Evidence on Common Features and Business Cycle Synchronization in Mercosur

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Abstract

The aim of this work is to analyze the business cycles of the Mercosur’s member countries in order to investigate their degree of synchronization. The estimation used is the Beveridge-Nelson-Stock-Watson multivariate trend-cycle decomposition, taking into account the presence of common features such as common trend and common cycle. Once the business cycles are estimated, their degree of synchronization is analyzed by means of coherence and phase in frequency domain. Despite the evidence of common features, the results suggest that the business cycles are not synchronized. This may generate an enormous difficulty to intensify the agreements into Mercosur.

Key-words: Mercosur, business cycles, trend-cycle decomposition, common features, spectral analysis.

Jel Codes: C32, E32, F02, F23.

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1 Introduction

The design of economic blocks, such as the European Union and the Mercosur, has the purpose to amplify society welfare through the unification of economic policies and commercial agreements. According to Backus and Kehoe (1992) and Chistodoulakis and Dimelis (1995), the success of these policies depends on the similarities of the business cycles of the member states. A business cycle is a periodic but irregular up-and-down movement in economic activity, measured by fluctuations in real GDP and other macroeconomic variables. However, in compliance with Lucas (1977), many authors focus the analysis on GDP, defining business cycles as the difference between the actual GDP and its long-run trend.

The aim of this paper is to analyze the business cycles of the Mercosur member countries. The Mercosur or southern common market is a regional trade agreement created in 1991 by the treaty of Asunción. Its members are: Argentina, Brazil, Paraguay, Uruguay and Venezuela, welcomed as the fifth member in 2006. These countries differ in their institutions, economic policies and industrial structures, creating an enormous internal asymmetry in Mercosur (Flores, 2005). Although the block was created in 1991, we will analyze a broader period, from 1951 to 2003. Therefore, if we find evidence in favor of similarity we can safely assume that it cannot be attributed only to Mercosur\(^1\). In fact, an inverse causality is investigated: if the similarities among the countries lead to commercial integration.

In the empirical literature, there is no consensus on how to estimate the trend-cycle components of economic time series and how to analyze the so-called co-movements\(^2\) in their business cycles. In the past decades a rich debate on the abilities of different statistical methods to decompose time series in long-run and short-run fluctuations has taken place (Baxter and King, 1995; Guay and St-Amant, 1996). The Hodrick-Prescott (HP) filter and the linear detrending are the usual univariate methodologies applied. However, these methodologies do not take in account the existence of common features among the economic series. In addition to that, as shown by Harvey and Jaeger (1993), the HP filter can induce spurious cyclicality when applied to integrated data. Therefore, in order to obtain a measure of the business cycles, we employ the Beveridge-Nelson-Stock-Watson (BNSW) multivariate trend-cycle decomposition, considering the occurrence of cointegration and serial correlation common feature among the variables.

It is worth noting that the existence of a common cyclical feature neither implies nor is implied by the existence of similar business cycles, as observed by Quah (Engle and Kozicki, 1993-comment) and Cubadda (1999). Therefore, to investigate the degree of synchronization or co-movement of their business cycles an extra effort is necessary. In this sense, many authors have used the linear correlations across cycles; however, this analysis gives a static measure of the co-movements since it is not a simultaneous analysis of the persistence of co-movement (Engle and Kozick, 1993). To avoid this critique, the measures of coherence

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\(^1\)Besides, there is not a consensus that Mercosur led to an increase in the flow of commerce among its integrated parts.

\(^2\)Two countries present co-movements when their real GDP expansions and downturns are simultaneous.
and phase in frequency domain are applied in order to investigate how synchronized the business cycles are (Wang, 2003). These frequency domain techniques constitute a straightforward way to represent economic cycles, once they provide information for all frequencies.

Finally, the results indicate the existence of common trends and common cycles among the economies studied. Thus, we confirm the need to use a multivariate approach, which is our first contribution. Frequency domain results identified synchronization in two sub-groups: Argentina-Venezuela and Brazil-Paraguay. Thus, in general, the countries of the Mercosur are not synchronized.

Besides this introduction, the paper is organized as following. Section 2 presents the econometric methodology. Section 3 reports the estimation and test and section 4 the results. Finally, the conclusions are summarized in the last section.

2 Econometric Model

Common features may be seen as restrictions over the dynamics of the countries and, consequently, over the dynamics of their business cycles. While cointegration refers to long-run relationships, common cyclical restrictions refer to short-run dynamics. Engle and Kozicki (1993) and Vahid and Engle (1993) proposed the serial correlation common feature (SCCF) as a measure of common cyclical feature in the short-run, which is applied in many empirical works. For example, Gouriéroux and Peaucelle (1993) analyzed some questions on purchase power parity; Campbell and Mankiw (1990) found a common cycle between consumption and income for most G-7 countries; Engle and Kozicki (1993) found common international cycles in GNP data for OECD countries; Engle and Issler (2001) found common cycles among sectorial output for US; and Candelon and Hécq (2000) tested the Okun’s law.

To implement the BNSW decomposition, taking into account the common features restrictions, a VAR model is estimated and the existence of long-run and short-run common dynamics is tested. Consider a Gaussian Vector Autoregression of finite order $p$, VAR($p$):

$$ y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \ldots + \phi_p y_{t-p} + \varepsilon_t $$

(1)

where $y_t$ is a vector of $n$ first order integrated series, $I(1)$, and $\phi_i$, $i = 1, \ldots, p$ are matrices of dimension $n \times n$ and $\varepsilon_t \sim \text{Normal}(0, \Omega)$, $E(\varepsilon_t) = 0$ and $E(\varepsilon_t \varepsilon_{t^\prime}) = \{\Omega, \; se \; t = t^\prime$ and $0_{n \times n}, \; se \; t \neq t^\prime; \; where \; \Omega$ is no singular}. The model (1) can be written equivalently as:

$$ \Pi(L) y_t = \varepsilon_t $$

(2)

where $\Pi(L) = I_n - \sum_{i=1}^{p} \phi_i L^i$ and $L$ represents the lag operator. $\Pi(1) = I_n - \sum_{i=1}^{p} \phi_i$ when $L = 1$. 

3
2.1 Long run restrictions (Cointegration)

The following assumptions are assumed:

**Assumption 1**: The \((n \times n)\) matrix \(\Pi(1)\) satisfies:

1. Rank \((\Pi(1)) = r, 0 < r < n, such that \Pi(1) can be expressed as \(\Pi(1) = -\alpha\beta'\), where \(\alpha\) and \(\beta\) are \((n \times r)\) matrices with full column rank \(r\).
2. The characteristic equation \(|\Pi(L)| = 0\) has \(n - r\) roots equal to 1 and all other are outside the unit circle.

Assumption 1 implies that \(y_t\) is cointegrated of order \((1, 1)\). The elements of \(\alpha\) are the adjustment coefficients and the columns of \(\beta\) span the cointegration space. Decompounding the polynomial matrix \(\Pi(L) = \Pi(1) L + \Pi^*(L) \Delta\), where \(\Delta \equiv (1 - L)\) is the difference operator, a Vector Error Correction (VEC) model is obtained:

\[
\Delta y_t = \alpha\beta' y_{t-1} + \sum_{j=1}^{p-1} \Gamma_j \Delta y_{t-j} + \varepsilon_t
\]

where \(\alpha\beta' = -\Pi(1), \Gamma_j = -\sum_{k=j+1}^p \phi_k\ (j = 1, ..., p - 1)\) and \(\Gamma_0 = I_n\).

2.2 Common cycles restrictions

The VAR(p) model can have short-run restrictions as shown by Vahid and Engel (1993).

**Definition 1** Serial Correlation Common Feature holds in (3) if there is a \((n \times s)\) matrix \(\tilde{\beta}\) of rank \(s\), whose column span the cofeature space, such as \(\tilde{\beta}'\Delta y_t = \tilde{\beta}'\varepsilon_t\), where \(\tilde{\beta}'\varepsilon_t\) is a \(s\)-dimensional vector that constitute an innovation process with respect to all information prior to period \(t\).

Consequently, the SCCF restrictions occur if there is a cofeature matrix \(\tilde{\beta}\) that satisfies the following assumption:

**Assumption 2** \(\tilde{\beta}' \Gamma_j = 0_{s \times n}, \quad j = 1, ..., p - 1\)

**Assumption 3** \(\tilde{\beta}' \alpha\beta' = 0_{s \times n}\)
2.3 Trend-Cycle decomposition

The BNSW trend-cycle decomposition can be introduced by means of the Wold representation of the stationary vector $\Delta y_t$ given by:

$$\Delta y_t = C(L)\varepsilon_t$$

(4)

where $C(L) = \sum_{i=0}^{\infty} C_i L^i$ is polynomial matrix in the lag operator, $C_0 = I_n$ and $\sum_{i=1}^{\infty} j |C_j| < \infty$. Using the following polynomial factorization $C(L) = C(1) + \Delta C^*(L)$, it is possible to decompose $\Delta y_t$ such that:

$$\Delta y_t = C(1) \varepsilon_t + \Delta C^*(L) \varepsilon_t$$

(5)

where $C^*_i = \sum_{j>0}^{\infty} (-C_j)$, $i \geq 0$, and $C^*_0 = I_n - C(1)$. Ignoring the initial value $y_0$ and integrating both sides of (5), we obtain:

$$y_t = C(1) \sum_{j=1}^{T} \varepsilon_t + C^*(L) \varepsilon_t = \tau_t + c_t$$

(6)

Equation (6) represents the BNSW decomposition where $y_t$ is decomposed in an “$n$” random walk process named “stochastic trend” and an “$n$” stationary process named “cycles”. Thus, $\tau_t = C(1) \sum_{j=1}^{T} \varepsilon_t$ and $c_t = C^*(L) \varepsilon_t$ represent the trend and cycle components, respectively. Assuming that long-run restrictions exist, then $r$ cointegration vectors exist ($r < n$). These vectors eliminate the trend component which implies that $\beta'C(1) = 0$. Thus, $C(1)$ has dimension $n - r$, which means that there are $n - r$ common trends. Analogously, assuming short-run restrictions, there are $s$ cofeature vectors that eliminate the cycles, $\beta' C^*(L) = 0$, which implies that $C^*(L)$ has dimension $n - s$, which is the number of common cycles. It is worth noting that $r + s \leq n$ and the cointegration and cofeatures vectors are linearly independent (Vahid and Engle, 1993). In order to obtain the common trends, it is necessary (and sufficient) to multiply equation (6) by $\tilde{\beta}'$, such that

$$\tilde{\beta}' y_t = \tilde{\beta}' C(1) \sum_{j=1}^{T} \varepsilon_t = \tilde{\beta}' \tau_t$$

This linear combination does not contain cycles because the cofeature vectors eliminate them. In the same way, to get the common cycles it is necessary to multiply equation (6) by $\beta'$, and so

$$\beta' y_t = \beta' C^*(L) \varepsilon_t = \beta' c_t$$

This linear combination doesn’t contain the stochastic trend because the cointegration vectors eliminate the trend component. A special case emerges when $r + s = n$. In this case, it is extremely easy to estimate the trend and cycle components of $y_t$. Once $\tilde{\beta}'$ and $\beta'$ are linearly independent matrices, it is possible to build a matrix $A$, such as $A_{n \times n} = (\tilde{\beta}', \beta')'$ has full rank and, therefore, is invertible. Notice that, the inverse matrix
can be partitioned as $A^{-1} = (\tilde{\beta}^- \quad \beta^-)$ and the trend and cycle components can be obtained as follows:

$$y_t = A^{-1} Ay_t = \tilde{\beta}^- (\tilde{\beta}' y_t) + \beta^- (\beta' y_t) = \tau_t + c_t$$

(7)

This implies that $\tau_t = \tilde{\beta}^- \tilde{\beta}' y_t$ and $c_t = \beta^- \beta' y_t$. Therefore, trend and cycle are linear combinations of $y_t$. Note that $\tau_t$ is generated by a linear combination of $y_t$ using the cofeature vectors, containing the long-run component (because $\tilde{\beta}' y_t$ is a random walk component) while $c_t$ is generated by a linear combination of $y_t$ using the cointegration vectors, containing the short-run component (because $\beta' y_t$ is $I(0)$ and serially correlated).

2.4 Estimation and testing

Considering the SCCF and the cointegration restrictions, we can rewrite the vector error correction as a model of reduced-rank structure. In (3) we define a vector $X_{t-1} = [y_{t-1}, \beta' \Delta y_{t-1}, \ldots, \Delta y_{t-p+1}]'$ of dimension $(n(p - 1) + r) \times 1$ and a $n \times (n(p - 1) + r)$ matrix $\Phi = [\alpha, \Gamma_1, \ldots, \Gamma_{p-1}]$. Therefore (3) is written as:

$$\Delta y_t = \Phi X_{t-1} + \varepsilon_t$$

(8)

If assumptions (1), (2) and (3) hold, then the matrices $\Gamma_i, i = 1, \ldots, p - 1$ are all of reduced rank $(n - s)$ and they can be written as $\Phi = A[\Psi_0, \Psi_1, \ldots, \Psi_{p-1}] = A\Psi$, where $A$ is $n \times (n - s)$ full column rank matrix and $\Psi$ has dimension $(n - s) \times (n(p - 1) + r)$ and $\tilde{\beta}' A\Psi = 0$, that is, $\tilde{\beta} \in sp(A_\perp)$ where $A_\perp$ is the orthogonal complement of $A$. Therefore, let $A = \tilde{\beta}_\perp$. Hence the model (8) can be expressed as a dynamic factor model with $n - s$ factor, given by $\Psi X_{t-1}$, which are linear combinations of the right hand side variables in (3).

$$\Delta y_t = \tilde{\beta}_\perp (\Psi_0, \Psi_1, \ldots, \Psi_{p-1}) X_{t-1} + \varepsilon_t$$

(9)

$$= \tilde{\beta}_\perp \Psi X_{t-1} + \varepsilon_t$$

(10)

To estimate the coefficient matrices $\tilde{\beta}_\perp$ and $\Psi$ in the reduced rank model (10) we use the Anderson’s (1951) procedure (see additionally Anderson, 1988, Johansen, 1995). This procedure is based in a canonical analysis, which is a special case of a reduced-rank regression. More specifically, the maximum-likelihood estimation of the parameters of the reduced-rank regression model may result a problem of canonical analysis. Therefore, we can use the expression $\text{CanCorr}\{X_t, Z_t|W_t\}$ that denotes the partial canonical correlations between $X_t$ and $Z_t$: both sets concentrate out the effect of $W_t$ that allows us to obtain canonical correlation, represented by the eigenvalues $\hat{\lambda}_1 > \hat{\lambda}_2 > \hat{\lambda}_3 \ldots \ldots > \hat{\lambda}_n$.

The orthogonal complement of the $n \times s$ matrix $B$, $n > s$ and rank($B$) = $s$, is the $n \times (n - s)$ matrix $B_\perp$ such that $B'_\perp B = 0$ and rank($B : B_\perp$) = $n$. Hence, $B_\perp$ spans the null space of $B$ and $B'$ spans the left null space of $B_\perp$. The space is denoted by $sp$.

This estimation is referred as Full Information Maximum Likelihood - FIML.
The Johansen test statistic is based on canonical correlation. In model (3) we can use the expression
\[ \text{CanCorr}\{\Delta y_t, y_{t-1}|W_t\} \]
where \( W_t = [\Delta y_{t-1}, \Delta y_{t-2}, \ldots, \Delta y_{t+p-1}] \) that summarizes the reduced-rank regression procedure used in the Johansen approach. It means that one extracts the canonical correlations between \( \Delta y_t \) and \( y_{t-1} \); both sets concentrated out the effect of lags of \( W_t \).

Moreover, we could also use a canonical correlation approach to determine the rank of the common features space due to SCCF restrictions. It is a test for the existence of cofeatures in the form of linear combinations of the variables in first differences, which are white noise (i.e., \( \tilde{\beta}' \Delta y_t = \tilde{\beta}' \varepsilon_t \) where \( \tilde{\beta}' \varepsilon_t \) is a white noise). Based on Tiao and Tsay (1985), Vahid and Engle (1993) proposed a sequential test for SCCF, assuming that the rank of \( \beta \) is known. The sequence of hypotheses to be tested are: \( H_0: \text{rank} (\tilde{\beta}) \geq s \) against \( H_a: \text{rank} (\tilde{\beta}) < s \), (see Lütkepohl, 1991; Velu et al, 1986) starting with \( s = 1 \) against the alternative model with \( s = 0 \) (there is no common cycle). If the null hypotheses is not rejected, we implement the test for \( s = 2 \), and so on.

In the VEC model the significance of the \( s \) smallest eigenvalues is determined through the following statistic:
\[
\xi_s = -T \sum_{i=1}^{s} \ln (1 - \lambda_i^2) \sim \chi^2_v, \quad s = 1, \ldots, n - r
\]  
\[
\lambda_1 < \lambda_2, \ldots, < \lambda_{n-r} < 1, \text{ with } v = s \left[ n(p-1) + r \right] - s(n-s) \text{ degrees of freedom}^5, \text{ where } n \text{ is the dimension of the system and } p \text{ the lag order of the VAR model. Suppose that the statistical test (11) has found } s \text{ independent linear combinations of the elements of } \Delta y_t \text{ unpredictable. This implies that there is an } n \times s \text{ matrix } \tilde{\beta} \text{ of full rank } s \text{ with } s \text{ eigenvectors associated with the } s \text{ smallest eigenvalues. Reinsel and Ahn (1992) propose a correction in statistic (11) in small samples } \xi_s \text{ corr} = \frac{T - n(p-1) - r}{T} \xi_s, \text{ where } T \text{ is the real number of observations after the deduction of initial points in regressions containing lags.}

3 Empirical results

3.1 Database

The database used was extracted from Penn World Table\(^6\), corresponding to Real GDP per capita series of Mercosur countries. The frequency is annual, ranging from 1951 to 2003. We consider the model
\[ Y_t = T_t C_t, \]
where \( C_t \) is the cycle and \( T_t \) the trend of the series. Define \( y_t = \log Y_t, \tau_t \equiv \log T_t \) and \( c_t \equiv \log C_t \). Then, \( y_t = \tau_t + c_t \). The Figure I reports the GDP expressed in log terms. After 1975,  

\(^5\)For \( p = 1 \) the degrees of freedom is \( (r+s)^2 \). Notice in the model \( \Delta y_t = \alpha \beta' y_{t-1} + \varepsilon_t \), the \( \text{rank}(\alpha \beta') = \hat{r} = n - s - r \), hence \( \nu = (n - \hat{r}) \times (np - \hat{r}) = (n - (n-s-r))^2 = (r+s)^2 \).

\(^6\)Heston, Robert Summers and Bettina Aten, Penn World Table Version 6.1, Center for International Comparisons at the University of Pennsylvania (CICUP), October 2002. Real GDP per capita (Constant Prices: Chain series) http : //pwt.econ.upenn.edu/php_site/pwt_index.php

7
in general, the series become closer - a behavior that may be generated by a common trend. Figure II displays the growth rates of real gross domestic products. It is possible to see the recession in Argentina, in 1989-1990.

Figure I. Real GDP (in log) per capita series of Mercosur countries (1951-2003)

Figure II. The growth rates of the real GDP (in log) per capita series of Mercosur countries (1951-2003)
3.2 Common Features results

To implement the methodology previously stated, a hierarchical procedure is followed to estimate the parameter of the model (see, Vahid and Engle, 1993). First, the VAR order, $p$, is estimated via information criteria: Akaike (AIC), Schwarz (SC) and Hannan-Quinn (HQ) (Lütkepohl, 1993). After that, we identify the number of long-run restrictions, $r$, through Johansen cointegration test. Then the number of short-run restrictions due to SCCF, $s$, is estimated using $\chi^2$ test. Finally, the matricial parameters are estimated in model (3) using the FIML procedure (Vahid and Issler (1993)).

Since BNSW decomposition assumes that the series are $I(1)$, we begin the analysis using the augmented Dickey-Fuller (ADF), Phillips-Perron (PP) and DF-GLS unit root tests. In all cases, the null hypothesis is the presence of an unit root. The results for all countries are reported in Table I. The three tests do not reject the unit root null hypothesis, at 5% level of significance, for all countries\(^7\).

<table>
<thead>
<tr>
<th>Country</th>
<th>ADF Statistic</th>
<th>Critical value</th>
<th>PP Statistic</th>
<th>Critical value</th>
<th>DF-GLS Statistic</th>
<th>Critical value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina</td>
<td>-1.8691</td>
<td>-3.4986</td>
<td>-1.9276</td>
<td>-3.4986</td>
<td>-1.9447</td>
<td>-3.1836</td>
</tr>
<tr>
<td>Brazil</td>
<td>-0.2404</td>
<td>-3.4986</td>
<td>-0.4308</td>
<td>-3.4986</td>
<td>-0.5998</td>
<td>-3.1868</td>
</tr>
<tr>
<td>Paraguay</td>
<td>-0.5757</td>
<td>-3.5004</td>
<td>-0.7392</td>
<td>-3.4986</td>
<td>-1.0805</td>
<td>-3.1868</td>
</tr>
<tr>
<td>Venezuela</td>
<td>-1.0972</td>
<td>-3.4986</td>
<td>-1.0780</td>
<td>-3.4986</td>
<td>-0.8094</td>
<td>-3.1836</td>
</tr>
</tbody>
</table>

To estimate the order of the VAR, the AIC, HQ and SC information criteria are used. Table II shows the results for $p \in \{1, 2, 3, 4, 5\}$. As the data are annual we consider that an upper bound of 5 lags is sufficient. We observe that the three criteria suggest $p = 1$, indicating a VAR(1) model. Although the $p$ selected by the criteria was one, to check the robustness of the results, we additionally test the model for $p = 2$ and $p = 3$.

\(^7\)In the case of ADF and DF-GLS tests, the choice of lags of the dependent variable in the right side of the test equation is based on the Schwarz criterion. In the PP test we use the nucleus of Bartlett and the window of Newey-West. All test equations have a constant and a linear trend. In any case, the results are robust to exclude of the linear trend.
Table II. Identification of the VAR order

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Lag 1</th>
<th>Lag 2</th>
<th>Lag 3</th>
<th>Lag 4</th>
<th>Lag 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>HQ</td>
<td>-17.98521*</td>
<td>-17.51349</td>
<td>-17.10059</td>
<td>-16.77532</td>
<td>-16.38107</td>
</tr>
</tbody>
</table>

Note: * indicates the lag suggested by information criteria

Considering $p = 1$, $p = 2$ and $p = 3$ the usual diagnostic tests are applied in order to verify if these specifications are suitable. For $p = 1$ and $p = 2$ the LM test does not indicate the presence of serial autocorrelation in the residuals, at 5% level of significance. On the other hand, for $p = 3$ the opposite result is obtained. The White heteroskedasticity test (without cross terms) does not find evidence of heteroskedasticity, at 5% level of significance, for $p = 1, 2, 3$. The Jarque-Bera normality test does not reject the null hypothesis of normal distribution of residuals only for $p = 1$, at 5% level of significance. Consequently, the best specification is obtained when $p = 1$.

To test if the series are cointegrated, the Johansen’s (1988) procedure is used. For each value of $p$ we consider two cases. First, we introduce a constant in the cointegration equation and, after, we add a linear trend. As the linear trend in the cointegration equation is significant (at 5% level), we consider this case in all subsequent analysis. In Table III the results for the cointegration test are shown for the case with constant and trend in the cointegration vector. The trace test indicates $r = 2$ for $p = 1, 2, 3$ while the maximum eigenvalue test suggest $r = 2$ for $p = 1, 2$ and $r = 1$ for $p = 3$. Thus, for $p = 1, 2$ both tests generate the same result and for $p = 3$ we opt for $r = 2$.

Table IV shows the SCCF test for $p = 1, 2$ using the correction given by Reinsel and Ahn (1992). For $p = 1$ the test indicates that $s = 4$, at 5% level of significance, but as the p-value is close to 5% we may assume $s = 3$ without trouble (see Table IV (a)). For $p = 2, 3$ the test indicates $s = 3$ (see Table IV (b) e (c)). Therefore, in all cases $s + r = n$. These results confirm the necessity to use a multivariate approach to identify the business cycles. In the next section we analyze the economic cycles obtained from the BNSW decomposition, considering the common cycles and the common trend restrictions. Once $s + r = n$, it is possible to find the trend and cycle components as shown above. Figure III shows the common cycles for each value of $p$. We observe that for $p = 1, 2$ common cycles are very similar.

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8 The null hypothesis of the LM test is the absence of serial correlation until the lag $h$. We consider $h$ from 1 to 5.
9 The normality test uses the orthogonalization of Cholesky.
Table III. Johansen’s cointegration test

<table>
<thead>
<tr>
<th>Null hypothesis</th>
<th>Statistical Test</th>
<th>Statistical Test</th>
<th>p-value</th>
<th>Statistical Test</th>
<th>Statistical Test</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Trace Test</td>
<td>Maximum Eigenvalue Test</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r = 0$</td>
<td>$130.023^*$</td>
<td>$88.803$</td>
<td>0.0000</td>
<td>$53.863^*$</td>
<td>$38.331$</td>
<td>0.0004</td>
</tr>
<tr>
<td>$r \leq 1$</td>
<td>$76.159^*$</td>
<td>$63.876$</td>
<td>0.0033</td>
<td>$38.998^*$</td>
<td>$32.118$</td>
<td>0.0062</td>
</tr>
<tr>
<td>$r \leq 2$</td>
<td>$37.161$</td>
<td>$42.915$</td>
<td>0.1670</td>
<td>$21.449$</td>
<td>$25.823$</td>
<td>0.1704</td>
</tr>
<tr>
<td>$r \leq 3$</td>
<td>$15.712$</td>
<td>$25.872$</td>
<td>0.5158</td>
<td>$11.634$</td>
<td>$19.387$</td>
<td>0.4500</td>
</tr>
<tr>
<td>$r \leq 4$</td>
<td>$4.078$</td>
<td>$12.517$</td>
<td>0.7308</td>
<td>$4.078$</td>
<td>$12.517$</td>
<td>0.7308</td>
</tr>
</tbody>
</table>

Note: *indicates rejection of null hypothesis, at 5% level of significance

<table>
<thead>
<tr>
<th>Null hypothesis</th>
<th>Statistical Test</th>
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<th>p-value</th>
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<tr>
<td></td>
<td>Trace Test</td>
<td>Maximum Eigenvalue Test</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r = 0$</td>
<td>$117.602^*$</td>
<td>$88.803$</td>
<td>0.0001</td>
<td>$38.861^*$</td>
<td>$38.331$</td>
<td>0.0434</td>
</tr>
<tr>
<td>$r \leq 1$</td>
<td>$78.7409^*$</td>
<td>$63.876$</td>
<td>0.0017</td>
<td>$36.844^*$</td>
<td>$32.118$</td>
<td>0.0122</td>
</tr>
<tr>
<td>$r \leq 2$</td>
<td>$41.896$</td>
<td>$42.915$</td>
<td>0.0630</td>
<td>$25.503$</td>
<td>$25.823$</td>
<td>0.0550</td>
</tr>
<tr>
<td>$r \leq 3$</td>
<td>$16.393$</td>
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<td>0.4614</td>
<td>$9.6343$</td>
<td>$19.387$</td>
<td>0.6579</td>
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<tr>
<td>$r \leq 4$</td>
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<td>$12.517$</td>
<td>0.3703</td>
<td>$6.759$</td>
<td>$12.517$</td>
<td>0.3703</td>
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</table>

Note: *indicates rejection of null hypothesis, at 5% level of significance

<table>
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<tr>
<th>Null hypothesis</th>
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<th>Statistical Test</th>
<th>p-value</th>
<th>Statistical Test</th>
<th>Statistical Test</th>
<th>p-value</th>
</tr>
</thead>
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<td>$19.387$</td>
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<tr>
<td>$r \leq 4$</td>
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<td>$12.517$</td>
<td>0.3694</td>
<td>$6.767$</td>
<td>$12.517$</td>
<td>0.3694</td>
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</tbody>
</table>

Note: *indicates rejection of null hypothesis, at 5% level of significance
Table IV. Common cycle test.

<table>
<thead>
<tr>
<th>Null hypothesis</th>
<th>$\lambda^2$</th>
<th>$\xi_{(p,s)}$</th>
<th>$[r + s]^2$</th>
<th>p-value</th>
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</thead>
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<tr>
<td>$s &gt; 0$</td>
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<td>$s &gt; 1$</td>
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<td>5.3875</td>
<td>16</td>
<td>0.9935</td>
</tr>
<tr>
<td>$s &gt; 2$</td>
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<td>25</td>
<td>0.8761</td>
</tr>
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<td>$s &gt; 3$</td>
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<td>49</td>
<td>0.0000</td>
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</tbody>
</table>

Note: *indicates rejection of null hypothesis, at 5% level of significance

<table>
<thead>
<tr>
<th>Null hypothesis</th>
<th>$\lambda^2$</th>
<th>$\xi_{(s,p)}$</th>
<th>$s (n(p - 1) + r) + s^2 - sn$</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s &gt; 0$</td>
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<td>5.8898</td>
<td>8</td>
<td>0.6596</td>
</tr>
<tr>
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<td>0.1819</td>
<td>14.7231</td>
<td>15</td>
<td>0.4715</td>
</tr>
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<td>106.2583</td>
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<td>0.0000</td>
</tr>
</tbody>
</table>

Note: *indicates rejection of null hypothesis, at 5% level of significance

<table>
<thead>
<tr>
<th>Null hypothesis</th>
<th>$\lambda^2$</th>
<th>$\xi_{(s,p)}$</th>
<th>$s (n(p - 1) + r) + s^2 - sn$</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
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</tbody>
</table>

Note: *indicates rejection of null hypothesis, at 5% level of significance
Figure III- Common Cycles.

Figure IV. Cyclical components for $p = 1$, $s = 3$ and $r = 2$. 

13
Figure IV shows the business cycle components for our best estimative; \( p = 1, s = 3 \) and \( r = 2 \). We notice an enormous contraction in Argentina in 1990’s, as expected. As for Brazil, the period of the economic miracle is apparent. To analyze the robustness of the results we estimate business cycles for each country for \( p = 1, 2, 3 \). Figure V shows the business cycle for each country. It is possible to see that the business cycles obtained from different \( p \) are similar.

Figure V. Cyclic components in each country for \( p = 1, p = 2 \) and \( p = 3 \).

4 Business cycle analysis

The degree of association among the contemporaneous movements may be measured through the pairwise linear correlation as reported in Table VI for \( p = 1, 2, 3 \) (see appendix A). We can observe that for \( p = 1 \) and \( p = 2 \) Brazil and Argentina have low positive correlation and, for \( p = 3 \), they have a negative correlation. Paraguay and Uruguay present a negative correlation while Brazil and Uruguay show a positive correlation. As for the common cycles, it is possible to see that the economic cycle of Argentina is more influenced by
common cycle 1, whereas Venezuela is influenced negatively by common cycle 2. Lastly, notice that for some values of $p$ the correlation is positive and for some others it is negative, which is an indication that the linear correlation is not an ideal measure to identify co-movements.

Once the analysis through linear correlation gives a static measure of the co-movements - as noted by Engle and Kozick (1993) -, to capture the simultaneous persistence of co-movements we use techniques based on the frequency domain. This analysis is an alternative method to analyze the data and a straightforward way to represent the economic cycles. Two measures are employed in frequency domain: coherence and phase. The coherence is a measure that corresponds to linear correlation in time domain, for instance, the coherence between two variables is a measure of the degree to which these variables are jointly influenced by cycles of specific frequency. The phase of the cross spectrum indicates if cycles in each frequency are synchronized or not. When the phase is null, it means that there are synchronized cycles in that frequency. Figures VII to X show the coherence and phase between pairs of the business cycles of the Mercosur members. These pictures show values of coherence varying between zero and one (vertical axis) for each value of frequency (horizontal axis). Values of phase (vertical axis) are calculated for each value of frequency (horizontal axis). At the final point of the horizontal axis, the frequency 0.5 corresponds to period of two years, the point 0.25 corresponds to four years, frequency 0.1 corresponds to ten years, and so on.

\[10^{th}\text{See Appendix B.}\]
\[11^{th}\text{To estimate coherence it is used a MSCOHERE function of Matlab 7.0 which considers smoothed with Hamming window of 30 with 50% overlap.}\]
As mentioned, to check the robustness of the frequency domain results we also consider the $VAR(2)$ and $VAR(3)$ models. In the first row of Figure VI the ideal values of coherence and phase are shown, that is, coherence one and phase zero in all frequencies. For example, this picture shows results for synchronization of business cycle of Argentina with himself at each value $p$, and, after, the same is made for Brazil and Paraguay. The same results are valid for Uruguay and Venezuela, but they are not reported. Figure VI also shows that Argentina and Paraguay have reasonable values for coherence and phase only for $p = 3$. 

Figure VI. Coherence and Phase
Figure VII reports interesting results for coherence and phase. First, Argentina and Uruguay appear to have high values of coherence for $p = 2$ and $p = 3$, but values of phases close to zero only for $p = 3$. This means that these countries appear to have non-synchronized cycles. After that, Argentina and Venezuela present values of coherence and phase very similar to the ideal values of synchronization. Similar results occur between Brazil and Paraguay, that is, for $p = 1$ and $p = 2$ values of coherence and phase are close to one and zero, respectively. Hence, Brazil and Paraguay compose another group with synchronized cycle.
In Figure VIII is showed that Brazil and Uruguay are synchronized only for \( p = 3 \) and Paraguay and Uruguay only for \( p = 1 \). Finally, Figure IX shows that Uruguay and Venezuela have values of coherence near to one, but their phase are different from zero in almost all frequencies, indicating that these couples of countries are not synchronized.

In summary the major evidence of synchronization is given in two group of countries; Argentina-Venezuela and Brazil-Paraguay. In general, all couples of countries present values of coherence less than one and their phase are generally different from zero.
Therefore, the lack of synchronization among the business cycles confirms that the presence of common cycles does not imply synchronization and corroborates the importance to conduct this analysis in frequency domain.

5 Conclusion

The design of economic blocks is based on the harmonization of economic and commercial policies. However, as argued by Backus and Kehoe (1992) and Chistodoulakis and Dimelis (1995), this harmonization is well succeeded when the member states are sufficiently similar. If this is true, it is of utmost importance to analyze the dynamics of the members and investigate the degree of synchronization of their business cycles. Regarding the Mercosur, it is common to see in the media discussions on the intensification of this economic block. However, it is not usual to argue which the necessary conditions for this intensification are and if they are valid. Considering the members of Mercosur (Argentina, Brazil, Paraguay, Uruguay and Venezuela),
this paper analyzes if there are any common dynamic in their economies and if their business cycles are syn-
chronized. To implement the analysis we estimate a VAR model and test the presence of common trends and
common cycles. Using the BNSW trend-cycle decomposition, the business cycles were estimated, taking in
account the cointegration and serial correlation common feature restrictions. Then, measures of coherence
and phase, in the frequency domain, are used to examine the degree of co-movements in business cycles.

The results suggest that there are three common trends and two common cycles among the countries.
These results confirm the necessity to use a multivariate approach to obtain the business cycles, the first con-
tribution of this work. Frequency domain results identified evidence of synchronization in two sub-groups;
Argentina-Venezuela and Brazil-Paraguay, but, in general, the countries are not synchronized. Hence, the
lack of synchronism or symmetry in the business cycle of Mercosur makes difficult a greater integration into
this economic block.
References


Table VI. Linear correlations in business cycles and in common cycle

<table>
<thead>
<tr>
<th>Countries</th>
<th>Argentina</th>
<th>Brazil</th>
<th>Paraguay</th>
<th>Uruguay</th>
<th>Venezuela</th>
<th>C. Cycle 1</th>
<th>C. Cycle 2</th>
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APPENDIX B : COHERENCE AND PHASE

Consider a vector of two stationary variables \( y_t = (X_t, Y_t) \). Let \( S_{YX}(w) \) represent the population spectrum of \( Y \) and \( S_{XY}(w) \) the population cross spectrum between \( X, Y \). The population cross spectrum can be written in term of its real and imaginary components as \( S_{XY}(w) = C_{XY}(w) + i Q_{XY}(w) \), where \( C_{XY}(w) \) and \( Q_{XY}(w) \) are labeled the population cospectrum and population quadrature spectrum between \( X, Y \) respectively.

The population coherence between \( X \) and \( Y \) is a measure of the degree to which \( X \) and \( Y \) are jointly influenced by cycles of frequency \( w \).

\[
h_{YX}(w) = \frac{|C_{XY}(w)|^2 + |Q_{XY}(w)|^2}{S_{YY}(w) S_{XX}(w)}
\]

Coherence takes values in \( 0 \leq h_{YX}(w) \leq 1 \). A value of one for coherence at a particular point means the two series are altogether in common at that frequency or cycle; if coherence is one over the whole spectrum then the two series are common at all frequencies or cycles.

The cross spectrum is in general complex, and may express in its polar form as:

\[
S_{XY}(w) = C_{XY}(w) + i Q_{XY}(w) = R(w) \exp(i \theta(w))
\]

where \( R(w) = \left\{ [C_{XY}(w)]^2 + [Q_{XY}(w)]^2 \right\}^{\frac{1}{2}} \) and \( \theta(w) \) represent the gain and the angle in radians at the frequency \( w \). The angle satisfies \( \tan(\theta(w)) = \frac{Q_{XY}(w)}{C_{XY}(w)} \). More details in Hamilton (1994).