Takeover bids, the free-rider problem, and the theory of the corporation

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It is commonly thought that a widely held corporation that is not being run in the interest of its shareholders will be vulnerable to a takeover bid. We show that this is false, since shareholders can free ride on the raider's improvement of the corporation, thereby seriously limiting the raider's profit. We analyze exclusionary devices that can be built into the corporate charter to overcome this free-rider problem. We study privately and socially optimal corporate charters under the alternative assumptions of competition and monopoly in the market for corporate control.

1. Introduction

■ In all but the smallest groups social choice takes place via the delegation of power from many to few. A fundamental problem with this delegation is that no individual has a large enough incentive to devote resources to ensuring that the representatives are acting in the interest of the represented. Since the representatives serve the Public Good, the social benefit to monitoring their activities is far larger than the private benefit to any individual. That is, the Public Good is a public good and each person attempts to be a free rider in its production.

It is often suggested that in a corporation the free-rider problem can be avoided by use of the takeover bid mechanism. Suppose that the current directors of the corporation are not acting in the shareholders' interest, but that each shareholder is too small for it to be in his interest to devote resources to overthrowing management.[1] It is argued that this situation will not persist because an entrepreneur (i.e., a "raider") can make a takeover bid: he can buy

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[1] See Williamson (1964) for a discussion of the separation of ownership and control in the corporation.

the company at a low price, manage it well, and then sell it back at a high price.[2] We show that this argument is false. Any profit a raider can make from the price appreciation of shares he purchases represents a profit shareholders could have made if they had not tendered their shares to the raider. In particular, suppose each shareholder is so small that his tender decision will not affect the outcome of the raid. Then, if a shareholder thinks that the raid will succeed and that the raider will improve the firm, he will not tender his shares, but will instead retain them, because he anticipates a profit from their price appreciation. As a result, a takeover bid may not be profitable even though current management is not acting in the interest of shareholders. Hence, even in a corporation, the public good (of the shareholders) is a public good.[3]

There is a real resource cost in operating the takeover mechanism. A raid should take place if and only if the social benefit is larger than the social cost. The raider bears the full social cost, but because shareholders attempt to free ride by not tendering their shares, he may be able to get only a small part of the social benefit. As a result, there may be many raids which should take place, but which do not, because it is not profitable for a raider to execute them.

Shareholders can overcome this free-rider problem. Specifically, they can write a constitution for the firm which permits the raider to exclude minority shareholders (i.e., shareholders who do not tender their shares to the raider and who hold shares in the postraid company) from sharing in all the improvements in the firm brought about by the raider. One method is for the shareholders to permit a successful raider to sell the firm's assets or output to another company owned by the raider at terms which are disadvantageous to minority shareholders. The raider then receives more from the raid than just his share of the company's (increased) profits. This compensation comes at the expense of other shareholders and represents a voluntary dilution of their property rights.

In Section 2 of the paper, we discuss the role of such dilutions and focus on how the extent of permitted dilution affects the tender price the raider must pay, and hence the profitability of raids. The ease with which raids can take place will, of course, influence the actions of the incumbent management. We study this in Section 3. In Sections 4 and 5, we consider the optimal amount of dilution from the point of view of the firm's shareholders and from

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[2] See Marris (1964) and Manne (1965) for a discussion of the role of takeover bids in ensuring that a director serves the interests of shareholders.

[3] The problem is similar to that which occurs in real estate deals involving urban renewal. It is sometimes suggested that if a few city blocks are dilapidated (because, e.g., there are filthy streets that are dark and unsafe, filled with houses that have unpainted ugly exteriors and uncut lawns, etc.), then an entrepreneur can buy the houses at a low price, clean the streets, etc., then sell the houses at a high price and make a profit. (The entrepreneur "internalizes" the externality.) However, if we own one of the houses on this street, then we shall surely not sell to the entrepreneur at a low price because, after the street is improved, our house will rise in value. In the case of real estate, this "unexcludability" is avoided by the use of secrecy or by making each house purchase contingent on the entrepreneur's acquiring *all* of the houses on the street (i.e., unanimity). The use of secrecy is difficult for firms traded in a stock market because: (A) in the United States the Williams Act requires potential raiders to disclose their intent publically after they purchase 5 percent of the stock and (B) the stock market is full of traders constantly looking for information that a stock will appreciate in value and thus secrecy is hard to maintain. The use of unanimity never occurs in takeovers of widely held companies, presumably because each shareholder would attempt to be the only "holdout" and thus anticipate a secret payment from the raider for his shares in addition to the tender price. With many small shareholders it would be very difficult to enforce a unanimous contract for the above reason.

the point of view of society. Section 6 comments on the role of competition among raiders. Finally, Section 7 contains concluding remarks. The Appendix contains the proofs of all the lemmas and propositions which are stated without proof in the text.

2. The role of dilution in takeover bids

■ In the Introduction we indicated that takeover bids may not ensure good management of corporations because of shareholders' attempts to free ride. The purpose of this section is to develop a formal model of this free-rider problem.

We assume that the profit of a typical firm is given by a function $f(a)$, where a is a description of activities engaged in by the firm (e.g., investment decisions, hiring decisions, and managerial effort). We suppose that there is no uncertainty about the firm's profit once the activity a has been selected. The number $f(a)$ may also be interpreted as the net present value of the future stream of profit generated by activity a or the market value of the firm's shares. (In this paper we shall not distinguish among profit, net present value, or market value.)[4] Let A denote the set of all feasible activities for the firm.

Consider a firm which is using activity $a_0 \in A$. Let $q = f(a_0)$ denote the current profit of the firm. Suppose now that an individual (henceforth known as the raider) announces his intention to take over the firm and announces a *tender price* p at which he is willing to buy unconditionally all shares tendered to him.

How will the shareholders of the target firm react to the tender offer? One of the most important factors influencing this reaction is the extent to which the shareholders believe the raider will improve the firm, should his takeover bid be successful. Because we wish to study the efficiency of the takeover bid process in the purest case, we shall assume that the raider is a profit maximizer.

For the sake of generality, however, we shall not suppose that the raider and the firm's current manager necessarily have the same ability in running the firm. Let $\max_{a \in A} f(a)$ be the maximum profit of the firm under current management. We write the profit of the firm under the raider's management as $v = \max_{a \in A} f(a) + \epsilon$, where ϵ is a measure of the differences in ability between the raider and the *status quo* manager. (We shall sometimes treat ϵ and thus v as random variables.) If the raider's bid is successful, the new profit of the firm is assumed to be given by v. We shall assume that both the raider and the shareholders know $\max_{a \in A} f(a)$, ϵ, and hence v at the time of the raid. This is a strong assumption, but it seems to provide a reasonable starting point for an analysis of takeover bids. Further, one might expect the takeover bid mechanism to work best when the shareholders and the raider are under no illusions about the quality of management.

The raider's takeover bid will be deemed successful if more than 50 percent of the shares are tendered to the raider. Suppose that the firm is owned by a large number of shareholders, each of whom owns a very small proportion of the firm. Under these conditions the probability that any shareholder's tender decision

[4] We shall assume that the market value of the firm's shares represents the total benefits which the shareholders get out of the firm. In other words, we assume that the demand curve for the firm's shares is infinitely elastic and hence shareholders get no consumer surplus from holding the firm's shares. A result of this is that the firm's initial shareholders will be unanimous that the firm should choose an action a to maximize $f(a)$.

will be decisive in determining the success or failure of the bid is negligible. Thus, each shareholder will ignore his impact on the outcome of the bid in making his tender decision.

We shall also assume that shareholders and the raider have rational expectations about the outcome of a bid. In this paper we shall not consider bids with stochastic outcomes, i.e., bids which succeed some fraction of the time and fail the remaining fraction of the time. Thus the only successful bids are those which are expected to be successful *with certainty*.

It is straightforward to show that under the above assumptions the commonly made argument that a poorly managed and hence low-priced firm can be taken over, managed well, and resold at a profit by an entrepreneur is incorrect. Suppose that the raider's tender price is p. Then any shareholder who thinks that the raid will succeed with certainty will *not* tender his shares if $p < v$. This results because, given that his tender decision has no impact on the outcome of the bid, he can do better by holding on (he gets v) than by tendering (he gets p). Since no shareholder tenders his shares, a bid with tender price $p < v$ will, of course, fail. It follows that for a raid to succeed, given that shareholders anticipate that it will succeed (this is the assumption of rational expectations), it is necessary that

$$p \geq v. \tag{1}$$

But under these conditions the raider makes no profit, since he pays at least as much for the firm's shares as they are worth to him. In fact, if, as we would expect, raids are costly, the raider actually makes a loss! Thus no raids will take place, even if q is very low relative to v.[5]

Raids are unprofitable because each shareholder is in a position to free ride on a potentially successful raid. Any profit the raider can expect from the price appreciation of the shares he purchases can be captured by a shareholder if he does *not* tender. Therefore, individual rationality by shareholders concerning the tender decision leads to an outcome which is highly undesirable for all shareholders—there are no takeover bids and bad management is not penalized.[6]

In practice the free-rider problem is not so severe as in the model described above, and raids do take place. What changes are necessary to allow for this possibility? The most obvious modification is to introduce differences in valuation of the firm by the raider and the shareholders—as a result, say, of differences in risk preferences or information. Let us continue to assume that the raider values the raider-controlled firm at v, but let the shareholders' valuation be v_s, which may be different from v. That is, v_s is the value prospective minority shareholders put on 100 percent of the dividends in the raider-controlled corporation. Then, by the argument above, the lowest tender price at which the raider can get control of the firm is v_s. At this price we may assume that all shares are tendered to the raider, since shareholders are indifferent between

[5] We are assuming that the raider is not a shareholder before he makes the raid. Clearly, if the raider owns a fraction of the firm larger than $c/(v - q)$ before a successful raid is announced, where c is the cost of the raid, then the raider can make a profit. See footnote 3 for the reasons it is difficult to make purchases secretly.

[6] In Grossman and Hart (1979b), we analyze bids at prices p satisfying $q < p < v$ whose outcomes are stochastic. We show that stochastic bids do not change the nature of the free-rider problem. In particular, the result that the raider cannot make a profit from a takeover bid without dilution still holds (where profit must be interpreted in expected terms).

tendering and not tendering. Let c be the cost of the raid. The raider's profit is then

$$\pi = v - v_s - c. \tag{2}$$

If v_s is sufficiently small relative to v, raids will now occur.

While differences in valuation are undoubtedly important in permitting raids to take place, it would be unwise for shareholders to rely on luck to bring a raider who, for some exogenous reason, values the dividend stream of the firm more than shareholders do. It is better for shareholders to *create* a divergence between the value of the dividend stream to a raider and its value to shareholders who attempt to free ride on the raider's improvements of the firm.

This can be achieved as follows. Let initial shareholders write a corporate constitution or charter permitting any successful raider to reduce the value of the postraid company by a certain amount, which the raider is permitted to pay to himself. There are a number of ways to achieve such a reduction in value. For example, the raider can be allowed to pay himself a large salary or to issue a number of new shares to himself. Alternatively, the raider can be permitted to sell the target firm's assets at below their true value, via a merger or liquidation, to another company owned by the raider. (See Section 7 for further comments on this method.) A third possibility is for the raider to sell the target firm's output to one of the raider's other companies at an artificially low price. For example, suppose that company A, which produces automobiles, obtains control of company B, which produces automobile tires. Then the new directors of company B might sell tires to A at a low price so that most of the profits of the tire company accrue to the automobile company.

Whichever method is used, the result is the same: the value to shareholders of not tendering their shares to the raider and of becoming minority shareholders in the raider-run firm is reduced. A divergence between the shareholders' valuation and the raider's valuation of the postraid firm is introduced, and shareholders are excluded to some extent from free riding on the improvements brought about by the raider.

It is important to realize that this divergence corresponds to a voluntary dilution of shareholder property rights. When the firm is well managed, it is worth v, and hence the shareholders could with some justification claim that v is the "true" or "fair" value of their shares. By permitting the raider to reduce the value of the company and to pay the excess to himself, shareholders are depriving themselves of the full worth of their shares: that is, they are voluntarily diluting their property rights. Much of takeover bid law implicitly assumes that such dilutions are undesirable. The point of view taken in this paper, however, is that dilutions of this kind are essential if the takeover bid mechanism is to be effective in penalizing bad managers.

Let us consider how dilutions affect the price at which the raider can acquire control. Assume that the initial shareholders, when writing a corporate charter, can enforce a maximal level of dilution, given by ϕ dollars.[7] For simplicity, in what follows we shall ignore any exogenous differences in raider

[7] Though actual corporate charters do not specify a monetary limit on dilution, they do specify the extent to which minority shareholders are protected from dilution. See Grossman and Hart (1980) for an analysis of how monetary levels of dilution are related to the stringency of disclosure and appraisal requirements.

and shareholder valuations. Given the dilution factor ϕ, the value to a shareholder from retaining his shares in the event that a raid succeeds is $v_s \equiv v - \phi$. Thus, if the raider offers the tender price p, and shareholders think that the raid will succeed, they will tender as long as

$$p \geq v - \phi. \tag{3}$$

In this case, of course, the raid will indeed succeed (we assume as above that shareholders who are indifferent between tendering and not tendering do tender).

Equation (3) implies that if $v - \phi < q$, bids can take place at below the *status quo* market value $q = f(a_0)$. Note, however, that such bids will fail if they are expected to fail (shareholders will not tender, since tendering means getting p if the bid is unconditional, whereas not tendering means getting q).[8] For this reason, and also because bids at below market value are rarely observed in practice, we shall henceforth rule out bids at $p < q$. That is, we shall impose the condition $p \geq q$ in addition to (3).

It follows that the lowest tender price which enables the raider to get control, when dilution is restricted to ϕ, is

$$p = \max(v - \phi, q). \tag{4}$$

Thus, if the cost of the raid is c, the raider's profit will be

$$v - p - c = v - \max(v - \phi, q) - c = \min(\phi, v - q) - c. \tag{5}$$

This will be positive if ϕ and $(v - q)$ both exceed c, and raids will take place under these conditions.

It is important to realize that in our model the only effect of dilution is to reduce the price the raider has to pay the shareholders to get control of the firm. Given our assumptions, the raider who pays $p = \max(v - \phi, q)$ will get complete control of the firm, i.e., he will acquire 100 percent of the shares. (Since (3) holds, no shareholder will wish to retain any shares.) Once he owns 100 percent of the firm, the dilution which he extracts is a matter of complete indifference—every extra dollar he receives in dilution is one dollar less received in dividends. The point, however, is that it is precisely the *threat* that the raider can dilute up to ϕ, which reduces the value to shareholders of retaining their shares and allows the raider to get control. (In a more complicated model in which some shares are tendered and some are not, the raider will find it in his interest to carry out dilution.)

In the next section, we study the effect of dilutions on the current management's choice of *status quo* profit, q. Then in Sections 4 and 5 we analyze the optimal level of dilution for shareholders and for society.

3. The influence of takeover bids on the manager's choice of the status quo

■ We take as a premise that corporations have the following sort of life cycle. Initially, a very small group of shareholders writes the corporate charter and

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[8] In other words, if $v - \phi < p < q$, there are two rational expectations equilibria. If the bid is anticipated to succeed, it will succeed. On the other hand, if the bid is anticipated to fail, it will fail. In contrast, when $p \geq \max(v - \phi, q)$, there is a single rational expectations equilibrium in which the bid succeeds.

decides to "go public," i.e., they sell part of their right to the company's earnings stream. Since these initial shareholders are large shareholders, who desire to sell their shares at the highest possible price, it is in their interest to devote resources to see that the corporation is organized to maximize the (expected) return to all potential shareholders. The initial shareholders realize that they will sell most of their shares in the future and that eventually the corporation will be owned by many small shareholders, none of whom will find it in his interest to collect information about the corporation. Hence, at the time the charter is written, initial shareholders try to devise self-enforcing mechanisms to ensure good management. One mechanism available to initial shareholders is to give directors salary incentives, e.g., stock options, warrants, etc. The initial shareholders recognize, however, that salary incentive schemes will not be perfect, because perfection would require the director's salary to be contingent on events which it would be quite costly for any small shareholder to verify. (For example, an optimal incentive scheme would attempt to distinguish between low earnings due to poor management and low earnings due to a general decline in the industry.)

Recognizing that there are many future states of nature in which the managerial salary incentive scheme will be so ineffective that directors will deviate significantly from profit maximization, initial shareholders write a corporate charter which encourages takeover bids. Initial shareholders realize that if deviations from profit maximization occur and if dilution is permitted, then a raider will find it in his interest to collect the appropriate information to discover how to revise the managerial incentive scheme so that profit maximization is again encouraged. That is, the raider can take over the firm at price $p = \max(v - \phi, q)$ and change the incentive scheme to incorporate all the new information available about the probability distribution of the firm's returns. The raider can then sell the company with the new incentive scheme in place at price v, and make a profit through the price appreciation of the target company's shares (assuming dilution is permitted).

When the raider takes over, he is in the same position as the initial shareholders were. In addition to revising the managerial salary incentive scheme, he may also modify the corporate charter. He then sells a large fraction of his shares so that the corporation is once again in the hands of many small shareholders. The whole life cycle then begins anew.

In this paper we shall analyze only one realization of this process; namely, assume the following sequence of events: (1) A single director-manager is assigned to the firm. (2) This manager chooses an action a_0 with resulting profit $q = f(a_0)$. (3) A potential raider arrives and decides whether or not to raid. (For the moment, we ignore the possibility that there is more than one raider; see, however, Section 6.) (4) If a raid is successfully carried out, the raider fires the current manager and replaces the action a_0 by a profit-maximizing action (it is assumed that the current manager has not yet had time to make any irreversible decisions); if no raid takes place, or if the raid is unsuccessful, the action a_0 is retained.

As in Section 2, the value of the firm under the raider's management is $v = \max_{a \in A} f(a) + \epsilon$. For reasons which will become clear, we assume that ϵ, and hence v, are stochastic. To indicate this we shall write $\tilde{v}$. We shall also assume that the cost of the raid is a random variable, denoted by $\tilde{c}$. The manager does not know the realizations of $\tilde{v}$ and $\tilde{c}$ when he chooses a_0. However, we assume that at the time of the raid, shareholders know the realization of $\tilde{v}$ and the raider knows the realization of $(\tilde{v},\tilde{c})$.

The manager has well-defined preferences over the set of feasible actions of the firm, A. These preferences will be assumed to be representable by a utility function $\hat{U}: A \rightarrow R$, where R is the real line.[9] It is useful to express the manager's utility in terms of the profit q that he must achieve instead of the action a he takes. We denote this by $U(q)$ and assume that U is continuous in q.[10] We assume that the initial shareholders who write the corporate charter know the derived or indirect utility function $U(q)$.[11]

In this section we take the dilution factor ϕ to be fixed, and we analyze how the manager's choice of *status quo* profit q depends on ϕ.

Suppose the manager chooses a_0, giving rise to the profit $q = f(a_0)$. Let (v,c) be the realization of $(\tilde{v},\tilde{c})$. We saw in Section 2 that if the raider decides to make a raid, he will have to offer at least the tender price $\max(v - \phi, q)$ to be successful, so his profit will be $v - \max(v - \phi, q) - c = \min(\phi, v - q) - c$, where c is the cost of the raid. Thus, a raid will occur for realizations of $(\tilde{v},\tilde{c})$ such that $\min(\phi, v - q) - c$ is positive and will not occur otherwise.

Let $\bar{U}$ be the utility that the manager receives if he is fired by the raider and must seek a job elsewhere. Without loss of generality, we set $\bar{U} = 0$. Then the manager's utility from the profit q is given by

$$\begin{cases} U(q) & \text{if} \quad \min(\phi, v - q) - c \leq 0, \quad \text{i.e., in the event of no raid;} \\ 0 & \text{if} \quad \min(\phi, v - q) - c > 0, \quad \text{i.e., in the event of a raid.} \end{cases} \tag{6}$$

We shall assume that the manager maximizes expected utility. Let $F(c,v)$ denote the distribution function of $(\tilde{c},\tilde{v})$ and let $\pi(\phi,q) \equiv \text{Prob}[\min(\phi, \tilde{v} - q) > \tilde{c}]$ denote the probability of a raid.[12] Since the manager's final utility is given by (6) for the particular realization (c,v) of $(\tilde{c},\tilde{v})$, it follows that the manager's expected utility from profit q is

$$W(q) = U(q)(1 - \pi(\phi,q)). \tag{7}$$

Hence, an optimal action for the manager is one that maximizes $W(q)$.

[9] The utility function $\hat{U}$ is understood to incorporate such factors as the salary of the manager, the amount of managerial effort required to implement the particular action a, the size of the firm, the manager's shareholding in the firm, and any salary incentive scheme that initial shareholders have devised for the manager. (For a discussion of these and other determinants of managerial "utility," see Baumol (1959), Marris (1964), and Williamson (1964). See also Ross (1977) and Mirrlees (1976) for discussion of salary incentive schemes.)

[10] Assume that the set of feasible profit levels $\{q \mid f(a) = q \text{ for some } a \in A\}$ equals the closed interval $[q_{\min}, \max_{a\in A} f(a)]$ for some nonnegative numbers $q_{\min}$, $\max_{a\in A} f(a)$. Define $U(q) = \max_{a\in A} \hat{U}(a)$ subject to $f(a) = q$, where $q_{\min} \leq q \leq \max_{a\in A} f(a)$. Note that management may be attempting to maximize profit, but it may lack the ability or information to do so. This can be incorporated into our model if we think of $U(q)$ as being very low for values of q which the manager lacks the ability or information to produce. Thus, in particular, set $U(q) = -\infty$ for $q > \max_{a\in A} f(a)$.

[11] For simplicity, we have taken the profit function $q = f(a)$ to be deterministic. Of course, if this were really the case, there would exist a salary incentive scheme which gets the manager to maximize profit: "produce maximum profit $\max_{a\in A} f(a)$ or you are fired." A perfect incentive scheme of this sort would not exist, and none of our results would change, if we assumed that $q = f(a,\theta)$, where θ is a parameter unobserved by shareholders, but known to the manager and the raider. The need for dilution, furthermore, is just as great as in the deterministic model studied in this paper.

[12] We shall assume that $\tilde{v}$ is bounded from above and that $\tilde{c}$ and $\tilde{v}$ are nonnegative.

Note that the probability of a raid $\pi(\phi,q)$ is a nonincreasing function of q. Hence, the tradeoff for the manager is between choosing a high profit action with an associated low chance of being raided and choosing an action which provides high managerial utility but which is likely to lead to a successful takeover bid.

If $\tilde{v}$ and $\tilde{c}$ are nonstochastic, then for a given choice of q, either a raid occurs with certainty or no raid occurs, i.e., $\pi(\phi,q) = 1$ or $\pi(\phi,q) = 0$. It follows that as long as $U(v) > 0$, the manager will always choose q large enough so that no raids ever take place. However, when $\tilde{v}$ and $\tilde{c}$ *are* stochastic, it will not in general be optimal for the manager to choose q such that $\pi(\phi,q) = 0$, and hence takeover bids will generally occur.

4. Shareholders' optimal choice of the dilution factor, ϕ

■ In the last section we studied the manager's action for a fixed dilution factor ϕ. We consider now the optimal value of the dilution factor, ϕ, for the initial shareholders of the firm.

We shall assume that the market values the firm according to its expected return; that is, the market is risk-neutral with respect to the firm's activities.[13] Hence, the initial shareholders—who wish to get as high a value for their shares as possible—will choose a value of ϕ which maximizes the expected return from the firm's operations. This expected return is given by

$$r(\phi) \equiv q(1 - \pi(\phi,q)) + E[\max(\tilde{v} - \phi, q) \mid \min(\phi, \tilde{v} - q) > \tilde{c}]\pi(\phi,q), \quad (8)$$

since if there is no raid (which occurs when $\min(\phi, \tilde{v} - q) \leq \tilde{c}$), the market value of the firm equals the profit of the firm, q; while if there is a raid (which occurs when $\min(\phi, \tilde{v} - q) > \tilde{c}$), the market value of shareholders' shares equals the tender price announced by the raider, $\max(\tilde{v} - \phi, q)$. It should be emphasized that $r(\phi)$ is the value at which the firm's shares sell in the market *before* it is known whether or not a takeover bid is going to occur.

Consider how changes in ϕ affect $r(\phi)$. As ϕ increases, the value of shares in the event of a raid (the tender price), $\max(\tilde{v} - \phi, q)$, decreases. At the same time, however, the probability of a raid, $\pi(\phi,q)$, increases. Hence an increase in ϕ reduces the *amount* shareholders gain from any particular raid, but it increases the *number* (the probability) of raids.

There is a further effect caused by a variation in ϕ which results from the fact that the *status quo* profit q depends on ϕ. In the last section we showed that the manager chooses q to maximize $W(q) = U(q)(1 - \pi(\phi,q))$. In general, we might expect that an increase in ϕ, by making raids easier, will lead the manager to choose a higher *status quo* profit. We shall demonstrate that this is indeed the case.

A difficulty that arises in the analysis of the relationship between *status quo* profit and ϕ is that there may be several actions which are optimal for the manager, and hence several possible optimal *status quo* profit levels, even if $U(q)$ is strictly concave. We shall assume that if the manager is indifferent between two actions, then he chooses the one with the higher profit level. This

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[13] This is a reasonable assumption if the firm's return is independent of the returns of other firms, and shareholders hold well-diversified portfolios.

enables us to write *status quo* profit q as a function of ϕ, $q(\phi)$, and in the Appendix we prove:

Proposition 1: $q(\phi)$ is nondecreasing in ϕ.

An increase in ϕ therefore has three effects: (1) for a given *status quo* profit q, it reduces the tender price offered by the raider if he chooses to raid, and hence the amount that the shareholders receive from any particular raid; (2) it increases the number of raids that take place for any *status quo* profit q; (3) it increases the return to shareholders in the event there is no raid, i.e., it increases the *status quo* profit q. From the point of view of the shareholders, (2) and (3) are goods, while (1) is a bad.

Because of these three effects, general analysis of the optimal ϕ is quite difficult. However, much insight can be gained from the analysis of special cases. Consider first the case where $\tilde{c}$ is nonstochastic. Then the following can be established:

Proposition 2: Suppose $\tilde{c}$ is nonstochastic, i.e., $\tilde{c} = c$ with probability one.

(a) If $\tilde{v}$ is nonstochastic, i.e., $\tilde{v} \equiv v$, and $U(v) > 0$, then it is optimal for the initial shareholders to choose any ϕ such that $\phi > c$. At any optimal ϕ no raids take place.

(b) If $\tilde{v}$ is stochastic (i.e., the marginal distribution of $\tilde{v}$ is not degenerate), then: (i) Initial shareholders will want to choose ϕ to maximize the tender price in the event of a raid subject to $\phi > c$. This is accomplished by setting $\phi > c$, but as close to c as possible.[14] (ii) Raids will generally take place. (iii) $q(\phi)$ is constant for all $\phi > c$.

Proof:

(a) From (5), raids take place if and only if min $(\phi, v - q) > c$. Since $\tilde{c}$ and $\tilde{v}$ are nonstochastic, this means that a raid takes place with probability one or probability zero. The assumption $U(v) > 0$ implies that the manager would rather profit maximize—and hence avoid a raid—than choose an action which would lead him to lose his job with certainty. It follows that the manager will act so that no raids take place. Hence, the return to shareholders is given by *status quo* profit, $q(\phi)$. From Proposition 1 it follows that it is optimal for shareholders to set ϕ as large as possible. However, if $\phi > c$, then min $(\phi, v - q) > c$ iff $v - q > c$. Hence $\pi(\phi,q)$ is constant for all $\phi > c$, and therefore $r(\phi) = q(\phi)$ is constant for all $\phi > c$.

Note that since in this case there are never any raids, the factor which makes a large ϕ unattractive to shareholders (namely that a large ϕ will lead to a low tender price in the event of a raid) is absent.

(b) If $\phi \leq c$, then by (5) no raids take place so that in this case $r(\phi) = q(\phi)$. Since $r(\phi) \geq q(\phi)$ for $\phi > c$, it follows from Proposition 1 that $\phi \leq c$ is not optimal. If $\phi > c$, then min $(\phi, \tilde{v} - q) > c$ iff $\tilde{v} - q > c$. Hence $\pi(\phi,q)$ is constant for all $\phi > c$. Therefore, the manager's action $q(\phi)$ will be a constant for all $\phi > c$. However, for all $\tilde{v}$ such that $\tilde{v} - \phi > q$, the tender price $p = \max(\tilde{v} - \phi, q)$ will be reduced when ϕ is raised. Thus, by making ϕ as small as possible *but* larger than c, shareholders maximize the tender price without

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[14] We denote such a situation by $\phi \cong c$. In general there is a trivial "openness problem" and no optimal ϕ exists in this case.

lowering managerial effort. In general, it will not be optimal for the manager to set q so high that the probability of a raid is zero (since with $\tilde{v}$ stochastic he can trade off low probability of raids against high managerial utility). Thus raids do in general take place. *Q.E.D.*

Proposition 2a illustrates the fact that permitting large levels of dilution, and thus a low tender price in the event of a raid, is good for shareholders if no raids ever occur. In Proposition 2b, however, raids do occur, and so shareholders wish to limit dilution to obtain a high tender price. In the case where $\tilde{c}$ is nonstochastic, this is achieved without any reduction in managerial effort because the threat of a raid, $\pi(\phi,q)$, will be constant for all $\phi > c$. The next proposition shows that if $\tilde{c}$ is stochastic, then there is a real tradeoff between the achievement of a high tender price and managerial efficiency.

Proposition 3: Suppose that $\tilde{v}$ is nonstochastic, i.e., $\tilde{v} = v$ with probability one, but that $\tilde{c}$ is stochastic (i.e., the marginal distribution of $\tilde{c}$ is not degenerate). Then it is optimal for the shareholders either (1) to put no restrictions on the raider's ability to dilute, i.e., to set $\phi = \infty$, which means that $p = \max(\tilde{v} - \phi, q) = q$; or (2) to choose ϕ to maximize

$$(v - \phi) \text{ Prob } (\phi > \tilde{c}) + q^* \text{ Prob } (\phi \leq \tilde{c}), \tag{9}$$

where q^* is the unconstrained utility-maximizing profit for the manager, i.e., q^* solves $\max_q U(q)$. (In case (2), the manager ignores the possibility of a raid and sets $q(\phi) = q^*$.)

The idea of the proof (which is given in the Appendix) can be seen by noting that if $\tilde{v} = v$ with certainty, then the raider's tender price is

$$p = \max(\tilde{v} - \phi, q) = \begin{cases} v - \phi & \text{if} \quad q \leq v - \phi \\ q & \text{if} \quad q > v - \phi. \end{cases}$$

Consider a given value of ϕ. Then for each q, p is independent of either ϕ or q. Let ϕ^* be the optimal choice of ϕ for the shareholders. If $p = q$ at the optimum, then ϕ can be set equal to $+\infty$ without changing anything. This is case (1) of Proposition 3, and raids are encouraged as much as possible. In this case the manager maximizes $U(q)$ Prob $(v - q \geq \tilde{c})$. Let $\hat{q}$ denote the solution to this problem.

On the other hand, if $p = v - \phi > q(\phi)$ at the optimal ϕ, then by lowering q the manager will not increase the probability of a raid. Hence, he chooses $q = q^*$—the maximizer of $U(q)$. This is case (2) of Proposition 3. In this case shareholders know that for all ϕ such that $v - \phi > q^*$, the manager's action is unchanged, and hence their tradeoff is between increasing ϕ to increase the probability of a raid, Prob $(\phi > \tilde{c})$, or decreasing ϕ to increase the tender price $(v - \phi)$.

Case (1) of Proposition 3 will apply when $\hat{q}$ is close to v and $\hat{q}$ is much larger than q^*: if the threat of a raid provides very strong incentives for good management and if the salary incentive scheme embodied in $U(q)$ is not very effective. Case (2) applies, in contrast, when $\hat{q}$ is substantially lower than v and $\hat{q}$ is close to q^*: when the threat of a raid provides a weak incentive for managerial effort relative to the salary incentive scheme. In the latter case it is not optimal for the shareholders to set $\phi = \infty$ and to sacrifice the chance of getting a high tender price in the event of a raid. In fact, it is preferable for them to dispense with the takeover threat altogether.

When $\tilde{v}$ is nonstochastic and $\tilde{c}$ is stochastic, shareholders choose only between the profit levels $\hat{q}$ and q^*. If $\tilde{v}$ and $\tilde{c}$ are both stochastic, however, then $q(\phi)$ will take on more than two values as ϕ varies. In fact, it will vary smoothly with ϕ because for each ϕ and q, the tender price $p = \max(\tilde{v} - \phi, q)$ will equal $\tilde{v} - \phi$ for some realizations of $\tilde{v}$, and it will equal q for other realizations of $\tilde{v}$. Hence, changes in ϕ and q will always have some impact on the probability of a raid. In other respects, though, the case where $\tilde{v}$ and $\tilde{c}$ are both stochastic is very similar to the case where $\tilde{c}$ alone is stochastic. In particular, there is a real tradeoff between achieving a high tender price and inducing managerial efficiency; shareholders' pursuit of the former leads to a (partial) sacrifice of the latter.

In summary, if shareholders know the costs of a takeover bid, then by setting $\phi > c$ they can compensate the raider for these costs. Proposition 2 shows that under these conditions it is optimal for shareholders to exploit fully the threat of raids: the optimal choice of ϕ leads to maximization of the *status quo* profit $q(\phi)$.[15] In contrast, when $\tilde{c}$ is stochastic, Proposition 3(2) shows that shareholders may limit the disciplinary role of raids considerably in their efforts to ensure a high tender price.

Since the case where $\tilde{c}$ is stochastic yields rather different results from that in which $\tilde{c}$ is nonstochastic, it is worth considering the nature of takeover bid costs. A raider must face four main costs. The first is the cost of collecting information about possible improvements in the firm. Second, there is the cost of raising the funds to finance the purchase of the firm. Third, there are the administrative and litigation expenses of the takeover bid itself. Finally, there is the cost of reorganizing the firm if the raid is successful.[16] While it may be possible to estimate some of these costs quite accurately (e.g., the cost of running the tender offer), it may be very difficult to estimate others (e.g., the cost of information collection or reorganization). Furthermore, some of these costs may be raider-specific; they may be high for some raiders and low for others. Since the characteristics of the particular raider who will make a bid are not known *a priori*, this may create considerable uncertainty about cost levels. For these reasons it seems likely that the initial shareholders writing the corporate charter will perceive the raider's cost—his reservation price for carrying out a raid—as a random variable (with possibly high variance) rather than a determinate number. In other words, the case where $\tilde{c}$ is stochastic would seem to be of greater practical significance than the case where $\tilde{c}$ is nonstochastic.[17]

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[15] In particular, $q(\phi) = q(\infty)$. One possibility which we have not considered is that shareholders subsidize raids. Under some conditions this can lead to higher *status quo* profit levels than $q(\infty)$. For example, suppose that $\tilde{c}$ and $\tilde{v}$ are nonstochastic. Let it be written into the corporate charter that anybody who carries out a raid will be given a subsidy equal to c. Then this forces the manager to set $q = v$, if he wants to avoid a raid. In contrast, if there is no subsidy and $\phi = \infty$, the manager can avoid a raid by setting $q = v - c$. There are obvious moral hazards involved with this scheme.

[16] For example, the raider may have to spend time working out how to devise an incentive scheme which will ensure that future managers of the firm carry out the profit-maximizing action.

[17] In the analysis we have lumped together the four above outlined costs. From a formal point of view, however, this is not really legitimate, since the cost of collecting information about the firm differs from the other costs in that it is a *sunk* cost by the time the raid occurs. Thus, while this cost will influence whether a raider investigates the firm in the first place, it will have no effect on the raider's decision to raid, once he has decided to become informed. See Grossman and Hart (1979a) for a model of takeovers which carefully distinguishes between the sunk and nonsunk costs.

5. The optimal choice of the dilution factor ϕ for society

■ So far we have looked at the initial shareholders' choice of the dilution factor. However, it is also interesting and important to consider society's perspective on this decision. This section will analyze the divergence between the shareholders' and society's views that is generated by the assumption that there is only one raider in the event of a takeover bid. The next section explains the reasons for this assumption.

We shall assume that society, like the shareholders, is risk neutral with respect to the activities of the firm. For a given value of ϕ, the return to society from the firm's activities is

$$R(\phi) \equiv q(\phi)(1 - \pi(\phi,q)) + E[\tilde{v} - \tilde{c} \,|\, \min(\phi, \tilde{v} - q(\phi)) > \tilde{c}]\pi(\phi,q)$$
$$= r(\phi) + E[\min(\phi, \tilde{v} - q(\phi)) - \tilde{c} \,|\, \min(\phi, \tilde{v} - q(\phi)) > \tilde{c}] \times \pi(\phi,q) \quad (10)$$

because when there is a raid, the efficiency gain is the increase in the profit of the firm minus the cost of resources used up in the raid. We are ignoring distributional effects—how much the raider gets versus how much the shareholders get—and we are also assuming that the social cost of resources consumed in the raid equals the private cost. Finally, we assume that perfectly competitive conditions prevail in the market for the firm's output(s), so that the social contribution of the firm to the economy is represented by its profit. (Thus we are not considering raids which take place to restrict competition.)

From (5) the raider's expected profit is $E[\min(\phi, \tilde{v} - q) - \tilde{c} \,|\, \min(\phi, \tilde{v} - q) > \tilde{c}] \times \pi(\phi,q)$. Hence, from (10) it is clear that the social return $R(\phi)$ will equal the private return $r(\phi)$ if the raider makes zero profit.[18] This will occur if there is competition by other raiders at the time of the raid. It will also occur if there are no realizations of $(\tilde{v},\tilde{c})$ for which a raid takes place. Recall from Proposition 2 that if $\tilde{v}$ and $\tilde{c}$ are nonstochastic, shareholders choose ϕ such that no raids take place. Hence we have:

Proposition 4: If $\tilde{c}$ and $\tilde{v}$ are nonstochastic and $U(v) > 0$, then it is optimal for society to choose any ϕ such that $\phi > c$.

Proposition 4 shows that, in particular, it is optimal for society to set $\phi = \infty$ when $\tilde{c}$ and $\tilde{v}$ are nonstochastic. We shall now show that, under our assumptions, this result generalizes: $\phi = \infty$ is socially optimal even when $\tilde{c}$ and $\tilde{v}$ are both stochastic. Recall from Section 4 that an increase in ϕ has three

[18] In measuring the firm's return to society, we have ignored the welfare of managers. In general, the manager gets consumer surplus out of his job. In fact, Calvo (1977) has emphasized that giving an employee consumer surplus in his present job is necessary if the threat of removal is to provide an incentive for good work. In our model this surplus depends on ϕ: $U(q)$ Prob $[\min(\phi, \tilde{v} - q) \leq \tilde{c}]$. One justification for ignoring managerial welfare is that (1) shareholders remove managerial surplus by charging an entry fee; and (2) managers are sufficiently risk averse with respect to the possibility of losing their job that the maximum entry fee that they are prepared to pay is negligibly small. A second justification for ignoring managers is that managerial surplus is dissipated in the competition between prospective managers to get the managerial position. For example, managers may have to incur (opportunity) costs queueing for managerial positions, with the length of the queue adjusting until individuals are just indifferent between joining the queue and looking for a nonmanagerial position. It is important to note that, if managerial welfare is significant and is included in $R(\phi)$, then, while some of our specific results are altered, the principal conclusion reached in this section—that there is a deviation between the privately and socially optimal levels of dilution—stands.

effects: (1) it reduces the raider's tender price for a given value of q; (2) it increases the number of raids that take place for each value of q; (3) it increases *status quo* profit q. From the point of view of society, (2) and (3) are goods (the former because a raid only occurs if $\tilde{v} - \tilde{c} > p \geq q(\phi)$, i.e., if the firm's return under the raider is higher than under current management), while (1) is irrelevant since distributional effects are being ignored. Therefore, in contrast to shareholders, society will never wish to limit dilutions to increase the raider's tender price. In the Appendix we prove:

Proposition 5: $R(\phi)$ is nondecreasing in ϕ. In particular, $R(\phi)$ achieves a maximum at $\phi = \infty$. Furthermore, if Prob $[\tilde{v} - q_{\min} - \tilde{c} > 0] < 1$, a necessary condition for $R(\phi)$ to achieve a maximum also at $\phi' < \infty$ is that $q(\phi)$ achieves a maximum at $\phi' = \phi$.

As noted, Proposition 5 depends on the fact that from an efficiency point of view the division of the spoils between the raider and the shareholders is irrelevant. This is true, however, only because we have implicitly assumed that the amount invested by initial shareholders is independent of the rate of return which they earn, i.e., investment is interest-inelastic. We now show that if a low expected rate of return on investment leads shareholders to withhold investment funds, it is no longer true that society will desire to set $\phi = \infty$.

Suppose the profit function $f(a)$ of previous sections applies to a firm of unit scale. Assume that if an aggregate amount of investment or capital, K, is forthcoming, then it will be possible to set up exactly $g(K)$ firms of unit scale so that aggregate profits are $g(K)f(a)$, where a is the action of a typical firm. The function $g(K)$ is assumed to be twice differentiable and increasing with $g(0) = 0$, $g(1) = 1$, $\lim_{K\to\infty} g'(K) = 0$, $g'(0) = \infty$, and $g''(K) < 0$. The strict concavity of g indicates that there are decreasing returns to scale in establishing new firms.

Let $s > 0$ be the opportunity cost of capital, i.e., the social rate of return which can be earned from investing in the unincorporated sector. Then the social return from investing in the corporate sector is given by

$$g(K)R(\phi) - sK. \tag{11}$$

In a private ownership economy, however, K will be chosen by private investors not to maximize (11) but instead to maximize the private return on investment

$$g(K)r(\phi) - sK. \tag{12}$$

Note that we assume that the private rate of return from investing in the unincorporated sector equals the social rate of return, s.

Assume that the economy is decentralized so that the government can set legal limits on the dilution of property rights but cannot control investment directly.[19] Thus, the government attempts to maximize (11) subject to K's being chosen by private investors. If $r(\phi) \geq 0$, let $K(\phi)$ denote the unique K which maximizes $g(K)r(\phi) - sK$ so that $K(\phi)$ is the level of private investment which is forthcoming if the government sets the maximum amount of dilution equal to ϕ. The government's objective is to choose ϕ to maximize $g(K(\phi))R(\phi)$

[19] If the government controlled both K and ϕ directly, then from (11) ϕ would be chosen to maximize $R(\phi)$ and K would be chosen to maximize $g(K)R(\phi) - sK$.

$- sK(\phi)$.[20] We shall compare the resulting value of ϕ with what emerges when initial shareholders have control over dilution, in which case they choose ϕ and K simultaneously to maximize (12). In particular, this means that an optimal ϕ for shareholders is one which maximizes $r(\phi)$: our earlier analysis applies to the shareholders' choice of ϕ even when investment is interest-elastic.

It is clear that, in general, it will now no longer be optimal for the government to set $\phi = \infty$. Although permitting unlimited dilutions maximizes the social return *per unit* of investment, this entails diluting the property rights of initial shareholders by reducing the expected rate of return to them, which in turn reduces their incentive to invest. In Proposition 6 we show that while the government *will* wish to limit dilution when investment is interest-elastic, it will never wish to limit dilution to a greater extent than shareholders would: the socially optimal ϕ is no smaller than the privately optimal ϕ. The distortion caused by the existence of only a single raider implies that the government should encourage raids at least as much as the private sector would.

Proposition 6: Let ϕ_p be a maximizer of $r(\phi)$. Assume $r(\phi_p) > 0$. Then (i) there is a ϕ_s satisfying $\phi_s \geq \phi_p$ which maximizes $g(K(\phi))R(\phi) - sK(\phi)$. (ii) If ϕ_p is the unique maximizer of $r(\phi)$, then every ϕ_s which maximizes $g(K(\phi))R(\phi) - sK(\phi)$ satisfies $\phi_s \geq \phi_p$. (iii) A sufficient condition that ϕ_s can be chosen equal to ϕ_p is that ϕ_p be a maximizer of $R(\phi)$. (iv) If ϕ_p is a unique maximizer of $r(\phi)$, a sufficient condition that $\phi_s > \phi_p$ is that $r(\phi)$, $R(\phi)$ are differentiable functions of ϕ and that $R'(\phi_p) > 0$.

Parts (i)–(iii) of Proposition 6 are proved in the Appendix. Part (iv), which says that the socially optimal level of dilution exceeds the privately optimal level if $R'(\phi_p) > 0$, is sufficiently simple to establish here. We shall show that at $\phi = \phi_p$, the government's objective function $g(K(\phi))R(\phi) - sK(\phi)$ is increasing. The derivative of the objective function is $(g'R - s)K' + gR'$. Shareholders choose K to maximize $g(K)r(\phi) - sK$ for $\phi = \phi_p$, so that

$$g'(K)r(\phi_p) = s.$$

Differentiating, we get

$$g''(K)K'(\phi_p)r(\phi_p) + g'(K)r'(\phi_p) = 0.$$

But $\phi = \phi_p$ maximizes $r(\phi)$ so that $r'(\phi_p) = 0$ and hence $K'(\phi_p) = 0$. Thus, the derivative of the government's objective function becomes gR', which is positive because $R'(\phi_p) > 0$. Thus $\phi_s > \phi_p$, and the government will want to make raids easier than the private sector does.

Let us interpret the condition $R'(\phi_p) > 0$. It is clear from Proposition 5 that this condition will generally hold as long as ϕ_p is not a maximizer of $R(\phi)$, i.e., as long as there is a divergence between private and social optimality in the interest-*inelastic* investment case. But we know that there is generally such a divergence, for we showed in Section 4 that when $\tilde{c}$ is

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[20] It is easy to show that if the distribution function of (c,v), $F(c,v)$, is a continuous function (this implies that $\tilde{c}$ and $\tilde{v}$ are both stochastic), then an optimal choice of ϕ exists for the government. Under the same conditions, it can be shown that there is a solution to the shareholders' problem: $\max_\phi r(\phi)$. The only difficulty in proving these results is that $q(\phi)$ may not be continuous in ϕ. This results from our assumption that if the manager is indifferent between two values of q, he chooses the higher one. However, $q(\phi)$ will be upper semicontinuous in ϕ, and this is all that is needed in the proofs.

stochastic, shareholders will in general limit the disciplinary role of raids in their efforts to ensure a high tender price, i.e., $q(\phi)$ will not be maximized at $\phi = \phi_p$ (see, in particular, Proposition 3(2)). It follows from Proposition 5 that $R(\phi)$ is also not maximized at $\phi = \phi_p$.[21]

If there is no divergence between private and social optimality in the interest-inelastic investment case, then Proposition 6(iii) tells us that there will also be no divergence in the interest-elastic investment case. Therefore, Proposition 6 may be summarized as follows: if privately and socially optimal levels of dilution are equal in the inelastic investment case, then they will also be equal in the elastic investment case; however, if the socially optimal level of dilution exceeds the privately optimal level of dilution in the inelastic investment case, then it will also in general exceed it (and certainly never fall short of it) in the elastic investment case.

6. Competition and the costs involved in a takeover bid

■ In this section we analyze the consequences of permitting competing raids. Suppose there are no sunk costs of a raid and there is perfect competition among raiders in the sense that the $(\tilde{v},\tilde{c})$ pairs of different raiders are perfectly correlated. Then in the event $(\tilde{v},\tilde{c}) = (v,c)$, raiders will compete and drive the tender price up to $v - c$ as long as $\phi > c$. Thus, if $\phi > c$, the raider's profit is zero, and the social and private benefits from a raid are equal. It follows that shareholders acting in their own (private) interests will choose the socially optimal amount of dilution, which is $\phi = \infty$. Note that the presence of competition among raiders does not in any way alter our conclusion that dilution is essential in permitting takeovers to occur.

This argument assumes that shareholders can rely on enough competition to protect them. Before giving some theoretical arguments why initial shareholders may not be able to rely on the existence of competing raiders at the time of a takeover bid, it is worth mentioning some empirical evidence in favor of our conclusion that ϕ is, in general, set sufficiently low to restrict raids. First, there is a class of corporations called "closed end mutual funds" for which it is clear that $v - c > q$.[22] These corporations have had many 5-year periods where if $\phi = \infty$, a raider could have made a profit of at least 15 percent (assuming $c/v = 5$ percent) by taking over at a price of q and then liquidating the corporation's assets for v. Second, there are many other companies for which

[21] Although a divergence between private and social optimality generally exists in the interest-inelastic investment case, and hence, by the argument just given, also in the interest-elastic investment case, in some special cases the divergence is absent. We already know that one such case is when $\tilde{c}$ and $\tilde{v}$ are both nonstochastic (see Propositions 2(a) and 4). In fact, the divergence is absent even when $\tilde{v}$ is stochastic, as long as $\tilde{c}$ is nonstochastic. For, by Proposition 2(b), $\phi_p \cong c$ under these conditions (see Proposition 2(b) and footnote 14). Furthermore, Proposition 2(b) shows that raising ϕ above ϕ_p neither increases *status quo* profit nor enlarges the set of raiding states, and hence has no effect on $R(\phi)$. Therefore, $\phi_p \cong c$ is also a maximizer of $R(\phi)$. Finally, if $\tilde{c}$ is stochastic, but $\tilde{v}$ is nonstochastic, Proposition 3(1) shows that it will sometimes be the case that $\phi = \infty$ is privately optimal. Again the divergence is absent in such cases.

[22] These corporations' only assets are shares of other corporations. The value of a particular closed end fund's assets can be calculated by checking the value of the shares it holds; the value of its assets is called its "net asset value." These corporations often sell at substantial discounts, i.e., the price of 100 percent of the closed end fund is often 20 percent lower than its net asset value; see Sharpe and Sosin (1974), Malkiel (1977), or any Monday issue of the *Wall Street Journal*.

there is some evidence that $v - c > q$.[23] The fact that companies can persist for long periods, operating publicly at profit levels substantially below maximum profit, is stong evidence in favor of the hypothesis that shareholders do not allow large levels of dilution; for, if they did, then surely some entrepreneur would take over these "discounted" companies. Presumably shareholders did not set $\phi = \infty$ because they were afraid that their property rights would be massively diluted because of a lack of competition at the time of a takeover bid.

As we noted at the end of Section 4, there are several types of costs which arise in a takeover bid. Some of these, e.g., the financing, administrative, litigation, and reorganization costs, are consistent with competition among raiders, since they are incurred at the time of the raid. However, the presence of *ex ante* costs of research and information collection, which are sunk by the time the raid takes place, will tend to limit *ex post* competition. This is so because generally one raider will be first to discover what changes should be made in a corporation, and since other raiders do not have this knowledge, they will not be able to compete effectively with the informed raider.

In fact, perfect competition, *ex post*, among raiders is inconsistent with any of their earning a return on their "sunk," *ex ante*, information costs. Therefore, not only will initial shareholders be unable to rely on *ex post* competition, but *ex post* competition will not always be desirable.[24] In particular, a corporate charter which (a) requires a raider to make public his information and intentions (thus transmitting the information for free to other potential raiders and encouraging competition) or (b) requires that the raider keep his tender offer outstanding for some fixed minimum amount of time (so shareholders can wait for a better offer from a competing raider) may be undesirable for initial shareholders. Instead, shareholders may prefer to protect their property rights by limiting the dilution level, ϕ. The Williams Act now makes (a) and (b) the law in the United States. The *ex post* competition it generates may in the long run discourage many raids from ever taking place. The decrease in managerial efficiency which is a consequence of the government-induced decrease in the probability of raids will not in general be socially desirable.[25]

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[23] The *Value Line Survey*, published monthly by Arnold Bernhard Co., New York, lists companies which have stock market prices which are as much as 50 percent smaller than the value of liquid assets (like vault cash) less the value of debt liabilities. The *Value Line Survey* lists these companies as good investments because they are ripe for a takeover bid! Hearsay evidence indicates that these firms are rarely taken over. See Hindley (1970) for empirical work on a similar problem.

[24] This problem is similar to competition in product innovation. See Loury (1977) and Dasgupta and Stiglitz (1977) for an analysis of *ex ante* competition and its relationship to *ex post* monopoly power.

[25] Recently, incumbent directors have used the courts to delay a raider's takeover bid while they search for a "White Knight" (i.e., another firm which will offer a higher price for the target than did the initial raider, as well as a better position for incumbent directors in the reorganized firm). This appears to be good for both shareholders and the directors. However, the price paid by the "White Knight" is often only slightly higher than that offered by the original raider. Thus the *ex post* gain in the event of a raid may not be very large. The *ex ante* loss, however, may be enormous, because a potential raider (who considers incurring research and information costs) cannot possibly hope to earn a return if he knows that after he announces his takeover bid, the directors will find a competitor and freely give the competitor all the information that the original raider had to pay for. Thus, a regime conducive to the use of "White Knights" is not necessarily in shareholders' or society's best interest, since it may lower the long-run frequency of takeover bids. It gives incumbent directors an enormous threat to use against potential raiders.

7. Conclusions

■ The proper management of a common property is a public good to all the owners of the property. Our fundamental hypothesis is that there are significant costs in ensuring that directors/managers act in the interest of the owners. If one small shareholder devotes resources to improving management, then all shareholders benefit. This is the externality that the takeover bid mechanism attempts to "internalize."

The only way to create proper incentives for the production of a public good is to exclude nonpayers from enjoying the benefits of the public good.[26] A simple takeover bid does not exclude shareholders from benefiting from the improvements in their corporation. Any profit a raider can make through the price appreciation of the shares he purchases can also be made by a shareholder who free rides and does not sell his shares to the raider. Thus, a raider faces the same sort of externality that any shareholder would face if he devoted resources to improving management.

We are thus led to the conclusion that the initial shareholders who write the corporate charter will create some exclusionary device so that a raider can benefit from a takeover bid other than through the price appreciation of the shares he purchases. This can be accomplished by permitting the raider to treat the shares of those who have not tendered differently from the shares he owns. In practice, this is often achieved as follows. After a raid succeeds, the raider has voting control and can vote to liquidate or merge the corporation with a parent wholly owned by the raider. The raider sets the price of this merger or liquidation at a value he determines as "fair" to all shareholders. Of course, it is in his interest to underestimate the value of the corporation's assets, since in that case the parent company, which he wholly owns, gets the target corporation's assets at a discount.

The nontendering shareholders are in a minority after a successful raid and a merger or liquidation at an unfavorable price represents a dilution of their property rights. The law in some states in the United States (e.g., Delaware) sanctions this dilution, because it recognizes that the raider has no fiduciary responsibility to act in all of the shareholders' interests (Brudney and Chirelstein, 1978, p. 1367). However, it is essential to note that *ex ante*, the initial shareholders could have prevented this *ex post* dilution. For example, the corporate charter could require that outside appraisers, who would be approved by the *minority* shareholders, estimate the value of the corporation's assets before a merger or liquidation. Another *ex ante* method the initial shareholders could have used to prevent dilutions would require approval of say ⅔ of the *minority* shareholders in the event that a raider attempts to merge or liquidate the corporation after a successful raid.[27]

We have shown that if initial shareholders cannot rely on competition among raiders, then they will tend to choose low levels of dilution. Shareholders realize that by permitting more dilution they increase the threat of a raid, which

[26] The strengthening of class action laws so that the person bringing the class action can get a large portion of the improvements would be helpful in insuring good management of common property.

[27] These are the actual "antitakeover" provisions (Articles 8 and 9) of the Brunswick Corporation's (a corporation registered in Delaware) corporate charter.

is good because it makes *status quo* management more efficient, but they also lower the tender price they receive in the event of a raid, which is bad for them. From a social welfare point of view, permissible dilutions should be large, because this produces a large threat of a raid, which makes current management very efficient. The fact that this large threat is sometimes exercised and shareholders get a low price has no welfare consequence if investment is *interest-inelastic* because it then represents a mere redistribution of the gains from improvement from the shareholders to the raider. From this analysis we concluded that shareholders will tend to make takeover bids more difficult than they should be from a social welfare point of view. Where investment by initial shareholders is *interest-elastic* and where the low tender price received by shareholders has the socially undesirable consequence of reducing investment, the government will want to restrict raids to some extent. But the government will still want to encourage raids more than the private sector will.

What policy implications can be drawn from our analysis? This can be answered on two levels. On one level we can conclude that U.S. government policy on takeover bids from the Williams Act in 1968 to the present may have had certain undesirable consequences. The Act may have made raids more difficult, contrary to what our analysis suggests the government's objective should be. Alleged securities disclosure law violations are often used by current management to stall the raider in the courts and in general to increase the cost of takeover bids. Alleged antitrust violations are also used by *status quo* management to increase the costs of a raid via costly litigation and delays. *Status quo* management uses the shareholders' money to impose these litigation costs on raiders. Further, the disclosure provisions of the Williams Act, which force a raider to announce his intentions after buying 5 percent of the company, may be good in that competition from other raiders is encouraged, but may be bad in that shareholders in an attempt to free ride will compete against the raider.[28] Of course, to the extent that there are social benefits from preventing monopoly and increasing shareholder information via disclosure, the undesirable consequences of making takeover bids more difficult must be traded off against these benefits. A discussion of some of the benefits and costs of disclosure laws may be found in Grossman and Hart (1980) and Ross (1978).

On another level, however, the positive analysis of this paper has implications for the management of common property generally. In particular, we have developed a model which can predict how much deviation can persist between the *potential* benefits of collective action and the *actual* benefits of collective action. If the corporation is properly managed, shareholders get a benefit, which we have represented by $\max_{a \in A} f(a)$, from pooling their resources to take advantage of increasing returns to scale—this is a *potential* benefit. The *actual* benefit of collective action, which we have represented by $\max_{\phi} r(\phi)$, will be smaller because, in general, directors will not act in the shareholders' interest. We have suggested that the deviation depends on the amount of unpredictability in the benefits and cost of making takeover bids.

Throughout the paper we have used the stock market corporation as our example of common property. However, the fact that the Public Good is a public

[28] Recently the Securities and Exchange Commission has proposed amending the Williams Act, so that a raider must disclose intentions even *before* he buys up 5 percent of the shares (*Wall Street Journal*, February 1, 1979, p. 12). This will further exacerbate the free-rider problem.

good is true for all forms of common property and collective action. Further, in some forms of common property, for example local public goods, there are mechanisms which are analogous to takeover bids (see footnote 3). Our analysis of the resulting deviation between the actual benefits and the potential benefits of collective action is therefore likely to generalize beyond the stock market corporation.

Appendix

Proofs of theorems

□ **Proposition 1:** $q(\phi)$ is nondecreasing.

Proof (by contradiction): Suppose $\phi < \phi'$ and $q(\phi) \equiv q > q' \equiv q(\phi')$. By revealed preference and the assumption that a manager chooses the higher profit action if he is indifferent between two actions, we get

$$U(q) \text{ Prob } [\min (\phi, \tilde{v} - q) \leq \tilde{c}] \geq U(q') \text{ Prob } [\min (\phi, \tilde{v} - q') \leq \tilde{c}], \quad \text{(A1)}$$

and

$$U(q') \text{ Prob } [\min (\phi', \tilde{v} - q') \leq \tilde{c}] > U(q) \text{ Prob } [\min (\phi', \tilde{v} - q) \leq \tilde{c}]. \quad \text{(A2)}$$

Clearly, the strict inequality in (A2) implies that Prob $[\min (\phi', \tilde{v} - q') \leq \tilde{c}] > 0$. Hence Prob $[\min (\phi, \tilde{v} - q') \leq \tilde{c}] \geq$ Prob $[\min (\phi', \tilde{v} - q') \leq \tilde{c}] > 0$, since $\phi < \phi'$. Therefore, we may divide (A1) and (A2) to get

$$\frac{\text{Prob } [\min (\phi, \tilde{v} - q) \leq \tilde{c}]}{\text{Prob } [\min (\phi, \tilde{v} - q') \leq \tilde{c}]} \geq \frac{U(q')}{U(q)} > \frac{\text{Prob } [\min (\phi', \tilde{v} - q) \leq \tilde{c}]}{\text{Prob } [\min (\phi', \tilde{v} - q') \leq \tilde{c}]}. \quad \text{(A3)}$$

Define $A =$ Prob $[\min (\phi, \tilde{v} - q) \leq \tilde{c}]$, $A' =$ Prob $[\min (\phi', \tilde{v} - q) \leq \tilde{c}]$, $B =$ Prob $[\min (\phi, \tilde{v} - q') \leq \tilde{c}]$, $B' =$ Prob $[\min (\phi', \tilde{v} - q') \leq \tilde{c}]$, and $\Delta_1 = A - A'$, $\Delta_2 = B - B'$. Then we can rewrite (A3) as

$$\frac{A}{B} > \frac{A'}{B'} = \frac{A - \Delta_1}{B - \Delta_2}. \quad \text{(A4)}$$

Now $A \geq B$, since $q > q'$. Also

$$\begin{aligned} \Delta_1 &= \text{Prob } [\min (\phi', \tilde{v} - q) > \tilde{c} \quad \text{and} \quad \min (\phi, \tilde{v} - q) \leq \tilde{c}] \\ &= \text{Prob } [\phi \leq \tilde{c} < \phi' \quad \text{and} \quad \tilde{v} - q > \tilde{c}] \leq \text{Prob } [\phi \leq \tilde{c} < \phi' \\ &\quad \text{and} \quad \tilde{v} - q' > \tilde{c}] \\ &= \text{Prob } [\min (\phi', \tilde{v} - q') > \tilde{c} \quad \text{and} \quad \min (\phi, \tilde{v} - q') \leq \tilde{c}] = \Delta_2. \end{aligned}$$

But, by (A4), $AB - A\Delta_2 > AB - B\Delta_1$, i.e., $B\Delta_1 > A\Delta_2$, which is impossible if $A \geq B$, $0 \leq \Delta_1 \leq \Delta_2$. *Q.E.D.*

□ **Proposition 3:** For statement of Proposition 3, see Section 4 of the text.

Proof: Let ϕ^0 be an optimal value of ϕ, and let $q^0 = q(\phi^0)$. Either $v - \phi^0 \leq q^0$ or $v - \phi^0 > q^0$. In the first case, the tender price will equal q^0 for all $\phi \geq \phi^0$. Thus $\pi(\phi, q^0)$ is constant for $\phi \geq \phi^0$, and hence the *status quo* profit $q(\phi)$ and $r(\phi)$ are constant for $\phi \geq \phi^0$.

On the other hand, if $v - \phi^0 > q^0$, then q^0 must maximize $U(q)$. Suppose there is a q' such that $U(q') > U(q^0)$. Then

$$U(q') \text{ Prob } [\min(\phi^0, v - q') \leqq \tilde{c}] > U(q^0) \text{ Prob } [\min(\phi^0, v - q') \leqq \tilde{c}]$$

$$\geqq U(q^0) \text{ Prob } [\phi^0 \leqq \tilde{c}] = U(q^0) \text{ Prob } [\min(\phi^0, v - q^0) \leqq \tilde{c}],$$

which contradicts the fact that $q^0 = q(\phi^0)$. Thus $q^0 \equiv q^*$. Then (9) is equivalent to the shareholder objective given in (8) in Section 4. *Q.E.D.*

□ **Proposition 5:** For statement of Proposition 5, see Section 5 of the text.

Proof of Proposition 5: We show first that R is nondecreasing in ϕ *for a given value of* q. We then show that R is nondecreasing in q *for a given value of* ϕ. These two facts combined with Proposition 1 will prove that R is nondecreasing in ϕ.

(a) For a given q, R is nondecreasing in ϕ. Let $S(\phi) = \{(c,v) | c < \min(\phi, v - q)\}$ and $Q(\phi) = \{(c,v) | c \geq \min(\phi, v - q)\}$. Assume $\phi' > \phi$; then noting that $Q(\phi) \supset Q(\phi')$ and $S(\phi') \supset S(\phi)$:

$$R(\phi') \equiv \int_{Q(\phi')} q dF(c,v) + \int_{S(\phi')} (v - c) dF(c,v);$$

$$R(\phi') \equiv \int_{Q(\phi)} q dF(c,v) - \int_{Q(\phi) \cap S(\phi')} q dF(c,v) + \int_{S(\phi)} (v - c) dF(c,v)$$
$$+ \int_{Q(\phi) \cap S(\phi')} (v - c) dF(c,v);$$

$$R(\phi') \equiv R(\phi) + \int_{Q(\phi) \cap S(\phi')} (v - c - q) dF(c,v) \geq R(\phi). \tag{A5}$$

(b) For a given ϕ, R is nondecreasing in q. Let $s(q) = \{(c,v) | c < \min(\phi, v - q)\}$ and $f(q) = \{(c,v) | c \geq \min(\phi, v - q)\}$. Assume $q' > q$. Then noting that $f(q') \supset f(q)$ and $s(q) \supset s(q')$:

$$R(q') = \int_{f(q')} q' dF(c,v) + \int_{s(q')} (v - c) d(c,v);$$

$$R(q') = \int_{f(q)} q' dF(c,v) + \int_{s(q) \cap f(q')} q' dF(c,v) + \int_{s(q)} (v - c) dF(c,v)$$
$$- \int_{s(q) \cap f(q')} (v - c) dF(c,v);$$

$$R(q') = \int_{f(q)} q' dF(c,v) + \int_{s(q)} (v - c) dF(c,v)$$
$$+ \int_{s(q) \cap f(q')} [q' - (v - c)] dF(c,v). \tag{A6}$$

Note that if $(c,v) \in s(q) \cap f(q')$, then $c < \min(\phi, v - q)$ and hence $c < \phi$; but $c \geq \min(\phi, v - q')$ since $(c,v) \in f(q')$; therefore $c \geq v - q'$. Thus $(c,v) \in s(q) \cap f(q')$ implies $q' - (v - c) \geq 0$. Whence from (A6)

$$R(q') \geq \int_{f(q)} q dF(c,v) + \int_{s(q)} (v - c) dF(c,v) = R(q).$$

To prove the last part of Proposition 5, note that $R(q') > R(q)$ as long as Prob $f(q) > 0$. This is guaranteed by the condition Prob $[\tilde{v} - q_{\min} - \tilde{c} > 0] < 1$.

☐ **Proposition 6:** For statement of Proposition 6, see Section 6.

Proof of Proposition 6: By contradiction. Suppose that ϕ_s is any optimal dilution for the government and that $\phi_s < \phi_p$. Then, by definition of ϕ_s, $g(K_s)R(\phi_s) - sK_s \geq g(K_p)R(\phi_p) - sK_p$, where $K_s \equiv K(\phi_s)$ and $K_p \equiv K(\phi_p)$. Therefore,

$$s(K_p - K_s) \geq g(K_p)R(\phi_p) - g(K_s)R(\phi_s) \geq g(K_p)R(\phi_p) - g(K_s)R(\phi_p), \quad \text{(A7)}$$

where the last inequality follows from Proposition 5 and $\phi_p > \phi_s$. Hence

$$s(K_p - K_s) \geq [g(K_p) - g(K_s)]R(\phi_p) \geq [g(K_p) - g(K_s)]r(\phi_p), \quad \text{(A8)}$$

where the last inequality follows from the fact that $R(\phi) \geq r(\phi)$ (i.e., the social return is always as large as the private return) as can be seen from equations (8) and (10). Now note that there are two possibilities:

(A) $r(\phi_s) < r(\phi_p)$ or (B) $r(\phi_s) = r(\phi_p)$, since ϕ_p is a maximum of $r(\phi)$.

(A) If $r(\phi_s) < r(\phi_p)$, then $K_s < K_p$, since investment has a higher nominal return under $r(\phi_p)$ for a given K. Hence $g(K_p)r(\phi_p) - sK_p > g(K_s)r(\phi_p) - sK_s$ or equivalently $[g(K_p) - g(K_s)]r(\phi_p) > s(K_p - K_s)$, which contradicts (A8).

(B) If $r(\phi_p) = r(\phi_s)$, then, by the definition of $K(\phi)$ in the statement of the proposition, $K_s = K_p$. Since $R(\phi)$ is nondecreasing, $R(\phi_p) \geq R(\phi_s)$. Therefore, ϕ_p would yield a social return at least as high as ϕ_s. That is, ϕ_p is an optimal dilution for the government to set and this contradicts the assumption that every optimal dilution for the government is strictly smaller than the private dilution ϕ_p.

This proves the first part of the proposition.

The proof of the second part follows from the fact that if there is *any* $\phi_s < \phi_p$, then by the argument in the above proof either (A) or (B) above obtains. But (A) is impossible by the same argument as above. (B) is impossible because $r(\phi)$ has a unique maximizer. Part (iii) follows from the fact that if ϕ_p is a maximizer of $R(\phi)$ and $\phi_s > \phi_p$, then $r(\phi_s) \leq r(\phi_p)$ and $R(\phi_p) = R(\phi_s)$. Hence, reducing ϕ_s to ϕ_p can only increase the private return on investment, which will improve things. Finally, Part (iv) was proved in the text. *Q.E.D.*

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