technologies. As there are no capital markets to finance these investments due to moral hazard problems, the government should intervene to correct this imperfection through transfers. Second, investments in local infra-structure may be necessary to allow individuals to adopt new technologies. Government intervention through transfers may be necessary to complement private investments.

Two papers are related to this one. Fernandez and Rogerson (1996) also address the issue of transfers across locations affecting the economic welfare. Benabou (2000) develops a model to explain how countries with similar economic and political fundamentals can sustain such different systems of fiscal redistribution, and education finance. Also, Barro and Sala-i-Martin (1995) argue that transfers can affect the convergence or divergence process, and tend to reduce the cross-state dispersion of per capita income.[2] However, neither they examine how barriers to the adoption of new technologies affect the convergence process, nor they correlate the optimal amount of transfer with the income disparity across locations and the degree of the country's economic development.

In the next section, the model is presented. In section 3, we solve the competitive equilbrium without migration and transfers and find the conditions for

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[2]However, they do not consider it the main source of the long-run decline in income dispersion across the states.

the existence of income convergence across locations. We also examine how barriers to the adoption of new technologies a¤ect the convergence process. In section 4, we introduce transfers in the competitive equilibrium without migration and analyse how they a¤ect convergence and economic growth. Next, we focus on the planner's problem to examine if transfers can be welfare improving and if the optimal amount of transfers varies with the degree of income disparity acroos regions and the country's level of economic development. In sections 5 and 6, we re-do the analysis in the two previous sections to investigate if the results alter when migration is allowed. Conclusions are discussed in the last section.

2   Model

In this section, we present a two-period model ($t = 0; 1$), with two locations and two types of individuals. The model is then used in the next sections to analyze the e¤ects of transfers across locations on convergence, growth and welfare.

At each point in time, there are two types of agents. They di¤er by their knowledge or capacity to operate more sophisticated technologies. The low type individual (l) has knowledge $A_{0;l}$ in the beginning of his life at time $t = 0$. The high type individual (h) has a greater level of initial knowledge which is equal to $A_{0;h}$, where $A_{0;h} > A_{0;l}$. One can think that the variable $A_{0;i}$ ($i = h; l$) represents the amount of human capital that each type of agent is born with. An individual with a higher level of human capital is able to use more sophisticated technologies, that is, he is more skilled. Therefore, it is assumed that the high type agent can perform all tasks that the low type agent is capable of doing, as well as more specialized ones that the low individuals do not have the ability to perform. The population is assumed to be constant over time and there is an in…nite number of agents, with measure one for each type of individual.

There are two locations: the rich (r) and the poor (p) ones. At $t = 0$, all \h" type agents are in location \r" and all \l" type agents are in location \p". This is not the only di¤erence between both locations. They are also asymmetric as it is assumed that the rich one has more specialized occupations. The high type individual can perform more sophisticated tasks only in location "r". This feature of the model captures the idea developed by Adam Smith that the degree of specialization is limited by the extent of the market. As it is going to be clear below, the rich region has a greater market.[3]

There is only one good (Y) in the economy, which can be produced in both locations, and it is non-storable. There is perfect competition in the production of good Y in both regions. However, the production functions are not the same in both locations, as there are more specialized occupations in location "r".

Total output produced in location "r" at time t ($Y_{t;r}$) is given by:

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[3] See Becker and Murphy (1993) for a theoretical discussion of how specialization and the division of labor are endogenously determined and depend on coordination costs, and also on the amount and extent of knowledge. See also Glaeser (1998) for a discussion of the connection of the division of labor and the city size.

$$Y_{t;r} = B(A_{t;h}H_{t;r})^{1-\circledR}(zA_{t;h}H_{t;r} + A_{t;l}L_{t;r})^{\circledR}; \qquad (1)$$

where:

(i) $A_{t;h}$ is the knowledge at time t of the "h" type agent;

(ii) $H_{t;r}$ is the number of high type individuals working at time t in location "r";

(iii) $L_{t;r}$ is the number of low type individuals working at time t in location "r";

(iv) B, z, and ® are parameters, with B > 1, 0 < ® < 1, and 0 < z $\leq$ 1.

Note that there are only two inputs in the production process: both types of labor. There is no physical capital in the economy and this single technology has constant returns to scale. The high type agent is able to perform two different tasks, while the low type agent can only perform one task. Both types of agents are substitutes inputs in the common task. The more sophisticated task performed only by the high type agent is a complement input to the less sophisticated one. This feature of the production function captures the idea mentioned above that the more skilled individual can perform more sophisticated services.[4]

The parameter \z" indicates the degree of substitutability of both inputs in the performance of the less sophisticated task. When z = 1, both inputs are perfect substitutes. Note that "z" has to be greater than zero in order to guarantee a positive output in location "r" either at time t = 0 when all low type agents are in location "p" or at time t = 1 in the competitive equilibrium when migration is not allowed.[5]

The production function in location "p" is the following:

$$Y_{t;p} = B(A_{t;h}H_{t;p} + A_{t;l}L_{t;p}); \qquad (2)$$

where:

(i) $Y_{t;p}$ is the output in location "p" at time t;

(ii) $L_{t;p}$ is the number of low type individuals working at time t in location "p";

(iii) $H_{t;p}$ is the number of high type individuals working at time t in location "p".

As in location "r", the single technology represented in equation (2) has constant returns to scale, and employs only both types of labor as inputs. An important di¤erence is that the high type agent can not perform the more sophisticated task in the poor region.

With perfect competition in the labor and product markets, wages are equal to the marginal productivity of each input. Wages per unit of knowledge (or

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[4] This feature of the model is borrowed from Stokey (1996) in which "labor provides two distinct productive services, which we may think of as physical e¤ort ("brawn") and mental e¤ort ("brains")". See also Andrade (1998) for a similar framework.

[5] In the analysis of the competitive equilibrium with migration, we will show that the paramter \z" has to be lower than 1 to guarantee that a fraction of the low type individuals will migrate to location "r" at time t = 1. When we do not allow migration, we consider z = 1 to simplify the analysis, as this parameter does not play any important role.

level of human capital) of individual type i in location j ($w_{i;j}$) at time $t = 1$ are as follows[6]:

$$w_{h;r} = B\left(\frac{(1-\alpha)\,[zA_{1;h}H_{1;r} + A_{1;l}L_{1;r}]^{\alpha}}{A_{1;h}^{\alpha}} + \alpha A_{1;h}^{1-\alpha}\,(zA_{1;h}H_{1;r} + A_{1;l}L_{1;r})^{\alpha-1} \right);$$

$$w_{l;r} = B\alpha A_{1;h}^{1-\alpha}\,(zA_{1;h}H_{1;r} + A_{1;l}L_{1;r})^{\alpha-1};$$

$$w_{l;p} = w_{l;r} = B.$$

Note that $w_{h;r}$ is always greater than $w_{l;r}$. Moreover, in the equilibrium with migration, as there are no costs to move to another location, we must have the following equality: $w_{l;r} = w_{l;p}$. Therefore, $w_{h;r}$ is always greater than $w_{h;p}$, which means that the high type agents have no incentive to migrate to location "p". We can then set $H_{1;r} = 1$ (and $H_{1;p} = 0$).[7] Using this condition, we can rewrite the production functions (1) and (2) in both locations as:

$$Y_{t;r} = B(A_{t;h})^{1-\alpha}\,(zA_{t;h} + A_{t;l}L_{t;r})^{\alpha};$$

$$Y_{t;p} = BA_{t;l}L_{t;p}.$$

Country's total output at time t is equal to:

$$Y_t = Y_{t;r} + Y_{t;p}.$$

Recall that all "l" type individuals are in location "p" and all "h" type individuals are in location "r" at time $t = 0$. Their wages at $t = 0$ can be seen as their initial endowments and are equal to:

$$E_h = Bz^{\alpha}A_{0;h};$$

$$E_l = BA_{0;l},$$

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[6] Without loss of generality, it is assumed that the price of good Y is equal to 1 in both periods.

[7] Theoretically, it is possible that all low type agents decide to stay in location "p" ($w_{l;r} < w_{l;p}$) and it is better for all high type agents to move to location "p" ($w_{h;r} < w_{h;p}$). In this case, population would be zero in the rich region. As this case is of no interest, we impose some condition to avoid this possibility when we deal with the competitive equilibrium with migraton.

and it is assumed that $z^{\circledR}A_{0;h} > A_{0;l}$ to guarantee that the more skilled individual has a greater initial income.

We now turn to the individual's problem. At time $t = 0$, each individual works, receives wages, and decides how much to invest to increase his knowledge to use new technologies (or his ability to adopt new new technologies) in the next period and how much to consume in this period. At time $t = 1$, we consider two possibilities. When migration is allowed, each agent has to decide where to live and to work at time $t = 1$, receive wages and consume in this period. When migration is not allowed, all low type agents stay in location "p" and all high type agents stay in location "r" at time $t = 1$. They work, receive wages and consume in this period.

When migration is allowed, individual type i's problem ($i = h; l$) is the following:

$$\max f V_r^i ; V_p^i g;$$

where,

$$V_j^i = \max_{C_{1;i};C_{2;i};X_{A;i}} u(C_{0;i}) + \bar{\ }u(C_{1;i})$$

such that,

$$C_{0;i} + X_{A;i} \quad E_i \ ¡ \ ¿ i, \tag{3}$$

$$A_{1;i} = f(A_{0;i}; W; \frac{1}{4})(X_{A;i} + T_i)^\cdot, \tag{4}$$

$$C_{1;i} \quad A_{1;i}w_{i;j}; \tag{5}$$

and,

(i) V_j^i is the lifetime utility if individual type i lives in location j at time $t = 1$;

(ii) $u(C_{t;i})$ is the individual type i's utility function with $u^0 > 0$ and $u^{00} < 0$;

(iii) $C_{0;i}$ is the consumption at time $t = 0$ of individual type i;

(iv) $C_{1;i}$ is the consumption at time $t = 1$ of individual type i;

(v) $A_{1;i}$ is the knowledge at time $t = 1$ of individual type i;

(vi) $X_{A;i}$ is the amount invested by individual type i to increase his knowledge to adopt new technologies;

(vii) $w_{i;j}$ is the wage per unit of human capital of individual type i in location j;

(viii) T_i is the government transfer to individuals type i at time $t = 0$ to complement the investments to acquire new technology;

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(ix) $¿_i$ is a lump-sum tax imposed on the individual type i at time $t = 0$;

(x) W is the world technology level;

(xi) $¯$ and $¸$ are parameters, with $0 < ¯ < 1$ and $0 < ¸ < 1$;

(xii) ¼ is also a parameter that indicates the size of barriers to technology adoption in the country, with $0 \le ¼ \le 1$.

Equation (3) is individual type i's budget constraint at time $t = 0$. His total income net of taxes ($E_i − ¿_i$) can be used either to consume ($C_{0;i}$) or to invest to increase his knowledge to adopt new technologies ($X_{A;i}$). Equation (5) is type i agent's budget constraint at time $t = 1$: his total income ($A_{1;i} w_{i;j}$) is devoted for consumption ($C_{1;i}$).

Equation (4) indicates the law of motion for the individual type i's knowledge to adopt new technologies (A). Obviously, it depends on the amount invested to increase his knowledge, either employing his own resources ($X_{A;i}$) or resources received from the government (T_i).

The returns of these investments depend on the function f which is assumed to have the following properties: (i) $f_1 > 0$; (ii) $f_2 > 0$; (iii) $f_{11} < 0$; (iv) $f_{22} < 0$; and (v) $f_{12} > 0$. It means that these returns depend positively on both arguments of the function: the level of each region's knowledge (measured by the initial knowledge of its population, $A_{0;i}$) and the level of the world knowledge assumed to be available to all (W). However, the second derivative in both arguments are negative, indicating diminishing returns. Moreover, the region's knowledge and the world's knowledge are complements in the process of new technologies' acquisition.

The properties of function f imply that the technology for adoption is such that it takes fewer resources to move from a lower level to a greater level as greater is the level of world technology.[8] This feature allows the possibility that location "p" grows faster than location "r", that is, income per capita convergence across locations can occur.

There is another important feature in the technology adoption. Given the region's technology level and the world technology level, the investment technology is characterized by diminishing returns. It occurs because it is assumed that $0 < ¸ < 1$.

We also allow the possibility that a country can impose barriers to the adoption of new technologies from abroad. We incorporate this aspect in the model by assuming that the function f depends on the parameter ¼ in the following way: the lower the parameter ¼, the greater are the barriers to the adoption of new technologies from abroad. We examine the implications of changing these barriers in the next section.[9]

In this paper, transfers are made only at time $t = 0$ and they are ...nanced through lump-sum taxes imposed in the same period. We consider only two types of transfers. Either the high type individuals who live in the rich location at time $t = 0$ are taxed and the resources are transfered to the low type

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[8] This is the same feature used in Parente and Prescott (1994).

[9] In a di¤erent theoretical set-up, Parente and Prescott (1994) use the disparity in technology adoption barriers to explain the di¤erence in income disparity across countries.

individuals who live in the poor location at time $t = 0$ or vice-versa. Transfers has to be used for the sole purpose of complementing the private investment to acquire new technology. Therefore, we only consider transfers from one location to another to finance the investments to adopt new technology. It can be seen as a mechanism to increase the region's knowledge. Obviously, the government budget constraint has to be in equilibrium, that is, $T_h + T_l = \iota_l + \iota_h$.

One can interpret $X_{A;i}$ as investment in education that allows individuals from a region to acquire a higher level of education. As there are no capital markets to finance these investments due to moral hazard problems, the government could intervene to correct this imperfection through transfers. Alternatively, $X_{A;i}$ can be seen as investments in infra-structure with the government responsible for it. These are possible justifications for the government intervention. It may be efficient to transfer resources to the rich region or the poor one. As it is going to be clear in the next section, it depends on the comparison of the returns on the investment to acquire new technologies in both locations.

At last, it is worth recalling the initial conditions of the model. First, $L_{0;r} = H_{0;p} = 0$, that is, all low type agents are in "p" and all high type agents are in "r" at time $t = 0$. Second, the initial knowledge of both types of individuals are given ($A_{0;h} > A_{0;l} > 0$). Finally, the level of the world technology (W) is exogenously given.

3   Equilibrium Without Migration and Transfers

In this section, we analyze the competitive equilibrium without migration and transfers.[10] Our aim is to present under which condition there is income convergence across locations. Moreover, we want to analyse what are the effects on convergence of the imposition of barriers to the adoption of new technologies available in the world.

Without migration, the low and high type agents stay, respectively, in the poor and rich location in the second period. Formally, we have: $L_{1;r} = H_{1;p} = 0$, and $H_{1;r} = L_{1;p} = 1$.

Definition 1 Given the initial conditions $\{A_{0;l}, A_{0;h}, W\}$, a competitive equilibrium without migration and transfers is defined by $X_{A;i}^{WO}$, $C_{t;i}^{WO}$, $w_{h;r}^{WO}$, and $w_{l;p}^{WO}$ $\{t = 0; 1 \text{ and } i = h; l\}$ such that: (i) given $w_{h;r}^{WO}$, $X_{A;h}^{WO}$, and $C_{t;h}^{WO}$ solve the high type individual's problem; (ii) given $w_{l;p}^{WO}$, $X_{A;l}^{WO}$, and $C_{t;l}^{WO}$ solve the low type individual's problem; (iii) $w_{h;r}^{WO}$ solves the firm's problem in location "r"; (iv) $w_{l;p}^{WO}$ solves the firm's problem in location "p"; (v) markets clear, that is, $Y_{1;r}^{WO} = C_{1;h}^{WO}$ and $Y_{1;p}^{WO} = C_{1;l}^{WO}$.

There are four unknown variables and four equations that characterize the competitive equilibrium without migration and transfers. The four variables are: $X_{A;l}^{WO}$, $X_{A;h}^{WO}$, $w_{h;r}^{WO}$, and $w_{l;p}^{WO}$. The four equations are: the first-order

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[10] For simplicity, it is assumed that $z = 1$ in this section and in the next one.

condition to each type of agent's problem and the ...rst-order condition to the ...rm's problem in each location. We can combine these equations and obtain the following equation for each type of agent ($i = h, l$):

$$u^{0}(C_{0;i}^{WO}) = {}^{-}Bf(A_{0;i}; W; \frac{1}{4})_{,}{}^{i}X_{A;i}^{WO}{}^{\mathbb{C}_{,i}\,{}^{1}}u^{0}(C_{1;i}^{WO}); \qquad (6)$$

where $C_{0;i}^{WO} = E_{i\;i}\;X_{A;i}^{WO}$, and $C_{1;i}^{WO} = Bf(A_{0;i}; W; \frac{1}{4})^{i}X_{A;i}^{WO}{}^{\mathbb{C}}{}_{,}$.

The above equation indicates that each type of agent equates the marginal utility of consumption at $t = 0$ with the discounted product of the marginal productivity of investment to acquire new technology and the marginal utility of consumption at $t = 1$.

In order to prove that there is a solution to the competitive equilibrium without migration and transfer, we need to show that there is a solution to equation (6) for each type of agent.

Proposition 1 There exists an unique competitive equilibrium when migration is not allowed.

Assumption 1: $u(c) = \ln(c)$.

Using assumption 1, we can obtain a closed form solution to each type of agent's problem. The investment to adopt new technologies and the amount of human capital at $t = 1$ in the competitive equilibrium without migration and transfers are, respectively, given by:

$$X_{A;i}^{WO} = \frac{{}^{-}_{,}}{(1 + {}^{-}_{,})}E_{i},$$

and

$$A_{1;i}^{WO} = f(A_{0;i}; W; \frac{1}{4})\left[\frac{\mu \quad {}^{-}_{,}}{1 + {}^{-}_{,}}\right]^{\P_{,}}B^{,}A_{\hat{0};i}.$$

Let $(1 + g_{j}^{WO})$ be the income per capita growth rate in location j in the competitive equilibrium without migration and transfers.[11]

Proposition 2 With assumption 1, we have:

$$^{i}1 + g_{p}^{WO}{}^{\mathbb{C}} = \frac{A_{1;l}^{WO}}{A_{0;l}} = \frac{f(A_{0;l}; W; \frac{1}{4})^{3}\,X_{A;l}^{WO}{}^{,}}{A_{0;l}};$$

and

$$^{i}1 + g_{r}^{WO}{}^{\mathbb{C}} = \frac{A_{1;h}^{WO}}{A_{0;h}} = \frac{f(A_{0;h}; W; \frac{1}{4})^{3}\,X_{A;h}^{WO}{}^{,}}{A_{0;h}}.$$

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[11]As the population is constant and equal to 1 and there is no migration, the income is equal to the income per capita.

Without migration, the growth rate in each location is equal to the rate that its residents' knowledge increases. This, in turn, depends on the amount that they invest to acquire new technology.[12]

In order to analyze the factors that a¤ect the convergence process, we specify the following functional form for the function f:

Assumption 2: $f(A_{0;i}; W; ¼) = (W)^{»¼} A_{0;i}^{1i »¼}$, where $0 < » < 1$ and $0 ¼ 1$:

As greater the parameter ¼ is, the lower are the barriers to the adoption of the world technology available to both locations. When ¼ is equal to 0, the interpretation is that the useable knowledge available in the world can not be exploited in any location. In this case, one can think that there are technology adoption barriers which preclude any location to bene…t from the use of a more developed technology available in the world. When ¼ is equal to 1, the interpretation is that there are no barriers to such adoption.

The following proposition indicates which conditions have to prevail, when there is no migration, for the model to generate a higher growth rate in location \p" in comparison with location "r".

Proposition 3 With assumptions 1, and 2, there is income per capita convergence (divergence) across locations if »¼ > ¸ (»¼ < ¸).

There are two possible cases. In the …rst case, there is convergence when »¼ > ¸. When this inequality holds, the returns on the investment to adopt new technologies are greater in the poorest location (\p") and, therefore, it grows faster than the richest location (\p"). Note that location "p" has the advantage of not having yet exploited a greater stock of useable knowledge available in the world. The parameter »¼ indicates the importance of the adoption of the world technology to the process of increasing the local knowledge. When the term »¼ is big enough, that is, it is greater than ¸, income convergence across locations occurs.

In the second case, when »¼ < ¸, the richest location grows faster than the poorest location. The disadvantage that location "p" has of possessing a lower stock of initial knowledge is not compensated by the advantage mentioned above of having a greater stock of world knowledge unexploited. As a result, there is income divergence across locations.

Barriers to the adoption of new technology available in the world can constrain the convergence process as it harms in greater length the poorest location. It blocks exactly the advantage that the poorest location has in the process of adopting new technologies. As the poorest location can bene…t the most from the elimination of the barriers to the adoption of new technologies, the implication is that the reduction of these barriers can speed up the convergence process, as showed in the following proposition.

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[12]We assume that the initial conditions are such that guarantees that $\min[1 + g_p^{WO}; 1 + g_r^{WO}] > 1$, that is, both regions have a positive growth rate. Formally, this implies that $f(A_{0;i}; W; ¼) \frac{-}{1+} \degree B \cdot A_{0;i}^{i 1} > 1$, where the left-hand side is the growth rate in the slowest-growth location.

Proposition 4 Using assumptions 1, and 2, we have: $\dfrac{d\left(\frac{1+g_p^{WO}}{1+g_r^{WO}}\right)}{d\frac14} > 0$.

Recall that as closer to 1 the parameter ¼ is, the lower are the barriers to the adoption of technologies from abroad. When ¼ increases, the di¤erence in the growth rate in the poorest location with respect to the one in the richest location increases. Note that the barriers can even revert the process of income convergence across locations. If it is su¢cient big, the growth rate in the poorest location, $1 + g_p^{WO}$, can even become lower than the correspondent one in the richest location,$1 + g_r^{WO}$:[13]

4   Equilibrium with Transfers and Without Migration

In this section, we maintain the restriction on labor mobility by not allowing migration and analyze the e¤ects of transfers on economic growth, convergence, and welfare.[14]

We begin our analysis by focusing on the competitive equilibrium and check if there is any trade-o¤ between equity across locations and the country's economic growth when a transfer policy is introduced. In other words, can transfers to the poor location be detrimental to growth?

We then turn to the planner's problem and our aim is to answer the following questions. Can transfers be welfare improving? If so, should they be directed to the poor location or the rich location? Should the optimum amount of transfers be dependent on the level of economic development, that is, how rich the country is? Finally, should the optimum amount of transfers be a function of the level of income disparity across locations?

We consider two types of transfers. Either taxes are imposed on type "h" agents and revenues are transferred to type "l" agents to …nance investments to acquire new technologies or vice-versa. Formally, the two alternatives are: $T_l = \dot\iota_h > 0$ and $T_h = \dot\iota_l = 0$, or $T_h = \dot\iota_l > 0$ and $T_l = \dot\iota_h = 0$.

4.1   Competitive Equilibrium

De…nition 2 Given the initial conditions $\{A_{0;l}, A_{0;h}, W\}$, a competitive equilibrium with transfers and without migration is de…ned by $X_{A;i}^T$, $C_{t;i}^T$, $w_{h;r}^T$, $w_{l;p}^T$, T_i, and $0 \quad \dot\iota_i \quad E_i$ $\{t = 0; 1$ and $i = h; l\}$ such that: (i) given $\dot\iota_h$, T_h, and $w_{h;r}^T$, $X_{A;h}^T$, and $C_{t;h}^T$ solve the high type individual's problem; (ii) given $\dot\iota_l$, T_h, and $w_{l;p}^T$, $X_{A;l}^T$, and $C_{t;l}^T$ solve the low type individual's problem; (iii) $w_{h;r}^T$ solves

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[13]For the rest of this paper, we assume for simpli…cation that ¼ = 1. Therefore, I will omit the paramater ¼ whenever the function f is employed from now on.

[14]As there is no migration, we continue to have the following: $L_{1;r} = H_{1;p} = 0$, and $H_{1;r} = L_{1;p} = 1$.

the ...rm's problem in location "r"; (iv) $w_{l;p}^T$ solves the ...rm's problem in location "p"; (v) markets clear, that is, $Y_{1;r}^T = C_{1;h}^T$ and $Y_{1;p}^T = C_{1;l}^T$; and (vi) the government's budget constraint is in equilibrium, that is, $T_h + T_l = \dot{c}_l + \dot{c}_h$.

For a given transfer policy, there are four unknown variables and four equations that characterize the competitive equilibrium without migration, as in the system that characterizes the competitive equilibrium without transfers covered in the previous section. The four variables are: $X_{A;l}^T$, $X_{A;h}^T$, $w_{h;r}^T$, and $w_{l;p}^T$. The four equations are: the ...rst-order condition to each type of agent's problem and the ...rst-order condition to the ...rm's problem in each location. We can combine these equations and obtain the following equation for each type of agent $(i = h; l)$:

$$u'(C_{0;i}^T) = {}^-Bf(A_{0;i}; W)_{_,} {}^i X_{A;i}^T + T_i {}^{\phi_{_,i} 1} u'(C_{1;i}^T), \qquad (7)$$

where $C_{0;i}^T = E_i \; i \; X_{A;i}^T \; i \; \dot{c}_i$, and $C_{1;i}^T = Bf(A_{0;i}; W)^i X_{A;i}^T + T_i{}^\phi$.

The above equation is similar to the one in the competitive equilibrium without transfers and migration. As before, each type of agent equates the marginal utility of consumption at $t = 0$ with the discounted product of the marginal productivity of investment to acquire new technology and the marginal utility of consumption at $t = 1$.

In order to prove that there is a solution to the competitive equilibrium with transfer, we need to show that there is a solution to equation (7) for each type of agent.

Proposition 5 There exists an unique competitive equilibrium when transfers are introduced and migration is not allowed.

Using assumption 1, we can obtain a closed form solution to each type of agent's problem. When $T = T_l = \dot{c}_h > 0$ and $T_h = \dot{c}_l = 0$, the amount invested to adopt new technologies and the amount of human capital at $t = 1$ are:

$$X_{A;h}^T = \frac{^-_{_,}}{(1 + {}^-_{_,})}(E_h \; i \; T) = X_{A;h}^{WO} \; i \; \frac{^-_{_,}}{(1 + {}^-_{_,})}T;$$

$$X_{A;l}^T + T = \frac{^-_{_,}}{(1 + {}^-_{_,})}(E_l + T) = X_{A;l}^{WO} + \frac{^-_{_,}}{(1 + {}^-_{_,})}T;$$

$$A_{1;h}^T = f(A_{0;h}; W)\left[X_{A;h}^{WO} \; i \; \frac{^-_{_,}}{(1 + {}^-_{_,})}T\right]^{\P_{_,}};$$

$$A_{1;l}^T = f(A_{0;l}; W)\left[X_{A;l}^{WO} + \frac{^-_{_,}}{(1 + {}^-_{_,})}T\right]^{\P_{_,}}:$$

These solutions are analogous when the transfer policy is characterized by $T_h = \imath_l > 0$ and $T_l = \imath_h = 0$:

From the above equations, one can see that $X_{A;l}^T + T + X_{A;h}^T = X_{A;l}^{WO} + X_{A;h}^{WO}$. It means that the total amount invested to acquire new technology in the economy does not change when transfers are introduced. There is a change in the composition in favor of the location that receives the transfers.

Proposition 6 With assumption 1, we obtain the following results: (i) for any transfer policy with $T_l > 0$ and $T_h = 0$, $\frac{y_{1;r}^{WO}}{y_{1;p}^{WO}} > \frac{y_{1;r}^T}{y_{1;p}^T}$; (ii) for any transfer policy with $T_h > 0$ and $T_l = 0$, $\frac{y_{1;r}^{WO}}{y_{1;p}^{WO}} < \frac{y_{1;r}^T}{y_{1;p}^T}$.

The above result shows that the income disparity across locations can be reduced if the poor location receives transfers from the rich location. Moreover, if the ‡ow of transfers goes in the another direction, bene...ting location "r", the income disparity increases. This is an obvious result but it is of interest because it can be used to analyze if there is any trade-o¤ between equity and growth when transfers are implemented. In other words, we want to check if transfers to the poor location to reduce the income disparity can hurt the country's economic performance, measured by its growth rate. In order to pursue this objective, we now turn to the e¤ects of transfers on the economic growth.

Proposition 7 With assumptions 1 and 2, there is a positive amount of transfers that maximizes the country's income per capita growth rate. If » > ‚, then $T_l = \imath_h > 0$ and $T_h = \imath_l = 0$. If » < ‚, then $T_h = \imath_l > 0$ and $T_l = \imath_h = 0$:

This last result indicates that the maximum growth rate is obtained when the marginal productivity to invest one additional unit to adopt new technologies is equalized across locations. Moreover, transfers can bene...t growth up to a certain point. Transfers beyond a certain level reduces the growth rate.

When there is income convergence without transfers, that is, when » > ‚, the marginal productivity of one additional unit invested to adopt new technology is greater in the poor location[15]. To reach the necessary equalization to maximize growth, transfers should be made toward location "p".

In contrast, when there is no income convergence in the competitive equilibrium without transfers (when » < ‚), the marginal productivity of one additional unit invested to adopt new technology is greater in the rich location[16]. The transfer policy that maximizes economic growth is the one in which resources should be transfered from the poor location to the rich location. In this case, it is clear that there is a trade-o¤ between growth and equity. Transfers to the rich location implies a greater growth rate and, at the same time, leads to greater income disparity, as the result in the previous proposition shows.

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[15] See proposition 3.
[16] See proposition 3.

4.2 Planner's Problem

We now focus our analysis on the e¤ect of transfers on the economic welfare. We turn to the planner's problem ($i = fl; hg$ and $j = fl; jg$):

$$\max u(C_{0;i}) + {}^- u(C_{1;i})$$

such that,

$$C_{0;i} + X_{A;i} = E_i; \tag{8}$$

$$C_{0;j} + X_{A;j} = E_j \; {}_i \; T; \tag{9}$$

$$C_{1;i} + C_{1;j} = B \left[f(A_{0;j}; W)(X_{A;j})^{\,*} + f(A_{0;i}; W)(X_{A;i} + T)^{\,*} \right]; \tag{10}$$

$$u(C_{0;j}) + {}^- u(C_{1;j}) \; \downarrow \; \overline{U} = u^{\,i} C_{0;j}^{WO} {}^{¢} + {}^- u^{\,i} C_{1;j}^{WO} {}^{¢}: \tag{11}$$

We want to examine if the solution to the planner's problem can increase the \i" type individual's utility (the recipient of the transfers) without reducing the utility level that the \j" type individual obtain in the competitive equilibrium without transfers, which is equal to $\overline{U}$. Individual \j" is taxed to ...nance the transfers to individual \i". Recall that we consider two types of transfers: either $T_l = \dot{\iota}_h > 0$ and $T_h = \dot{\iota}_l = 0$ or $T_h = \dot{\iota}_l > 0$ and $T_l = \dot{\iota}_h = 0$. Therefore, when \i = h" than \j = l" and vice-versa.

In the set-up speci...ed above, we consider that ouput at time $t = 1$ can be distributed to both types of agents independently where it was produced. In other words, we allow side payments to compensate the agent that pays the taxes at time $t = 0$. The objective is to analyse if the introduction of transfers can generate enough additional output at time $t = 1$ to compensate the individuals that ...nance the transfers. In this case, we can guarantee that transfers are e¢cient as the bene...ciary's utility increases by a greater amount than the reduction in the other agent's utility when compensation is not possible.

Using assumption 1, we can rewrite the planner's problem in the following way:

$$\max \ln[E_i \; {}_i \; X_{A;i}] + {}^- \ln[Bf(A_{0;j}; W)(X_{A;j})^{\,*} + Bf(A_{0;i}; W)(X_{A;i} + T_i)^{\,*} \; {}_i$$

$${}_i \; C_{1;j}] \; {}_i \; {}^1 f\overline{U} \; {}_i \; \ln[E_j \; {}_i \; T \; {}_i \; X_{A;j}] \; {}_i \; {}^- \ln[C_{1;j}]g;$$

where \¹" is the lagrangean multiplier.

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The ...rst-order conditions of this problem are:

$$\frac{1}{(E_i - X_{A;i})} = \frac{-\beta f(A_{0;i};W)_\gamma (X_{A;i} + T_i)^{\gamma-1}}{\beta f(A_{0;j};W)(X_{A;j})^\gamma + \beta f(A_{0;i};W)(X_{A;i} + T_i)^\gamma - C_{1;j}} \quad \tag{12}$$

$$\frac{-}{\beta f(A_{0;j};W)(X_{A;j})^\gamma + \beta f(A_{0;i};W)(X_{A;i} + T_i)^\gamma - C_{1;j}} = \frac{1}{C_{1;j}}; \quad \tag{13}$$

$$\frac{-\beta f(A_{0;i};W)_\gamma (X_{A;i} + T_i)^{\gamma-1}}{\beta f(A_{0;j};W)(X_{A;j})^\gamma + \beta f(A_{0;i};W)(X_{A;i} + T_i)^\gamma - C_{1;j}} = \frac{1}{(E_j - T - X_{A;j})}; \quad \tag{14}$$

$$\frac{-\beta f(A_{0;j};W)_\gamma (X_{A;j})^{\gamma-1}}{\beta f(A_{0;j};W)(X_{A;j})^\gamma + \beta f(A_{0;i};W)(X_{A;i} + T_i)^\gamma - C_{1;j}} = \frac{1}{(E_j - T - X_{A;j})}; \quad \tag{15}$$

$$\overline{U} = \ln[E_j - T - X_{A;j}] - \ln[C_{1;j}]: \quad \tag{16}$$

Using (15) in (13), we get rid of λ and obtain the following expression for $C_{1;j}$:

$$C_{1;j} = -\beta f(A_{0;j};W)_\gamma (X_{A;j})^{\gamma-1}(E_j - T - X_{A;j}): \quad \tag{17}$$

Using (17) in (12) to get rid of $C_{1;j}$, we obtain the following:

$$\beta f(A_{0;j};W)(X_{A;j})^\gamma + \beta f(A_{0;i};W)(X_{A;i} + T_i)^\gamma =$$

$$= -\beta f(A_{0;j};W)_\gamma (X_{A;j})^{\gamma-1}(E_j - T - X_{A;j}) + -\beta f(A_{0;i};W)_\gamma (X_{A;i} + T_i)^{\gamma-1}(E_i - X_{A;i}): \quad \tag{18}$$

Using equations (14) and (15), we obtain the condition that the marginal producvitity to invest in the adoption of new technology has to be the same in both locations:

$$\frac{f(A_{0;i};W)}{f(A_{0;j};W)} = \left[\frac{X_{A;i} + T_i}{X_{A;j}}\right]^{\gamma-1}: \quad \tag{19}$$

The solution to the system of three equations ((16), (18), and (19)) and three unknowns ($X_{A;i}, X_{A;j}$, and T_i) solves the planner's problem.

The next proposition shows that transfers can be welfare improving. The explanation is that tranfers can equate the marginal productivity to invest in the adoption of new technology in both locations. Therefore, a greater output can be produced at time $t = 1$ by increasing the investment in location with greater productivity and reducing in the another one, that is, wihtout the need to have a greater amount invested at time $t = 0$. Therefore, enough additional resources are generated to compensate the individual that bears the cost of ...nancing the transfer policy.

Proposition 8 With assumptions 1 and 2, we obtain the following results: (i) if $\gg > \gamma$, then there is a $T_l^* = I_h^* > 0$, with $T_h^* = I_l^* = 0$, that is welfare improving; and (ii) if $\gg < \gamma$, then there is a $T_h^* = I_l^* > 0$, with $T_l^* = I_h^* = 0$, that is welfare improving.

Using the planner's problem, we can show that the optimal amount of transfers does not depend on the level of the country's economic development. It means that it does not change if we make the experiment of increasing the initial technology level in both locations (multiplying by a constant "m") without altering the degree of income disparity in both locations.

Proposition 9 With assumptions 1 and 2, the optimal amount of transfer does not change if we multiply by "m" ($8m$, $m > 0$) the initial technology level in both locations ($A_{0;l}$ and $A_{0;h}$).

The implication of this result is that the optimal amount of transfers as a fraction of the country's total output declines with the level of economic development. Controlling for the variable that measures the income disparity across locations, one should observe countries making less transfers as a fraction of GDP as they become richer.

Finally, we analyse how the optimal amount of transfers should vary with the degree of income disparity across regions, while maitaining constant the level of the country's economic development.

Proposition 10 With assumptions 1 and 2, the optimal amount of transfer increases with "v", with $v = \frac{A_{0;h}}{A_{0;l}}$ ($v > 1$), while maintaining $\overline{A} = \frac{A_{0;h} + A_{0;l}}{2}$ constant.

The above result indicates that two countries with the same income per capita and di¤erent degrees of income inequality across regions should pursue di¤erent transfer policy. The one with greater income disparity should follow a more agressive transfer policy. The reason is that, ceteris paribus, a greater disparity indicates a a greater di¤erence in the marginal productivity to invest in the adoption of new technology across locations. Hence, a greater amount of transfer is necessary to generate the equalization that maximizes the economic welfare.

5 Equilibrium With Migration and Without Transfers

In this section, we analyse the competitive equilibrium without transfers and with migration. This equilibrium will serve as a benchmark to the analysis developed in the next section where we investigate the e¤ects of transfer in a framework in which migration is allowed.

De…nition 3 Given the initial conditions $\{A_{0;l},\ A_{0;h},\ W,\ L_{0;r} = H_{0;p} = 0\}$, a competitive equilibrium with migration is de…ned by $X_{A;i}^M$, $C_{t;i}^M$, $L_{t;r}^M$, $L_{t;p}^M$, $w_{h;r}^M$, $w_{l;r}^M$ and $w_{l;p}^M$ $\{t = 0;1$ and $i = h;l\}$ such that: (i) given $w_{h;r}^M$, $X_{A;h}^M$, and $C_{t;h}^M$ solve the high type individual's problem; (ii) given $w_{l;p}^M$ and $w_{l;r}^M$, $X_{A;l}^M$, and $C_{t;l}^M$ solve the low type individual's problem; (iii) $w_{h;r}^M$ and $w_{l;r}^M$ solve the …rm's problem in location "r"; (iv) $w_{l;p}^M$ solves the …rm's problem in location "p"; (v) markets clear, that is, $L_{1;r}^M + L_{1;p}^M = 1$.

We restrict our analysis to the competitive equilibrium with migration in which there is a positive fraction of low type individuals in both locations. It means that $0 < L_{1;r}^M < 1$. As there is no cost to migration, we have that $w_l^M = w_{l;r}^M = w_{l;p}^M$ in this speci…c equilibrium.

Therefore, there are …ve unknown variables and …ve equations that characterize the competitive equilibrium with migration. The …ve variables are: $X_{A;l}^M$, $X_{A;h}^M$, $w_{h;r}^M$, $L_{1;r}^M$, and w_l^M. The …ve equations are:

$$u^0(C_{0;l}^M) = -{}^{\text{£}} w_l^{M}{}^{\text{¤}} [f(A_{0;l}; W) \, _{,} \, {}^{i} X_{A;l}^M{}^{\text{¢}}{}_{,i\ 1}] u^0(C_{1;l}^M); \qquad (20)$$

$$u^0(C_{0;h}^M) = -{}^{\text{£}} w_{h;r}^{M}{}^{\text{¤}}{}^{\text{h}} f(A_{0;h}; W) \, _{,} \, {}^{i} X_{A;h}^M{}^{\text{¢}}{}_{,i\ 1}{}^{i} u^0(C_{1;h}^M); \qquad (21)$$

$$w_l^M = B = w_{l;p}^M; \qquad (22)$$

$$w_l^M = B^{\circledR} \, {}^{i} A_{1;h}^M{}^{\text{¢}}{}_{1i} \, {}^{i} z A_{1;h}^M + A_{1;l}^M L_{1;r}^M{}^{\text{¢}}{}_{\circledR i\ 1} = w_{l;r}^M \qquad (23)$$

$$w_{h;r} = B \left\{ \frac{(1\ _{\text{i}}\ {}^{\circledR})\ {}^{\text{h}} z A_{1;h}^M + A_{1;l}^M L_{1;r}^M{}^{\text{i}}{}_{\circledR}}{A_{1;h}^M}{}^{\circledR} + {}^{\circledR} \, {}^{i} A_{1;h}^M{}^{\text{¢}}{}_{1i} \, {}^{i} z A_{1;h}^M + A_{1;l}^M L_{1;r}^M{}^{\text{¢}}{}_{\circledR i\ 1} \right\};$$

$$ \qquad (24)$$

where: $A_{1;h}^M = f(A_{0;h}; W) \left[X_{A;h}^M \right]^{1-i}$, $A_{1;l}^M = f(A_{0;l}; W) \left[X_{A;l}^M \right]^{1-i}$, $C_{0;i}^M = E_i - X_{A;i}^M$, $C_{1;l}^M = w_l^M f(A_{0;l}; W) \left[X_{A;l}^M \right]^{\delta}$, and $C_{1;h}^M = w_{h;r}^M f(A_{0;h}; W) \left[X_{A;h}^M \right]^{\delta}$.

Equations (20) and (21) are the ...rst-order condition, respectively, to the low and high type individual's problems. As in the other equilibria examined in the previous sections, each type of agent equates the marginal utility of consumption at t = 0 with the discounted product of the marginal productivity of investment to acquire new technology and the marginal utility of consumption at t = 1. Equation (22) is the ...rst-order condition to the ...rm's problem in location "p" and equations (23) and (24) are the ...rst-order conditions to the ...rm's problem in location "r".

In order to prove that there is an unique solution to the competitive equilibrium with migration, we need to show that there is an unique solution to the system formed by equations (20) through (24).

Assumption 3: $z < \left[\beta^{\frac{1}{1-\delta}} \alpha \right]$ and $\left(\frac{A_{0;l}}{A_{0;h}} \right)^{\frac{1+\delta i}{1-\delta}} > \left[\beta^{\frac{1}{1-\delta}} \alpha \right] - z$.

Proposition 11 With assumptions 1, 2, and 3, there exists an unique competitive equilibrium with migration and with $0 < L_{1;r}^M < 1$.

In order to exist a competitive equilibrium in which there are low type individuals living in both locations at t = 1 (that is, $0 < L_{1;r}^M < 1$), the above proposition shows that two conditions have to be satis...ed. First, if all low type agents are in location "p", then their wage has to be greater in location "r". Therefore, it is in the best interest of these individuals to migrate to the richest location. Hence, we can guarantee that not all low type agents stay in location "p" at t = 1, that is, $L_{1;r}^M > 0$. Formally, we need to have the following: $w_{l;p}(L_{1;r} = 0) < w_{l;r}(L_{1;r} = 0)$. Second, if all low type agents are in location "r", then their wages are greater in location "p". This condition prevents that all low type individuals move to location "r". As more individuals move to the richest location, their marginal productivity reduces there and it becomes less attractive to migrate. To guarantee that this migration movement halts before we reach the point in which $L_{1;r}^M = 1$, we need the following formal condition: $w_{l;r}(L_{1;r} = 1) < w_{l;p}(L_{1;r} = 1)$.

Assumption 3 guarantees that the competitive equilibrium does not have a corner solution, neither one with $L_{1;r}^M = 0$ (the case of no migration) nor another with $L_{1;r}^M = 1$ (the case of no population in the poorest region).

Using assumptions 1 and 3, we can get a closed form solution to the competitive equilibrium with migration. The amount invested to adopt new technologies and the amount of knowledge at t = 1 are, respectively, given by (i = h; l) (they are equal to the ones in the competitive equilibrium without migration):

$$X_{A;i}^M = \frac{\delta}{(1 + \delta)} E_i = X_{A;i}^{WO};$$

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and

$$A_{1;i}^M = f(A_{0;i}; W; \frac{1}{4}) \left(\frac{\text{-}}{1+\text{-}} \right) B \cdot A_{\hat{0};i} = A_{1;i}^{WO}:$$

The fraction of low type agents living and working in location "r" at t = 1 is equal to:

$$L_{1;r}^M = \left(\frac{A_{0;h}}{A_{0;l}} \right)^{1+\text{-};i\,»} \, ^3 \, \overset{\sim}{\underset{}{}} \, \textcircled{}^{\frac{1}{T_i\textcircled{}}} \, i \, z \; :$$

Using assumption 3 and the above equation, one can easily see that $0 < L_{1;r}^M < 1$. As $\textcircled{}^{\frac{1}{T_i\textcircled{}}} i \, z \, > 0$, then $L_{1;r}^M > 0$. As $\left(\frac{A_{0;l}}{A_{0;h}} \right)^{1+\text{-};i\,»} > \textcircled{}^{\frac{1}{T_i\textcircled{}}} i \, z$, then $L_{1;r}^M < 1$.

The following proposition shows that migration can reduce the income disparity across locations. The latter measured by the di¤erence in income per capita is lower in the competitive equilibrium with migration (when $0 < L_{1;r}^M < 1$) in comparison with the competitive equilibrium without migration.

Proposition 12 With assumptions 1 through 3, $\frac{y_{1;r}^{WO}}{y_{1;p}^{WO}}$ may be lower than $\frac{y_{1;r}^M}{y_{1;p}^M}$.

Finally, let $\left(1 + g_j^M\right)$ be location "j"'s income per capita growth rate with migration. We obtain the following:

Proposition 13 With assumptions 1 through 3, $\left(1 + g_p^M\right)$ may be greater than $\left(1 + g_r^M\right)$:

The above proposition shows the necessary conditions for the existence of income per capita convergence across locations when migration is allowed.

6 Equilibrium With Migration and Transfers

In this section, we re-do the analysis implemented in section 4, and consider the e¤ects of transfers on convergence, growth, and economic welfare. The di¤erence in this section is that the restriction on labor mobility is eliminated.

We begin our analysis by introducing transfers in the competitive equilibrium with migration ($0 < L_{1;r}^M < 1$), and verifying if transfers to the poor location can be detrimental to growth. We then turn to the planner's problem and investigate if transfers can be welfare improving and if the optimal amount of transfers should depend on the income disparity across location and on the country's level of economic development.[17]

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[17] Either taxes are imposed on type "h" agents and revenues are transferred to type "l" agents to …nance investments to acquire new technologies or vice-versa. Formally, the two alternatives are: $T_l = \dot{\iota}_h > 0$ and $T_h = \dot{\iota}_l = 0$, or $T_h = \dot{\iota}_l > 0$ and $T_l = \dot{\iota}_h = 0$.

6.1 Competitive Equilibrium

Definition 4 Given the initial conditions $\{A_{0;l}, A_{0;h}, W\}$, a competitive equilibrium with transfers and migration is defined by $X_{A;i}^{MT}$, $C_{t;i}^{MT}$, $w_{h;r}^{MT}$, $w_{l;p}^{MT}$, $L_{1;r}^{MT}$, $L_{1;p}^{MT}$, T_i, and $0 \leq \iota_i \leq E_i$ $\{t = 0; 1$ and $i = h; l\}$ such that: (i) given ι_h, T_h, and $w_{h;r}^{MT}$, $X_{A;h}^{MT}$, and $C_{t;h}^{MT}$ solve the high type individual's problem; (ii) given ι_l, T_h, and $w_{l;p}^{MT}$, $X_{A;l}^{MT}$, and $C_{t;l}^{MT}$ solve the low type individual's problem; (iii) $w_{h;r}^{MT}$ and $w_{l;r}^{MT}$ solves the firm's problem in location "r"; (iv) $w_{l;p}^{MT}$ solves the firm's problem in location "p"; (v) markets clear, that is, $Y_{1;r}^{MT} + Y_{1;p}^{MT} = C_{1;h}^{MT} + C_{1;l}^{MT}$; and (vi) the government's budget constraint is in equilibrium, that is, $T_h + T_l = \iota_l + \iota_h$.

When $T_l = \iota_h > 0$ and $T_h = \iota_l = 0$, and using assumptions 1 through 3, the closed formed solution to this competitive equilibrium with transfers and migration is given by:[18]

$$X_{A;h}^{MT} = X_{A;h}^{M} - \frac{\text{-}}{(1 + \text{-})} T,$$

$$A_{1;h}^{MT} = f(A_{0;h}; W)\,[X_{A;h}^{M} - \frac{\text{-}}{(1 + \text{-})} T]^{\cdot},$$

$$X_{A;l}^{MT} + T = X_{A;l}^{M} + \frac{\text{-}}{(1 + \text{-})} T,$$

$$A_{1;l}^{MT} = f(A_{0;l}; W)\,[X_{A;l}^{M} + \frac{\text{-}}{(1 + \text{-})} T]^{\cdot};$$

$$L_{1;r}^{MT} = \left[\frac{1}{\text{-}} \right]^{\frac{1}{\text{-}}} \cdot z \left[\frac{A_{1;h}^{MT}}{A_{1;l}^{MT}} \right].$$

Note that the total amount invested in adopting new technologies in the economy does not change with the introduction of transfers, as $X_{A;l}^{MT} + T + X_{A;h}^{MT} = X_{A;h}^{M} + X_{A;l}^{M}$. Similarly to the case when tranfers were introduced in the competitive equilibrium without migration, there is only a change in the composition of the investment in favor of the location that receives the tranfers.

Proposition 14 With assumption 1, $\frac{dL_{1;r}^{MT}}{dT_l} < 0$, and $\frac{dL_{1;r}^{MT}}{dT_h} > 0$:

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[18] The solution is analogous for the another type of transfers, that is, when $T_h = \iota_l > 0$ and $T_l = \iota_h = 0$.

The above result indicates that transfers to location "p" reduce the fraction of low type individuals who move to location \r". In other words, transfers to the poorest location inhibits migration to the richest one. It occurs because the marginal productivity of the low type individuals in the richest location gets reduced when there is a change in the composition of the investment to acquire new technology in favor of the poorest region. The ‡ow of low type individuals to location "p" increases the marginal productivity until the wages are equalized again across locations. The opposite result is obtained when the transfers are directed to the richest location. The explanation is analogous.

Recall that when transfers were introduced in the competitive equilibrium without migration, we reached the result that transfers to the poorest location lead to a reduction in the income disparity across locations. We also concluded that if the ‡ow of transfers goes in the another direction, bene…ting location "r", the income disparity increases. The same result is obtained when migration is allowed as the following proposition shows.

Proposition 15 With assumption 1, we obtain the following results: (i) for any transfer policy with $T_l > 0$ and $T_h = 0$, $\frac{y_{1;r}^M}{y_{1;p}^M} > \frac{y_{1;r}^{MT}}{y_{1;p}^{MT}}$; and (ii) for any transfer policy with $T_h > 0$ and $T_l = 0$, $\frac{y_{1;r}^M}{y_{1;p}^M} < \frac{y_{1;r}^{MT}}{y_{1;p}^{MT}}$.

Let $d = \left[\rho^{\frac{1}{1-\rho}} - z \right]$. Hence:

Proposition 16 With assumptions 1 and 2, there is a positive amount of transfers (T_j^π) that maximizes the country's income per capita growth rate. If $\left(\frac{A_{0;h}}{A_{0;l}} \right)^{1-\rho} < \frac{1}{[(z+d)^\rho - d]}$, then $T_l^\pi = \iota_h > 0$ and $T_h^\pi = \iota_l = 0$. If $\left(\frac{A_{0;h}}{A_{0;l}} \right)^{1-\rho} > \frac{1}{[(z+d)^\rho - d]}$, then $T_h^\pi = \iota_l > 0$ and $T_l^\pi = \iota_h = 0$:

The above proposition shows that the maximum growth rate is reached when the marginal productivity of one additional investment to acquire new technology is equalized across locations. When $\left(\frac{A_{0;h}}{A_{0;l}} \right)^{1-\rho} < \frac{1}{[(z+d)^\rho - d]}$, the marginal productivity is greater in the poorest location. Transfer to location "p" reduces the marginal productivity there as more investment is implemented and increases in location "r". In order to maximize the country's growth rate, transfer should be increased until the marginal productivity equalization occurs. When $\left(\frac{A_{0;h}}{A_{0;l}} \right)^{1-\rho} > \frac{1}{[(z+d)^\rho - d]}$, the marginal productivity is greater in the richest location. Therefore, the opposite result holds, that is, transfers should be directed to location "r".

Note that this is precisely the same qualitative result obtained in proposition 7, when transfer was introduced in the competitive equilibrium without migration. The introduction of migration did not alter the conclusion.

6.2 Planner's Problem

We now analyze the effects of transfers on the economic welfare when migration is allowed. The planner's problem is the following[19]:

$$\max u\,(C_{0;i}) + \bar{u}\,(C_{1;i})$$

such that,

$$C_{0;i} + X_{A;i} = E_i;\tag{25}$$

$$C_{0;j} + X_{A;j} = E_j - T;\tag{26}$$

$$C_{1;i} + C_{1;j} = Y_{1;r} + Y_{1;p};\tag{27}$$

$$u\,(C_{0;j}) + \bar{u}\,(C_{1;j}) \geq \overline{U} = u^{\,i}\!\left(C_{0;j}^{M}\right)^{\!\!\!\!\!\Cent} + \bar{u}^{\,i}\!\left(C_{1;j}^{M}\right)^{\!\!\!\!\!\Cent};\tag{28}$$

where,

$$Y_{1;r} + Y_{1;p} = B \overset{3}{} f\,(A_{0;h};W)\,(X_{A;h}(T))^{\cdot}\,^{\displaystyle ^{1-\circledR}}$$

$$\overset{3}{} z f\,(A_{0;h};W)\,(X_{A;h}(T))^{\cdot} + f\,(A_{0;l};W)\,(X_{A;l}(T))^{\cdot}\,L_{1;r}\,^{\displaystyle ^{\circledR}} +$$

$$+ f\,(A_{0;l};W)\,(X_{A;l}(T))^{\cdot}\,(1 - L_{1;r});\tag{29}$$

and: (i) $X_{A;h}(T) = X_{A;h}$ and $X_{A;l}(T) = X_{A;l} + T$ when \i" is individual type "l" and "j" is individual type "h", and (ii) $X_{A;h}(T) = X_{A;h} + T$ and $X_{A;l}(T) = X_{A;l}$ when \i" is individual type "h" and "j" is individual type "l".

In this section, we want to examine if the solution to the planner's problem can increase the \i" type individual's utility (the recipient of the transfers) without reducing the utility level that the \j" type individual obtain in the competitive equilibrium with transfers, which is equal to $\overline{U}$. Individual \j" is taxed to ...nance the transfers to individual \i".[20]

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[19] Recall that we consider two types of transfers: either $T_l = \phi_h > 0$ and $T_h = \phi_l = 0$ or $T_h = \phi_l > 0$ and $T_l = \phi_h = 0$.

[20] For the same reasons explained in section 4, we allow the possibility of side payments at time $t = 1$.

The ...rst-order conditions of the planner's problem are (where \¹" is the lagrangean multiplier):

$$\frac{1}{(E_i \; \text{i} \; X_{A;i})} = \frac{\frac{d(Y_{1;r} + Y_{1;p})}{dX_{A;i}}}{[Y_{1;r} + Y_{1;p} \; \text{i} \; C_{1;j}]}; \tag{30}$$

$$\frac{-}{[Y_{1;r} + Y_{1;p} \; \text{i} \; C_{1;j}]} = \frac{1}{C_{1;j}}; \tag{31}$$

$$\frac{\frac{d(Y_{1;r} + Y_{1;p})}{dT}}{[Y_{1;r} + Y_{1;p} \; \text{i} \; C_{1;j}]} = \frac{1}{(E_j \; \text{i} \; T \; \text{i} \; X_{A;j})}; \tag{32}$$

$$\frac{\frac{d(Y_{1;r} + Y_{1;p})}{dX_{A;j}}}{[Y_{1;r} + Y_{1;p} \; \text{i} \; C_{1;j}]} = \frac{1}{(E_j \; \text{i} \; T \; \text{i} \; X_{A;j})}; \tag{33}$$

$$\overline{U} = \ln[E_j \; \text{i} \; T \; \text{i} \; X_{A;j}] \; \text{i} \; {}^{-} \ln[C_{1;j}]; \tag{34}$$

$$L_{1;r} = \frac{{}^{\circledR \frac{1}{T_i} \circledR} \; \text{i} \; z \; f \; (A_{0;h}; W) \; (X_{A;h}(T))}{f \; (A_{0;l}; W) \; (X_{A;l}(T))}: \tag{35}$$

Using (33) in (31), we get rid of ¹ and obtain the following expression for $C_{1;j}$:

$$C_{1;j} = \frac{d(Y_{1;r} + Y_{1;p})}{dX_{A;j}} (E_j \; \text{i} \; T \; \text{i} \; X_{A;j}): \tag{36}$$

Using (36) in (30) to get rid of $C_{1;j}$, we obtain the following:

$$Y_{1;r} + Y_{1;p} = \frac{d(Y_{1;r} + Y_{1;p})}{dX_{A;i}} (E_i \; \text{i} \; X_{A;i}) + \frac{d(Y_{1;r} + Y_{1;p})}{dX_{A;j}} (E_j \; \text{i} \; T \; \text{i} \; X_{A;j}): \tag{37}$$

Using equations (32) and (33), we obtain the following condition:

$$\frac{d(Y_{1;r} + Y_{1;p})}{dT} = \frac{d(Y_{1;r} + Y_{1;p})}{dX_{A;j}}: \tag{38}$$

The solution to the system of four equations ((34), (35), (37) and (38)) and four unknowns ($X_{A;i}, X_{A;j}$, $L_{1;r}$ and T_i) solves the planner's problem when migration is allowed.

The following propositions con...rm that the results obtained in section 4:2 are also valid when we allow migration, with the same economic interpretation.

The next proposition shows that transfers can be welfare improving when migration is allowed.

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Proposition 17 With assumptions 1 and 2, we obtain the following results when migration is allowed: (i) if $\left(\frac{A_{0;h}}{A_{0;l}}\right)^{3 - i} < \frac{1}{[(z+d)^{\circledast} - d]}$, then there is a $T_l^{\alpha} = \iota_h^{\alpha} > 0$, with $T_h^{\alpha} = \iota_l^{\alpha} = 0$, that is welfare improving; and (ii) if $\left(\frac{A_{0;h}}{A_{0;l}}\right)^{3 - i} > \frac{1}{[(z+d)^{\circledast} - d]}$, then there is a $T_h^{\alpha} = \iota_l^{\alpha} > 0$, with $T_l^{\alpha} = \iota_h^{\alpha} = 0$, that is welfare improving.

The following proposition shows that the optimal amount of transfers does not depend on the level of the country's economic development, even when migration is allowed.

Proposition 18 With assumptions 1 and 2, the optimal amount of transfer when migration is allowed does not change if we multiply by "m" (8m, m > 0) the initial technology level in both locations ($A_{0;l}$ and $A_{0;h}$).

Finally, the last proposition con...rms that the optimal amount of transfers should vary with the degree of income disparity across regions, while maitaining constant the level of the country's economic development, even when migration is allowed.

Proposition 19 With assumptions 1 and 2, the optimal amount of transfer when migration is allowed increases with "v", with $v = \frac{A_{0;h}}{A_{0;l}}$ (v > 1), while maintaining $\overline{A} = \frac{A_{0;h} + A_{0;l}}{2}$ constant.

7 Conclusion

This paper developed a two-period model with heterogeneous agents to analyze the e¤ects of transfers across locations on convergence, growth and welfare. One important feature of the model is that returns on the investment to acquire new technology depend positively on the level of each region's knowledge and on the level of the world knowledge assumed to be available to all. These returns are fundamental to determine each location's economic growth. In one hand, the poorest region has a disadvantage as it has a lower stock of knowledge. On the other hand, it has the advantage of not having yet exploited a greater stock of useable knowledge available in the world.

When the returns are greater in the poor region, we obtained the following results. First, there is income convergence across locations. Second, transfers to the poor location increases the country's economic growth, but only up to a certain point. Third, the introduction of transfers can increase the country's output in order to compensate the individuals that ...nance them. In other words, we can guarantee that transfers are e¢cient as the bene...ciary's utility increases by a greater amount than the reduction in the other agent's utility when compensation is not possible. When the returns are lower in the poor region, the ...rst two results are reversed, and the type of transfers that increase the economic welfare is the one directed to the rich location.

Another important result is that barriers to the adoption of new technology available in the world can constrain the convergence process as it harms in greater length the poorest region.

We also obtained the result that the optimal amount of transfers as a fraction of the country's total output declines with the level of economic development. Controlling for the variable that measures the income disparity across locations, one should observe countries making less transfers as a fraction of GDP as they become richer.

Finally, we conclude that two countries with the same income per capita and di¤erent degrees of income inequality across regions should pursue di¤erent transfer policy. The one with greater income disparity should follow a more agressive transfer policy. The reason is that, ceteris paribus, a greater disparity indicates a a greater di¤erence in the marginal productivity to invest in the adoption of new technology across locations. Hence, a greater amount of transfer is necessary to generate the equalization that maximizes the economic welfare.

8 Appendix

Proposition 1: There exists an unique competitive equilibrium when migration is not allowed.

Proof. It is straightforward to show that there is an unique $X_{A;i}^{WO}$ that solves equation (6), $(i = h; l)$. ∎

Proposition 2: With assumption 1, we have:

$$\left(1 + g_p^{WO}\right) = \frac{A_{1;l}^{WO}}{A_{0;l}} = \frac{f\left(A_{0;l}; W; \frac{1}{4}\right)\left(X_{A;l}^{WO}\right)^3}{A_{0;l}};$$

and

$$\left(1 + g_r^{WO}\right) = \frac{A_{1;h}^{WO}}{A_{0;h}} = \frac{f\left(A_{0;h}; W; \frac{1}{4}\right)\left(X_{A;h}^{WO}\right)^3}{A_{0;h}}:$$

Proof. We have that: $\left(1 + g_p^{WO}\right) = \frac{BA_{1;l}^{WO}}{BA_{0;l}} = \frac{f\left(A_{0;l}; W; \frac{1}{4}\right)\left(X_{A;l}^{WO}\right)^3}{A_{0;l}}:$ The proof is similar for the rich location. ∎

Proposition 3: With assumptions 1, and 2, there is income per capita convergence (divergence) across locations if $\frac{1}{4} > \,$ ($\frac{1}{4} < \,$).

Proof. Using assumption 2, there is income convergence across location if and only if:

$$\frac{\left(1 + g_p^{WO}\right)}{(1 + g_r^{WO})} = \left(\frac{A_{0;l}}{A_{0;h}}\right)^{\frac{1}{4}} > 1:$$

The above inequality holds i¤ $\frac{1}{4} > \,$: The proof is similar for the case of divergence. ∎

Proposition 4: Using assumptions 1, and 2, we have: $\dfrac{d\left(\frac{1+g_p^{wo}}{1+g_r^{wo}}\right)}{d\mu} > 0.$

Proof. As $\dfrac{(1+g_p^{WO})}{(1+g_r^{WO})} = \left(\dfrac{A_{0;l}}{A_{0;h}}\right)^{\mu}$ and, $A_{0;h} > A_{0;l}$, the proposition holds. ∎

Proposition 5: There exists an unique competitive equilibrium when transfers are introduced and migration is not allowed.

Proof. For any T_i, it is straightforward to show that there is an unique $X_{A;i}^T$ that solves equation (7), $(i = h; l)$. ∎

Proposition 6: With assumption 1, we obtain the following results: (i) for any transfer policy with $T_l > 0$ and $T_h = 0$, $\dfrac{y_{1;r}^{wo}}{y_{1;p}^{wo}} > \dfrac{y_{1;r}^{T}}{y_{1;p}^{T}}$; (ii) for any transfer policy with $T_h > 0$ and $T_l = 0$, $\dfrac{y_{1;r}^{wo}}{y_{1;p}^{wo}} < \dfrac{y_{1;r}^{T}}{y_{1;p}^{T}}$.

Proof. Note that $\dfrac{y_{1;r}^{wo}}{y_{1;p}^{wo}} = \dfrac{f(A_{0;h};W)(X_{A;h}^{WO})}{f(A_{0;l};W)(X_{A;l}^{WO})}$. When $T_l > 0$ and $T_h = 0$, we have: $\dfrac{y_{1;r}^{T}}{y_{1;p}^{T}} = \dfrac{f(A_{0;h};W)(X_{A;h}^{WO} i\, gT)}{f(A_{0;l};W)(X_{A;l}^{WO}+gT)}$, where $g = \frac{-}{1+}$: It is straightforward to show that $\dfrac{y_{1;r}^{wo}}{y_{1;p}^{wo}} > \dfrac{y_{1;r}^{T}}{y_{1;p}^{T}}$ when $T_l > 0$ and $T_h = 0$. The proof is analogous when $T_h > 0$ and $T_l = 0$. ∎

Proposition 7: With assumptions 1 and 2, there is a positive amount of transfers that maximizes the country's income per capita growth rate. If $\gg >$ ₃, then $T_l = \dot{\iota}h > 0$ and $T_h = \dot{\iota}l = 0$. If $\gg <$ ₃, then $T_h = \dot{\iota}l > 0$ and $T_l = \dot{\iota}h = 0$:

Proof. The total output with transfers and without migration at time $t = 1$ is equal to:

$$Y_1 = Y_{1;r} + Y_{1;p} = B\left[f(A_{0;l};W)\left(X_{A;l}^T(T)\right)^{\text{¢}} + f(A_{0;h};W)\left(X_{A;h}^T(T)\right)^{\text{¢}} \right];$$

where $X_{A;l}^T$ is a function of T. Taking the derivative of Y_1 with respect to T, and using assumption 2, we obtain the following condition to maximize the country's income per capita growth rate:

$$\left(\frac{A_{0;l}}{A_{0;h}}\right)^{1_i \gg} = \left(\frac{X_{A;l}^T(T)}{X_{A;h}^T(T)}\right)^{1_i} : \tag{39}$$

If ₃ $\gg >$ ₃ then the left-hand side is greater than the right-hand side when there is no transfer. Therefore, we need $T_l^{\text{¤}} = \dot{\iota}h > 0$ and $T_h^{\text{¤}} = \dot{\iota}l = 0$ to satisfy equation (39) and obtain the maximum income per capita growth rate. If $\gg <$ ₃, then we need $T_h^{\text{¤}} = \dot{\iota}l > 0$ and $T_l^{\text{¤}} = \dot{\iota}h = 0$ to satisfy equation (39). ∎

Proposition 8: With assumptions 1 and 2, we obtain the following results: (i) if $\gg >$ ₃, then there is a $T_l^{\text{¤}} = \dot{\iota}_h^{\text{¤}} > 0$, with $T_h^{\text{¤}} = \dot{\iota}_l^{\text{¤}} = 0$, that is welfare improving; and (ii) if $\gg <$ ₃, then there is a $T_h^{\text{¤}} = \dot{\iota}_l^{\text{¤}} > 0$, with $T_l^{\text{¤}} = \dot{\iota}_h^{\text{¤}} = 0$, that is welfare improving.

Proof. It is only necessary to show that there is a $X_{A;l}^{\pi}$, $X_{A;h}^{\pi}$ and T_i^{π} $(i = h; l)$ that increase both types of individual's utility in comparison with the competitive equilibrium without transfers characterized by $X_{A;l}^{WO}$, $X_{A;h}^{WO}$ and $T = 0$. Suppose that $\gg > \cdot$. Let $X_{A;l}^{\pi} = X_{A;l}^{WO}$, and choose $X_{A;h}^{\pi}$ and T_l^{π} such that: $X_{A;h}^{\pi} = X_{A;h}^{WO} - T_l$ and equation (19) be satis...ed. With this allocation, the amount invested is equal to the one in the competitive equilibrium without transfers. As a result, both individuals do not have their utility at time $t = 0$ altered. Moroever, with the new distribution of investments across locations, total output at time $t = 1$ is greater in comparison with the competitive equilibrium without transfers. Hence, consumption can be greater for both types of agents at time $t = 1$. The proof is analogous when $\ll < \cdot$. ∎

Proposition 9: With assumptions 1 and 2, the optimal amount of transfer does not change if we multiply by "m" ($\otimes m$, $m > 0$) the initial technology level in both locations ($A_{0;l}$ and $A_{0;h}$).

Proof. Using assumption 2, and the expressions for $X_{A;l}^{WO}$ and $X_{A;h}^{WO}$ obtained in the competitive equilibrium without transfers and migration, the left-hand side (LHS) and the right-hand side (RHS) of equation (19) are, respectively, equal to LHS $= \left(\frac{A_{0;l}}{A_{0;h}}\right)^{1-\gg}$ and RHS $= \left(\frac{A_{0;l}}{A_{0;h}}\right)^{1-\cdot}$. Equation (19) is only satis...ed in the competitive equilibrium without migration and transfers when $\cdot = \gg$. In this case the optimum amount of transfer is equal to zero. When $\cdot \ne \gg$, the optimal amount of transfers is positive. The LHS and RHS do not alter if we multiply $A_{0;l}$ and $A_{0;h}$ by \m". Therefore, the amount of transfers necessary to equate the LHS with the RHS does not change with "m". ∎

Proposition 10: With assumptions 1 and 2, the optimal amount of transfer increases with "v", with $v = \frac{A_{0;h}}{A_{0;l}}$, while maintaining $\overline{A} = \frac{A_{0;h}+A_{0;l}}{2}$ constant.

Proof. Using assumption 2, and the expressions for $X_{A;l}^{WO}$ and $X_{A;h}^{WO}$ obtained in the competitive equilibrium without transfers and migration, the left-hand side (LHS) and the right-hand side (RHS) of equation (19) are, respectively, equal to LHS $= \left(\frac{A_{0;l}}{A_{0;h}}\right)^{1-\gg}$ and RHS $= \left(\frac{A_{0;l}}{A_{0;h}}\right)^{1-\cdot}$. If $A_{0;l} = A_{0;h} = A$, then $\overline{A} = A$, LHS $=$ RHS and the optimal amount of transfers is zero. Suppose that we multiply $A_{0;l} = A$ by "s" ($s > 1$) and $A_{0;h} = A$ by "$(1-s)$". Note that: $v = \frac{A_{0;h}}{A_{0;l}} = \frac{sA}{(1-s)A} = \frac{s}{(1-s)}$ and $\overline{A} = \frac{A_{0;h}+A_{0;l}}{2} = \frac{sA+(1-s)A}{2} = A$. Hence, "v" increases with "s" and $\overline{A}$ does not change with "s". Note that the di¤erence between the LHS and the RHS increases with "v" and a greater amount of transfers is necessary to equate both sides. ∎

Proposition 11: With assumptions 1, 2, and 3, there exists an unique competitive equilibrium with migration and with $0 < L_{1;r}^M < 1$.

Proof. With assumption 1, we obtain $X_{A;l}^M$ and $X_{A;h}^M$, respectively, from equations (20) and (21). Using assumption 2, when we equate (22) and (23), we obtain $L_{1;r}^M$. Assumption 3 guarantees that $0 < L_{1;r}^M < 0$. From equation (24), we get $w_{h;r}$. ∎

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Proposition 12: With assumptions 1 through 3, $\frac{y_{1;r}^{WO}}{y_{1;p}^{WO}}$ may be lower than $\frac{y_{1;r}^{M}}{y_{1;p}^{M}}$ $\left(0 < L_{1;r}^{M} < 1\right)$.

Proof. We have the following using, respectively, the closed formed solution to the competitive equilibrium without and with migration:

$$\frac{y_{1;r}^{WO}}{y_{1;p}^{WO}} = z^{\circledR} \left(\frac{A_{0;h}}{A_{0;l}}\right)^{1+\,_{\flat}i\,\gg};$$

and

$$\frac{y_{1;r}^{M}}{y_{1;p}^{M}} = \left(\frac{(z+d)^{\circledR} A_{0;h}^{1+\,_{\flat}i\,\gg}}{A_{0;l}^{1+\,_{\flat}i\,\gg} + dA_{0;h}^{1+\,_{\flat}i\,\gg}}\right);$$

where $d = \left(\circledR \frac{1}{\mathbb{1}_{i}\,\circledR}\right) \, z$.

Using the above equations, we get the following result:

$$\frac{y_{1;r}^{WO}}{y_{1;p}^{WO}} \, R \, \frac{y_{1;r}^{M}}{y_{1;p}^{M}}$$

if and only if,

$$\frac{(z+d)^{\circledR} + z^{\circledR}}{z^{\circledR}d} \, R \, \left(\frac{A_{0;h}}{A_{0;l}}\right)^{1+\,_{\flat}i\,\gg}.$$

∎

Proposition 13: With assumptions 1 through 3, $\left(1 + g_p^M\right)$ may be greater than $\left(1 + g_r^M\right)$:

Proof. We have the following using, respectively, the closed formed solution to the competitive equilibrium without and with migration:

$$\left(1 + g_p^M\right) = \left(\frac{-_{\flat}}{1 + -_{\flat}}\right) B \cdot W^{\gg} A_{0;l}^{\flat i\,\gg};$$

and,

$$\left(1 + g_r^M\right) = \frac{\left(\frac{-_{\flat}}{1 + -_{\flat}}\right) B \cdot W^{\gg} \left(1 + \frac{d}{z}^{\circledR} A_{0;h}^{\flat i\,\gg}\right)}{1 + d \left(\frac{A_{0;h}}{A_{0;l}}\right)^{1+\,_{\flat}i\,\gg}};$$

Using the above equations, we get the following result:

$$\left(1 + g_r^M\right) \, R \, \left(1 + g_p^M\right);$$

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if and only if,

$$\mu_{\frac{A_{0;h}}{A_{0;l}}} \P_{,i} \gg R \frac{1 + d\left(\frac{A_{0;h}}{A_{0;l}}\right)^{1+,i \gg}}{1 + \frac{d}{z}} :$$

∎

Proposition 14: With assumption 1, $\frac{dL_{1;r}^{MT}}{dT_l} < 0$, and $\frac{dL_{1;r}^{MT}}{dT_h} > 0$:

Proof. First, let's consider the case in which $T = T_l = \d_h > 0$ and $T_h = \d_l = 0$. We have:

$$L_{1;r}^{MT} = \left(\frac{1}{\d_i}\right)^{\frac{1}{\d_i}} \d z \left[\frac{f(A_{0;h};W)}{f(A_{0;l};W)}\left(\frac{X_{A;h}^M \d \frac{1}{(1+)}T}{X_{A;l}^M + \frac{1}{(1+)}T}\right)\right]:$$

It is straightforward to see that $\frac{dL_{1;r}^{MT}}{dT_l} < 0$. The proof is analogous if $T = T_h = \d_l > 0$ and $T_l = \d_h = 0$. ∎

Proposition 15: With assumption 1, we obtain the following results: (i) for any transfer policy with $T_l > 0$ and $T_h = 0$, $\frac{y_{1;r}^M}{y_{1;p}^M} > \frac{y_{1;r}^{MT}}{y_{1;p}^{MT}}$; and (ii) for any transfer policy with $T_h > 0$ and $T_l = 0$, $\frac{y_{1;r}^M}{y_{1;p}^M} < \frac{y_{1;r}^{MT}}{y_{1;p}^{MT}}$.

Proof. First, let's consider the case in which $T = T_l = \d_h > 0$ and $T_h = \d_l = 0$. Using the closed formed solutions for $A_{1;h}^{MT}$, $A_{1;l}^{MT}$, and $L_{1;r}^{MT}$, we obtain the following:

$$\frac{y_{1;p}^{MT}}{y_{1;r}^{MT}} = \frac{f(A_{0;l};W)\left(X_{A;l}^M + \frac{1}{(1+)}T\right) + df(A_{0;h};W)\left(X_{A;h}^M \d \frac{1}{(1+)}T\right)}{(z+d)\left(f(A_{0;h};W)\left(X_{A;h}^M \d \frac{1}{(1+)}T\right)\right)})$$

$$) \quad \frac{y_{1;p}^{MT}}{y_{1;r}^{MT}} = \frac{f(A_{0;l};W)\left(X_{A;l}^M + \frac{1}{(1+)}T\right)}{(z+d)\left(f(A_{0;h};W)\left(X_{A;h}^M \d \frac{1}{(1+)}T\right)\right)} + \frac{d}{(z+d)};$$

where, $d = \left(\frac{1}{\d_i}\right) \d z :$

Note that when $T = 0$, $\frac{y_{1;p}^{MT}}{y_{1;r}^{MT}} = \frac{y_{1;p}^M}{y_{1;r}^M}$. As $\frac{d\left(\frac{y_{1;p}^{MT}}{y_{1;r}^{MT}}\right)}{dT} > 0$, any positive transfer to the poor location reduces the income disparity across locations. The proof is analogous if $T = T_h = \d_l > 0$ and $T_l = \d_h = 0$. ∎

Proposition 16: With assumptions 1 and 2, there is a positive amount of transfers (T_j^a) that maximizes the country's income per capita growth rate. If

$\left(\frac{A_{0;h}}{A_{0;l}}\right)^{\frac{1}{1-\alpha}} < \frac{1}{[(z+d)^{\beta}-d]}$, then $T_l^{\ast} = \iota_h > 0$ and $T_h^{\ast} = \iota_l = 0$. If $\left(\frac{A_{0;h}}{A_{0;l}}\right)^{\frac{1}{1-\alpha}} > \frac{1}{[(z+d)^{\beta}-d]}$, then $T_h^{\ast} = \iota_l > 0$ and $T_l^{\ast} = \iota_h = 0$:

Proof. Using the closed formed solution to $L_{1;r}^{MT}$, the total output with transfers and migration at time $t = 1$ is equal to:

$$Y_1 = Y_{1;r} + Y_{1;p} = B^{\theta}\left([(z+d)^{\beta} - d]A_{1;h}^{MT}(T) + A_{1;l}^{MT}(T)\right)^{a};$$

where $d = \rho^{\frac{1}{1-\rho}} - z$ and $A_{1;i}^{MT}$ is a function of T. Taking the derivative of Y_1 with respect to T, and using assumption 2, we obtain the following condition to maximize the country's income per capita growth rate:

$$[(z+d)^{\beta} - d]\left(\frac{A_{0;h}}{A_{0;l}}\right)^{1-\alpha} = \left(\frac{A_{1;h}^{MT}(T)}{A_{1;l}^{MT}(T)}\right)^{1-\alpha}: \qquad (40)$$

If $\left(\frac{A_{0;h}}{A_{0;l}}\right)^{\frac{1}{1-\alpha}} < \frac{1}{[(z+d)^{\beta}-d]}$, then the left-hand side is lower than the right-hand side when there is no transfer. Therefore, we need $T_l^{\ast} = \iota_h > 0$ and $T_h^{\ast} = \iota_l = 0$ to satisfy equation (40) and obtain the maximum income per capita growth rate. If $\left(\frac{A_{0;h}}{A_{0;l}}\right)^{\frac{1}{1-\alpha}} > \frac{1}{[(z+d)^{\beta}-d]}$, then we need $T_h^{\ast} = \iota_l > 0$ and $T_l^{\ast} = \iota_h = 0$ to satisfy equation (40). ∎

Proposition 17: With assumptions 1 and 2, we obtain the following results when migration is allowed: (i) if $\left(\frac{A_{0;h}}{A_{0;l}}\right)^{\frac{1}{1-\alpha}} < \frac{1}{[(z+d)^{\beta}-d]}$, then there is a $T_l^{\ast} = \iota_h^{\ast} > 0$, with $T_h^{\ast} = \iota_l^{\ast} = 0$, that is welfare improving; and (ii) if $\left(\frac{A_{0;h}}{A_{0;l}}\right)^{\frac{1}{1-\alpha}} > \frac{1}{[(z+d)^{\beta}-d]}$, then there is a $T_h^{\ast} = \iota_l^{\ast} > 0$, with $T_l^{\ast} = \iota_h^{\ast} = 0$, that is welfare improving.

Proof. The proof is analogous to the one in proposition 8. ∎

Proposition 18: With assumptions 1 and 2, the optimal amount of transfer when migration is allowed does not change if we multiply by "m" (8m, m > 0) the initial technology level in both locations ($A_{0;l}$ and $A_{0;h}$).

Proof. Using equation (38), the proof is analogous to the one in proposition 9. ∎

Proposition 19: With assumptions 1 and 2, the optimal amount of transfer when migration is allowed increases with "v", with $v = \frac{A_{0;h}}{A_{0;l}}$ (v > 1), while maintaining $\overline{A} = \frac{A_{0;h} + A_{0;l}}{2}$ constant.

Proof. Using equation (38), the proof is analogous to the one in proposition 10. ∎

9 References

Andrade, E., "Growth, Distribution and School Policy", PhD Dissertation, University of Chicago, 1998.

Barro, R., and Sala-i-Martin, X., "Economic Growth", McGraw Hill Inc, 1995.

Becker, G. and Murphy, K., "The Division of Labor Coordination Costs and Knowledge", Quarterly Journal of Economics, 107:4, 1992.

Benabou, R., "Unequal Societies: Income Distribution and the Social Contract", American Economic Review, March, 2000.

Fernandez, R., Rogerson, R., "Income Distribution, Communities, and the Quality of Education", Quarterly Journal of Economics, February, 1996.

Glaeser, E., "Are Cities Dying?", Journal of Economic Perspectives, Spring, 1998.

Parente, S., and Prescott, E., "Barriers to Technology Adoption and Development", Journal of Political Economy, vol. 102, no. 2, 1994.

Quah, D., "Regional Convergence Clusters across Europe", European Economic Review, April, 1996a

Quah, D., "Convergence Empirics across Economies with (Some) Capital Mobility", Journal of Economic Growth, March, 1996b.

Sala-i-Martin, X., "The Classical Approach to Convergence of Growth Analysis", Economic Journal, July, 1996

Stokey, N., "Free Trade, Factor Returns, and Factor Accumulation", Journal of Economic Growth, , v. 1, iss. 4, December, 1996.

Ter-Minassian, T.,"Fiscal Federalism", IMF Press, 1996.