Credit risk
The case of First Interstate Bankcorp

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Abstract

Structural models for pricing credit risk can be used to forecast the spread on risky bonds and for hedging credit risk. This article examines the forecasting accuracy of the Black–Scholes–Merton (BSM) model of risky debt using a data set consisting of weekly bond data for First Interstate Bancorp over the period January 1986–August 1993. In addition, structural model hedge parameters and credit spread options are tested for their effectiveness in hedging the increasing credit risk premium on First Interstate Bancorp debt. Credit spread options in combination with a duration hedge offer the best hedging strategy, reducing the standard deviation of the hedging error by a minimum of 84%. © 2002 Elsevier Science Inc. All rights reserved.

Keywords: Risk management; Credit risk; Duration hedge; Credit spread option

JEL classification: G13; G21

1. Introduction

Credit risk is measured as the uncertainty of future credit losses around their expected levels. While Merton’s seminal article on the pricing of risky debt was published in 1974, interest in pricing models for credit risk has gathered pace in the past 7 years.¹ Concerns by

¹ Kao (2000) shows that the number of published articles devoted to credit risk pricing increased sharply in 1993 and continued on an upward trend since then.
banks and bank regulators that credit risk is accurately priced and that sufficient capital is held against it, along with the growth in the credit derivative market enabling more effective management of credit risk, have motivated much of the recent research.

How to effectively measure and hedge credit risk has been a central concern of financial institution regulators and practitioners. Because duration hedging becomes less efficient as credit risk increases, one area of particular concern is the nature of hedging strategies that might be adopted to control credit risk. The purpose of this article is to determine if alternative hedging techniques derived from the Black–Scholes–Merton (BSM) model can outperform traditional duration hedging strategies. We also use a simple model of a credit spread option to determine its hedging effectiveness. The tests are run on a unique set of data on First Interstate Bancorp, which includes both equity prices and weekly bond prices for a range of debt maturities.

Several issues are addressed in this article. First, we test whether the BSM model is able to accurately describe the credit spread movement. Second, we investigate whether a hedging strategy based on the BSM model outperforms the traditional duration method. Third, if hedging strategies, based solely on either duration or the BSM model, perform poorly, can better hedging strategies be formed by combining the two models? Last, we test the usefulness of a credit spread option for hedging credit spread risk.

The test of the BSM model is done in two ways in this study. Firstly, we use the BSM model to estimate the theoretical credit spreads. We then test the hedging performance of the BSM model. The tests that we construct are similar to those suggested by Jarrow and van Deventer (1998). In the credit derivative simulation exercises, theoretical credit spread call option prices are derived using a one-step binomial model. As expected, the results show that the best hedging strategy for a risky bond portfolio hedges both credit risk and interest rate risk. We find that the combination of credit spread options with a duration hedge outperforms all other hedging methods, with a minimum 84.87% reduction in the standard deviation of the hedging error.

The paper is structured as follows. Section 2 reviews the literature. Section 3 discusses the models that will be used in the empirical tests. In Section 4, the data and the hedging strategies are discussed. Section 5 presents both the credit spread estimation results and the hedging results. The conclusion is found in Section 6.

2. Literature review

Modeling credit risk can be classified according to the two predominant approaches used. The first approach uses structural models, which value credit risk based on equity market and accounting information of the risky bond issuers, treating equity as a call option on the assets of the firm. The second approach consists of using reduced-form models, which utilize rating information provided by rating agencies, such as Standard and Poor’s or Moody’s, to model default risk. Because the focus of this study is on using the BSM model, we focus on a description of structural models. Reduced-form models are briefly discussed with reference to risk-neutral pricing in Section 3.2.
Black and Scholes (1973) and Merton (1974) (BSM) were the first authors to introduce a contingent claims approach to value corporate risky debt. In the BSM model, the holder of corporate risky debt equivalently holds one unit of risk-free (or default-free) bond with face value ($F$) and a short position in a put option on the firm’s value ($V$) with strike price equal to the face value of the debt. This implies that the terminal date value of the risky debt $B_T$ is equal to

$$B_T = F - \max[F - V_T, 0]$$

Various authors have extended the BSM model. However, there is no strong evidence to suggest that the empirical performance of the structural model can be improved by imposing more realistic or simply more complicated assumptions (Wei & Guo, 1997).

One approach to testing the BSM model is to use it to estimate future spreads. Jarrow and van Deventer (1998) argue that this approach is inappropriate because it is always possible to calibrate a model’s parameters to guarantee accurate pricing. Measuring the hedging performance provides the most robust test as the best credit risk pricing model’s hedging error should have the smallest standard deviation compared with other models. Jarrow and van Deventer test the hedging performance of the BSM model in comparison with a traditional duration hedge. The test results show that all the models are able to reduce the hedge errors significantly. However, the traditional hedging approach dominates the BSM model.

One difficulty with the approach used by Jarrow and van Deventer (1998) is that they combine the BSM model’s hedge ratio with the duration hedge ratio. Because in the BSM model equity is modelled as an option written on the assets of the firm, the risk can be hedged by trading equity. Hence, if the objective of the hedging is to test the BSM model, a position in U.S. Treasuries will not allow us to isolate the hedging performance of the BSM model. The combination of duration and the BSM model hedge may be practical and give a better hedging result, but it does not give a clear idea about the BSM model’s performance.

Duration hedging has been the simplest hedging technique adopted by practitioners. Empirical evidence finds that duration-based hedging techniques are as good as sophisticated regression-based methods. However, their efficiency declines with the bonds’ credit rating, as illustrated in Chance (1990). This is because duration measures the change in the value of the bond solely due to changes in the interest rate. It implicitly assumes that there is no credit risk. This assumption may be reasonable for government bonds and investment grade bonds but not for risky bonds.

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2 The assumptions of the BSM model are constant risk-free rate, a single zero-coupon bond liability maturing at time $T$, absence of arbitrage and transaction costs, zero bankruptcy costs and enforced protection of priority in bankruptcy, rational wealth maximizing shareholders, and assets of the firm follow geometric Brownian motion.

3 Black and Cox (1976) extend the BSM model by allowing bankruptcy before maturity. Longstaff and Schwartz (1995a, 1995b) further improve the Black and Cox’s (1976) model by introducing a dynamic risk-free interest rate and an exogenous level of return given default into the model.

4 Even though Jarrow and van Deventer (1998) point out that the BSM model hedge should not include a position in U.S. Treasuries, they argue that a combination of positions in both stock and U.S. Treasuries provides a more practical approach.
Longstaff and Schwartz (1995b) show that Macaulay duration can be negative for high-risk bonds and Leland and Toft (1996) show that the price/interest rate relationship can exhibit concavity rather than convexity for such bonds. It is thus necessary to consider underlying default risk in the hedging strategy. In empirical tests, Ilmanen, Mcguiire, and Warga (1994) show that duration is able to explain 90% of the variation in price of Aaa- and Aa-rated bonds and explains only around 35% in A- and Baa-rated bonds. These theoretical and empirical results suggest that traditional duration hedging may not be appropriate for bonds subject to credit risk and that alternative techniques that also account for credit risk should be used.

3. Models for credit risk

3.1. The estimation of the credit spread

The BSM model states that risky bond holders equivalently hold a long position in a risk-free bond and a short position in a put option. The price of the put option with payoff $\max(F - V, 0)$ is given by the Black and Scholes (1973) option pricing formula.

$$p(t, T) = F e^{-r(T-t)} N(-d + \sigma \sqrt{T-t}) - VN(-d),$$  \hspace{1cm} (1)

with

$$d = \frac{\ln \frac{F}{P} + \left(r + \frac{1}{2} \sigma^2\right)(T-t)}{\sigma \sqrt{T-t}}$$  \hspace{1cm} (2)

where $p(t, T)$ is the put option price, $t$ is current time, $T$ is the maturity date of the bond and put option, $r$ is the risk-free rate, $V$ is current market value of the firm, $F$ is the face value of the debt, $F(t, T)$ is the current market value of the risk-free debt so that $F(t, T) = F e^{-r(T-t)}$ is the instantaneous variance of the return on the firm’s assets, and $N(\cdot)$ is the univariate cumulative normal distribution function (Eqs. (1) and (2)). Hence, the risky bond value is given by

$$B(t, T) = F(t, T) - p(t, T) = F(t, T) N(d - \sigma \sqrt{T-t}) + VN(-d).$$  \hspace{1cm} (3)

Viewing the yield on the risky bond as being given by a premium over the risk-free rate, Eq. (3) can be written equivalently as

$$X = \ln \left[ N(d - \sigma \sqrt{T-t}) + \frac{V}{F(t, T)} N(-d) \right].$$  \hspace{1cm} (4)

where $X$ is defined as the credit spread and defines the relationship between the price of the risky bond and the yield on the risky bond as $B(t, T) = F e^{(r + X)(T-t)}$ (Eq. (4)).

The risk-free interest rate ($r$) is assumed to equal the U.S. Treasury zero-coupon bond rate for a bond maturing at the same time as the option. The value of the firm ($V$) and the volatility
of \( V(\sigma_V) \) can be estimated through the option nature of the firm’s equity. Because equity can be modelled as a call option on the firm’s assets, if there are \( n \) shares outstanding with \( S \) as the market price of the stock, then

\[
nS = VN(d) - Fe^{-r(T-t)}N(d - \sigma_V \sqrt{T-t})
\]

and

\[
\sigma_e = \frac{V}{nS} \frac{\partial nS}{\partial V} \sigma_V = \frac{V}{nS} N(d) \sigma_V.
\]

The volatility of the return on equity \( \sigma_e \) can be estimated using historical data. \( V \) and \( \sigma_V^2 \) can be found by simultaneously solving Eqs. (5) and (6).\(^5\)

The point (\( F \)) is such that when the firm’s value falls below \( F \) the firm defaults. In the option pricing formula, \( F \) is the strike price, which triggers the exercise of the option. The default point is difficult to determine when the firm has more than one kind of debt. In this study, \( F \) is assumed to be equal to the sum of the total short-term liabilities and half of the amount of the long-term liabilities.\(^6\)

3.2. Credit risk and credit spread options

A credit spread option is an option where the underlying asset is the credit spread. Kijima and Komoribayashi (1997), Longstaff and Schwartz (1995a), and Schonbucher (1997) discuss the pricing of credit spread options. Models of the credit spread option typically require estimation of dynamic interest rate parameters and the correlation coefficient between standard Wiener processes of the risk-free interest rate and the credit spread. For example, using Moody’s corporate bond yield data, Longstaff and Schwartz (1995b) find that credit spreads are negatively related to interest rates.

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\(^5\) The Newton–Raphson method, which is an iterative procedure for solving nonlinear equations, can be used to solve for \( V \) and \( \sigma^2 \). The procedure begins with an estimate of \( V \) and \( \sigma^2 \) and successively adjusts the values so that they satisfy the two equations.

\(^6\) This is suggested by Bohn (1999) and Crosbie (1999).
Pierides (1997) derives a valuation equation for the credit spread option by using the Black and Scholes (1973) option pricing formula. The fact that the value of the firm is neither

Fig. 1. Credit spreads for First Interstate Bancorp.

Fig. 2. First Interstate Bancorp share prices.
observable nor traded is usually overcome by assuming that it is possible to trade a security that is perfectly correlated with the value of the firm’s assets (e.g. Leland, 1994; Toft & Prucyk, 1997). Then, dynamic completeness of the market is used to invoke the existence of the equivalent martingale measure. Therefore, in both structural and reduced-form approaches to the modeling and pricing of default risky securities, an assumption of risk neutrality is often used so that expected cash flows can be discounted at the risk-free rate. For example, Saa-Requejo and Santa-Clara (1999) argue that in frictionless markets the existence of a risk-neutral probability measure can be viewed as roughly equivalent to the nonexistence of arbitrage opportunities. They develop a structural model of default risk, where the variable of interest is the ratio of firm value to face value of debt. What matters in the model is the risk-neutral probability of this ratio hitting one. Expectations are calculated under the risk-neutral measure. Longstaff and

Table 2
Hedge parameters for each of the strategies adopted

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Hedging instrument</th>
<th>Hedge parameter Number of hedge instrument per corporate bond</th>
</tr>
</thead>
<tbody>
<tr>
<td>Do nothing</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1:1 Treasury hedge</td>
<td>T-Bond</td>
<td></td>
</tr>
<tr>
<td>Duration hedge</td>
<td>T-Bond</td>
<td></td>
</tr>
<tr>
<td>BSM model hedge</td>
<td>Equity</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$e^{-X(T-t)}$ \left( \frac{1}{N(d)} - 1 \right)$</td>
</tr>
<tr>
<td>Credit spread call option hedge</td>
<td>Call option on the credit</td>
<td>$1/\Delta$ where $\Delta = \frac{f_u - f_d}{F e^{-(r+Xu)(T-t)} - F e^{-(r+Xd)(T-t)}}$</td>
</tr>
<tr>
<td></td>
<td>spread</td>
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<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BSM model and duration hedge</td>
<td>T-Bond and equity</td>
<td>$\text{Equity: } (1/N(d) - 1)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\text{T-Bond: } e^{-XT-\eta}$</td>
</tr>
<tr>
<td>Credit spread call option and</td>
<td>Call option on the credit</td>
<td>$\text{Call option: } 1/\Delta$</td>
</tr>
<tr>
<td>duration hedge</td>
<td>spread and T-Bond</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\text{T-Bond: } e^{-XT-\eta}$</td>
</tr>
</tbody>
</table>

The hedge parameter indicates the number of units of the hedging instrument that must be purchased or sold in order to hedge the exposure to $S$ of First Interstate Bancorp bonds. For example, for the duration hedge, the equation represents the sensitivity of the risky bond $B$ to the treasury bond $TB$. The risky bond price is given by $B = e^{-XT-\eta}e^{-r(T-t)} = e^{-XT-\eta}(TB)$. That is, $\partial B = e^{-XT-\eta}(TB)$. So, $e^{-XT-\eta}$ units of Treasury bond futures are purchased for every unit of corporate bond held long. Using the BSM model, a long position in $(1/N(d) - 1)$ units of equity is held for every unit of corporate bond held long. For the credit spread call options, $\Delta$ represents the number of units of risky bond held for each credit spread call option. Therefore, $1/\Delta$ is the number of call spread options sold for every risky bond held long. For the combination hedge, the sensitivity of the corporate bond price is given by the total differential, $\partial B = (\partial B)/(\partial nS)\partial nS + ((\partial B)/(\partial (TB)))\partial (TB) = ((1/(N(d)) - 1) - \partial nS + e^{-XT-\eta}\partial (TB)$. So a position is taken in equity and treasury bonds in order to hedge. See footnote 9 for an explanation of the positions held for the simulation exercise.

a The derivation for this hedge ratio can be found in Appendix B.
Schwartz (1995b) use the probability of default under the risk-neutral measure to value a risky bond.

Jarrow, Lando, and Turnbull (1997) provide a method for estimating the risk-neutral probabilities of default and credit-rating changes from a cross section of default-free and risky bonds for subsequent use in reduced-form models. In these models, default is characterized using a “default intensity,” which under certain conditions on diversification of credit risk can be shown to be identical under the martingale (risk-neutral) and the empirical measure (Jarrow, Lando, & Yu, 2000). Arvanitis, Gregory, and Laurent (1999) and Delianedis and Geske (1999) also use risk-neutral probabilities in modeling default risky securities.

The credit spread options used in this study are priced using a one-step binomial model and risk-neutral valuation. Details can be found in Appendix B. As previously discussed, approaching the valuation of the credit spread option using discounted expected payoff under the equivalent martingale measure rests on the assumption that markets are dynamically complete. Whilst recognizing that this assumption is unlikely to be satisfied for a credit derivative given the infancy of the market, it is an approach that has been extensively used in valuation. Lack of completeness and illiquidity in the credit derivative market may drive actual market prices away from prices produced by theoretical models based on risk-neutral pricing. Nevertheless, the pricing method will yield a benchmark or approximate price, even when markets are not dynamically complete. The one-period nature of the model is a

Fig. 3. The 2-year credit spread estimation.
simplification used to investigate whether the model can improve hedging performance as compared to the duration model (in the spirit of Jarrow & van Deventer, 1998).

Fig. 4. The 5-year credit spread estimation.

Fig. 5. The 10-year credit spread estimation.
The value of the option in the binomial model with up and down factors “u” and “d,” respectively, is given by Eq. (7):

\[ c = e^{-rt} (pf_u + (1 - p)fd) \]  

(7)

where \( p = \frac{e^{-XT} - e^{-Xd(T-t)}}{(e^{-Xu(T-t)} - e^{-Xd(T-t)})} \). Payoffs in the up state \( (f_u) \) and down state \( (fd) \) are given by Eq. (8):

\[ f_u = \max((Xu - \chi), 0) \times T \times Fe^{-(r+\chi)T}, \text{ and} \]
\[ f_d = \max((Xd - \chi), 0) \times T \times Fe^{-(r+\chi)T}. \]  

(8)

In this model, \( p \) is the risk-neutral probability and \( \chi \) is the exercise price of the spread option. This simple discrete model ignores some of the properties of the credit spread. The model assumes that the expected spread in the next period is the current spread and constructs a portfolio that is free of credit risk using risk-neutral probabilities. This construction guarantees the absence of arbitrage. However, it neglects any relationship between the processes governing the risk-free rate and the credit spread.

A similar approach is used by Das (1997) who notes that credit spreads tend to be mean reverting. He builds a simple one-period risk-neutral model of the credit spread that is a discretization of the square root process. Equal probabilities of 0.5 are assigned to the

Table 3
Statistical summaries of theoretical and observed spreads

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>n</th>
<th>S.D.</th>
<th>S.E. mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-year observed spreads</td>
<td>110.0129</td>
<td>387</td>
<td>56.5248</td>
<td>2.8733</td>
</tr>
<tr>
<td>2-year theoretical spreads</td>
<td>15.5385</td>
<td>387</td>
<td>27.0525</td>
<td>1.3752</td>
</tr>
<tr>
<td>5-year observed spreads</td>
<td>135.7571</td>
<td>387</td>
<td>77.1056</td>
<td>3.9195</td>
</tr>
<tr>
<td>5-year theoretical spreads</td>
<td>62.0179</td>
<td>387</td>
<td>94.7439</td>
<td>4.8161</td>
</tr>
<tr>
<td>10-year observed spreads</td>
<td>157.2119</td>
<td>387</td>
<td>92.4502</td>
<td>4.6995</td>
</tr>
<tr>
<td>10-year theoretical spreads</td>
<td>111.9017</td>
<td>387</td>
<td>155.3491</td>
<td>7.8968</td>
</tr>
</tbody>
</table>
lattice, and a risk adjustment guarantees that the risk-neutral process is consistent with the absence of arbitrage.

4. Data and hedging strategies

4.1. Data

First Interstate Bancorp collected a data set over the period January 1986–August 1993 for risk management purposes. First Interstate Bancorp polled six major investment banking firms. Each firm was asked for its weekly quotation on the issue of $100 million worth of bonds at a maturity of 2, 3, 5, 7, and 10 years. The final data set for each maturity was obtained by averaging four quotes after eliminating the highest and lowest quotes. Credit ratings for First Interstate Bancorp given by Standard and Poor’s are listed in Table 1.

The data set is divided into three periods based on the rating. The 2-, 5-, and 10-year risk-free rates were collected from Bloomberg. Credit spreads are shown in Fig. 1 and share prices of First Interstate Bancorp over the period are shown in Fig. 2.

4.2. The hedging strategies

The bond portfolio contains market (interest rate) risk and credit risk. An investor holding First Interstate Bancorp bonds may adopt hedging strategies that are concerned with either interest rate risk, credit risk, or a combination of the two risks. Table 2 documents the strategies and the hedge parameters.

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Table 5

<table>
<thead>
<tr>
<th>The differences between observed spreads and theoretical spreads</th>
<th>Paired differences</th>
<th>S.E. mean</th>
<th>95% Confidence interval of the difference</th>
<th>( t )</th>
<th>df</th>
<th>Sig. (two-tailed)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>S.D.</td>
<td></td>
<td>Lower</td>
<td>Upper</td>
<td></td>
</tr>
<tr>
<td>2 years</td>
<td>94.4744</td>
<td>38.6721</td>
<td>1.9658</td>
<td>90.6094</td>
<td>98.3395</td>
<td>48.059</td>
</tr>
<tr>
<td>5 years</td>
<td>73.7392</td>
<td>56.6528</td>
<td>2.8798</td>
<td>68.0771</td>
<td>79.4013</td>
<td>25.605</td>
</tr>
<tr>
<td>10 years</td>
<td>45.3102</td>
<td>96.6225</td>
<td>4.9116</td>
<td>35.6533</td>
<td>54.9670</td>
<td>9.225</td>
</tr>
</tbody>
</table>

A comparison of the actual spreads with those predicted by the BSM model for 2-, 5- and 10-year bonds.

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7 Background information on First Interstate Bancorp can be found in Appendix A.
8 Only one data point, which was on 14 August 1992, was considered as an outlier. The credit spread for 2 years suddenly jumped from 90 to 350 and then fell back to 88 and for 5 years jumped from 107 to 250 then fell to 103. There was no major event detected around that time. Therefore, that quotation was replaced by the average of the previous and the next quotes.
The simulation for hedging risk is constructed as follows:

1. At the start of each period, it is assumed that $1 million principal amount of First Interstate Bancorp 2-year zero-coupon bonds is purchased.
2. Simultaneously, the abovementioned hedging strategies are constructed.
3. After 1 week, all the positions are closed and the payoff is calculated.\(^9\)
4. Steps 1–3 are repeated for each week from 25 October 1985 to 20 August 1993.
5. The standard deviations of the payoffs are calculated.
6. The equivalent bond value at end of each week and corresponding credit spread are calculated.

The credit spread option used here is a call option on the credit spread with a maturity of 3 months and with the strike spread set equal to the current credit spread.\(^10\) It is assumed that

\(^9\) It is assumed that the bond can be sold at the rate quoted at the beginning of each week. Therefore, the risk faced by an investor holding the risky bond portfolio is the risk that the interest rate and/or the credit spread will decrease at the beginning of the next week (because we have to spend more to buy back the bonds with the same face value). Hence, the hedging strategy will be to buy Treasury bond futures (at the current rate) or sell the credit spread call option.

\(^10\) Note that for each calculation of the credit spread option the volatility of the spread is estimated using an historical standard deviation on the preceding 52 weekly observations of the credit spread.

<table>
<thead>
<tr>
<th>Hedging strategies</th>
<th>S.D.</th>
<th>Percent reduction in S.D.</th>
<th>Period by rating</th>
<th>S.D.</th>
<th>Percent reduction in S.D.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without hedge</td>
<td>3370.81</td>
<td>–</td>
<td>AA 3447.05</td>
<td>–</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>A 3202.46</td>
<td>–</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>BBB 3372.53</td>
<td>–</td>
<td></td>
</tr>
<tr>
<td>One to one hedge</td>
<td>1851.07</td>
<td>45.09</td>
<td>AA 1397.28</td>
<td>59.46</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>A 615.007</td>
<td>80.80</td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td>BBB 2350.36</td>
<td>30.31</td>
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<tr>
<td>Duration hedge</td>
<td>1844.86</td>
<td>45.27</td>
<td>AA 1379.27</td>
<td>59.99</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>A 613.074</td>
<td>80.86</td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td>BBB 2347.3</td>
<td>30.40</td>
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<tr>
<td>The BSM hedge</td>
<td>3359.98</td>
<td>0.32</td>
<td>AA 3447.03</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td></td>
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<td>A 3200.25</td>
<td>0.07</td>
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<td></td>
<td>BBB 3758.02</td>
<td>−11.43</td>
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<tr>
<td>Credit spread call option hedge</td>
<td>2956.159</td>
<td>12.30</td>
<td>AA 3478.719</td>
<td>−0.92</td>
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<tr>
<td></td>
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<td></td>
<td>A 3150.553</td>
<td>1.62</td>
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<td>BBB 2460.259</td>
<td>27.05</td>
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<tr>
<td>The duration and the BSM hedge</td>
<td>2278.34</td>
<td>32.41</td>
<td>AA 1380.74</td>
<td>59.94</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>A 612.732</td>
<td>80.86</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>BBB 3035.75</td>
<td>9.99</td>
<td></td>
</tr>
<tr>
<td>The duration and the credit spread call option hedge</td>
<td>510.029</td>
<td>84.87</td>
<td>AA 288.482</td>
<td>91.63</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>A 337.560</td>
<td>89.46</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>BBB 659.312</td>
<td>80.45</td>
<td></td>
</tr>
</tbody>
</table>
the options sold can be bought back at their theoretical price after 1 week. This treatment assumes away any liquidity problems and reduces the problem of mispricing (or model risk).

5. Results

5.1. The 2-, 5-, and 10-year credit spread estimation

The 2-, 5-, and 10-year credit spreads are estimated with the BSM model and are shown in Figs. 3–5, respectively. The statistical summaries of the tests are in Table 3 and the correlations between the observed spreads and the theoretical spreads are shown in Table 4. Table 5 gives the tests results of comparison of the theoretical spreads and the observed spreads.

Figs. 3–5 show that the BSM model seems to correctly predict the shape of the credit spread but does not give a good estimation of the observed spreads, a statement that is reinforced with the correlation results in Table 4. The BSM model systematically underestimates the credit spreads for 2-year spreads, where the mean difference is 94.47 basis points. However, the BSM model sometimes overestimates the credit spreads when the maturity is longer, as can be seen in Figs. 4 and 5.

Table 7
5-year bond hedging results

<table>
<thead>
<tr>
<th>Hedging strategies</th>
<th>S.D.</th>
<th>Percent reduction in S.D.</th>
<th>Period by rating</th>
<th>S.D.</th>
<th>Percent reduction in S.D.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without hedge</td>
<td>6192.14</td>
<td></td>
<td>AA</td>
<td>6829.78</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>A</td>
<td>5078.89</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>BBB</td>
<td>6117.22</td>
<td></td>
</tr>
<tr>
<td>One to one hedge</td>
<td>3369.04</td>
<td>45.59</td>
<td>AA</td>
<td>2508.49</td>
<td>63.27</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>A</td>
<td>1202.6</td>
<td>76.32</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>BBB</td>
<td>4295.74</td>
<td>29.78</td>
</tr>
<tr>
<td>Duration hedge</td>
<td>3320.58</td>
<td>46.37</td>
<td>AA</td>
<td>2416.103</td>
<td>64.62</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>A</td>
<td>1204.463</td>
<td>76.28</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>BBB</td>
<td>4253.053</td>
<td>30.47</td>
</tr>
<tr>
<td>The BSM hedge</td>
<td>6735.767</td>
<td>−8.78</td>
<td>AA</td>
<td>6838.2</td>
<td>−0.12</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>A</td>
<td>5057.03</td>
<td>0.43</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>BBB</td>
<td>7209.93</td>
<td>−17.86</td>
</tr>
<tr>
<td>Credit spread call option hedge</td>
<td>5477.092</td>
<td>11.55</td>
<td>AA</td>
<td>6882.48</td>
<td>−0.77</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>A</td>
<td>4873.275</td>
<td>4.05</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>BBB</td>
<td>4538.162</td>
<td>25.81</td>
</tr>
<tr>
<td>The duration and the BSM hedge</td>
<td>4491.42</td>
<td>27.47</td>
<td>AA</td>
<td>2455.32</td>
<td>64.05</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>A</td>
<td>1202.79</td>
<td>76.32</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>BBB</td>
<td>6073.37</td>
<td>0.72</td>
</tr>
<tr>
<td>The duration and the credit spread call option hedge</td>
<td>549.727</td>
<td>91.12</td>
<td>AA</td>
<td>353.164</td>
<td>94.83</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>A</td>
<td>286.6198</td>
<td>94.36</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>BBB</td>
<td>710.093</td>
<td>88.39</td>
</tr>
</tbody>
</table>
In sum, the statistical tests reject the BSM model. The results are consistent with previous studies such as Jones, Mason, and Rosenfeld (1984) and Ogden (1987). The BSM model systematically underestimates the credit spreads for the 2-year credit spread. However, the 5- and 10-year estimations during the period starting from the beginning of 1990 were greater than the observed spreads in certain periods.

5.2. Hedging results

The hedging results for 2-, 5-, and 10-year bonds are shown in Tables 6–8, respectively. The first column lists all the hedging strategies. The second column is the standard deviation of the final payoff for the whole period, measured in dollars. The standard deviations for each period are listed in Column 5. Columns 3 and 6 are the reduction in standard deviation of each hedging strategy.

The results show that the combination of the duration hedge and the credit spread option hedge performs the best among all the other hedging strategies in all three types of bonds. The reduction in standard deviation for 2-, 5-, and 10-year bonds are 84.87%, 91.12%, and 92.63%, respectively. This result is illustrated in Fig. 6,\textsuperscript{11} where the hedging error from the

\begin{table}[h]
\centering
\caption{10-year bonds hedging results}
\begin{tabular}{llllll}
\hline
\hline
Without hedge & 7192.09 & – & AA & 8376.29 & – \\
 & & & A & 5081.53 & – \\
 & & & BBB & 6986.41 & – \\
One to one hedge & 4289.84 & 40.35 & AA & 3415.12 & 59.23 \\
 & & & A & 1687.49 & 66.79 \\
 & & & BBB & 5358.07 & 23.31 \\
Duration hedge & 4126.99 & 42.62 & AA & 3193.26 & 61.88 \\
 & & & A & 1631.34 & 67.90 \\
 & & & BBB & 5191.05 & 25.70 \\
The BSM hedge & 7191.336 & 0.01 & AA & 8343.14 & 0.40 \\
 & & & A & 5017.99 & 1.25 \\
 & & & BBB & 7028.59 & –0.60 \\
Credit spread call option hedge & 6120.268 & 14.90 & AA & 8108.75 & 3.19 \\
 & & & A & 4814.2 & 5.26 \\
 & & & BBB & 4847.07 & 30.62 \\
The duration and the BSM hedge & 4323.09 & 39.89 & AA & 3266.11 & 61.01 \\
 & & & A & 1599.11 & 68.53 \\
 & & & BBB & 5486.18 & 21.47 \\
The duration and the credit spread call option hedge & 530.41 & 92.63 & AA & 424.31 & 94.93 \\
 & & & A & 200.15 & 96.06 \\
 & & & BBB & 663.02 & 90.51 \\
\hline
\end{tabular}
\end{table}

\textsuperscript{11} Figs. 6 and 7 show only a portion of the data, so that the results can be clearly seen. Results for the whole period are similar.
combined duration and credit option hedge is shown as the solid line. Clearly, managing the two risks, interest rate and credit risk, has resulted in an almost perfect hedge as evidenced by the lack of variability in the hedging outcomes. That this combination hedge works so well is explained by the fact that the credit option hedging errors are negatively correlated with the hedging errors from the duration hedge, as is illustrated in Fig. 7.

The combination hedge significantly dominates the second best hedging strategy, the duration hedge, which can reduce the risk by only about 30%. The duration hedge itself works fairly well in the first and the second periods (refer to Table 1), when the bonds are
rated as AA to A−. The standard deviation can be reduced by up to 80%. However, the effectiveness of duration hedge reduces sharply in the last period, when the bonds are rated as BBB. For example, for 2-year bonds, the duration hedge reduces the standard deviation by 59.99% and 80.86% in the first two periods, but only by 30.40% for the last period. In sum, the results are consistent with previous studies, which show that the duration hedge is not as effective when the bond is subject to credit risk. Furthermore, the effectiveness of the duration hedge also reduces when the maturity of the bond increases.

The credit spread call option hedge works poorly by itself, especially in the first two periods. The average risk reductions overall are less than 5%. The hedge increases rather than reduces the risk of the portfolio in the first period for the 2- and 5-year bonds. This is because the credit spread option hedges only credit risk and ignores interest rate risk. The performance of the option hedging improves significantly in the last period to 27.05%, 25.81%, and 30.62% for the three types of bonds, respectively.

Summarizing, the results present evidence that in order to hedge the total risk of the risky bond portfolio, both interest rate risk and credit risk management methods are important and both risks need to be hedged. The BSM hedging strategy is not able to reduce the standard deviation but adds more volatility to the portfolio in the hedging simulation. These results suggest that the standard deviation reductions of the BSM model hedging in the tests done by Jarrow and van Deventer (1998) may be due to the additional duration hedging. The reductions in standard deviations for the BSM hedge in the first two periods are not significantly different from zero. This reflects the fact that the BSM model underestimates the credit spreads from 1985 to the end of 1990, where most of theoretical spreads approach zero for 2- and 5-year bonds. Consequently, the hedge ratios in these two periods are almost zero. The performance of the BSM model worsens in the last period for all three bonds. The standard deviations actually increase by 11.43%, 17.76%, and 0.6%, respectively. Jarrow and van Deventer also note that on average the direction of the Merton hedge appears incorrect.

6. Conclusions

This study has shown that the BSM model generates credit spreads, which are significantly different from, even though highly correlated with, observed credit spreads. The hedging simulation results show that the BSM model cannot reduce but rather increase the underlying risk of the bond portfolio.

In order to test the effectiveness of a credit derivative in hedging the credit risk of First Interstate Bancorp, a one-step binomial model is used to price a credit spread call option. While it is acknowledged that the model will give less than accurate pricing, the usefulness of the option as a hedging instrument can be tested. Our results show that using this model, the combination of the traditional duration hedging strategy with the credit spread call option

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12 This relationship, as seen in Figs. 3–5, has been the basis for using the BSM model to predict credit spreads by calibrating the model’s predictions to empirically observed credit spreads.
hedge significantly dominates all the other models in hedging the total risk of a bond portfolio. This hedging strategy gives a reduction in the standard deviation of the hedging errors of 84.9%, 91.1%, and 92.6% for 2-, 5-, and 10-year bonds, respectively. Our results show that a combination of duration hedging with a credit spread option hedge effectively can significantly reduce both the interest rate risk and the credit risk of a subinvestment grade bond portfolio.

Our results confirm a number of common perceptions concerning the hedging of interest rate and credit risk. In particular, the results highlight the need for an integrated approach to hedging interest rate risk and credit risk for securities subject to default risk. We have used a simple binomial model for the credit spread option. This approach is useful for demonstrating the possible risk reduction achievable through hedging using a credit derivative. However, other models of the credit spread are needed for accurate pricing. Given the illiquidity in the market for credit derivatives, it is likely that the cost of a credit spread option to hedge the exposure of an individual credit would be prohibitive. A topic for future research is to investigate the more interesting question of possible hedging approaches for mitigating the risk on a portfolio of risky debt using credit derivatives on appropriate indexes.

Acknowledgments

The authors are grateful to Kamakura and Dr. Donald van Deventer for supplying the First Interstate Bancorp data set used in this study. We thank Kevin Davis and Iain Maclachlan for helpful comments.

Appendix A. Background information on First Interstate Bancorp

First Interstate Bancorp operated the largest retail banking system in the US with 1040 branches at the end of the 1980s. In 1989, the bank lost $388 million in Texas and $154 million in Arizona. As a result, First Interstate Bancorp as a whole lost $124.5 million or $3.30 a share. When the financial statement was released at the beginning of 1990, the size of the loan loss reserves and timing surprised the market and professional analysts. After writing off the losses, First Interstate Bancorp consequently had an equity to asset ratio of 2.9%, excluding goodwill. An equity to asset ratio below 4.0% is considered unhealthy.

To offset the losses and push the capital ratio back to the required level, First Interstate Bancorp tried to raise $400 million by issuing new shares. However, after announcing the new share issue at the beginning of January 1990, investors’ concerns drove the bank’s stock price down about $20 a share to $33, forcing the company to issue more shares than originally planned. Eventually, the bank could only raise $280 million. This was well below its original target. In order to search for more capital, the bank decided to sell one of its Colorado branches and the New Mexico branch.

The bank’s share price fell 30% from the beginning of 1990 to June of that year, making it an attractive acquisition target. First Interstate Bancorp stocks became very speculative when the Kohlberg Kravis Roberts and Co. purchased about half of First Interstate Bancorp’s new
issues, giving it a holding of 9.98% in the bank. Kohlberg Kravis Roberts and Co is a New York merchant bank best known for leveraged buyouts. The activities by arbitragers further increased the trading volume and volatility in its stock after takeover rumours spread. From the beginning of 1990, about 42% of the shares have changed hands within 5 months. All of the abovementioned reasons significantly increased the volatility of the First Interstate Bancorp stocks.

Appendix B. The derivation of the value of a call option on the credit spread

The value of the credit spread options can be derived based on risk-neutral valuation principles. Construct a portfolio consisting of a long position in a number $\Delta$ of the risky bonds and a short position in one credit spread call option. The risky bond is further assumed to have a face value of $F$ and maturity of $T$.

A simple binomial model is used for the spread assuming that the credit spread $X$ can either move up from $X$ to $X_u$ or down from $X$ to $X_d$.

The call option ($f$) on the credit spread has payoffs at the end of period, given by $f_u$ and $f_d$ corresponding to the movement of the credit spread. Because $-((\partial B)/(\partial X))=TFe^{-(r+X)T}$,

$$f_u = \max((X_u - \chi),0) \times T \times Fe^{-(r+X_u)T},$$

and

$$f_d = \max((X_d - \chi),0) \times T \times Fe^{-(r+X_d)T},$$

where $\chi$ is the strike spread (Eq. (A2.1)). We assume that the expected value of the credit spread at the end of the period is equal to the current spread. Although this may not be a good approximation to the way that credit spreads actually behave, it provides a useful benchmark for the hedging exercise. This gives

$$X = qX_u + (1 - q)X_d \text{ or } 1 = qu + (1 - q)d$$

where $q$ is the statistical probability of an up movement. The stochastic process for the credit spread implies that the variance of the proportional change in the credit spread in the small

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13 We assume that the risky bond on which the credit spread is written is a zero-coupon security.
time interval will be \((\sigma_X^2 \Delta t)\), where \(\sigma_X\) is the volatility of the credit spread. Also, because variance is given by \([E(X^2) - [E(X)]^2]\), it follows that
\[
qu^2 + (1 - q)d^2 - [qu + (1 - q)d]^2 = \sigma_X^2 \Delta t \quad (A2.3)
\]
Because \(u = 1/d\), from Eq. (A2.2), \(q = (1 - d)/(u - d)\). Substitute into Eq. (A2.3), the value of \(u\) can be found from the following equation: \(\sigma_X^2 \Delta t = u + (1/u) - 2\). Hence, Eq. (A2.4):
\[
u = \frac{\sigma_X^2 \Delta t + 2 + \sigma_X \sqrt{\sigma_X^2 \Delta t^2 + 4 \Delta t}}{2} \quad \text{and } d = 1/u \quad (A2.4)
\]
The payoff on the portfolio consisting of \(D\) units of risky bond and a short position in one credit spread option will be given by:
\[
\Delta F e^{-(r + Xu)(T-t)} - f_u
\]
\[
\Delta F e^{-(r + Xd)(T-t)} - f_d
\]
The \(\Delta\) is chosen so that the portfolio earns the same return at the end of the period: \(\Delta F e^{-(r + Xu)(T-t)} - f_u = \Delta F e^{-(r + Xd)(T-t)} - f_d\), so
\[
\Delta = \frac{f_u - f_d}{F e^{-(r + Xu)(T-t)} - F e^{-(r + Xd)(T-t)}} \quad (A2.5)
\]
Because \(F e^{-(r + Xu)(T-t)} < F e^{-(r + Xd)(T-t)}\), the hedge ratio (\(\Delta\)) for the risky bond is negative. To see this, note that when the credit spread increases, the value of the bond falls, but the value of the call option on the credit spread increases. Therefore, if one is holding the risky bond, to hedge the underlying credit risk, the call option on the credit spread should be bought.

The present value of the portfolio will be \(e^{-rt}(\Delta F e^{-(r + Xu)(T-t)} - f_u)\), which must be equal to the initial cost of setting up the portfolio if there are to be no arbitrage opportunities: \(\Delta F e^{-(r + Xu)(T-t)} - f_u = e^{-rt}(\Delta F e^{-(r + Xd)(T-t)} - f_d)\). Substituting \(\Delta\) from Eq. (A2.5),
\[
f = e^{-rt} \left( \frac{e^{-XT} - e^{-Xd(T-t)}}{e^{-Xu(T-t)} - e^{-Xd(T-t)}} f_u - \frac{e^{-XT} - e^{-Xu(T-t)}}{e^{-Xu(T-t)} - e^{-Xd(T-t)}} f_d \right)
\]
The price for a credit spread call option therefore is
\[
f = e^{-rt} (pf_u + (1-p)f_d) \quad (A2.6)
\]
where \(p = ((e^{-XT} - e^{-Xd(T-t)})/(e^{-Xu(T-t)} - e^{-Xd(T-t)}))\) is the risk-neutral probability of an up movement in the binomial lattice (Eq. (A2.6)).
References


