Incentives to Innovate and the Decision to Go Public or Private*

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April 2011

Abstract
We model the impact of public and private ownership structures on firms’ incentives to choose innovative projects. Innovation requires the exploration of new ideas with potential advantages but unknown probability of success. We show that it is optimal to go public when firms wish to exploit the current technology and to go private when firms wish to explore new ideas. This result follows from the fact that privately-held firms are less transparent to outside investors than publicly-held firms. In private firms, insiders can time the market by choosing an early exit strategy when they learn bad news. This option makes insiders more tolerant of failures and thus more inclined to choose innovative projects. In public firms, an early exit strategy is less valuable because there is less information asymmetry about cash flows. In such firms, prices of publicly-traded securities react quickly to good news, providing insiders with incentives to choose conventional but safer projects in order to cash in early when good news arrive. Extensions to the model allow us to incorporate other drivers of the decision to go public or private, such as liquidity and cost of capital. Our model rationalizes recent evidence linking private equity to innovation and creative destruction and also generates new predictions concerning the determinants of going public and going private decisions.

JEL Classification: G2, G3, O3.

Keywords: innovation, private equity, capital ownership structure, going public.

*We thank Paul Laux, Stewart Myers, Walter Novaes, Jean-Charles Rochet, and seminar participants at LSE, Zurich, Pompeu Fabra, Queen Mary, Lausanne, Exeter, Stockholm School of Economics, Stockholm University, and Universidade Nova de Lisboa for useful comments and suggestions. Part of this paper was written while Silva was visiting the MIT Sloan School of Management; he is grateful for its hospitality. Silva acknowledges financial support from INOVA, NOVA FORUM, and FCT PTDC/ECO/72021/2006. E-mail addresses: d.ferreira@lse.ac.uk, manso@mit.edu, acsilva@novasbe.pt.
1. Introduction

There is evidence that private firms are more innovative than publicly-traded firms. Using patent citation data, Lerner, Sorensen and Strömberg (2010) find that firms invest in more influential innovations after being acquired by private equity funds. Popov and Roosenboom (2009) find that private equity investment increases the number of patents in a panel sample of firms from 21 European countries. Private equity is also associated with corporate restructuring, changes in strategic direction, and creative destruction. For example, Davis et al. (2009) find that firms acquired by private equity funds fire workers and shut down existing establishments, but also engage more in mergers and acquisitions and create jobs in new establishments. They conclude that private equity is a catalyst for creative destruction.\(^1\)

We introduce a model in which the choice of ownership structure of the firm—either public or private—affects managers’ incentives to innovate. Our main contribution is to show that private ownership creates incentives for innovation, while public ownership creates incentives against innovation. Because we allow for an endogenous choice of ownership structure, the model also provides a novel explanation for the decision to go public or private. We find that the decision to go public or private is affected by the relative profitability of innovative versus conventional projects.

In our model, a risk-neutral insider chooses between a conventional project and an innovative project \((\text{exploitation of existing ideas or exploration of new ideas}; \text{March, 1991})\). Both projects generate cash flow streams in two consecutive periods. The insider has an option to liquidate his stake early (i.e. before final cash flows are realized) by trading on the basis of his private information. We first show that, under private ownership, if the insider can time the market by choosing an early exit strategy after bad news, the insider becomes more tolerant of early failures and thus more inclined to choose the innovative project. This\(^2\) tolerance-for-failure effect is the key driver of innovation in private companies.

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\(^1\)See Strömberg (2009) for a review of the literature on private equity and innovation.

\(^2\)Evidence consistent with the tolerance-for-failure effect is provided by Acharya and Subramanian (2009), who show empirically that innovation is more prevalent in countries with debtor-friendly bankruptcy codes,
Exiting early constitutes a real option to the insider. This option is valuable only when the insider’s actions do not fully reveal his private information. An early exit does not fully reveal the insider’s private information for two reasons. First, cash flows of private firms may not be fully observable, thus outside investors may be poorly informed about interim cash flows generated by projects. Second, the insider may suffer a liquidity shock, thus outside investors cannot know whether trading is motivated by information or liquidity.

Under public ownership, cash flows are observable and thus an early exit after bad news is not profitable. Therefore, there is no tolerance for failures in public companies. Furthermore, market prices of public securities react quickly to good news. This is known to create incentives for short-termist behavior (e.g., Stein, 1989). A rational pressure towards quick results arises in our model because good news are quickly incorporated into market prices. Thus, the insider may prefer the conventional project because it has a higher expected probability of an early success. We show that the unique equilibrium under public ownership involves choosing the conventional project with some positive probability, even when innovation is ex ante efficient.

Our model shows that incentives in public firms are biased towards conventional projects, while incentives in private firms are biased towards innovative projects. Consequently, the optimal structure of ownership—public or private—changes with the firm’s life cycle depending on whether exploitation of existing ideas or exploration of new ideas is optimal.

We interpret our model as a theory of the evolution of ownership structures. It is usually believed that exploration is important early in a firm’s life. Our model thus predicts that it is optimal to start private to maximize incentives to explore. Our model also views going private decisions as complements to risky restructurings. Every time a firm needs to reinvent itself, it makes sense to do it out of the public eye. Major company restructurings involving radical changes in strategy are departures from the conventional, and thus more properly
motivated under private equity.\textsuperscript{3}

Our model sheds light on a number of controversial issues raised in the empirical literature on the real effects of venture capital and buyout investments. Kaplan and Strömberg (2009) review this evidence and conclude that private equity investment creates value not only because of tax benefits and the exploitation of mispricings in debt and equity markets, but also by affecting corporate behavior, such as operations and investments. But the evidence on the role of private equity on innovation remains controversial. For example, while there is some agreement that venture capital investment positively affects patenting activity,\textsuperscript{4} others argue that this does not imply increases in productivity (Ueda and Hirukawa, 2008). Furthermore, there is the question of whether “innovation follows VC investment” or “VC investment follows innovation” (Mollica and Zingales, 2007; Hirukawa and Ueda, 2008).

Our model’s predictions can reconcile many of such disparate results. In particular, our model suggests that, when studying the relation between private equity and innovation, it is important to distinguish between venture capital and public-to-private buyouts. Our model’s main empirical implications are as follows:

- Firms become more innovative after public-to-private transitions, as the evidence suggests (Lerner, Strömberg, Sorensen, 2010; Davis et al., 2009). But, because the ownership mode is endogenous in the model, it is also true that more innovative industries attract more private equity investment.

- There can be too much innovation in private companies: innovation does not necessarily increase productivity or profitability in privately-held companies (Ueda and Hirukawa, 2008).

- VC investment does not necessarily lead to an increase in innovation activities, because

\textsuperscript{3}For an alternative incentive-based theory of the life cycle of speculative industries, see Biais, Rochet, and Woolley (2009).

\textsuperscript{4}Kortum and Lerner (2000) provide evidence that venture capital backing is positively related to patent count. In their data, VC backing has a much larger impact on patents than corporate R&D. Ueda and Hirukawa (2008) confirm such findings in an updated sample.
venture capitalists usually invest in companies that are already private.

- On the other hand, in our model, public-to-private transitions are necessary for innovation to occur; in this case, innovation follows PE (buyout) investment. Furthermore, if PE firms have unique skills in identifying promising companies, PE-backed private companies are more successful in their innovations than non-PE-backed ones.

Our theory also has implications for the empirical work on private equity and innovation. First, it highlights that controlling for the type of the transition (public-to-private versus private-to-private) is at least as important as controlling for the type of investment (buyout versus venture capital). Second, although the discussion in the empirical literature usually focuses on sorting out the direction of causality, our theory suggests a unifying explanation for both hypotheses ("innovation follows private equity" and "private equity follows innovation"). It might be more fruitful to pursue empirical strategies that allow for the testing of these two possibilities simultaneously.

There is an emerging theoretical and empirical literature on the role of ownership structures and financing choices on corporate innovation. An early example is Aghion and Tirole (1994); more recent works include Aghion, Van Reenen, and Zingales (2009), Atanassov, Nanda, and Seru (2007), Belenzon, Berkovitz, and Bolton (2009), Bhattacharya and Guriev (2006, 2009), and Fulghieri and Sevilir (2009). These papers focus on related but different questions, such as the impact of capital structure, governance, organization, and ownership concentration on corporate innovation.

Our model is also closely related to two different theoretical literatures: (1) models of interactions between stock prices and incentives in firms and (2) models of the decision to

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5Boucly, Sraer and Thesmar (2009) provide evidence that it is relevant to distinguish private-to-private and public-to-private transitions. They find that LBOs are followed by growth when the targets are financially constrained. In public-to-private LBOs (and in private-to-private LBOs of not financially constrained targets), they find that LBOs are not followed by growth. We do not derive explicit implications for firm growth. However, if growth is related to periods of exploitation of existing technologies, then private-to-public transitions would be followed by growth. Moreover, compatible with the evidence in Boucly et al., public-to-private LBOs would be followed by restructuring or experimentation of an innovative process, but no immediate growth.

Our model is also related to the work of Stein (1989), who develops a model of rational managerial short-termism driven by the stock market. In his model, firms take actions to boost current earnings at the cost of lower future earnings in an attempt to mislead the market. In equilibrium managers are stuck with an inefficient strategy. The same logic is present in our model. If the firm is public, a manager may choose the conventional project even when the innovative project has a higher net present value, because the former has a higher expected probability of generating high earnings in the short run. But our model also shows the other side of the story. If the firm is private and thus free from pressure to boost current earnings, it puts too much emphasis on future cash flows. Without the stock market punishing short-run falls in earnings, managers rationally become biased towards innovative projects, which are risky but very profitable if successful. This bias may lead them to inefficient long-termism: innovation may be chosen even when it is inferior to conventional methods. Thus, our model provides a more balanced view of market incentives: while managers of public firms may focus excessively on current earnings, managers of private firms may focus excessively on future earnings. The best structure thus depends on the nature of the projects available to the firm at a particular time.

Our paper is also related to a large literature on the choice between public and private structures, including Boot, Gopalan, and Thakor (2006), Shah and Thakor (1988), Chemmanur and Fulghieri (1999), Zingales (1995), and Pagano and Roel (1998). None of these papers consider incentives for innovation as a determinant of ownership structures.

More closely related to our model is the work of Maksimovic and Pichler (2001). In their
model, firms may choose between a new or an existing technology and then decide whether to finance future rounds of investment with either public or private offerings. Public offerings are assumed to be cheaper, but they reveal information about the industry prospects to potential competitors. Thus, firms may strategically delay finance or resort to private offerings to prevent entry. Their model is concerned with the effect of technological uncertainty at the industry level on the mode and timing of financing. Our model is concerned with the effect of the financing mode (private or public) on firms’ internal incentives to choose between different technologies. Thus, our model allows us to address a different question: Should the decision to go public or private depend on the profitability of new versus old technologies?

The paper is structured as follows. We present the basic model in Section 2, discuss the going public or private decision in Section 3, develop extensions in Section 4, and conclude with a discussion of empirical applications in Section 5. All proofs are in the Appendix.

2. The model

2.1. Setup

A risk-neutral insider initially holds all shares of a firm. The insider has no initial wealth, is protected by limited liability, and has outside utility normalized to zero. We view the insider as a manager-entrepreneur who founded the firm and owns it fully. Because the identity of the manager making the decisions is not important in our model, we assume that the founder stays as manager regardless of how many shares he sells to other investors. The results are identical if the founder is replaced by a newly-hired professional manager.

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### 2.1.1. Technology

The insider has to decide between two projects, projects 1 and 2, at two consecutive dates, dates 0 and 1. Each project has two possible outcomes: success or failure. Success yields earnings $S$ and failure yields earnings $F$, $S > F$. We call project 1 *exploitation of existing ideas* and project 2 *exploration of new ideas*. This setup is similar to Manso (2009).

If the insider chooses project 1, the conventional project, there is a probability $p$ of success. The probability $p$ is known to everyone. If the insider chooses project 2, the innovative project, the probability of success is $q$, which is unknown. It is only possible to learn about $q$ if the insider chooses project 2. We assume that

$$E[q|F] < E[q] < E[q|S].$$

The expectation of success $q$ increases if project 2 is successful in period 1 and decreases if project 2 fails in period 1.

The insider will only consider choosing the innovative project if it has a chance of improving upon the old method. Thus, we assume also that $E[q|S] > p$ to eliminate the trivial case in which project 1 always strictly dominates project 2. On the other hand, the insider always chooses the innovative project if $E[q]$, the unconditional probability of success before ever trying the project, is higher than $p$. The interesting case is when $E[q] < p$. To economize on algebra and notation, define $\delta$ and $\theta$ such that $\delta p = E[q]$ and $\theta p = E[q|S]$. These assumptions imply that $0 < \delta < 1$ and $1 < \theta < 1/p$. To summarize,

$$\delta p = E[q] < p < E[q|S] = \theta p.$$  \hspace{1cm} (2)

Equations (1) and (2) encapsulate all characteristics of project 2. From (1), project 2 is exploratory because it is only possible to know more about the new method by trying it out. From (2), project 2 is promising because its probability of success is higher than
the probability of success of project 1 if project 2 is successful in period 1. We can think of radical methods that look unlikely to work but that would greatly improve the current method if they do work. The interpretation of $\delta$ and $\theta$ is that a method is more radical the smaller $\delta$ is and the higher $\theta$ is.

Total profits (gross of any initial investment costs) are given by the undiscounted sum of earnings of the two dates, $\pi = x_1 + x_2$, where $x_t$ is equal to $F$ or $S$. We assume that earnings are only liquid at date 2. That is, earnings $x_1$ are realized at date 1 but dividends based on $x_1$ are paid at date 2 (as when sales are on trade credit so that earnings $x_1$ are simply accounts receivables). More generally, we wish to capture a situation in which it is possible to learn a signal $x_1$ at date 1 about future profits of the firm, although such cash flows have not yet materialized. We call $x_1$ earnings at date 1 for simplicity of exposition, but it can also be understood as “a signal at date 1 about earnings at date 2.”

The insider makes an initial investment $I$, paid in cash, to produce positive earnings by investing in either project. Without this initial investment, all earnings are equal to zero regardless of the project chosen.

The insider may switch from one project to the other after observing $x_1$. If the insider initially chooses to exploit the old method, the option to switch has zero value. If the initial choice is to explore the new method, however, to maximize firm value the insider switches to project 1 after observing $x_1 = F$. The option to switch is valuable under exploration. If the new method is tried out but fails, the insider returns to the old method. Figure 1 provides a visual summary of the technology taking into account the option to switch.

To simplify notation, we make $F = 0$ and $S = 1$, without loss of generality. Under exploitation (project 1), the expected market value of the firm, gross of initial investment costs, is $v_1 = p (1 + p) + (1 - p) p$. This expression implies

$$v_1 = 2p.$$  \hfill (3)
We always write the value of the firm gross of initial investment costs, unless we say otherwise. The value of the firm takes into account the two periods of operation.

![Diagram showing the decision process between two projects over three dates: Date 0, Date 1, and Date 2. The diagram includes nodes labeled Project 1 and Project 2 with probabilities p and 1-p at each date. The outcomes S (successful) and F (failure) are connected with arrows, indicating the flow of decisions and outcomes.]  

**Figure 1.** Technology: earnings and probabilities associated with each project choice.

If the insider chooses exploration (project 2), the firm maintains the use of the innovative method in case of success at date 1. In case of failure, the firm returns to the old method (project 1). The expected market value of the firm under exploration is then

\[ v_2 = \delta p (1 + \theta p) + (1 - \delta p) p, \]  
or

\[ v_2 = p \{1 + \delta \{1 + p (\theta - 1)\}\}. \]  

(4)

The innovative project (project 2) is ex ante preferable to the conventional project (project 1) if and only if \( v_2 - v_1 \geq 0 \). We have

\[ v_2 - v_1 > 0 \text{ if and only if } \delta \{1 + p (\theta - 1)\} > 1. \]  

(5)
2.1.2. Liquidity and financial market frictions

The key financial market friction in our model is the existence of a demand for liquid assets caused by (unmodeled) borrowing constraints. We model the insider’s preference for liquid assets by assuming that he has a utility function as in Diamond and Dybvig (1983),

\[ U(c_1, c_2) = \begin{cases} 
  c_1 & \text{with probability } \mu, \\
  c_2 & \text{with probability } 1 - \mu, 
\end{cases} \tag{6} \]

where \( c_t \) is consumption at date \( t \). This reduced-form approach is common in microeconomic models of liquidity shocks (see e.g. Freixas and Rochet, 1997). With probability \( \mu \), a liquidity shock forces the insider to consume at date 1. With probability \( 1 - \mu \), there is no liquidity shock and dividends and consumption are synchronized at date 2. We can think of liquidity shocks as representing different types of consumers. Insiders that do not suffer a liquidity shock are called late consumers. Insiders that suffer a liquidity shock are early consumers.\(^7\)

For liquidity shocks to have a real impact on decisions, we need to assume that the insider faces borrowing constraints. The assumption of limited liability eliminates uncollateralized borrowing. The assumption of zero initial wealth implies that the insider has no initial collateral. We need to assume further that the insider cannot borrow using the securities issued against the firm’s cash flows as collateral.\(^8\)

Liquid securities such as cash can be stored from one period to the following at no cost. There is no discounting nor systematic risk in the economy.

2.1.3. Project financing

The insider must sell securities backed by future cash flows to finance the initial investment \( I \), as the insider has no initial wealth. The insider may sell securities to private or public

\(^7\)We interpret the liquidity shock as any reason for the insider to sell other than private information, including portfolio rebalancing, tax considerations and behavioral biases. For evidence of such motives to trade, see Kallunki, Nilsson, and Hellström (2009).

\(^8\)Although we state this as an assumption, it is possible to endogenize borrowing constraints fully by introducing additional moral hazard considerations to the problem.
investors. The initial investment $I$ is observable to all and contractible. Thus, the investment $I$ must occur for sure if the insider sells securities to raise funds for investments.

We initially assume that the only securities available are share contracts. This is for simplicity of exposition. Capital structure choices are relevant in our model (that is, the model is not in a Modigliani-Miller world), but they do not change the qualitative results about the choice between private and public ownership structures. In Section 4 we discuss the robustness of the results to different contracting assumptions and to the introduction of other securities, such as debt.

2.1.4. Differences between private and public ownership structures

The key results of our model depend only on one difference between private and public ownership: the ability of outsiders to observe the interim earnings $x_1$ in a public firm but not in a private firm. Under public ownership, we assume that interim earnings $x_1$ are observable by everyone. Under private ownership, in contrast, only the insider and the incumbent private investors observe $x_1$. Future private investors do not observe $x_1$ either. These assumptions capture the fact that public companies are more transparent. Public companies are subject, for example, to tighter regulatory disclosure requirements such as quarterly earnings reporting and comprehensive annual reports, to analyst coverage and, perhaps most importantly, to the aggregation of dispersed information into the stock price.

For the sake of realism and to permit the analysis of different trade-offs, we also allow for other differences between the two structures, such as the cost of capital and liquidity costs. These enrich the model but are not necessary for any of the qualitative results linking innovation incentives and going public or private decisions.

We assume that there are transaction costs associated with raising funds for investment through an IPO.\footnote{Lee et al. (1996) estimate that administrative and underwriting costs are usually about 11\% of the IPO proceeds. More importantly, IPO underpricing can create much higher costs, with total costs reaching the 20-30\% range (Ritter, 1987). Seasoned Equity Offerings (SEOs) are less costly, but discounts are also common, with a typical negative stock price reaction after announcements of equity offerings of 3\% (Asquith} We capture the costs of issuing public equity by a parameter $c_{\text{pub}} \in (0, 1)$,
such that each dollar sold in public offerings yields only $c_{pub}$ to the firm. A high $c_{pub}$ means a low discount.

Raising capital through private equity also involves transaction costs. We denote by $c_{priv} \in (0, 1)$ the discount factor associated with private securities. This parameter is likely to change with changes in the institutional environment and the state of the economy. For example, when interest rates are relatively low, private equity funds can borrow cheaply and thus going private becomes less costly for the firm, as required returns fall. Private equity booms are thus associated with high levels of $c_{priv}$.

We make no assumptions with respect to the relative cost of public equity capital $c_{priv} - c_{pub}$. Thus, our model allows for situations in which funds for investment are cheaper if financed by public securities ($c_{pub} > c_{priv}$) as well as cases in which being private reduces the cost of capital ($c_{pub} < c_{priv}$).\(^{10}\)

A traditional justification for going public is to create liquidity for insiders’ shareholdings (see for example Chemmanur and Fulghieri, 1999, and Ritter and Welch, 2002). For example, a founder may value the option of selling his stake quickly on the market should the need arise. If the firm is privately held, the founder may have to negotiate with a few private investors. Especially in cases in which the founder suffers a liquidity shock and needs to sell quickly, the bargaining power of the founder may be compromised if the firm is private. In contrast, in public markets the founder may be able to sell more easily his own shares through organized markets (provided compliance with insider trading regulations). To capture a potential liquidity advantage of public equity, we assume that each dollar in shares sold by the insider at date 1 (the liquidity shock period) yields only $k \leq 1$ if the company is private. No such discount happens if the firm is public. To focus on the main mechanism on the decisions of innovation and public or private structures, we assume initially no liquidity discount when

\(^{10}\)In Chemmanur and Fulghieri’s (1999) model of the decision to go public, the cost of capital in public firms reflects the trade-off between the liquidity of public securities and the information production costs associated with the duplication of monitoring efforts by public investors. Our model can incorporate such effects in reduced form by changing $c_{pub}$ and $c_{priv}$ according to which effect dominates.
the insider sells his own shares, $k = 1$. In Section 4, we analyze the case in which $k < 1$.

2.1.5. The structure of information and timing of events

At date 0, the insider decides to sell a fraction $1 - \alpha_{\text{priv}}$ of the shares to private investors or $1 - \alpha_{\text{pub}}$ to public markets. The insider needs to raise at least $I$ in cash to pay for the initial investment cost. After paying $I$, the insider chooses project 1 or 2. The outside investors cannot observe which project was chosen.

At date 1, the insider observes the first realization of earnings $x_1 \in \{0, 1\}$ and then chooses project 1 or project 2, which again is unobservable by outsiders. The insider then learns about the preference for liquid assets. If the insider is an early consumer, he sells all shares he owns regardless of the their market valuation. If the insider is a late consumer, he may sell some of the shares or keep them until date 2. After observing whether the insider places orders to sell or to keep the shares, the market forms a price for the shares.

At date 2, the second-period earnings $x_2 \in \{0, 1\}$ are realized, the shareholders receive dividends $x_1 + x_2$, and the firm is liquidated. The liquidation value is normalized to zero. Figure 2 shows the time line.

2.2. Private ownership

Consider first the case of private ownership, that is, at date 0, the insider sells $1 - \alpha_{\text{priv}}$ shares to private investors. We take $\alpha_{\text{priv}}$ as exogenous for now and then work backwards to
find the optimal $\alpha_{priv}$.

After $1 - \alpha_{priv}$ shares are sold, at the end of date 0, the insider chooses either project 1 or 2. Recall that the project choice is private information to the insider. The intuition is that, although investments may be observable, the insider has unique information or expertise that allows him to assess the characteristics of the available projects. This is a natural assumption, which is consistent with the view that managers’ unique expertise may be essential for investment decisions.

Let $\sigma_{priv} \in [0, 1]$ be the probability that the insider chooses project 2 (innovation). We allow from the outset for the possibility of equilibria involving mixed strategies. Intuitively, an equilibrium with strictly mixed strategies could also be interpreted as the choice of an “intermediate project,” which is more innovative than project 1 but not as radical as project 2. Our goal in this section is to compute the equilibrium project choice $\sigma_{priv}^*$ under private ownership.

### 2.2.1. Selling behavior at date 1

At the end of date 1, after observing $x_1$, the insider chooses whether to keep or sell the shares of the firm. We assume that the current private investors cannot buy out the insider.\textsuperscript{11} Thus, if the insider sells, the buyers are either new private investors or public investors in an IPO. Because the identity of the new investors is irrelevant in our model, we simply say that the insider sells shares to the market.

We first consider how the market updates its beliefs after observing the insider selling shares at date 1. Let $m$ be the posterior probability that the insider had a liquidity shock

\textsuperscript{11}Our results do not change qualitatively under the weaker assumption that there is some positive probability that current private investors cannot offer liquidity insurance to insiders. There are many reasons for that being the case. One possibility is that all capital committed to a private equity fund has already been used. Even if there is still capital available, fund covenants may prevent the investment of more than a certain fraction of fund capital in a single firm (Kaplan and Strömberg, 2009). Fund covenants and restrictions to raising additional capital can be rationalized as potential solutions to agency conflicts between general partners (fund managers) and limited partners (Axelson, Strömberg, and Weisbach, 2009). Finally, it could also be that funds need to exit early in order to produce evidence of good performance and raise more capital (Gompers, 1996).
conditional on the insider selling shares at date 1. A low $m$ means that the market believes that the insider is selling shares because the firm is overvalued, while a high $m$ means that the market believes that a liquidity shock is probably the main reason why the insider sells.$^{12}$

A liquidity shock forces the insider to sell his shares. Without a liquidity shock, the insider chooses whether to sell or not to sell. We thus need to characterize when an insider without a liquidity shock chooses to sell. The following lemma describes the insider’s behavior when earnings are $x_1 = 1$.

**Lemma 1** In the private ownership case, a late-consumer insider never sells shares at date 1 after observing a success ($x_1 = 1$).

A late-consumer insider would only sell shares at date 1 after a success if the insider believes that the shares are overvalued. That is, from the perspective of the insider, if the share price is higher than $1+p$ or $1+\theta p$, the values of the firm in case of success under project 1 or 2. The lemma shows that the prices at date 1 are always less than $1+p$. The market expects that the insider is at least more likely to sell after a failure than after a success. Market rationality then rules out share prices not compatible with the selling behavior of the insider. Prices at date 1 are never high enough to make an insider with good news sell rather than keep his shares.

As the market does not observe the outcome of the firm, the market assigns a positive probability of failure even if the project is successful. That decreases the share price and encourages the insider to keep the shares in case of success. For the public case, as the market observes the outcome, we will see that the insider has incentives to sell after observing a success if the insider has undertaken the conventional project.

The asymmetry of information, nevertheless, causes some price overvaluation, as the market cannot assign zero probability for failure either. But the overvaluation is never high

$^{12}$If we treat $m$ as an exogenous parameter, we can perform comparative statics with respect to market beliefs; a low $m$ is equivalent to a “cold market” while a high $m$ is equivalent to a “hot market.” On the other hand, by treating $m$ as endogenous, as we do in this paper, hot and cold markets still exist, but they are driven by fundamentals rather than sentiment.
enough to make an insider to sell after a success. As we see now, the overvaluation is high enough to make the insider sell after a failure.

Let \( b \in [0, 1] \) be the probability that a late-consumer insider sells shares after observing a failure, \( x_1 = 0 \).\(^{13}\) Let us construct the value of the probability \( m(b) = \Pr(Sale \mid Shock) \) in terms of \( \mu, b, \) and the technology parameters. By Bayes’s rule, rational market beliefs imply that

\[
m(b) = \frac{\Pr(Sale \mid Shock) \Pr(Shock)}{\Pr(Sale)}.
\]  

(7)

The inputs for this formula are as follows. In an equilibrium in which the probability of choosing project 2 is \( \sigma_{priv} \), the unconditional probability of selling shares at date 1 is

\[
\Pr(Sale) = \mu + (1 - \mu) [\sigma_{priv} (1 - \delta p) b + (1 - \sigma_{priv}) (1 - p) b].
\]  

(8)

That is, the probability is given by the sum of the probabilities of selling after a liquidity shock and the probability of selling without a liquidity shock. With a liquidity shock, this probability is given by \( \mu \). Without a liquidity shock, the probability of selling is \( \Pr(No\ Shock \cap Sale) \), which is equal to the probability of not having a shock, \( 1 - \mu \), times the probability of selling given that there was not a shock, \( \sigma_{priv} (1 - \delta p) b + (1 - \sigma_{priv}) (1 - p) b \). This last term takes into account that the insider never sells in case of success, as obtained with lemma 1, and takes into account the different probabilities of failures in the cases of innovative and conventional projects.

Conditional on having a liquidity shock, the insider sells with probability \( \Pr(Sale \mid Shock) = 1 \). Since the probability of a liquidity shock is \( \mu \), we have

\[
m(b) = \frac{\mu}{\mu + (1 - \mu) [\sigma_{priv} (1 - \delta p) b + (1 - \sigma_{priv}) (1 - p) b]}.
\]  

(9)

\(^{13}\)Because \( b \) can only be non-zero if \( x_1 = 0 \), whether project 1 or 2 were chosen is immaterial for the decision to sell, thus \( b \) does not need to be conditional on project choice. For brevity, we omit the proof of this claim; this proof is available upon request.
The equilibrium value of shares if the market holds rational beliefs is

\[ V_{\text{priv}} = m(b) \left[ \sigma_{\text{priv}} v_2 + (1 - \sigma_{\text{priv}}) v_1 \right] + (1 - m(b)) p. \]  \hspace{1cm} (10)

If the sale is stimulated by the liquidity shock, in the first term of the right-hand side, then the insider cannot sell strategically after a failure. Then, the market value of the share is given by a weighted average of the values of the innovative and the conventional projects, \( \sigma_{\text{priv}} v_2 + (1 - \sigma_{\text{priv}}) v_1 \). The weights are the probabilities of undertaking each kind of project. If the sale is not stimulated by the liquidity shock, then, by lemma 1, the market knows that the insider does not sell in case of success. As the optimal action after a failure is to switch to the conventional project, then the value of the firm in case of failure is given by \( p \).

As the expected value of the firm after a failure is \( p \), the condition for the insider to sell the firm’s shares after a failure is

\[ V_{\text{priv}} \geq p. \]  \hspace{1cm} (11)

The next lemma shows that \( V_{\text{priv}} > p \). Therefore, the insider always sells after a failure.

**Lemma 2** In the private ownership case, a late-consumer insider sells shares with probability \( b = 1 \) at date 1 after observing a failure \( (x_1 = 0) \).

The insider always sells after a failure because the market assigns a positive probability for a success. This belief is rational because the insider could have received a liquidity shock and be forced to sell.

A key aspect of the private ownership case is the possibility of selling shares at date 1 after observing a failure. A late-consumer insider only sells shares at date 1 if they are overvalued. This happens in equilibrium in the case of a failure because the market does not observe \( x_1 \) and thus cannot distinguish between a liquidity-motivated sale and an opportunistic sale. This information asymmetry creates a valuable option for a late-consumer insider.

Let \( T (\sigma_{\text{priv}}) \equiv V_{\text{priv}} - p \) denote the intrinsic value of the option to exit early for a late-consumer insider conditional on \( x_1 = 0 \). Selling shares is a real option to the insider. The
value of the underlying asset is the market value of shares in equilibrium, \( V_{\text{priv}} \), while the exercise price of the option is \( p \). The function \( T(\sigma_{\text{priv}}) \) depends on both the fundamental parameters and on the equilibrium strategy and beliefs. Lemma 2 implies that \( T(\sigma_{\text{priv}}) > 0 \).

### 2.2.2. Project choice at date 0

Now that we know how the insider behaves and how the market sets the price of shares at date 1, we can go back to date 0 to analyze the choice between the projects 1 and 2 (the conventional or the innovative project). Suppose that the market expects project 2 to be chosen with probability \( \sigma_{\text{priv}} \). In this case, the expected value as of date 0 of each share held by the insider if the insider chooses project 2 is given by

\[
u_{\text{priv},2} \equiv \mu V_{\text{priv}} + (1 - \mu) [(1 - \delta p) V_{\text{priv}} + \delta p (1 + \theta p)]. \tag{12}
\]

This expression takes into account the fact that, at date 0, the insider does not know yet his type. With probability \( \mu \), the insider will be an early consumer and be forced to sell. With probability \( 1 - \mu \) the insider will be a late consumer and will have the option to sell voluntarily. From lemmas 1 and 2, we know that the insider will sell after a failure and will not sell after a success.

Similarly, the expected value of one share when the insider chooses project 1 at date 0 while the market expects that project 2 is chosen with probability \( \sigma_{\text{priv}} \) is

\[
u_{\text{priv},1} \equiv \mu V_{\text{priv}} + (1 - \mu) [(1 - p) V_{\text{priv}} + p (1 + p)]. \tag{13}
\]

An equilibrium with positive probability of exploration, \( \sigma_{\text{priv}} > 0 \), exists only if \( \nu_{\text{priv},2} \geq \nu_{\text{priv},1} \). That is, choosing project 2 at date 0 must be incentive compatible for the insider. With the expressions of \( \nu_{\text{priv},2} \) and \( \nu_{\text{priv},1} \), substituting \( V_{\text{priv}} = T(\sigma_{\text{priv}}) + p \), we obtain

\[
u_{\text{priv},2} \geq \nu_{\text{priv},1} \Leftrightarrow v_2 - v_1 + p(1 - \delta)T(\sigma_{\text{priv}}) \geq 0. \tag{IC}
\]
Equation (IC) is the incentive compatibility condition for the insider to explore with positive probability. An equilibrium in which the insider chooses project 2 with $\sigma_{\text{priv}} > 0$ exists only if the incentive compatibility condition (IC) holds. In the same way, we have a positive probability of choosing project 1, $\sigma_{\text{priv}} < 1$, only if $v_2 - v_1 + p(1 - \delta)T(\sigma_{\text{priv}}) \leq 0$. We have a mixed strategy equilibrium, $0 < \sigma_{\text{priv}} < 1$, if and only if equation (IC) holds with equality.

The intuition for the incentive effects of private ownership on innovation can be grasped from the incentive compatibility condition (IC). The term $v_2 - v_1$ is the efficiency incentives for choosing the innovative project. We call this the efficiency effect. This effect is fully determined by the technology. In a first best world, the efficiency effect would determine which project is chosen.

The term $p(1 - \delta)T(\sigma_{\text{priv}})$ is a force pushing towards innovation. We have

$$p(1 - \delta)T(\sigma_{\text{priv}}) = (1 - \delta p)T(\sigma_{\text{priv}}) - (1 - p)T(\sigma_{\text{priv}}) > 0. \quad (14)$$

Because the innovative project has a higher probability of failure than the conventional project, the expected value of the option to exit early is higher with innovation, $(1 - \delta p)T(\sigma_{\text{priv}}) > (1 - p)T(\sigma_{\text{priv}})$.

The value of the option to exit early $T(\sigma_{\text{priv}})$ reflects the fact that the private ownership structure displays a higher degree of tolerance for failure than the first-best benchmark. Tolerance for failure has been shown to be a key feature of optimal incentive schemes for innovation (Manso, 2009). The key insight of our model is that tolerance for failure is more valuable for innovation because the option to exit early is exercised more often when exploration is chosen. Thus, the tolerance-for-failure effect is always positive.

The option to exit early nudges the insider towards choosing the innovative project. When innovation is efficient from a technological perspective ($v_2 - v_1 \geq 0$), this extra incentive for innovation is not necessary; the incentive compatibility condition is not binding.
More surprising are the effects when $v_2 - v_1 < 0$. In this case, innovation is inefficient. We would then have $\sigma_{priv}^* = 0$ without the effects on the tolerance for failure. However, as the option value of an early exit is positive, we can have $\sigma_{priv}^* > 0$ or even $\sigma_{priv}^* = 1$. Innovation may be chosen with certainty despite being inefficient. If the tolerance-for-failure effect dominates the negative efficiency effect, the private ownership structure creates incentives to innovate even when it would be optimal to choose the conventional project.

The next proposition characterizes the equilibrium value of $\sigma_{priv}$ under all possible pure strategy and mixed strategy equilibria. In particular, there is a unique $\sigma_{priv}^*$ for a given set of parameters $(p, \delta, \theta, \mu)$. The proposition follows from the incentive compatibility condition (IC) and from the properties of $T(\sigma_{priv})$.

**Proposition 1** For each set of parameters $(p, \delta, \theta, \mu)$, there exists a unique equilibrium probability of exploration for the private ownership case $\sigma_{priv}^* \in [0,1]$ given by:

1. If $v_2 \geq v_1$, then $\sigma_{priv}^* = 1$ (exploration for sure if innovation is efficient).

2. If $v_2 < v_1$, then

$$\sigma_{priv}^* = \begin{cases} 1, & \text{if } \frac{v_1 - v_2}{p(1-\delta)} \leq T(1), \\ \sigma_{priv}^* \text{ such that } T(\sigma_{priv}^*) = \frac{v_1 - v_2}{p(1-\delta)}, & \text{if } T(1) < \frac{v_1 - v_2}{p(1-\delta)} < T(0), \\ 0, & \text{if } T(0) \leq \frac{v_1 - v_2}{p(1-\delta)}, \end{cases} \tag{15}$$

where $T(\sigma_{priv}) \equiv V_{priv} - p$, the option to sell at $t = 1$, is a decreasing function of $\sigma_{priv}$.

Figure (3) shows the three possible cases when $v_1 - v_2 > 0$. The horizontal dashed lines represent different values for $\frac{v_1 - v_2}{p(1-\delta)}$. Consider, for example, decreasing values of $v_1 - v_2$ keeping $p(1-\delta)$ fixed, achieved by increasing $\theta$. The $R_1$ line represents a case in which $v_1 - v_2$ is high. In such a case, the efficiency effect towards $v_1$ is large and dominates the tolerance-for-failure effect, which implies the first-best action $\sigma_{priv}^* = 0$ to be chosen in equilibrium. The $R_2$ line represents an intermediate value of $v_1 - v_2$, for which a probability
of innovation $\sigma_{\text{priv}}^* \in (0, 1)$ makes the insider indifferent between projects 1 and 2. That means that the efficiency effect is fully offset by the tolerance-for-failure effect, and that the equilibrium involves some inefficient amount of innovation. The figure also shows that $\sigma_{\text{priv}}^*$ increases if the probability of the shock, $\mu$, increases, as $T(\sigma_{\text{priv}})$ increases with $\mu$ (the proposition below proves this property). The $R_3$ line is a case where $v_1 - v_2$ is positive but small, so that the option to exit early is so valuable to the insider that the insider makes the inefficient project choice in equilibrium, $\sigma_{\text{priv}}^* = 1$.

**Figure 3.** Equilibrium probability of undertaking the innovative project, $\sigma_{\text{priv}}^*$, when $v_1 > v_2$, for different values of $R = \frac{v_1 - v_2}{p(1-\delta)}$. If $R$ is high, then $\sigma_{\text{priv}}^* = 0$ ($R_1$). If $v_2$ gets closer to $v_1$, $\sigma_{\text{priv}}^*$ increases. For intermediate values of $R$, $0 < \sigma_{\text{priv}}^* < 1$ ($R_2$). We may have $\sigma_{\text{priv}}^* = 1$ even though $v_1 > v_2$ ($R_3$). The figure also shows that $\sigma_{\text{priv}}^*$ increases when $\mu$ increases.

In sum, our model shows that the private ownership structure is biased towards innovation. This bias is welcome when $v_2 > v_1$ but may lead to inefficiencies if $v_1 > v_2$. Earnings opacity, typical in privately-owned firms, gives an exit option to the insider. It is profitable
to sell before a bad signal about the value of the firm becomes public. The exit option is available regardless of the project chosen, innovative or conventional. But the exit option is more valuable under innovation because the probability of failure is higher.

The effects of $\theta$, $\delta$, and $\mu$ on $\sigma_{priv}^*$ are given by proposition 2. When $v_2 - v_1 \geq 0$, then $\sigma_{priv}^* = 1$ and so changes in the fundamental parameters do not affect the equilibrium. Therefore, proposition 2 focus on the case $\sigma_{priv}^* \in (0,1)$, for which $v_2 - v_1 < 0$. This is the case $R_2$ in Figure 3.

**Proposition 2** When $\sigma_{priv}^* \in (0,1)$, $\frac{\partial \sigma_{priv}^*}{\partial \theta} > 0$, $\frac{\partial \sigma_{priv}^*}{\partial \delta} > 0$, and $\frac{\partial \sigma_{priv}^*}{\partial \mu} > 0$.

Increases in $\delta$ and $\theta$ increase the NPV of innovation. Thus, the equilibrium amount of innovation increases.

Moreover, an increase in $\mu$ makes easier for a late-consumer insider to disguise a trade after $x_1 = 0$ as a liquidity shock. As a result, innovation becomes more attractive and, in equilibrium, there is more innovation, $\sigma_{priv}^*$ increases.

**2.2.3. The value of being private**

We now compute the expected value of the firm to the insider at $t = 0$, just after raising capital through private investors to fund the investment $I$. The expected value of the firm depends on the fraction of shares that the insider keeps after raising capital, $\alpha_{priv}$. Let $x_{priv}$ be the value for which the insider sells each share at date $t = 0$ and let $u_{priv} \equiv \sigma_{priv} u_{2,priv} + (1 - \sigma_{priv}) u_{1,priv}$ denote the expected value of the firm’s share for the insider. The value of the firm to the insider under private ownership is then given by

$$W_{priv} = \alpha_{priv} u_{priv} + (1 - \alpha_{priv}) c_{priv} x_{priv} - I. \quad (16)$$

The value of the shares for the investors can be $x_{priv} = u_{priv}$ or $x_{priv} \neq u_{priv}$ depending on whether the investors are subject to the liquidity shock and on the discount $k \leq 1$ for a sale from private to public.
If the investors are not subject to the liquidity shock, then the investors can wait until \( t = 2 \) to obtain the true value of the firm. In this case, \( x_{priv} \) is equal to the expected output of the firm in the case of private ownership, \( x_{priv} = v_{priv} \), where \( v_{priv} = \sigma_{priv} v_2 + (1 - \sigma_{priv}) v_1 \). If \( v_2 > v_1 \), then the investors expect \( \sigma_{priv} = 1 \) and so \( v_{priv} = v_2 \). If \( v_1 > v_2 \) and the investors expect \( \sigma_{priv} > 0 \), then \( v_{priv} < v_1 \). Even so, the investors do not discount the value of the firm because of the liquidity shock, as they can wait until \( t = 2 \). The value of \( \mu \) only affects \( u_{priv} \) indirectly by its influence on \( \sigma_{priv} \).

If the investors are subject to the liquidity shock together with the insider, they receive \( kV_{priv} \) at \( t = 1 \) for the sale of the shares, where \( k \leq 1 \) are the transaction costs from the sale from private to public. As the investors are subject to the same conditions as the insider, they exhibit the same strategic behavior at \( t = 1 \) as the insider. As a result, they value each share of the firm at the same value as the insider, \( x_{priv} = u_{priv} \). To understand how \( u_{priv} \) can be different than \( v_{priv} \), consider, for example, the extreme case with \( k = 0 \) and a liquidity shock for sure, \( \mu = 1 \). In this case, we have \( u_{priv} = 0 \) but \( v_{priv} > 0 \). Therefore, it can make difference if the investors are able to secure their shares until \( t = 2 \), or if the investors are subject to the shock together with the insider.\(^{14}\)

With \( k = 1 \), as we have in this section, the two cases are identical. We have \( u_{priv} = v_{priv} \) for any \( (p, \delta, \theta, \mu) \) if \( k = 1 \). The reason is that a sale from private to public does not imply transaction costs when \( k = 1 \). The market adjusts the values of \( \sigma_{priv} \) and \( u_{priv} \) to make \( u_{priv} = v_{priv} \). To prove that \( u_{priv} = v_{priv} \) if \( k = 1 \), substitute the expressions of \( u_{priv,1} \), \( u_{priv,2} \), and \( V_{priv} \) above on the definition of \( u_{priv} \). We obtain

\[
    u_{priv} = \mu v_{priv} + (1 - \mu) \left\{ \sigma_{priv} \left[ \delta p (1 + \theta p) + (1 - \delta p) p \right] + (1 - \sigma_{priv}) 2p \right\}. \tag{17}
\]

\(^{14}\)In principle, there could be a third possible case, in which the investors engaged in strategic behavior after a failure independently from the insider. However, the market could easily observe a sale from a private equity fund made independently from the insider. As the market antitipates when the investors would sell, an independent strategic behavior would identify the type of the firm, which would reveal the true value of the firm. As a result, it is optimal for the investors to behave in the same way as the insider to avoid having the type of the firm identified. This situation would then be identical to the case in which the investors and the insider receive the liquidity shock.
As $v_1 = 2p$ and $v_2 = \delta p (1 + \theta) + (1 - \delta) p$, then $u_{\text{priv}} = v_{\text{priv}}$. If, for example, $v_2 > v_1$, then proposition 1 implies $\sigma_{\text{priv}} = 1$ and so $u_{\text{priv}} = u_{\text{priv,2}} = v_2$. If $v_1 > v_2$ and $0 < \sigma_{\text{priv}} < 1$, then $v_2 < u_{\text{priv}} = v_{\text{priv}} < v_1$. The market takes the behavior of the insider into account to adjust the expected output of the firm at $v_{\text{priv}} < v_1$. In any case, as $k = 1$, $u_{\text{priv}} = v_{\text{priv}}$.

When $k < 1$, the same steps imply that $u_{\text{priv}} < v_{\text{priv}}$. Therefore, we will discuss the effects of valuing the shares at $u_{\text{priv}}$ or at $v_{\text{priv}}$ in section 4, when we discuss the case $k < 1$. Here, as $u_{\text{priv}} = v_{\text{priv}}$, we do not need to consider the two cases about the effects of the liquidity shock separately.

Given the valuation of the private investors, the revenue from selling shares must satisfy $(1 - \alpha_{\text{priv}}) c_{\text{priv}} v_{\text{priv}} \geq I$, substituting $x_{\text{priv}} = v_{\text{priv}}$. Because of the trading costs implied by $c_{\text{priv}} < 1$, the insider will sell the minimum number of shares for the investment. That is, $\alpha_{\text{priv}}$ is such that

$$(1 - \alpha_{\text{priv}}) c_{\text{priv}} v_{\text{priv}} = I. \quad (18)$$

To avoid uninteresting cases in which the investment can never be financed, let $I \in (0, c_{\text{priv}} \min\{v_1, v_2\})$. That is, the firm’s cost of capital is never so high to imply that funds for investment cannot be raised. The equilibrium insider’s stake is then $\alpha_{\text{priv}} = 1 - \frac{I}{c_{\text{priv}} v_{\text{priv}}}$.

Substituting the equilibrium value of $\alpha_{\text{priv}}$ in the expression of $W_{\text{priv}}$, we obtain $W_{\text{priv}} = u_{\text{priv}} - \frac{I}{c_{\text{priv}} v_{\text{priv}}}$. As $u_{\text{priv}} = v_{\text{priv}}$ for $k = 1$, the ex ante value of the firm to the insider under private ownership is then

$$W_{\text{priv}} = v_{\text{priv}} - \frac{I}{c_{\text{priv}}}. \quad (19)$$

The first term represents the expected outcome from the project decision and the second term is the initial investment cost. $W_{\text{priv}}$ is different from the first best in a frictionless economy because raising funds for investing is costly, $c_{\text{priv}} < 1$, which generates deadweight costs. Moreover, a surprising result is that $W_{\text{priv}}$ may differ from the first best in a full information economy because the equilibrium level of innovation $\sigma_{\text{priv}}^*$ may be excessive as compared to the first best. That is, we can have $v_{\text{priv}} < v_1$ even though $v_1 > v_2$. On the
other hand, under private ownership there is never too little innovation.

2.3. Public ownership

Consider now the case of public ownership, in which the insider funds the investment $I$ on the innovative or conventional projects by selling $1 - \alpha_{pub}$ shares at date 0 to the public market. As in the case of private ownership, the insider sells his remaining shares at date 1 if there is a liquidity shock. The difference between the public case and the private case is the transparency of earnings. In the case of public ownership, the earnings $x_1$ can be observed by all investors.

2.3.1. Selling behavior at date 1

The behavior of the insider after a success is different under public ownership. We obtained that the insider never sells voluntarily after a success under private ownership. Under public ownership, the insider always sells after a success if the insider undertakes the conventional project.

The steps to analyze the equilibrium are similar to the case of private ownership. Let $\sigma_{pub} \in [0, 1]$ denote the probability with which the insider chooses project 2, exploration.

Earnings transparency means that the market always knows when the firm experienced a failure, $x_1 = 0$. The market also knows that project 1 is always chosen after $x_1 = 0$. Therefore, although the market does not know which project was chosen at date 0, that knowledge is not relevant for the value of the firm when $x_1 = 0$. Regardless of the project chosen, the expected market value of the firm when $x_1 = 0$ is $p$ because there is no information asymmetry between the insider and the market. Thus, shares are always fairly valued when $x_1 = 0$ and the insider gains nothing by selling shares. (Comparing to the private case, $V_{pub} = p$ when $x_1 = 0$ and the insider is indifferent between selling or not after a failure.) We can assume that the insider sells or keeps his shares when $x_1 = 0$. The equilibrium payoffs are not affected by this choice.
The insider may however choose to sell shares after a success, \( x_1 = 1 \). Although the market knows that \( x_1 = 1 \), the market does not know which project was chosen at date 0. If project 1 was chosen, the expected value of the firm is \( 1 + p \). If project 2 was chosen, the expected value of the firm is \( 1 + \theta p \). Thus, the insider is weakly better off when the market believes that project 2 was initially chosen. That creates a value-relevant information asymmetry, which may distort the incentives of the insider when making project choice decisions.

The next lemma characterizes the behavior of a late-consumer insider after \( x_1 = 1 \).

**Lemma 3** In the public ownership case, a late-consumer insider at date 1 after observing a success, \( x_1 = 1 \):

1. Never sells shares if the innovative project was chosen.
2. Weakly prefers to sell shares if the conventional project was chosen.

Therefore, according to point 1, the insider never sells voluntarily at date 1 after exploration. The intuition is that, if project 2 was chosen, after \( x_1 = 1 \) the firm is always sold with a discount because the market can never be certain that project 2 was chosen.

On the other hand, according to point 2, the insider trades with probability 1 if the insider chooses the conventional project and obtains a success (to simplify the exposition, we assume that the insider sells in case of indifference). Selling after \( x_1 = 1 \) if the insider chooses project 1 is always profitable as long as the market assigns some probability to project 2.

In the case of private ownership, in contrast, the insider does not sell in case of success for either the conventional or the innovative projects. The reason for the difference in behavior is the fact that the outsiders can observe the success in the case of a public firm, but not in the case of a private firm. In the private case, a firm may have had a success, but the market always assigns a positive probability for a failure. As a result, it is never optimal to sell after a success. In the public case, the market can observe a success but still cannot
observe the project. So, with public ownership, it is optimal to sell after a success when the conventional project was chosen.

Given lemma 3, without the possibility of a liquidity shock, trading after $x_1 = 1$ would reveal the choice of project. Liquidity shocks allow insiders who choose project 1 to trade after $x_1 = 1$ without revealing the choice of the project. In equilibrium, late consumer insiders who have chosen project 1 pool with early consumer insiders.

2.3.2. Project choice at date 0

In equilibrium, the market must have correct beliefs and thus must assign probability $\sigma_{pub}$ to project 2 being chosen. When the market observes a success and the insider sells shares, the market assigns probability $s$ that project 2 has been chosen. The difference between $\sigma_{pub}$ and $s$ is that $\sigma_{pub}$ is the unconditional probability of choosing project 2 while $s$ is the probability of project 2 being chosen given that the insider sells shares and the market observes $x_1 = 1$:

$$s \equiv \Pr(\text{Project 2} \mid \text{Sale, } x_1 = 1) = \frac{\Pr(\text{Sale, } x_1 = 1 \mid \text{Project 2}) \Pr(\text{Project 2})}{\Pr(\text{Sale, } x_1 = 1)}. \quad (20)$$

The inputs for this formula are as follows. From lemma 3, the insider only sells after choosing project 2 and $x_1 = 1$ if there is a liquidity shock:

$$\Pr(\text{Sale, } x_1 = 1 \mid \text{Project 2}) = \mu \delta p. \quad (21)$$

From lemma 3, the probability of selling and $x_1 = 1$ is

$$\Pr(\text{Sale, } x_1 = 1) = (1 - \sigma_{pub}) p + \sigma_{pub} \delta p \mu. \quad (22)$$

Finally, the unconditional probability of project 2 is $\sigma_{pub}$. Therefore, equilibrium beliefs must be

$$s(\sigma_{pub}) = \frac{\sigma_{pub} \mu \delta}{(1 - \sigma_{pub}) + \sigma_{pub} \mu \delta}. \quad (23)$$
Given such beliefs, the market value of shares sold at $t = 1$ after a success is

$$V_{pub}^S = 1 + s (\sigma_{pub}) \theta p + [1 - s (\sigma_{pub})] p.$$  \hspace{1cm} (24)

We now calculate the expected gains for the insider from choosing either project 1 or project 2. The expected value of one share if the insider chooses project 1 is given by

$$u_{pub,1} = p V_{pub}^S + (1 - p) p.$$  \hspace{1cm} (25)

If the insider chooses project 1, the probability of success is $p$. In the case of a success, the insider sells and obtains $V_{pub}^S$. If there is a failure, the market value of the firm becomes $p$ as the best project to choose at date 1 is project 1, again with probability $p$ of success.

The expected gain per share for the insider from choosing project 2, exploration, is

$$u_{pub,2} = \delta p [ \mu V_{pub}^S + (1 - \mu) (1 + \theta p)] + (1 - \delta p) p.$$  \hspace{1cm} (26)

At date 1, the probability of success is $\delta p$. In case of success, the insider only sells if there is a liquidity shock, which happens with probability $\mu$. If a liquidity shock does not force the insider to sell, the insider keeps the shares until date 2 and continues to undertake project 2, now with probability of success equal to $\theta p$. If $x_1 = 0$, which happens with probability $(1 - \delta p)$, the insider obtains $p$ whether the insider keeps the shares or sells them.

The next proposition fully characterizes the equilibrium $\sigma_{pub}^*$ for all mixed strategy and pure strategy equilibria. For a given a set of parameters, the equilibrium $\sigma_{pub}^*$ is unique.

**Proposition 3** For each set of parameters $(p, \delta, \theta, \mu)$, there exists a unique equilibrium probability of exploration for the public ownership case $\sigma_{pub}^* \in [0, 1)$ given by

$$\sigma_{pub}^* = \frac{s^*}{\mu \delta + s^* (1 - \mu \delta)},$$  \hspace{1cm} (27)
where
\[
 s^* = \max \left\{ \frac{(v_2 - v_1) - \delta \mu p^2 (\theta - 1)}{p^2 (\theta - 1) (1 - \delta \mu)}, 0 \right\}.
\]  
(28)

Moreover, $\sigma^{*}_{pub}$ is such that

1. If $v_1 \geq v_2 - \delta \mu p^2 (\theta - 1)$, then $\sigma^{*}_{pub} = 0$ (in particular, exploitation for sure if $v_1 > v_2$).

2. If $v_1 < v_2 - \delta \mu p^2 (\theta - 1)$, then $\sigma^{*}_{pub} \in (0, 1)$.

Proposition 3 shows that an equilibrium with full innovation, $\sigma^{*}_{pub} = 1$, is never possible.

If the market expects exploration with probability 1, then choosing exploitation becomes a dominant strategy. By exploiting, the insider increases the probability of success and, if successful, makes a profit by selling the shares at date 1. The strategy $\sigma_{pub} = 1$ could only be an equilibrium without liquidity shocks, $\mu = 0$. In this case, the market would know that there are sales after a success only if the insider exploited; the insider would not be able to disguise exploitation. So, the insider sets $\sigma^{*}_{pub} = 1$, only if $\mu = 0$. For $\mu > 0$, the insider always sets $\sigma^{*}_{pub} < 1$.

The proposition also shows that the public ownership implies the use of project 1 when project 1 is efficient. When $v_1 > v_2$ then the insider chooses the conventional project with probability 1. This contrasts with the case of private ownership, in which the insider may choose the innovative project when the conventional project is efficient. On the other hand, if $v_2 > v_1$, however greater $v_2$ is, the insider never chooses to explore with probability one under public ownership. In fact, the insider may choose project 1 with probability 1 even though $v_2 > v_1$.

These results show that public ownership creates a bias against innovation. But it always induces the efficient choice of project when $v_1 > v_2$.

The effects of $\theta$, $\delta$, and $\mu$ on $\sigma^{*}_{pub}$ are given by Proposition 4. When $v_1 \geq v_2$, $\sigma^{*}_{pub} = 0$. Therefore, the proposition focus on the case $\sigma^{*}_{pub} \in (0, 1)$, for which $v_2 > v_1$.

**Proposition 4** When $\sigma^{*}_{pub} \in (0, 1)$, $\frac{\partial \sigma^{*}_{pub}}{\partial \theta} > 0$, $\frac{\partial \sigma^{*}_{pub}}{\partial \delta} > 0$, and $\frac{\partial \sigma^{*}_{pub}}{\partial \mu} < 0$. 


The innovative project is more valuable when $\theta$ or $\delta$ increase as these parameters increase the probability of success of the innovative project. The parameter $\delta$ increases the expected probability of success at $t = 1$, and $\theta$ increases the expected probability of success at $t = 2$ given that the project was successful at $t = 1$. As the innovative project is more valuable when $\theta$ or $\delta$ increase, an increase in one of these parameters make innovation increase. In the model, $\sigma_{pub}^*$ increases.

On the other hand, innovation decreases with an increase in the probability of the shock, $\frac{\partial \sigma_{pub}^*}{\partial \mu} < 0$. Recall that the proposition considers the case $v_2 > v_1$, as $\sigma_{pub}^* \in (0, 1)$. Therefore, $\sigma_{pub}^* < 1$ means that the insider undertakes the conventional project with positive probability although the conventional project is inefficient. The insider has this behavior because the probability of success of the conventional project is higher than the probability of success of the innovative project at $t = 1$, $p > \delta p$. The projects are such that $v_2 > v_1$ because $\theta p$ compensates the small probability of success at $t = 1$ of the innovative project. Given that the market does not observe the project choice but observes $x_1$, the insider sets $\sigma_{pub}^* < 1$ to increase the probability of a successful outcome at $t = 1$.

If liquidity shocks occur very often, it is easier for the insider to hide the choice of project 1. Frequent liquidity shocks make the market believe that the insider is selling because of a liquidity shock. Not because of success under exploitation. As it is easier to hide the choice of exploitation, the incentives to choose innovation decrease.

In contrast to $\frac{\partial \sigma_{pub}^*}{\partial \mu} < 0$, we have $\frac{\partial \sigma_{priv}^*}{\partial \mu} > 0$ under private ownership. However, under private ownership, $\sigma_{priv}^* \in (0, 1)$ for $v_1 > v_2$. An increase in $\mu$ lets the insider hide the choice of project 2 in the private case. In the private and public cases, the increase in $\mu$ gives incentives to the insider to undertake the inefficient choice.

In sum, proposition 4 says that exploiting the old method is better when $\theta$ or $\delta$ are low, or when $\mu$ is high.
2.3.3. The value of being public

Analogously to the case of private ownership, the ex ante value of the firm to the insider is given by

\[ W_{pub} = \alpha_{pub} u_{pub} + (1 - \alpha_{pub}) c_{pub} x_{pub} - I, \]  

where now the variables reflect the fact that the shares to fund the investment are sold to public investors. So, \( u_{pub} = \sigma_{pub} u_{pub,2} + (1 - \sigma_{pub}) u_{pub,1} \) and \( x_{pub} \) is the price paid by the public investors for each share of the firm at \( t = 0 \). Let \( v_{pub} = \sigma_{pub}^* v_2 + (1 - \sigma_{pub}^*) v_1 \).

As in the private case, we may have \( x_{pub} = u_{pub} \) or \( x_{pub} = v_{pub} \) if the investors are subject to the liquidity shock together with the insider or not. However, substituting the equilibrium values of \( \sigma_{pub}, u_{pub,1}, \) and \( u_{pub,2} \) on \( u_{pub} \), we obtain that \( u_{pub} = v_{pub} \). The reason in that there is no transaction cost after a liquidity shock when the firm starts public. A liquidity shock obliges the insider to sell the shares, but the sale is made through the stock market where the shares are already being traded. This situation is equivalent to case of private ownership with \( k = 1 \). Therefore, \( u_{pub} = v_{pub} \).

Working in a similar way as for the case in which the insider sells for private investors, the value of \( \alpha_{pub} \) is such that

\[ (1 - \alpha_{pub}) c_{pub} v_{pub} = I, \]  

as \( x_{pub} = v_{pub} \). Solving for \( \alpha_{pub} \) and substituting in the expression of \( W_{pub} \), we obtain

\[ W_{pub} = v_{pub} - \frac{I}{c_{pub}}. \]  

In the public ownership case, we have \( \sigma_{pub} < 1 \) even when \( v_2 > v_1 \), which implies \( v_{pub} < v_2 \). Therefore, there can be inefficiencies in equilibrium because of too little innovation. This kind of inefficiencies disappear when \( v_1 > v_2 \). In this case, \( \sigma_{pub} = 0 \) and \( v_{pub} = v_1 \). Also, there can be inefficiencies because raising funds through equity offerings is costly, as \( c_{pub} < 1 \). Having \( v_{pub} = v_1 \) when \( v_1 > v_2 \) and \( v_{pub} < v_2 \) when \( v_2 > v_1 \) implies that the public ownership
is especially effective in giving incentives to undertake the conventional technology.

3. The decision to go public or private

The insider chooses to go private or public according to the values of $W_{priv}$ and $W_{pub}$, the expected values of the firm for the insider under private and public ownership. If $W_{priv} > W_{pub}$ then the insider chooses the private ownership. That is, the insider chooses to raise funds through private investors such as the investors in a private equity fund (we include the indifference $W_{priv} = W_{pub}$ under this case). In the same way, if $W_{pub} > W_{priv}$ then the insider chooses the public ownership.

To simplify notation, define the relative cost advantage of public offerings compared to private offerings as

$$a \equiv \frac{1}{c_{priv}} - \frac{1}{c_{pub}} = \frac{c_{pub} - c_{priv}}{c_{priv} c_{pub}}.$$  \hspace{1cm} (32)

If public offerings are cheaper than private offerings ($c_{pub} > c_{priv}$), then $a > 0$.

Proposition 5 follows from the comparison of $W_{priv}$ with $W_{pub}$.

**Proposition 5** The private ownership structure is preferable to the public ownership structure if and only if

$$(\sigma_{priv} - \sigma_{pub}) (v_2 - v_1) \geq a I,$$  \hspace{1cm} (33)

for a given set of parameters $(p, \delta, \theta, \mu)$, where $\sigma_{priv}$ and $\sigma_{pub}$, are given by propositions 1 and 3.

The choice between public and private is driven by two considerations. The first consideration is the main novelty of our model: the choice between public versus private depends on the relative efficiency of innovative and conventional projects, $v_2 - v_1$. The second consideration, summarized in $a$, is the relative cost of capital advantage of public offerings compared to private offerings.
We have $\sigma_{\text{priv}} - \sigma_{\text{pub}} \geq 0$ for any $(p, \delta, \theta, \mu)$. Thus, going private is more attractive than going public when innovation is efficient ($v_2 - v_1 > 0$). In fact, if we shut down the effect of the cost of capital by setting $a = 0$, whether innovation is efficient or not is the only consideration for the choice of ownership structure, as shown in the next corollary.

**Corollary 1** Let $a = 0$, so that the private ownership structure is preferable to the public ownership structure if and only if $(\sigma_{\text{priv}} - \sigma_{\text{pub}})(v_2 - v_1) \geq 0$ for a given set of parameters $(p, \delta, \theta, \mu)$. Then:

1. If innovation is efficient ($v_2 > v_1$), the insider goes private.

2. If the conventional project is efficient ($v_1 > v_2$), the insider strictly prefers to go public if $\frac{v_1 - v_2}{p(1-\delta)} < T(0)$ and is indifferent between going public or private if $\frac{v_1 - v_2}{p(1-\delta)} \geq T(0)$.

3. If both projects are equivalent ($v_2 = v_1$), the insider is indifferent between going public or private.

If $v_2 > v_1$ then $\sigma_{\text{priv}} = 1$ and $\sigma_{\text{pub}} < 0$ by propositions 1 and 3. Therefore, the condition to go private is satisfied. If $v_1 \geq v_2$ then the insider assigns $\sigma_{\text{priv}} > 0$ in an inefficient way in some cases under private ownership. At the best, the insider assigns $\sigma_{\text{priv}} = 0$ when $v_1 \geq v_2$. At the most, when $\sigma_{\text{priv}} = 0$, the insider is indifferent between the public and the private ownership. When $\sigma_{\text{priv}} > 0$ and $v_1 > v_2$, the insider strictly prefers the public structure.

### 4. Robustness and extensions

While presenting our main results, we have made many simplifying assumptions to facilitate the exposition. In this section, we discuss the robustness of our model to relaxing some of these assumptions. We also show that some simple extensions lead to additional implications that also have empirical content.
4.1. Contracting

We have assumed that the firm can only issue one type of securities: straight share contracts. This assumption is not essential for the qualitative results of the paper. Although expanding the contracting space increases the number of instruments the insider can use to maximize firm value, they do not fully eliminate inefficiencies that may arise when the insider cannot choose between public and private ownership forms. We illustrate this fact with an example that allows the company to have debt in its capital structure.

4.1.1. Debt

Our goal here is to understand whether the availability of debt securities makes the choice of ownership irrelevant. For the sake of brevity, we focus only on the public case when \( v_2 - v_1 > 0 \); that is, there is a public market for the firm’s shares but the firm can also borrow to finance some or all of its investments (debt can be either public or private). In this case, debt is likely to have an impact on incentives to innovate. The asset substitution effect (Jensen and Meckling, 1976) makes risky projects more attractive when there is debt. Therefore, this effect could offset the public ownership bias against innovation. We investigate here whether this conjecture is true.

Suppose that the firm finances its investment fully with debt with face value \( D \), a zero-coupon long-term bond, to be paid in the end of period 2.\(^{15}\) If debt is not paid in full, bondholders seize the company’s cash flows. It is trivial to show that nothing changes from the previous analysis if \( D \leq S = 1 \), thus here we focus on the interesting case in which \( D \in (1, 2) \). In this case, default occurs unless the firm observes two successes in a row.

Suppose that the insider chooses project 2 with probability \( \sigma' \). Given that in equilibrium the market’s belief that the insider has chosen 2 must be \( \sigma' \), the insider will not trade voluntarily after choosing project 2 and \( x_1 = 1 \). However, the insider will trade with probability

\(^{15}\)As it will become clear, financing the initial investment fully with debt is the optimal financing choice unless the cost of debt capital is higher than the cost of equity capital.
1 if he used project 1 and \(x_1 = 1\). When the market observes \(x_1 = 1\) and there are shares being sold, market prices in equilibrium are

\[
V'(\sigma') = s'(\sigma') \theta p (2 - D) + (1 - s'(\sigma')) p (2 - D).
\]

(34)

Simplifying,

\[
V'(\sigma') = [1 + s'(\sigma') (\theta - 1)] p (2 - D).
\]

(35)

Thus, the per share expected utility from choosing project 1 is

\[
u_1'(\sigma') = p V'(\sigma'),
\]

(36)

while the per share expected utility from project 2 is

\[
u_2'(\sigma') = \mu \delta p V'(\sigma') + (1 - \mu) \delta \theta p^2 (2 - D).
\]

(37)

For the insider to be willing to randomize between 1 and 2, we need \(s'(\sigma') = s'\) where

\[
s' \equiv \frac{(1 - \mu) \delta \theta - (1 - \mu \delta)}{(\theta - 1) (1 - \mu \delta)} > 0.
\]

(38)

Crucially, we have \(s' < 1\), implying that the first best \((s = 1)\) cannot be implemented in this case.

To find out the effect of debt on the probability of innovation, we need to compare \(s'\) with

\[
s^* = \max \left\{ \frac{\delta [1 + p (\theta - 1) (1 - \mu)] - 1}{p(\theta - 1) (1 - \delta \mu)}, 0 \right\}.
\]

(39)

Algebra shows that

\[
s' - s^* = \max \left\{ \frac{(1 - \delta) (1 - p)}{p(\theta - 1) (1 - \delta \mu)}, 0 \right\} > 0,
\]

(40)

which implies that debt increases the amount of innovation in the public case when \(v_2 > v_1\).
Thus, if firms want to innovate more but remain public, it is optimal to lever up. However, we also find that $s' < 1$, so the public ownership structure with debt is still inferior to the private ownership case when $v_2 > v_1$. In sum, the capital structure is not a perfect substitute for the ownership structure in providing incentives to innovation.

### 4.1.2. Other contractual arrangements

In our model, as long as the insider can choose between public and private structures ex ante, the first-best outcomes are always achieved. Thus, any other contractual solution can at best replicate what the choice of ownership mode does. Although a full analysis of the optimal contractual solution is beyond the scope of this paper, the case in which debt contracts are allowed illustrates the limitations of contractual solutions that do not involve an optimal choice of ownership mode.

### 4.2. Illiquid private securities

As discussed in Subsection 2.1.4, private securities are probably more difficult to unload than public securities because private securities are not traded in centralized markets. To capture the relative illiquidity of private securities, we now assume that, for each dollar sold in shares at date 1 if the firm is private, the insider only pockets $k < 1$.

Most of the analysis of the private case remains unchanged. In particular, Lemma 1 is not affected: also for $k < 1$, an insider never sells if there is a success.

The probability of selling after a failure, however, changes. With $k < 1$, the necessary condition for selling shares after a failure changes to

\[
  k V_{\text{pri}o} \geq p. \tag{41}
\]

---

16Such an analysis can be done as in Manso (2009), with three important modifications: (i) Diamond-Dybvig preferences, (ii) free trading of securities at date 1 (i.e. the possibility of exiting the contract at date 1), and (iii) different levels of transparency of date 1 cash flows (public versus private).
Because $V_{priv} > p$, we have $kV_{priv} > p$ for $k$ close enough to 1. As a result, the insider sells for sure after a failure if the market for private securities is liquid enough: as $k$ approaches 1, eventually we get $b = 1$. On the other hand, if the market at date 1 is very illiquid ($k$ close to zero), then a late-consumer insider never sells: $b = 0$. For intermediate values of $k$, the equilibrium is in strictly mixed strategies, with $b \in (0, 1)$ and $b$ increasing in $k$. The next lemma formalizes these results.

**Lemma 4** *In the private ownership case with $k \in (0, 1)$, a late-consumer insider sells shares with equilibrium probability $b(k, \sigma_{priv})$ at date 1 after observing $x_1 = 0$, where*

$$
\begin{align*}
    b(k, \sigma_{priv}) &= \begin{cases} 
        1, & \text{if } k \geq k_1(\sigma_{priv}), \\
        \frac{k[\mu + \sigma_{priv}(v_2 - v_1)] - p}{\mu(1 - \mu)[1 - p + \sigma_{priv}(1 - \delta)]p}, & \text{if } k_2(\sigma_{priv}) < k < k_1(\sigma_{priv}), \\
        0, & \text{if } k \leq k_2(\sigma_{priv}), 
    \end{cases}
\end{align*}
$$

(42)

$$
\begin{align*}
    k_1(\sigma_{priv}) &\equiv \frac{\mu p + (1 - \mu)[1 - p + \sigma_{priv}(1 - \delta)]p}{\mu[v_1 + \sigma_{priv}(v_2 - v_1)] + (1 - \mu)[1 - p + \sigma_{priv}(1 - \delta)]p}, \\
    k_2(\sigma_{priv}) &\equiv \frac{p}{v_1 + \sigma_{priv}(v_2 - v_1)}. 
\end{align*}
$$

The threshold values $k_1$ and $k_2$ define three regions for the behavior of the insider, as shown in Figure 4. In Region 3, the insider never sells shares. In Region 2, the insider plays a strictly mixed strategy on selling shares. The probability $b$ of selling after failure increases with market liquidity. If the market for private securities is liquid enough, $k \geq k_1$, as shown in Region 1, then the insider sells after a failure with probability 1.

Figure 4 also illustrates the effect of the liquidity shock on the equilibrium strategy. If $\mu$ increases, $k_1$ decreases: a late-consumer insider sells shares with probability 1 for a larger set of values for $k$. That is, the insider sells even with a less liquid market. Intuitively, if $\mu$ increases, it becomes easier for the insider to disguise a failure behind a liquidity shock. The insider has more incentives to sell.
Figure 4. $b(k)$: probability of a late-consumer insider selling shares after $x_1 = 0$.

We redefine $T$ (the intrinsic value of the option to exit early for a late-consumer insider) as

$$T(\sigma_{\text{priv}}, k) = \max\{k V_{\text{priv}} - p, 0\}.$$  \hfill (43)

This option has zero value if the underlying $k V_{\text{priv}}$ is low, which may happen because the market for private securities is very illiquid (low $k$) or because the market is “cold,” i.e. the market believes that when an insider sells shares, $x_1 = 0$ is very likely ($\mu$ is low). In terms of the regions in Figure 4, $T(\sigma_{\text{priv}}, k)$ is strictly positive in Region 1, while zero in Regions 2 and 3.

The next proposition generalizes our results in proposition 2 to the case where $k \leq 1$.

**Proposition 6** In the private ownership case with $k \in [0, 1]$, for each set of parameters $(o, \delta, \theta, \mu, k)$, the equilibrium $\sigma^*_{\text{priv}}(k)$ is given by:

1. If $v_2 > v_1$, then $\sigma^*_{\text{priv}}(k) = 1$ (exploration for sure if innovation is efficient, for all $k$).
2. If $v_2 < v_1$, then $\sigma^*_\text{priv} (k)$ is given by

$$\sigma^*_\text{priv}(k) = \begin{cases} 1, & \text{if } \frac{v_1 - v_2}{p(1-\delta)} \leq T(1,k), \\ \sigma^*_\text{priv} \text{ such that } T(\sigma^*_\text{priv}, k) = \frac{v_1 - v_2}{p(1-\delta)}, & \text{if } T(1,k) < \frac{v_1 - v_2}{p(1-\delta)} < T(0,k), \\ 0, & \text{if } T(0,k) \leq \frac{v_1 - v_2}{p(1-\delta)}. \end{cases}$$ (44)

3. If $v_2 = v_1$, then $\sigma^*_\text{priv} (k) \in \arg\min_{\sigma_{\text{priv}} \in [0,1]} T(\sigma_{\text{priv}}, k)$.

The bias of the private ownership structure towards innovation is also present here: we can have $\sigma^*_\text{priv} = 1$ with $v_1 > v_2$ and $k < 1$. That is, the insider may choose the innovative project with certainty even though the conventional project is more efficient and the market for private securities is illiquid.

Starting from an equilibrium with $\sigma^*_\text{priv} = 1$ and $v_1 > v_2$, as $k$ decreases towards zero, the insider eventually chooses a mixed strategy between the innovative and the conventional project ($0 < \sigma^*_\text{priv} < 1$). As $k$ continues to decrease, the insider sooner or later chooses the conventional project with certainty ($\sigma^*_\text{priv} = 0$). If $v_1 = v_2$, the insider may be indifferent among several strategies and we can have multiple $\sigma^*_\text{priv}$ in equilibrium. When $v_2 > v_1$, for any $k$, the insider always chooses the innovative project with certainty ($\sigma^*_\text{priv} = 1$).

4.2.1. The decision to go private or public with $k < 1$

In the same way as with $k = 1$, the insider chooses to go private or public to obtain the maximum ex ante value of the firm to the insider. The value of $W_{\text{pub}}$ is unchanged, as there are no liquidity costs associated with selling shares at date 1 in the public case, $W_{\text{pub}} = v_{\text{pub}} - \frac{L}{c_{\text{pub}}}$. In the private case, on the other hand, the share value of the firm must now take into account the discount $k < 1$ when there is a sale from private to public. For the private case, as the fraction of shares of the insider $\alpha_{\text{priv}}$ and the expected share value $u_{\text{priv}}$ change with $k$, the expression of $W_{\text{priv}}$ changes.

As before, the fraction of shares of the insider is obtained by $(1 - \alpha_{\text{priv}}(k))c_{\text{priv}}x_{\text{priv}}(k) =$
where $x_{\text{priv}}(k) = u_{\text{priv}}(k)$ if the investors receive the liquidity shock with the insider (and, therefore, act in the same way as the insider), or $x_{\text{priv}}(k) = v_{\text{priv}}(k)$ if the investors can wait until $t = 2$ to obtain the output of the firm. We have now $u_{\text{priv}}(k) = \sigma_{\text{priv}}(k) u_{\text{priv},2}(k) + [1 - \sigma_{\text{priv}}(k)] u_{\text{priv},1}(k)$, where

\begin{align*}
    u_{\text{priv},1}(k) &= \mu k V_{\text{priv}} + (1 - \mu) \{ (1 - \sigma_{\text{priv}})(1 - \delta p) b + (1 - \sigma_{\text{priv}}) (1 - p) b \} V_{\text{priv}}(k), \\
    u_{\text{priv},2}(k) &= \mu k V_{\text{priv}} + (1 - \mu) \{ (1 - \delta p) b k V_{\text{priv}} + (1 - b) p + \delta p (1 + \theta p) \} V_{\text{priv}}(k),
\end{align*}

and $v_{\text{priv}}(k) = \sigma_{\text{priv}}(k) v_2 + [1 - \sigma_{\text{priv}}(k)] v_1$. As $(1 - \alpha_{\text{priv}}(k)) c_{\text{priv}} x_{\text{priv}}(k) = I$, then $W_{\text{priv}}(k) = \alpha_{\text{priv}}(k) u_{\text{priv}}(k)$. Substituting $\alpha_{\text{priv}}(k) = 1 - I c_{\text{priv}} x_{\text{priv}}(k)$ implies

$$W_{\text{priv}}(k) = u_{\text{priv}}(k) - \frac{I}{c_{\text{priv}} x_{\text{priv}}(k)}.$$  

We saw in section 2.2.4 that $u_{\text{priv}} = v_{\text{priv}}$ when $k = 1$ and that $u_{\text{priv}}(k) < v_{\text{priv}}(k)$ if $k < 1$. Therefore, the case $x_{\text{priv}}(k) = u_{\text{priv}}(k)$ is more stringent for the insider, as the investors give a lower valuation for the firm shares. It is harder for the insider to raise $I$. As a result, $W_{\text{priv}}(k)$ is smaller when $x_{\text{priv}}(k) = u_{\text{priv}}(k)$ and $k < 1$. It is more likely that insider chooses the private ownership if $x_{\text{priv}}(k) = v_{\text{priv}}(k)$. However, the conclusions about the incentives to innovate with private ownership are unchanged whether $x_{\text{priv}}(k) = v_{\text{priv}}(k)$ or $x_{\text{priv}}(k) = u_{\text{priv}}(k)$.

Substituting the expressions of $\sigma_{\text{priv}}(k)$, $u_{\text{priv},1}(k)$, and $u_{\text{priv},2}(k)$ on the definition of $u_{\text{priv}}(k)$ implies

$$u_{\text{priv}}(k) = v_{\text{priv}}(k) - (1 - k) \{ \mu + (1 - \mu) [\sigma_{\text{priv}} (1 - \delta p) b + (1 - \sigma_{\text{priv}}) (1 - p) b] \} V_{\text{priv}}(k).$$  

Consider first the case $x_{\text{priv}}(k) = u_{\text{priv}}(k)$. Then, equation (47) implies

$$W_{\text{priv}}(k) = v_{\text{priv}}(k) - \frac{I}{c_{\text{priv}}} - L(k),$$  

(49)
where

\[ L(k) \equiv (1 - k) \{ \mu + (1 - \mu) [\sigma_{\text{priv}} (1 - \delta p) b(k) + (1 - \sigma_{\text{priv}}) (1 - p) b(k)] \} V_{\text{priv}}. \] (50)

The new term \( L(k) \) represents the expected costs of illiquidity associated with the sale of shares at date 1. This represents another source of deadweight losses associated with private ownership: the trading of shares because of liquidity shocks or privileged information is costly because private securities are illiquid.

The choice between public and private is then modified to include this cost. Now the private ownership structure is preferable to the public ownership structure if and only if

\[ [\sigma_{\text{priv}}(k) - \sigma_{\text{pub}}] (v_2 - v_1) \geq a I + L(k). \] (51)

where \( a \) is as defined in (32). When \( c_{\text{priv}} = c_{\text{pub}} (a = 0) \), we have the following proposition.

**Proposition 7** When \( c_{\text{priv}} = c_{\text{pub}} \), the decision of the insider about the ownership structure is given by:

1. If \( v_1 \geq v_2 \), the insider always chooses the public structure.

2. If \( v_2 > v_1 \), there is a unique \( k^* \in (0, 1) \) such that the insider chooses the public structure if \( k < k^* \) and chooses the private structure if \( k \geq k^* \).

If private securities are less liquid than public securities \((k < 1)\), the insider faces a trade-off when \( v_2 > v_1 \): the private structure provides appropriate incentives to innovate but imposes liquidity costs. If the liquidity costs are large \((k \text{ small})\), the insider prefers to choose the public structure even though it leads to less innovation. If we think of \( k < 1 \) as representing the costs of selling private securities, such as IPO costs, our model suggests that innovation is fostered by the development of IPO markets. In the model, an increase in \( k \).
The conclusions are the same for the case in which $x_{\text{priv}}(k) = v_{\text{priv}}(k)$. The only change is that the insider will be willing to choose the private ownership for a smaller $k$, as $W_{\text{priv}}(k)$ is higher for $x_{\text{priv}}(k) = v_{\text{priv}}(k)$. So, there will be a $k^{**} < k^*$ that satisfies the condition 2 of proposition 7.

5. Conclusions

Our results suggest that public and private firms invest in fundamentally different ways. Private firms take more risks, invest more in new products and technologies, and pursue more radical innovations. Private firms are more likely to choose projects that are complex, difficult to describe, and untested. Organizational change is also more likely under private ownership. Mergers and acquisitions, divestitures, and changes in organizational structure and management practices are more easily motivated under private ownership.

On the other hand, public firms choose more conventional projects. Their managers appear short-sighted; they care too much about current earnings. They find it difficult to pursue complex projects that the market does not appear to understand well. Public firms go private after bad shocks, when it is clear that their business models are no longer working and there is need for restructuring.

Anecdotal and systematic evidence corroborates the link between private ownership and innovative change. Firms that go private pursue more influential innovations (Lerner, Sorensen, and Strömberg, 2010) and engage more in organizational change (Davis et al., 2009). There is also some evidence that private equity owned firms introduce innovations in management practices (Bloom, Sadun, and Van Reneen, 2008).

Moon (2006) describes the acquisition by Morgan Stanley Capital Partners of an oil and gas subsidiary of a utility that was undergoing a restructuring. The company had good long term-prospects according to independent analysts, but faced several years of negative cash flows due to the restructuring efforts. Although finding strategic buyers for the company
seemed the most logical solution, none of the public firms in the industry appeared to be willing to deal with the complexity of the business and with its negative cash flows. Private equity investors, on the other hand, were keen to deal with this uncertainty and with the prospect of negative cash flows in the short run.

There are still some untested implications of our model. Our model predicts that cash-flow volatility should be higher in private than public firms. Private firms should be more profitable during technological revolutions, while public firms should be more valuable in mature but growing industries.

Our model also has implications for the decision to go public or private. Firms are likely to go public after a technological breakthrough, that is, when it makes sense to exploit a newly discovered technology. Firms are likely to go private after suffering permanent negative productivity shocks, that is, when their existing technologies or business models become permanently unprofitable. Chemmanur, He, and Nandy (2007) find that firms go public at the peak of their productivity and then performance declines after going public. This is consistent with firms going public only after perfecting a new technology; they become public in the “harvesting” period. The model also explains why companies go private when performance is particularly poor.

Finally, we note that there are many directions to which the model can be extended. Our model emphasizes two important effects—short termism and (lack of ) tolerance for failures—that make public firms ill suited to pursue innovations. But one could also argue, along the lines of Burkart, Gromb, and Panunzi (1997), that the “hands-off” approach of public shareholders is necessary to foster managerial initiative, and may counteract the effects we emphasize here. This is a promising avenue for future theoretical and empirical explorations.
6. Appendix—Proofs

Lemma 1.

Proof. Let \( b^F \) denote the probability of selling given a failure and \( b^S \) denote the probability of selling given a success, both for the case of no liquidity shock. We will prove that \( b^S = 0 \).

First, we have \( b^F \geq b^S \). The reason is that the condition to sell in case of success is more stringent than the condition to sell in case of failure. We need \( V_{priv} \geq 1 + p \) for the insider to sell in case of success if the insider undertakes the conventional project, or \( V_{priv} \geq 1 + \theta p \) if the insider undertakes the innovative project. For the insider to sell in case of failure, we need \( V_{priv} \geq p \). Any firm that satisfies a condition for selling after a success also satisfies the condition for selling after a failure. In the same way, fewer firms that satisfy the condition to sell after a failure also satisfy the condition to sell after a success. Therefore, the probability of selling after a failure must be at least as large as the probability of selling after a success, \( b^F \geq b^S \).

To prove that \( b^S = 0 \), we show that \( V_{priv} < 1 + p \) always. As \( 1 + p < 1 + \theta p \), the insider never sells after a success whether the insider undertakes the conventional or the innovative project.

Let \( h \) denote the probability that the project failed, given that the insider is selling the shares, \( h \equiv \Pr (F \mid Sale) \), and let \( s \) denote the probability that the insider has chosen the innovative project given the sale of shares, \( s \equiv \Pr (Project \ 2 \mid Sale) \). Both \( h \) and \( s \) are probabilities assigned by the market. A rational market values each share at \( t = 1 \) at

\[
V_{priv} = hp + (1 - h) [s (1 + \theta p) + (1 - s) (1 + p)] .
\]  

(52)

The share’s value increases if \( s \) increases, that is, if the market puts a higher probability of undertaking the innovative project. Let \( s = 1 \) to imply the highest \( V_{priv} \) given the other
parameters. \( s = 1 \) implies
\[
V_{\text{priv}} \leq h p + (1 - h)(1 + \theta p).
\]  
(53)

Similarly, \( V_{\text{priv}} \) is high if the market puts a small probability on the sale being motivated by a failure, that is, if \( h \) is low. If it were possible to set \( h = 0 \), then \( V_{\text{priv}} = 1 + \theta p > 1 + p \) and the insider would sell, fooling a market that expected the innovative project. But the market cannot observe the output of the firm at \( t = 1 \) and so \( h > 0 \).

To find the lowest possible value of \( h \), use Bayes’s rule to write 
\[
h = \frac{\Pr(Sale | F) \Pr(F)}{\Pr(Sale)}.
\]
So 
\[
h \geq \Pr(F) \iff \Pr(Sale | F) \geq \Pr(Sale).
\]
The value of \( \Pr(Sale) \) is given by 
\[
\Pr(Sale) = \mu + (1 - \mu) \{\sigma_{\text{priv}}[\delta pb^S + (1 - \delta p)b^F] + (1 - \sigma_{\text{priv}})[pb^S + (1 - p)b^F]\}.
\]
On the other hand, 
\[
\Pr(Sale | F) = \frac{\Pr(Sale \cap F)}{\Pr(F)},
\]
where 
\[
\Pr(Sale \cap F) = \mu[\sigma(1 - \delta p) + (1 - \sigma_{\text{priv}})(1 - p)] + (1 - \mu)[\sigma_{\text{priv}}(1 - \delta p) + (1 - \sigma_{\text{priv}})(1 - p)]b^F
\]
and 
\[
\Pr(F) = \sigma_{\text{priv}}(1 - \delta p) + (1 - \sigma_{\text{priv}})(1 - p).
\]
So, as \( \mu < 1 \), 
\[
\Pr(Sale | F) \geq \Pr(Sale) \iff b^F \geq b^S,
\]
which holds by the discussion above. Therefore, 
\[
h = \Pr(F | Sale) \geq \Pr(F).
\]
Depending on the project undertaken, \( \Pr(F) = 1 - p \) or \( \Pr(F) = 1 - \delta p \), with \( 1 - \delta p > 1 - p \). So, we always have \( h \geq 1 - p \).

One way of understanding that \( h \geq 1 - p \) is that, if \( \mu = 1 \), then \( \Pr(F | Sale) = \Pr(F) \) as all firms are sold anyway. So, the fraction of firms with \( F \) is equal to the unconditional fraction. If \( \mu < 1 \), then some firms will not be sold. The firms that are not sold are more likely to be the successful firms (as \( b^F \geq b^S \)). So, it will be easier to find \( F \) firms in the pool of firms that were sold. As a result, \( \Pr(F | Sale) > \Pr(F) \geq 1 - p \).

Substituting \( s = 1 \) and the lowest possible value of \( h \), \( h = 1 - p \), the values for which \( V_{\text{priv}} \) is highest, we obtain 
\[
V_{\text{priv}} \leq (1 - p)p + p(1 + \theta p).
\]
Moreover, \( \theta < 1/p \). So, 
\[
V_{\text{priv}} < p(1 - p)(1 + 1) = p(3 - p).
\]
We have 
\[
p(3 - p) < 1 + p \iff (p - 1)^2 > 0,
\]
which is always true as \( p = 1 \) is ruled out by assumption. Therefore,
\[
V_{\text{priv}} < 1 + p
\]  
(54)
and the condition for selling never holds. As \( b^S = 0 \), we denote the probability of selling after a failure by \( b \) instead of \( b^F \) to simplify notation. ■

**Lemma 2.**

**Proof.** For any given pair of market beliefs \((\sigma_{\text{priv}}, b)\), the insider sells with probability 1 after a failure if \( V_{\text{priv}} > p \iff m[\sigma_{\text{priv}} v_2 + (1 - \sigma_{\text{priv}}) v_1] + (1 - m) p > p \). As \( \mu > 0 \), we have \( m > 0 \). So, \( V_{\text{priv}} > p \) holds for any \((\sigma_{\text{priv}}, b)\) because \( v_1 > p \) and \( v_2 > p \). ■

**Proposition 1.**

**Proof.** It is sufficient to obtain the value of \( \sigma_{\text{priv}} \) that satisfies the incentive compatibility (IC) constraints for \( v_2 - v_1 \geq 0 \) and for \( v_2 - v_1 < 0 \). From \( T(\sigma_{\text{priv}}) \equiv V_{\text{priv}} - p \), the expression of \( T(\sigma_{\text{priv}}) \) is \( T(\sigma_{\text{priv}}) = m(\sigma_{\text{priv}})[\sigma_{\text{priv}} v_2 + (1 - \sigma_{\text{priv}}) v_1 - p] \). We have \( T(\sigma_{\text{priv}}) > 0 \) as \( m > 0 \), \( v_1 > p \) and \( v_2 > p \).

**Case 1.** If \( v_2 - v_1 \geq 0 \), then the IC condition for project 2, \( v_2 - v_1 + p(1 - \delta) T(\sigma_{\text{priv}}) > 0 \), is trivially satisfied as \( T(\sigma_{\text{priv}}) > 0 \). On the other hand, the IC condition for project 1 cannot be satisfied. Therefore, \( \sigma_{\text{priv}}^* = 1 \) is the only equilibrium.

**Case 2.** If \( v_1 - v_2 > 0 \), then, as \( V_{\text{priv}} - p > 0 \),

\[
\frac{\partial T(\sigma_{\text{priv}})}{\partial \sigma_{\text{priv}}} = \frac{\partial m(\sigma_{\text{priv}})}{\partial \sigma_{\text{priv}}} [\sigma v_2 + (1 - \sigma) v_1 - p] - (v_1 - v_2) m(\sigma_{\text{priv}}) < 0 \tag{55}
\]

because \( \frac{\partial m(\sigma_{\text{priv}})}{\partial \sigma_{\text{priv}}} < 0 \) and \( [\sigma v_2 + (1 - \sigma) v_1 - p] > 0 \). Therefore, the highest value for the option to exit is \( T(0) = \frac{\mu p}{\mu \vdash (1 - \mu)(1 - p)} \). As a result, if \( T(0) \leq \frac{v_1 - v_2}{p(1 - \delta)} \) then the IC condition for project 1 is satisfied and the IC condition for project 2 cannot be satisfied. So, \( u_{\text{priv},1} > u_{\text{priv},2} \) and \( \sigma_{\text{priv}}^* = 0 \).

Now, \( T \) is minimized at \( \sigma_{\text{priv}} = 1 \), \( T(1) = \frac{\mu p([1 + p(\theta - 1)])}{\mu \vdash (1 - \mu)(1 - \delta)} \). Therefore, as \( T(\sigma) \) is decreasing in \( \sigma \in (0, 1) \), there exists a unique \( \sigma^* \in (0, 1) \) such that \( T(\sigma_{\text{priv}}^*) = \frac{v_1 - v_2}{p(1 - \delta)} \) if \( T(1) < \frac{v_1 - v_2}{p(1 - \delta)} < T(0) \). In this case, the IC condition holds with equality and \( u_{\text{priv},1} = u_{\text{priv},2} \).

On the other hand, if \( \frac{v_1 - v_2}{p(1 - \delta)} \leq T(1) \) then the IC condition for project 2 is satisfied and the IC condition for project cannot be satisfied. In this case, \( u_{\text{priv},2} > u_{\text{priv},1} \) and \( \sigma_{\text{priv}}^* = 1 \).
Lemma 3.

Proof. Point 1. Rational market beliefs imply that shares sold after \( x_1 = 1 \) can be valued at most at \( 1 + \theta p \). Therefore, the late-consumer insider strictly prefers to keep his shares unless the market believes that \( \sigma_{pub} = 1 \). However, \( \sigma_{pub} = 1 \) cannot be an equilibrium. If the market believes that \( \sigma_{pub} = 1 \), then the insider would instead exploit, sell in case of success, and obtain an expected payoff \( p (1 + \theta p) + (1 - p) p > \delta p (1 + \theta p) + (1 - \delta p) p \). (Recall that the market observes \( x_1 = 1 \) but cannot observe the project.) Therefore, \( \sigma_{pub} = 1 \) cannot be an equilibrium. Thus, if an equilibrium exists, it must be that \( \sigma_{pub} < 1 \). As \( \sigma_{pub} < 1 \), the insider in the public structure never sells in case of success.

Point 2. Rational market beliefs imply that shares sold after \( x_1 = 1 \) can be valued at least at \( 1 + p \). The late-consumer insider then strictly prefers to sell his shares unless the market believes that \( \sigma_{pub} = 0 \), in which case he is indifferent between selling or not selling.

Proposition 2.

Proof. Suppose that \( \sigma^*_{priv} \in (0, 1) \). In this case, the IC condition implies that \( \sigma^*_{priv} \) is defined implicitly by \( v_2 - v_1 + p(1 - \delta) T(\sigma^*_ {priv}) = 0 \). Define \( G(\sigma_{priv}, \mu, \theta, \delta) = v_2 - v_1 + p(1 - \delta) T(\sigma^*_ {priv}) \). Substituting \( T(\sigma_{priv}) = V_{priv} - p \) implies \( G(\sigma_{priv}, \mu, \theta, \delta) = v_2 - v_1 + p(1 - \delta) \{m(\sigma_{priv})[\sigma v_2 + (1 - \sigma) v_1 - p] \} \). Using the implicit function theorem, \( \frac{\partial \sigma_{priv}}{\partial x} = -\frac{\partial G}{\partial x} \frac{\partial x}{\partial \sigma_{priv}} \), where \( x \) is the parameter of interest and \( \frac{\partial G}{\partial \sigma_{priv}} < 0 \), as \( \frac{\partial G(\sigma_{priv}, \mu, \theta, \delta)}{\partial \sigma_{priv}} = p(1 - \delta) \{ \frac{\partial m(\sigma_{priv})}{\partial \sigma_{priv}}[\sigma v_2 + (1 - \sigma) v_1 - p] + (v_2 - v_1) m(\sigma_{priv}) \} \), \( \frac{\partial m}{\partial \sigma_{priv}} < 0 \), and \( v_2 - v_1 < 0 \). We have \( \frac{\partial G(\sigma^*_ {priv}, \mu, \theta, \delta)}{\partial \theta} = p(1 - \delta)[p^2 \delta + p (1 - \delta) m(\sigma^*_ {priv}) p^2 \delta \sigma^*_ {priv}] > 0 \) which implies \( \frac{\partial \sigma^*_ {priv}}{\partial \theta} > 0 \). Moreover, after some algebra, it can be shown that

\[
\frac{\partial G(\sigma^*_ {priv}, \mu, \theta, \delta)}{\partial \delta} = \left[ 1 + p (\theta - 1) \right] - \left[ p m(\sigma^*_ {priv})[p + \sigma^*_ {priv} (v_2 - v_1)] + p(1 - \delta) \left\{ \frac{\partial m(\sigma^*_ {priv})}{\partial \delta} \left[ p + \sigma^*_ {priv} (v_2 - v_1) \right] + \sigma^*_ {priv} p [1 + p (\theta - 1)] \right\} > 0 \right],
\]

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which implies \( \frac{\partial \sigma_{\text{priv}}^*}{\partial \mu} > 0 \). For the effect of \( \mu \) on \( \sigma_{\text{priv}}^* \), we have

\[
\frac{\partial G(\sigma_{\text{priv}}^*; \mu, \theta, \delta)}{\partial \mu} = p(1 - \delta) \frac{\partial m}{\partial \mu}[p + \sigma_{\text{priv}}^*(v_2 - v_1)] > 0, \text{ because } \frac{\partial m}{\partial \mu} > 0. \text{ So, } \frac{\partial \sigma_{\text{priv}}^*}{\partial \mu} > 0. \]

**Proposition 3.**

**Proof.** For the insider to be willing to randomize between projects 1 and 2, we must have equal expected gains from both projects, that is

\[
pV_{\text{pub}}(\sigma_{\text{pub}}) + (1 - p)p = \delta p[\mu V_{\text{pub}}(\sigma_{\text{pub}}) + (1 - \mu)(1 + \theta p)] + (1 - \delta p)p. \tag{57}
\]

Solving for \( s \),

\[
s^* = \frac{\delta[1 + p(\theta - 1)] - \delta \mu p(\theta - 1) - 1}{p(\theta - 1)(1 - \delta \mu)}, \tag{58}
\]

as long as the numerator is positive. If negative, the equilibrium \( s^* \) is zero because project 1 always gives higher payoffs than project 2. In any case, by (58), \( s^* < 1 \). Using (23), \( \sigma_{\text{pub}}^* = s^*/[\mu \delta + s (1 - \mu \delta)] \) when \( s^* > 0 \), and \( \sigma_{\text{pub}}^* = 0 \) when \( s^* = 0 \), there is a one-to-one mapping between \( \sigma_{\text{pub}}^* \) and \( s^* \).

To prove that \( v_1 \geq v_2 - \delta \mu p^2(\theta - 1) \Rightarrow \sigma_{\text{pub}}^* = 0 \) and that \( v_1 < v_2 - \delta \mu p^2(\theta - 1) \Rightarrow \sigma_{\text{pub}}^* \in (0, 1) \), write \( s^* \) as \( s^* = \frac{v_2 - v_1 - \delta \mu p^2(\theta - 1)}{p^2(\theta - 1)(1 - \delta \mu)}. \)

**Proposition 4.**

**Proof.** From Proposition 3, \( \sigma_{\text{pub}}^* \) is strictly increasing in \( s^* \) when \( \sigma_{\text{pub}}^* \in (0, 1) \). Therefore, we can obtain the effect of each parameter on \( \sigma_{\text{pub}}^* \) by its effect on \( s^* \) using \( \frac{\partial \sigma_{\text{pub}}^*}{\partial x} = \frac{\partial s^*}{\partial \sigma_{\text{pub}}^*} \frac{\partial s^*}{\partial x} \), where \( x \) is the parameter of interest and \( \frac{\partial s^*}{\partial \sigma_{\text{pub}}^*} > 0 \). From \( s^* = \frac{\delta[1+(1-\mu)p(\theta-1)]-1}{p(\theta-1)(1-\delta \mu)} \), we have

\[
\frac{\partial s^*}{\partial \theta} = \frac{1-\delta}{p(1-\delta \mu)(\theta-1)^2} > 0, \quad \frac{\partial s^*}{\partial \delta} = \frac{(1-\mu)p(\theta-1)+1}{p(\theta-1)(1-\delta \mu)^2} > 0, \quad \text{and} \quad \frac{\partial s^*}{\partial \mu} = -\delta(1-\delta)\frac{1+p(\theta-1)}{p(\theta-1)(1-\delta \mu)^2} < 0. \]

**Proposition 5.**

**Proof.** From the expressions of \( W_{\text{priv}} \) and \( W_{\text{pub}} \) in (19) and (31), we obtain \( W_{\text{priv}} \geq W_{\text{pub}} \iff v_{\text{priv}} - v_{\text{pub}} \geq aI \iff (\sigma_{\text{priv}} - \sigma_{\text{pub}})(v_2 - v_1) \geq aI. \)

**Corollary 1.**

**Proof.** By proposition 5, when \( a = 0 \), the insiders prefers the private ownership structure if \( (\sigma_{\text{priv}} - \sigma_{\text{pub}})(v_2 - v_1) \geq 0 \). So, for the three cases in the corollary, we have. (1) If \( v_2 > v_1, \)
the condition reduces to $\sigma_{\text{priv}} \geq \sigma_{\text{pub}}$. From propositions 1 and 3, $\sigma_{\text{priv}} = 1$ and $\sigma_{\text{pub}} < 1$.

Therefore, $\sigma_{\text{priv}} > \sigma_{\text{pub}}$ and the insider goes private. (2) If $v_1 > v_2$, then the condition to go private reduces to $\sigma_{\text{priv}} \leq \sigma_{\text{pub}}$. By proposition 1, $\sigma_{\text{priv}} > 0$ if $\frac{v_1 - v_2}{p(1-\delta)} < T(0)$ and $\sigma_{\text{priv}} = 0$ if $\frac{v_1 - v_2}{p(1-\delta)} \geq T(0)$. By proposition 3, $\sigma_{\text{pub}} = 0$. So, we have $\sigma_{\text{priv}} > \sigma_{\text{pub}}$ when $\frac{v_1 - v_2}{p(1-\delta)} < T(0)$ (the insider then goes public), and $\sigma_{\text{priv}} = \sigma_{\text{pub}}$ otherwise (the insider is then indifferent between going public or private). (3) If $v_1 = v_2$, then $(\sigma_{\text{priv}} - \sigma_{\text{pub}})(v_2 - v_1) = 0$ and the insider is indifferent between going public or private.

**Lemma 4.**

**Proof.** From $V_{\text{priv}}(b) = m(b)[\sigma_{\text{priv}}v_2 + (1 - \sigma_{\text{priv}})v_1] + (1 - m)p$ and

$$m(b) = \frac{\mu}{\mu + (1 - \mu)[\sigma_{\text{priv}}(1 - \delta)p + (1 - \sigma_{\text{priv}})(1 - p)]b},$$

we obtain

$$V_{\text{priv}}(b) = \frac{\mu[v_1 + \sigma_{\text{priv}}(v_2 - v_1)] + (1 - \mu)[1 - p + \sigma_{\text{priv}}p(1 - \delta)]pb}{\mu + (1 - \mu)[1 - p + \sigma p(1 - \delta)]b}. \quad (59)$$

Separate the proof in three parts, for $b = 1$, $b = 0$, and $0 < b < 1$.

(1) For $b = 1$ to be an equilibrium strategy for the insider, we need $kV_{\text{priv}}(1) \geq p$. Substituting the expression of $V_{\text{priv}}(1)$, the condition for selling is

$$k \geq \frac{\mu p + (1 - \mu)[1 - p + \sigma_{\text{priv}}p(1 - \delta)]p}{\mu [v_1 + \sigma_{\text{priv}}(v_2 - v_1)] + (1 - \mu)[1 - p + \sigma p(1 - \delta)]p} \equiv k_1(\sigma_{\text{priv}}). \quad (60)$$

As $v_1 + \sigma_{\text{priv}}(v_2 - v_1) > p$, $k_1 < 1$. So, if $k > k_1(\sigma_{\text{priv}})$, $b = 1$ is an equilibrium strategy.

(2) For $b = 0$ to be an equilibrium strategy for the insider, we need $kV_{\text{priv}}(0) \leq p$. Similar algebra shows that this condition is equivalent to

$$k \leq \frac{p}{v_1 + \sigma_{\text{priv}}(v_2 - v_1)} \equiv k_2(\sigma_{\text{priv}}), \quad (61)$$
where \( 0 < k_2(\sigma_{\text{priv}}) < k_1(\sigma_{\text{priv}}) \).

(3) If \( k \in (k_2, k_1) \), an equilibrium must be in strictly mixed strategies. Imposing the condition \( kV(b(\sigma_{\text{priv}})) = p \) leads to

\[
b(\sigma_{\text{priv}}) = \mu \frac{k[v_1 + \sigma_{\text{priv}}(v_2 - v_1)] - p}{(1-k)(1-\mu)(1-p + \sigma_{\text{priv}}p(1-\delta))}. \tag{62}
\]

Substituting in (62) shows that \( b(\sigma_{\text{priv}}) = 0 \) if \( k = k_2 \), and that \( b(\sigma_{\text{priv}}) = 1 \) if \( k = k_1 \). Furthermore, \( b(\sigma_{\text{priv}}) \) is strictly increasing in \( k \), as \( \frac{\partial b}{\partial k} = \mu \frac{v_1 + \sigma(v_2 - v_1) - p}{p(1-\mu)(1-k)(1-p + \sigma p(1-\delta))} > 0 \).

Therefore, \( b(k) \in (0, 1) \) for \( k \in (k_2, k_1) \). ■

**Proposition 6.**

**Proof.** To prove each case it is sufficient to find out the values of \( \sigma_{\text{priv}} \) that satisfy the incentive compatibility (IC) conditions. The IC condition for \( \sigma_{\text{priv}}^* \geq 0 \), which holds when \( u_{2,\text{priv}}(k) \geq u_{2,\text{priv}}(k) \), is \( IC \equiv v_2 - v_1 + p(1-\delta)T(\sigma_{\text{priv}}, k) \geq 0 \), where \( T(\sigma_{\text{priv}}, k) = \max\{0, kV_{\text{priv}} - p\} \). We have \( \sigma_{\text{priv}}^* = 1 \iff IC > 0, 0 < \sigma_{\text{priv}}^* < 1 \iff IC = 0 \), and \( \sigma_{\text{priv}}^* = 0 \iff IC < 0 \).

**Case 1.** \( v_2 > v_1 \). Because \( T(\sigma_{\text{priv}}, k) \geq 0 \), then the IC condition for project 1 cannot be satisfied while the IC condition for project 2 is trivially satisfied, \( IC > 0 \) for all \( k \). Thus, \( \sigma_{\text{priv}}^* = 1 \).

**Case 2.** \( v_1 > v_2 \). Suppose that there is an equilibrium with \( \sigma_{\text{priv}} > 0 \). From the IC condition, it must be that \( T(\sigma_{\text{priv}}, k) > 0 \), which implies \( b(\sigma_{\text{priv}}) = 1 \). Then \( T(\sigma_{\text{priv}}, k) = kV_{\text{priv}}(\sigma_{\text{priv}}) - p > 0 \) and \( \frac{\partial T(\sigma_{\text{priv}}, k)}{\partial \sigma_{\text{priv}}} = \frac{\partial kV_{\text{priv}}(\sigma_{\text{priv}})}{\partial \sigma_{\text{priv}}} \). As \( V_{\text{priv}} = m[v_1 + \sigma_{\text{priv}}(v_2 - v_1)] + (1-m)p \) and \( \frac{\partial V_{\text{priv}}(\sigma_{\text{priv}})}{\partial \sigma_{\text{priv}}} < 0 \), because \( v_1 > v_2 \) and \( \frac{\partial m(\sigma_{\text{priv}})}{\partial \sigma_{\text{priv}}} < 0 \), then \( T(\sigma_{\text{priv}}, k) \) is decreasing in \( \sigma_{\text{priv}} \). The minimum value of \( T(\sigma_{\text{priv}}, k) \) is \( T(1, k) \). Therefore, if \( T(1, k) = \frac{v_1 - v_2}{p(1-\delta)} \) then \( IC \geq 0 \) for all \( \sigma_{\text{priv}} \). So, \( u_{2,\text{priv}}(k) \geq u_{1,\text{priv}}(k) \) and that the equilibrium is given by \( \sigma_{\text{priv}}^* = 1 \). If \( T(0, k) = \frac{v_1 - v_2}{p(1-\delta)} < T(1, k) \), then, as \( T(\sigma_{\text{priv}}, k) \) is decreasing in \( \sigma_{\text{priv}} \), there is a unique \( \sigma_{\text{priv}}^* \in (0, 1) \) such that \( T(\sigma_{\text{priv}}^*, k) = \frac{v_1 - v_2}{p(1-\delta)} \). For this \( \sigma_{\text{priv}}^* \), the IC condition holds with equality and \( u_{2,\text{priv}} = u_{1,\text{priv}} \). If \( \frac{v_1 - v_2}{p(1-\delta)} \leq T(0, k) \), then \( IC \leq 0 \) for all \( \sigma_{\text{priv}} \), which
implies that \( u_{2, \text{priv}} (k) \leq u_{1, \text{priv}} (k) \) and the equilibrium is given by \( \sigma_{\text{priv}}^* = 0 \).

**Case 3.** \( v_1 = v_2 \). The IC condition reduces to \( p (1 - \delta) T (\sigma_{\text{priv}}, k) \geq 0 \). For \( \sigma_{\text{priv}} > 0 \), we still have \( b = 1 \) and that \( T \) is decreasing in \( \sigma_{\text{priv}} \). Define \( \sigma_L \) as \( \{ \sigma_L = 1, \text{ if } kV_{\text{priv}} (1) > p; \sigma_L = V_{\text{priv}}^{-1} (p/k), \text{ if } kV_{\text{priv}} (1) < p < kV_{\text{priv}} (0); \text{ and } \sigma_L = 0 \text{ if } kV_{\text{priv}} (0) < p \}. \) If \( \sigma_L = 1 \), then for any \( \sigma_{\text{priv}} < 1 \) we have \( T (\sigma, k) > 0 \), thus the only equilibrium is \( \sigma_{\text{priv}}^* = 1 \). If \( \sigma_L \in (0, 1) \), then \( T (\sigma_{\text{priv}}, k) = \max \{ kV_{\text{priv}} (\sigma_{\text{priv}}) - p, 0 \} = 0 \) for any \( \sigma_{\text{priv}}^* \in [\sigma_L, 1] \), which implies that the insider is indifferent between any \( \sigma_{\text{priv}}^* \in [\sigma_L, 1] \), proving the result. \( \blacksquare \)

**Proposition 7.**

**Proof.** Let \( w (k) \equiv W_{\text{priv}} (k) - W_{\text{pub}} \). The insider chooses the private structure if \( w (k) > 0 \). With \( c_{\text{priv}} = c_{\text{pub}} \), the expression of \( w (k) \) simplifies to \( w (k) = u_{\text{priv}} (k) - u_{\text{pub}} \). \( u_{\text{pub}} \) does not depend on \( k \), as \( k \) affects the sale of shares only in the private case.

**Part 1.** By proposition 3, \( \sigma_{\text{pub}} = 0 \), then \( u_{\text{pub}} = v_1 \). Moreover, if \( k < 1 \), then \( u_{\text{priv}} (k) < v_{\text{priv}} (k) = \sigma_{\text{priv}} (k) v_2 + (1 - \sigma_{\text{priv}} (k)) v_1 \). As \( v_{\text{priv}} (k) < v_1 \), then \( w (k) < 0 \) and the insider chooses the public structure. If \( k = 1 \), then we return to the conditions of corollary 1. In particular, when \( \sigma_{\text{priv}} (1) = 0 \), the insider is indifferent between the public and the private structures, or the insider chooses the public structure if \( \frac{v_1 - v_2}{p (1 - \delta)} < T (0, 1) \). In general, \( w (k) \leq 0 \).

**Part 2.** If \( v_2 > v_1 \) then, by proposition 3, after some algebra,

\[
\sigma_{\text{pub}} = \begin{cases} 
\frac{(v_2 - v_1) - \delta \mu p^2 (\theta - 1)}{(v_2 - v_1) (1 - \delta \mu)}, & \text{if } \mu < \mu_L \equiv \frac{v_2 - v_1}{\delta p^2 (\theta - 1)}, \\
0, & \text{if } \mu \geq \mu_L.
\end{cases}
\]  

(63)

By proposition 6, \( \sigma_{\text{priv}} = 1 \). So, \( u_{\text{priv}} = v_2 - L (k) \). To show that there exists a \( k^* \in (0, 1) \) such that the insider chooses the private structure iff \( k \geq k^* \), it suffices to show that the function \( w (k) = W_{\text{priv}} (k) - W_{\text{pub}} \) has the following properties: \( w (0) \leq 0 \), \( w (k) \) is nondecreasing and continuous, and \( w (1) \geq 0 \).
(i) \( w(0) \leq 0 \). Consider first the case of \( \mu \geq \mu_L \). In such a case,

\[
  w(0) = W_{\text{priv}}(0) - W_{\text{pub}} = (1 - \mu) v_2 - v_1. 
\]

Because this function is decreasing in \( \mu \), it achieves a maximum at \( \mu = \mu_L \), in which case it becomes

\[
  \left(1 - \frac{v_2 - v_1}{\delta p^2 (\theta - 1)}\right) v_2 - v_1 = -(v_2 - v_1) \frac{(1 + \delta)}{\delta p(\theta - 1)} < 0. 
\]

Thus \( w(0) \) is also negative for any \( \mu \geq \mu_L \).

What about \( \mu < \mu_L \)? In this case, we have

\[
  w(0) = (1 - \mu) v_2 - v_1 + \frac{(v_2 - v_1) - \delta \mu p^2 (\theta - 1)}{1 - \delta \mu}. 
\]

Differentiating with respect to \( \mu \) yields

\[
  \frac{\partial w(0)}{\partial \mu} = -v_2 + \frac{\delta p^2 (\theta - 1) (1 - \delta \mu) - \delta [(v_2 - v_1) - \delta p^2 (\theta - 1) \mu]}{(1 - \delta \mu)^2} \]

\[
  = \frac{p}{(1 - \delta \mu)^2} \frac{\delta (p + 1)(\theta - 1) [(1 - \delta) - (1 - \delta \mu)^2] - (1 - \delta \mu)^2}{(1 - \delta \mu)^2} < 0. 
\]

Thus, the highest value of \( w(0) \) occurs when \( \mu \to 0 \). As \( \lim_{\mu \to 0} w(0) = v_2 - v_2 = 0 \), then \( w(0) < 0 \) for all \( \mu > 0 \).

(ii) \( w(k) \) is nondecreasing and continuous. We have to consider the different regions in which \( b = 0 \), \( 0 < b < 1 \), and \( b = 1 \). In Region 3 (\( k < k_2(1) \)), we have \( u_2(k, \mu) = \mu k v_2 + (1 - \mu) v_2 \), which is increasing in \( k \). In Region 2 (\( k_2(1) \leq k \leq k_1(1) \)), we have \( u_2(k) = \mu p + (1 - \mu) v_2 \), which is constant in \( k \). In Region 1 (\( k > k_1(1) \)), we have \( u_2(k) = k v_2 + (1 - \mu) \delta p (1 + \theta p) (1 - k) \), which is increasing in \( k \). Thus, \( u_2(k) \) is increasing in regions 1 and 3, and constant in region 2. Therefore, \( w(k) \) is nondecreasing (continuity is easily verified).

(iii) \( w(1) \geq 0 \). This is trivially verified: \( w(1) = v_2 - \sigma_{\text{pub}} v_2 - (1 - \sigma_{\text{pub}}) v_1 \geq 0 \).
As a result there exists a \( k^* \) such that \( w(k^*) = 0 \).

To prove uniqueness, we have to rule out \( w(k) = 0 \) for \( k \in [k_2, k_1] \). As \( u_{\text{priv}}(k) \) is constant in this region, we only need to show that \( w(k_2) < 0 \). If \( \mu \geq \mu_L \), then \( u_{\text{pub}} = v_1 \). So, \( w(k_2, \mu) = \mu p + (1 - \mu) v_2 - v_1 \), which is decreasing in \( \mu \). Substituting the expression of \( \mu_L \), we have that \( w(k_2, \mu_L) = \frac{-v_2 - v_1}{p(\theta - 1)} < 0 \) and so \( w(k_2, \mu) < 0 \). If \( \mu < \mu_L \), then \( w(k_2, \mu) = \mu p + (1 - \mu) v_2 - v_1 - \frac{(v_2 - v_1) - \delta p^2 (\theta - 1) \mu}{1 - \delta \mu} \). We have \( w(k_2, \mu) = 0 \) trivially if \( \mu = 0 \), which is ruled out by assumption. For \( \mu > 0 \), we have \( w(k_2, \mu) < 0 \Leftrightarrow \mu < 1 \), which is always true. Therefore, \( w(k_2, \mu) < 0 \) for all \( \mu \), which implies that \( k^* \notin [k_2, k_1] \). As \( w(k, \mu) \) is nondecreasing for \( k \leq k_1 (1) \) and is increasing for \( k > k_1 (1) \), we have a unique \( k^* \) and \( k^* > k_1 (1) \).

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