Longevity bond premiums: The extreme value approach and risk cubic pricing

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A R T I C L E   I N F O

Article history:
Received December 2008
Received in revised form September 2009
Accepted 21 September 2009

Insurance Branch Category:
IB13

JEL classification:
G12
G22

Subject Category:
IM01
IM10
IE10

Keywords:
Securitization
Longevity risk
Extreme value theory
Bond spreads

A B S T R A C T

The purpose of this study is to analyze the securitization of longevity risk with an emphasis on longevity risk modeling and longevity bond premium pricing. Various longevity derivatives have been proposed, and the capital market has experienced one unsuccessful attempt by the European Investment Bank (EIB) in 2004. After carefully analyzing the pros and cons of previous securitizations, we present our proposed longevity bonds, whose payoffs are structured as a series of put option spreads. We utilize a random walk model with drift to fit small variations of mortality improvements and employ extreme value theory to model rare longevity events. Our method is a new approach in longevity risk securitization, which has the advantage of both capturing mortality improvements within sample and extrapolating rare, out-of-sample longevity events. We demonstrate that the risk cubic model developed for pricing catastrophe bonds can be applied to mortality and longevity bond pricing and use the model to calculate risk premiums for longevity bonds.

1. Introduction

The purpose of this study is to analyze the securitization of longevity risk with an emphasis on the longevity risk modeling and longevity bond premium pricing. We analyze the pros and cons of previous attempts to securitize longevity and mortality risk and propose a new design for longevity bonds where payoffs are structured as a series of put option spreads, contingent on a mortality index. We adopt a random walk model with drift to fit small variations of mortality improvements and employ extreme value theory to model rare longevity events. Integrating these two parts in a unified framework, our model has the advantages of both capturing mortality improvements within the sample and extrapolating extreme longevity events out of the sample. We also contribute to the existing literature by applying our mortality model in conjunction with the risk cubic model proposed by Lane and Movchan (1999) to calculate premiums for our proposed longevity bonds.

Longevity risk is defined as the uncertainty of mortality improvements in the future. In the aggregate sense, longevity risk is the risk that people of a certain population might live longer, on average, than expected. Longevity risk is created through otherwise favorable developments such as improvements in nutrition and medical care. For instance, Hardy (2005) points out that the life expectancy for men aged 60 was 5 years’ longer in 2005 than anticipated in mortality projections made in the 1980s. Individuals face the longevity risk of outliving their resources, and hedge this risk through social security systems, pension plans, and private annuity products. The governments and private annuity providers issuing these products become exposed to longevity risk. Providers can diversify longevity risk at the individual level following the law of large numbers, assuming insured lives are independent. However, aggregate longevity risk breaks down the risk pooling mechanism and is non-diversifiable, making the provision of risk management tools increasingly difficult.
Due to the retirement of baby-boomers and the reduction of death rates among the elderly, both social security systems and private annuity providers are facing an “aging-population tsunami,” creating potentially significant future imbalances between contributions and benefits. The shift of pension plans from defined benefit to defined contribution raises the demand for private annuity products even further. As the demand for individual annuities increases, insurers will have to manage the potential longevity risk created by issuing new annuity policies.

The theory of alternative risk transfer (ART) provides the theoretical fundamentals for a capital market solution to the longevity risk problem. Insurers and reinsurers are supplementing the traditional risk warehousing function with risk intermediation, which enables them to operate more efficiently and increases their underwriting capacities (Cowley and Cummins, 2005; Cox and Lin, 2007; Froot, 2001). In the past few decades, insurance companies have successfully transferred catastrophic risks in their property-liability business to financial markets by issuing catastrophe (CAT) bonds (Cummins et al., 2004; Cummins and Weiss, 2009). More recently, various mortality and longevity bonds have been designed and/or issued to transfer mortality (or longevity) risk into the capital market.

The idea of mortality securitization was first proposed by Samuel H. Cox in a talk in 1998. Cox et al. (2000) mention this idea in their paper. However, Blake and Burrows (2001) may be the first to propose an explicit structural design for longevity bonds. Their bonds are designed so that coupon payments are contingent on the percentage of a certain population cohort surviving some future period. Longevity bonds allow annuity providers to hedge against aggregate longevity risk: if annuitants live longer than anticipated, the insurers would incur losses due to longer payment periods, but they would also receive more coupon payments for holding longevity bonds to offset their losses (Denault et al., 2007). A more extensive discussion of mortality-linked derivatives is provided by Blake et al. (2006a), who present various forms of longevity bonds, swaps, futures, and options, and investigate their potential uses.

Swiss Re issued the first pure mortality bond in December 2003, and (re)insurers subsequently issued five additional mortality bond transactions to reduce their exposure to extreme longevity risk, with a total volume exceeding $2 billion (GC Securities, 2008; Swiss Re, 2009). The European Investment Bank (EIB) offered the first longevity bonds in November 2004 to hedge longevity risk for pension funds and annuity providers. Although this particular deal failed to be launched because of insufficient demand, it did attract public attention and provided an instructive case study (Blake et al., 2006a). A general question is why mortality securitizations such as the Swiss Re deal sold well but the EIB bonds failed. We argue that the EIB failure was attributable to design flaws, and we propose an alternative design for longevity bonds, modeled on the successful designs employed in the mortality securitization market.

Mortality models are crucial as a means of quantifying mortality and longevity risks and providing the basis for pricing and reserving. Cairns et al. (2006) provide a detailed overview of stochastic mortality models. The recent development of discrete-time or continuous-time jump models represents a main stream of research on mortality securitization modeling (see Biffis, 2005; Bauer and Kramer, 2007; Cox et al., 2008; Chen et al., 2009). Lin and Cox (2008) argue that mortality jumps need to be taken into account because the rationale behind mortality securitization is to hedge extreme mortality risks. Cox et al. (2006) propose a geometric Brownian motion with jumps to model the age-adjusted mortality rates. Chen and Cox (2009) explore models with permanent versus transitory jump effects. They find that the model with transitory jump effects is more appropriate for mortality securitization, because most adverse mortality jumps are caused by catastrophic events and only have short-term effects. They also argue that the model with permanent jump effects is applicable in the longevity market, since longevity risks usually have persistent, long-term effects on future mortality rates.

Another important line of research on extreme events modeling is through extreme value theory (EVT). One challenge in longevity risk modeling is the imprecise knowledge about rare longevity events. We have very few episodes of extreme mortality improvements, restricting the ability to learn from past experience. The evolution of mortality improvement is a slow but persistent process, and is influenced by socioeconomic, biological, environmental and behavioral developments (Lin and Cox, 2007). Sometimes the small change of one factor can affect mortality improvements dramatically. The inaccuracy and unavailability of mortality data at the very old ages also make mortality modeling difficult (Buettner, 2002; Li et al., 2008). Fortunately, EVT offers an attractive solution to these problems.

In this paper, we extend traditional parametric mortality modeling and integrate it with EVT. Basically, we use a random walk model with drift for mortality fitting before some threshold, and assume that mortality exceedances over the threshold follow a generalized Pareto distribution, as postulated in EVT (e.g., Embrechts et al., 2008). By this means, we can not only model the mortality data within our samples but can also rationally extrapolate more extreme, out-of-sample longevity events.

Longevity securities involve significant valuation problems. Financial pricing approaches emphasize the creation of replicating portfolios (Black and Scholes, 1973; Merton, 1973) or the existence of a unique risk-neutral measure (Cox and Ross, 1976; Harrison and Kreps, 1979; Harrison and Pliska, 1981) in a complete market. Even in an incomplete market, the non-arbitrage condition ensures the existence of at least one risk-neutral measure, or we can use a distortion operator such as the Wang transform (Wang, 2000, 2002) to create a risk-adjusted measure, under which financial securities can be priced. However, we usually do not have clues regarding the appropriate choice of the risk-neutral measure or distortion operator. The alternative insurance pricing approaches usually assume investors are risk averse, thereby adding a risk loading to expected losses to compensate investors for the risk of the insurance contract.

The Lane Financial Corporation (LFC) suggests a cubic model to calculate risk premiums of CAT bonds (Lane and Movchan, 1999; Lane, 2000). The model considers the frequency and severity of the loss distribution and uses the Cobb–Douglas function to capture the trade-off between these two factors in the insurance pricing framework. Longevity (or mortality) bonds are analogous to CAT bonds, which motivizes us to extend the CAT bond pricing methodology to the longevity (or mortality) securitization markets. We find that the LFC’s cubic model can exactly replicate the risk premium of the Swiss Re mortality bond issued in 2003. We therefore employ the cubic model to explicitly derive risk premia for our proposed longevity bonds.

The remainder of this paper proceeds as follows. In Section 2, we provide an overview of the contract designs of previous securitizations. In Section 3, we point out the problems of the EIB bond by comparing it with the Swiss Re mortality bond, and propose our hypothetical longevity bonds parallel to the Swiss Re deal. In Section 4, we model the mortality dynamics using a random walk with

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3 Extreme Value Theory has been used to model adverse mortality jumps in mortality securitization. Beelders and Colarossi (2004) apply EVT to the Swiss Re mortality bond issued in 2003 and review some of the benefits of this approach.
2. Overview of the Swiss Re bond and the EIB bond

2.1. The Swiss Re Vita I mortality bond

The first bond that we discuss here is not a longevity bond but rather a mortality bond designed to hedge extreme mortality risk (or brevity risk, see MacMinn and Richter, 2004). We analyze mortality bonds to help us understand why mortality securitizations have been successful and to provide some ideas for potential designs of longevity bonds.

In order to reduce its exposure to adverse mortality risks, Swiss Re issued the first pure mortality bonds in 2003 through a special purpose vehicle (SPV) called Vita Capital I.\(^4\) The Vita I transaction is diagrammed in Fig. 1. The issue size was $400 million. It is a three-year deal that was issued in December 2003 and matured on 1 January 2007. Coupons were paid quarterly at a rate of three-month US dollar LIBOR plus a spread of 135 basis points (bps). Vita Capital I executed a swap transaction to swap Swiss Re’s fixed premium payments for LIBOR. The principal repayment is at risk and depends on a weighted mortality index based on five countries, males and females, and different age-groups.\(^5\) If the mortality index did not exceed 1.3 times the 2002 base level (denoted as \(l_0\) hereafter) during any of the three years, the principal would be fully repayable. Otherwise, the investors would receive a reduced principal repayment if the mortality index exceeded this threshold, and would get nothing back if the index were above 1.5 times the base level. The bond matured on schedule without any loss of principal to the investors, and the coverage capacity was replaced on 1 January 2007 with a new bond, Vita Capital III.

From Swiss Re’s point of view, the payoffs from the Vita Capital I transaction were structured as call option spreads with a lower strike price \(M = 1.3l_0\) and an upper strike price \(U = 1.5l_0\).\(^6\) Swiss Re pays premiums to Vita Capital I in return for payoffs contingent on the mortality index. If the mortality index exceeded the lower strike, Swiss Re would withdraw funds from Vita Capital I. The full amount of principal would flow to Swiss Re if the mortality index were to exceed the upper strike. Denote \(l_t\) the mortality index for year \(t\). The option payoff for year \(t\) as a percentage of the principal is expressed as:

\[
\text{Payoff}_t = \frac{\text{Max}(l_t - M, 0) - \text{Max}(l_t - U, 0)}{U - M}.
\]

In issuing catastrophe securities, there is always a tradeoff between moral hazard and basis risk. Linking payoffs of mortality bonds to public mortality indices can reduce investors’ concern about moral hazard problems, but it also introduces a basis risk since the insurer’s mortality experience could deteriorate significantly more or less than that of the index. For this reason, mortality bonds are likely to appeal to large, diversified insurers or reinsurers (Cowley and Cummins, 2005), and the same is likely to be true of longevity bonds.

2.2. The EIB/BNP Paribas longevity bond

An interesting experiment in long-term longevity bonds was attempted by the European Investment Bank (EIB) in November 2004, about one year after Swiss Re’s first mortality bond issue. It was targeted at pension plans and annuity providers, and was much closer in nature to the survivor bond proposed by Blake and Burrows (2001). This particular security failed to be launched because of insufficient demand, and was withdrawn in late 2005.

As can be seen from Fig. 2, this bond is the product of the co-operation between EIB as the issuer, BNP Paribas as the designer/originator and Partner Re as the longevity risk reinsurer. The total value of the issue was £540 million and the term of this bond was 25 years. Investors would receive an annual coupon payment of £50 million multiplied by a realized survivor index (denoted as \(S_t\)) of the English and Welsh male population aged 65 in 2002, as published annually by the UK Office for National Statistics. In simple terms, the bond would have committed EIB to make longevity-linked payments in sterling to investors. Since EIB also wished to pay a floating rate in euros, the bond incorporated a cross-currency (i.e., fixed-sterling-for-floating-euro) interest-rate swap with BNP. BNP would have taken on longevity exposure

\(^4\) Swiss Re is the world’s largest reinsurer (A.M. Best, 2008). The SPV is a passive financial intermediary that exists to insulate investors from the sponsor’s credit risk, provide transparent servicing of assets and liabilities, structure tranches of debt to appeal to different classes of investors, insulate investors from agency costs of issuer, creating a “pure play” security, and provide tax and accounting benefits to the sponsor.

\(^5\) The five countries are US, UK, France, Italy and Switzerland. The weights assigned to each country are: US 70%, UK 15%, France 7.5%, Italy 5%, and the Switzerland 2.5%.

\(^6\) The later Vita Capital transactions have a similar structure.
in this transaction, and thereby structured a mortality swap with Partner Re, in which Partner Re would exchange the fixed sterling payment for the floating sterling payments based on a projected survivor index for the same cohort (denoted as $S'_f$). The mortality forecasts were based on the UK Government Actuary’s Department’s 2002-based central projection of mortality, adjusted for Partner Re’s own internal revisions to these forecasts.\footnote{Further details are provided in Blake et al. (2006a).}

### 3. Designs for longevity bonds

#### 3.1. Lessons from previous securitizations

A generally asked question is why the Swiss Re bond was attractive to investors but the EIB bond was unsuccessful. Lin and Cox (2008) propose a mortality securitization model in an incomplete market framework, and calculate the market prices of mortality risk for the Swiss Re mortality bond and the EIB longevity bond, respectively. They find that the Swiss Re deal offers a higher risk premium to investors than CAT bonds, while the EIB bond charges a very high risk premium to take longevity risks in the UK pension plans, explaining the opposite market outcomes of these two securities.

Blake et al. (2006b) compare the main characteristics of these two bonds, and draw out the analogies between longevity bonds and conventional debt securities. They point out that longevity bonds might be financially engineered by existing securities. Table 1 demonstrates their comparison results. Blake et al. (2006a) also discuss the possible drawbacks existing in the EIB bond design. For example, they argue that the maturity should be longer to provide an effective hedge, the degree of model and parameter risk is high, and the level of basis risk is too high relative to the price being charged.

From our point of view, there are more reasons underlying the relative success of the bond designs. First, the principal in the EIB bond design is provided by hedgers (e.g., annuity providers), imposing a substantial capital requirement for the hedgers. By contrast, in the Swiss Re design, the hedger (insurer or reinsurer), pays only an annual premium to hedge the extreme mortality risk. Second, the longevity risk embedded in the EIB bond is transferred to a reinsurer, Partner Re. However, the mortality risk in the Swiss Re bond is passed on directly to financial markets, thus accessing a broader capital base and creating new risk-bearing capacity. Third, the EIB bond design exposed the hedger to the direct credit risk of the EIB and the indirect credit risk of BNP Paribas and Partner Re, while the Swiss Re deal was fully collateralized. Fourth, the EIB bond provides “ground up” protection, covering the entire annuity payments. The Swiss Re mortality bond is more attractive because it only covers payments in excess of some strike level.

#### 3.2. A proposed new design for longevity bonds

Based on the above analysis, the simplest way to securitize longevity risk would be to parallel the design of the Swiss Re Vital I bond. A proposed new design for longevity bonds based on the Swiss Re model is presented in Fig. 3. Consider an annuity provider that is exposed to potential longevity risks. To hedge the longevity risk, it sets up an SPV to issue longevity bonds. The bonds are purchased by debt investors in the capital market and have a long maturity, say, 20 years. The bonds carry a premium of x basis points over 3-month US LIBOR.\footnote{The risk premium can be determined either by financial pricing approaches (creating a risk-neutral measure) or by insurance pricing approaches (adding a risk loading to expected losses). Embrechts (2000) provides an overview and comparison of these approaches. In this paper, we adopt an insurance pricing approach—risk cubic model.} The principal in and the term to maturity of the bond are $T$, and the coupon rate is $r$. The coupon payments to investors are reduced as the principal is used up.

The index we set up here is similar to that in the Swiss Re deal. It is a weighted average of mortality rates based on several big countries, males and females, and different age groups, defined as follows:

$$ I_t = \sum_j C_j \sum_i A_i (G^m q_{i,j,t}^m + G^f q_{i,j,t}^f), $$

(2)

where $I_t$ is the mortality index for period $t$, $q_{i,j,t}^m$ and $q_{i,j,t}^f$ are mortality rates in the age group $i$ for country $j$ for males and females, respectively. $G^m$ and $G^f$ are corresponding gender weights, $C_j$ is the weight attached to country $j$, and $A_i$ is the weight attributed to age group $i$.

If the mortality index decreases substantially because of unexpected longevity events, annuity providers will suffer losses from the mortality improvements. They could hedge this risk through longevity bond transactions. Once the mortality index falls
Table 1
Comparison of the Swiss Re mortality bond and the EIB/BNP Paribas longevity bond. Source: Summarized from Blake et al. (2006b).

<table>
<thead>
<tr>
<th>The Swiss Re bond</th>
<th>The EIB/BNP bond</th>
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<tbody>
<tr>
<td>A hedge to the issuer</td>
<td>A hedge to the investors</td>
</tr>
<tr>
<td>The issuer gains if mortality index is extremely high</td>
<td>The issuer gains if the survivor index is lower than expected</td>
</tr>
<tr>
<td>A hedge against a portfolio dominated by life insurance/reinsurance policies</td>
<td>A hedge against a portfolio dominated by annuity policies</td>
</tr>
<tr>
<td>Short-term (3-year)</td>
<td>Long-term (25-year)</td>
</tr>
<tr>
<td>Mortality index is weighted over five countries, males and females, and a range of ages</td>
<td>Survivor index is based on a single country</td>
</tr>
<tr>
<td>The principal is at risk from mortality deterioration</td>
<td>Coupon payments are at risk from longevity shocks</td>
</tr>
</tbody>
</table>

![Diagram of swap counterparty, fixed return, annuity issuers, premiums, longevity ltd, debt investors, put option spread, principal repayment, mortality index, swap counterparty, fixed return, annuity issuers, premiums, longevity ltd, debt investors, put option spread, principal repayment, mortality index]

Fig. 3. The mechanism design of our hypothetical longevity bond.

![Graph showing cumulative payoff of 20-year longevity bond payoffs: Assume the index finishes above the lower strike.](image1)

Fig. 4. 20-year longevity bond payoffs: Assume the index finishes above the lower strike.

![Graph showing cumulative payoff of 20-year longevity bond payoffs: Assume the index reaches the lower strike.](image2)

Fig. 5. 20-year longevity bond payoff: Assume the index reaches the lower strike.

below a predetermined upper strike threshold, part of the principal would be withdrawn from the SPV to help the annuity providers make annuity payments. All of the principal will be released to the sponsor if an extreme longevity event occurs such that the mortality index reaches the lower strike threshold. In other words, payoffs of the longevity bonds to the issuer would be structured as a series of put option spreads with an original upper strike and a lower strike. Considering the fact that mortality generally is improving over time, once the mortality index in year $t$ falls below the upper strike, we need to reset the upper strike in year $t + 1$ to this index value.

Mathematically, the payoff of the longevity bonds for year $t$ as a percentage of the original principal can be written as:

$$\text{Payoff}_t = \frac{\max(U_t - I_t, 0) - \max(M - I_t, 0)}{U - M},$$

where $M$ is the lower strike, $U$ is the original upper strike, $U_t = \min(U_{t-1}, I_{t-1})$ is the revised upper strike for year $t$.

The longevity bonds terminate at the earlier of a predetermined maturity date (20 years in our example) and the time when the revised upper strike reaches the lower strike. Suppose the upper and lower strikes ($U$ and $M$) are 90% and 50% of the initial index value, respectively. Fig. 4 graphs the cumulative payoff from the longevity bonds, assuming the mortality index is well above the lower strike for the bond’s entire lifetime. Fig. 5 demonstrates the situation where the mortality index hits the lower strike in year 20 and all the principal goes to the issuer.

Among the important advantages of this bond design to the issuer are that funds are released gradually as needed to respond
to declines in the mortality index and that the bond coverage is provided for a significant period of time.

4. Mortality modeling

4.1. The classic Lee–Carter model

The Lee–Carter model has become the leading statistical model of mortality forecasting in the demographic literature (Deaton and Paxson, 2004). It serves as a benchmark for official forecasts of the US life expectancy by the Census Bureau population forecast. The two recent Social Security Technical Advisory Panels have suggested the use of this model or other methods consistent with it (Lee and Miller, 2001).

Denote \( m_{x,t} \) the central death rate for age \( x \) at time \( t \). Lee and Carter (1992) model the death rate to be log-linearly correlated with a time-varying factor \( k_t \), and adjust the age-specific effects through two sets of coefficients \( a_x \) and \( b_x \), i.e.,

\[
\ln(m_{x,t}) = a_x + b_x k_t + e_{x,t},
\]

where \( a_x \) represents the age pattern of death rates, \( b_x \) represents age-specific reactions to \( k_t \), and \( e_{x,t} \) is the zero mean disturbance term. In this form, the Lee–Carter model captures the mortality evolution in mutually exclusive age groups via \( a_x \) and \( b_x \), and links all age groups together using the common risk factor \( k_t \). Lee and Carter referred to \( k_t \) as the mortality index. In order to distinguish \( k_t \) from the mortality index defined in the securitization contract, we name \( k_t \) the mortality factor hereafter.

The solution to the Lee–Carter model is not unique and is determined up to a linear transformation. Following Lee and Carter (1992), we impose the normalization conditions for identification:

\[
\sum_x b_x = 1 \quad \text{and} \quad \sum_t k_t = 0.
\]

These conditions immediately imply that the parameter estimates \( \hat{a}_x \) is simply the empirical average of \( \ln(m_{x,t}) \) over time,

\[
\hat{a}_x = \frac{1}{T} \sum_{t=1}^{T} \ln(m_{x,t}),
\]

where \( T \) is the length of the time series of mortality data.

As for the estimates \( \hat{b}_x \) and \( \hat{k}_t \), Lee and Carter suggest a two-stage procedure to solve them. In the first stage, the singular value decomposition (SVD) method is applied to the matrix of \( \ln(m_{x,t}) - \hat{a}_x \) to obtain \( \hat{b}_x \) and \( \hat{k}_t \). In the second stage, the \( \hat{k}_t \) factors are re-estimated such that the number of deaths recorded in each year is equal to that estimated from the model,

\[
D_t = \sum_x \left( \text{Pop}_{x,t} \exp(\hat{a}_x + \hat{b}_x \hat{k}_t) \right)
\]

where \( D_t \) is the actual total number of deaths at time \( t \), and \( \text{Pop}_{x,t} \) is the population in age group \( x \) at time \( t \).

Based on the US age-specific death rates per 100,000 population from 1900 to 2006 (National Center for Health Statistics), we implement this two-stage procedure. We report the fitted values \( \hat{a}_x \) and \( \hat{b}_x \) for 11 age groups in Panel 1 of Table 2 and plot the final estimates of the mortality factor \( \hat{k}_t \) in Fig. 6. We can see that generally the mortality rates of young age groups respond more rapidly when the mortality factor changes. As expected, the mortality factor \( \hat{k}_t \) is decreasing over time, which shows the trend of mortality improvements.

4.2. Modeling \( k_t \) with EVT

To forecast the future distribution of \( k_t \), we need to choose a suitable model to fit \( k_t \). Lee and Carter (1992) find a random walk with drift that describes \( k_t \) well, and they employ an intervention model to remove the impact of the 1918 influenza pandemic. Chen and Cox (2009) model \( k_t \) with a jump process, and discuss the difference between adverse mortality jumps and longevity jumps. Other studies on mortality jump models include Bauer and Kramer (2007); Hainaut and Devolder (2007); Cox et al. (2008); Lin and Cox (2008); and Chen et al. (2009).

The rationale for incorporating a jump process in mortality models is to capture the extreme mortality risk (either adverse mortality risk or longevity risk) and distinguish it from the small random variations. However, the potential values of a risk have a probability distribution which is never observed exactly, although past losses may provide partial information about that distribution. Traditional parametric methods are ill-suited to extreme probabilities, since parametric modeling produces a good fit in the central regions at the expense of good fit in the tail (Diebold et al., 1999). In addition, we do not have enough historical data on the rare longevity risk to fit the tail.

Fortunately, extreme value theory (EVT) provides us with guidance to achieve reliable estimates of extreme probabilities. The key idea in the EVT is to “estimate extreme probabilities by fitting a model to the empirical survival function of a set of data using only the extreme event data rather than all the data, thereby fitting the tail, and only the tail” (Sanders, 2005). In this sense, the advantage of the EVT approach can also be viewed as a disadvantage: it focuses on the tail distribution and ignores the central regions of the data. Therefore, we propose to integrate parametric modeling with EVT in a unified modeling framework. Our model not only has a good fit on the historical data, but also is able to extrapolate more extreme, out of sample data. In other words, it provides probability estimates not only within the boundary of observed data, but also beyond the boundary.

Small mortality variations: Random walk with drift

As suggested by Lee and Carter (1992), we choose a random walk model with drift to describe the small variations of the mortality factor \( k_t \), i.e.,

\[
k_{t+1} = k_t + \mu + \sigma Z_{t+1},
\]

where \( \mu \) and \( \sigma \) are the expected change and the volatility of the mortality factor \( k_t \), respectively, and \( Z_t \) is a standard normal random variable.

Let \( X_{t+1} = k_t - k_{t+1} \). We can interpret \( X_{t+1} \) as the mortality improvement in year \( t + 1 \). The random variable \( X \) is normally distributed with mean \( -\mu \) and variance \( \sigma^2 \). Therefore, \( X \) has a cumulative distribution function (CDF) for small values of \( x \).

\[
F_1(x) = \Phi \left( \frac{x + \mu}{\sigma} \right),
\]

and probability density function (PDF),

\[
f_1(x) = \frac{1}{\sigma} \phi \left( \frac{x + \mu}{\sigma} \right) = \frac{1}{\sqrt{2\pi}\sigma} \exp \left( -\frac{(x + \mu)^2}{2\sigma^2} \right).
\]

We also test the significance of adding mortality jumps in our model. We exclude high values of \( X \) (since we only consider the small random variations right now) and fit the remaining data using the mortality model with jumps (see Chen and Cox, 2009) and the model without jumps (the random walk with drift), respectively. We find that the likelihood ratio test does not reject the model without jumps. To avoid the problem of over-fitting and minimize the number of parameters being estimated, we choose a simpler model (without jumps) for the non-extreme mortality improvements, and reserve our modeling efforts to the estimation of the tail distribution.

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10 Source: http://www.cdc.gov/nchs/data/hus/2009/n27/data12.pdf. The National Center for Health Statistics reported the age-specific death rates per 100,000 population from 1900 to 2006 (National Center for Health Statistics). We implement this two-stage procedure. We report the fitted values \( \hat{a}_x \) and \( \hat{b}_x \) for 11 age groups in Panel 1 of Table 2 and plot the final estimates of the mortality factor \( \hat{k}_t \) in Fig. 6. We can see that generally the mortality rates of young age groups respond more rapidly when the mortality factor changes. As expected, the mortality factor \( \hat{k}_t \) is decreasing over time, which shows the trend of mortality improvements.

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Parameter estimates, $\hat{a}_k$ and $\hat{b}_k$, derived from the Lee–Carter model.

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<tr>
<td>25–34</td>
<td>$-6.0884$</td>
<td>$-5.9837$</td>
<td>$-5.9574$</td>
<td>$-5.9296$</td>
</tr>
<tr>
<td>35–44</td>
<td>$-5.4964$</td>
<td>$-5.4745$</td>
<td>$-5.4529$</td>
<td>$-5.4294$</td>
</tr>
<tr>
<td>45–54</td>
<td>$-4.7925$</td>
<td>$-4.7734$</td>
<td>$-4.7534$</td>
<td>$-4.7316$</td>
</tr>
<tr>
<td>55–64</td>
<td>$-4.0267$</td>
<td>$-4.0071$</td>
<td>$-3.9879$</td>
<td>$-3.9693$</td>
</tr>
<tr>
<td>75–84</td>
<td>$-2.4297$</td>
<td>$-2.4146$</td>
<td>$-2.4003$</td>
<td>$-2.3861$</td>
</tr>
<tr>
<td>85 over</td>
<td>$-1.8192$</td>
<td>$-1.6084$</td>
<td>$-1.5995$</td>
<td>$-1.5912$</td>
</tr>
</tbody>
</table>

Table 2

where $\Phi$ and $\phi$ are the CDF and PDF of a standard normal random variable, respectively.

We can easily estimate the mean (0.2172) and the standard deviation (0.6072) from the whole sample set. Fig. 7 compares the empirical CDF of the mortality improvement $X$ and the CDF of a normal random variable with mean 0.2172 and standard deviation 0.6072. The upper tail of the mortality improvement is fat tailed, which indicates that a simple random walk model with drift cannot capture the extreme events.

The tail distribution: Extreme value theory

We now turn to fitting the tail distribution via EVT. Broadly speaking, there are two types of models for extreme values. Block maxima models apply to maxima of a sequence of observations and the peaks-over-threshold (POT) models work with exceedances over a high threshold (Embrechts et al., 2008). We are interested in the exceedances in the tail distribution because the payoff of our hypothetical longevity bonds is linked with the mortality improvement $X$ above a certain level.

Let $x_0$ be the finite or infinite right endpoint of the distribution $F(x)$. We define the conditional distribution of the exceedances over a high threshold $u$ by

$$F_\xi(u) = P[X - u \leq x | X > u] = \frac{F(x + u) - F(u)}{1 - F(u)}, \quad (11)$$

for $0 \leq x < x_0 - u$.

According to the Pickands–Balkema–de Haan Theorem (Balkema and de Haan, 1974; Pickands, 1975), for a sufficiently high threshold $u$, the excess distribution function $F_\xi(u)$ may be approximated by the generalized Pareto distribution (GPD), $G_{\xi,\theta}(x)$, for some value of $\xi$ and $\theta$.

According to Pickands–Balkema–de Haan Theorem (Balkema and de Haan, 1974; Pickands, 1975), for a sufficiently high threshold $u$, the excess distribution function $F_\xi(u)$ may be approximated by the generalized Pareto distribution (GPD), $G_{\xi,\theta}(x)$, for some value of $\xi$ and $\theta$. Here, $G_{\xi,\theta}(x)$ is defined as

$$G_{\xi,\theta}(x) = \begin{cases} 1 - (1 + \xi x / \theta)^{-1/\xi}, & \text{if } \xi \neq 0 \\ 1 - \exp(-x/\theta), & \text{if } \xi = 0, \end{cases} \quad (12)$$

where $\theta > 0$, and the support is $x \geq 0$ when $\xi \geq 0$ and $0 \leq x \leq -\theta / \xi$ when $\xi < 0$. $\xi$ is the shape parameter of the distribution and $\theta$ is an additional scaling parameter. If $\xi > 0$ we have a reparameterized version of the ordinary Pareto distribution. $\xi = 0$ corresponds to the exponential distribution and $\xi < 0$ is known as a type II Pareto distribution.

We can extend the GPD family by adding a location parameter $\gamma$. The GPD $G_{\xi,\theta}(x)$ is defined to be $G_{\xi,\theta}(x - \gamma)$. Therefore, for $x - \gamma \geq 0$, the distribution function of the ground-up exceedances $F_\xi(x - \gamma)$ may be approximated by $G_{\xi,\theta}(x - \gamma) = G_{\xi,\theta}(x)(\gamma)$.

The choice of the threshold $u$ is basically a tradeoff between bias and variance. On the one hand, we need to choose a high enough $u$ so that the GPD can be applied asymptotically here (reduce the bias). On the other hand, we need to keep $u$ low so that we have...
Fig. 7. Empirical Cumulative Distribution Function (CDF) of mortality improvements $X$.

enough data to estimate the parameters (control the variance). A typical choice of $u$ is around the 90th to 95th percentile (Sanderson, 2005).

For $x \geq u$, the distribution function can be written as:

$$F_2(x) = P[X \leq x] = (1 - F(u))F_0(x - u) + F(u).$$  

We now know that we can approximate $F_2(x - u)$ by $G_{\xi, \theta}(x - u)$ for a large $u$. We can also estimate $F(u)$ from the data by $F_0(u)$, the empirical distribution evaluated at $u$. This means that for $x \geq u$ we can use the tail estimate

$$\hat{F}_2(x) = (1 - F_0(u))G_{\xi, \theta}(x) + F_0(u)$$

$$= 1 - (1 - F_0(u))\left(1 + \xi \left(\frac{x - u}{\theta}\right)\right)^{-\frac{1}{\xi}},$$  

(14)

to approximate the distribution function $F(x)$. Taking derivatives w.r.t. $x$, we obtain

$$\hat{f}_2(x) = \frac{1 - F_0(u)}{\theta} \left(1 + \xi \left(\frac{x - u}{\theta}\right)\right)^{-\frac{1}{\xi} - 1},$$  

(15)

4.3. Algorithm

In the preceding discussion, we have specified our mortality model. We briefly review the model here. When $x < u$, the mortality improvement $X$ can be written as a random walk with drift. The probability density function is:

$$f_1(x) = \frac{1}{\sqrt{2\pi} \sigma} \exp \left(-\frac{(x + \mu)^2}{2\sigma^2}\right).$$  

(16)

When $x \geq u$, the mortality improvement $X$ can be approximated by a generalized Pareto distribution, with the probability density function

$$f_2(x) = \frac{1 - F_0(u)}{\theta} \left(1 + \xi \left(\frac{x - u}{\theta}\right)\right)^{-\frac{1}{\xi} - 1}.$$

(17)

If we have a time series of $K$ observations of $k_i$, there will be $K - 1$ observations of $X$’s values. For a fixed threshold $u$, these $K - 1$ observations of $X$ can be divided into two groups: one group with $X$ values less than $u$, the other group with $X$ values greater than or equal to $u$. Assuming that there are $K_1$ observations in the first group and $K_2$ observations in the second group ($K_1 + K_2 = K - 1$), the log-likelihood function is

$$l(\mu, \sigma, \xi, \theta; u) = l_1(\mu, \sigma; u) + l_2(\xi, \theta; u),$$

(18)

where

$$l_1(\mu, \sigma; u) = \sum_{i=1}^{K_1} \ln f_1(x_i; u),$$

(19)

and

$$l_2(\xi, \theta; u) = \sum_{i=1}^{K_2} \ln f_2(x_i; u).$$

(20)

Note that we use $F_0(u)$ to approximate $F(u)$ at the threshold $u$ such that $F_0(u) = F(u)$. To ensure the distribution function is continuous at the threshold point, we add a constraint $F_1(u) = \Phi(\frac{u - \mu}{\sigma}) = F_0(u)$. For a fixed $u$, parameter estimation for the parametric modeling part and the generalized Pareto part can be done separately by maximizing $l_1$ and $l_2$, respectively. The choice of $u$ depends on the maximization of the profile log-likelihood

$$l_p(u) = l(\hat{\mu}(u), \hat{\sigma}(u), \hat{\xi}(u), \hat{\theta}(u); u),$$

(21)

where $\hat{\mu}(u), \hat{\sigma}(u), \hat{\xi}(u)$ and $\hat{\theta}(u)$ are the maximum likelihood estimates of $\mu, \sigma, \xi$ and $\theta$ for a fixed $u$, respectively.

The algorithm for estimating the threshold $u$ and other parameters in our model can be summarized as follows:

1. For $u = x_{0.95}$ (the 95th percentile of $X$).
   a. find the values of $\hat{\mu}$ and $\hat{\sigma}$ that maximize $l_1$;
   b. find the values of $\hat{\xi}$ and $\hat{\theta}$ that maximize $l_2$;
   c. compute the value of the profile log-likelihood, $l_p = l_1 + l_2$.
2. Repeat step 1 for $u = x_{0.95}, \ldots, x_{0.85}$;
3. Find the value of $u$ that gives the maximum profile log-likelihood.

The value of $u$ obtained in step 3 is the optimal threshold. The maximum likelihood estimates $\hat{\mu}, \hat{\sigma}, \hat{\xi}$ and $\hat{\theta}$ under the optimal threshold are the optimal parameter estimates.

4.4. Parameter estimates

Following the algorithm in Section 4.3, we obtain the parameter estimates in Panel 1 of Table 3, where the threshold $u$ changes from the 85th percentile to 95th percentile of $X$. The profile log-likelihood function reaches its highest value when we set the threshold to be the 93rd percentile of $X$. Therefore, the optimal threshold equals $x_{0.93}$, which is 0.8646. For the mortality improvement data lower than this threshold, our maximum likelihood estimation suggests that the mortality factor improves by 0.1606 every year, on average. The volatility of the mortality improvement is equal to 0.4770. For the mortality improvement data higher than this threshold, we obtain an estimate of $\hat{\xi}$ that is greater than 0, which is consistent with the fat upper tail of the empirical distribution of mortality improvements (see Fig. 7). The probability of large decreases in mortality, and risk to bondholders, would be underestimated should one simply use a normal distribution to model the data.

12 Threshold selection is the “Achilles’ heel” of the POT method. There is no theoretical approach with a satisfactory performance suitable for this purpose (Christoffersen, 2003). A usual approach for estimating the threshold is based on the plot of average exceedances (Mean Excess Graph). The threshold ‘u’ can be identified as the value corresponding to the kink from where it is verified as an upward slope (McNeil and Saladin, 1997). However this technique is subjective and cannot identify a unique value of the threshold. In this paper, we use the profile log-likelihood maximization as an alternative method.
5. Risk premiums for longevity bonds: A numerical example

In this section, an insurance pricing approach is used to calculate the risk premiums of longevity bonds. Recall that in Section 3, we designed the longevity bonds to parallel existing mortality bonds. Both longevity bonds and mortality bonds are similar to CAT bonds. While CAT bonds may depend on an industry loss index, longevity (or mortality) bonds are contingent on a public mortality index of a certain population. If no trigger event occurs, the full principal of the bond is paid back to investors at maturity. Otherwise, part or all of the principal will be released to the sponsor and the investors will receive less than their full investment at maturity. Therefore, it is natural to price longevity bonds using a CAT bond pricing methodology.

5.1. CAT bond pricing

CAT bonds normally offer investors higher yields than the risk-free interest rate, e.g., the LIBOR (Wang, 2004; Cummins and Weiss, 2009). The premium spread over LIBOR reflects the price of insurance risk and can be decomposed into two parts. The first part compensates investors for expected losses (EL), while the second part compensates investors for the uncertainty associated with the investment, which is called expected excess return (EER).

Premium = EL + γEER. \hspace{1cm} (22)

In a perfect, riskless market, there would be no expected excess return, provided that insurance risk is uncorrelated with the movement of financial markets. However, insurance markets are not riskless and are far from perfect. Therefore, investors normally require a risk loading to compensate for risk bearing in the market for catastrophe securities (Lane, 2000). Several approaches have been applied in the insurance and actuarial literature to model EER. The remainder of this section discusses the standard actuarial approaches and compares them to the relatively new cubic model proposed by the Lane Financial Corporation (LFC).

**Expected value principle**

Under the expected value principle, the expected excess return is proportional to the expected losses with a percentage γ. The premium spread is then determined as:

Premium = EL + γEL. \hspace{1cm} (23)

This approach is not risk sensitive since it considers only the expected value. The advantage is that it requires only the first moment of the loss distribution and thus can be easily implemented.

**Standard deviation principle**

A widely used risk measure is the standard deviation, which leads to the standard deviation principle. The premium can be obtained by loading a portion γ of the standard deviation σ(L) on the expected losses EL:

Premium = EL + γσ(L). \hspace{1cm} (24)

**LFC’s cubic model**

The standard deviation principle works well for symmetric random outcomes. However, insurance risk is often skewed: most of the probability mass is centered at zero loss, while there is a small probability of potentially large negative returns. In order to capture the asymmetry of insurance risk, the Lane Financial Corporation began to explore the conditional expected loss (CEL) as a better risk measure in 1997. It is noteworthy that CEL only represents the severity of the loss distribution. The frequency of loss also should be taken into account in the insurance risk pricing framework. The loss frequency is usually measured by the probability of first dollar loss (PFL). EL, PFL and CEL are governed by the following relation:

EL = E[I|L > 0] Pr(L > 0) + E[I|L = 0] Pr(L = 0)

= E[I|L > 0] Pr(L > 0)

= CEL * PFL. \hspace{1cm} (25)

The LFC suggest using the Cobb–Douglas function to capture the tradeoff between CEL and PFL empirically (Lane, 2000), i.e.,

EER = γ(PFL)^α(CEL)^β. \hspace{1cm} (26)

The premium spread can therefore be written as

Premium = EL + γ(PFL)^α(CEL)^β. \hspace{1cm} (27)
5.2. Application of the LFC cubic model to longevity bond pricing

As mentioned above, we believe that the CAT bond pricing methodology can be applied to longevity (or mortality) bonds. Accordingly, we utilize CAT bond transaction data to estimate the parameters, $\alpha$, $\beta$, $\gamma$, in the Cobb–Douglas function, and use the LFC’s cubic model to calculate the premiums of longevity (or mortality) bonds.

In order to check the feasibility of this approach, we use the Swiss Re Vita I mortality bond issued in 2003 as an example. CAT bond data are available from the LFC’s annual report of insurance-linked securities financial statistics. We collected the variables $EER$, $EL$, $PFL$, and $CEL$ for each CAT bond transaction from 1997 to 2003, and run the log-linear regression:

$$\ln \text{(EER)} = c + \alpha \ln \text{(PFL)} + \beta \ln \text{(CEL)},$$

where $c = \ln \gamma$.

The parameter estimates are shown in Table 4. All parameters are statistically significant. The reported $R^2$ is a healthy 67%, which indicates an appropriateness of the fitted model. Based on these parameter estimates, we substitute $EL$, $PFL$ and $CEL$ of the Swiss Re mortality bond in 2003, obtained from (Lane and Beckwith, 2004), into the LFC’s cubic model. We get exactly the same premium (135 bps) as announced by Swiss Re (see Table 5).

Now we are confident to use the fitted parameters to predict risk premiums that should be attached to our hypothetical longevity bonds. For simplicity, we assume the mortality index $I_t$ only depends on the US death rates, and is a weighted average across different age groups

$$I_t = \sum_x \omega_x m_{x,t},$$

where $\omega_x$ is the weight assigned to age group $x$ based on the year 2000 standard population.

We set the lower strike $M$ and the initial upper strike $U$ equal to 50% and 70% of the base level of the mortality index, respectively, i.e.,

$$M = 0.5I_0, \quad U = 0.7I_0,$$

where $I_0$ denotes the 2005 base level of the mortality index.

The bond principal starts to diminish when the mortality index drops below the upper strike. The loss ratio in year $t$ is given by

$$\text{Loss ratio}_t = \frac{\max(U_t - I_t, 0) - \max(M - I_t, 0)}{U - M},$$

and $U_t = \min(U_{t-1}, l_{t-1})$ is the adjusted upper strike for year $t$. The bond has a maximum life of 20 years, and it terminates when the adjusted upper strike $U_t$ reaches the lower strike $M$.

We can simulate the mortality index and calculate the risk premium based on the LFC’s cubic model. The algorithm is developed as follows.

1. Simulate a uniform random variable $R_t$ from $[0, 1)$, $t = 1, 2, \ldots, 20$;
2. If $R_t < \Phi\left(\frac{\ln \gamma - \alpha I_t}{\sigma}ight)$, the mortality improvement $X_t$ is normally distributed with mean $-\mu$ and variance $\sigma^2$, and the simulated $X_t = -\mu + \sigma \Phi^{-1}(R)$. Otherwise, the mortality improvement $X_t$ has the GPD distribution:

$$X_t = \frac{\hat{\theta}}{\xi} \left[\left(\frac{1 - R}{1 - F_\hat{R}(u)}\right)^{-\xi} - 1\right] + u;$$
3. Update the mortality factor $k_{t+1} = k_t - X_t$
4. Calculate the central mortality rates for different age groups $m_{x,t} = \exp(\hat{a}_x + \hat{b}_x)$, and the mortality index $I_t = \sum_x \omega_x m_{x,t}$
5. Replicate step 1 to step 4 for 10,000 times, and calculate $EL$, $PFL$ and $CEL$ based on the loss ratio function.
6. Calculate the spread premium over LIBOR based on the LFC’s cubic model.

The simulation results are illustrated in Panel 1 of Table 6. We fix the lower strike equal to the 50% of the base level of the mortality index, and change the initial upper strike from 70% to 90% of the base level. The premium spread over LIBOR increases from 321 bps to 821 bps, since it is more likely for investors to lose their principal when the initial upper strike increases.

5.3. Sensitivity tests

It is noteworthy that the risk premium of a longevity bond is determined based on all available information up to the time of bond issuance. Once a given bond has been issued, its premiums will remain constant over the life of the bond. However, as mortality evolves over time, these changes would impact the secondary market prices of the bonds, once an active secondary market has been developed. Evolving mortality also would affect the premiums of bonds issued after additional mortality experience has become available. To provide an indication of the potential effects of changing mortality on bond premiums (and by implication secondary market prices), we conduct a set of sensitivity tests.

Specifically, we perform backtesting analysis using mortality data during four different historical time periods: 1900–2006 (our benchmark period), 1900–2003, 1900–2000, and 1900–1997. Thus, the tests are designed to show the sensitivity of bond premiums to the emergence of new mortality information over the period 1998 through 2006. For each historical time period, we first estimate the parameters $\hat{a}_x$, $\hat{b}_x$, and $\hat{k}_t$ in the Lee–Carter model. We then fit the $\hat{k}_t$ series with our proposed mortality model, determine the optimal threshold $\hat{m}$ and estimate the parameters $\hat{\mu}$, $\hat{\sigma}$, $\hat{\xi}$ and $\hat{\theta}$.

We set the base level of the mortality index, $I_0$, to be the mortality index one year before the end of the examined period (for example, if we examine the period 1900–2003, $I_0$ is set at the level in year 2002) and calculate the longevity risk premiums.

The sensitivity results are reported in Fig. 6 and Tables 2, 3 and 6. Fig. 6 plots the estimated mortality factor $\hat{k}_t$ for the four time periods. This graph shows that $\hat{k}_t$ is not very sensitive to the emergence of new mortality data. The $\hat{k}_t$ curve moves upwards a little bit when we consider a longer period, due to the fact that mortality is generally increasing over time combined with the normalization constraint ($\hat{k}_t$’s sum to zero). The parameter estimates $\hat{a}_x$ and $\hat{b}_x$ are shown in Table 2 for the four historical time periods and the age groups used in this study. For each age group, the parameters seem quite stable over the four historical time periods. The parameter estimates $\hat{\mu}$, $\hat{\sigma}$, $\hat{\xi}$ and $\hat{\theta}$ are shown in Table 6. There is some variation in these parameters for a given threshold level. However, the threshold level that maximizes the likelihood function remains very stable, at the 93rd percentile for the baseline period and the period 1900–1997, and the 94th percentile for the periods 1900–2003 and 1900–2000.
The bottom line, of course, is the effect of the sensitivity analysis on the bond premia. These results are shown in Table 6. The results show that emerging mortality does affect the bond premiums over time, as expected. In general, the changes are greatest between the results in Panel 4 for 1900–1997 and the results in Panel 1 for 1900–2006. Averaging the premium changes across the five combinations of upper and lower strikes is shown in Table 6, we find that the bond premiums increase on average by 5.7% between the periods 1900–2006 and 1900–2003, and by 10.7% and 13.8%, respectively, between 1900–2006 and the two earlier periods 1900–2000 and 1900–1997. The premium indexes exhibit a slight variation when the initial upper strike is 70% of the base level since it is less likely to have longevity events driving the mortality index to such a low level, while the premiums become more volatile as the initial upper strike increases. Nevertheless, we consider the premiums to be reasonably stable over time, and it is noteworthy that we would expect some variation in bond premiums over time as mortality continues to evolve.

6. Conclusions and discussion

Capital markets act as the ultimate solution to diversifying and transferring risks. In the last few years, the risks of property catastrophes and extreme mortality have been successfully transferred to financial markets using CAT bonds and mortality bonds, respectively. In contrast with the rapid growth of the CAT bond and mortality securitization markets, longevity securitization still remains in the experimental stage, although various longevity securities have been proposed. In our view, the failure of the EIB longevity bond was mainly due to poor design rather than mispricing. It provides "ground-up" protection to annuity providers and pension plans, which is not necessary, and requires a substantial capital commitment by the hedgers in order to obtain protection. The EIB bond also would have transferred the longevity risk to a reinsurer rather than to the capital market. The successful experience of Swiss Re’s mortality bonds suggests that a parallel contract design would also be applicable to longevity bonds. We propose a longevity bond whose payoff to the issuer is a series of put option spreads written on a weighted mortality index, with a fixed lower strike and increasing upper strikes. Therefore, we link the payoff to the tail distribution of longevity risk.

Modeling the tail of the longevity distribution is one of the novel aspects of this paper. Traditional parametric modeling sacrifices a good fit at the tail in exchange for a good fit in the center region. In addition, longevity jumps are extremely rare events, and we have little historical data to use in regard to estimation purposes. Extreme value theory provides a solution to these problems. In this paper, we make the first attempt to integrate a parametric mortality model with extreme value theory. We observe that a random walk with drift nicely explains most of the mortality improvement data, while the exceedances above a certain threshold have a fat tailed distribution. Our model has the advantage of not only fitting the existing mortality data but also extrapolating more extreme longevity jump events.

Our paper also contributes to the existing literature by applying the LFC’s CAT bond pricing methodology to calculate risk premiums of longevity bonds. As stated by Lane (2000), “A risk is a risk is a risk, in whatever market it appears.” The pricing of longevity or mortality risk therefore should be related to the pricing of reinsurance catastrophic risk. The LFC’s cubic model captures the trade-off between the frequency of and severity of adverse events. It is easy to understand and implement, and thereby may be favored by risk modeling firms and investors.

References


