The integrated impact of credit and interest rate risk on banks: A dynamic framework and stress testing application

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Abstract

Credit and interest rate risk are the two most important risks faced by commercial banks in their banking book. In this paper we derive a consistent and comprehensive framework to measure the integrated impact of both risks. By taking account of the repricing characteristics of assets, liabilities and off balance sheet items, we assess the integrated impact of credit and interest rate risk on banks' economic value and capital adequacy. We then stress test a hypothetical but realistic bank using our framework and show that it is fundamental to measure the impact of credit and interest rate risk jointly.

1. Introduction

One of the defining characteristics of banks is that they borrow short and lend long. Following Diamond and Dybvig's seminal work (Diamond and Dybvig, 1983) most of the banking literature has tended to focus on the liquidity implications of such a maturity transformation function. The maturity mismatch – or more precisely the repricing mismatch – is also the key source of interest rate risk in the banking book. According to banks, interest rate risk is the most significant source of market risk for commercial banks (see IFRI-CRO, 2007). And, hence, after credit risk it is the second most important source of risk for the capital adequacy of these institutions. Banks and regulators are aware of the importance of both risks. But because of a lack of models available they tend to manage these risks separately even though, as Jarrow and Turnbull (2000) point out, 'economic theory tells us that market and credit risk are intrinsically related to each other and not separable'.

In this paper we propose a general framework to measure the riskiness of banks which are subject to correlated interest rate and credit shocks. We focus only on the banking book, but extend existing work by modelling both sides of the balance sheets: assets, liabilities and off balance sheet items whilst, crucially, accounting for their repricing characteristics. To analyse the riskiness of the bank we apply a stress test. The results of the stress-test are judged by two criteria.

Our first criterion is an economic value condition. It is based on whether the mark to model value of the bank's assets is large enough relative to the value of its liabilities. This is, for example, one of the condition for insolvencies under FDIC rules (see Section 1821(c) (5), US, 2002). It uses perfect foresight to capture how current and future changes due to the stress scenario affect the value of all of the bank's positions instantaneously. Hence, it provides a long-term view of the impact of a shock.

Our second criterion – the capital adequacy condition – reflects current general regulatory approaches. Although new accounting rules...
standards allow banks to use the fair value option for some securities, most assets and liabilities in the banking book are valued at book value as banks hold them to maturity. Therefore, we assume that the banking book in our implementation is valued using book-value accounting. Under book-value accounting, profits and losses are accounted for only when they materialise, i.e. what matters are realised net cash flows and not changes in the economic value. Hence, a particular path of profits may lead a bank to be undercapitalised in the short run because of severe losses which are expected to be outweighed by future profits. From an economic value perspective the bank would be solvent but because of market or regulatory constraints the bank may find it difficult to continue to operate. Therefore, we also assess whether a bank is sufficiently well capitalised through time which requires us to project the banks’ write-offs, net interest income and capital requirements in a consistent fashion.

Through our methodology we capture (a) the impact of credit risk on the whole portfolio, (b) interest rate risk stemming from the repricing mismatch between assets, liabilities and off balance sheet net positions as well as basis and yield curve risk, and (c) the interdependence between credit and interest rate risk.

We illustrate our framework in the context of a stress test by applying an adverse macroeconomic shock to a portfolio of a hypothetical but realistic bank. In recent years, stress tests have become an increasingly important risk management tool – both on a macro as well as on a bank specific level. The main difference between the two types of stress tests is that central banks use macro stress tests to assess vulnerabilities not only of individual banks but also of the banking systems as whole, whilst stress tests used by banks for risk management purposes tend to focus only on the impact of the stress test on their own exposures. Our framework – which could be used for both macro and bank specific stress tests – significantly improves on most stress testing models. Macro stress tests tend to focus primarily on credit risk (see Sorge and Virolainen, 2006), whilst the lack of integration of market and credit risk has been identified as one failure of stress testing practices at banks in the run-up to the recent crisis (Basel Committee on Banking Supervision, 2009).

The implementation of the stress test requires the use of a comprehensive approach. At the beginning of the simulation the bank's balance sheet is priced to reflect initial market conditions. Using a macro model, we then trace the impact of a stress scenario on macro variables, which, through a term structure model, affect the government yield curve. These results are used to determine the bank's exposure to credit risk on its asset as well as liability side. And by modelling the repricing characteristics of the bank's assets, liabilities and off balance sheet items we estimate the bank's exposure to interest rate risk as well as to the interaction between credit and interest rate risk. Finally, the bank's balance sheet is rebalanced every quarter to guarantee that fundamental accounting identities are satisfied.

Our simulation shows that interest rate and credit risk have to be assessed simultaneously as well as jointly for the whole portfolio including assets, liabilities and off balance sheet items. Importantly, we show that it is important to consider the disaggregated repricing characteristics of assets, liabilities and off balance sheet items to fully capture the combined impact of interest and credit risk on banks' net interest income.

Our stress test application illustrates the key transmission channels from shock to impact. Following the stress, credit risk crystallises as macro conditions deteriorate. The significant increase in interest rates leads to a compression of interest margins given the repricing mismatch between short-term borrowing and long-term lending. This decreases profits even further in the first year than would be the case if only credit risk is assessed. However, over time the bank can start to reprice assets to reflect both the change in the default-free yield curve and the deterioration in credit quality. The offsetting effect of higher net interest income implies that net profits start to recover before write-offs peak.

We find that the margin compression and subsequent expansion due to the repricing mismatch is quantitatively material. Were we to ignore interest rate risk, we would underestimate the fall in net profits of the bank by over 50% in the first year in our stress scenario. But we would overestimate the fall by nearly 100% in the third year. We also decompose the impact of the shock into three components: ‘pure credit risk’ i.e. the impact of credit risk from non-interest rate factors, ‘pure interest rate risk’, i.e. the impact of interest rate risk excluding the effect on credit risk, and the interaction of both risks. We show that the interaction term is a significant driver of net-profitability and capital adequacy. The contribution of each component and the speed at which profits return back to equilibrium depend on various assumptions including the repricing characteristic of the bank's balance sheet and its cost of funding. Nonetheless, regardless of variations in the main assumptions all simulations show that the interaction between credit and interest rate risk plays a crucial role.

The remainder of the paper is structured as follows. In Section 2 we review the literature and discuss the contributions of this paper. In Section 3 we propose a general framework to derive the economic value and capital adequacy conditions for a bank which is subject to credit and interest rate risk in the banking book. In Section 4 we discuss our empirical strategy to capture credit and interest rate risk for a hypothetical bank. In Section 5 we present the results of the stress test and in Section 6 we investigate the importance of interest rate, credit risk and their interaction. Our results are evaluated against a number of sensitivity tests in Section 7. Finally, we summarise the main conclusions of the paper in Section 8.

2. Literature review

There exist vast literatures on modelling credit risk (for an overview see Duffie and Singleton, 2003) and interest rate risk (for an overview see Hull, 2008). One of the simplest risk management tools to measure interest rate risk in the banking book is gap analysis, where banks or regulators assess interest rate risk by purely looking at the net repricing mismatch between assets, liabilities and off balance sheet items. There are several problems with standard and more sophisticated gap analysis (see StaiIouras, 2006). Most importantly these tests implicitly assume that shocks to the risk-free yield curve have no impact on the credit quality of assets.

In general, there are not many papers that attempt to model credit and interest rate risk together. Most of the literature tends to focus on the impact of shocks on assets without modelling liabilities and the repricing characteristics of the whole portfolio. For

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3. Interest rate risk also arises from differences in embedded options of assets and liabilities. While in some countries (e.g. in the US) prepayment risk plays an important role, it is not a major source of risk in other countries (e.g. in the UK) due to high prepayment penalties. Even though our framework could be extended to capture optionality, we do not consider them in this paper.

4. By focusing on the net repricing mismatch, gap analysis also fails to consider non-linearities which could lead to the underestimation of interest rate risk. For example, some short-term customer deposit rates track the risk-free rate plus a negative spread. Hence, banks may be unable to lower deposit rates in line with large falls in risk-free rates given they are bounded by zero. By modelling the whole portfolio we capture this compression in banks' net margins.

5. An exception is the operations research literature discussing asset and liability management. However, only a few models allow for the possibility of default (e.g. see Jobst et al., 2006). In this case they only consider a corporate bond portfolio funded by a simple cash account thereby ignoring the repricing mismatch between assets and liabilities as the most important source of interest rate risk.
example, Jarrow and Turnbull (2000) are among the first to show theoretically how to integrate interest rate (among other market risks) and credit risk using a factor modelling approach. There is also strong empirical evidence that interest rate changes have an impact on the credit quality of assets (e.g. see Carling et al., 2007).

Barnhill and Maxwell (2002) aim to measure credit and market risk for the whole portfolio of a bank. They develop a simulation framework to revalue assets and liabilities depending on the state of several systematic risk factors. They assess the stability of a bank, based on the distribution of economic value. Relative to this paper, our contribution is to capture one of the most important sources of interest rate risk – repricing mismatches between assets and liabilities. Furthermore, we also take into account off balance sheet items and the impact of credit and interest rate risk on future earnings and capital adequacy.

Following the initiation of the Financial Stability Assessment Programmes by the IMF and the World Bank in 1999, macro stress tests have become a central tool for assessing the vulnerability of bank loan. Based on the distribution of economic value. Relative to this paper, our contribution is to capture one of the most important sources of interest rate risk – repricing mismatches between assets and liabilities. Furthermore, we also take into account off balance sheet items and the impact of credit and interest rate risk on future earnings and capital adequacy.

Whilst the outlook of macro and bank specific stress tests may differ, they essentially share the same conceptual underpinnings (see Summer, 2008). In contrast to our paper, few stress tests, whether on a macro or on a bank specific level, take an integrated perspective across risks. Most banks do not have an economic capital model capable of integrating credit and interest rate risk (Basel Committee on Banking Supervision, 2006). This lack of integration of risks was also identified as one failure of stress testing practices at banks prior to the outbreak of the recent crisis (Basel Committee on Banking Supervision, 2009).

Similarly, most macro stress tests used by policymakers focus primarily on credit risk (see Sorge and Virolainen, 2006). Relative to previous papers which rely on reduced form underpinnings (see Haldane et al., 2007), our contribution is to capture the complicated and fundamental relationship between interest rate changes and interest rate income. The most sophisticated stress testing framework developed so far (Elsinger et al., 2006) captures default and interest rate risk in an integrated fashion by conditioning both risks on the underlying systematic factors. But while their approach is in the spirit of gap analysis, we account for the disaggregated repricing characteristics of assets and liabilities necessary to fully capture the combined impact of interest and credit risk.

Another contribution of our paper relative to earlier stress testing applications is to ensure that the balance sheet of the bank balances at any point in time. While this is a fundamental accounting identity that must hold, most, if not all, stress testing models do not impose this restriction.

3. The framework

In this section we first discuss the integration of interest rate and credit risk for a generic asset. We then apply the insight from the generic asset to derive the economic value and capital adequacy conditions for a bank with a portfolio of assets and liabilities with different risk and repricing characteristics.

3.1. A generic asset

The economic value \( EVA_i \) of a generic asset \( i \) with maturity \( T \) is simply the risk-adjusted discounted value of future coupon payments \( C \) and the principal \( A \). Hence

\[
EVA_i = \sum_{t=1}^{T} D_{i,t} C_o A_t + D_{i,1} A_i
\]

(1)

For simplicity we assume that all assets are equivalent to bullet bonds – i.e. repay the principal only at maturity and pay a constant coupon \( C_o \) priced at time \( t = 0 \). For example, such an asset could be a fixed-interest rate bond with no embedded options or a simple bank loan.

The discount function is given by:

\[
D_{i,1:k} = \prod_{l=1}^{k} D_{i,t-l,1-t,l}
\]

(2)

where \( d \) is the period by period risk-adjusted discount factor which is equal to the inverse of \( 1 + R \), the risk-adjusted interest rate. In continuous time, \( R \) equals the risk-free rate plus a credit risk premium. However, as our application is set up in discrete time we follow Duffie and Singleton (2003, p. 134):

\[
R_{t+1,1-t,1} = E_t \left( \frac{r_{t+1,1-t,1} + PD_{i,t+1,1-t,1} \times LGD_{i,t+1,1-t,1}}{1 - PD_{i,t+1,1-t,1} \times LGD_{i,t+1,1-t,1}} \right) \Omega_t \]

(3)

where \( r_{t+1,1-t,1} \) is the forward risk-free interest rate between \( t + 1 \) and \( t + 1 \) known at time \( t \). \( LGD_i \) is the expected loss given default for borrower \( i \) which we initially assume to be constant. \( PD_{i,t+1,1-t,1} \) is the risk-neutral probability of default of borrower \( i \) between \( t + 1 \) and \( t + 1 \) conditional on surviving until \( t + 1 \). Expectations are taken subject to the time \( t \) information set \( \Omega_t \), which, importantly, contains information on the development of systematic risk drivers of PDs and interest rates.

We do not observe empirical coupon rates and need to reprice assets and liabilities according to their contractual repricing characteristics. To do so we assume that at the time of issuance the economic value equals the face value of the asset. This implies that \( EVA_{i,t-0} \Omega_0 = A_i \) in Eq. (1). Solving for \( C_{o,i} \), we obtain:

\[
C_{o,i} = \frac{1 - D_{i,t}}{\sum_{l=1}^{T} D_{i,l}}
\]

(4)

Eqs. (3) and (4) are crucial for understanding the channels through which credit and interest rate risk affect a generic asset. First, both the expected credit risk premium and the expected risk-free yield curve depend on a common set of macroeconomic risk factors. Hence, unexpected changes in these risk factors impact on both credit and interest rate risk. Second, unexpected movements in the risk-free yield curve do change borrowers’ credit risk. When economic conditions change, the yield curve and PDs of the asset will adjust instantaneously and hence the discount factors, \( D_{i,1:k} \), will also adjust immediately. But as coupon rates remain fixed up to the time of repricing, the economic value of the asset will remain fixed.

\[^6\] Their paper also looks at a maturity mismatch of +/− one year and conclude that this is important. But +/− one year is clearly too simplistic to capture the full impact of the maturity mismatch on the riskiness of banks.

\[^7\] Assuming that all assets are equivalent to bullet bonds is convenient for computational purposes. It has no implication on the bank’s economic value condition. Although the assumption matters for total cash in- and out-flows, it has no material consequences for the bank’s profitability and capital adequacy over our forecast horizon as long as paid back principals are reinvested in similar assets in line with the assumptions described later in Section 3.2.3.

\[^8\] The formula assumes that the same \( LGD_i \) applies to both coupons and principal and a zero liquidity premium.

\[^9\] There is also a feedback from credit risk to interest rates. Such an effect is partially embedded in the macro-model, which we use to simulate the systematic risk factors in the following sections. But this channel is hard to quantify formally and we do not explicitly consider it in this paper.
diverge from its face value. Once the asset can be repriced, coupon payments will reflect the new economic conditions and the economic value will equal once again the face value. Applying this insight to a bank portfolio implies that while the economic value always reflects all future and current economic conditions instantaneously, income will only adjust sluggishly depending on the repricing mismatch of assets and liabilities.

3.2. A generic bank

As described in the introduction, in this paper we assess the riskiness of a bank along two dimensions – economic value and capital adequacy. These two conditions closely mirror actual regulatory practices. For example, Bliss and Kaufman (2007) show that from an economic perspective three distinct forms of insolvency for banks can be distinguished under US law (see Section 1821(c) (5), US, 2002): ‘book-value insolvency defined by book values for banks can be distinguished under US law (see Section 1821(c)) from an economic perspective three distinct forms of insolvency determined according to appropriate accounting standards; regulatory insolvency, also defined in terms of book values but set at a higher threshold; and economic insolvency, determined by the market value of assets and face value of liabilities’ (Bliss and Kaufman, 2007, p. 156). Given that the regulatory insolvency is more stringent than book-value insolvency, we focus on economic and regulatory insolvencies or, as we define them, economic value and capital adequacy conditions.

Throughout the framework discussion we use a generic bank. Any bank can be seen as a large portfolio of assets and liabilities. In particular, we will look at N asset classes, and M liability classes \( L_j \) where all exposures in an asset (liability) class \( i (j) \) have the same risk characteristics.\(^{10}\)

3.2.1. Condition 1: the economic value perspective

A firm’s economic value is generally defined as the economic value of its assets minus the economic value of its liabilities. However, looking at the economic value of liabilities for banks may not be desirable from a regulatory perspective since it is not the economic value of liabilities but the banks’ ability to repay liabilities at par which matters most. Hence, our first condition to assess the stability of a bank is to see whether the economic value of assets (EVA) conditional on credit and interest rate risk is greater than the face value of all its liabilities (FVL), or equivalently, whether the (adjusted) economic value of the bank (EVB) is greater than zero.

Condition 1 – economic value:

\[
EVB_t = EVA_t - FVL_t > 0
\]  

(5)

3.2.2. Condition 2: the capital adequacy perspective

As discussed, the economic value always reflects all future and current economic conditions instantaneously thereby providing a long-term view of the impact of a stress test scenario. The capital adequacy perspective determines whether a bank is sufficiently well capitalised in all future states of the world. This provides an important dimension to risk assessment for the banking book as an undercapitalised bank will be subject to regulatory interventions. Under book-value accounting, profits and losses are only accounted for once they materialise, i.e. what matters are realised cash flows and defaults rather than changes in the economic value. It is therefore crucial to assess whether a bank’s capital adequacy given its net profits profile remains above the regulatory minimum \( k \) for all periods in the medium term \( W \). Hence, our second condition is:

\[
\frac{SF_t}{RWA_t} > k \quad \forall \ t < W
\]

(6)

where \( RWA \) denotes risk-weighted assets and \( SF \) shareholder funds. Shareholder funds are equal to the book value of equity at time \( t \) and are assumed to be the only capital of the bank. After time \( t \), shareholders funds will vary according to the banks’ net profit profile (see Section 3.2.3).

Risk-weighted assets are calculated under two different approaches. We first take risk weights to be constant over time. This could be seen as an approximation of the Basel I framework. Under this approach, risk-weighted assets are simply the weighted sum of exposures to asset \( j \) at time \( t \) with risk weights \( \varpi \) differing across asset classes. Hence Condition 2 under this approach is:

Condition 2a – capital adequacy with constant risk weights:

\[
\frac{SF_t}{RWA_{IRB}^{t}} > k \quad \forall \ t < W \\text{with} \ RWA_{IRB}^{t} = \sum_{i} \varpi_i A_{i}^t
\]  

(7)

Following a severe adverse shock, the constant risk weight approach may underestimate the risks to the capital adequacy of the bank. Therefore we also use the Basel II internal rating based approach to derive time-varying risk weights \( w_i^t \) for different asset classes (see Basel Committee on Banking Supervision, 2006). Hence Condition 2 under this approach becomes:

Condition 2b – capital adequacy with time-varying risk weights:

\[
\frac{SF_t}{RWA_{IRB}^{t}} > k \quad \forall \ t < W \\text{with} \ RWA_{IRB}^{t} = \sum_{i} w_i^t A_{i}^t
\]  

(8)

3.2.3. Forecasting shareholder funds

Were it possible to observe the profile of all coupon rates, the economic value condition would only require determining the appropriate risk-adjusted discount rate. As this is not the case, we need to assume that the economic value of assets equals their face value in order to derive initial coupon payments. Such an assumption is also applied every time an asset or a liability is repriced. In addition, we add four more assumptions in order to forecast shareholder funds and ensure that the bank’s balance sheet balances at each point in time.

First, we assume that exposures within an asset class are infinitely fine grained, i.e. individual exposures within an asset class are small.\(^{11}\) We also assume that conditional on the realisation of systematic risk factors, defaults are independent.\(^{12}\) These common assumptions, in line with the basic Basel II formula, imply that, conditional on a specific path of systematic risk factors, unexpected losses are zero.

Second, we assume that depositors are passive: once deposits mature, depositors are willing to roll over their deposits with the same repricing characteristics as long as both the capital adequacy and the economic value conditions are met.

Third, we assume that the bank does not actively manage its portfolio composition: once assets mature, the bank continues to hold (e.g. see Egloff et al., 2007).

\(^{10}\) Although we do not distinguish whether assets and liabilities are on or off balance sheet items in this section, we will model them separately in our stress test application.

\(^{11}\) For a discussion of this assumption see Alessandrini and Drehmann (2010).

\(^{12}\) This is a standard assumption in credit risk models implemented for day to day risk management, even though recent research has shown that it does not necessarily hold (e.g. see Egloff et al., 2007).
Fourth, we assume that the bank uses its free cash flows to pay back the most costly liabilities that matured rather than invest into new assets or expand the balance sheet. If shareholder funds decrease by more than write-offs, we assume that the bank is able to attract new interbank deposits.13

Our behavioural assumptions are to a certain degree arbitrary. But we restrict ourselves to the simplest behavioural rule rather than re-optimising the bank’s portfolio in a mean-variance sense in each period as this would be beyond the scope of this paper.

To enhance readability for a multi-asset and multi-liability bank we will drop the expectation operator and will do so for the remainder of the paper. All calculations are, however, based on expectations conditional on the information set available at the time of pricing. Furthermore, for stock variables, for example the economic value of a loan, we use the subscript t to indicate the value of the variable at time t. For flow variables, for example a bank’s interest receivables, we use the subscript t to indicate the accrued value of the variable between t – 1 and t.

Deriving shareholder funds SF at each future period requires tracking net profits which either grow by retained earnings (i.e. profits after taxes and dividend payouts) or decrease by losses, in which case no taxes and dividends are paid.14 Hence, shareholder funds can be computed as

\[
SF_t = \theta \max(0; NP_t) + \min(0; NP_t) + SF_{t-1}
\]  

with \( \theta < 1 \) given that the bank pays taxes as well as dividends.

Net profits (NP) between period t – 1 and t are the sum of net interest income plus other income (Ot) minus write-offs (WR) and costs (C). Net interest income in turn is the sum of the total cash flows the bank receives from its assets (CFA), minus total cash flows it pays on its liabilities (CFL).

\[
NP_t = (CFA_t - CFL_t) - WR_t + Ot_t - Cost_t
\]  

For simplicity, we assume that other income and costs are driven by a constant exogenous process which we set to zero in the simulation.

### 3.2.4. Forecasting cash flows and the balance sheet for a general bank

We consider a bank with N asset classes \( A \) which have different PDs and LGDs. Within each asset class, exposures can be in different repricing buckets. Following the behavioural assumption outlined above, the bank invests in new projects with the same repricing and risk characteristics as the matured assets. However, it will not replace defaulted assets. Hence, the evolution of each asset class i from time t – 1 to t adjusting for default is:

\[
A^i_{t} = A^i_{t-1} (1 - PD^i_{t} \cdot LGD^i)
\]

and the total cash flows from assets (CFA) between t – 1 and t is:

\[
CFA_t = \sum_{i=1}^{N} \left( \sum_{b=1}^{C} C_{0}^{i,b} A_{t-1}^{i,b} + \sum_{b=1}^{C} \sum_{l=1}^{L} \sum_{i=1}^{N} I_{l}^{i,b} A_{t-1}^{i,b} \right)
\]  

with

\[
I_{l,i} = 1 \text{ in period } l \text{ when assets in bucket } b \text{ have been repriced the last time prior to } t.
\]

\[
I_{l,i} = 0 \text{ otherwise.}
\]

The interpretation of Eq. (12) is relatively straightforward. The first term in the brackets sums the coupon payments \( C \) of asset classes which have not been repriced at time t, and the second term sums coupon payments \( C \) of asset classes which were last repriced in period l prior to time t. All coupons are priced using Eqs. (1)–(4). Finally, Eq. (12) sums over the N different asset classes.

Given the evolution of assets, write-offs are given by:

\[
WR_t = \sum_{i=1}^{N} LGD^i PD^i A_{t-1}^{i,b}
\]

Eqs. (12) and (13) highlight how profits are driven by changes in write-offs, exposures and cash-flow contributions to net interest income. For example, if economic conditions deteriorate, write-offs will increase. Such an increase will also decrease \( A^i \) and in turn CFA collected between time t – 1 and t, ultimately reducing NP. On the other hand, the bank also receives higher coupon payments \( C \) from non-defaulted assets which have been repriced to reflect the increase in credit risk and risk-free interest rates.

Similarly, given that we assume that borrowers are willing to roll over the bank’s liabilities, the total cash flows paid on liabilities (CFL) between t – 1 and t is:

\[
CFL_t = \sum_{j=1}^{M} \left( \sum_{b=1}^{C} C_{j}^{b} t_{b}^{b} + \sum_{b=1}^{C} \sum_{l=1}^{L} \sum_{i=1}^{N} I_{l}^{i,b} t_{l-1}^{b} \right)
\]

with

\[
I_{l,i} = 1 \text{ in period } l \text{ when liabilities in bucket } b \text{ have been repriced the last time prior to } t.
\]

\[
I_{l,i} = 0 \text{ otherwise.}
\]

In line with Eqs. (12), (14) sums over all liability classes with the first term in brackets summing the coupon payments \( C \) of liability classes which have not been repriced at time t, and the second term summing coupon payments \( C \) of liability classes which were last repriced in period l prior to time t.

Eqs. (10)–(14) allow us to forecast net profits, which together with the evolution of write-offs (Eq. (13)) determine the change in shareholder funds. Assets on the other hand only vary in line with write-offs (Eq. (11)). To balance the bank’s balance sheet, liabilities have to adjust. In line with our forth assumption at the beginning of Section 3.2.3 this implies that:

\[
\Delta A_t = \Delta A_t - \Delta SF_t = -WR_t - \Delta SF_t
\]

### 4. Stress testing credit and interest rate risk for a stylised bank

Based on the framework outlined in the previous section, we propose a stress test that captures the impact of a stress on assets, liabilities and off balance sheet items. Moreover, the stress test accounts for the complex nature of the repricing characteristics of a bank’s balance sheet. From a stress testing perspective, it is essential that underlying interdependencies are captured – between PDs of different asset classes and between PDs and the risk-free yield curve and macro factors.

#### 4.1. The stress test

As pointed out by Summer (2008), current stress testing models can be characterised as a chain of models starting with a specific shock to systematic risk factors, followed by a data generating process which captures how systematic risk drivers interact between...
each other and across time, and finally a model computing the impact on banks' balance sheets. Our approach to stress testing follows this logic (see Fig. A1 in Appendix).

At the beginning of the simulation the bank's balance sheet is priced to reflect losses in line with the baseline scenario. Afterwards, the stress scenario crystallises in the first quarter of the simulation. And similarly to standard impulse response analysis we assume that no further shocks occur in the future. Using the macro model, we trace the impact of the scenario on all macro variables over a three year horizon, after which variables are assumed to revert gradually back to equilibrium levels. We then project the yield curves and default probabilities conditional on our macroeconomic forecast in each quarter. This information is sufficient to calculate the economic value condition.

To assess whether the capital condition is satisfied, we need to project net profits and also time-varying weights under the internal approach. We assume that after the shock hits in quarter one, the bank can reprice all assets and liabilities in the first repricing bucket, then defaults materialise and income and net profits are realised. At this point we assess the economic capital condition of the bank. At the end of the quarter the bank rebalances its balance sheet in line with our behavioural assumptions. The remaining forecast periods follow the same structure in each quarter, except that no further shocks occur. However, the repricing mechanism becomes increasingly complex to track over time as different assets and liabilities are repriced at different points in time (see Fig. A2).

4.2. The hypothetical bank

We construct a hypothetical bank with a stylised balance sheet (see Table A1 in Appendix). We restrict ourselves to domestic exposures only. This reduces the number of systematic risk drivers we have to capture dramatically, without changing the key insights of this paper. All exposures are assumed to be held to maturity unless a default event occurs.

We distinguish six asset classes: interbank lending, flexible as well as fixed rate mortgages, credit card lending, lending to private non-financial corporations (PNFC), and risk-free debt securities including treasuries. On the liability side we initially consider three different types of liabilities: interbank borrowing, borrowing from households and borrowing from PNFCs. Later we also include debt instruments issued by the bank. Additionally we account for shareholder funds and interest rate derivatives as off balance sheet items.

In line with accounting standards, assets, liabilities and off balance sheet items are allocated to five repricing buckets as shown in Table A1 in Appendix. In our analysis, we assume that assets, liabilities and off-balance sheet items in the last three buckets are uniformly distributed across quarters within each bucket. For the last bucket we assume that the maximum time to repricing is 10 years. The repricing mismatch between assets, liabilities and off-balance sheet items is referred to as interest rate sensitivity gap.

We use repricing buckets rather than maturity buckets. This means that, for example, a flexible mortgage with a 20-year maturity that reprices every three months is allocated in the three-month repricing bucket. We use repricing buckets in order to correctly capture the impact of changes in the macroeconomic environment on the bank's net-interest income and hence profits.

Although our balance sheet is a hypothetical construct, it is calibrated to reflect the key characteristics of a realistic commercial bank. For example, shareholder funds, profitability (both in terms of return on equity and on assets), the cost-income ratio and the amount of assets and liabilities in each repricing bucket match a realistic commercial bank. And we also use off balance sheet items to make sure that the interest rate sensitivity gap reflects that of a realistic commercial bank.

4.3. The scenario

As baseline scenario in our stress test we use the central projection presented in the Bank of England February 2005 Inflation Report where interest rates are assumed to follow the market forward curve (see Bank of England, 2005). As it has often been the case in the United Kingdom, it is downward sloping. In this scenario, short-term interest rates fluctuate around 4.5% and output growth remains around 3% throughout the whole forecast period.

As stress scenario we consider the simultaneous occurrence of three shocks originally used for the IMF ‘Financial Stability Assessment Programme’ in 2002: a 12% decline in UK residential and commercial property prices, an inflationary shock driven by a 1.5% unanticipated increase in UK average earnings growth and a 15% unanticipated depreciation in the trade-weighted sterling exchange rate. The size of the shocks to each series broadly corresponds to an event three standard deviations away from the mean. This constellation of shocks provides what was regarded as a severe but not too extreme scenario around the time when we implemented the stress scenario. A similar combination of shocks has, for example, also been applied to analyse financial stability risks in the UK (Haldane et al., 2007). For illustrative purposes, we therefore use this scenario for our simulations. The stress scenario is run from 2005 Q1 and forecasted over a three year horizon.

4.4. The macroeconomic model, the yield curve model, PDs and LGDs

To be able to forecast PDs and yield curves we need forecasts of macroeconomic variables in the scenarios, which we obtain by using the Bank of England’s macro model. During the forecast of the stress scenario no judgements are applied to the models and we simply feed the shocks mechanically first through the macro model and then through the other modules discussed below. As will become apparent, and at the heart of the integration of credit and interest rate risk, the key macroeconomic variable is the interest rate. Hence, modelling the monetary policy reaction to the initial shock is crucial. In line with general macro stress-testing practices we assume a mechanical Taylor rule.

The term-structure model we employ is based on Diebold et al. (2006) with three latent factors and three observable macroeconomic variables. We apply it to UK interest rates with maturities from three months to ten years extracted from the Bank of England yield curve data set. The three observable macroeconomic variables are the output gap, inflation and the Bank Rate. The three latent factors have the usual interpretation as the level, slope and curvature of the yield curve. The estimated term-structure model

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15 In order to reflect the interest rate sensitivity gap of a realistic commercial bank we use the accounting standard known in the UK as FRS 13. Under this accounting standard commercial banks disclosed in their published accounts the repricing mismatch between interest-sensitive assets and liabilities taking into account interest rate swaps, forward contracts and other interest rate derivatives whose effect is to alter the bank’s interest rate sensitivity gap.

16 The evolution of the risk free term structure over a three year horizon in the base and the stress scenario is shown in the working paper version (see Drehmann et al., 2008, p. 35).

17 A Taylor rule is, of course, not representative of the way in which the Monetary Policy Committee sets interest rates. Committee members use a range of models and judgements in forming their assessments.

18 We are grateful to Chris Kubelec who has estimated this model using monthly data between 1986 and 2005.
enables us to forecast the default-free yield curves across maturities up to ten years conditional on a given macro scenario. Libor is then forecasted by assuming a constant spread over the default-free term structure of 30 basis points.

There seems to be common agreement that macroeconomic factors are important drivers of credit risk (for an overview see Duffie and Singleton, 2003). In contrast to most credit risk models, our adopted approach has the benefit that it explicitly identifies the systematic risk drivers of credit risk as macroeconomic factors. More generally, the models we use are based on standard regression analysis linking aggregate default probabilities to macroeconomic variables. They are described, including coefficient estimates, in Bunn et al. (2005) and can be summarised as follows.

The corporate PD is modelled as a function of own lagged values, GDP growth, corporate income gearing, the change in commercial property capital values, change in real interest rates and the ratio of net debt of PNFCs to nominal GDP. The PD on mortgage loans is modelled as a function of mortgage income gearing, unemployment, undrawn housing equity and loan to value ratio of first-time buyers. Finally, the PD on credit card loans is modelled as a function of household income gearing and the number of active credit balances.

For all types of household and corporate lending, income gearing – the ratio of interest payments relative to disposable income – is found to be an important driver of PDs. Income gearing in turn is highly sensitive to changes in interest rates as higher interest rates translate into higher interest payments. This implies that, for example, an unexpected increase in interest rates will, ceteris paribus, not only affect net profits through the interest rate sensitivity gap but also through higher default rates due to an increase in income gearing.

Although we relax the assumption in our sensitivity analysis, we assume that LGDs are fixed in the base case simulation. Slightly worse than average industry numbers suggest, we assume that the LGD on interbank loans is 40%, the LGD on mortgage loans to be 30%, the LGD on credit cards to be 80% and the LGD on corporate loans to be 60%.

4.5. Pricing

Below we describe the pricing of assets, liabilities and off balance sheet items.

4.5.1. Pricing of assets

In Section 3 we proposed a risk-neutral pricing framework to derive coupon rates which we do not observe. It is well known that there is no simple mapping from actual PDs, which we simulate, into risk-neutral PDs, which we require for pricing (see e.g. Duffie and Singleton, 2003). Given that the core of our framework is to assess the riskiness of banks which are subject to both credit and interest rate risk, an explicit model of risk premia is beyond the scope of this paper. In all our simulations we assume that the risk premium is stable over time and equal to zero. This assumption may introduce a downward bias in net interest income as risk-adjusted coupon rates would be higher than those in our simulation.19 Hence, our economic value and capital adequacy conditions are more likely to be violated in line with a conservative approach to risk management.

4.5.2. Pricing of liabilities

Eqs. (1)–(4) should, in theory, apply to the pricing of all liabilities using the bank’s own PD and LGD. However, it is well known that shorter-term customer deposit rates generally are below the risk-free interest rate even when accounting for non-interest costs net of fees. This may be the result of deposit insurance schemes or barriers to entry limiting competition. While an economic rationalisation of negative spreads can be found for short maturities it is less convincing for medium to long maturities. We assume that as the time to repricing increases the interest paid by the bank on deposits gradually converges to the risk-free interest rate. We model the deposit rate on household deposits with one quarter to repricing to be 2% below the Bank Rate and the corporate deposit rate to be 1% below the Libor rate. The negative spreads are then assumed to decline linearly reaching zero at a one year repricing maturity.

For most of our simulation we assume that all liabilities of the hypothetical bank are in the form of deposits and interbank lending. In Section 7.5 we modify the hypothetical bank’s portfolio by introducing debt instruments, where spreads reflect the bank’s own credit risk. However, there is a circularity problem as a bank’s own credit risk depends, inter alia, on the spread that the bank pays on its debt instruments which in turn depends on the bank’s own credit risk.

We therefore use an indirect method: starting with an initial rating (A+) we forecast its evolution by applying the rating model of Pagratis and Stringa (2009). This model is an ordered probit model that predicts ratings based on factors which can be forecasted by our framework such as capital adequacy, profits before tax and write-offs, write-offs relative to net interest income, cost-income ratio, interest rates, bank’s size, GDP, and country. We then map ratings to spreads where spreads are obtained from the average credit spread term structure of sterling corporate bonds over the 2003–06 period for each rating category.20 We use corporate spreads as we do not observe sufficient bank-specific spreads for all ratings in the United Kingdom.

4.5.3. Pricing of interest sensitive off balance sheet items

The bank uses off balance sheet items to manage its exposure to interest rate risk. In particular, it uses interest rate derivatives to decrease its exposure to increases in short-term interest rates. Hence, the bank has a lower interest rate sensitivity gap in the zero to three month bucket than implied by its on balance sheet exposures.

In order to capture how the bank’s hedging strategy modifies the interest rate sensitivity gap we simply assume that a positive (negative) net off balance sheet position in a given repricing bucket increases (decreases) the bank’s assets in that bucket and that the bank receives (pays) a risk-free coupon rate. This is equivalent to assuming that counterparty risk is eliminated without costs by a clearing house.

4.6. Defaults, net interest income, write-offs and the re-balancing of the balance sheet

As we assume an infinitely fine grained portfolio, no unexpected credit risk losses materialise conditional on the macro scenario. Net-interest income is simply the difference between total cash flows from assets (Eq. (12)) and total cash flows paid on liabilities (Eq. (14)). At the end of each quarter the balance sheet changes in line with our behavioural assumption taking account of write-offs and profits (Eq. (15)).

19 Including risk premia would affect both assets and liabilities. However given that a large proportion of liabilities carries neither any coupons or negative spreads (see Section 4.5.2) our treatment of risk premia is likely to introduce on average a downward bias in the bank’s net interest rate income. To analyse the sensitivity of our results, we assessed the impact of a positive risk premium. In all cases, net interest income increases. Results are available on request.

20 For a detailed description see Drehmann et al. (2008, p. 21).
4.7. The economic value and the capital conditions

No further assumptions are needed to implement the economic value condition. The economic value condition (Eq. (5)) is only calculated twice; once at the beginning of the simulation and once after the stress scenario materialises. As discussed, we assume that no other shocks occur throughout our simulation. And given that the economic value perspective captures all current and future implications of the shock instantaneously, it will not be breached in the future, unless the economic value conditions is violated immediately after the shock occurred.

For the capital conditions (Eqs. (6) and (7)) we assume throughout the simulation that capital can be proxied by shareholder funds. The current Tier 1 minimum capital requirement relative to risk-weighted assets is 4%. Therefore, we set 4% as our threshold k. For Condition 2a we set the following constant risk weights: 0.5 for interbank lending, 0.35 for mortgage lending, 0.75 for unsecured lending and 1 for corporate loans. For the capital adequacy Condition 2b we use the Basel II formula (see Basel Committee on Banking Supervision, 2006) to derive time-varying risk weights.

5. Results

In this section we first analyse the impact of the stress scenario on macroeconomic condition and then assess the economic value and the capital adequacy conditions.

5.1. Risk-free and credit spread yield curves

Given the strong macroeconomic fundamentals at the end of 2005, the impact of the shocks on the economy is less severe than the early 1990s recession. For example, GDP continues to increase, albeit growth falls significantly. Following the stress the risk-free yield curve shifts upwards with the short end of the curve around 5.5% in the first quarter increasing steadily over the three years reaching almost 10% three years after the shock.

In the stress scenario, the largest increase in spreads on loans occurs for mortgages.21 Although the spread on credit cards does not rise by as much, it remains higher relative to that on mortgages. The corporate spread is least affected by the macroeconomic shock. The main reason for the subdued rise in the corporate spread is consistent with the relatively high credit quality of the banks’ corporate lending book and with the nature of the chosen shock.

5.2. Condition 1: the economic value perspective

As discussed in the framework section the economic value perspective measures the potential long-term impact of the shock on the bank. The net economic value of our hypothetical bank in the baseline scenario is calibrated to 7.3% of the face value of assets. This equals the book value of assets net of liabilities and off balance sheet items. Immediately after the shock crystallises the economic value falls to 5.7%. Notwithstanding that this represents a 21% fall, the long-term combined impact of credit and interest rate risk is not large enough to threaten the stability of the hypothetical bank.

5.3. Condition 2: the capital adequacy perspective

Even though the economic value condition is not violated, it may still be the case that in the short or medium term the bank makes losses which could threaten its capital. For this reason it is important to investigate whether Condition 2 is satisfied, that is whether the bank’s capital adequacy remains above the regulatory minimum in all periods for the next three years.

As described in Section 3, Condition 2 depends, inter alia, on the evolution of net profits, shareholder funds and risk-weighted assets. In turn the key two determinants of net profits are net interest income and write-offs. In line with Bunn et al. (2005) write-offs are significantly higher in the stress scenario and peak towards the end of the final year (dotted lines in Fig. 1). However, the trough in the bank’s net profits in the stress scenario occurs after only two years. This is because net interest income initially falls slightly due to a rise in borrowers defaulting as well as to the margin compression between short-term borrowing and long-term lending rates (solid lines in Fig. 1). But, after one and a half years net interest income starts to increase. This follows the gradual repricing of assets reflecting the higher credit risk in the stress scenario, which the bank passes on to borrowers.

The combined impact of write-offs and net interest income imply that net profits fall by more than 50% in the eighth quarter but then start to recover (dashed lines in Fig. 2). As will be discussed in more detail in the next section, it is clear from Fig. 2 that interest rate and credit risk have to be assessed jointly.

The impact of the shock can also be summarised in terms of return on equity (RoE) as illustrated in Fig. 2 (dotted line). Compared to an initial RoE of around 20% in the baseline scenario, the shock nearly halves the bank’s RoE in the worst quarter two years after the shock. But it is also evident that the bank remains profitable in every quarter over the three-year horizon. Given our assumption that after tax profits and dividends are retained as capital, shareholder funds increase in each quarter. And given that under the standardised approach risk weights do not adjust to the decrease in credit quality, Condition 2a improves in both scenarios as shown in Fig. 3 (Panel A).

Conversely, under the internal approach the increase in shareholder funds is more than offset by the increase in risk weights reflecting the rise in credit risk (Fig. 3, Panel B). However, the overall fall does not threaten the stability of the bank as the capital ratio always remains well above the regulatory minimum. Therefore Condition 2a is also satisfied in all periods.

21 Credit spread curves for all types of lending in the base and stress scenario are shown in the working paper version (see Drehmann et al., 2008, p. 23). In the base line scenario, mortgage spreads are in line with average those observed in the market place. As default rates and LGDs on credit cards are highest, spreads on credit card lending are much wider for any other type of lending. Spreads on corporates compare to a BBB spread which is slightly above the average quality (BB) of the corporate portfolio of a typical G10 bank (see Catarineu-Rabell et al., 2005).
Overall, independently of whether we look at the short or long-run indicators, the stress scenario would weaken our hypothetical bank but it would not threaten its stability.

6. Integration of interest rate and credit risk

Given that interest rate and credit risk are intrinsically related, this section investigates which risk is the main driver of the fall in profits and capital in the stress scenario. To do so, we disentangle the impact of the shock into three components:

(A) The impact of credit risk from non-interest rate factors.
(B) The impact of interest rate risk but excluding the effect of changes in interest rates on credit risk.
(C) The impact of the interaction of credit risk and interest rate risk.

To assess (A) we calculate PDs conditional on all systematic risk factors changing to their stressed levels and interest rates remaining at their base case scenario level. Hence, (A) highlights the importance of all non-direct interest rate factors. We refer to this component as ‘pure credit risk’. (B) is similar to interest sensitivity analyses run by banks and therefore we refer to it as ‘pure interest rate risk’. As discussed previously, these tests look at shifts (often only parallel) in the yield curve but ignore any implications this may have on PDs and credit risk. (C) is calculated as the difference between the impact of the overall shock, as described in the previous section, and the combined impact of (A) and (B).

6.1. Write-offs, net interest income and profits

The impact of the shock on write-offs, net interest income and profits is shown in Fig. 4. Due to the sharp increase in interest rates households’ and corporates’ income gearing increases materially. Hence, in comparison to other macroeconomic factors, interest rates are the key drivers of write-offs in our scenario as shown in Fig. 4 (Panel A).

Fig. 4 (Panel B) disentangles the complex effects of interest rate and credit risk on net interest income. As gap analysis suggests, ‘pure interest rate risk’ decreases net interest income as margins are compressed (shaded area). However, ‘pure interest rate risk’ does not take account of the impact of interest rates on credit quality nor the interaction between interest rates and other credit risk drivers in a stressed scenario. As already discussed, the increase in credit risk has two opposing effects on net interest income. On the one hand, higher write-offs decrease net interest income as borrowers default on coupon payments and the bank’s exposures decline over time. On the other hand, there is a positive impact of credit risk on net interest income because, over time, banks adjust the credit spread on loans that are repriced. It is apparent from Fig. 4 (Panel B) that the latter effect dominates.

Looking at the overall impact on profits (Fig. 4, Panel C) it is evident that in our scenario the rise in interest rates is the main cause of the fall in net profits as it drives both the squeeze in net margins and the rise in write-offs. But, more important, Fig. 4 clearly shows why credit and interest rate risk have to be assessed jointly and simultaneously for the whole portfolio. In our stress scenario, were the bank to focus only on the impact of credit risk on write-offs (Panel A) without taking net interest income into account – as it is often the case for a standard stress-test analysis – it would overestimate the overall negative impact of the scenario on net profits by around 25%. Interestingly, the effects are not symmetric over time. In the first year, focusing only on write-offs would lead to the underestimation of the negative impact on net profits by over 50% as the decrease in net interest income (solid line in Panel B) is...
not taken into account. However, by the third year, the bank has repriced a large proportion of its assets leading to an increase in net interest income. Therefore, a bank focusing solely on write-offs would ignore this positive effect and overestimate the negative impact on net profits by nearly 100% in the third year. Conversely, if the bank were to assess the impact of higher interest rates on its book by purely undertaking a sensitivity analysis based on its repricing mismatch (shaded area in Panel B), it would underestimate the negative impact of the shock by around 30% over the three-year period.

6.2. Capital adequacy

The contribution of credit risk, interest rate risk and their interaction to the fall in profits in the stress scenario is automatically reflected in the change to the bank's adequacy with constant risk weights. As risk weights do not adjust to the decrease in credit quality, the driver of the lower profile of the bank's capital adequacy (Fig. 3, Panel A) is simply the fall in profits (Fig. 4, Panel C). On the other hand, under the internal ratings based approach, risk weights change to reflect the increase in credit risk. Indeed
as discussed in Section 5.3, the increase in weights leads to a material fall in the bank’s capital adequacy. This implies that under time-varying risk weights overall credit risk should play an even more central role.

Fig. 5 shows the cumulative impact of the shock on the bank’s capital adequacy with time-varying risk weights. These results, when compared to those of Fig. 4 (Panel C), highlight the importance of having different perspectives to assess a bank’s riskiness. ‘Pure interest rate risk’ is an important driver of the fall in net profits, which in turn contribute to shareholder funds over time. The latter also determine the numerator of the capital adequacy condition. However, under time-varying risk weights the denominator of the capital condition is driven by credit risk. Overall, and in line with the above intuition credit risk plays the key role. But even more interesting Fig. 5 shows that the interaction of credit and interest rate risk accounts for about 60% of the fall in the bank’s capital adequacy.

7. Sensitivity analysis

In the previous sections we showed that the shock does weaken our hypothetical bank but it does not threaten its stability. And we also highlighted that the interaction of credit and interest rate risk plays a key role. In this section we analyse the sensitivity of these results to some of the bank’s balance sheet characteristics and some of our main assumptions.

7.1. Changes in the bank’s interest rate sensitivity gap

To assess how sensitive our results are to the bank’s interest rate sensitive gap we consider the case where the bank has completely hedged its exposure to interest rate risk over the next five years. In particular, we assume that the bank has set the first four repricing buckets to zero by a greater use of off balance sheet items as shown in Table A1 in Appendix.

By assuming a zero interest rate sensitivity gap we eliminate the bank’s exposure to ‘pure interest rate risk’ over the forecast horizon. More interestingly, setting the interest rate sensitivity gap to zero does not affect the importance of the interaction between credit and interest rate risk. First, higher credit risk is still passed on to borrowers once assets can be repriced according to the assets repricing characteristics (Fig. 6, Panel A, shaded areas). Second, setting the interest sensitivity to zero does not materially change the profile of the bank’s capital adequacy under time-varying risk weights (Fig. 6, Panel B) as the change in the interest rate gap neither affects ‘pure credit risk’, nor the importance of the interaction between credit and interest rate risk, which is the main driver of the fall in the bank’s capital.

7.2. Fixed versus flexible mortgages

Our hypothetical bank has a larger proportion of flexible than fixed rate mortgages. Although this is the result of our calibration, in some countries such as the US the opposite is normally the case. We therefore arbitrarily remove all flexible rate mortgages and allocated these assets to fixed rated mortgages as shown in Table A1 in Appendix. This would, ceteris paribus, increase the bank’s net interest rate sensitivity gap. Given that the consequences of varying the interest rate sensitivity gap have already been discussed above, we increase the bank’s use of off balance sheet items to restore the original gap. This allows us to focus on the impact of mortgages with different repricing characteristics.

The analysis so far has abstracted from the possibility that interest rates could have a different impact on the default likelihood of fixed and floating rate mortgages. Hence, we have implicitly assumed that the stress scenario, and in particular the increase in interest rates, affects the PD of fixed rate mortgages as much as the PD of floating rate ones. As mortgage servicing costs for fixed rate borrowers take longer to reflect the increase in credit risk than those for floating rate ones, this assumption may not hold. In general, whether we differentiate between fixed and flexible mortgage PD is unlikely to have a material impact on our results given that in all sections in this paper around 88% of the mortgages are repriced over our forecast horizon. But this section is the exception given that only about a quarter of the total mortgages are repriced over the forecast horizon.

To approximate for different default rates across variable and fixed rate mortgages, we simulate the stress scenario assuming that only non-interest rate risk factors, such as the unemployment rate, drive the PD of fixed rate mortgages until the loan gets repriced. Note that this is an extreme assumption. Even if mortgage interest payments remain constant, higher interest rates may still affect the default probability of fixed rate borrowers because servicing costs of other debt may, for example, increase.

22 This is because the mortgage PD model does not differentiate between fixed and flexible mortgages. Ideally, the model should be estimated separately for each type, but this was not possible because of data limitation.
On the one hand assuming no impact of interest rates on fixed rate mortgages causes write-offs during the stress to increase by less in comparison to the base case simulation in Section 6.1 (Fig. 7, Panel A). On the other hand, the size and duration of the fall in net interest rate income is larger. Net interest income only recovers fully after eleven quarters rather than six. This may at first appear surprising given that we have maintained the original interest rate sensitivity gap and overall defaults are lower. The cause is not an increase in ‘pure interest rate risk’ given that both banks have the same interest rate sensitivity gap. But it is because over our forecast horizon the bank can only adjust credit spreads for a much lower proportion of mortgages to reflect the higher credit risk due to non-interest rate factors such as unemployment.

Overall, the decrease in net interest income is larger than the decrease in write-offs. In particular in the last year of our forecast horizon, the fall in net profits is seven percentage points larger than in the base case simulation (Fig. 7, Panel B). Nonetheless, neither the capital adequacy nor the economic value condition is threatened.

These results highlight that not only do banks take on more interest rate risk but also more credit risk when providing fixed rate mortgages, even when assuming that default rates of fixed rate mortgages are much lower than for flexible ones as the bank does not have the option to increase mortgage credit spreads as the economic environment deteriorates. Hence, the bank cannot fully hedge the consequences of the increased proportion of fixed mortgages by simply maintaining the original interest rate sensitivity gap.

7.3. Constant spread on variable-rate mortgage loans

We have assumed that the bank can change the lending rate on those mortgages that are repriced reflecting both the changes in the risk-free interest rate and in credit risk. However, depending on the legal characteristics and external constraints, banks may not be able to pass on higher credit risk but only reprice flexible rate mortgages in line with changes in the default-free interest rate. Here we consider the opposite case. We return to our original balance sheet but now assume that the bank can adjust mortgage rates in line with risk-free interest rates but must hold a constant spread on the variable-rate mortgages for the first three years.

Comparing the dotted and continuous lines in Panel A in Fig. 8, it is clear that net interest income adjusts more slowly when spreads are held constant. Hence, net interest income is substantially lower than in the base case simulation because the bank cannot pass on the higher credit risk to borrowers. But higher interest
rates are fully reflected in higher default rates in contrast to Section 7.2. Therefore, net profits decrease substantially and for longer than in the base case simulation, reaching a minimum of around 50 in the eleventh quarter as shown in Panel B in Fig. 8. However, the bank continues to make positive net profits and satisfies the economic value as well as both capital adequacy conditions. Even when using time-varying risk weights, capital adequacy always remains above 5.5%.

Overall this result highlights that, even when risk characteristics in terms of PDs and LGDs dynamics remain the same, repricing characteristics of exposures can have a substantial impact on the financial strength of a bank.

7.4. Cyclical LGD

A recent book edited by Altman et al. (2005) provides strong evidence that recovery rates are low when aggregate default rates are high. For example, Frye (2005) finds that the LGD in high default years exceeds LGD in low default years by around 15% points.

Even though we start with relatively high LGDs in the main section, we assume that they do not change in the stress scenario. We test the sensitivity of our results to this assumption by decreasing the recovery rates by 15% points in the stress scenario as suggested by Frye. Given that it is unrealistic that LGDs remain at the new higher level forever, we assume that they gradually revert to their baseline levels over the following 10 years.

One of the effects of higher LGDs is a rise in discount rates and hence a larger fall in the economic value in the stressed condition (24% in comparison to 21% in the base case simulation). But the economic value conditions remain satisfied.

Higher LGDs also imply a significant rise in write-offs (Fig. 9, Panel A). Although credit spreads widen in response to higher LGDs, they do not fully offset the higher write-offs over the forecast horizon. The result is a further fall in net profits in comparison to non-cyclical LGDs. However, net profits continue to be positive and shareholder funds rise over time. Therefore, the LGD assumption does not have a material impact on shareholder funds as a proportion of RWA, under the constant risk weights approach. But under the internal rating based approach, RWA do increase and the capital ratio falls to a minimum of 4.4% (dotted line in Fig. 9, Panel B) versus a minimum of 5.5% with constant spreads. This is a clear indication that higher LGDs ‘hurt’ the bank twice: first higher LGDs lead to higher losses and therefore a slower accumulation of shareholder funds. Second, and more importantly, higher LGDs increase risk weights significantly and hence lower capital adequacy ratios for the internal-based approach.
The above LGD stress test is implemented rather mechanically. In principle it would be best to link LGDs directly to macro factors and especially interest rates. However, data for the UK on loss given default are scarce and not publicly accessible, so it is therefore not possible to estimate the response of LGDs to the stress scenario. As an alternative, we simply linked the increase in LGDs – one for one – to the fall in house prices (which initially decrease by 12% in our stress scenario). We find that the bank’s capital adequacy falls by more than when LGDs are kept constant but by less than when LGDs are increased in line with Frye (2005).

7.5. Including debt instruments

In the previous sections we assumed that all liabilities of the hypothetical bank are in form of customer or interbank deposits. However, debt instruments usually account for a sizable proportion of banks’ liabilities. In this section we modify the hypothetical bank’s portfolio by substituting 50% of the interbank liabilities with debt instruments (see Table A1 in Appendix). Debt instruments should be priced according to Eqs. (1)–(4) by taking the bank’s own credit risk into account. Given the circularity between a bank’s own PD and its cost of debt, we use the indirect method described in Section 4.5.2 to forecast the bank’s rating which in turn determines the spread the bank has to pay on its debt instruments.

When we simulate the bank with debt instruments we find that the deterioration in the bank’s financial ratios is not large enough to trigger a downgrade (i.e. the bank’s rating does not change from the initial A+ level).

To explore a possible worst case of the impact of the shock on the bank’s balance sheet, we combine the last three sensitivity tests: constant spreads for mortgages, cyclical LGDs and debt instruments outstanding. In comparison to the impact of the stress in the base case simulation profits are significantly lower. As a consequence the bank gets downgraded twice after the second year, which in turn implies that the bank has to pay higher spreads in the debt market. Following an increase in the cost of debt the bank does make losses throughout the third year (Fig. 10, Panel A). However, even in this more extreme case the impact of the shock is still not sufficient to push the bank below the 4% capital adequacy threshold (Fig. 10, Panel B).

Nevertheless it is important to stress that the above downgrade illustrates the importance of considering the combined impact of interest rate risk and credit risk. Neither the impact of credit risk from non-interest rate factors nor the ‘pure interest rate risk’ are large enough to trigger the downgrade of the bank. The bank is only downgraded when their combined impact is taken into account. Were we simply to add the results of a standard gap analysis model and a separate credit risk model we would likely fail to capture the downgrade which in turn leads to an increase in the bank’s funding costs.

It may be worth to summarise the key impact of our sensitivity analysis. We have seen that changing the interest rate sensitivity gap with off balance sheet items solely affects the ‘pure interest rate risk’ component. Pricing characteristics on the other hand matter for the impact of the interaction term and ‘pure credit risk’ on net interest income, whilst higher LGDs are important not only for write-offs but also for net interest rate income. Our analysis with subordinate debt combines these elements and shows in addition that lower profitability can lead to downgrades which in turn can decrease net interest income further. Overall, it is also clear that ‘pure interest rate risk’ is an important driver of the fall in profits – especially in the short run. Nonetheless, the impact of interaction of credit and interest rate risk is crucial, especially in second and third year, emphasising the importance of assessing both risks jointly. And while ‘pure interest rate risk’ is less relevant under time-varying risk weights, the interaction term continue to play a key role for capital adequacy.23

8. Conclusion

Credit and interest rate risk are the two most important risks faced by commercial banks in their banking book. And given that they are intrinsically related, they cannot be measured separately. In this paper we derive a comprehensive and consistent framework to measure the integrated impact of both risks on the banking book.

To illustrate our framework we analyse the impact of a macro stress on a hypothetical but realistic bank. This requires a modelling approach that is more comprehensive than generally employed. First, we take account of the fact that both credit and interest rate risk are driven by a set of common macroeconomic

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23 A detailed table comparing the impact of different risk components in each simulation is available on request.
risk factors. Second, we incorporate the dependency of credit risk on unexpected changes in interest rates. Third, and a key contribution of the paper, we show that it is crucial to model the whole balance sheet including assets, liabilities, off-balance sheet items as well as their respective repricing characteristics.

Hence we are able to capture (a) the impact of ‘pure credit risk’ on the whole portfolio, and (b) the impact of ‘pure interest rate risk’ in line with advanced gap analysis. But we go beyond this. In reality it is not only interest rates but also credit risk which determines the bank’s customers’ borrowing rates and its cost of funding. Hence, through our framework and by considering the individual repricing characteristics of assets and liabilities, we are able to capture (c) the impact of the interaction between credit and interest rate risk on net-interest income.

Our simulations show that the interaction between credit and interest rate risk is quantitatively significant. Were we to ignore interest rate risk – as generally done in the stress testing literature – we would underestimate the fall in net profits of the bank by over 50% in the first year of our stress scenario, but we would overestimate the fall by nearly 100% in the third year. The magnitude of these effects and the speed at which profits return back to equilibrium depend on a number of assumptions. We show that whilst the interest rate sensitivity gap determines the level of ‘pure interest rate risk’, it does not determine the impact of ‘pure credit risk’ or the importance of the interaction of credit and interest rate risk on net profits. Indeed, the individual repricing characteristics of assets and liabilities are crucial in determining the speed at which profits return to normality by, for example, establishing when higher credit risk can be passed on to borrowers.

We also show that a deterioration in a bank’s fundamentals can increase the bank’s funding costs thereby further lowering its profitability in a potential vicious circle. Such risks are usually not captured by standard macro stress tests. Our framework can therefore be seen as a new comprehensive framework for conducting macro-economic stress tests.

Whilst the implementation of our framework relies on some behavioural assumptions, stress testing models generally have similar assumptions, albeit implicit. For example credit risk is often assessed over a one year horizon independently of the repricing characteristics of assets, which is the same as assuming that the bank is passive in the face of any shock. Furthermore, we ensure that the bank’s balance sheet balances at each point in time; most if not all stress testing models ignore this fundamental accounting identity. Improving our behavioural rules in the implementation of the framework could be an avenue for future research. It could also be useful to look at more disaggregated and sophisticated credit risk models. Furthermore, it would be interesting to explore the sensitivity of LGDs to systematic risk factors in greater depth. Even though the literature is expanding in this area, data limitations on recovery rates are an obstacle.

Although we expect that the above extensions would refine the exact importance of the interaction between credit and interest rate risk, we think that they will not alter the main message of this paper: that a complete risk assessment needs to measure the com-

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Table A1
Balance sheet of the hypothetical bank.

<table>
<thead>
<tr>
<th>Time buckets</th>
<th>0–3 months</th>
<th>3–6 months</th>
<th>6–12 months</th>
<th>1–5 years</th>
<th>&gt;5 years</th>
<th>Non-interest bearing funds</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Assets</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total deposits by banks</td>
<td>32,300</td>
<td>16,400</td>
<td>7600</td>
<td>21,300</td>
<td>15,600</td>
<td></td>
<td>160,000</td>
</tr>
<tr>
<td>Total loans and advances to banks</td>
<td>32,300</td>
<td>16,400</td>
<td>7600</td>
<td>21,300</td>
<td>15,600</td>
<td></td>
<td>160,000</td>
</tr>
<tr>
<td>Total loans and advances to customers</td>
<td>98,000</td>
<td>3400</td>
<td>4300</td>
<td>3000</td>
<td>6000</td>
<td></td>
<td>116,300</td>
</tr>
<tr>
<td>Total households</td>
<td>49,000</td>
<td>1700</td>
<td>2150</td>
<td>1500</td>
<td>3000</td>
<td></td>
<td>58,150</td>
</tr>
<tr>
<td>Mortgages</td>
<td>(44,000)</td>
<td>(1305)</td>
<td>(1500)</td>
<td>(1350)</td>
<td>(1000)</td>
<td>(3000)</td>
<td>(51255)</td>
</tr>
<tr>
<td>Fixed Rate Mortgages</td>
<td>49,000</td>
<td>1700</td>
<td>2150</td>
<td>1500</td>
<td>3000</td>
<td></td>
<td>58,150</td>
</tr>
<tr>
<td>Variable Rate Mortgages</td>
<td>49,000</td>
<td>1700</td>
<td>2150</td>
<td>1500</td>
<td>3000</td>
<td></td>
<td>58,150</td>
</tr>
<tr>
<td><strong>Liabilities</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total liabilities</td>
<td>130,300</td>
<td>5000</td>
<td>4900</td>
<td>4400</td>
<td>600</td>
<td>14,800</td>
<td>160,000</td>
</tr>
<tr>
<td>Total deposits by banks</td>
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<td>7600</td>
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<td>2150</td>
<td>1500</td>
<td>3000</td>
<td></td>
<td>58,150</td>
</tr>
</tbody>
</table>

Note: The balance sheet applies to all simulations unless particular items are replaced for a specific sensitivity test, which are shown as numbers in brackets and in italics. All assets and liabilities are assumed to be domestic exposures. For the actual analysis, the exposure of the bank to an asset/liability/off-balance sheet item in a particular repricing bucket is equally split between the number of quarters within the repricing bucket. For the last bucket we assume that the maximum time to repricing is ten years.

For a definition of repricing buckets see Footnote 2.
combined impact of interest and credit risk jointly. And it is also crucial to adopt a comprehensive approach capturing the key systematic risk factors as well as the whole portfolio, including the disaggregated repricing characteristics of assets, liabilities and off balance sheet items.

Acknowledgements

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Appendix A

See Table A1, Figs. A1 and A2.

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**Fig. A1.** Stylised overview over the stress testing model.

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**Fig. A2.** Time line of the simulation.
References


