Modelling and forecasting commodity prices: Exploiting regime shifts and macroeconomic linkages

Melisso Boschi† and Luca Pieroni‡
University of Perugia

June 2008

Abstract

In this paper we assess the predictive potential of commodity prices and macroeconomic variables for each other's dynamics exploiting the possibility of non-linearities, in the specific form of regime-switchings, in the joint stochastic process. We develop a structural model of the relationship between metal commodities prices and stocks and the macroeconomy that is a slightly modified version of Boschi and Pieroni (2008) and Pieroni and Ricciarelli (2005). We then generalize the empirical version of this model to allow for Markov-regime switches in the intercept, in the autoregressive coefficients and in the covariance matrix terms.

The possibility of a time-varying structure of the relationship between a vast class of asset prices and macroeconomic variables has started to be investigated in the last years, with a special reference to forecasting properties of the proposed models. For example Jaditz et al. (1998) compare linear and non-linear models of US industrial production and asset prices and conclude that non-linear models improve upon their linear counterparts. Similarly, Tkacz (2001) find that non-linear models improve upon linear ones in forecasting Canadian GDP. On the other hand Galbraith and Tkacz (2000) find that limited non-linearity in the output-term spread relationship at the international level, while Swanson and White (1997) find that linear VAR models generally outperform multivariate neural network forecasts of US output and inflation. Stock and Watson (2003) draw the conclusion that this poor performance of non-linear model might be due to the difficulty to specify the "right" non-linear model. In fact, the non-linear model proposed by Guidolin and Ono (2006) improves the forecasting performance of asset prices.

† Corresponding author. Department of Economics, Finance, and Statistics, University of Perugia, Via A. Pascoli, 20, 06123, Perugia, Italy. E-mail: mboschi@stat.unipg.it.

‡ Department of Economics, Finance, and Statistics, University of Perugia, Via A. Pascoli, 20, 06123, Perugia, Italy. E-mail: lpieroni@unipg.it..

Keywords: Commodities, Forecasting, Markov-switching VAR.
JEL Classification: .

1 Introduction

A large body of research has investigated the possibility that macroeconomic aggregates and asset prices may help predict each other's dynamics. In fact, researchers have recently considered as a priority the reconciliation of the Efficient Market Hypothesis (EMH), by which asset prices should follow a random walk, with the predictive power of macroeconomic dynamics for rational investors. However, the evidence on the predictive potential of macroeconomic variables for asset prices is mixed. Neely and Weller (2000) show that VAR models are outperformed by a simple benchmark model in which expected excess returns in stock and exchange markets are assumed to be constant and dividend yields and forward premiums are assumed to follow random walks. They suggest that the poor forecasting performance is due to underlying structural changes, which points to some role for regime switching models. This confirm the suspicion put forward by Bossaerts and Hillion (1999) who argue that the poor out-of-sample forecasting performance of even the best linear models may be due to the time-varying nature of parameters, thus suggesting that the correct underlying model is possibly of a regime switching type. Recent papers (e.g. Guidolin and Timmermann (2005a) for excess stock and bond returns or Guidolin and Timmermann (2005b) for short term interest rates) have found that regime switching models may prove extremely useful to forecast over intermediate frequencies, such as monthly data. On a longer monthly data set, Guidolin and Ono (2006) find overwhelming evidence of regime switching in the joint process for asset prices and macroeconomic variables. They also find that modelling explicitly the presence of such regimes improves considerably the out-of-sample performance of a model of the linkages between asset prices and the macroeconomy.

A parallel literature has focused on the help asset returns can provide when forecasting macroeconomic variables, particularly output and inflation (see Stock and Watson (2003) for a detailed review and evaluation of this research). A number of authors in the late 1980s (e.g. Harvey (1988, 1989), Estrella and Hardouvelis (1991)) formalized empirically the concept that an inverted yield curve signals a recession, though subsequent work, by looking closer to the US evidence, has led to the conclusion that the predictive content of the term spread for output is time-varying (e.g. Ang, Piazzesi, and Wei (2003)). Although the theoretical link between stock prices and economic activity is firmly grounded, the empirical predictive content of these asset prices has proved to be rather poor (see Cambell (1999) for a review).

In this paper we assess the predictive potential of commodity prices and macroeconomic variables for each other's dynamics exploiting the possibility of non-linearities, in the specific form of regime-switchings, in the joint stochastic process. We develop a structural model of the relationship between metal commodities prices and stocks and the macroeconomy that is a slightly modified version of Boschi and Pieroni (2008) and Pieroni and Ricciarelli (2005). We then generalize the empirical version of this model to allow for Markov-regime switches in the intercept, in the autoregressive coefficients and in the covariance matrix terms.

The possibility of a time-varying structure of the relationship between a vast class of asset prices and macroeconomic variables has started to be investigated in the last years, with a special reference to forecasting properties of the proposed models. For example Jaditz et al. (1998) compare linear and non-linear models of US industrial production and asset prices and conclude that non-linear models improve upon their linear counterparts. Similarly, Tkacz (2001) find that non-linear models improve upon linear ones in forecasting Canadian GDP. On the other hand Galbraith and Tkacz (2000) find that limited non-linearity in the output-term spread relationship at the international level, while Swanson and White (1997) find that linear VAR models generally outperform multivariate neural network forecasts of US output and inflation. Stock and Watson (2003) draw the conclusion that this poor performance of non-linear model might be due to the difficulty to specify the "right" non-linear model. In fact, the non-linear model proposed by Guidolin and Ono (2006) improves the forecasting performance of asset prices.

Evidence of an effect of business cycle conditions on the behaviour of commodities inventories and prices is documented by Fama and French (1988), who derive from this relationship an indirect test of commodity theory of storage. Evidence of regime shifting in non-ferrous price dynamics is found by Heaney (2006) who tests a model of commodity pricing accounting for both stocks-out and convenience yield as an explanation of the relationship between prices and stockholdings. He finds that two fairly stable states exist that relate to the level of the underlying stocks.

1.1 Markov-switching VAR models

The Markov-switching model has been popularized in time series analysis by Hamilton (1988, 1989, 1994). Its multivariate version extends a standard linear VAR model by allowing its parameters to be subject to regime shifts. In such situation, rather than time-varying, the VAR process can be modelled as time-invariant conditional on an unobservable regime variable s_t which indicates the regime prevailing at time t. Therefore, Markov-switching vector autoregressions are generalizations of the basic VAR model of order p:

$$\mathbf{y}_t = \boldsymbol{\nu} + \sum_{i=1}^{p} \mathbf{A}_i \mathbf{y}_{t-i} + \mathbf{u}_t \tag{1}$$

where $\mathbf{y}_t = (y_{1t}, ..., y_{Kt})'$ is a K-dimensional vector, $\boldsymbol{\nu}$ is an intercept term, $\mathbf{A}_i$, for $i = 1, ..., p$, are $K \times K$ matrices of coefficients, and $\mathbf{u}_t$ is a vector of residuals. Denoting $\mathbf{A}(L) = \mathbf{I}_K - \mathbf{A}_1 L + ... + \mathbf{A}_p L^p$ as the lag polynomial of dimension $K \times K$, I assume that there are no roots on or inside the unit circle $|\mathbf{A}(z)| \neq 0$ for $|z| \leq 1$, where L is the lag operator. Under the additional assumption that $\mathbf{u}_t \sim NID(\mathbf{0}, \boldsymbol{\Sigma})$, equation (1) is the intercept form of a stable *Gaussian* VAR model of order p.

Since I assume that the parameters of the *observed* time series vector $\mathbf{y}_t$ depend on the *unobservable* regime variable s_t, a model for the regime generating process is required. In the Markov-switching VAR model the regime $s_t \in \{1, ...M\}$ is assumed to be governed by a discrete time, discrete state Markov stochastic process characterized by the following transition probabilities:

$$p_{ij} = \Pr(s_{t+1} = j \mid s_t = i), \ \sum_{j=1}^{M} p_{ij} = 1 \ \forall i, j \in \{1, ...M\}\,. \tag{2}$$

The transition probabilities can be represented by the following transition matrix:

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1M} \\ p_{21} & p_{22} & \cdots & p_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ p_{M1} & p_{M2} & \cdots & p_{MM} \end{bmatrix} \tag{3}$$

where $p_{iM} = 1 - p_{i1} - ... - p_{i,M-1}$ for $i = 1, ..., M$. A crucial assumption for the theoretical properties of MS-VAR models is that s_t follows an irreducible ergodic M state Markov process with transition matrix given by (3).

Therefore, if time series are subject to shifts in regime, the M regimes Markov-switching form of the VAR(p) model of equation (1) is given by:

$$\mathbf{y}_t = \boldsymbol{\nu}(s_t) + \sum_{i=1}^{p} \mathbf{A}_i(s_t)\mathbf{y}_{t-i} + \mathbf{u}_t. \tag{4}$$

where $\mathbf{u}_t \sim NID(\mathbf{0}, \boldsymbol{\Sigma}(s_t))$ and $\mathbf{A}_1(s_t), ..., \mathbf{A}_p(s_t), \boldsymbol{\Sigma}(s_t)$ are shifts functions describing the dependence of the parameters $\mathbf{A}_1, ..., \mathbf{A}_p, \boldsymbol{\Sigma}$ on the realized regime s_t.

The parameters of the model are estimated with the maximum likelihood method (see Hamilton (1989, 1994)). The maximization of the likelihood function of an MS-VAR requires an iterative estimation of the parameters of the autoregression and the transition probabilities governing the Markov chain of the unobserved states. This is usually obtained through the implementation of the *Expectation Maximization* (EM) algorithm introduced by Dempster *et al.* (1977) and proposed by Hamilton (1990) for this class of models.

2 Data

The data are monthly, with range varying from one commodity market to another, the largest sample spanning the horizon 1988:1-2007:3. Commodity prices and stocks are from DATASTREAM, while macroeconomic variables are taken from the IFS Database.

The following transformations are performed. The commodity prices are cash quotations in US dollars, except for copper and lead for which pound sterling quotations are available. Copper and lead prices are first transformed in US dollars using the monthly UK£/US$ exchange rate. US dollar-denominated prices are divided by the US PPI (production price index – code 63BB.ZF of IFS), to obtain real values. The price variable entering the estimation is a rate of return relative to inflation. This practice implicitly assumes that rational investors take into account real rather than nominal prices in financial decisions. Similarly, stock amounts are entered as rates of growth as approximated by log-differences.

As for macroeconomic variables, we use an index of industrial production, the real effective exchange rate, and the real interest rate. The index of industrial production (code 66..CZF) is first divided by the CPI (Consumer Price Index - code 64. . . ZF) and then transformed in log-difference to obtain approximations of the rate of growth. The real effective exchange rate, as well, enters the model in log-difference. Finally, the change in the real interest rate is obtained by taking the first difference of the Federal Funds rate (code 60B..ZF) net of the expected rate of inflation as proxied by log-difference of the CPI one period ahead. All data are seasonally adjusted.

The following formulas clarify the data transformation procedure.

$$\Delta p_{it} = \ln(P_{it}/PPI_t) - \ln(P_{it-1}/PPI_{t-1})$$

where P_{it} is the i-th commodity price at time t with i = aluminium, copper, lead, nickel, tin, and zinc, and PPI_t is the US production price index at time t. Further:

$$\Delta s_{it} = \ln(S_{it}) - \ln(S_{it-1})$$

where S_{it} is the amount of stock of commodity i at time t.

$$\Delta ip_{it} = \ln(IP_t/CPI_t) - \ln(IP_{t-1}/CPI_{t-1})$$

where IP_t is the US industrial production index and CPI_t is the US consumer price index, both at time t.

$$\Delta reer_t = \ln(REER_t) - \ln(REER_{t-1})$$

where $REER_{it}$ is the US real effective exchange index at time t. Finally:

$$\Delta rir_t = (R_t - \pi_{t+1}) - (R_{t-1} - \pi_t)$$

where R_t is the US Federal Funds rate and $\pi_t = \ln(CPI_t) - \ln(CPI_{t-1})$, both at time t.

3 Estimation results

Since the theory is inconclusive as to the specification of the model most appropriate to obtain the best predictive accuracy of the joint behaviour of commodity and macroeconomic variables, we estimate a large number of models and resort to a series of criteria to select the correct specification.

1. The first criterion refers to testing the null hypothesis of model linearity against the alternative of multiple regimes. We employ a likelihood ratio (LR) test but we are aware that in the presence of Markov processes

such testing procedure suffer from non-standard asymptotic distributions of the test statistics due to the existence of nuisance parameters under the null hypothesis (see Krolzig (1997) and Garcia (1998) for a detailed discussion). In such a case, it is common practice (e.g. Guidolin and Ono (2005)) to use the upper bound for the significance level of the LR test under nuisance parameters proposed by Davies (1977). Therefore we use the LR test statistic to test the null of against the alternative where the number of states is indicated by the corresponding model specification.

2. Three standard information criteria, i.e. the Akaike (AIC), Bayes Schwartz (BIC), and Hannan-Quinn (HQ) criteria. These statistics compare the in-sample fit, which is measured by the residual variance, against the number of estimated parameters. Since these criteria penalize models with a large number of parameters, thus identifying the model ex-ante potential out-of-sample, they are supposed to trade-off in-sample fit with prediction accuracy. The model minimizing each of these information criteria is the best performing one. Further, since these criteria do not explicitly suffer from nuisance parameters issues, they can be used to compare models with different number of regimes, as well as nested models with different within the same class of regimes.

All the models estimated are reported in Tables 1-6, which also reports the outcome of the tests and information criteria discussed above. Empirical evidence in strongly in favour of non-linearity in the dynamic relationship between commodity prices and storage levels on one hand and macroeconomic variables on the other. This is implied by the Davies' test p-value which approximates zero in all commodities markets and for all model specification. Once established that the number of regimes $k > 1$ we need to select an appropriate model within the general regime switching class MSIAH (k, p) using the information criteria, along the lines of Guidolin and Timmermann (2005b) and Guidolin and Ono (2006). Tables 1-6 show that there is some tension between the AIC and the other two information criteria. In fact, while the former select almost invariably the most richly parameterized model, the other two tend to prefer more parsimonious specification. As noted by Guidolin and Ono (2006) this is relatively unsurprising since the literature has already stressed the tendency of AIC to select relatively larger models in non-linear frameworks (e.g. Fenton and Gallant (1996)). The HQ and SBC criteria, instead, tend to concentrate on a farly parsimonious model, the MSIH $(2, 1)$, i.e. a VAR of order 1 with regime-shifting intercept and covariance matrix. We use this model in two out of six commodities markets, aluminium and copper. These are the models for which we have very partial forecating results.

4 Forecasting performance (work in progress - very partial results)

To assess the forecasting performance of our MSIH $(2,1)$ model of aluminium and copper market, we implement a 'pseudo out-of-sample' recursive strategy along the lines of Guidolin and Ono (2006). We choose a number of observations for each series amounting to about 25 percent of the entire data sample over which we compute models' forecasts. Specifically, we estimate recursively our MSIH $(2,1)$ model over expanding samples starting with 1989:7 - 2002:12, 1989:7 - 2003:1, ... up to 1989:7 - 2007:2. For each estimation we obtain a set of parameter estimates. For each model estimation, we compute 1-month ahead forecasts for each of the 5 variables (related to both the commodity market and the macroeconomy) included in the system. The forecast generated for variable h by model M 1-month ahead is indicated by $\widehat{y}_{t+1}^{(M,h)}$. Then we calculate the resulting forecast error defined as $e_{t+1}^{(M,h)} \equiv y_{t+1}^{h} - \widehat{y}_{t+1}^{(M,h)}$. We follow the same procedure for the first benchmark we consider in the analysis, i.e. a VAR (1). The first statistic we consider to evaluate predictive accuracy of the two competing models is the root mean squared forecast error (RMSFE) defined by:

$$RMSFE^{(M,h)} \equiv \sqrt{\frac{1}{51}\sum_{t=2003:1}^{2007:2}\left(y_{t+1}^{h} - \widehat{y}_{t+1}^{(M,h)}\right)^{2}}.$$

These statistics for the models of aluminium and copper are reported in Table 7. It is clear that, though very similar, the predictive perfomrmance of the linear model is generally superior, with the only exceptions of copper price and the real exchange rate in the model of copper.

References

[1] Bossaerts, P. and P. Hillion (1999) “Implementing statistical criteria to select return forecasting models: What do we learn?”, Review of Financial Studies, 12, 405–428.

[2] Fama, E. F. and K. R. French (1988) “Business Cycles and the Behavior of Metals Prices”, The Journal of Finance, 43, 1075-1093.

[3] Galbraith, J. W. and G. Tkacz (2000) “Testing for Asymmetry in the Link Between the Yield Spread and Output in the G-7 Countries”, Journal of International Money and Finance, 19, 657-672.

[4] Guidolin, M. and S. Ono (2006) “Are the dynamic linkages between the macroeconomy and asset prices time-varying?”, Journal of Economics and Business, 58, 480–518.

[5] Guidolin, M. and A. Timmermann (2005a) “Economic implications of bull and bear regimes in UK stock and bond returns”, Economic Journal, 115, 111–143.

[6] Guidolin, M. and A. Timmermann (2005b) “Optimal forecast combination weights under regime shifts: An application to US interest rates”, mimeo, Federal Reserve Bank of St. Louis and UCSD.

[7] Hamilton, J. D. (1988) “Rational expectations econometric analysis of changes in regime. An investigation of the term structure of interest rates” *Journal of Economic Dynamics and Control*, **12**, 385-423.

[8] Hamilton, J. D. (1989) “A new approach to the economic analysis of nonstationary time series and the business cycle” *Econometrica*, **57**, 357-384.

[9] Hamilton, J. D. (1990) “Analysis of time series subject to changes in regime” *Journal of Econometrics*, **45**, 39-70.

[10] Hamilton, J. D. (1994) *Time Series Analysis*, Princeton University Press, Princeton.

[11] Harvey, C. R. (1988) “The Real Term Structure and Consumption Growth”, Journal of Financial Economics, 22, 305-333.

[12] Harvey, C. R. (1989) “Forecasts of Economic Growth from the Bond and Stock Markets”, Financial Analysts Journal, 45, 38-45.

[13] Heaney, R. (2006) “An empirical analysis of commodity pricing”, The Journal of Futures Markets, 26, 391-415.

[14] Jaditz, T., L. A. Riddick, and C. L. Sayers (1998) “Multivariate Nonlinear Forecasting: Using Financial Information to Forecast the Real Sector”, Macroeconomic Dynamics, 2, 369-382.

[15] Neely, C. and P. Weller (2000) "Predictability in international asset returns: Reexamination", Journal of Financial and Quantitative Analysis, 35, 601–620.

[16] Stock, J. and M. Watson (2003) "Forecasting output and inflation: The role of asset prices", Journal of Economic Literature, 41, 788-829.

[17] Tkacz, G. (2001) "Neural Network Forecasting of Canadian GDP Growth", International Journal of Forecasting, 17, 57-69.

Table 7. Out-of-sample performance

Aluminium		
	RMSFE	
	MSIH(2,1)	**VAR(1)**
p_al	0.0501	0.0485
s_al	0.0574	0.0553
ip	0.0060	0.0058
rir	0.4120	0.4027
reer	0.0116	0.0115
Copper		
	RMSFE	
	MSIH(2,1)	**VAR(1)**
p_co	0.0859	0.0870
s_co	0.1857	0.1844
ip	0.0059	0.0059
rir	0.4225	0.4102
reer	0.0114	0.0118