Does adding up of economic capital for market- and credit risk amount to conservative risk assessment? ∗

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1. Introduction

The distinction between market and credit risk and their independent analysis has a certain tradition in the banking industry. Both practitioners and regulators have traditionally thought credit risk to be mainly relevant for the banking book and market risk to be mainly relevant for the trading book. In this way, the separation into market and credit risk mimics the traditional organization within banks into credit departments and market investment departments.

The traditional association of credit risk to the banking book and market risk to the trading book may have inspired arguments to the effect that keeping separate economic capital for market and credit risk is conservative. Implicitly these arguments follow the following pattern:

Premise 1 ‘Diversification’: Under a subadditive risk measure the risk of the total portfolio will be smaller than or at most equal to the sum of the risk of the banking book and of the trading book.

Premise 2 Credit risk is only relevant to the banking book and market risk is only relevant to the trading book.

Conclusion Under all subadditive risk measures total risk will be smaller or at most equal to the sum of market risk and credit risk.

This is a valid argument. If the premises are true, the conclusion must also necessarily be true. The conclusion can be wrong only if at least one of the premises is wrong. Premise 1 is not disputable; it is the definition of subadditivity. Premise 2 is usually not accepted literally, but it is considered to be a good approximation. So the Conclusion need not necessarily be true—at least not based on the argument. Still it is a popular view among regulators and many practitioners.
We will show in Section 2 that the inverse of the above argument also holds. Assuming Premise 1, the Conclusion holds only if Premise 2 holds: only if the portfolio is separable into a market subportfolio depending exclusively on market risk but not on credit risk factors, and a credit subportfolio depending just on credit but not on market risk factors, will integrated risk capital be smaller than the sum of market risk and credit risk capital. In other words, underestimation of risk is possible if the portfolio cannot be separated into a market and a credit subportfolio.

In this paper, we challenge the traditional view that integrated risk capital will always be smaller than the sum of market and credit risk capital. We reject this conclusion, both in its literal form, and as an approximation. We argue that in many situations a split into credit market portfolio is not possible because some positions will simultaneously depend on market risk and credit risk factors. If, in such a situation, a separation into two subportfolios is enforced, this will necessarily lead to wrong assessment of the true risk. Using the example of foreign currency loans, we illustrate this compounding effect.

Our results apply to all situations where market risk and credit risk are computed for a common time horizon. In practice market and credit risk are often computed for different time horizons: market risk is often computed over a time horizon of ten days whereas credit risk is usually computed across the one year horizon. Many economic capital calculations exist where it makes sense to use a common time horizon.

In credit risk analysis, PDs may depend on market prices. On the level of individual loans, payment obligations depend on market prices like interest rates or exchange rates. A counterparty with given payment ability is more likely to default if its payment obligation increases. On the portfolio level, additional risk is introduced by the common exposure of borrowers to interest or exchange rates. An adverse move in these risk factors drives up default correlations. If collateral is marked to market, LGD also may depend on market risk factors, illustrating another potential channel for market risk and credit risk interaction over a common time horizon. Our example of FX loans also illustrates the problem of “wrong way exposure”. We have wrong way exposure if PDs are high when exposure is high. This is a case of the interaction between market and credit risk via the channel of market risk factors influencing PD and LGD. Finger (1999) proposes some measure of exposure that, when multiplied by the counterparty default probability, gives the expected default loss on the transaction. Our integrated market–credit risk model captures wrong way exposure via the payment obligation process. In fact, it captures not only the dependence between PD and exposure, but also the dependence of default correlations on macro risk factors.

Defaults may increase exposure to market risk. Positions hedged against market risk might suddenly be exposed because one side of the hedge defaults. For example, in the Russian crisis of 1998 some Western banks suffered severe losses from dollar–rouble futures. They held opposite future positions with Russian and Western counterparties. These hedged positions suddenly became unhedged when Russian counterparties defaulted, and the open exposure was severely hit by the simultaneous devaluation of the rouble. PDs can also affect market prices of collaterals. If default frequencies increase and banks have to liquidate collaterals, this may reduce their value. The loop from credit risk to market risk and back to credit risk can be closed. For example, an increase in PDs might give rise to a drop of collateral values, which in turn increases LGDs. A credit risk model which neglects the bidirectional link to the collateral market could therefore fail to describe important sources of risk.

Our analysis studies the interaction between market risk and credit risk over a common time horizon both theoretically and by an illustrative example. It has implications for supervisory practice and principles. On the level of supervisory practice caution is required when in a risk assessment problem market and credit risk are analysed separately when in fact significant interaction between market and credit risk can be expected from the nature of the problem. For instance, a bank can pass on the interest rate risk (the market risk component) of a loan to the borrower. But since the loan repayment capacity of a borrower is a function of the interest rate, by doing so the bank may in fact only substitute a market risk for a credit risk. While the market risk is passed on to the borrower it still remains relevant for the bank indirectly as a potential credit risk. In such a situation an integrated analysis of market risk and credit risk is essential for an adequate calculation of economic capital. In terms of supervisory principles, our analysis suggests that we need to rethink the approach to look at risk classes along the dimensions of market–credit and maybe also operational risk and then analyse these risks separately and in isolation. The principle should be to analyse all the relevant risk factors for a given risk assessment problem, no matter whether they might be considered as market- or a credit risk, and to study their simultaneous impact on portfolio risk.

Related research. The literature on integration of market and credit risk seems to take different perspectives on the risk integration problem. One strand of the literature takes a critical view of the traditional categorization. Jarrow and Turnbull (2000) is an early paper that develops a reduced form model for incorporating stochastic interest rates into traditional credit risk models. Medova and Smith (2005) develop a credit risk framework that incorporates stochastic interest rates but which is based on a structural credit risk model. Barnhill and Maxwell (2002) propose a simulation framework for an integrated market and credit risk analysis for fixed income portfolios. In contrast to these papers, which all concentrate on modelling issues, our paper works with a model that is stripped down to the conceptual essentials while focussing on the aspect of comparing an integrated risk analysis with a separate analysis of credit and market risk.

Duffy and Singleton (2003, Chap. 13) report on Duffy and Pan (2001) and compare pure market risk (in the absence of credit risk) to integrated risk of a loan portfolio and find that integrated risk is higher than pure credit risk. In contrast this paper compares integrated risk to the sum of pure market risk and pure credit risk.

Another strand of the recent literature about integrated risk modelling seems to take a different perspective (see Rosenberg and Schuermann, 2006; Dimakos and Aas, 2004). These papers do not take issue with the traditional categorization, but instead point out that the portfolios analysed under the different categories market and credit risk, can be understood as risks of subportfolios of the total bank portfolio. Clearly when subportfolios can be constructed the only issue that remains to be discussed is quantifying the diversification effect if these subportfolios are merged into an overall portfolio. This is exactly what these authors do in their papers. In contrast, we argue that the issue of an integrated market and credit risk analysis is not a diversification issue. The problem is often that the subportfolio construction along market risk and credit risk factors is not possible. If, in such a situation the portfolio value is approximated by subportfolios of market and credit risk, a valuation error will usually lead to a risk assessment error and if worse comes to worst to a significant underestimation of the true risk.

The paper is organized as follows: Section 2 gives a theoretical analysis where the traditional approach is contrasted with an integrated analysis, Section 3 analyses foreign currency loans by means of a toy example. Section 4 extends the toy model to a real world simulation of a hypothetical foreign currency loan portfolio. Section 5 concludes with a discussion of regulatory implications. All proofs are collected in Appendix A.
2. Integrated versus separate analysis of market and credit risk

Market risk is defined as the risk that a financial position changes its value due to the change of an underlying market risk factor, like a stock price, an exchange rate or an interest rate. Credit risk is defined as the risk not receiving the promised payment on an outstanding claim. Market risk factors, the determinants of market risk, are usually market prices, or are derived from them. Credit risk factors, the determinants of the components of default losses, like default probabilities, losses given default, exposures at default, may be idiosyncratic properties of individual obligors, or macroeconomic variables influencing all obligors in the same way. Some risk factors may influence both market risk and credit risk. Interest rates, for example, are market prices determining the values of various fixed income instruments, but they also have an influence on default probabilities.

Assume a separation of risk factors into market and credit risk factors is given. It is not important for our argument which risk factors are actually seen as market or as credit risk factors. What matters is that one such separation is made. This assumption excludes situations where some or all risk factors affect both market and credit risk. In Section 5 we will give an alternative interpretation of our results with implications in this situation.

Risk assessment is based on portfolio valuation. Let us thus start with this aspect first. Assume a function \( V : \Omega_t \times \Omega_e \rightarrow \mathbb{R} \) is given, which specifies the value of a portfolio in dependence of some vectors \( a \in \Omega_t \) and \( e \in \Omega_e \) of credit and market risk factors, respectively.

\( \text{Market risk} \) deals with the value change of a portfolio which arises from movements in market risk factors, assuming that credit risk factors remain constant at some \( a_0 \):

\[ \Delta M(e) := V(a_0, e) - V(a_0, e_0). \]

Value changes are calculated in comparison with the portfolio value \( V(a_0, e_0) \) in a reference scenario \( (a_0, e_0) \). \( \text{Credit risk} \) deals with value changes caused by movements in credit risk factors, assuming all market risk factors are constant at \( e_0 \):

\[ \Delta C(a) := V(a, e_0) - V(a_0, e_0). \]

\( \text{Integrated risk} \) relates to the value change caused by simultaneous moves of market and credit risk factors:

\[ \Delta V(a, e) := V(a, e) - V(a_0, e_0). \]

By definition integrated risk analysis requires market and credit risk moves to be across the same time horizon. Therefore, integrated risk can only be compared to the sum of market and credit risk if market risk and credit risk are measured over the same time horizon.

Adding up economic capital for market and credit risk implicitly rests on the assumption that value changes can be approximated by the sum of market plus credit risk factor related value changes:

\[ \Delta V(a, e) \approx \Delta C(a) + \Delta M(e). \]

This corresponds to the approximation

\[ V(a, e) \approx V(a_0, e_0) + \Delta C(a) + \Delta M(e). \]

Clearly for a general portfolio valuation function \( V(a, e) \) the approximation \( \Delta C(a) + \Delta M(e) \) does not always overestimate but sometimes underestimates the true \( \Delta V(a, e) \). If in some scenario \((a, e)\) the approximation error

\[ D(a, e) := \Delta V(a, e) - \Delta C(a) - \Delta M(e) \]

is negative, we have malign risk interaction. Accordingly, if \( D \) is non-negative in all scenarios, we say we have benign interaction of credit and market risk.

Fig. 1 shows a situation with \( D < 0 \) where true integrated risk is underestimated. This negative interaction of risk is caused by the non-additivity of the value function \( V \). The following proposition classifies the functions \( V \) for which the approximation error is zero everywhere.
Proposition 1. The approximation is exact, that is
\[ \Delta V(a,e) = \Delta C(a) + \Delta M(e), \]
if and only if \( V \) has the form
\[ V(a,e) = V_1(a) + V_2(e). \] (2)

In this case the portfolio can be separated into two subportfolios, one depending purely on credit risk factors, the other depending solely on market risk factors.

This proposition is technically simple but conceptually important. In particular the “only if” part is interesting. Linear value functions \( V \) fulfil condition (2) and are therefore exactly approximated.

Moving on from valuation to risk assessment, the properties of the value change functions in various scenarios \((a,e)\) carry over to risk measures and risk capital. Define the functions \( A \) and \( E \) on \( \Omega_A \times \Omega_E \) by \( A(a,e) := a \) and \( E(a,e) := e \). If the parameter space \( \Omega_A \times \Omega_E \) is equipped with a probability measure, functions \( A \) and \( E \) can be regarded as random vectors, and functions \( V, \Delta V, \Delta C, \Delta M \), etc. can be regarded as random variables. Random vector \( A \) describes the credit risk factors and is the marginal distribution resulting from integrating over the market risk factors. Random vector \( E \) describes the market risk factors and is the marginal distribution resulting integrating over the credit risk factors. Credit, market, and integrated risk can be written as
\[ \Delta C = V(A,e_0) - V(a_0,e_0), \]
\[ \Delta M = V(a_0,E) - V(a_0,e_0), \]
\[ \Delta V = V(A,E) - V(a_0,e_0). \]

These functions can be identified with random variables describing the profit changes resulting in a pure credit risk, pure market risk, and an integrated analysis. They are random variables on the same probability space and can therefore be compared by any risk measure \( \rho \).

Defining as we did \( \Delta C \) and \( \Delta M \) on the basis of the marginal, instead of the conditional distributions conforms to the intuition that in a pure credit (resp. market) risk analysis the distribution of credit (resp. market) risk factors is determined empirically and without reference to the market (resp. credit) risk factors.\(^1\) Any coherent risk measure \( \rho \) can be applied to these random variables.

We measure the effect of an integrated analysis of market and credit risk by the index
\[ I := \frac{\rho(\Delta V)}{\rho(\Delta C) + \rho(\Delta M)}, \]
which we define if \( \rho(\Delta C) + \rho(\Delta M) > 0 \) and \( \rho(\Delta V) > 0 \). In the case of negative inter-risk interaction \( I > 1 \). Integrated risk is larger than the sum of market risk and credit risk. \( I = 1.2 \) means that total risk is 20% larger than the sum of credit and market risk. If \( I < 1 \) we have positive interaction of credit and market risk. Integrated risk will be smaller than the sum of credit and market risk.

Proposition 2. In the case of benign interaction of risk \((D \geq 0)\) separate analysis of market and credit risk overestimates true risk:
\[ \rho(\Delta V) \leq \rho(\Delta C) + \rho(\Delta M). \] (3)

This holds for all subadditive risk measures \( \rho \). Otherwise, in the case of malign interaction of risk \((D < 0 \text{ somewhere})\), there exists a coherent risk measure \( \rho \) for which separate analysis of market and credit risk overestimates true risk:
\[ \rho(\Delta V) > \rho(\Delta C) + \rho(\Delta M). \]

Propositions 1 and 2 together establish the inverse of the argument in the introduction. The Conclusion (“Under all subadditive risk measures the risk total risk is smaller than or at most equal to, the sum of market risk and credit risk.”) implies Premise 2. (“The portfolio is separable into a credit subportfolio and a market subportfolio.”)

Portfolios where credit and market risk are separated into different subportfolios were considered by Dimakos and Aas (2004) and Rosenberg and Schuermann (2006). In this case \( V \) is of the form
\[ V(a,e) = V_1(a) + V_2(e). \]
For such a portfolio by Proposition 1 the approximation is exact, i.e.,
\[ \Delta V(a,e) = \Delta C(a) + \Delta M(e) \]
and \( \Delta V = \Delta M + \Delta C \). Thus \( \rho(\Delta V) = \rho(\Delta C + \Delta M) \leq \rho(\Delta C) + \rho(\Delta M) \) and \( I < 1 \) for any subadditive risk measure \( \rho \). This implies that inter-risk interaction is always positive for a portfolio with credit and market risk separated into different subportfolios. For such portfolios the determination of risk capital as the sum of market and credit risk capital will necessarily be conservative. The above papers analyse the size of the diversification effect in the special case of separable portfolios.

3. A toy example of underestimation of the true risk

As an example where the need for an integrated analysis of market and credit risk is obvious we now analyse foreign currency loans. (For variable rate loans a similar effect was described in Breuer et al. (2008).) In order to understand the compounding effect for this particular example we first use a toy model that has been stripped to the bare essentials to reveal the fundamental mechanisms.

Foreign currency loans caught the attention (and raised the concern) of supervisory authorities because these instruments have recently become highly popular among private households to take out home mortgages. This form of mortgage financing has been especially popular in Austria and in Central and Eastern Europe. Foreign currency loans can be seen as a carry-trade. In the carry-trade, an investor borrows money from one country, where the borrowing cost is low, and invests it in another country, where investments yield a high rate of return. The flip-side of the advantage of a low borrowing rate is an exchange rate risk. Since the debt service capacity of a borrower is a function of the exchange rate, his credit risk is a direct function of market risk factor changes. Foreign currency loans are therefore a clear case where market and credit risk factors have to be studied simultaneously.

To formalise a foreign currency loan in a toy model, consider a single obligor who has taken out a Swiss Franc loan of 1 Euro. At the current exchange rate of \( f(0) \) this amounts to a Swiss Franc loan of \( 1/f(0) \), where \( f(0) \) is the home currency value of the foreign currency at time 0. After 1 year the loan expires and the payment obligation in the introduce. The Conclusion (“Under all subadditive risk measures the risk total risk is smaller than or at most equal to, the sum of market risk and credit risk.”) implies Premise 2. (“The portfolio is separable into a credit subportfolio and a market subportfolio.”)

\[ V(a,e) = \min(a,e) - e = - \max(e - a, 0). \] (5)

Now let us fix some reference scenario \((a_0, e_0)\). Credit risk, the profit or loss of the bank arising from moves in the credit risk factor

\[ \rho(\Delta V) \leq \rho(\Delta C) + \rho(\Delta M). \]

If \( \Delta C \) (resp. \( \Delta M \)) were to be defined on basis of the conditional distribution of credit (resp. market) risk factors given the values \( e_0 \) (resp. \( a_0 \)), \( \Delta C = \{ V(E - e_0) - V(a_0, e_0) \} \) (resp. \( \Delta M = \{ V(A - a_0) - V(a_0, e_0) \} \)), then even for a linear portfolio and normally distributed risk factors integrated risk might be larger than the sum of credit and market risk. Proposition 2 does not hold for \( \Delta C \) and \( \Delta M \) defined on the basis of the conditional distributions. This was pointed out to us by Piergiorgio Alessandrini.

\(^1\) If \( \Delta C \) (resp. \( \Delta M \)) were to be defined on basis of the conditional distribution of credit (resp. market) risk factors given the values \( e_0 \) (resp. \( a_0 \)), \( \Delta C = \{ V(E - e_0) - V(a_0, e_0) \} \) (resp. \( \Delta M = \{ V(A - a_0) - V(a_0, e_0) \} \)), then even for a linear portfolio and normally distributed risk factors integrated risk might be larger than the sum of credit and market risk. Proposition 2 does not hold for \( \Delta C \) and \( \Delta M \) defined on the basis of the conditional distributions. This was pointed out to us by Piergiorgio Alessandrini.
alone, assuming the payment obligation of the obligor will have the value $e_0$ with certainty, is

$$D_C(a) = \frac{V(a, e_0)}{C_0} \left(\frac{V(a, e_0)}{C_0} - \max\left(e_0 \big/ C_0, 0\right) + \max\left(0, \frac{e_0 - a_0}{C_0}\right)\right).$$

The profit or loss of the bank arising from moves in the market risk factor $e$ alone, assuming the payment ability of the obligor will have the value $a_0$ with certainty, is

$$D_M(e) = \frac{V(a_0, e)}{C_0} \left(\frac{V(a_0, e)}{C_0} - \max\left(e \big/ C_0, 0\right) + \max\left(0, \frac{e_0 - a_0}{C_0}\right)\right).$$

Assuming that no defaults are possible would amount to choosing $a_0 = 1.5$. But any other choice of $a_0$ would also be possible. The smaller is $a_0$ the more defaults will occur in the market risk scenarios. This increases market risk and decreases the negative inter-risk diversification effect. Still it is justified to call this a market risk analysis, because the credit risk factor is assumed to be constant and therefore is not a source of uncertainty.

Fig. 2 plots credit risk $\Delta C(a)$ (left) and market risk $\Delta M(e)$ (right) for the toy model with $a_0 = 1.5, e_0 = 0.9$.

Fig. 3. Plot of the function $D$ in the toy model for $a_0 = 1.5, e_0 = 0.9$. 
the payoff profile of a short put on the payment ability \(a\) with strike \(e_0\), which reflects Merton’s key idea of structural credit risk models, regarding a loan as a short put on the payment ability. Market risk has the payoff profile of a short call on the exchange rate \(e\) with strike \(a_0\).

Does the separate calculation of credit and market risk overestimate or underestimate integrated risk? Fig. 3 shows plots of the function \(D\) for \(a_0 = 1.5\) and \(e_0 = 0.9\). The function \(D\) is negative in some regions. For scenarios in this region, integrated risk is larger than the sum of credit risk plus market risk. This is an example of the malign compounding effect between credit risk and market risk. One can easily show analytically that \(D\) is negative in some region whenever \(a_0 \neq e_0\). Only in the special case \(a_0 = e_0\) is \(D\) everywhere non-negative and an integrated analysis always leads to lower risk capital than a separate analysis.

4. A real world example

We have analysed the logic of risk underestimation effects in theory and within the context of a toy example of a foreign currency loan. But do these effects matter in real world examples? We want to use the final section to extend the toy model to a real world model that can be brought to the data. Our integrated market risk and credit risk model is a structural model with a stochastic default barrier defined by the payment obligation of the customer, which in turns depends on market risk factors like interest and exchange rates. This model can capture the functional relation between interest rate and exchange rate fluctuations on the one hand and default probabilities and losses given default on the other hand. A relation of this kind, namely an increase in default risk triggered by adverse interest rate and exchange rate moves, is one key risk of FX loans. It cannot be captured by a pure credit risk model or a pure market risk model. Nor can a simple integrated model assuming some ("wrong way") correlations between default probabilities and FX rates capture the non-linear functional relation between these variables. The analysis of this section will give us some insight into the possible quantitative dimension of the problem.

Consider a portfolio of foreign currency loans with \(N\) obligors indexed by \(i = 1, \ldots, N\). All loans are underwritten at the initial time \(t = 0\). In order to receive the home currency amount \(l\), an obligor takes a loan of \(l/f(0)\) units in the foreign currency. The obligor borrows \(l/f(0)\) units of the foreign currency on the interbank market. After one period, at time \(t = 1\), which we take to be 1 year, the loan expires and the bank repays the foreign currency on the interbank market with an interest rate \(r\), and it claims a home currency amount from the customer which is exchanged at the rate \(f(1)\) to the foreign currency amount \((l/f(0))(1 + r + s_f)\), which is the original loan plus interest \(r\) rolled over from four quarters plus a spread \(s_f\). So the customer’s payment obligation to the bank at time 1 in the home currency is

\[ a_i = l_i(1 + r_f) f(1)/f(0) + l_i s_f f(1)/f(0). \]

(6)

The first term on the right hand side is what the bank has to repay on the interbank market, the second term is the spread profit of the bank. For a home currency loan the payment obligation would be

\[ a_i = l_i(1 + r_f + s_h), \]

where \(s_h\) is the interest rate in the home currency and \(s_f\) is the spread to be paid by the customer on a home currency loan.

Whether an obligor will be able to meet this obligation depends on his payment ability \(a_i\). Like in a structural credit risk model, we assume that an obligor defaults if his payment ability at the end of the period is smaller than his payment obligation.

**Assumption 1.** Obligors default in the event that their payment ability \(a_i\) at the expiry of the loan is smaller than their payment obligation \(o_i\). In the event of a default the customer pays \(a_i\) instead of \(o_i\).

The profit of the bank with obligor \(i\) is therefore

\[ V_i := \min(a_i, o_i) - l_i(1 + r_f) f(1)/f(0). \]

(7)

\(f(0)\) is the known exchange rate at time \(t = 0\). \(f(1)\) and \(r\) are random variables. In the profit function \(V_i\) the first term is what the obligor repays and the second term is what the bank has to pay on the interbank market.

We model the ability of an obligor to repay his obligations as a function of macroeconomic conditions and an idiosyncratic risk component. The form of our payment ability process resembles firm value process in the model of Merton (1974) but it is adapted to incorporate the macroeconomic influence as in Pesaran et al. (2005).

**Assumption 2.** The payment ability at final time 1 for each single obligor \(i\) is distributed according to

\[ a_i(1) = a_i(0) \cdot \frac{\text{GDP}(1)}{\text{GDP}(0)} \cdot \epsilon, \]

(8)

\[ \log(\epsilon) \sim N(\mu, \sigma), \]

(9)
where \(a(0)\) is a constant, and \(\mu = -\sigma^2/2\) ensuring \(\mathbb{E}(\epsilon) = 1\). For different obligors the realisations \(\epsilon_i\) are independent of each other and of GDP.

GDP(0) is the known GDP at time \(t = 0\), GDP(1) is a random variable. The distribution of \(\epsilon_i\) reflects obligor specific random events, like losing or changing job. The support of \(\epsilon_i\) is \((0, \infty)\) reflecting the fact that the amount \(a_i\) available for repayment of the loan cannot be less than zero if the obligor has no lines of credit open with the bank. Since the expected value of \(\epsilon_i\) is 1 and \(\epsilon\) is independent of GDP, the expectation of \(\epsilon(1)\) is \(\mu(1)\) times the expectation of GDP(1)/GDP(0). Pesaran et al. (2005) use a model of this type for the returns of firm value. Assumption 2 amounts to taking the predictable mean of the log-returns in their model to be \(\log(GDP(1)/GDP(0))\). A GDP increase shifts the payment ability distribution to the right. This is shown in Fig. 4. It increases distance to default and reduces default probabilities, provided the payment obligation is unchanged.

Assuming that the realizations of \(\epsilon_i\) are independent for different customers is the doubly stochastic hypothesis.\(^2\) Conditional on the path of macro and market risk factors which determine the default intensities of all customers, customer defaults are independent.

The initial payment ability \(a(0)\) is a customer specific parameter determined in the loan approval procedure. For example, to be on the safe side the bank can extend loans only to customers with \(a(0)\) equal to 1.2 times the loan amount. This extra margin is taken into account in the rating. From a rating system the bank determined from these two conditions. For example, to be determined in the loan approval procedure. For example, to be on the safe side the bank can extend loans only to customers with \(a(0)\) equal to 1.2 times the loan amount. This extra margin is taken into account in the rating. From a rating system the bank determined from these two conditions. For example, to be on the safe side the bank can extend loans only to customers with \(a(0)\) equal to 1.2 times the loan amount. This extra margin is taken into account in the rating. From a rating system the bank determined from these two conditions. For example, to be on the safe side the bank can extend loans only to customers with \(a(0)\) equal to 1.2 times the loan amount. This extra margin is taken into account in the rating. From a rating system the bank determined from these two conditions. For example, to be on the safe side the bank can extend loans only to customers with \(a(0)\) equal to 1.2 times the loan amount. This extra margin is taken into account in the rating. From a rating system the bank determined from these two conditions. For example, to be on the safe side the bank can extend loans only to customers with \(a(0)\) equal to 1.2 times the loan amount. This extra margin is taken into account in the rating. From a rating system the bank determined from these two conditions. For example, to be on the safe side the bank can extend loans only to customers with \(a(0)\) equal to 1.2 times the loan amount. This extra margin is taken into account in the rating. From a rating system the bank determined from these two conditions. For example, to be on the safe side the bank can extend loans only to customers with \(a(0)\) equal to 1.2 times the loan amount. This extra margin is taken into account in the rating. From a rating system the bank determined from these two conditions. For example, to be on the safe side the bank can extend loans only to customers with \(a(0)\) equal to 1.2 times the loan amount. This extra margin is taken into account in the rating. From a rating system the bank determined from these two conditions. For example, to be on the safe side the bank can extend loans only to customers with \(a(0)\) equal to 1.2 times the loan amount. This extra margin is taken into account in the rating.

The payment ability distribution must satisfy the following condition:

\[
p_i = P[a(1) < a].
\]

\(a(1)\) is a function of \(\sigma\) and \(a\) is a function of the spreads. Spreads are set to achieve some target expected profit for each loan:

\[
\mathbb{E}(V_i(\sigma, s)) = EP_{\text{target}}.
\]

where \(V_i\) is the profit with obligor \(i\) and \(EP_{\text{target}}\) is some target expected profit. The two free parameters \(\sigma\) and \(s\) (\(s_h\) resp. \(s_g\)) are determined from these two conditions.

How do credit and market risk factors interact in this model? At the end of the period, at time 1, after the obligor has paid the bank, and the bank has met its obligation at the interbank market, the bank has a net open foreign currency position \(s_h(f(1)/f(0))\) for obligor \(i\). This is the only part of the position for which current regulation requires market risk capital. Default risk, on the other hand, is determined by the probability that payment ability falls below payment obligation. This is a function of both the interest rate and the exchange rate. Thus default risk is a function of market risk factors. Therefore an integrated risk analysis is necessary.

To model the probability law of risk factors we use the GVAR time series model due to Pesaran et al. (2006). The GVAR model is an error correction model that allows a parsimonious modelling of economic interdependence between countries or regions. This is exactly what we need in terms of risk factors, which involve exchange rate, interest rates and macroeconomic interactions between Austria and Switzerland. The basic idea of GVAR modelling is that it allows the global model to be built from separately estimated country models with foreign variables entering the equation as weakly exogenous trade weighted averages. Country models can be estimated separately and stacked into a global model without

<table>
<thead>
<tr>
<th></th>
<th>GDP</th>
<th>FEUR</th>
<th>FCYP</th>
<th>CHF/USD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>5.446</td>
<td>1.246</td>
<td>0.556</td>
<td>0.423</td>
</tr>
<tr>
<td>Std. dev.</td>
<td>0.097</td>
<td>1.870</td>
<td>6.301</td>
<td>0.387</td>
</tr>
</tbody>
</table>

**Correlations**

<table>
<thead>
<tr>
<th></th>
<th>GDP</th>
<th>FEUR</th>
<th>FCYP</th>
<th>CHF/USD</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>1.000</td>
<td>0.291</td>
<td>0.217</td>
<td>-0.040</td>
</tr>
<tr>
<td>FEUR</td>
<td>1.000</td>
<td>0.519</td>
<td>0.140</td>
<td></td>
</tr>
<tr>
<td>FCYP</td>
<td>1.000</td>
<td>0.100</td>
<td>0.007</td>
<td>1.000</td>
</tr>
</tbody>
</table>

The distribution of the profit \(V\) Eq. (7) was calculated by a Monte Carlo simulation of 100 000 draws from the distribution of \(f(1)\), GDP and \(r\). In each macro scenario defaults of the customers’ payment abilities were determined by draws from the distribution of the payment ability process \(8\). The relative importance of GDP shocks versus idiosyncratic shocks is displayed in Fig. 4. The distribution of the macro risk factors was estimated from quarterly data 1989–2005 from the IFS of the International Monetary Fund. The estimated values for means and covariances of logarithms of the macro risk factors are given in Table 1.

Let the portfolio be given as \(N = 100\) loans of \(€10000\) taken out in CHF by customers in the rating class B+, corresponding to a default probability of \(p_i = 2\%\), or in rating class BBB+, corresponding to a default probability of \(p_i = 0.1\%\). We assume that the bank extends loans only to customers with \(a(0)\) equal to 1.2 times the loan amount.

Our way to quantify the interaction effect between market and credit risk is to compare the sum of market risk plus credit risk capital to capital required to cover integrated risk. An integrated market and credit risk analysis requires a common time horizon. We take this to be the 1 year time horizon usual for credit risk. We make no statement about how to combine market and credit risk computed over different time horizons.

We assume the bank leaves its loan portfolio unchanged over the time horizon. Over a time horizon of 1 year, this might be a reasonable approximation for consumer loan portfolios which are rarely traded. (For actively traded portfolios one would need a model of the rebalancing strategy of the portfolio managers. But this is beyond the scope of this paper.) Another simplification is our assumption that all loans are underwritten at the initial time \(0\) and simultaneously expire at time \(T\). These simplifications are however not essential for the key question we have in mind here:

"Can we expect negative risk interaction to be quantitatively negligible or not?"

The spreads \(s_h\) and \(s_g\) for each rating class were set in such a way that a target expected profit of \(€160\) on each loan is achieved, which amounts to a 20% return on an assumed capital charge of \(€800\) for a loan of \(€10000\). The resulting spreads are given in Table 2.

\(^2\) See Duffie and Singleton (2003). Note also that there is some empirical evidence that the doubly stochastic hypothesis might be violated, cf. Das et al. (2007).

\(^3\) To perform estimations and simulations we use our own \(R\)-implementation of the GVAR model based on Pesaran et al. (2000, 2006). Our implementation builds on work done by Zeugner (2006) who wrote a Matlab implementation of Pesaran et al. (2000).
Table 2
Spreads of rating classes.

<table>
<thead>
<tr>
<th>Rating</th>
<th>Loan type</th>
<th>Loan spread</th>
<th>Interest rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>BBB+</td>
<td>Home</td>
<td>0.0491</td>
<td>160.15</td>
</tr>
<tr>
<td>B+</td>
<td>Home</td>
<td>0.0736</td>
<td>165.62</td>
</tr>
<tr>
<td>BBB+</td>
<td>Foreign</td>
<td>0.0363</td>
<td>162.29</td>
</tr>
<tr>
<td>B+</td>
<td>Foreign</td>
<td>0.0755</td>
<td>168.97</td>
</tr>
</tbody>
</table>

Table 3
Risk capital for market, credit, and integrated risks of the foreign currency loan portfolio. Market risk \( \Delta M \) assumes no defaults are possible and considers only value changes due to market risk factor changes. Credit risk \( \Delta C \) reflects the value changes due to credit risk factor changes disregarding the possibility that market risk factors could vary stochastically. Market risk factors are fixed at their expected values. Integrated risk \( \Delta V \) calculates the value change assuming both credit and market risk factors simultaneously influence the value of a position. The final column calculates the inter-risk diversification index \( I \). Standard deviations are shown in brackets. Initial income \( a(0) = 12,000 \).

<table>
<thead>
<tr>
<th>Rating</th>
<th>( a ) (%)</th>
<th>MR no CR</th>
<th>CR no MR</th>
<th>Integrated</th>
<th>Risk interaction</th>
</tr>
</thead>
<tbody>
<tr>
<td>BBB+</td>
<td>0</td>
<td>1059 (3)</td>
<td>0 (0)</td>
<td>1.13</td>
<td>1.13</td>
</tr>
<tr>
<td>BBB+</td>
<td>5</td>
<td>234 (4)</td>
<td>0 (0)</td>
<td>1.23</td>
<td>1.23</td>
</tr>
<tr>
<td>BBB+</td>
<td>1</td>
<td>1576 (8)</td>
<td>0 (0)</td>
<td>1.94</td>
<td>1.94</td>
</tr>
<tr>
<td>BBB+</td>
<td>0.5</td>
<td>1698 (10)</td>
<td>1 (0)</td>
<td>2.73</td>
<td>2.73</td>
</tr>
<tr>
<td>BBB+</td>
<td>0.1</td>
<td>1951 (21)</td>
<td>3 (2)</td>
<td>8.22</td>
<td>8.22</td>
</tr>
<tr>
<td>B+</td>
<td>10</td>
<td>1012 (4)</td>
<td>795 (4)</td>
<td>1.43</td>
<td>1.43</td>
</tr>
<tr>
<td>B+</td>
<td>5</td>
<td>1285 (5)</td>
<td>1022 (6)</td>
<td>1.92</td>
<td>1.92</td>
</tr>
<tr>
<td>B+</td>
<td>1</td>
<td>1641 (8)</td>
<td>1523 (14)</td>
<td>3.54</td>
<td>3.54</td>
</tr>
<tr>
<td>B+</td>
<td>0.5</td>
<td>1768 (11)</td>
<td>1730 (19)</td>
<td>4.48</td>
<td>4.48</td>
</tr>
<tr>
<td>B+</td>
<td>0.1</td>
<td>2032 (22)</td>
<td>2257 (45)</td>
<td>7.59</td>
<td>7.59</td>
</tr>
</tbody>
</table>

Note that in the same rating class spreads for FX loans are slightly higher than for home currency loans. This reflects the higher risk of FX loans, emerging from the subsequent analysis (compare Tables 3 and 4). Additionally, in order to achieve e.g. BBB+ for a home currency loan, a customer only needs a \( \sigma \) of 0.0491, whereas a FX loan customer needs to achieve a much smaller \( \sigma \) of 0.0363. In other words, a customer with a given standard deviation \( \sigma \) in his payment ability will be in a higher rating class for a FX loan.

Does the separate calculation of credit and market risk capital overestimate or underestimate integrated risk capital? The market risk factors are \( e := (r_t, f_t, r_t) \) for foreign currency loans and \( e := (r_t) \) for home currency loans. Credit risk factors are \( a := (\text{GDP}, \text{inter}, \text{interest}) \) for FX loans and \( a := (\text{GDP, inter}) \) for home currency loans. The portfolio value function is \( V = \sum_{i=1}^{N} V_i \). As a reference scenario we take the expected values \( a_0 := \mathbb{E}(e) \) of the market risk factors and \( a_0 := (\mathbb{E}(\text{GDP}), \text{inter})_{i=1..N} \), which implies that no obligor defaults.

We compare the distributions of integrated risk \( \Delta V(a, e) \) to the sum \( \Delta C(a) + \Delta M(e) \). The distributions of credit and market risk by their Expected Shortfall (ES) at different quantiles \( \alpha \). In order to exclude non-subadditivity of the risk measure as a possible explanation for the negative inter-risk diversification effect, we calculate risk capital intended to cover unexpected losses as measured by Expected Shortfall (ES). For a profit loss distribution \( X \) risk capital is \( \mathbb{E}_c(X) := \mathbb{E}(X) - \mathbb{E}_S(X) \).

\[ \mathbb{E}_S(X) := \mathbb{E}(X) - \mathbb{E}_S(X) \]

(12)

where \( ES_\alpha \) is Expected Shortfall at some confidence level \( \alpha \), as defined, e.g. in Acerbi and Tasche (2002, Def. 2.6). Standard deviations of approximation errors of \( ES \) are calculated using the method of Manistre and Hancock (2005).

Table 3 displays the risk capital for market, credit, and integrated risk. The key result of the simulation are found in the last two columns, which display the index \( I \). They show malign risk interaction consistently, for all quantiles \( \alpha \), and in both rating classes. Integrated risk capital is significantly higher than the sum of credit risk and market risk capital. Separate analysis underestimates true risk by factors between 1.13 and 8.22 for BBB+ and factors between 1.43 and 7.59 for the B+ portfolio.

This dramatic effect clearly reflects a malign interaction of market and credit risk which cannot be captured by providing separately for market and credit risk capital. Holding separate risk capital for market risk and for credit risk is by far not sufficient to cover true risk. This does not come as a surprise. The main risk of foreign currency loans, namely the danger of increased defaults triggered by adverse exchange rate moves, is neither captured by pure market risk nor by pure credit risk models.

Table 4 shows that negative risk interaction also occurs for home currency loans. This can be explained by the dependence of default rates on the home interest rate. Home interest rate changes are reflected in payment obligation changes. An increase of this market risk factor therefore triggers an increase in default rates. But the effect for home currency loans is much smaller than for foreign currency loans. Separate analysis underestimates true risk by factors between 1.25 and 2.75, depending on quantile and rating class. Negative risk interaction is weaker because the payment obligation of home currency loans depends much less sensitively on market factor changes.

Note that pure market risk is zero for the home currency loan portfolio. The reason for this assuming that no customer defaults, profits of the bank are not affected by exchange or interest rate changes.

How sensitively do these results depend on the choice of initial payment ability \( a(0) \)? In our model the only two exogenous input parameters are \( a_0 \) and the rating class (resp. default probability). Table 5 shows the results for initial payment ability \( a(0) = 11,000 \) and \( a(0) = 13,000 \) at a confidence level of \( \alpha = 1% \). We observe that for lower \( a(0) \) the negative integration effect is considerably stronger than for higher \( a(0) \). For example,subprime loans are more exposed to dangerous interaction between market and credit risk. Borrowers having difficulties meet-
ing payment obligations that were rising with interest rates let to the onset of the subprime crises.

5. Conclusions

In this paper we challenge the traditional approach of dividing risks into market risk and credit risk. We argue that this approach is conceptually problematic because many portfolios are not separable into a market subportfolio and a credit subportfolio. We argue that as a consequence separate risk assessment can be seriously flawed. Only if a portfolio can be separated into a market and a credit subportfolio, can we be sure that calculating economic capital independently for market risk and credit risk will always calculate an upper bound for the necessary risk capital. If portfolio positions depend simultaneously on market risk and credit risk factors the nature of the risk assessment problem changes. If for such a portfolio market risk and credit risk are calculated separately, this is based on a wrong portfolio valuation and leads to a wrong assessment of true portfolio risk.

Before discussing the implications of our results for regulators and banks, let us turn to an alternative interpretation of our results.5 In the example of the foreign currency loan portfolio the exchange rate can be considered both a market risk and a credit risk factor. It is a market risk factor because it has an effect on the portfolio value in case no defaults happen. At the same time it is a credit risk factor. It is a market risk factor because it has an effect on the portfolio value changes caused by changes in default probabilities, default correlations or losses given default, even if default does not actually happen. This amounts to a proper market risk model being in fact an integrated model.

A proper model of credit risk has to take into account all risk factors which have an effect on default losses. For the foreign currency loan portfolio this means that the credit risk model has to take into account exchange rate movements. This amounts to a proper credit risk model being in fact an integrated model. Similarly, a proper market risk model has to take into account market value changes caused by changes in default probabilities, default correlations or losses given default, even if default does not actually happen. This amounts to a proper market risk model being in fact an integrated model.

In this interpretational framework our results show that an approximate credit risk analysis assuming fixed values of the market risk factors can dramatically underestimate true credit risk if market and credit risk interact. Both interpretations of our analysis imply that a separate calculation of pure market risk and pure credit risk is not an admissible approximation to integrated risk if market and credit risk interact.

What can regulators and banks learn from our results? Banks are not necessarily safe, even if they hold the sum of market risk plus credit risk capital as calculated in the current framework. Regulators are only able to ensure that economic capital is sufficient if they take into account possible interactions of market risk and credit risk. When analysing capital calculations banks and regulators should ask themselves several questions in order to determine whether additional capital is necessary in addition to the sum of market risk and credit risk capital calculated by the bank.

Do borrowers’ payment obligations depend on market risk factors? The answer would be yes, for example, for adjustable rate loans and foreign currency loans. The payment obligations of borrowers depend on interest rates or exchange rates. If the answer is yes, does the rating system reflect the sensitivity of payment obligations and thus of PDs to interest rate and exchange rate moves? If no, additional capital on top of market risk and credit risk capital might be required to protect against losses arising from PD increases triggered by interest or exchange rate moves.

Is the risk of collateral price changes reflected in credit risk capital calculations? It is not sufficient to enter market values of collaterals in LGD estimates. Neither is it sufficient to include market prices of collaterals as additional independent risk factors. In case PDs depend on interest rates, exchange rates, or other market risk factors, the effect of collateral value changes depends on the level of PDs, and thus of the other market risk factors. For low PDs collateral prices are less relevant than in situations of high PDs.

Additionally, market risk calculations should reflect influences from credit events. If counterparties of hedges default, what is the additional market risk to which a bank is exposed? Do market risk capital calculations reflect the risk of adverse market risk moves happening together with credit events of hedge counterparties? If no, additional economic capital might be required.

Is there a substantial proportion in the collateral market serving as collateral for transactions exposed to similar risk factors? If yes, there is a risk that increased PDs for those transactions lead to increased sales of collaterals and thus to a drop in collateral values. This risk has to be covered by additional capital if it is not reflected in credit risk calculations. A historical example is provided by US real estate markets, where substantial proportions served as collateral for adjustable rate mortgages. The initial PD shock led to additional sales of real estate, which in turn brought down real estate

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5 We thank Peter Raupbach, Nikola Tarashev, and an anonymous referee for pointing this out.

Table 5
Sensitivity of results on the choice of initial income $a_0$. The final column calculates the risk interaction indices $I$ and $I^*$. The second line shows that a customer with $a_0 = 11,000$ cannot achieve rating BBB+ on a foreign currency loan. Even if the idiosyncratic variation $\sigma$ were zero, the variation in the payment obligation is too large for the required default probability of $0.01\%$.

<table>
<thead>
<tr>
<th>$a_0$ (€)</th>
<th>Rating</th>
<th>Currency</th>
<th>Spread</th>
<th>$\sigma$</th>
<th>MR no CR</th>
<th>RC</th>
<th>CR no MR RC</th>
<th>Integrated MR&amp;CR</th>
<th>Risk interaction $I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>11,000</td>
<td>BBB+</td>
<td>Home</td>
<td>160.06</td>
<td>0.0197</td>
<td>0</td>
<td>58</td>
<td>269</td>
<td>4.64</td>
<td></td>
</tr>
<tr>
<td>11,000</td>
<td>BBB+</td>
<td>Foreign</td>
<td>Impossible</td>
<td>0.0316</td>
<td>0</td>
<td>824</td>
<td>1775</td>
<td>2.15</td>
<td></td>
</tr>
<tr>
<td>11,000</td>
<td>B+</td>
<td>Home</td>
<td>162.53</td>
<td>0.0150</td>
<td>1618</td>
<td>0</td>
<td>31,569</td>
<td>19.51</td>
<td></td>
</tr>
<tr>
<td>12,000</td>
<td>BBB+</td>
<td>Home</td>
<td>160.15</td>
<td>0.0491</td>
<td>0</td>
<td>310</td>
<td>462</td>
<td>1.49</td>
<td></td>
</tr>
<tr>
<td>12,000</td>
<td>BBB+</td>
<td>Foreign</td>
<td>162.29</td>
<td>0.0363</td>
<td>1576</td>
<td>0</td>
<td>3056</td>
<td>1.94</td>
<td></td>
</tr>
<tr>
<td>12,000</td>
<td>B+</td>
<td>Home</td>
<td>165.62</td>
<td>0.0736</td>
<td>0</td>
<td>1805</td>
<td>2299</td>
<td>1.27</td>
<td></td>
</tr>
<tr>
<td>12,000</td>
<td>B+</td>
<td>Foreign</td>
<td>168.97</td>
<td>0.0755</td>
<td>1641</td>
<td>1523</td>
<td>11,201</td>
<td>3.54</td>
<td></td>
</tr>
<tr>
<td>13,000</td>
<td>BBB+</td>
<td>Home</td>
<td>160.22</td>
<td>0.0745</td>
<td>0</td>
<td>515</td>
<td>662</td>
<td>1.29</td>
<td></td>
</tr>
<tr>
<td>13,000</td>
<td>BBB+</td>
<td>Foreign</td>
<td>162.29</td>
<td>0.0711</td>
<td>1576</td>
<td>236</td>
<td>2015</td>
<td>1.11</td>
<td></td>
</tr>
<tr>
<td>13,000</td>
<td>B+</td>
<td>Home</td>
<td>168.27</td>
<td>0.1109</td>
<td>0</td>
<td>2705</td>
<td>3167</td>
<td>1.17</td>
<td></td>
</tr>
<tr>
<td>13,000</td>
<td>B+</td>
<td>Foreign</td>
<td>171.58</td>
<td>0.1163</td>
<td>1666</td>
<td>2663</td>
<td>7921</td>
<td>1.83</td>
<td></td>
</tr>
</tbody>
</table>
prices, thereby increasing LGDs, reducing LTVs and triggering new defaults.

Appendix A. Proof of Proposition 1

If \( V \) has the form \( V(a, e) = V_1(a) + V_2(e) \), we have: \( \Delta C(a) + \Delta M(e) = V(a_0, e) + V(a, e_0) - V(a_0, e_0) \)

\( = V_1(a_0) + V_2(e) + V_1(a) + V_2(e_0) - 2V_1(a_0) - 2V_2(e_0) \)

\( = V_1(a) + V_2(e) - V_1(a_0) - V_2(e_0) \)

\( = \Delta V(a, e) \).

Conversely, if \( \Delta V(a, e) = \Delta C(a) + \Delta M(e) \), then \( V(a, e) = V(a_0, e) + V(a, e_0) - V(a_0, e_0) \) which equals \( V_1(a) + V_2(e) \) for \( V_1(a) := V(a, e_0) - V(a_0, e_0) \) and \( V_2(e) := V(a_0, e) \). \( \square \)

Appendix B. Proof of Proposition 2

Since \( D = \Delta V - \Delta C - \Delta M \geq 0 \) for all scenarios, \( \Delta V \geq \Delta M + \Delta C \) holds for the corresponding random variables. Monotonicity and subadditivity of a coherent risk measure \( \rho \) imply \( \rho(\Delta V) \leq \rho(\Delta C + \Delta M) \leq \rho(\Delta C) + \rho(\Delta M) \). Conversely, assume there is some scenario \((a', e')\) for which \( 0 > D(a', e') = \Delta V(a', e') - \Delta C(a') - \Delta M(e') \). Now consider the following risk measure \( \rho \). To each random variable \( F \) arising from a portfolio function \( f : \Omega \times \Omega \to \mathbb{R} \) by \( F := f(A, E) \) the risk number \( \rho(F) := -\hat{f}(a', e') \) is assigned. This is a coherent risk measure. We have \( \rho(\Delta C) = \rho(V(A, e_0) - V(a_0, e_0)) - V(a', e_0) + V(a_0, e_0) \), and similarly for \( \Delta M \) and \( \Delta V \). This implies \( \rho(\Delta V) = -V(a', e') + V(a_0, e_0) > -V(a', e_0) + V(a_0, e_0) \).

\( \square \)

References


Zeugner, S., 2006. Implementing Pesaran-Shin-Smith, manuscript, Institute for Advances Studies, Vienna.