TRIPs under no enforcement of the National Treatment Commitment Rule

Rafael Pinho Senra de Morais

Getulio Vargas Foundation

February 2009

JEL classification: O38; F53; O31; O34

Keywords: TRIPs; National Treatment; Intellectual Property Rights; Patents

Phone: +55 21 3799-6065 Fax: +55 21 3799-5459 Email: rpinhodemorais@gmail.com
Abstract

In the World Trade Organization (WTO) Agreement on Trade Related Aspects of Intellectual Property Rights (TRIPs), the National Treatment Commitment Rule (NTCR) obliges countries to treat equally national and foreign intellectual property right (IPR) applicants and holders. The related literature takes the respect to the NTCR as granted, despite of clear empirical evidence on many countries’ discrimination in favour of residents as concerns IPR enforcement.

Using the setup of Lai and Qiu (2003), we relax the NTCR enforcement assumption. We adopt a different approach to their paper, focusing on equilibrium and first best outcomes. Conditions on countries’ market sizes and innovative capabilities determine interior and corner solutions. Social optimum requires only global incentives to innovation, no matter who provides them. Moreover, if IPR implementation is costlier in South, efficiency requires only North to protect IPRs and more than at equilibrium. Our results, at the same time as complementary, contradict Lai and Qiu (2003) to some extent.
I - Introduction

Lai and Qiu (2003) provided a framework to deal with a two-country trade model where both North and South have some innovative capability and simultaneously choose protection levels for intellectual property rights (IPRs). However, they take the respect to the National Treatment Commitment Rule (NTCR) as granted, which is presented as a realistic and inoffensive assumption, as concerns the obtained results. The aim of our paper is to show the opposite.2

The NTCR has been present in International Law since the Paris Convention for the Protection of Industrial Property (May 1883). Its objective was to guarantee expatriated inventors the same IPR protection in their new country of residence as native inventors. According to the United Nations Secretary General (1964), most countries were compliant with this rule, including most developing economies. The principle is present in the WTO Agreement on Trade Related Aspects of Intellectual Property Rights (TRIPs) articles 1.3 and 3 but its interpretation has been broadened, requiring countries to recognize and enforce to that same extent IPRs arisen from R&D effort exerted abroad. Countries are, however, left with great discretion in the adoption of enforcement measures.3

In practice, this extended NTCR is commonly respected in legal texts but not effectively implemented, OCDE countries being the only arguable exception4. Examples of the disrespect of legally-protected IPRs are piracy, imitations and illegal copying concerning all sorts of differentiated goods – obviously known to some extent by local authorities. In what concerns the NTCR, our claim is that governments – in particular in developing countries – commonly neglect ensuring that their local authorities do enforce the NTCR. This means that in many occasions there is discrimination in favour of nationals when enforcing IPRs. This includes police officers who are looser in punishing imitation of trademarks or copyrights if those are foreign; judges who consistently decide in favour

2 Grossman and Lai (2004) go even further, assuming explicitly the full enforcement of the NTCR.
3 Morais (2002) provides a more thorough analysis of this issue.
4 The underlying idea is that countries do not enforce the NTCR in order to avoid sending huge royalties abroad. Data from 1990 indicate that 97.6% of the US$ 33 billions of global IPRs royalties receipts were transferred between OCDE countries (Hoekman (1994)).
of national firms and against international conglomerates in patent and trade secret lawsuits; patent offices which consistently turn proportionally less applications into grants for non-residents than for residents (we provide evidence on the latter in the Appendix). Moreover, as concerns pharmaceutical patents, compulsory licensing of drugs should be part of the list of NTCR non-enforcement in spite of being a legal loophole under TRIPs rules, as it is also a way for South not to send the due royalties to the North.

Like in Lai and Qiu (2003), our main concern is the positive externalities generated by one country when it enforces patents, particularly in terms of consumers’ access to a larger range of differentiated products. When country X enforces patents to products developed in Y, on top of X consumers’ loss due to the exercise of a temporary monopoly power, there is a direct positive impact on Y firms’ profits; additionally more products are going to be invented in Y, which will benefit consumers in both X and Y at steady state. Therefore there is a positive externality from X patenting Y’s innovations not only on the profits of firms in Y but also on the surplus of consumers in Y in the long run, through the increase in the number of products available to consumers in Y. On the other hand, when country X grants patents to its domestic innovations, firms are statically better off at the expense of consumers in X but consumers (in both X and Y) dynamically benefit from a greater variety of products. So, there is also a positive externality here, but it only benefits consumers in Y (and not firms in Y).

In Lai and Qiu (2003)\textsuperscript{5}, the full enforcement of the NTCR is assumed and therefore government X is constrained to provide the same protection to its domestic innovations and to Y’s innovations. Thus, all the above effects are added up and therefore undistinguishable. In our framework, each set of effects is the consequence of a change in a distinct choice variable, as described in the previous paragraph. Once countries are allowed to enforce the two protection levels of IPRs independently (one for domestically developed goods and one for goods invented abroad), we obtain complementary results to LQ which contradict theirs to some extent. LQ for example advocates an increase in the level of IPR protection in the South, as this is efficiency-enhancing in their setup – given the level of protection in the North, which is a quite misleading result. We prefer, instead, to

\textsuperscript{5} From now on, we shall refer to Lai and Qiu (2003) as LQ.
focus on the Pareto efficient solution and the conditions under which each region is willing to enforce protection on IPRs.

The remainder of the paper is organized as follows. Section II presents the changes implied by the non-enforcement of the NTCR to LQ’s model, while in Section III we solve it for obtaining the Nash equilibrium enforced patent durations. Section IV shows the welfare analysis and Section V concludes the paper. In the Appendix A, we discuss some data on patent offices’ activities worldwide to support our claim that the NTCR is not enforced in practice, while in Appendix B we report tables with numeric computations which are important for one of our results.

II - Model setting

We keep the whole structure of the North-South model in Lai and Qiu (2003), except that there is no *a priori* assumption on the adoption of the national-treatment commitment rule (NTCR). We however take a different approach. LQ imposes a needed condition for obtaining an interior solution and studies the features of this Nash equilibrium. Moreover, they focus on the welfare impact of keeping North at Nash equilibrium and increasing the level of protection in the South. Here, we do not assume an interior solution but focus instead on the conditions for it and the features of the different possible corner solutions. Moreover, we compute the Pareto optimal enforced patent lengths.

In our model, TRIPs imposes the NTCR but its enforcement is unobservable. Or, if ever observable, it is not contractible and can not be opposed at international courts. Regions are consequently free to cheat. Regions’ choice variables are the enforced patent durations: \( T_{jk} = w_{jk} - \tau \), where \( w_{jk} \) is the enforcement level chosen by region j to patents on goods invented in k and \( \tau \) is the minimum duration imposed by TRIPs, such as 20 years for patents (of all origins).

Therefore, region k’s government sets two possibly distinct patent lengths: \( T_{kk} \) and \( T_{-kk} \) for \( k \in \{S, N\} \), where the first subscript denotes the region where the good has been invented and

---

6 Some evidence of the non-contractibility is the great freedom countries enjoy in the adoption of IPR enforcement measures in the TRIPs Agreement, since it only provide general non-enforceable guidelines.

7 Alternatively, we could have the enforcement level \( w_{jk} \) explicitly incorporated to the model, but this would change nothing in the analysis. Our approach to enforcement eases the exposition and is similar to the one in Grossman and Lai (2004), except that they assume a fully enforced NTCR (\( w_{kk} = w_{-kk} \)), just like LQ.
developed and the second one denotes the region where it is sold and consumed. Consequently, the second subscript determines whose choice variable a $T_{jk}$ is: k’s government sets the duration of the patents it issues, no matter the origin j of the products.

For the remainder, we keep the same setup and notation (wherever possible) as LQ. T is a product life-time; $x_{jk}(i)^\alpha$ is the utility from the consumption of a quantity x of good i invented in region j and consumed in region k. The tilde identifies those products whose patent has expired. The parameter $\alpha$ is between 0 and 1 and is the direct elasticity of consumption. N represents the market size while M is the number of products.

We report LQ’s market of traditional goods z and the numeraire y in the computations\(^8\) but focus exclusively on the market for differentiated goods x, those bearing a highly innovative component and therefore where IPR protection plays a crucial role.

The steady state flow utility of the representative consumer in region k comes from consuming on-patent and off-patent goods produced in both regions, plus utility from z and y:

$$u_k(t) = \sum_{i \in \{N,S\}} T_{jk} \left[ x_{jk}(i)^\alpha \right] + \sum_{j \in \{N,S\}} (T - T_{jk}) \left[ \tilde{x}_{jk}(i)^\alpha \right] di + \left( a z_k - \frac{1}{2} e_k^T z_k^2 \right) + y_k$$

$$= T_{kk} M_k (1 - \alpha) \alpha^{2\alpha} + T_{kk} M_{-k} (1 - \alpha) \alpha^{2\alpha} + \alpha (T - T_{kk}) M_k (1 - \alpha) \alpha^{2\alpha} +$$
$$+ (T - T_{kk}) M_{-k} (1 - \alpha) \alpha^{2\alpha} + \left[ a - p_k(z) \right] \left[ 2 e_k \right] + I_k$$

Since demands are independent of patent lengths, individual and aggregate demands are the same as in LQ. Individual demands are: $x_{jk}(i) = \left[ \frac{p_{jk}(i)}{\alpha} \right]^{-\frac{1}{1-\alpha}}$ and $\tilde{x}_{jk}(i) = \left[ \frac{\tilde{p}_{jk}(i)}{\alpha} \right]^{-\frac{1}{1-\alpha}}$

The aggregate demands are obtained as: $X_{jk}(i) = N_k x_{jk}(i)$ and $\tilde{X}_{jk}(i) = N_k \tilde{x}_{jk}(i)$

The individual firm’s problem is not changed at all. The firm still maximizes its per-period operating profit, as a monopolist when it is got a patent, or facing competition once the patent has expired. We then obtain:

$$p_{jk}(i) = \frac{1}{\alpha} \quad \tilde{p}_{jk}(i) = 1$$

\(^8\) The terms related to z and y are irrelevant for our purposes and disappear in the first-order conditions.
\( X_{jk} (i) = N_k \alpha^{1-\alpha} \quad \tilde{X}_{jk} (i) = N_k \alpha^{1-\alpha} \)

\( \pi_{jk} = N_k A \quad \tilde{\pi}_{jk} = 0 \)

where the first column represents goods with valid patents and the second one represents those with expired patents.

The individual firm’s problem is unchanged, and we get the same price structure and period operating profit as in LQ. The new life-time profit of firm i based in region k is \(9\):

\[
\Pi_k (i) = \int_0^{T_{kS}} \pi_{kS} (i) dt + \int_0^{T_{kN}} \pi_{kN} (i) dt - a_k \frac{1}{b_k} = (N_k T_{kS} + N_k T_{kN}) A - a_k \frac{1}{b_k}
\]

where \( A \equiv (1-\alpha) \alpha^{i(1+\alpha)} \varepsilon \equiv \frac{1}{1-\alpha} \) and \(-a_k \frac{1}{b_k}\) is the innovation cost, as in LQ.

Products are indexed in an ascending order of innovation costs, which take this specific shape, what does not impact the results qualitatively. The parameter \( b_k \) is between 0 and 1 and is equal to the inverse of the elasticity of the innovation cost with respect to the position of the good in the ordering. So, \(1/b_k\) is the (percentage) increase in the innovation cost when there is a (percentage) increase in the position of the good in the list, i.e., when we move from a good easy to develop to another with a higher innovation cost. For each position in the list (for each i), it is intuitive to imagine that this elasticity is smaller for North than South, i.e., that \(1/b_N\) is smaller than \(1/b_S\), or \(b_S\) is smaller than \(b_N\).

The marginal firms both in South and in North as the last ones to invest profitably in R&D, and as a consequence, as the ones earning zero profits. Thanks to this definition and the ascending order assumed for the innovation costs, one gets the number of products developed in each period in both South (\(M_S\)) and North (\(M_N\)). Here we also assume the sets of products developed in North and in South to be non-intersecting.

From this, we obtain the number of products developed in each period in each region:

\[
M_S = \left[ (N_k T_{NS} + N_k T_{SN}) \frac{A}{a_S} \right]^{b_S} \quad \text{and} \quad M_N = \left[ (N_k T_{NS} + N_k T_{NN}) \frac{A}{a_N} \right]^{b_N}
\]

The steady state welfare flow in region k is then given by \(10\):

\[
W_k (T_{SS}, T_{SN}, T_{NN}, T_{NS}, M_S, M_N) = N_k u_k (t) + \int_0^{M_k} \Pi_k (i) di + R_{2k}
\]
\[= N_k T_{kk} M_k (1-\alpha)\alpha^{2\alpha} + N_k T_{-kk} M_{-k} (1-\alpha)\alpha^{2\alpha} + N_k (T - T_{kk}) M_k (1-\alpha)\alpha^{2\alpha} +
+ N_k (T - T_{-kk}) M_{-k} (1-\alpha)\alpha^{2\alpha} + N_k I_k + M_k (N_S T_{kS} + N_N T_{kn}) A - \frac{b_k}{1+b_k} M_k^{1+b_k} + U_{zk}\]

III - The Choice of Patent Durations

Country \(k\) maximizes its welfare by setting \(T_{kk}\) and \(T_{-kk}\) in a simultaneous game played against country \(-k\).

III.1) Interior Equilibrium

We assume an interior solution for the time being and will derive the necessary conditions later. We first compute the South’s optimal choice for the protection level of its domestic patents:

\[
\frac{dW_S}{dT_{SS}} = \frac{\partial W_S}{\partial T_{SS}} + \frac{\partial W_S}{\partial M_S} \frac{\partial M_S}{\partial T_{SS}} + \frac{\partial W_S}{\partial M_N} \frac{\partial M_N}{\partial T_{SS}} = 0
\]

where

\[
\frac{\partial W_S}{\partial T_{SS}} = -N_S M_S \left[1 - (1 + \alpha)\alpha^{\alpha}\right] \left[1 - \alpha\right]\alpha^{\alpha} < 0
\]

\[
\frac{\partial W_S}{\partial M_S} \frac{\partial M_S}{\partial T_{SS}} = \left\{N_S \left[T - T_{SS} \left(1 - \alpha^{\alpha}\right)\right] \left[1 - \alpha\right]\alpha^{\alpha}\right\} \left\{\frac{N_S}{(N_S T_{SS} + N_N T_{SN})} b_M M_S\right\} > 0
\]

\[
\frac{\partial W_S}{\partial M_N} \frac{\partial M_N}{\partial T_{SS}} = 0
\]

There are several differences with respect to LQ – where \(T_{SS} = T_{NS}\) had to hold. The third term here is zero since the protection level of one region’s own patents has no impact on the number of products invented in the other region, once we assume no \textit{a priori} obligation to respect the NTCR.

Consider now the first term. The direct decrease in the South’s welfare, given by consumer losses and producer gains is here smaller than when \(T_{SS} = T_{NS}\). In the LQ setup, by strengthening \(T_{SS}\), South also strengthens \(T_{NS}\) because of the NTCR, and therefore there is an additional negative impact on the South’s consumer surplus coming from northern goods, which is absent here.

From such computation we obtain the South’s reaction function for southern products:
By symmetry, the North’s reaction function for northern goods is:

\[
T_{NN}(T_{NS}) = \frac{b_N T}{1 - (1 + \alpha)N_s T_{NS}} - \frac{1 - (1 + \alpha)\alpha^{ac}}{1 - (1 + \alpha)\alpha^{ac} + (1 - \alpha^{ac})/b_N} N_s T_{NS}
\]  

(2)

Concerning the protection granted by South to goods invented in North, we set:

\[
\frac{dW_s}{dT_{NS}} = \frac{\partial W_s}{\partial T_{NS}} + \frac{\partial W_s}{\partial M_s} \frac{\partial M_s}{\partial T_{NS}} + \frac{\partial W_s}{\partial M_N} \frac{\partial M_N}{\partial T_{NS}} = 0
\]

where

\[
\frac{\partial W_s}{\partial T_{NS}} = -N_s M_N (1 - \alpha^{ac}) (1 - \alpha) / b_N > 0
\]

\[
\frac{\partial W_s}{\partial M_N} \frac{\partial M_N}{\partial T_{NS}} = 0
\]

\[
\frac{\partial W_s}{\partial M_s} \frac{\partial M_s}{\partial T_{NS}} = \left\{N_s \left[ T - T_{NS} \left(1 - \alpha^{ac}\right) (1 - \alpha) \alpha^{ac}\right] \right\} \left\{ \frac{N_s}{(N_s T_{NS} + N_N T_{NN})} b_N M_N \right\} > 0
\]

The effects present in LQ’s differentiation of \( T_S \) and absent our differentiation of \( T_{SS} \) appear here. Note that LQ’s \( \frac{dW_s}{dT_S} \) is precisely equal to \( \frac{dW_s}{dT_{SS}} + \frac{dW_s}{dT_{NS}} \) in our setting and the same holds for each of the three terms of these differentials individually. We are in fact disentangling the effects of the IPR protection setting game which were put together in Lai and Qiu (2003) due to the NTCR.

We then obtain the South’s reaction function for northern goods:

\[
T_{NS}(T_{NN}) = \frac{b_N T}{(1 - \alpha^{ac}) (1 + b_N)} - \frac{N_s}{(1 + b_N) N_N} T_{NN}
\]  

(3)

Once again, by symmetry we obtain for North:

\[
T_{SN}(T_{SS}) = \frac{b_S T}{(1 - \alpha^{ac}) (1 + b_S)} - \frac{N_S}{(1 + b_S) N_N} T_{SS}
\]  

(4)

**Lemma 1:** There are two distinct games, defined according to the product origin. In one game, South and North interact for determining the protection for products developed in South
equations 1 and 4); in the other game, both regions strategically decide on the protection for northern
products (see equations 2 and 3).

Comment:

Such non-interaction between variables concerning goods of different origins is driven by the
zeros in the partial derivatives: $\frac{\partial M_N}{\partial T_{SS}} = \frac{\partial M_S}{\partial T_{NS}} = \frac{\partial M_S}{\partial T_{NN}} = \frac{\partial M_N}{\partial T_{SN}} = 0$. These zeros, in turn, are due to
the violation of the NTCR and the assumption on the non-intersection between $M_S$ and $M_N$. (which
comes from GL’s setup). Consequently, protection to goods invented in one region does not affect the
number of goods invented in the other one, no matter who provides the protection. Therefore, for a
given country there is no strategic interaction between its two choice variables. The inter-regional
externality, however, is still present due to the interaction between the protection level for goods
developed in $k$ chosen by $k$ itself and the protection level for goods developed in $k$ chosen by the other
region (-$k$).

Note that the term with a minus sign in each of equations 1 to 4 stands for the strategic
substitutability between patent lengths for a given product origin. The magnitude of such (negative)
reaction is clearly increasing in $N_N/N_S$ for the South’s choice variables (equations 1 and 3) and
decreasing for the North’s (equations 2 and 4), which is intuitive.

From Lemma 1, we have four equations displayed in two systems of two equations each (one
for $T_{SS}$ and $T_{SN}$ and another one for $T_{NN}$ and $T_{NS}$).\(^{11}\) The equilibrium for southern goods requires
solving the system containing equations 1 and 4. It can be easily shown that, for all values of the ratio
between market sizes, in a graph of $T_{SS}$ and $T_{SN}$ the South’s reaction function is steeper than the
North’s reaction function\(^{12}\). Consequently, for ensuring $T_{SS} > 0$ at equilibrium, the necessary and
sufficient condition requires $T_{SN}$ (given that $T_{SS} = 0$) in the North’s reaction function to be smaller than
$T_{SN}$ (given that $T_{SS} = 0$) in the South’s one, or:

$$\frac{N_N}{N_S} \leq \frac{(1-\alpha^{\text{def}})(1+b_S)}{1-(1+\alpha)\alpha^{\text{def}}} = B$$

\(^{11}\) In LQ there was only one system of two equations to determine $T_S$ and $T_N$.
\(^{12}\) The opposite holds for $T_{NS}$ as a function of $T_{NN}$: the South’s reaction function is flatter than the North’s one.
If we isolate $T_{SS}$ instead, we get $T_{SS}(T_{SN})$. For ensuring $T_{SN} > 0$ at equilibrium, we need $T_{SS}$ (given that $T_{SN} = 0$) in the North’s reaction function to be larger than $T_{SS}$ (given that $T_{SN} = 0$) in the South’s one, or:

$$\frac{N_N}{N_S} > \frac{1 - \alpha^{\text{ae}}}{1 - (1 + \alpha)\alpha^{\text{ae}} + (1 - \alpha^{\text{ae}})b_S} \overset{\text{def}}{=} C$$

If we solve the other system of two reaction functions (equations 2 and 3), we will obtain analogous conditions for the other two variables. So, $T_{NS} > 0$ requires:

$$\frac{N_N}{N_S} < \frac{1 - (1 + \alpha)\alpha^{\text{ae}} + (1 - \alpha^{\text{ae}})b_N}{1 - \alpha^{\text{ae}}} = 1 + b_N - \frac{\alpha^{\epsilon}}{1 - \alpha^{\text{ae}}} \overset{\text{def}}{=} D$$

Finally, the necessary and sufficient condition for $T_{NN} > 0$ is:

$$\frac{N_N}{N_S} > \frac{1 - (1 + \alpha)\alpha^{\text{ae}}}{(1 - \alpha^{\text{ae}})(1 + b_N)} = \frac{1}{1 + b_N} - \frac{\alpha^{\epsilon}}{(1 - \alpha^{\text{ae}})(1 + b_N)}$$

For this equation, the term on the right-hand side is strictly smaller than one. Therefore, the assumption $N_N > N_S$ ensures that $T_{NN}$ is always strictly larger than zero at equilibrium.

**Lemma 2:** The country holding the largest market for differentiated products always sets a positive level of patent enforcement for innovations developed domestically.

*Comment:*

In Lai and Qiu (2003), the statement was shown to be true under the NTCR. In our setting (no NTCR), countries have different incentives to protect their own patents, since the choice of $T_{kk}$ is dissociated from the choice of $T_{kk}$. Setting a positive $T_{kk}$ is more beneficial than a positive $T_{kk}$ because the country also gains through the increase in its firms’ profits, additionally to the gains from increasing consumers’ surplus thanks to the increase in product variety. On the other hand, since there is no NTCR, a positive $T_{kk}$ has no positive impact through $M_{kk}$, in such a way that we can not state a priori that a country has more incentives to protect $T_{kk}$ than it had to protect $T_k$ in LQ. As such, solving the model was required to prove that $N_N > N_S$ implies $T_{NN} > 0$.

This result ensures that no matter the magnitude of the free-riding by the small region on the large one, it will never lead the large one to completely neglect protecting IPRs at equilibrium.
**Proposition 1:** Under no enforcement of the national treatment commitment rule, an equilibrium where both North and South enforce strictly positive patent lengths for both their own patents and patents of foreign origin requires conditions (I) and (II) below to hold and the ratio \( \frac{N_N}{N_S} \) to fall in the following interval: \( \max \{I; C\} < \frac{N_N}{N_S} < \min \{B; D\} \)

Condition (I): \( b_N > \frac{\alpha^e}{1 - \alpha^{ae}} \)

Condition (II): \( \left( 1 + b_N - \frac{\alpha^e}{1 - \alpha^{ae}} \right) \left( 1 + b_S - \frac{\alpha^e}{1 - \alpha^{ae}} \right) > 1 \)

**Proof:**

The existence of an interior solution for the equilibrium (where all the four patent lengths – the choice variables – are set at strictly positive levels) is subject to the simultaneous respect of the four conditions we computed before, which reduce to the following three, since \( N_N > N_S \) (Lemma 2):

\[
\frac{N_N}{N_S} < B \quad ; \quad \frac{N_N}{N_S} > C \quad ; \quad \frac{N_N}{N_S} < D
\]

However, allowing the above three conditions to hold at the same time requires:

\( B > 1 \); \( D > 1 \); \( B > C \) and \( D > C \)

The first one always holds.

The second only holds if \( b_N > \frac{\alpha^e}{1 - \alpha^{ae}} \), which we call condition (I).

The third one always holds. (This is stated below as Lemma 3).

The forth one only holds if \( \left( 1 + b_N - \frac{\alpha^e}{1 - \alpha^{ae}} \right) \left( 1 + b_S - \frac{\alpha^e}{1 - \alpha^{ae}} \right) > 1 \), which is condition (II).

Consequently, conditions (I) and (II) are necessary (but not sufficient) for an interior solution.

**Comment:**

Under no NTCR, an equilibrium where both North and South set strictly positive patent lengths for both own patents and patents of foreign origin is hard to obtain. It requires, additionally to
a moderate level of the ratio between market sizes, the innovation cost in North to be small enough (I) and the one in South to be not too large (II). One should notice that condition (I) tells us that if North is skilled enough in generating innovation, the optimal choice of South is to issue IPRs on northern inventions instead of free-riding, provided the South’s relevant market is not too small.

**Lemma 3:** In spite of the non-enforcement of the national treatment commitment rule, Research and Development in South is always stimulated.

*Comment:*

Protection is always provided for northern products since $N_N > N_S$ ensures that $T_{NN}$ is strictly positive at equilibrium (Lemma 2). As regards southern products, we do not need any extra assumption: the fact that condition $B > C$ always holds guarantees that if either $T_{SS}$ or $T_{SN}$ is zero at equilibrium, the other one is strictly positive.

In Lai and Qiu (2003), an additional assumption was needed to ensure that $T_S$ is strictly larger than zero. It concerned the relative size of the markets and required $N_N$ not to be too much larger than $N_S$, otherwise South would not have any incentives to protect IPRs.

In our framework, South also has no incentive to enforce southern patents if North has a much larger market for differentiated products. However, if it is so, North himself has incentives to protect southern products, because no protection would represent a huge loss for northern consumers since they are numerous to benefit from more products being invented in the South, no matter how good in producing innovation South is. Therefore North enforces IPRs on southern innovations in the case South herself lacks interest in it.

**III.2) Corner Solutions**
Since $T_{NN} > 0$ in any case (Lemma 2), we have *a priori* 7 possibilities of corner solutions, according to the combinations of $T_{SN}$, $T_{SS}$ and $T_{NS}$ being positive or zero\(^\text{13}\). However, we also know that we cannot have $T_{SS}$ and $T_{SN}$ simultaneously equal to zero (Lemma 3). Consequently, the case where $T_{SS} = T_{SN} = 0$ and $T_{NS} > 0$ is excluded, as well as the one where $T_{SS} = T_{SN} = T_{NS} = 0$. We are then left with five cases.

**Proposition 2:** There are five possibilities for a corner solution where at least one of the regions does not protect IPRs of (at least) some origin. These cases are determined by the market sizes and the innovative capabilities of North and South, and go beyond the classical free-riding of South on northern innovation.

The five cases are defined according to the value of the ratio between market sizes *vis-à-vis* the combinations of thresholds $B$, $C$ and $D$ we defined previously.

1\(^{st}\) Case: $\max\{C, D\} < \frac{N_N}{N_S} < B$. Here, $T_{NS} = 0$ as $D$ is too small because North does not have a large enough comparative advantage in generating innovations (low $b_N$) *vis-à-vis* the South’s costs of protecting northern IPRs. Consequently, South prefers to protect only its domestic patents and free-ride on northern innovations, as Lemma 2 ensures $T_{NN} > 0$.

2\(^{nd}\) Case: $\frac{N_N}{N_S} < \min\{C, D\}$. Here, $T_{SN} = 0$ as $C$ attains a large value *vis-à-vis* the ratio of market sizes, as South has a quite developed innovative capability (large $b_S$) and/or a quite large market size (large $N_S$). As a consequence, North knows that South will protect its domestic innovations (Lemma 3) and prefers to have its consumers free-riding on southern products.

3\(^{rd}\) Case: $D < \frac{N_N}{N_S} < C$. Here, $T_{NS} = 0$ and $T_{SN} = 0$ as the violations of the conditions for an interior solution stated in the first two cases hold at a time. A reciprocal misbehavior comes up as Nash equilibrium.

\(^\text{13}\) There are $2^3 = 8$ possible combinations, but we exclude the one where all three variables are positive since this stands for the interior solution (previously analysed).
4th Case: $B < \frac{N_N}{N_S} < D$. This is a peculiar case, as South protects northern innovations in spite of neglecting its domestic ones: $T_{SS} = 0$ and $T_{NS} > 0$. It requires a low elasticity of consumption of the representative consumer in each country (low $\alpha$) and North to have a much greater innovative capacity than South ($b_N \gg b_S$). Computation (available in Appendix B) shows that $B < D$ only when $\alpha$ is roughly smaller than 0.2 and $b_S$ is extremely close to 0 and $b_N$ is extremely close to 1.

5th Case: $\max\{B, D\} < \frac{N_N}{N_S}$. Here, $T_{SS} = 0$ and $T_{NS} = 0$ as the market in North is so much larger than the one in South that the latter has no incentive at all to protect IPRs at equilibrium.

IV - Welfare Analysis

As in LQ, we define the world (or global) welfare as the sum of the North’s and the South’s welfare. Consequently, after regrouping terms, we obtain:

$$W = W_S\left(T_{SS}, T_{SN}, T_{NN}, T_{NS}, M_S, M_N\right) + W_N\left(T_{SS}, T_{SN}, T_{NN}, T_{NS}, M_S, M_N\right)$$

$$= N_S u_S(t) + \int_0^{M_S} \Pi_S(i) di + R_{SS} + N_N u_N(t) + \int_0^{M_N} \Pi_N(i) di + R_{SN}$$

$$= \left[M_S\left(N_S T_{SS} + N_N T_{SN}\right) + M_N\left(N_S T_{NS} + N_N T_{NN}\right)\right](1-\alpha)\alpha^{2\epsilon} +$$

$$+ \left[T_{SS}\left(M_S + M_N\right)\left(N_S + N_N\right) - M_S\left(N_S T_{SS} + N_N T_{SN}\right) - M_N\left(N_S T_{NS} + N_N T_{NN}\right)\right](1-\alpha)\alpha^{\epsilon} +$$

$$+ M_S\left(N_S T_{SS} + N_N T_{SN}\right)A - \frac{b_S}{1 + b_S} M_S^{1+b_S} + U_{SS} + M_N\left(N_N T_{NN} + N_S T_{NS}\right)A - \frac{b_N}{1 + b_N} M_N^{1+b_N} + U_{SN}$$

**Proposition 3:** From an efficiency perspective, as every externality is incorporated, it is irrelevant which country protects the IPR on a good invented in a specific region. Only the overall incentives to innovation in that region matter.\(^{14}\)

**Proof:**

Take $Q_S = N_S T_{SS} + N_N T_{SN}$ and $Q_N = N_S T_{NS} + N_N T_{NN}$. From the above rearrangement it is easier to notice that the total welfare is a function of $Q_S$ and $Q_N$, and not of the patent lengths.

\(^{14}\) Grossman and Lai (2004) find exactly the same result, although in a different setting and assuming the NTCR.
individually. The steady state number of goods invented per period can also be written as a function of

Solving the maximization of W with respect to $Q_S$ and $Q_N$ yields (for an interior solution):

$$\frac{N_S T_{SS}^* + N_N T_{SN}^*}{N_S + N_N} = \frac{b_S T}{1 - (1 + \alpha)\alpha^{ae} + (1 - \alpha^{ae})b_S}$$

$$\frac{N_N T_{SN}^* + N_S T_{NS}^*}{N_S + N_N} = \frac{b_N T}{1 - (1 + \alpha)\alpha^{ae} + (1 - \alpha^{ae})b_N}$$

The terms on the left represent a weighted average of the patent lengths for products of a given origin, weighted by the destination country’s market size and averaged by the sum of the market sizes.

Note that $Q_N^* > Q_S^*$ since $b_N > b_S$. The North’s comparative advantage in generating innovations determines how larger should be the incentives to northern innovation and, consequently, the optimal split of R&D activity between North and the South.

However, if we were to introduce in the model some implementation cost of IPR protection which varies across regions as a convex function of the patent length, we stick to the interior Pareto solution just described. On the other hand, if we introduce some non-increasing marginal implementation cost, the efficient outcome will be a corner solution where only the country where it is cheaper to enforce IPR protection provides it. If it is cheaper to enforce IPRs in North due to available expertise and lower cost of public funds, the social optimum requires South not to protect any IPRs. Moreover, North should protect them to a larger extent than it does at Nash equilibrium:

$$T_{SN}^* = \frac{b_S T}{1 - (1 + \alpha)\alpha^{ae} + (1 - \alpha^{ae})b_S} \cdot \frac{N_S + N_N}{N_N} > T_{SN}^{\text{max}} = \frac{b_S T}{(1 - \alpha^{ae})(1 + b_S)}$$

$$T_{NN}^* = \frac{b_N T}{1 - (1 + \alpha)\alpha^{ae} + (1 - \alpha^{ae})b_N} \cdot \frac{N_S + N_N}{N_N} > T_{NN}^{\text{max}} = \frac{b_N T}{1 - (1 + \alpha)\alpha^{ae} + (1 - \alpha^{ae})b_N}$$

In LQ, multi-sectoral negotiations were required for giving South incentives to increase IPR protection, which was efficiency-enhancing given North sticks to its Nash equilibrium level of protection. Here, it is North who should increase its patent protection for efficiency matters. When the NTCR is not enforced, there is no role for multi-sectoral agreements to make it incentive-compatible.
for the poor countries to protect global innovation at efficient levels, as they are not the ones supposed to protect it at first best.

V - Conclusion

The main intention of this work is to analyse an IPR policy setting game where the respect to the National Treatment Commitment Rule (NTCR) is not taken as granted, since in the real world this rule is not properly enforced. Even if asymmetric information is not an issue, observable cheating is not always punishable. Using the model of Lai and Qiu (2003) and violating the NTCR, we first show the disconnection between the choice of patent enforcement levels for southern and northern innovations (Lemma 1). Two further results prove that R&D is always stimulated at equilibrium: North always protects northern IPRs (Lemma 2) and some protection is always provided for goods invented in South (Lemma 3).

We then state the conditions under which an interior solution for the game is obtained (Proposition 1). It requires the innovation cost in North to be small enough and the one in South to be not that large, as well as a market in North not much larger than the one in the South. The results strengthen the argument that the imposition of the NTCR and its effective enforcement is not a necessary tool neither for ensuring protection of northern IPRs in South (avoiding the classical free-riding of southern countries on northern innovation), nor for guaranteeing incentives for innovation in South or for stimulating R&D in the world. From our modeling, a natural move towards an interior solution seems to be more related to fundamental social policy in poor countries than to international IPR enforcement. By increasing the southern market for differentiated goods and its innovative capacity, which could be attained by polices targeted at increasing the purchase power of poor consumers and through investment in education, South will have proper incentives to protect IPRs at equilibrium. We will then satisfy the conditions for an interior solution.

In the last part, however, we show that the social optimum requires only overall incentives to R&D, no matter who provides them. However, if we further assume it is cheaper for North to provide IPR protection (due to existing expertise and lower cost of public funds), we move to a corner Pareto
solution where only North should protect IPRs for both northern and southern goods, and to a larger extent than North would do at Nash equilibrium. We are then able to obtain results to some extent contradictory to Lai and Qiu (2003): increasing welfare here requires rich countries to strengthen IPR protection.

We do not claim any straight policy recommendation from this paper, as the model here, just like Lai and Qiu (2003), lacks realism. It does not incorporate neither the possible positive externalities from IPR enforcement on the local economy (expertise development, institutional security), nor other incentives countries might have to protect IPRs on foreign innovations (international reputation, FDI attraction). Our main point was to challenge the assumption on the full enforcement of the National Treatment Commitment Rule and prove this assumption is not innocuous. Its violation furthermore allowed us to obtain insightful and intuitive results concerning the enforced patent lengths at equilibrium and first best.

Future research will hopefully fill some of those gaps.

References


United Nations Secretary General, 1964. The role of patents in the transfer of technology to developing countries: report of the Secretary-General. UN, New York.
Appendix A

In this Appendix A we provide some quick data analysis in support of our claim that the NTCR is not enforced in practice. Data comes from the WIPO website (http://www.wipo.int/ipstats/en/statistics/patents/) and concerns patent applications and patent grants per country and per year, disentangled between residents and non-residents. We wish to emphasize that patent offices worldwide have great discretion in the way they treat applications and that, in particular, they can discriminate against non-residents.

The first point to emphasize is that most African and South Asian countries consistently do not report their data to the WIPO. For the remaining developing countries, gaps for many years occur. This can possibly be pointed out as a cheating strategy for not providing evidence of the non-respect of the NTCR. The crucial point is that although in many cases non-compliance is observed, such cheating cannot be opposed in courts.

We report here a measure of the relative approval of patent applications of residents relative to non-residents: the relative approval ratio (RAR) from 1995 onwards (since TRIPs was signed in 1994). The RAR is the ratio between the approval ratio for residents and the one for non-residents, where the approval ratio is the ratio between total applications and total grants for a number of years, assuming on average it takes 2 (in the first column of the table below) or 3 (in the second column) or 4 years for an application to reach a decision. Although this is obviously not a very precise measure, it turns out that repeating the exercise under alternative granting lags has very little impact on the RAR for most countries (as shown below, Latvia and Slovenia being the exceptions). In any case, we do not dare providing any estimation in this Appendix, as just this quick raw data analysis is enough for making our point – which is basically to show that numbers for different countries differ enormously and that for some countries the ratio is consistently above 1.

A RAR equal to 1 means the country grants proportionally the same amount of patents to foreigners and nationals. If x % of the applications of residents succeed and x % of the applications of non-residents succeed, then RAR = 1. If it is the case, it only means that the country does not discriminate if on average both kinds of applications bear the same quality when submitted. And this relative quality of applications are probably correlated with the difficulty of the language, quality of IPR law cabinets and translators in the country, among others. However, numbers are sometimes so impressive that these other influences on the RAR lose importance.

The following are the relative approval ratios for the countries for which the database has no gaps in the TRIPs era.
The USA is the benchmark and its RAR is consistently close to 1, no matter the granting lag adopted. Australia consistently grants 5 times more patents to non-residents than to residents, and the UK 3.33 times. It sounds reasonable to assert that this is probably due to the truthful respect of the national treatment commitment rule, and thus the recognition of the better quality of foreign patents, which mainly originate in the USA. Also, as those countries have all the same official language, no translation is needed, which reduces the cost for foreigners to apply and also facilitates communication, increasing the precision of foreign applications to English-speaking countries as compared to translated ones.

The point to emphasize is that many countries (and even the European Patent Office) show RAR larger than 1 for any granting lag assumed: Czech Republic, Hong Kong, Japan, Monaco, Moldova, Poland, Romania and Russian Federation. Strikingly, Russia consistently grants proportionally twice as many patents to residents than to non-residents. Apart from possibly Hong Kong and Japan, it would be hard to justify this preference for residents on the basis of a better average quality of the residents’ patent applications. Surely language plays a
role, but our claim is that much of this gap comes from a lack of official concern towards the equal treatment of foreign applicants as compared to national ones.

Appendix B

In this Appendix we compute the values of B, C and D (labelled mistakenly A, B and C in the tables below). In particular, B and D are necessary for the consideration of the 4th case of corner solution explored in the text. This is the peculiar case where South prefers not to protect southern innovations but enforces patents on northern ones. It only possibly happens when $B < D$. As shown by the tables below, this requires the elasticity $\alpha$ to be roughly smaller than 0.2 and $b_S$ to be very close to 0 and $b_N$ to be very close to 1. For moderate values of $\alpha$ it is not even possible to have such case, as for example with $\alpha$ equal to $\frac{1}{2}$. 
\[
\frac{(1 - \alpha^\omega) (1 + b_2)}{1 - (1 + \alpha) \alpha^\omega} = A
\]

| $b_2$ | 0.01 | 0.05 | 0.10 | 0.15 | 0.20 | 0.25 | 0.30 | 0.35 | 0.40 | 0.45 | 0.50 | 0.55 | 0.60 | 0.65 | 0.70 | 0.75 | 0.80 | 0.85 | 0.90 | 0.95 | 0.96 | 0.97 | 0.98 | 0.99 |
|-------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| 0.005 | 1.34 | 1.48 | 1.63 | 1.70 | 1.77 | 1.84 | 1.91 | 1.98 | 2.06 | 2.12 | 2.19 | 2.26 | 2.33 | 2.40 | 2.47 | 2.55 | 2.62 | 2.69 | 2.76 | 2.81 |
| 0.01  | 1.54 | 1.60 | 1.67 | 1.75 | 1.83 | 1.90 | 1.96 | 2.05 | 2.13 | 2.21 | 2.28 | 2.36 | 2.44 | 2.51 | 2.59 | 2.66 | 2.74 | 2.82 | 2.89 | 2.97 | 3.03 |
| 0.015 | 1.62 | 1.69 | 1.77 | 1.86 | 1.93 | 2.01 | 2.09 | 2.17 | 2.25 | 2.33 | 2.41 | 2.49 | 2.57 | 2.65 | 2.73 | 2.81 | 2.89 | 2.97 | 3.06 | 3.13 | 3.20 |
| 0.02  | 1.69 | 1.76 | 1.84 | 1.93 | 2.01 | 2.10 | 2.18 | 2.26 | 2.35 | 2.43 | 2.52 | 2.60 | 2.69 | 2.77 | 2.85 | 2.94 | 3.02 | 3.10 | 3.19 | 3.27 | 3.34 |
| 0.025 | 1.76 | 1.83 | 1.92 | 2.00 | 2.09 | 2.18 | 2.26 | 2.35 | 2.44 | 2.52 | 2.61 | 2.70 | 2.79 | 2.87 | 2.96 | 3.05 | 3.13 | 3.22 | 3.31 | 3.39 | 3.46 |
| 0.03  | 1.82 | 1.89 | 2.00 | 2.10 | 2.19 | 2.29 | 2.39 | 2.49 | 2.59 | 2.69 | 2.79 | 2.89 | 2.99 | 3.09 | 3.19 | 3.29 | 3.39 | 3.49 | 3.59 | 3.69 | 3.79 |
| 0.035 | 1.87 | 1.96 | 2.06 | 2.16 | 2.26 | 2.37 | 2.48 | 2.59 | 2.70 | 2.82 | 2.93 | 3.06 | 3.19 | 3.34 | 3.49 | 3.65 | 3.81 | 3.98 | 4.15 | 4.32 | 4.50 |
| 0.04  | 1.92 | 2.00 | 2.10 | 2.20 | 2.32 | 2.44 | 2.57 | 2.70 | 2.84 | 2.99 | 3.13 | 3.29 | 3.46 | 3.65 | 3.85 | 4.07 | 4.30 | 4.53 | 4.77 | 5.02 | 5.28 |
| 0.045 | 1.97 | 2.05 | 2.15 | 2.25 | 2.38 | 2.54 | 2.73 | 2.83 | 3.03 | 3.22 | 3.42 | 3.62 | 3.84 | 4.07 | 4.32 | 4.58 | 4.86 | 5.14 | 5.44 | 5.76 | 6.10 |
| 0.05  | 2.02 | 2.10 | 2.20 | 2.30 | 2.40 | 2.60 | 2.80 | 3.00 | 3.20 | 3.40 | 3.60 | 3.80 | 4.05 | 4.32 | 4.60 | 4.90 | 5.21 | 5.53 | 5.86 | 6.21 | 6.58 |
| 0.055 | 2.07 | 2.15 | 2.25 | 2.35 | 2.45 | 2.66 | 2.86 | 3.06 | 3.28 | 3.50 | 3.72 | 3.96 | 4.22 | 4.50 | 4.80 | 5.11 | 5.43 | 5.77 | 6.13 | 6.51 | 6.91 |
| 0.06  | 2.11 | 2.19 | 2.30 | 2.40 | 2.51 | 2.71 | 2.92 | 3.13 | 3.35 | 3.58 | 3.83 | 4.10 | 4.39 | 4.70 | 5.02 | 5.36 | 5.72 | 6.10 | 6.50 | 6.91 | 7.34 |
| 0.065 | 2.16 | 2.24 | 2.34 | 2.45 | 2.56 | 2.77 | 2.99 | 3.22 | 3.47 | 3.74 | 4.03 | 4.34 | 4.68 | 5.04 | 5.42 | 5.82 | 6.25 | 6.72 | 7.22 | 7.74 | 8.29 |
| 0.07  | 2.19 | 2.28 | 2.39 | 2.50 | 2.76 | 3.02 | 3.32 | 3.65 | 4.01 | 4.39 | 4.80 | 5.24 | 5.71 | 6.21 | 6.75 | 7.32 | 7.92 | 8.56 | 9.24 | 9.96 | 10.72 |
| 0.075 | 2.23 | 2.32 | 2.43 | 2.54 | 2.66 | 2.87 | 3.12 | 3.40 | 3.79 | 4.21 | 4.66 | 5.15 | 5.68 | 6.25 | 6.86 | 7.51 | 8.20 | 8.93 | 9.71 | 10.53 | 11.39 |
| 0.08  | 2.27 | 2.36 | 2.47 | 2.58 | 2.70 | 2.93 | 3.20 | 3.51 | 3.85 | 4.23 | 4.66 | 5.15 | 5.68 | 6.25 | 6.86 | 7.51 | 8.20 | 8.93 | 9.71 | 10.53 | 11.39 |
| 0.085 | 2.31 | 2.40 | 2.51 | 2.63 | 2.74 | 2.97 | 3.24 | 3.57 | 3.94 | 4.35 | 4.80 | 5.30 | 5.85 | 6.45 | 7.11 | 7.82 | 8.57 | 9.37 | 10.22 | 11.12 | 12.08 |
| 0.09  | 2.34 | 2.44 | 2.56 | 2.67 | 2.79 | 3.02 | 3.27 | 3.56 | 3.90 | 4.30 | 4.75 | 5.25 | 5.79 | 6.38 | 7.02 | 7.70 | 8.43 | 9.22 | 10.07 | 10.98 | 11.95 |
| 0.095 | 2.38 | 2.47 | 2.59 | 2.71 | 2.83 | 3.06 | 3.30 | 3.62 | 3.97 | 4.34 | 4.74 | 5.20 | 5.71 | 6.30 | 6.99 | 7.74 | 8.53 | 9.38 | 10.29 | 11.26 | 12.30 | 13.42 |
\[
\frac{1 - \alpha^{\alpha e}}{1 - (1 + \alpha)\alpha^{\alpha e} + (1 - \alpha^{\alpha e})} = B
\]

<table>
<thead>
<tr>
<th>b_ε</th>
<th>0.01</th>
<th>0.05</th>
<th>0.10</th>
<th>0.15</th>
<th>0.20</th>
<th>0.25</th>
<th>0.30</th>
<th>0.35</th>
<th>0.40</th>
<th>0.45</th>
<th>0.50</th>
<th>0.55</th>
<th>0.60</th>
<th>0.65</th>
<th>0.70</th>
<th>0.75</th>
<th>0.80</th>
<th>0.85</th>
<th>0.90</th>
<th>0.95</th>
<th>0.99</th>
</tr>
</thead>
<tbody>
<tr>
<td>α</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.05</td>
<td>1.39</td>
<td>1.32</td>
<td>1.24</td>
<td>1.17</td>
<td>1.10</td>
<td>1.04</td>
<td>0.99</td>
<td>0.95</td>
<td>0.90</td>
<td>0.86</td>
<td>0.83</td>
<td>0.80</td>
<td>0.76</td>
<td>0.74</td>
<td>0.71</td>
<td>0.69</td>
<td>0.66</td>
<td>0.64</td>
<td>0.62</td>
<td>0.60</td>
<td>0.59</td>
</tr>
<tr>
<td>0.10</td>
<td>1.50</td>
<td>1.41</td>
<td>1.32</td>
<td>1.24</td>
<td>1.17</td>
<td>1.10</td>
<td>1.04</td>
<td>0.99</td>
<td>0.95</td>
<td>0.90</td>
<td>0.86</td>
<td>0.83</td>
<td>0.80</td>
<td>0.76</td>
<td>0.74</td>
<td>0.71</td>
<td>0.69</td>
<td>0.66</td>
<td>0.64</td>
<td>0.62</td>
<td>0.61</td>
</tr>
<tr>
<td>0.15</td>
<td>1.58</td>
<td>1.49</td>
<td>1.38</td>
<td>1.29</td>
<td>1.22</td>
<td>1.15</td>
<td>1.08</td>
<td>1.03</td>
<td>0.98</td>
<td>0.93</td>
<td>0.89</td>
<td>0.85</td>
<td>0.82</td>
<td>0.79</td>
<td>0.76</td>
<td>0.73</td>
<td>0.70</td>
<td>0.68</td>
<td>0.66</td>
<td>0.64</td>
<td>0.62</td>
</tr>
<tr>
<td>0.20</td>
<td>1.65</td>
<td>1.55</td>
<td>1.44</td>
<td>1.34</td>
<td>1.26</td>
<td>1.18</td>
<td>1.12</td>
<td>1.06</td>
<td>1.00</td>
<td>0.96</td>
<td>0.91</td>
<td>0.87</td>
<td>0.84</td>
<td>0.80</td>
<td>0.77</td>
<td>0.74</td>
<td>0.72</td>
<td>0.69</td>
<td>0.67</td>
<td>0.65</td>
<td>0.63</td>
</tr>
<tr>
<td>0.25</td>
<td>1.71</td>
<td>1.60</td>
<td>1.48</td>
<td>1.38</td>
<td>1.29</td>
<td>1.21</td>
<td>1.14</td>
<td>1.08</td>
<td>1.03</td>
<td>0.98</td>
<td>0.93</td>
<td>0.89</td>
<td>0.85</td>
<td>0.82</td>
<td>0.79</td>
<td>0.76</td>
<td>0.73</td>
<td>0.70</td>
<td>0.68</td>
<td>0.66</td>
<td>0.64</td>
</tr>
<tr>
<td>0.30</td>
<td>1.77</td>
<td>1.65</td>
<td>1.52</td>
<td>1.42</td>
<td>1.32</td>
<td>1.24</td>
<td>1.17</td>
<td>1.10</td>
<td>1.05</td>
<td>0.99</td>
<td>0.95</td>
<td>0.90</td>
<td>0.87</td>
<td>0.83</td>
<td>0.80</td>
<td>0.77</td>
<td>0.74</td>
<td>0.71</td>
<td>0.69</td>
<td>0.67</td>
<td>0.65</td>
</tr>
<tr>
<td>0.35</td>
<td>1.82</td>
<td>1.70</td>
<td>1.56</td>
<td>1.45</td>
<td>1.35</td>
<td>1.27</td>
<td>1.19</td>
<td>1.12</td>
<td>1.06</td>
<td>1.01</td>
<td>0.96</td>
<td>0.92</td>
<td>0.88</td>
<td>0.84</td>
<td>0.81</td>
<td>0.78</td>
<td>0.75</td>
<td>0.72</td>
<td>0.70</td>
<td>0.69</td>
<td>0.67</td>
</tr>
<tr>
<td>0.40</td>
<td>1.87</td>
<td>1.74</td>
<td>1.60</td>
<td>1.48</td>
<td>1.38</td>
<td>1.29</td>
<td>1.21</td>
<td>1.14</td>
<td>1.08</td>
<td>1.03</td>
<td>0.98</td>
<td>0.93</td>
<td>0.89</td>
<td>0.85</td>
<td>0.82</td>
<td>0.79</td>
<td>0.75</td>
<td>0.73</td>
<td>0.70</td>
<td>0.69</td>
<td>0.68</td>
</tr>
<tr>
<td>0.45</td>
<td>1.92</td>
<td>1.78</td>
<td>1.63</td>
<td>1.51</td>
<td>1.40</td>
<td>1.31</td>
<td>1.23</td>
<td>1.16</td>
<td>1.10</td>
<td>1.04</td>
<td>0.99</td>
<td>0.94</td>
<td>0.90</td>
<td>0.86</td>
<td>0.83</td>
<td>0.79</td>
<td>0.76</td>
<td>0.73</td>
<td>0.71</td>
<td>0.69</td>
<td>0.68</td>
</tr>
<tr>
<td>0.50</td>
<td>1.96</td>
<td>1.82</td>
<td>1.67</td>
<td>1.54</td>
<td>1.43</td>
<td>1.33</td>
<td>1.25</td>
<td>1.18</td>
<td>1.11</td>
<td>1.05</td>
<td>1.00</td>
<td>0.95</td>
<td>0.91</td>
<td>0.87</td>
<td>0.83</td>
<td>0.80</td>
<td>0.77</td>
<td>0.74</td>
<td>0.71</td>
<td>0.69</td>
<td>0.68</td>
</tr>
<tr>
<td>0.55</td>
<td>2.00</td>
<td>1.85</td>
<td>1.70</td>
<td>1.56</td>
<td>1.45</td>
<td>1.35</td>
<td>1.27</td>
<td>1.19</td>
<td>1.12</td>
<td>1.06</td>
<td>1.01</td>
<td>0.96</td>
<td>0.92</td>
<td>0.88</td>
<td>0.84</td>
<td>0.81</td>
<td>0.78</td>
<td>0.75</td>
<td>0.72</td>
<td>0.70</td>
<td>0.69</td>
</tr>
<tr>
<td>0.60</td>
<td>2.04</td>
<td>1.89</td>
<td>1.73</td>
<td>1.59</td>
<td>1.47</td>
<td>1.37</td>
<td>1.29</td>
<td>1.21</td>
<td>1.14</td>
<td>1.08</td>
<td>1.02</td>
<td>0.97</td>
<td>0.93</td>
<td>0.89</td>
<td>0.85</td>
<td>0.81</td>
<td>0.78</td>
<td>0.75</td>
<td>0.72</td>
<td>0.70</td>
<td>0.69</td>
</tr>
<tr>
<td>0.65</td>
<td>2.08</td>
<td>1.92</td>
<td>1.76</td>
<td>1.61</td>
<td>1.49</td>
<td>1.39</td>
<td>1.33</td>
<td>1.22</td>
<td>1.15</td>
<td>1.09</td>
<td>1.03</td>
<td>0.98</td>
<td>0.93</td>
<td>0.89</td>
<td>0.85</td>
<td>0.82</td>
<td>0.79</td>
<td>0.76</td>
<td>0.73</td>
<td>0.70</td>
<td>0.69</td>
</tr>
<tr>
<td>0.70</td>
<td>2.12</td>
<td>1.96</td>
<td>1.80</td>
<td>1.64</td>
<td>1.51</td>
<td>1.41</td>
<td>1.31</td>
<td>1.23</td>
<td>1.16</td>
<td>1.10</td>
<td>1.04</td>
<td>0.99</td>
<td>0.94</td>
<td>0.90</td>
<td>0.86</td>
<td>0.83</td>
<td>0.79</td>
<td>0.76</td>
<td>0.73</td>
<td>0.70</td>
<td>0.69</td>
</tr>
<tr>
<td>0.75</td>
<td>2.16</td>
<td>1.99</td>
<td>1.81</td>
<td>1.66</td>
<td>1.53</td>
<td>1.42</td>
<td>1.33</td>
<td>1.25</td>
<td>1.17</td>
<td>1.11</td>
<td>1.05</td>
<td>1.00</td>
<td>0.95</td>
<td>0.91</td>
<td>0.87</td>
<td>0.83</td>
<td>0.80</td>
<td>0.77</td>
<td>0.74</td>
<td>0.71</td>
<td>0.69</td>
</tr>
<tr>
<td>0.80</td>
<td>2.20</td>
<td>2.02</td>
<td>1.83</td>
<td>1.68</td>
<td>1.55</td>
<td>1.44</td>
<td>1.34</td>
<td>1.26</td>
<td>1.18</td>
<td>1.12</td>
<td>1.06</td>
<td>1.01</td>
<td>0.96</td>
<td>0.92</td>
<td>0.88</td>
<td>0.84</td>
<td>0.81</td>
<td>0.78</td>
<td>0.75</td>
<td>0.72</td>
<td>0.70</td>
</tr>
<tr>
<td>0.85</td>
<td>2.23</td>
<td>2.05</td>
<td>1.86</td>
<td>1.70</td>
<td>1.57</td>
<td>1.45</td>
<td>1.35</td>
<td>1.27</td>
<td>1.19</td>
<td>1.13</td>
<td>1.07</td>
<td>1.01</td>
<td>0.96</td>
<td>0.92</td>
<td>0.88</td>
<td>0.84</td>
<td>0.81</td>
<td>0.78</td>
<td>0.75</td>
<td>0.72</td>
<td>0.70</td>
</tr>
<tr>
<td>0.90</td>
<td>2.27</td>
<td>2.08</td>
<td>1.88</td>
<td>1.72</td>
<td>1.59</td>
<td>1.47</td>
<td>1.37</td>
<td>1.28</td>
<td>1.20</td>
<td>1.14</td>
<td>1.07</td>
<td>1.02</td>
<td>0.97</td>
<td>0.93</td>
<td>0.89</td>
<td>0.85</td>
<td>0.81</td>
<td>0.78</td>
<td>0.75</td>
<td>0.72</td>
<td>0.70</td>
</tr>
<tr>
<td>0.95</td>
<td>2.30</td>
<td>2.11</td>
<td>1.91</td>
<td>1.74</td>
<td>1.60</td>
<td>1.46</td>
<td>1.33</td>
<td>1.29</td>
<td>1.21</td>
<td>1.14</td>
<td>1.06</td>
<td>1.03</td>
<td>0.98</td>
<td>0.93</td>
<td>0.89</td>
<td>0.85</td>
<td>0.82</td>
<td>0.79</td>
<td>0.76</td>
<td>0.73</td>
<td>0.71</td>
</tr>
</tbody>
</table>
\[
\frac{1 - (1 + \alpha)\alpha^{\alpha \epsilon} + (1 - \alpha^{\alpha \epsilon})b_N}{1 - \alpha^{\alpha \epsilon}} = 1 + b_N - \frac{\alpha^{\epsilon}}{1 - \alpha^{\alpha \epsilon}} = C
\]

| \(b_N\) | 0.01 | 0.05 | 0.10 | 0.15 | 0.20 | 0.25 | 0.30 | 0.35 | 0.40 | 0.45 | 0.50 | 0.55 | 0.60 | 0.65 | 0.70 | 0.75 | 0.80 | 0.85 | 0.90 | 0.95 | 0.99 |
|-------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| \(\alpha\) | 0.05 | 0.72 | 0.76 | 0.81 | 0.86 | 0.91 | 0.96 | 1.01 | 1.06 | 1.11 | 1.16 | 1.21 | 1.26 | 1.31 | 1.36 | 1.41 | 1.46 | 1.51 | 1.56 | 1.61 | 1.66 | 1.70 |
|       | 0.10 | 0.67 | 0.71 | 0.76 | 0.81 | 0.86 | 0.91 | 0.96 | 1.01 | 1.06 | 1.11 | 1.16 | 1.21 | 1.26 | 1.31 | 1.36 | 1.41 | 1.46 | 1.51 | 1.56 | 1.61 | 1.66 |
|       | 0.15 | 0.63 | 0.67 | 0.72 | 0.77 | 0.82 | 0.87 | 0.92 | 0.97 | 1.02 | 1.07 | 1.12 | 1.17 | 1.22 | 1.27 | 1.32 | 1.37 | 1.42 | 1.47 | 1.52 | 1.57 | 1.61 |
|       | 0.20 | 0.61 | 0.66 | 0.70 | 0.75 | 0.80 | 0.86 | 0.90 | 0.96 | 1.00 | 1.06 | 1.10 | 1.15 | 1.20 | 1.25 | 1.30 | 1.36 | 1.40 | 1.45 | 1.50 | 1.55 | 1.60 |
|       | 0.25 | 0.58 | 0.62 | 0.67 | 0.72 | 0.77 | 0.82 | 0.87 | 0.92 | 0.97 | 1.02 | 1.07 | 1.12 | 1.17 | 1.22 | 1.27 | 1.32 | 1.37 | 1.42 | 1.47 | 1.52 | 1.57 |
|       | 0.30 | 0.57 | 0.61 | 0.66 | 0.71 | 0.76 | 0.81 | 0.86 | 0.91 | 0.96 | 1.01 | 1.06 | 1.11 | 1.16 | 1.21 | 1.26 | 1.31 | 1.36 | 1.41 | 1.46 | 1.51 | 1.56 |
|       | 0.35 | 0.55 | 0.59 | 0.64 | 0.69 | 0.74 | 0.79 | 0.84 | 0.89 | 0.94 | 0.99 | 1.04 | 1.09 | 1.14 | 1.19 | 1.24 | 1.29 | 1.34 | 1.39 | 1.44 | 1.49 | 1.53 |
|       | 0.40 | 0.53 | 0.57 | 0.62 | 0.67 | 0.72 | 0.77 | 0.82 | 0.87 | 0.92 | 0.97 | 1.02 | 1.07 | 1.12 | 1.17 | 1.22 | 1.27 | 1.32 | 1.37 | 1.42 | 1.47 | 1.51 |
|       | 0.45 | 0.52 | 0.56 | 0.61 | 0.66 | 0.71 | 0.76 | 0.81 | 0.86 | 0.91 | 0.96 | 1.01 | 1.06 | 1.11 | 1.16 | 1.21 | 1.26 | 1.31 | 1.36 | 1.41 | 1.46 | 1.50 |
|       | 0.50 | 0.51 | 0.56 | 0.60 | 0.65 | 0.70 | 0.75 | 0.80 | 0.85 | 0.90 | 0.95 | 1.00 | 1.05 | 1.10 | 1.15 | 1.20 | 1.25 | 1.30 | 1.35 | 1.40 | 1.45 | 1.50 |
|       | 0.55 | 0.50 | 0.54 | 0.59 | 0.64 | 0.69 | 0.74 | 0.79 | 0.84 | 0.89 | 0.94 | 0.99 | 1.04 | 1.09 | 1.14 | 1.19 | 1.24 | 1.29 | 1.34 | 1.39 | 1.44 | 1.49 |
|       | 0.60 | 0.49 | 0.53 | 0.58 | 0.63 | 0.68 | 0.73 | 0.78 | 0.83 | 0.88 | 0.93 | 0.98 | 1.03 | 1.08 | 1.13 | 1.18 | 1.23 | 1.28 | 1.33 | 1.38 | 1.43 | 1.47 |
|       | 0.65 | 0.48 | 0.52 | 0.57 | 0.62 | 0.67 | 0.72 | 0.77 | 0.82 | 0.87 | 0.92 | 0.97 | 1.02 | 1.07 | 1.12 | 1.17 | 1.22 | 1.27 | 1.32 | 1.37 | 1.42 | 1.47 |
|       | 0.70 | 0.47 | 0.51 | 0.56 | 0.61 | 0.66 | 0.71 | 0.76 | 0.81 | 0.86 | 0.91 | 0.96 | 1.01 | 1.06 | 1.11 | 1.16 | 1.21 | 1.26 | 1.31 | 1.36 | 1.41 | 1.46 |
|       | 0.75 | 0.46 | 0.50 | 0.55 | 0.60 | 0.65 | 0.70 | 0.75 | 0.80 | 0.85 | 0.90 | 0.95 | 1.00 | 1.05 | 1.10 | 1.15 | 1.20 | 1.25 | 1.30 | 1.35 | 1.40 | 1.45 |
|       | 0.80 | 0.45 | 0.49 | 0.54 | 0.59 | 0.64 | 0.69 | 0.74 | 0.79 | 0.84 | 0.89 | 0.94 | 0.99 | 1.04 | 1.09 | 1.14 | 1.19 | 1.24 | 1.29 | 1.34 | 1.39 | 1.43 |
|       | 0.85 | 0.45 | 0.49 | 0.54 | 0.59 | 0.64 | 0.69 | 0.74 | 0.79 | 0.84 | 0.89 | 0.94 | 0.99 | 1.04 | 1.09 | 1.14 | 1.19 | 1.24 | 1.29 | 1.34 | 1.39 | 1.43 |
|       | 0.90 | 0.44 | 0.48 | 0.53 | 0.58 | 0.63 | 0.68 | 0.73 | 0.78 | 0.83 | 0.88 | 0.93 | 0.98 | 1.03 | 1.08 | 1.13 | 1.18 | 1.23 | 1.28 | 1.33 | 1.38 | 1.42 |
|       | 0.95 | 0.43 | 0.47 | 0.52 | 0.57 | 0.62 | 0.67 | 0.72 | 0.77 | 0.82 | 0.87 | 0.92 | 0.97 | 1.02 | 1.07 | 1.12 | 1.17 | 1.22 | 1.27 | 1.32 | 1.37 | 1.41 |