Basel’s value-at-risk capital requirement regulation: An efficiency analysis

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Abstract

We analyze the optimal portfolio policies of expected utility maximizing agents under VaR Capital Requirement (VaR-CR) regulation in comparison to the optimal policy under exogenously-imposed VaR Limit (VaR-L) and Limited-Expected-Loss (LEL) regulations. With VaR-CR regulation the agent strategy consists of simultaneous decisions on both the portfolio VaR and on the implied amount of required eligible capital. As a result, the performance of VaR-CR regulation depends on its design (the parameter \( n \)) and the agent preferences. We show that an optimal VaR-CR regulation allows the regulator on the one hand, to completely eliminate the exposure to the largest losses, which may jeopardize the existence of the institution, and on the other hand, to restrain the portfolio exposure to all other losses. These results rationalize the current Basel regulations. However, the analysis shows also that there is an optimal level of required eligible capital from the regulator standpoint. Counter-intuitively, any requirement above this optimal level is inefficient as it leads to a smaller amount of actually maintained eligible capital and thereby to a larger exposure to the most adverse states of the world. Unfortunately, the current Basel’s range of required levels (\( n = 3–4 \)) is within this inefficient range. Moreover, with an inefficient regulation the agent might employ an inefficient reporting and disclosure procedure.

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1. Introduction

Value-at-risk (VaR) has become the standard measure for risk management and regulation.\(^1\) The theoretical analysis of VaR regulations can be divided roughly into two main categories. The first and the primarily analyzed category is the regulation which imposes a limit on the maximal allowed VaR of the portfolio. Several recent papers analyze this case. Vorst (2001), for example, shows that maximizing expected return under exogenously-imposed limit on the portfolio VaR (VaR-L) might lead to a larger exposure to extreme losses. Basak and Shapiro (2001) comprehensively analyze this case and explore optimal portfolio policies of expected utility maximizing agents under exogenously-imposed limit on their portfolio VaR. They, find that “VaR risk managers often optimally choose a larger exposure to risky assets than non-risk managers and consequently incur larger losses when losses occur”. Consequently, Basak and Shapiro (2001) suggest an alternative risk measure to VaR, the Limited-Expected-Loss (LEL) regulation. According to their pioneer analysis, a regulation that imposes a limit on the maximal allowed expected loss overcomes the VaR-L shortcoming. The current Basel Committee on Banking Supervision regulations introduces conceptually a different regulation. This regulation imposes a minimal level of eligible capital that the agents must maintain at all times as a function of the portfolio VaR. Of course, this regulation, like any imposed constraint, may be costly. This case is very important because it obligates financial institutions in all developed countries to maintain eligible capital as a function of their bi-weekly market-risk VaR.

Inspired by the study of Basak and Shapiro (2001), who analyze the VaR-L and the LEL limit regulations and using the techniques they employed, this paper analyzes the VaR Capital Requirement (VaR-CR) regulation. More precisely, this paper explores the agent’s optimal investment strategy under VaR-CR regulation. We would like to stress at the outset that the agent strategy under VaR-CR regulation differs substantially from the strategy in the case of VaR-L and LEL regulations. This is because in the VaR-CR regulation the exogenously-imposed parameter is the level of the required eligible capital rather than the maximal allowed VaR and the expected loss in VaR-L and LEL regulations, respectively. Consequently, in the VaR-CR case through deciding on the portfolio VaR the agent determines also the amount of the Actually Maintained Eligible Capital (hereafter AMEC), which at the same time affects the portfolio VaR. This is because this required reserved eligible capital is part of the regulated portfolio and thus influences directly the portfolio VaR.\(^2\) In other words, in the case of VaR-CR regulation the agent

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\(^1\) An introduction and overview of VaR can be found in Duffie and Pan (1997) and in the excellent book by Jorion (2000).

\(^2\) While we focus on the optimal strategy under VaR-CR regulation, Cuoco and Liu (2002) who also analyze aspects of the VaR-CR regulation, focus on the optimal reporting and disclosure strategy of a VaR-CR regulated agent. Therefore they focus on the current Basel Committee on Banking Supervision (1996) Amendment VaR-CR regulation reporting and Back-Testing sanction mechanism, showing that this specific procedure may lead the agent to curb the risk of the traded portfolio and to report the true VaR.
strategy should incorporate simultaneous decisions on the portfolio VaR and on the implied amount of AMEC while taking into consideration their mutual impacts. 3

The main results of this study are as follows: the agent may face two scenarios, which depend on the regulation level, \( n \). When the required level of eligible capital is relatively small the agent chooses to stay passive and does not respond to the VaR-CR regulation. Thus, the agent behaves in a similar manner to Basak’s (1995) portfolio insurer agent (see also Grossman and Vila (1989) and Grossman and Zhou (1996)) and maintains the terminal wealth under all possible states above a “semi” exogenously-imposed level, which may be a function also of the agent’s preferences. However, the higher the required level of eligible capital (the higher \( n \)) the higher the chances that the agent chooses to become active and to “arbitrage” the regulation and thereby to reduce the amount of the AMEC. This action is referred as arbitraging the regulation because in response to the VaR-CR regulation the agent changes the portfolio composition (by buying put options, for example) and thereby reduces the portfolio VaR and hence decreases rather than increases the AMEC. Thus, the agent can gain on the “expense” of the regulator who faces a decreased impact of the regulation and consequently an increased portfolio risk exposure. In other words, arbitraging the regulation, as we shall see below, may increase the risk exposure in some states, even though the VaR is decreased. Consequently, a higher level of required eligible capital, \( n \), might lead to a smaller amount of AMEC and hence to a larger risk exposure from the regulator standpoint. This outcome implies that there is an optimal level of required eligible capital (the parameter \( n \)), which should not be exceeded. Interestingly, the numerical analysis shows that the current Basel’s range of required levels (\( n \) equal 3–4 time the portfolio VaR) is above this optimal level and therefore is defined as inefficient. Namely, the current regulation is not optimal and therefore might employ also an inefficient reporting and disclosure procedure.4 Thus, by reducing \( n \) the same and even improved level of safety for the entire economy can be obtained accompanied by a reduction in the total cost of regulation to the economy.

The reminder of the paper is organized as follows. Section 2 briefly describes the VaR-CR regulation. Section 3 solves the VaR-CR agent’s optimization problem and compares the solution with the solution of the VaR-L agent. Section 4 discusses the properties of the VaR-CR regulation and its advantages over the VaR-L and LEL limit regulations. Section 5 provides a numerical analysis. Section 6 concludes the paper. A brief description of the economic setting is given in Appendix A.

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3 It is worth mentioning that the VaR-CR regulation analysis presented here is closely related also to the growing tendency of risk management relative to a performance of some benchmark. Roll (1992) has set the basis of this methodology by studying the mean–variance criterion in which the two measures are calculated relative to some benchmark (the so-called Tracking-Error-Variance criterion). This work was later complemented by Jorion (2003). Recently, Basak et al. (2006) used a dynamic setting, similar to the setting presented here, to study portfolio management and policy under benchmarking risk management. Although it is beyond the scope of this paper, the analysis and results presented here can be adapted for further studying of benchmarking risk management under various risk management criteria.

4 Another implied result is that the current Back-Testing mechanism encourages the agents to further arbitrage the regulation and thereby reduces the level of safety for the entire economy.
2. Modeling the VaR capital requirement regulation (VaR-CR)

In order to analyze and compare the VaR-CR regulation with the VaR-L and the LEL limit regulations we use a standard setting, which is identical to that of Basak and Shapiro (2001). Therefore, in the Appendix we provide only a brief description of the model and the reader can find the detailed assumptions of the model in their study. Yet, the general framework is as follows:

Each agent in the economy is endowed at time 0 with risky securities, providing him with an initial wealth of \( W(0) \). Each agent chooses a portfolio plan which induces a process denoted by \( p(t) \equiv (p_1(t), \ldots, p_N(t))^{\top} \), the vector of fractions of wealth invested in the \( N \) risky securities. The agent may hold also the riskless bond. Uncertainty is represented by a filtered probability space \( (\Omega, \mathcal{F}, \{\mathcal{F}_t\}, P) \), on which is defined \( N \)-dimensional Brownian motion \( \omega(t) = (\omega_1(t), \ldots, \omega_N(t))^{\top}, t \in [0, T] \), where \( t \) represents current time. All stochastic processes are assumed adapted to \( \{\mathcal{F}_t; t \in [0, T]\} \), the augmented filtration generated by \( \omega \) (see also Appendix A).\(^5\) The induced non-negative, terminal-horizon portfolio wealth is denoted by \( \tilde{W}(T, p) \), where \( T \) stands for the terminal date and \( \tilde{W}(T, p) \) is a random variable (for simplicity, hereafter we omit the notion \( p \)). As in Basak and Shapiro (2001), \( \tilde{W}(T) \) represent the terminal possible realizations of the selected portfolio plan. Each agent is assumed to maximize expected state-independent utility \( u(\tilde{W}(T)) \) over terminal wealth. The function \( u(\cdot) \) is assumed twice continuously differentiable, strictly increasing, strictly concave, and to satisfy \( \lim_{x \to -\infty} u'(x) = \infty \) and \( \lim_{x \to \infty} u'(x) = 0 \).

Using the standard industry definition of VaR, it is defined as the maximum loss corresponding to a given probability, \( \alpha \), over a given horizon. Assuming VaR horizon to coincide with the investment horizon, VaR is given by

\[
\Pr(W(0) - \tilde{W}(T) \leq VaR(\alpha)) \equiv 1 - \alpha, \quad \alpha \in [0, 1],
\]

(note that \( \tilde{W}(T) \) is a random variable). The agent is regulated and required to maintain eligible capital which is free of any claims at all times as a function of the risk exposure measured by VaR.\(^6\) Thus, the agent terminal wealth never falls below this minimal level. This setting is similar, yet not the same, as Basak and Shapiro’s (2001) VaR-L agent who must maintain the portfolio VaR below some pre-specified level. This regulation can be embedded in the economy as

\[
\min[\tilde{W}(T)] \geq \max[\nu VaR(\alpha), 0],
\]

where \( nVaR(\alpha) \) denotes the required amount of eligible capital and where both \( \tilde{W}(T) \) and \( VaR(\alpha) \) are a function of the selected portfolio plan, \( p \). The maximization argument is required for the trivial case in which VaR is negative. Obviously, in this trivial case regulation is redundant and therefore, hereafter VaR is assumed to be positive without repeating this assumption. According to Basel Committee on Banking Supervision (1996) regula-

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\(^5\) The agent’s wealth process at time \( t \) \((t < T)\) then follows: \( dW(t) = W(t)[r(t) + p(t)^{\top}(\mu(t) - r(t) I)]dt + W(t)p(t)^{\top} \sigma(t)d\omega(t) \), where \( I \equiv (1, \ldots, 1)^{\top} \), \( r \) is the interest rate, \( \mu \equiv (\mu_1, \ldots, \mu_N)^{\top} \) is the drift coefficients vector, and \( \sigma \equiv \{\sigma_{ij}, i = 1, \ldots, N; j = 1, \ldots, N\} \) the volatility matrix.

\(^6\) For example, according to the current Basel Committee on Banking Supervision (1996) Amendment the agent is required to maintained eligible capital which can be a combination of shareholder equity and retained earnings (tier 1 capital), supplementary capital (tier 2 capital) and short-term subordinated debt (tier 3 capital), which under certain scenarios can be default without exposing the institution to bankruptcy.
tions, $n$ is in between 3 and 4,\textsuperscript{7} depending on the past performance of the VaR evaluation model.\textsuperscript{8} Note that inequality (2) can also be written (ignoring the trivial case of a negative VaR) as

$$\text{VaR}(x) \leq \min[\tilde{W}(T)]/n.$$  

(2')

Thus, the regulation constraint in inequality (2') is identical in its structure to Basak and Shapiro's (2001) constraint on their VaR-L agent, $\text{VaR}(x) \leq W(0) - \tilde{W}^\prime$, where $\tilde{W}$ is the regulation deterministic pre-specified limit. Note that the variable $\min[\tilde{W}(T)]$ is also deterministic once the agent decides on the portfolio policy. Thus, the only, yet important, difference is that at time 0 the agent has an additional degree of freedom to indirectly choose through the portfolio policy the level $\min[\tilde{W}(T)]$. Multiplying inequality (2) by minus 1 and adding $W(0)$ to both sides yields $W(0) - \min[\tilde{W}(T)] \leq W(0) - n\text{VaR}(x)$. As $\text{VaR}(x) \leq W(0) - \min[\tilde{W}(T)]$, by replacing the left side of the inequality we obtain

$$\text{VaR}(x) \leq \frac{W(0)}{1 + n}.$$  

(3)

Inequality (3) sets an upper deterministic limit on the portfolio VaR, which is also pre-specified as in Basak and Shapiro's (2001) limit constraint. Needless to say, this does not imply a similar solution since inequality (3) is only nested in the general constraint (2), where the limit is over the entire terminal wealth $\tilde{W}(T)$. In other words, the general VaR-CR regulation nests inherently the VaR-L regulation and thus incorporates implicitly a deterministic pre-specified limit on the portfolio VaR. Proposition 1 below shows that this difference is critical and that the VaR-CR realistic approach may outdo the VaR-L and LEL regulations.

3. Optimal policies under VaR-CR regulation

In this section, we solve the optimization problem of the VaR-CR regulated agent and then discuss the properties of the solution in comparison with the solution of the VaR-L regulated agent. Using the martingale representation approach (see, Karatzas et al., 1987; Cox and Huang, 1989), the dynamic optimization problem of the VaR-CR regulated agent can be stated as

$$\begin{align*}
\max_{\tilde{W}(T)} & \quad E[u(\tilde{W}(T))] \\
\text{s.t.} & \quad E[\xi(T)\tilde{W}(T)] \leq \xi(0)W(0), \\
& \quad \min[\tilde{W}(T)] \geq n\text{VaR}(x) \equiv n(W(0) - \tilde{W}(T)),
\end{align*}$$

(4)

\textsuperscript{7} According to The New Basel Capital Accord (2003) (Part 2, p. 6) the capital requirement for market risk of $n$ times the portfolio VaR is multiplied by 12.5 (i.e. the reciprocal of the minimum capital ratio of 8%) and then it is added to the denominator of the capital ratio. This ratio, which is defined as the regulatory capital divided by the risk-weighted complied for credit risk plus 12.5 times the capital requirements for market risk and operational risk, should be equal to 8% as defined in Basel Committee on Banking Supervision (1988). Thus, the supplementary capital for market risk is equal to $12.5 \times n\text{VaR} \times 8\% = n\text{VaR}$ corresponding to inequality (2).

\textsuperscript{8} Another realistic assumption, which can be incorporated easily into the model, is that the institution may have claims on some portion $1 - \beta$ of the required amount of eligible capital. Under this additional assumption, the factor $n$ in constraint (2) should be replaced simply by $n\beta$. 
where $\zeta(T, \omega)$ is interpreted as the Arrow–Debreu price per unit probability $P$ of one consumption good in state $\omega \in \Omega$ at time $T$ (see Basak and Shapiro, 2001) and where $W(T)$ is given by the equation $\text{VaR}(x) \equiv W(0) - W(T)$ (namely, $W(T)$ is the minimum wealth with probability $1 - \alpha$, i.e. $\text{Pr}(W(T) \geq W(T)) = 1 - \alpha$) and thus it is part of the solution, and hence is determined simultaneously with $\text{VaR}(x)$.\footnote{Note the difference between $\text{min}[\hat{W}]$ and $W(T)$. $\text{min}[\hat{W}]$ is the lowest possible realization of $\hat{W}(T)$ whereas $W(T)$ is the minimum realization subject to a probability of $\alpha$ to have a lower realization.}

The definition of $W(T)$ by means of the portfolio VaR produces an expression which looks similar to Basak and Shapiro’s (2001) exogenously-imposed limit condition on VaR, $\text{VaR}(x) \leq W(0) - W$ (see Eq. (5) in their study). The source for this similarity is that the VaR-CR regulation nests a deterministic limit on VaR, as has been showed in inequality (3). Nevertheless, $W(T)$ is completely different from $W$ since it is determined by the agent herself as part of her optimal strategy. Thus, the eligible capital due to capital requirement (CR) and the VaR are determined simultaneously, hence is called VaR-CR strategy. This difference together with the difference in the constraint in (4), are revealed in Proposition 1, which characterizes the optimal solution of problem (4), assuming it converges as expected to Basak and Shapiro’s (2001) Benchmark unregulated agent (denoted as B-agent and shown in Fig. 1 as $W_B(T)$). The same result takes place when $I(y\bar{\zeta}) \geq W(0)$. In this case, since the probability of any loss in the unconstrained solution
is smaller than \( x \) then regulation is not binding from the first place and the agent chooses the unconstrained solution. Thus, \( W(T) = I(y, \xi) \), \( \xi = \xi \), \( \xi = \infty \) and the solution converges again to the unregulated B-agent. The later case is analogous to the case of Basak and Shapiro (2001) in which the VaR limit constraint is not binding (\( \xi \geq \xi \) there).

Fig. 1 reveals that when the regulation is binding there could be two solutions depending both on the level of the required eligible capital and on the agent’s preferences. This is because in our realistic model the actual amount of eligible capital the agent chooses to maintain is also a function of the portfolio policy and the implied VaR of this portfolio. Therefore, the agent can influence it practically by manipulating the portfolio VaR depending on her own utility function.

Panel A in Fig. 1 depicts the Passive Case in which the agent chooses not to respond to the imposed capital requirement regulation leaving the portfolio VaR to be determined as without regulation. This passive behavior takes place when the required level, \( n \), is sufficiently small such that the agent expected utility loss from decreasing the portfolio VaR is larger than the expected utility gain from the implied reduction of the amount of the AMEC. In this Passive Case, the solution converges into two subsets: the bad states, which the agent fully insure against, and the good states, which are left completely uninsured. Thus, in the Passive Case the agent behaves like the PI-agent and maintains the horizon
wealth above a “semi” exogenously-imposed level. However, there are two differences between the passive VaR-CR agent and the PI-agent. First, in the Passive Case it is the agent to decide to stay passive and to fully insure against all bad states. Therefore, this behavior, which may be desirable from the regulator standpoint, depends on the agent and her utility function. Second, the amount of the AMEC is only “semi” exogenously imposed since it might be also a function of the agent utility function. This result takes place when n is sufficiently small, denoted by n1, where “sufficiently” depends also on the agent’s utility function such that \( n \in n_1 \forall n < I(y_{\bar{z}})/(W(0) - I(y_{\bar{z}})) \).

In this case, assuming the agent is passive, \( W(T) = I(y_{\bar{z}}), \bar{z} = \bar{z}, \bar{\bar{z}} = u'(n(W(0) - I(y_{\bar{z}})))/y \) and the amount of the AMEC is equal to \( n(W(0) - I(y_{\bar{z}})) \), which is a function of the agent utility function. Hence, the agent insures against bad states in the range \( \bar{z}(T) > \bar{\bar{z}} = u'(n(W(0)/(1 + n))) \) and leaves the rest states uninsured. In contrast, if n is sufficiently large, denoted by n2, such that \( n \in n_2 \forall n \geq I(y_{\bar{z}})/(W(0) - I(y_{\bar{z}})) \) then \( W(T) = nW(0)/(1 + n), \bar{z} = u'(nW(0)/(1 + n))/y, \xi = \xi \) and the amount of the AMEC is no longer a function of the agent utility function and it is equal to \( nW(0)/(1 + n) \). Hence, in this case the agent fully insures against bad states in the range \( \xi(T) > \bar{\bar{z}} = u'(n(W(0)/(1 + n))) \) \( /y \) and leaves the rest states uninsured.

To sum up, in the Passive Case when the agent chooses not to respond to the imposed VaR-CR regulation, she behaves like a PI-agent and allegedly, the regulator can impose any level of insurance as \( \lim_{n \to \infty} W(T) = \lim_{n \to \infty} nW(0)/(1 + n) = W(0) \). This is allegedly true since the larger the n the higher the chances that the agent chooses to respond to the imposed regulation and not to remain passive.

Panel B of Fig. 1 depicts the more realistic Active Case in which the agent finds it profitable to decrease the portfolio VaR in order to reduce the amount of the AMEC. This active behavior takes place when n is sufficiently large such that the agent’s expected utility loss from decreasing the portfolio VaR is smaller than the expected utility gain from the implied reduction of the amount of the AMEC. Panel B of Fig. 1 reveals that in the Active Case when n is larger than n2 but not too large, denoted by n3, and satisfies \( n \in n_3 \forall n < I(y_{\bar{z}})/(W(0) - W(T, n)) \) the VaR-CR agent classifies the unfavorable states into three subsets: the worst states \( \bar{z}(T) \), the bad states \( \bar{\bar{z}} = \bar{\bar{z}}(T) \leq \bar{z}(T) \) and the intermediate states \( \bar{z} < \bar{z}(T) \leq \bar{\bar{z}}(T) \). As can be seen from Panel B of Fig. 1, the VaR-CR agent fully insures against the intermediate states and leaves the bad states completely uninsured like the VaR-L agent. However, in contrast to the VaR-L agent, the VaR-CR agent also partially insures against the worst states. Finally, when n is sufficiently large, denoted by n4, and satisfies \( n \in n_4 \forall n \geq I(y_{\bar{z}})/(W(0) - W(T, n)) \), the VaR-CR agent classifies the unfavorable states into two subsets: the bad states \( \bar{z} = \bar{z} < \bar{z}(T) \) and the intermediate states \( \bar{z} < \bar{z}(T) \). Again, like the VaR-L agent, the VaR-CR agent fully insures against the intermediate states. However, in contrast to the VaR-L agent the VaR-CR agent also partially insures against the bad states. Thus, the good news is that like the passive VaR-CR agent, the active VaR-CR agent always partially insures against the worst states. However, the less encouraging news is that the larger the required level of eligible capital (i.e. the larger the n) the higher the incentive for the agent to reduce the portfolio VaR by over-insuring intermediate states on the expense of the partial worst states insurance. This is because the larger the n the larger the reduction of the amount of AMEC from the same reduction in the portfolio VaR and thereby, the larger the gain in terms of expected utility from arbitraging the regulation. This action is referred as arbitraging the regulation because by manipulating the portfolio VaR in response to the VaR-CR regulation the
agent can gain on the “expense” of the regulator who faces the negative implications. In other words, by actively decreasing the portfolio VaR and thereby decreasing the amount of the AMEC, the agent, on the one hand, increases her expected utility and on the other hand, decreases the impact of the regulation and consequently increases the risk exposure of the portfolio. Surprisingly, the result of increased incentive to arbitraging the regulation implies the existence of a maximal amount of the AMEC that the regulator cannot exceed. Above this amount, the higher the \( n \), the smaller the eligible capital the agent actually maintained and thus the smaller the partial insurance in the worst states. Any attempt from the regulator to increase the required level of eligible capital above this limit leads to the opposite result of a smaller amount of the AMEC. Furthermore, this maximal amount of the AMEC implies also the existence of inefficient range of required levels of eligible capital. Accordingly, any value above \( n \), which leads to the maximal amount of the AMEC, reduces both the amount of the AMEC and the agent expected utility and thus hurts both the agent and the regulator.

From the above analysis it is observable that the VaR-CR can be seen as a compromise between a full insurance requirement and a VaR limit requirement. If \( n \) is sufficiently small, where sufficiency depends in this context also on the agent preferences, VaR-CR regulation has the characteristics of a portfolio insurance regulation. For sufficiently large \( n \), the VaR-CR regulation becomes more like a combination of the portfolio insurance regulation and the VaR-L regulation.

Fig. 2 juxtaposes the VaR-CR regulation agent under the various scenarios with the VaR-L agent and the unconstrained B-agent, assuming regulations are binding. (Otherwise, all solutions converge to the unconstrained B-agent). Panel A depicts the optimal terminal wealth of the VaR-CR agent under all possible behaviors \( W_{VaR\ CR}^{i}(T) \), where \( i = 1, 2, 3, 10 \) of the VaR-L agent, \( W_{VaR\ L}^{i}(T) \) and of the B-agent, \( W_{B}(T) \). Panel B depicts the shape of the probability density function of these agents’ optimal terminal wealth. For the sake of demonstration, the same terminal level, \( W(T; n) \), is assumed in all cases which is equal also to the imposed limit level of the VaR-L agent, \( W \). In other words, the imposed level of the required eligible capital, \( n \), is set on each agent such that all agents derive the same VaR which is equal also to the limit on the VaR-L agent. (We use this setting for presentation purposes only. Proposition 2 in Section 4 proves that if the agent has a CRRA preferences then the following results are valid in the general case and are not restricted only to those values of \( n \)). The noticeable result is that for some range of the worst states the optimal horizon wealth always obeys \( W_{VaR\ L}^{i}(T) < W_{B}(T) < W_{VaR\ CR}^{i}(T) \). By the same token, panel B reveals that in contrast to both the VaR-L agent and the B-agent, the VaR-CR agents’ probability to incur the largest possible losses, up to a certain level of loss, is always zero. Thus, the VaR-CR agents always insure against a certain range of the worst states and therefore less exposed to worst-case scenarios than both the VaR-L agent and the B-agent. This encouraging result is good news for regulators given that the VaR-CR regulation indeed prevents from the agent to be exposed to the largest losses which may cause credit and insolvency problems. This result is comparable to Basak and Shapiro’s (2001) result of the LEL regulated agent. However, the VaR-CR agent not only partially insures against high losses, as does the LEL agent, but she also bounds the maximal

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10 In Fig. 2, the passive behavior cases (namely, where \( n \) is either \( n_1 \) or \( n_2 \)) are considered as one case as they diverse only on technical grounds and have identical characteristics.
loss. This clear-cut result confirms the rational of using VaR as a capital requirement measure with only two reservations. First, from Fig. 2 it is also observable that the VaR-CR agent might compensate for the cost of insuring the largest losses by increasing the exposure to the intermediate losses either gradually \(W_{VaR}^{CR1}(T)\) or dichotomously \(W_{VaR}^{CR2}(T)\). This could be seen in panel B where there is a probability mass build-up on a certain threshold someplace between the largest losses and the smaller ones. Therefore, the regulator should be aware of that threshold and the possible implied risk overexposure. This is especially important, given our previous claim that this threshold depends non-linearly on the required capital level, \(n\), and might even shifted to the range of higher losses with \(n\) from a certain level. Second, in Fig. 2 there is an implicit assumption that the agent never reduces the portfolio VaR to a level for which the amount of the AMEC is strictly zero and thus eliminates completely the partial insurance against the worst states (namely, the agent never chooses \(W(T; n) = W(0)\)). Proposition 2 in the next section proves that if the agent has a CRRA preferences then this is indeed the case. Accordingly, the agent never reduces the portfolio VaR to a level for which the required eligible capital is strictly zero and always insures against the worst states.

### 4. Properties of the VaR-CR regulation optimal strategy

Basak and Shapiro (2001) shows that after adjusting for the initial endowment, the VaR-L agent wealth plan is equivalent to the PI-agent plan plus a short position in “binary” options, or to the B-agent plan plus an appropriate position in “corridor” options (see, Basak and Shapiro, 2001, p. 379). Analogously, the VaR-CR optimal wealth plan

**Fig. 2.** Optimal horizon wealth and the implied probability density of VaR-CR agent versus VaR-L agent. The figure plots the optimal horizon wealth of the VaR-CR agent and the VaR-L agent under the various scenarios as a function of the horizon state price density \(\xi(T)\), and the shape of the implied probability density. The dotted curve is for the unconstrained B-agent and the thin dashed curve is for the VaR-L agent. Panel A plots the optimal horizon wealth of the various agents. All VaR-CR agents partially insure against largest losses thus incur smaller losses than both the VaR-L agent and the B-agent under the worst states of nature. Panel B plots the shape of the optimal horizon wealth probability density function. The probability of all VaR-CR agents to fall into the worst states is zero.
in (5) can be expressed either as a simple PI-agent plan in the Passive agent case or in the Active agent case as

\[
\tilde{W}_{CR}(T; y(W(0))) = \tilde{W}_B(T; y_B(W_\ast)) - \max(\tilde{W}_B(T; y_B(W_\ast)), 0) - (W(T) - \max(\tilde{W}_B(T; y_B(W_\ast)), 0))1_{\{\xi < \xi_c(T)\}}
\]

where \(W_\ast\) is set such that \(y_B(W_\ast) = y(W(0))\) and the PI-agent insurance limit is set to be \(W = W(T)\). In other words, adjusting for the initial endowment and for the implied insured level, the VaR-CR active agent plan is equivalent to a PI-agent plan plus a short position in complex “binary” options. Correspondingly, it is equivalent to the B-agent plan plus a long position in “corridor” options and “binary” options.

Continue with the analogy, because

\[
W_\ast = W(0) - E\left[\frac{\xi(T)}{\xi(0)} \max(\tilde{W}(T) - \tilde{W}_B(T; y_B(W_\ast)), 0)\right]
\]

\[
+ E\left[\frac{\xi(T)}{\xi(0)} \left(\tilde{W}(T) - \max(\tilde{W}_B(T; y_B(W_\ast)), 0)\right)1_{\{\xi < \xi_c(T)\}}\right],
\]

then \(\tilde{W}_B(T; y_B(W_\ast))\) is the optimal policy of the unconstrained B-agent, whose initial endowment is simply \(W(0)\) decreased by the price of a put option which implements the PI policy and increased by the proceed of the short selling of the complex “binary” option.

In order to further analyze the properties of the optimal strategy of the VaR-CR agent and to compare it with the optimal strategy of the VaR-L agent, let us assume CRRA preferences, such that \(u(W) = \frac{W^{1-\gamma}}{1-\gamma}, \gamma > 0\), and lognormal state prices distribution with constant interest rate and market price for risk. Basak and Shapiro (2001) use two measures of loss, the expected future value of loss, \(L_1\), and its present value, \(L_2\), in order to show explicitly that under VaR-L regulation the expected extreme loss is larger than that incurred by an unregulated agent. In contrast, Proposition 2 below shows that there is always a range of losses for which the expected extreme loss under the VaR-CR regulation is equal to zero, and there is always a wider range of losses for which the expected extreme loss is smaller than that incurred by the VaR-L regulated agent, by the LEL regulated agent and by the unregulated agent. With this respect, the VaR-CR regulation is superior to the other regulations.

**Proposition 2.** Assume \(u(W) = \frac{W^{1-\gamma}}{1-\gamma}, \gamma > 0\) and the interest rate \(r\), the drift coefficients \(\mu\), and the volatility matrix \(\sigma\) (see Section 2) are constants. For a given terminal wealth \(W(T)\), define the following two measures of loss:

\[
L_1(W) = E\left[\frac{(W - \tilde{W}(T))1_{\{\tilde{W}(T) < \bar{W}\}}}{\bar{W}}\right] \quad \text{and} \quad L_2(W) = E\left[\frac{\xi(T)}{\xi(0)} \left(W - \tilde{W}(T)\right)1_{\{\tilde{W}(T) < \bar{W}\}}\right],
\]

where results below \(\bar{W}\) are classified as a loss. Then,

(i) there always exists \(W_1 > 0\) such that \(L_i(\tilde{W}_{CR}(T); W_1) = 0\); \(i = 1, 2\), except from the trivial cases of VaR_B(\(\bar{Z}\)) \(\leq 0\) and \(n = 0\) in which \(\bar{W}_1 \equiv 0\).
(ii) there always exists $W_2 > W_1 > 0$ such that

$$L_i(\tilde{W}_{\text{VaR}}^\text{CR}(T); W_2) \leq L_i(\tilde{W}_b(T); W_2) \leq L_i(\tilde{W}_L^\text{VaR}(T); W_2); \quad i = 1, 2.$$ 

(the proof is available from the authors on request).

According to Proposition 2, an agent with CRRA preferences always partially insures against the most adverse states for which $\tilde{W}(T) < W_1$ and thus entirely eliminates the exposure to the most extreme losses. At the same time, the agent expected loss is always smaller or equal to that of the VaR-L agent and the B-agent up to a certain level of loss $W_2$, which is above $W_1$. It can easily be shown that under loosen realistic assumptions about the agent’s utility function, $W_2$ is always above Basak and Shapiro’s (2001) original definition for loss $W = \min(\Pr(\tilde{W}_B(T) \leq W) \equiv x, W)$ where $W$ is the exogenously-imposed limit on the VaR-L agent. In other words, a well-designed VaR-CR regulation enables the regulator to completely eliminate the possibility of extreme losses, which might jeopardize the institution survival, and at the same time to restrain the expected loss of all-size losses. Naturally, designing an optimal regulation depends on both the regulator objective function as well as on some specific parameters including the agent’s utility function and the specific distribution of state prices. For example, if the regulator wishes to maximize the range of losses that are completely eliminated by the regulation, then he should solve for $\max_x W_1$ subject to $L_i(\tilde{W}_{\text{VaR}}^\text{CR}(T), W_1) = 0$, as is demonstrated in the next section. Similarly, if the regulator wishes to completely eliminate losses below $W_1$ and to minimize the expected loss up to a certain level, $W_2$, then he should try to solve for $\min_x L_i(\tilde{W}_{\text{VaR}}^\text{CR}(T), W_2)$ subject to $L_i(\tilde{W}_{\text{VaR}}^\text{CR}(T), W_1) = 0$. Likewise, the regulator can try to solve for any other desirable objective.

The results of Proposition 2 are valid also for the case of the LEL regulation in which the agent is subject to a limit on her portfolio LEL. Since the VaR-CR agent always partially insures against the most adverse states in the range $\tilde{W}(T) < W_1$ then there is always $W_2$ for which her expected loss is smaller or equal to that of the LEL limit agent. Naturally, $W_2$ in this case is smaller than that in the previous case of the VaR-L agent. Nevertheless, an optimal design of the VaR-CR regulation may better guarantee the survival of the institution.

5. Efficiency analysis of VaR-CR regulation

In this section we use the previous results to find and analyze the optimal level of required eligible capital from the regulator standpoint. Let us first define an optimal policy from the regulator point of view.

**Definition 1.** A regulation policy is optimal if it maximizes the range of the largest losses, which are eliminated entirely by the regulation.

Namely, given the induced optimal strategy of the agent, the optimal regulation identifies the level of required eligible capital that maximizes the amount of AMEC and thereby minimizes the maximal possible loss (i.e. $\max_x [W_2] \equiv \max_x [n(W(0) - \tilde{W}(T))]$ subject to $L_i(\tilde{W}_{\text{VaR}}^\text{CR}(T), W_1) = 0$, where the maximization argument from the regulator standpoint is $n$). Note that this range lies within the range of losses in which the VaR-CR agent’s expected future value of the loss and the present value of the loss, $L_i(\tilde{W}_{\text{VaR}}^\text{CR}(T), W_1)$,
i = 1, 2, are both equal to zero and thus outperform the VaR and LEL limit regulations (see Proposition 2). As has been previously said, this is only one possible goal and there could be other targets for this regulation. Below, we show that under reasonable assumptions about the agent’s preferences the current Basel’s range of required levels of eligible capital (namely, where n is between 3 and 4) is in the inefficient range. This is because the same amount of AMEC, which induced by this range, can be obtained by imposing lower levels of required capital. Furthermore, within this range increasing the required level of eligible capital has an insignificant impact on the amount of AMEC and in some cases even a negative impact on this amount.

**Proposition 1’**. Assume \( u(W) = \frac{W^{1-\gamma}}{1-\gamma}, \gamma > 0 \) and \( r, \mu \) and \( \sigma \) are constants as in Proposition 2, then the time-\( T \) wealth under optimal strategy of the VaR-CR agent is given by

\[
\tilde{W}_{\text{VaR,CR}_T}(T) = \begin{cases} 
(y\xi(T))^{-1/\gamma} & \text{if } \xi(T) \leq W^{-1}(T)/y, \\
W(T) & \text{if } W^{-1}(T)/y < \xi(T) \leq \xi, \\
(y\xi(T))^{-1/\gamma} & \text{if } \bar{\xi} < \xi(T) \leq \max(\{n(W(0) - W(T))\}^{-1}/y, \bar{\xi}), \\
n(W(0) - W(T)) & \text{if } \max(\{n(W(0) - W(T))\}^{-1}/y, \bar{\xi}) < \xi(T),
\end{cases}
\]

where \( \bar{\xi} \) is such that \( \Pr(\xi(T) > \bar{\xi}) = \alpha \), \( W(T) \) is the maximum between \( W(T) = nW(0)/(1+n) \) and the solution of

\[
W^{-1}(T)(F\left(\frac{W^{-1}(T)}{y}\right) - F(\xi)) - n^{1-\gamma}(W(0) - W(T))^{-1/\gamma}F\left(\frac{\{n(W(0) - W(T))\}^{-1}}{y}\right)
\]

\[
+ \frac{\gamma}{1-\gamma} \frac{1}{s}\xi^{\gamma-1} \left(-W^{-1}(T)\frac{W^{-1}(T)}{y} + (W(0) - W(T))^{-1}h\left(\frac{\{n(W(0) - W(T))\}^{-1}}{y}\right)\right)
\]

\[
+ \frac{\gamma}{1-\gamma} \frac{1}{s} \left(W^{-1}(T)f\left(\frac{W^{-1}(T)}{y}\right) - n^{1-\gamma}(W(0) - W(T))^{-1}f\left(\frac{\{n(W(0) - W(T))\}^{-1}}{y}\right)\right)
\]

\[
y\left(G\left(\frac{W^{-1}(T)}{y}\right) - G(\xi)\right) - nG\left(\frac{\{n(W(0) - W(T))\}^{-1}}{y}\right) + \frac{\gamma}{s}\xi^{\gamma-1} \left(-W^{-1}(T)h\left(\frac{W^{-1}(T)}{y}\right)\right)
\]

\[
+ (W(0) - W(T))^{-1}h\left(\frac{\{n(W(0) - W(T))\}^{-1}}{y}\right) + \frac{\gamma}{s}y\left(g\left(\frac{W^{-1}(T)}{y}\right) - ng\left(\frac{\{n(W(0) - W(T))\}^{-1}}{y}\right)\right),
\]

and \( y \) solves

\[
y^{-1/\gamma} \left(1 - H\left(\frac{W(T)^{-\gamma}}{y}\right) + H(\xi) - H\left(\max\left(\frac{\{n(W(0) - W(T))\}^{-\gamma}}{y}, \xi\right)\right)\right)
\]

\[
+ W(T) \left(G\left(\frac{W(T)^{-\gamma}}{y}\right) - G(\xi)\right) + n(W(0) - W(T))G\left(\max\left(\frac{\{n(W(0) - W(T))\}^{-\gamma}}{y}, \xi\right)\right)
\]

\[
= \xi(0)W(0),
\]

where

\[
F(X) = N\left(\frac{\ln \frac{1}{s} - m}{s}\right), \quad G(X) = e^{-m + \frac{\xi}{s}}N\left(\frac{\ln \frac{1}{s} - m}{s} + s\right)
\]
and

\[
H(X) = e^{-\frac{1-m+(\frac{1}{\gamma})^\frac{s^2}{2}}{2}} N\left( \frac{\ln \frac{1}{X} - m}{s} + \frac{\gamma - 1}{\gamma} s \right),
\]

and where \(N(\cdot)\) is the cumulative standard normal distribution and \(f(X), g(X)\) and \(h(X)\) are the corresponded lognormal density functions, respectively, and

\[
m = E\left[ \frac{1}{C_0} \ln n(T) \right] = r + \frac{|k|^2}{2}
\]

and

\[
s^2 = Var\left[ \frac{1}{C_0} \ln n(T) \right] = ||k||^2.
\]

(the proof is available from the authors on request.)

Before we continue with the efficiency analysis, we need the following definitions:

**Definition 2.** The VaR-CR efficient set, \(E\), is defined as

\[
E \equiv \{ n_i : n_i \in n \text{ such that } n_i(W(0) - W(T; n_i)) > n(W(0) - W(T; n)) \ \forall n < n_i \}. \quad (7)
\]

Then, VaR-CR efficiency suggests that the unavoidable reduction in the regulated agent’s expected utility, due to an increase in \(n\), is “compensated” to some extent from the regulator standpoint by an increase in the amount of the AMEC. Namely, the same or a larger amount of AMEC cannot be obtained by imposing a lower level of \(n\).

**Definition 2’.** The VaR-CR inefficient set, \(\bar{E}\), is defined as

\[
\bar{E} \equiv \{ n_i : n_i \in n \text{ such that } n_i(W(0) - W(T; n_i)) \leq n(W(0) - W(T; n)) \ \text{for at least one } n < n_i \}
\]

Thus, inefficiency suggests that the same amount of AMEC or a larger one can be obtained by imposing a lower \(n\).

The efficient set can be further divided into the efficient and the inefficient sets in terms of total expected utility. Thus, the efficient set in terms of total expected utility is within the range of \(n\) for which an increase in \(n\) increases the expected utility for the regulator by more than the decrease in the expected utility of the agent. Formally,

**Definition 3.** The VaR-CR utility-efficient set, \(E_u\), is a subset of the VaR-CR efficient set for which

\[
E_u \equiv \left\{ n_i \in E : \text{ such that } \frac{dE(U^R(n_i(W(0) - W(T; n_i))))}{dn} > -\frac{dE(U(W(T; n_i))))}{dn} \right\}, \quad (8)
\]

where \(U^R(\cdot)\) represents the regulator utility from the amount of the AMEC and \(U(\cdot)\) is the agent utility function.

Unfortunately, the definition in (8) is difficult to apply in practice because it requires the exact knowledge of the agent’s and the regulator’s utility functions.

**Definition 4.** The Maximum Level, \(n_{\text{max}}\), which implies the maximum feasible AMEC, is defined as

\[
n_{\text{max}} \equiv \{ n_i \in E \text{ such that } n_i(W(0) - W(T; n_i)) \geq n(W(0) - W(T; n)) \ \forall n > 0 \}. \quad (9)
\]
Namely, the Maximal Level is the level of required eligible capital that leads the agent to a policy of maintaining the maximum feasible amount of the AMEC.

5.1. Numerical demonstration of efficiency analysis

In what follows, we analyze numerically the VaR-CR regulation using the results of the previous sections and the above definitions. The analysis covers the Mid-term 60-trading-days as well as the Short-term 10-trading-days and the Long-term full-year horizons. As the results of all cases are very similar, we present here only the result of the 60-trading-days case which represents the typical horizon the regulation applies for (see, for example, the discussion in Hendricks and Hirtle, 1997). (The results of the other cases are available from the authors on request.)

Fig. 3, the Mid-term 60-trading-days parameters case, depicts the amount of the AMEC as a function of the required level of eligible capital under the CRRA agent’s optimal strategy given in (6). The state price density function is assumed to be lognormal with parameters corresponding approximately to the US market: a risk-free rate of \( r = 0.05 \times 60/252 = 0.0119 \) and \( \|k\| = 0.4\sqrt{60/252} = 0.1952 \) (where \( k(t) \equiv \sigma(t)^{-1}(\mu(t) - r(t) \bar{1}) \) is the Sharpe ratio, defined in Appendix A).

Generally, all agents, whose their risk aversion level \( \gamma \) is equal to 0.3, 0.6, 0.9 or 1.2, increase the amount of the AMEC up to the Maximal Level and from that level decrease the amount of the AMEC with the increase of, \( n \). The increase of the amount of AMEC hold for all levels of \( n \) below the Maximal Level except from one step where the amount of the AMEC steps down discontinuously with the increase of \( n \). In addition, for all agents the rate of the increase of the AMEC declines with \( n \) until it becomes negative above the Maximal Level. Fig. 4 indicates the type of the agents’ strategy as a function of \( n \) for each agent, separately. The figure depicts also the portfolio VaR, which is induced from the

![Image](image.png)

**Fig. 3.** Actually maintained eligible capital as a function of the required level of eligible capital – The Mid-term 60 days parameters case. The figure plots the amount of the actually maintained eligible capital as a function of the required level of eligible capital under the agent’s optimal strategy. The plots assume CRRA preferences with a relative risk aversion parameter, \( \gamma \), equals to 0.3 (dashed curve), 0.6 (solid curve), 0.9 (dashed-dotted curve) and 1.2 (dotted curve). The state price density function is assumed to be lognormal with 60 days parameters corresponding to the US market: 60 days risk-free rate of \( r = 0.05 \times 60/252 = 0.0119 \) and \( \|k\| = 0.4\sqrt{60/252} = 0.1952 \). The fixed parameters are \( \alpha = 0.01, \bar{W}(0) = 1 \) and \( \xi(0) = 1 \), then, \( \xi = 1.53 \).
agents’ strategy, as a function of $n$. The figure reveals that all agents stay passive and do not change their portfolio VaR in response to the VAR-CR regulation as long as the required level, $n$, is sufficiently small (namely, $n \in \{n_1, n_2\}$, as defined in Fig. 1). As a result, this range of $n$ is efficient. However, from a certain threshold, when $n$ shifts from $n_2$ to $n_3$ (see also Fig. 1), all agents become active and reduce their portfolio VaR and thereby reduce their portfolio amount of AMEC. Notably, when shifting from a passive strategy to an active strategy, the portfolio VaR dropped sharply and correspondingly the amount of the AMEC dropped at a rate, which is $n$ times greater. This decline creates a wide inefficient range of $n$ just after this threshold. Then, at some larger level, there may be an additional range of $n$ that is efficient again. However, the rate of increase of the amount of AMEC with $n$ within this range is much smaller. Finally, above the Maximal Level the amount of the AMEC is decreased with $n$ such that any level above this level is inefficient. Furthermore, the less risk averse the agent, the smaller the threshold of the required level, $n$, from which the agent decides to actively reduce the portfolio VaR and the sharper is the
abrupt reduction of this VaR. Thus, the riskier the agent’s portfolio, as a result of a smaller risk aversion, the larger the chances that this agent also actively reduces her portfolio VaR and the more aggressive is this reduction and therefore the larger is the impact of this response on the amount of the AMEC. Consequently, from a relatively low required level of approximately \( n > 1 \), the increase in \( n \) leads to a lower relative increase, if any, in the amount of the AMEC (i.e. the slope of the curve is less than 45°).

Comparing the Mid-term 60-trading-days with the Short-term 10-trading-days and the Long-term full-year horizons (not presented in the paper) shows that the longer the horizon the smaller the threshold from which the various agents decide to actively reduce their portfolio VaR and the sharper the decrease in VaR from that threshold.

To sum up, the efficient range, in which a higher required level of eligible capital induced a larger amount of AMEC, is considerably narrow. This outcome holds for all levels of risk aversion and for parameters which are typical to a broad range of time horizons from only 10 days, where \( n < 2 \), 60 days, where \( n \sim 1 \), and longer such as a full year where \( n < 1 \). In other words, for all levels of risk aversion discussed in this paper and for any time horizon from 10 days and longer the current Basel range of levels (i.e., \( n = 3–4 \)) is inefficient or at best induces a small impact on AMEC, as a higher \( n \) within this range implies a smaller amount of AMEC or a much smaller relative increase in this amount. Furthermore, the riskier the agent’s portfolio, as a result of a smaller risk aversion, the narrower is the efficient range.

These results reveal two major shortfalls of the current Basel’s regulations. The first is that the same and even a larger amount of the AMEC can be obtained by imposing a lower level of \( n \). Thus, the same and even higher level of safety for the entire economy can be obtained while at the same time the total cost of regulation to the economy is reduced. The second shortfall relies in the current Back-testing mechanism. Currently, poor performance of the previously reported VaR resulted in a larger \( n \), which in it turn, might lead the agent to further arbitrage the regulation and thereby reduce the amount of the AMEC at the current period and so forth. Furthermore, the riskier the agent’s portfolio, due to less risk aversion, the higher the incentive for the agent to further arbitrage the regulation. Naturally, when taking in account the cost for the agent from this witched circle she may decide to break it and to report the actual VaR correctly. However, from the above analysis it is evident that the current range of \( n \) implies a smaller cost for the agent than it is implied. For example, by increasing \( n \) by 33.3%, from 3 to 4, the cost for agent is much smaller than 33.3% due to the ability to reduce the amount of the AMEC. Thus, an alternative mechanism in which the additional required amount of capital due to previous poor performances is not a function of only the future VaR could be much more effective in increasing the amount of the AMEC and thereby to encourage the agent to avoid an additional regulation arbitrage.

6. Concluding remarks

The literature on VaR regulations covers mainly the case of VaR Limit (VaR-L) regulation, in which a maximal limit on the allowed VaR is exogenously imposed, and the LEL regulation suggested by Basak and Shapiro (2001). Inspired by the study of Basak and Shapiro (2001), this study analyzes the alternative regulation of VaR Capital Requirement (VaR-CR) regulation in which the agent is required to maintain eligible capital at all times as a function of the portfolio VaR, i.e. \( n \) times the VaR. This case is important because it is
the required regulation by the current Basel Committee on Banking Supervision. In the case of VaR-CR regulation the exogenously-imposed parameter is the level of the required eligible capital. Unlike the VaR-L regulation case, in VaR-CR regulation case the agent determines simultaneously the VaR of the portfolio and the implied Actually Maintained Eligible Capital (AMEC), where both affect each other. Specifically, the agent chooses between two possible behaviors depending on both the exogenously-imposed level of required eligible capital and on her preferences. In the first scenario the agent chooses not to respond to the imposed capital requirement regulation. In this passive behavior the agent behaves in a similar manner to Basak’s (1995) portfolio insurer agent and maintains the horizon wealth under all possible states above a “semi” exogenously-imposed level. Surprisingly and differently from the portfolio insurer agent, in the case of VaR-CR regulation this level is a function also of the agent utility function and cannot be determined solely by the regulator. In the second possible scenario, the agent finds it profitable to over-insure the intermediate unfavorable states and thus manipulate the portfolio VaR in order to reduce the amount of the AMEC. Therefore, in response to the VaR-CR regulation the agent changes the portfolio composition and thereby reduces the portfolio VaR while increasing the risk exposure in some states. More specifically, in this case the VaR-CR agent has some of the characteristic of the VaR-L agent whereas she fully insures against the intermediate unfavorable states and leaves some of the unfavorable states completely uninsured. However, the VaR-CR regulation is better than the VaR-L regulation from the regulator point of view because the VaR-CR agent also partially insures against the worst states.

The most important result of this paper which has practical implications to the current Basel regulations is that the VaR-CR agent strategy might lead her to “arbitrage the regulation” by decreasing the VaR and the implied amount of the AMEC while increasing at the same time the risk exposure. We show that the larger the required level of eligible capital, $n$, the greater the incentive for the agent to reduce the portfolio VaR at the expense of the worst states insurance. Consequently, a larger level of required eligible capital may lead to a smaller amount of the AMEC, hence the exposure to insolvency losses may be increased. This arbitraging of the regulation by the agent implies the less intuitive outcome of the existence of an inefficient range of levels of required eligible capital (i.e. an inefficient values of the parameter $n$) and correspondingly a maximal amount of AMEC held by the agent. Any requirement above this level, $n$, induces a smaller amount of AMEC, hence a possible larger risk exposure.

To sum up, VaR-CR can be seen as a compromise between full insurance requirement and VaR-L requirement. We find that this compromise is a two-edge sword and its efficiency in reducing the exposure to the largest possible losses depends on the value $n$ imposed by the regulator. Nevertheless, an optimal VaR-CR regulation (optimal $n$) produces better results from the regulator standpoint than the VaR-L regulation and even outperforms the LEL regulation, as it does not lead the agent to the undesirable result of increasing the exposure to largest possible losses (see Basak and Shapiro, 2001). This optimal VaR-CR regulation allows the regulator on the one hand, to completely eliminate the exposure to the largest losses, which jeopardize the existence of the institution, and on the other hand, to restrain the portfolio exposure to all other losses. These results rationalize the current Basel Committee on Banking Supervision regulations. However, the VaR-CR method has its drawbacks as it performance is sensitive to the specific context and frame. This is because, the higher the required level of eligible capital, $n$, the larger
the incentive for the agent to further decrease the portfolio VaR and thereby reducing the amount of the AMEC and increasing the risk exposure. This, dependency of the performance of VaR-CR regulation on its optimal design and the fact that this optimization is a function also of the agent preferences reveals two major shortfalls of the current Basel regulations. First, the current range of levels of required eligible capital \( n = 3–4 \) times of the VaR) is \textit{inefficient} because by reducing \( n \) the same and even a higher level of safety for the entire economy can be obtained accompanied by a reduction in the total cost of regulation to the economy. Secondly, the current Back-testing mechanism encourages the agents to further arbitrage the regulation and thereby reduce the level of safety for the entire economy. These drawbacks indicate that a different buffer, rather than the current simple multiplier, should be used to determine the required level of capital. This optimal buffer is a subject for another study.

Appendix A. The economic setting

The methodology used in Proposition 1 and in the proofs (which are available from the authors on request) are adopted from Basak and Shapiro (2001). Therefore, we provide only a brief description of the economy and a detailed description of it can be found in their study. We consider a finite-horizon, \([0, T]\), economy with a single consumption good (the numeraire). Uncertainty is represented by a filtered probability space \((\Omega, F, \{F_t\}, P)\), on which is defined \( N \)-dimensional Brownian motion \( \omega(t) = (\omega_1(t), \ldots, \omega_N(t))^T, t \in [0, T] \), where \( t \) represents current time. All stochastic processes are assumed adapted to \( \{F_t; t \in [0, T]\} \), the augmented filtration generated by \( \omega \). Namely, \( F_t \) represents the information available up to current time, \( t \). All stated (in)equalities involving random variables that hold \( P \)-almost surely. Hereafter, we assume that all states to be well defined without explicitly stating the regularity conditions ensuring this. Investment opportunities are represented by a risk-free bond in zero net supply and \( N \) risky stocks, each in constant net supply of 1.

Dynamic market completeness defines a unique state price process, \( \xi \), given by
\[
d\xi(t) = -\xi(t)[r(t)dt + k(t)\,d\omega(t)],
\]
where \( k(t) \equiv \sigma(t)^{-1}(\mu(t) - r(t)\hat{1}) \) is the market price of risk (the Sharpe ratio) \( \hat{1} \equiv (1, \ldots, 1)^T \), and the interest rate \( r \), the drift coefficients \( \mu \equiv (\mu_1, \ldots, \mu_N)^T \), and the volatility matrix \( \sigma \equiv \{\sigma_{ij}; i = 1, \ldots, N; j = 1, \ldots, N\} \) are possibly path-dependent. The quantity \( \xi(T, \omega) \) is interpreted as the Arrow–Debreu price per unit probability \( P \) of one consumption good in state \( \omega \in \Omega \) at time \( T \).

References

Basel Committee on Banking Supervision, Amendment to the Capital Accord to Incorporate Market Risks, January 1996.


