Default and Renegotiation: A Dynamic Model of Debt

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Introduction

- The model in this paper is part of the family of partial equilibrium models seen in the second half of our course.
- More specifically, it is a model very similar to Holmström and Tirole (1997, QJE), which we saw as an example of models with nonverifiable income.
- Hart and Moore’s model focus also assumes that the entrepreneur’s income can not be observed or audited and that he can consume his entire income.
- However, the threat of project termination can induce the entrepreneur to avoid default or to pay the investor a renegotiated amount, higher than the liquidation value.
The Framework

- Three time periods: \( t \in \{0, 1, 2\} \)
- Competitive supply of investors;
- One entrepreneur with initial wealth \( w \) and whose project needs financing and costs \( I > w \).
- Projects yields random returns \( R_1 \) and \( R_2 \) when assets are in place.
The Framework

- At $t = 1$, the project’s assets have a random liquidation value $L$, satisfying $L > 0$ and $EL < I$.
- Funds not paid to the investor can be reinvested at $t = 1$ and yield a random return $s$ at $t = 2$, satisfying $1 \leq s \leq R_2/L$.
- Uncertainty concerning $(s, L, R_1, R_2)$ ends at $t = 1$.
- At $t = 2$, the assets are worthless.
The Framework

The authors also assume that:

1. Assets are divisible at $t = 1$ (so the project can be partially terminated at this date);

2. Although realization of $(R_1, R_2, L, s)$ is known to both parties, it is not verifiable to outsiders. Therefore, it is not possible to write contracts directly conditioned on these variables.

3. The project would be carried out in a first-best world.
Debt Contracts \((P, T)\)

- A debt contract is a pair \((P, T)\) in which the investor lends \(I - w + T\) to the entrepreneur at \(t = 0\), and at \(t = 1\) the entrepreneur pays back the investors an amount \(P\). At \(t = 2\), no payments are made.

- At \(t = 1\), the entrepreneur chooses whether he will pay back the investor or not. Since he can use the project’s assets for this purpose, there are two possibilities:
  1. He can choose whether or not to default: \(T + R_1 + L \geq P\)
  2. He is forced to default: \(T + R_1 + L < P\).

- In the event of default, if liquidation value is too low, the investor may find preferable to renegotiate the contract.

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Payoff in case of default: a “non-wealthy” entrepreneur

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Renegotiation game

- With probability \((1 - \alpha)\) the entrepreneur makes a take-it-or-leave-it offer to the investor, and with probability \(\alpha\) the investor is the one who makes such an offer to the entrepreneur.

- To eliminate potential inefficiencies, the entrepreneur is allowed to make an offer prior to this game.

- If the entrepreneur gets to make the take-it-or-leave-it offer, the outcome will be at point \(X^0\), and the investor’s gross payoff will be \(L\).
### Renegotiation game

- If the **investor** gets to make the take-it-or-leave-it offer and the entrepreneur is “wealthy” \((T + R_1 > R_2)\), the investor can sell back the project’s assets to the entrepreneur in return for an amount of

\[
T + R_1 - \left( \frac{T + R_1 - R_2}{s} \right)
\]

- If the **investor** gets to make the take-it-or-leave-it offer and the entrepreneur is “not wealthy” \((T + R_1 < R_2)\), the investor can ask for \(T + R_1\) and make the entrepreneur liquidate a fraction \(f = 1 - ((T + R_1)/R_2)\) of the assets. The investor will then be handed

\[
T + R_1 + \left( 1 - \frac{T + R_1}{R_2} \right) L
\]
Investor’s payoff

- We can combine these subcases by rewriting the investor’s net payoff when he gets to make the offer, following default:

\[
\min \left\{ T + R_1 + \left( 1 - \frac{T + R_1}{R_2} \right) L, T + R_1 - \frac{T + R_1 - R_2}{s} \right\}
\]

- The investor’s overall expected payoff following default will be:

\[
\bar{P}(R_1, R_2, L, s; T) = (1 - \alpha)L + \alpha \min \left\{ T + R_1 + \left( 1 - \frac{T + R_1}{R_2} \right) L, T + R_1 - \frac{T + R_1 - R_2}{s} \right\}
\]

- The entrepreneur’s initial offer makes this the actual investor’s gross payoff, rather than only his expected gross payoff.

- Since the entrepreneur will pay his debt \( P \) if and only if \( P \leq \bar{P} \), the investor’s net payoff will be

\[
N(R_1, R_2, L, s; P, T) = \min \{ \bar{P} - T, P - T \}
\]
Entrepreneur’s payoff

- If $T + R_1 > T + N$, i.e., if the entrepreneur has enough liquid funds to pay the investor, the entrepreneur’s payoff will be

  $$R_2 + s(R_1 - N)$$

- If $T + R_1 < T + N$, then some fraction $(1 - f)$ of the assets must be sold, and the entrepreneur’s payoff will be

  $$fR_2 = \left(1 - \frac{(N - R_1)}{L}\right) R_2 = R_2 - \frac{(N - R_1)R_2}{L}$$

- By combining these two cases, we have:

  $$\Pi(R_1, R_2, s, L; P, T) = \min \left\{ R_2 - \frac{(N - R_1)R_2}{L}, R_2 + s(R_1 - N) \right\}$$
Optimal Debt Contracts

An optimal debt contract must solve:

\[
\begin{align*}
\max_{P,T \geq 0} & \quad \mathbb{E}[\Pi(R_1, R_2, s, L; P, T)] \\
\text{s.t.} & \quad \mathbb{E}[N(R_1, R_2, s, L; P, T)] \geq I - w
\end{align*}
\]

in which

\[
\begin{align*}
N &= \min\{P - T, P - T\} \\
\Pi &= \min \left\{ R_2 - (N - R_1) \frac{R_2}{L}, R_2 + (R_1 - N)s \right\} \\
\bar{P} &= (1 - \alpha) L + \alpha \min \left\{ T + R_1 + \left(1 - \frac{T + R_1}{R_2}\right) L, T + R_1 - \frac{T + R_1 - R_2}{s} \right\}
\end{align*}
\]
Proposition 1

Suppose that either (1) $R_1, R_2, L$ and $s$ are nonstochastic, or (2) $s \equiv R_2/L$ and $s$ is nonstochastic, or (3) $L$ is nonstochastic and $\alpha = 0$. Then all debt contracts that satisfy the entrepreneur’s break-even constraint (B-E) with equality are optimal.
Optimal Debt Contracts

Proof:

1. Notice that if \((R_1, R_2, s, L)\) is nonstochastic, then \(\bar{P}\) is also nonstochastic, which means that \(N\) is also nonstochastic. Since the investor breaks even, then \(N = I - w\) and the entrepreneur’s payoff does not depend on \((P, T)\).

2. If \(s \equiv R_2/L\), then \(\Pi = sL + (R_1 - N)s\). It is irrelevant whether the funds invested between dates 1 and 2 are used to prevent liquidation or used for reinvestment. Since the investor must break even, on average, we have that:

\[
E[\Pi] = sE(L) + sE(R_1) - s \underbrace{E(N)}_{I-w}
\]

Therefore, the entrepreneur's payoff does not depend on \((P, T)\).

3. If \(L\) is nonstochastic and \(\alpha = 0\), then both \(\bar{P}\) and \(N\) are nonstochastic. Since the investor breaks even, \(N = I - w\) and the entrepreneur’s payoff does not depend on \((P, T)\).
Optimal Debt Contracts

Definition
The fastest debt contract has $T = 0$.

Definition
The slowest debt contract has $P = \infty$.

Definition
A transfer $T \geq 0$ is said to be feasible if there exists a debt contract $(P, T)$ satisfying the investor’s participation constraint.
Proposition 2

Suppose that $s \equiv 1$. Then, among the class of debt contracts: (1) if only $R_1$ is stochastic, the slowest debt contract is optimal; (2) if only $R_2$ is stochastic, the fastest debt contract is optimal; and (3) if only $L$ is stochastic and $\alpha = 1$, the slowest debt contract is optimal.
Optimal Debt Contracts

Sketch of Proof (2):

1. Since \(s \equiv 1\), the sum of the payoffs of both parties is given by:
   \[
   \Pi + N = R_1 + fR_2 + (1 - f)L
   \]

2. Since the optimal contract must also maximize \(\mathbb{E}(\Pi) + \mathbb{E}(N)\) subject to \(\mathbb{E}(N) = I - w\), an optimal contract must also solve:
   \[
   \max_{P,T \geq 0} \mathbb{E}[f(R_2 - L)] \tag{3}
   \]
   \[
   s.t. \ \mathbb{E}(N) = I - w \tag{4}
   \]

3. Let us recall that the fraction of the initial project that the entrepreneur retains following renegotiation is given by a negative affine transformation of \(N(R_2, P(T), T)\), truncated above by 1 for low \(R_2\):
   \[
   f(R_1, R_2, s, L; P(T), T) = \min \left\{ 1, 1 + \frac{R_1 - N(R_2, P(T), T)}{L} \right\}
   \]

4. The truncation will make \(\mathbb{E}(f)\) fall when \(T\) rises, proving optimal the fastest debt contract.
Remark

If \( s \equiv R_2/L \), then the maximum feasible value of \( N \) (the net payoff of the investor) – which we’ll call \( M \) – is given by:

\[
M = L + \alpha R_1 (1 - 1/s)
\]

Definition

The distribution \( F(\cdot) \) first-order stochastically dominates \( G(\cdot) \) if for every nondecreasing function \( h : \mathbb{R} \rightarrow \mathbb{R} \), we have:

\[
\int h(x)dF(x) \geq \int h(x)dG(x)
\]
Proposition 3

Suppose that \( s \equiv \frac{R_2}{L} \) and that a higher value of \( s \) increases the distribution of \( M \) conditional on \( s \), in the sense of first-order stochastic dominance. Then among the class of debt contracts the fastest debt contract is optimal.
Optimal Debt Contracts

**Sketch of Proof:**
If $s \equiv R_2/L$, the entrepreneur's payoff is given by $\Pi = sL + sR_1 - sN$, which reduces the problem to:

$$\min_{P,T \geq 0} \mathbb{E}[sN] \quad (5)$$

subject to

$$\mathbb{E}(N) = I - w \quad (6)$$

And the renegotiation payoff of the investor is given by:

$$\overline{P} = L + \alpha R_1 \left(1 - \frac{1}{s}\right) + \alpha T \left(1 - \frac{1}{s}\right)_{M}$$

1. For any $(P, T)$ with $T > 0$, let $N$ denote the investor's payoff.

2. Consider replacing this contract by the fastest debt contract $(\hat{P}, 0)$, where $\hat{P}$ is the smallest solution to $\mathbb{E}[\min\{M, \hat{P}\}] = I - w$. We denote the investor's payoff under this contract by $\hat{N} = \min\{M, \hat{P}\}$. 

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Optimal Debt Contracts

Sketch of Proof:

3. \( \mathbb{E}[N(M, s)] \geq I - w \equiv \mathbb{E}[\hat{N}(M)] \)

4. \( \mathbb{E}[\hat{N}(M)] - \mathbb{E}[N(M, s)] \equiv \mathbb{E}[\Delta(M, s)] \leq 0. \)

5. By inspection, it is possible to show that, for some \( s^* \) such that \( 1 \leq s^* \leq \infty \), we have that:

\[ (s - s^*)\mathbb{E}[\Delta(M, s)|s] \leq 0 \quad \forall s \]

6. Taking expectations, we have:

\[ \mathbb{E}[s\Delta(M, s)] \leq s^*\mathbb{E}[\Delta(M, s)] \leq 0 \]

7. If follows that \( \mathbb{E}[s\hat{N}] \leq \mathbb{E}[sN(M, s)] \). And the fastest debt contract \((\hat{P}, 0)\) (weakly) dominates \((P, T)\).
More General Contracts

- If minor alterations are made to the model, numerical examples in which debt contracts lead to inefficiencies are easy to produce.
- In this case, the authors consider the use of more general contracts (message-game contracts) and obtain optimality conditions for these contracts.
- The authors also modify the model to allow for different choice of project scale $t = 0$, and obtain results for the optimal choice of $I$. 
Some directions for future (from 1998 on) research

- Adding more periods to study the maturity of debt contracts when there is uncertainty;
- Investigating which contracts would be optimal when the assumptions made in the propositions of this paper do not hold.
- Considering a scenario in which parties deal with each other multiple times, and may acquire a reputation for paying their debts other than defaulting, or for liquidating the assets other than renegotiating.

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