Fiscal and monetary policy interaction: a simulation based analysis of a two-country New Keynesian DSGE model with heterogeneous households‡

Marcos Valli*

Fabia Carvalho**

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Abstract

This paper presents a simulation-based analysis of a New-Keynesian DSGE model of two mixed economies where public and private capital combine to produce intermediate goods. The fiscal authority pursues a policy of primary surplus targets that features non Ricardian effects, using a multitude of policy instruments: current expenditures, taxation, distributional transfers, and investment. The economies are populated by heterogeneous agents that differ both as to their savings constraints and as to their labor skills. The model also features a number of nominal and real frictions in goods and labor markets. Our model builds on ECB’s New Area Wide Model (NAWM), presented in Coenen et. al. (2008) and Christoffel et. al. (2008), however, in addition to improvements in the modeling of the fiscal sector, we correct the specification of the final-goods price indices, which now become compatible with the assumption of perfect competition (zero-profits) in the final-goods markets. Additionally, we correct the recursive representation of the wage setting rule and the wage distortion index. We calibrate the model for the Brazilian economy and the rest of the world (USA+EURO) to assess the macroeconomic and distributional effects of domestic shocks to monetary policy, government investment, public transfers and to the primary surplus. We also adopt alternative types of monetary and fiscal policy reaction functions to analyze the interaction between fiscal and monetary policies. Comparisons are made through dynamic impulse-response functions and moment analysis.

Keywords: DSGE, fiscal policy, monetary policy, government investment, primary surplus, heterogeneous agents, market frictions

JEL Classification: E32; E62; E63

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* Executive Office of Special Studies, Central Bank of Brazil. Email: marcos.valli@bcb.gov.br (Corresponding author)

** Executive Office of Special Studies, Central Bank of Brazil. Email: fabia.carvalho@bcb.gov.br
1. Introduction

DSGE models are now part of the core set of tools used by major central banks to assess the widespread effects of policy making. Building mostly on the recent New Keynesian literature (Monacelli, 2005, Galí and Monacelli, 2008, Smets and Wouters, 2003, Adolfson et. al., 2007, among others), these models have been further enriched in several aspects by the inclusion of alternative pricing assumptions, imperfect competition in distinct economic sectors, international financial linkages, and financial frictions. However, as Ratto et. al. (2009) argue, “so far, not much work has been devoted towards exploring the role of fiscal policy in the (DSGE) New-Keynesian model”. ¹

DSGE models are a promising tool to understanding the final outcome of interactions between fiscal and monetary policies. The recent trend in modeling the fiscal sector in New Keynesian DSGE models includes non-Ricardian agents and activist fiscal policies (Gunter and Coenen, 2005, Mourougane and Vogel, 2008, and Ratto et. al., 2009) mostly to assess the effects of shocks to government consumption on aggregate consumption and output and also the distributional effects of fiscal policies. However, the practice of fiscal policy usually encompasses more than decisions on consumption expenditures. The government often intervenes in the economy through public investment with important externalities upon private investment.

Ratto et. al. (2009) are a recent attempt to account for the strategic role of public investment in policy decisions in a DSGE setup, introducing a rule for public investment that responds to the business cycle and assuming that public capital interferes in the productivity of private firms, but does not belong to factor decisions.

We also introduce public investment in our model, but in a different manner. We assume that firms can choose the optimal composition of private and public capital goods in their production of intermediate goods. Therefore, there is a market for physical capital, where investment strategies are discretionary to the capital owner.

Our model builds on ECB’s New Area Wide Model (NAWM) presented in Coenen et. al. (2008) and Christoffel et. al. (2008), hereinafter referred to as CMS and CCW respectively. However, there are important distinctions. First, in addition to having a government that uses consumption and transfers to fulfill its fiscal objectives, we introduce a fiscal policy rule that tracks primary surplus targets, but also reacts to

¹ Rato, Roeger and Veld (p.p. 222). The italics are ours.
economic conditions and to the level of the public debt. We understand that this framework better approximates the theoretical setting of these models to the current practice of fiscal policy in a number of countries. Second, monetary policy in the domestic economy is modeled with a forward looking rule to better approximate the conduct of policy to an inflation targeting framework.

Third, the labor market is augmented by introducing heterogeneity in labor skills. This modeling strategy is important to allow for wage differentiation amongst households that work the same amount of hours, and thus improve the theoretical treatment of inequality. Fourth, we derive different consumer and investment price indices to guarantee that the producers of final consumption and investment goods operate under perfect competition, a feature that the NAWM does not show under the price indices presented in CMS and CCW. Fifth, these modifications yield a representation of the economy’s resource constraint distinct from the one presented in CMS and CCW. Sixth, we correct the recursive representation of the wage setting rule and the wage distortion index. Finally, we introduce a steady-state spread between the interest rates of domestically and internationally traded bonds that accounts for the risk premium that can be significant in emerging economies.

We calibrate our model for the Brazilian economy and the rest of the world (USA+EURO), leaving the monetary and fiscal policy rules of the rest of the world as specified in CMS and CCW. We assess the impulse responses to shocks and analyze the implications of the interaction between fiscal and monetary policies. In particular, we assess the macroeconomic and distributional effects of shocks to government investment, primary surplus, transfers, and monetary policy, and analyze the effects of concomitant shocks to the fiscal and monetary policy rules. We also draw some conclusions on the impact of varying degrees of rigor in the implementation of the fiscal rule, of fiscal commitment to a sustainable public debt path, and of the commitment of the monetary policy to the inflation target.

A contractionist shock to monetary policy has the standard effect of reducing inflation and output to levels below the steady trend, with implications on the capital, labor and goods markets. In this model, the shock also implies a rise in public debt, with important effects on the primary surplus through the fiscal rule. An expansionist shock to the primary surplus implies an increase in government consumption and investment, with short-lived expansionist effects on the output and long-lasting upward pressure on inflation.
Our simulations indicated that temporary shocks to government investment and to government transfers, under a primary surplus target regime committed with debt stabilization, imply a general loss of economic dynamism, yet through distinct channels. The shock to government investment increases total investment and, in equilibrium, capital increases. However, the shock to government investment crowds out private investment, and, as it is financed through debt issuance and cuts in current government expenditures, firms reduce their capital utilization. In other words, demand effects predominate over those of supply. The distributional effects of the shock are not significant. On the other hand, a shock to government transfers is favorable to households with fewer alternatives to transfer wealth across periods. This distributional effect is short-lived, lasting for about a year, suggesting that contraction of government demand, due to the fiscal discipline, cannot be offset by transfers in the long run.

Different levels of commitment to stabilization of the public debt have important implications to the model’s responses, to the volatility of endogenous variables and to the participation of each shock in the variance of inflation and output growth. As to impulse responses, increasing commitment to the steady state debt strengthens the contractionist impact of the monetary shock onto consumer price inflation and output, and implies stronger distributive effects. The volatility of consumer price inflation increases, as does the correlation between inflation and output growth. We show that in an specific range of degrees of commitment to the steady state debt, monetary policy shocks have the least impact onto output variability and the most impact onto the variance of consumer price inflation.

A more rigorous implementation of the primary surplus rule implies lower variance of inflation and output growth, and significantly increases the influence of the monetary policy shock onto the variance of consumer price inflation and of the output growth. We also obtain that strongly (and negatively) correlated policy shocks dampen the contractionist effect of the monetary policy shock onto inflation and output.

Fixing the commitment to debt stabilization at a level that grants the least impact of the monetary policy shock onto the variance of consumer price inflation, we obtain that strongly (and negatively) correlated policy shocks reduce the variance of inflation and output growth. We also obtain that increasing the commitment to the inflation target in the monetary policy rule reduces the variance of inflation and output growth, and their correlation, with the drawback that the fiscal shock has a higher stake on the variance of inflation.
The model is also simulated under alternative monetary policy rules. Augmenting the rule to include an explicit reaction to the exchange rate variability or the output growth adds sluggishness to the reversal of inflation to the steady state after a monetary policy shock. However, the initial impact of the shock onto the economic activity is milder (yet more persistent). By activating the policy shocks only, the response to the exchange rate volatility reduces the variance of inflation, output growth and the exchange rate. The monetary policy shock has a smaller effect on output variation and gains influence on the volatility of inflation. The response to the output growth reduces output growth variance, but increases the variance of consumer price inflation and the exchange rate. Under this policy rule, the shock to the monetary policy loses its influence over inflation variance, but also reduces its participation on the variance of output growth and the exchange rate.

The paper is organized as follows. Section 2 provides an overview of the model, focusing on the extensions proposed to the NAWM. Section 3 details the calibration strategy and the normalization to attain stationary representations of the aggregated variables. Section 4 analyses the impulse responses of the model and experiments with distinct types of policy orientation. The last section concludes the paper.

2. The model

In the model, there are two economies of different sizes that interact in both goods and financial markets. Except for monetary and fiscal policy rules, both economies are symmetric with respect to the structural equations that govern their dynamics, but the structural parameters are allowed to differ across countries.

Each economy is composed of households, firms, and the government. Households are distributed in two continuous sets that differ as to their access to capital and financial markets, and also to their labor skills. Families in the less specialized group, hereinafter referred to as group $J$, can smooth consumption only through non-interest bearing money holdings, whilst the other group of households (group $I$), with more specialized skills, has full access to capital, and to domestic and international financial markets. Within their groups, households supply labor in a competitive monopolistic labor market to produce intermediate goods. There are Calvo-type wage rigidities combined with hybrid wage indexation rules.
Firms are distributed in two sets. The first produces intermediate goods for both
domestic and foreign markets, and operates under monopolistic competition with Calvo-
type price rigidities combined with hybrid price indexation. The other set is composed
of three firms, each one of them producing one single type of final good: private
consumption, public consumption, or investment goods. Final goods firms are assumed
to operate under perfect competition.

The government comprises a monetary authority that sets nominal interest rates,
and a fiscal authority that levies taxes on most economic activities, and decides on its
consumption expenditures, investment, and distributional transfers to achieve a primary
surplus target.

A detailed technical description of the model is left to appendix A. In the
remaining of this section, we correct important equations in CMS and CCW and model
a fiscal sector that is more in line with the current practice of fiscal policy in a wide
number of countries. Public investment has spillover effects over private investment and
affects the market for capital goods.

2.1. Wage setting

Households in group $I$ choose consumption $C_{i,t}$ and labor services $N_{i,t}$ to maximize the
separable intertemporal utility with habit formation

$$E_t \left\{ \sum_{k=0}^{\infty} \beta^k \left[ \frac{1}{1-\sigma} \left( C_{i,t+k} - \kappa C_{I,t+k-1} \right)^{1-\sigma} - \frac{1}{1+\xi} \left( N_{i,t+k}^{1+\xi} \right) \right] \right\} \quad (1)$$

subject to the budget constraint

$$\left(1 + \tau^c_i + \Gamma_i (v_i)\right) P_{C,i} C_{i,t} + P_{L,i} I_{i,H,t} + R_t^{-1} B_{i,t+1} + \left(1 - \Gamma_t \left(B_{i,t}^F \right) \right) \tau_{F,t} + \bar{\Xi}_{i,t} + \Phi_{i,t} \quad (2)$$

$$= \left(1 - \tau^k_i - \tau^w_i \right) W_{i,t} N_{i,t} + \left(1 - \tau^k_i \right) \left[ u_{i,t} R_{K,H,t} - \Gamma_i (u_{i,t}) P_{L,t} \right] K_{i,H,t} + \tau^k_i \delta P_{i,t} K_{i,H,t}$$

$$+ \left(1 - \tau^{\nu}_{i} \right) D_{i,t} + TR_{i,t} - T_{i,t} + B_{i,t} + S_{t} B_{i,t}^F + M_{i,t-1}$$

where $W_{i,t}$ is the wage earned by the household for one unit of labor services, $I_{i,H,t}$ is
private investment in capital goods, $B_{i,t+1}$ are domestic government bonds, $M_{i,t}$ is
money, $B_{i,t+1}^F$ are foreign private bonds, $S_{t}$ is the nominal exchange rate, $R_{F,t}$ is the
interest rate of the foreign bonds, $rp$ is the steady state spread between interest rates of
domestically and internationally traded bonds, \( \Gamma_B'B_t^F(B_{i,t}^F) \) is a risk premium when outside the steady state (detailed in appendix B), \( \Gamma_i(v_{i,t}) \) is a transaction cost, \( D_{i,t} \) are dividends, \( K_{i,H,t} \) is the private capital stock, \( u_{i,t} \) is capital utilization, \( \Gamma_{u}(u_{i,t}) \) is the cost on capital utilization, \( R_{K,H,t} \) is the gross rate of the return on private capital, \( TR_{i,t} \) are transfers from the government, \( \Xi_{i,t} \) is a lump sum rebate on the risk premium introduced in the negotiation of international bonds, and \( \Phi_{i,t} \) is the stock of contingent securities negotiated within group \( I \), which act as an insurance against risks on labor income. Taxes are \( \tau_i^c \) (consumption), \( \tau_i^N \) (labor income), \( \tau_i^W \) (social security), \( \tau_i^K \) (capital income), \( \tau_i^D \) (dividends) and \( T_{i,t} \) (lump sum, active only for the foreign economy). The parameter \( \kappa \) is the external habit persistence, \( \beta \) is the intertemporal discount factor, \( \sigma_1 \) is the intertemporal elasticity of consumption substitution, \( \zeta_1 \) is the elasticity of labor effort relative to the real wage, and \( \delta \) is the depreciation of capital. Price indices are \( P_{c,t} \) (final consumption good), is the price index of consumption goods, and \( P_{I,t} \) (investment goods). Cost functions are detailed in appendix B.

Within each group, households compete in a monopolistic competitive labor market. By setting wage \( W_{i,t} \), household \( i \) commits to meeting any labor demand \( N_{i,t} \). Wages are set à la Calvo, with a probability \( (1-\xi_t) \) of optimizing each period. Households that do not optimize readjust their wages based on a geometric average of realized and steady state inflation \( \bar{W}_{i,t} := \left( \frac{P_{c,t-1}}{P_{c,t-2}} \right)^{\xi_t} P_{c,t-1}^{1-\xi_t} W_{i,t-1} \). Optimizing households in group \( I \) choose the same wage \( \tilde{W}_{i,t} \), which we denote \( \tilde{W}_{I,t} \).

As in CMS, household \( i \)'s optimization with respect to the wage \( \tilde{W}_{I,t} \) yields the first order condition, which is the same for every optimizing household:

\[
E_t \left[ \sum_{k=0}^{\infty} \left( \xi_t \beta \right)^k N_{i,t+k} \left( \Lambda_{i,t+k} (1-\tau_{i,t+k}^{N} - \tau_{i,t+k}^{W}) \frac{\tilde{W}_{I,t}}{P_{c,t+k-1}} \frac{P_{c,t+k-2}}{P_{c,t+k-1}} \pi_{c,t+k}^{1-\xi_t} \right) \right] = 0
\]
where $\frac{\Lambda_{t,I}}{P_{C,t}}$ is the Lagrange multipliers for the budget constraint, and $\eta_t/(\eta_t - 1)$ is the after-tax real wage markup, in the absence of wage rigidity (when $\xi_t \rightarrow 0$), with respect to the marginal rate of substitution between consumption and leisure. The markup results from the worker’s market power to set wages.

Equation (3) can be expressed in a recursive form:

$$\left(1-\omega\right)^\xi \left(\frac{\hat{W}_{t,I}}{P_{C,t}}\right)^{1+\eta_t,\xi} = \frac{\eta_t}{\eta_t - 1} \frac{F_{t,I}}{G_{t,I}}$$

where

$$F_{t,I} := \left(\frac{W_{I,t}}{P_{C,t}}\right)^{n_t} N_t^{I} + \xi_t, \beta, E_t \left(\frac{\pi_{C,t+1}}{\pi_{C,t}^{1+\xi_t}}\right)^{\eta_t(1+\xi_t)} \cdot F_{t+1}$$

$$G_{t,I} := \Lambda_{t,I} \left(1 - \tau_{t,N} - \tau_{t,W}^{I}\right) \left(\frac{W_{I,t}}{P_{C,t}}\right)^{n_t} N_t^{I} + \xi_t, \beta, E_t \left(\frac{\pi_{C,t+1}}{\pi_{C,t}^{1+\xi_t}}\right)^{\eta_t-1} \cdot G_{t+1}$$

and $N_t^{I}$ is households group $I$ aggregate labor demanded by firms, and $W_{I,t}$ is household group $I$’s aggregate wage index.

This recursive representation corrects the one presented in CMS after including the multiplicative constant $(1-\omega)^\xi$ on the left hand side. This constant arises from the labor demand equation. The derivation of equation (4) is detailed in appendix C.

2.2. Production

There are two types of firms in the model: producers of tradable intermediate goods and producers of non-tradable final goods. All final goods producers combine domestic and foreign intermediate goods in the production, with the exception of the one producing public consumption goods, as, in practice, the greatest share of government consumption is composed of services, which are usually non-tradable.

2.2.1 Intermediate goods firms
A continuum of firms, indexed by \( f \in [0,1] \), produce tradable intermediate goods \( Y_{f,j} \) under monopolistic competition. We depart from the set-up in CMS by introducing mixed capital as an input to the production of these goods. We assume that firms competitively rent capital from the government, \( K_{f,j}^G \), and from households in group \( I \), \( K_{f,j}^H \), and transform it into the production input \( K_{f,j}^S \) through the following CES technology:

\[
K_{f,j}^S = \left[ (1 - \omega_s)^{-\eta_s} \left( K_{f,j}^H \right)^{\eta_h^{-1}} + (\omega_s)^{-\eta_s} \left( K_{f,j}^G \right)^{\eta_G^{-1}} \right]^{\eta_s^{-1}}
\]

where \( \omega_s \) is the economy’s degree of dependence on government investment, and \( \eta_s \) stands for the elasticity of substitution between private and public goods, and also relates to the sensitivity of demand to the cost variation in each type of capital.

In addition to renting capital services, intermediate goods firms hire labor \( N_{f,j} \) from all groups of households to produce the intermediate good \( Y_{f} \) using the technology.

\[
Y_{f,j} = z_i \left( u_{f,j} \cdot K_{f,j}^S \right) \left( \frac{z_i}{N_{f,j}} \right)^{\alpha} - \psi \cdot zn_i
\]

where \( \psi \cdot zn_i \) is a cost, which in steady state is constant relative to the output. The constant \( \psi \) is chosen to ensure zero profit in the steady state, and \( z_i \) and \( zn_i \) are respectively temporary and endogenous processes that follow the process:

\[
\ln(z_i) = (1 - \rho_z) \ln(z) + \rho_z \ln(z_{i-1}) + \varepsilon_{z,i} \tag{7}
\]

and

\[
\frac{zn_i}{zn_{i-1}} = (1 - \rho_{zn}) \cdot gy + \rho_{zn} \cdot \frac{zn_{i-1}}{zn_{i-2}} + \varepsilon_{zn,i} \tag{8}
\]

where \( z \) is the stationary level of total factor productivity, \( gy \) is the steady state growth rate of labor productivity, \( \rho_z \) and \( \rho_{zn} \) are parameters, and \( \varepsilon_{z,i} \) and \( \varepsilon_{zn,i} \) are exogenous white noise processes.

For a given level of production, intermediate goods firms take the cost of capital \( R_{K,j} \), the average per capita wage \( W_i \), and social security contribution \( \tau_i^W \) as given to
minimize the total cost of inputs \( R_{K,f}K_{f,t}^S + (1 + \tau^w_t)W_tN_{f,t} \) subject to the technology in (6). The firm’s marginal cost \( (MC_{f,t}) \) obtains from the first order conditions:

\[
MC_{f,t} = \frac{R_{K,f}K_{f,t}^S + (1 + \tau^w_t)W_tN_{f,t}}{Y_{f,t} + \psi z_n_t}
\]

which can also be expressed as a function of wages and capital remuneration. It can be shown that the marginal cost is equal across firms, i.e., \( MC_{f,t} = MC_t \), which implies

\[
MC_t = \frac{1}{z_n_t^{1-\alpha} \alpha (1-\alpha)^{-1-\alpha}} \left( R_{K,t}^\gamma \right)^\rho \left( (1 + \tau^w_t)W_t \right)^{-\alpha}
\]

For a given total demand for capital, the intermediate firm minimizes total cost of private and public capital, solving:

\[
\min_{K^H_{f,t}, K^G_{f,t}} R_{K,f}^H K_{f,t}^H + R_{K,f}^G K_{f,t}^G
\]

subject to (5).

First order conditions to this problem yield the average rate of return on capital

\[
R^K_{K,f} = \left(1 - \omega_g\right) \left( R_{K,f}^H \right)^{-\eta_g} + \omega_g \left( R_{K,f}^G \right)^{-\eta_g} \frac{1}{1-\eta_g}
\]

The aggregate demand functions for each type of capital goods are:

\[
K^G_{G,t} = \omega_g \left( \frac{R_{G,t}^G}{R_{K,t}^G} \right)^{-\eta_g} K_t^S
\]

\[
K^H_{H,t} = (1 - \omega_g) \left( \frac{R_{H,t}^H}{R_{K,t}^H} \right)^{-\eta_g} K_t^S
\]

All firms are identical since they solve the same optimization problem. Aggregate capital rented to intermediate goods firms can be restated as (15) by suppressing the subscript “\( f \)” from (5), using (12), and aggregating the different types of capital across firms:

\[
K_t^S = \left(1 - \omega_g\right)^{1/\eta_g} \left( K^H_{H,t} \right)^{\eta_g^{-1}} + \omega_g^{1/\eta_g} \left( K^G_{G,t} \right)^{\eta_g^{-1}} \frac{\eta_g}{\eta_g^{-1}}
\]

We also depart from CMS by introducing differentiated labor skills in the model. In many economies, banks require a minimum amount of deposits to open a savings account, which alone can leave a high share of the population unprotected from inflation. We thus reason that households with a lower degree of labor skills are exactly
those that are investment constrained in the model. This modeling strategy allows for an equilibrium where skillful workers earn more working the same amount of hours.

The labor input used by firm $f$ in the production of intermediate goods is a composite of labor demanded to both groups of households. In addition to the population-size adjustment ($\omega$) that CMS add to the firm’s labor demand, we add the parameter $\nu_\omega$ to introduce a bias in favor of more skilled workers. The resulting labor composite obtains from the following transformation technology

$$N_{f,t}^D := \left(1 - \nu_\omega \omega\right)^{1/\eta} \left(N_{f,t}^I\right)^{1-1/\eta} + \left(\nu_\omega \omega\right)^{1/\eta} \left(N_{f,t}^J\right)^{1-1/\eta} \eta^{(1-1/\eta)}$$

(16)

where

$$N_{f,t}^I := \left[\left(\frac{1}{1 - \omega}\right)^{1/\eta} \left(\int_0^{N_{f,t}^I} d_i \right)^{1-1/\eta} \right]^{1/\eta \eta(1-1/\eta)}$$

(17)

and

$$N_{f,t}^J := \left[\left(\frac{1}{\omega}\right)^{1/\eta} \left(\int_{1 - \omega}^{N_{f,t}^J} d_j \right)^{-1/\eta} \right]^{1/\eta \eta(1-1/\eta)}$$

(18)

and where $\frac{1}{\eta}$ is the elasticity of substitution between labor from households in group $I$ and $J$, $\eta_I$ is the inverse elasticity of substitution between members of group $I$, and $\eta_J$ is the inverse elasticity of substitution between members of group $J$. The special case when $\nu_\omega = 1$ corresponds to the equally skilled workers assumption, as in CMS.

Taking average wages ($W_{I,t}$ and $W_{J,t}$) in both groups as given, firms choose how much to hire from both groups of households by minimizing total labor cost $W_{f,t}N_{f,t}^I + W_{f,t}N_{f,t}^J$ subject to (16). It follows from first order conditions that the aggregate wage is:

$$W_t = \left[(1 - \nu_\omega \omega)W_{I,t}^{1-\eta} + \nu_\omega \omega W_{J,t}^{1-\eta}\right]^{1/(1-\eta)}$$

(19)

and the aggregate demand functions for each group of households are:

$$N_{I,t}^I = (1 - \nu_\omega \omega) \left(\frac{W_{I,t}}{W_t}\right)^{-\eta} N_{I,t}^D$$

(20)

$$N_{J,t}^J = \nu_\omega \omega \left(\frac{W_{J,t}}{W_t}\right)^{-\eta} N_{J,t}^D$$

(21)

The firm demands labor $N_{f,t}^I$ and $N_{f,t}^J$ from each individual in groups $I$ and $J$ taking individual wages $W_{I,t}$ and $W_{J,t}$ as given to minimize the average cost.
\[
\int_0^{1-\eta} W_i^i N_{i,j}^i di + \int_{1-\eta}^1 W_j^j N_{j,j}^j dj \quad \text{subject to aggregation constraints (17) and (18). From the}
\]
first order conditions, aggregate wages for each household group can be represented as a function of optimal and mechanically readjusted wages:

\[
W_{i,j} = \left(1 - \xi_i \right)(\tilde{W}_{i,j})^{1-\eta_i} + \xi_i \left(\tilde{W}_{i,j}\right)^{-\eta_i} \right]^{\theta_i(1-\eta_i)}
\]

\[
W_{j,j} = \left(1 - \xi_j \right)(\tilde{W}_{j,j})^{1-\eta_j} + \xi_j \left(\tilde{W}_{j,j}\right)^{-\eta_j} \right]^{\theta_j(1-\eta_j)}
\]

Prices are set under monopolistic competition, with Calvo-type price rigidities. As intermediate goods pricing does not differ from what CMS obtain, we leave the details for appendix A, including the first order conditions and their recursive representations.

Aggregating over firms, domestic and export intermediate goods prices are

\[
P_{H,i} = \left[1 - \xi_H \right) (\tilde{P}_{H,i})^{1-\theta} + \xi_H \left(\tilde{P}_{H,i}\right)^{-\theta} \right]^{\theta/(1-\theta)}
\]

\[
P_{X,i} = \left[1 - \xi_X \right) (\tilde{P}_{X,i})^{1-\theta} + \xi_X \left(\tilde{P}_{X,i}\right)^{-\theta} \right]^{\theta^*/(1-\theta^*)}
\]

where \(\xi_H\) and \(\xi_X\) are the Calvo parameters, the terms \(\theta/(1-\theta)\) and \(\theta^*/(1-\theta^*)\) denote the domestic and export price markups over nominal marginal costs, in the absence of price rigidities, where \(\theta\) is the elasticity of substitution between domestic intermediate goods and \(\theta^*\) is the analogue for export goods.

2.2.2 Final goods firms

As in CMS, there are three firms producing non-tradable final goods. One specializes in the production of private consumption goods, another in public consumption goods, and the third in investment goods. Except for the public consumption good, the production of final goods combines both foreign and domestic intermediate goods using a CES-type technology.

In the following subsections we correct the price index of private consumption goods and investment goods to ensure that final goods firms operate under perfect competition. The pricing of public consumption goods is exactly the same as in CMS.
2.2.2.a. Private consumption goods

To produce private consumption goods $Q_i^C$, the firm purchases bundles of domestic $H_i^C$ and foreign $IM_i^C$ intermediate goods. Whenever it adjusts its imported share of inputs, the firm faces a cost, $\Gamma_{IM_i^C}(IM_i^C / Q_i^C)$, detailed in appendix B. Letting $\nu_c$ denote the bias towards domestic intermediate goods, the technology to produce private consumption goods is

$$Q_i^C = \left\{ (\nu_c)^{1/\mu_c} [H_i^C]^{-1/\mu_c} + (1-\nu_c)^{1/\mu_c} \left[ (1-\Gamma_{IM_i^C}(IM_i^C / Q_i^C))IM_i^C \right]^{-1/\mu_c} \right\}^{\mu_c/(\mu_c-1)}$$  \hspace{1cm} (26)

where

$$H_i^C = \left\{ \int (H^C_{f,j})^{1-1/\theta} \, df \right\}^{\theta/(\theta-1)}$$

$$IM_i^C = \left\{ \int (IM^C_{f,j})^{1-1/\theta'} \, df' \right\}^{\theta'/(\theta'-1)}$$

The firm minimizes total input costs

$$\min_{H_i^C,IM_i^C} P_{H,i}H_i^C + P_{IM,i}IM_i^C$$  \hspace{1cm} (27)

subject to the technology constraint (26) taking intermediate goods prices as given.

The price index that results from solving this problem is:

$$P_{C,j} = (\Omega_i^C)^{-\mu_c} (\tilde{X}_i^C)^{\mu_c}$$  \hspace{1cm} (28)

where

$$\tilde{X}_i^C = \left\{ \nu_c P_{H,i}^{1-\mu_c} + (1-\nu_c) \left[ P_{IM,i} / \Gamma^3_{IM_i^C} (IM_i^C / Q_i^C) \right]^{1-\mu_c} \right\}^{1-\mu_c}$$  \hspace{1cm} (29)

$$\Omega_i^C = \left\{ \nu_c \left( P_{H,i} \right)^{1-\mu_c} + (1-\nu_c) \left( \frac{\Gamma^3_{IM_i^C} (IM_i^C / Q_i^C)}{(1-\Gamma_{IM_i^C}(IM_i^C / Q_i^C))} \right) \left[ P_{IM,i} / \Gamma^3_{IM_i^C} (IM_i^C / Q_i^C) \right]^{1-\mu_c} \right\}$$  \hspace{1cm} (30)

Details of the derivation of (28) are shown in appendix E.
In CMS, the multiplier $\lambda^C_t$ is assumed to be the price index for one unit of the consumption good. However, this result is not compatible with their assumption that final goods firms operate with zero profits.

Notice that only when $\Omega^C_t = \lambda^C_t$ do we obtain $P^C_t = \lambda^C_t = \Omega^C_t$. This requires \( \left( \frac{\Gamma^3_{IM} \left( IM^C_t / Q^C_t \right)}{\left( 1 - \Gamma^3_{IM} \left( IM^C_t / Q^C_t \right) \right)} \right) = 1 \), a very specific case.

In general, when this equality does not hold, the demand equations, as a function of the price index, should be

\[
H^C_t = V^C \left( \frac{P^H_t}{\Omega^C_t} \right)^{1-\mu_C} \left( \frac{P^H_t}{P_t} \right)^{-\mu_C} Q^C_t \tag{31}
\]

\[
IM^C_t = (1 - V^C) \left( \frac{P^H_t}{\Omega^C_t} \right)^{1-\mu_C} \left( \frac{P^H_t}{P_t} \right)^{-\mu_C} Q^C_t \frac{1}{1 - \Gamma^3_{IM} \left( IM^C_t / Q^C_t \right)} \tag{32}
\]

These demand equations are different from the ones in CMS, resulting in important differences in the market clearing equations, as we show in subsequent sessions. Equations (31) and (32) are obtained by combining first order conditions with equation (28).

2.2.2.b. Investment goods

The pricing problem of investment goods is analogous to that of consumer goods. The investment goods price index, which also differs from CMS, is

\[
P^I_t = \left( \Omega^I_t \right)^{1-\mu_I} \left( \lambda^I_t \right)^{\mu_I} \tag{33}
\]

where

\[
\Omega^I_t = \left[ V^I_t \left( P^H_t \right)^{1-\mu_I} + (1 - V^I_t) \left( \frac{\Gamma^3_{IM} \left( IM^I_t / Q^I_t \right)}{1 - \Gamma^3_{IM} \left( IM^I_t / Q^I_t \right)} \right) \frac{P^IM_t}{P^I_t} \right]^{\mu_I} \frac{1}{1-\mu_I} \tag{34}
\]

and

\[
\lambda^I_t = \left[ V^I_t P^H_t \left( P^H_t \right)^{1-\mu_I} + (1 - V^I_t) \left( \frac{\Gamma^3_{IM} \left( IM^I_t / Q^I_t \right)}{1 - \Gamma^3_{IM} \left( IM^I_t / Q^I_t \right)} \right) \frac{P^IM_t}{P^I_t} \right]^{1-\mu_I} \tag{35}
\]
2.3 Fiscal authority

The fiscal authority sets a primary surplus target, levies taxes on consumption, labor, capital and dividends, and adjusts expenditures and budget financing accordingly.

The primary surplus target $SP_t$ is defined as:

$$SP_t = \tau_c^D P_G, C_t + (\tau_N^D + \tau_w^D) W_t N_t^D + \tau_c^K H_t - (\tau_s^D u_t, i_t + \gamma) P_t, I_t + \tau_i^D D_t$$

where $\tau_c^c, \tau_N, \tau_w^w, \tau_i^i, \tau_c^K$ are tax rates levied on consumption, labor income, social security from workers, social security from firms, capital and dividends.

We assume that the domestic government has a primary surplus target, yet the realization of the primary surplus is affected by deviations of the public debt and economic growth from their steady-states:

$$\frac{SP_t}{P_{Y,t}, Y_t} = \rho_{1,sp} \frac{SP_{t-1}}{P_{Y,t-1}, Y_{t-1}} + \rho_{2,sp} \frac{SP_{t-2}}{P_{Y,t-2}, Y_{t-2}} (1 - \rho_{1,sp} - \rho_{2,sp}) \left[ sp + \phi_{BY_t} \left( \frac{B_t}{P_{Y,t-1}, Y_{t-1}} - B_Y \right) + \phi_{G} \left( \frac{G_{t-1}}{Y_{t-2}} - G_Y \right) + \varepsilon_{sp,t} \right]$$

The anti-cyclic component in the fiscal rule was identified in isolated estimations of fiscal rules for Brazil, and is also present, yet in a different manner, in Ratto et. al. (2009):

In our calibrations, the foreign economy is represented by the USA and the Euro area. Therefore, for the foreign economy, we adopt CMS’s assumption that the fiscal authority does not follow a primary surplus target, and government expenditures with consumption, $g_t = \left( \frac{P_{G,t}}{P_{Y,t}} \right) \left( \frac{G_t}{Y_t} \right)$, follow an autoregressive process:

$$g_t = (1 - \rho_g) g + \rho_g g_{t-1} + \varepsilon_{g,t}$$

where $g$ is the steady state value of government expenditures as a share of GDP and $\varepsilon_{g,t}$ is a white noise shock to government expenditures. Specifically for the foreign economy, we assume that lump sum taxes follow an autoregressive process of the type:
\[
\left( \frac{T_t}{P_{Y,t}Y_t} \right) := \phi_B \left( \frac{R_{t+1}B_{t+1}}{P_{Y,t}Y_t} - B_Y \right) \tag{39}
\]

where \( B_Y \) is the steady state value of government bonds.

For both economies, government transfers follow the autoregressive process:

\[
\left( \frac{TR_t}{P_{Y,t}Y_t} \right) = (1-\rho_{tr})TR_t + \rho_{tr} \left( \frac{TR_t}{P_{Y,t}Y_t} \right) + \epsilon_{tr,t} \tag{40}
\]

where \( tr \) is the steady state value of government transfers, and \( \epsilon_{tr,t} \) represents a white noise shock to government transfers.

Total transfers are distributed to each household group according to:

\[
TR_{t,j} := \frac{(1-\omega_{v_{tr}})}{1-\omega} TR_t \tag{41}
\]

\[
TR_{t,J} := v_{tr} TR_t \tag{42}
\]

where \( v_{tr} \) is the bias in transfers towards group \( J \).

Government investment follows an autoregressive rule of the form

\[
i_g_t = (1-\rho_{ig})i_g + \rho_{ig}i_g_{t-1} + \epsilon_{ig,t} \tag{43}
\]

and public capital accumulation follows the rule

\[
K_{G,t+1} = (1-\delta)K_{G,t} + \left( 1-\Gamma_t \left( \frac{I_{G,t}}{I_{G,t-1}} \right) \right) I_{G,t} \tag{44}
\]

The government budget constraint is thus

\[
\tau_c^C P_{C,t} C_t + (\tau_i^N + \tau_i^W + \tau_i^W) W_t . N_t^D + \tau_i^K (R_{K,t} u_{t,J} - (\Gamma_u(u_{t,J}) + \delta) P_{I,t}) K_t + \tau_i^D D_t + T_t + R_{t+1} B_{t+1} + M_t + u_{t,J} R_{G,t} K_{G,t} - P_{G,t} G_t - TR_t - B_t - M_{t-1} - P_{I,t} I_{G,t} = 0 \tag{45}
\]

2.4. Monetary authority

The domestic monetary authority follows a forward-looking interest rate rule that is compatible with an inflation targeting regime

\[
R_t^4 = \phi_{R1} R_{t-1}^4 + \phi_{R2} R_{t-2}^4 + (1-\phi_{R1} - \phi_{R2}) \left[ R_t^4 + \phi_{\Pi} \left( \frac{P_{C,t+3}}{P_{C,t-1}} - \Pi \right) \right] + \phi_{g_t} \left( \frac{Y_t}{Y_{t-1}} - g_Y \right) + \epsilon_{t,R} \tag{46}
\]
where $\Pi$ is the annual inflation target, $R^4$ is the annualized quarterly nominal equilibrium interest rate, which satisfies $R^4 = \beta^{-1}\Pi$, $g_Y$ is the steady state output growth rate, and $\varepsilon_{R,t}$ is a white noise shock to the interest rate rule.

For the foreign economy we adopt the representation in CMS:

$$ R_t^4 = \phi_R . R_{t-1}^4 + (1-\phi_R) \left[ R^4 + \phi_\Pi \left( \frac{P_{C,t}}{P_{C,t-3}} - \Pi_t \right) \right] + \phi_{g_Y} \left( \frac{Y_t}{Y_{t-1}} - g_Y \right) + \varepsilon_{R,t} $$  \hspace{1cm} (47)

2.5. Aggregation and market clearing

Any aggregated model variable $Z_i$ denoted in per capita terms results from the aggregation $Z_i := \int Z_{h,i} dh = (1-\omega)Z_{I,i} + \omega Z_{J,i}$, where $Z_{I,i}$ and $Z_{J,i}$ are the respective per capital values of $Z_i$ for families $I$ and $J$. Details on the aggregation that do not substantially differ from CMS are left for appendix A.

There are important distinctions in the aggregate relations that obtain from this model as compared to those in CMS. The first refers to the wage dispersion index, and the second to the economy’s resource constraint, which are detailed below.

2.5.1. Wage dispersion

The equilibrium between supply and demand for labor occurs at the individual level, as represented below:

$$ N_{i,t} = N_{i,t}^I := \int_0^1 N_{f,t}^I df $$ \hspace{1cm} (48)

$$ N_{j,t} = N_{j,t}^I := \int_0^1 N_{f,t}^J df $$ \hspace{1cm} (49)

Aggregating the demand of all firms for labor services yields

$$ N_{i,t} = \frac{1}{1-\omega} \left( \frac{W_{i,t}}{W_{i,t}} \right)^{-\eta_i} N_{i,t}^I $$ \hspace{1cm} (50)
which can also be represented, using the group-wise aggregated labor demand
equations, as a function of total demand for labor by the intermediate firms:

\[ N_{i,t} = \frac{1}{1 - \omega} \left( \frac{W_{i,t}}{W_{i,t}} \right)^{-\eta} \left( \frac{W_{i,t}}{W_{i,t}} \right)^{-\eta} N_t^D \]  

(52)

\[ N_{j,t} = \omega \left( \frac{W_{j,t}}{W_{j,t}} \right)^{-\eta} \left( \frac{W_{j,t}}{W_{j,t}} \right)^{-\eta} N_t^D \]  

(53)

Let \( N_{i,t} \) and \( N_{j,t} \) be the aggregate supply of labor from each household group,
and \( N_s,t \) the total supply of labor. Aggregating the supply of labor using equations (50)
and (51) yields

\[ N_{i,t} := \frac{1}{1 - \omega} \int_{0}^{1} N_{i,j} dj = \frac{\psi_{i,t}}{1 - \omega} N_t^I \]  

(54)

\[ N_{j,t} := \int_{1 - \omega}^{1} N_{j,t} dj = \frac{\psi_{j,t}}{\omega} N_t^J \]  

(55)

where \( \psi_{i,t} := \int_{0}^{1} \left( \frac{W_{i,t}}{W_{i,t}} \right)^{-\eta} \) \( dj \) and \( \psi_{j,t} := \int_{1 - \omega}^{1} \left( \frac{W_{j,t}}{W_{j,t}} \right)^{-\eta} \) \( dj \) are the dispersion
indices.

We show in appendix F that the wage dispersion indices \( \psi_{i,t} \) and \( \psi_{j,t} \) can be
stated in a recursive formulation that differs from the working paper version of CMS²:

\[ \psi_{i,t} := (1 - \xi) \left( \frac{\tilde{W}_{i,j}}{W_{i,j}} \right)^{-\eta} + \xi \left( \frac{\pi_{c,j-1}^{x_i-\xi_{i,j}}}{\pi_{w_{i,j}}} \right)^{-\eta} \psi_{j,t-1} \]  

(56)

\[ \psi_{j,t} := (1 - \xi) \left( \frac{\tilde{W}_{j,j}}{W_{j,j}} \right)^{-\eta} \left( \frac{W_{j,j}}{P_{j,j,Y_j}} \right)^{-1} + \xi \left( \frac{\pi_{c,j-1}^{x_{i,j}-\xi_{i,j}}}{\pi_{w_{j,j}}} \right)^{-\eta} \psi_{j,t-1} \]  

(57)

where \( \pi_{w_{i,j}} \) and \( \pi_{w_{i,j}} \) stand for household \( I \) and \( J \) wage inflation rates.

² Equation A.9, WPS 747/ECB.
Aggregating the demand for labor from household groups $I$ and $J$, using equations (54) and (55), results in
\[ N_{S,I} := \psi_{I,J} \cdot N^I_I + \psi_{J,J} \cdot N^J_J \]
which can be restated as a relation between total aggregate supply and demand that depends on the total wage dispersion index:
\[ N_{S,I} = \psi_{I,J} \cdot N^D_I \quad (58) \]
where total wage dispersion is
\[ \psi_{I,J} := \left\{ (1 - \omega) \left( \frac{W_{I,J}}{W_I} \right)^{-\eta} \psi_{I,J} + \omega \left( \frac{W_{J,J}}{W_J} \right)^{-\eta} \psi_{J,J} \right\} \]

2.5.2. Aggregate resource constraint

The price indices derived in the previous sessions entail representations for the aggregate resource constraint of the economy that are importantly different from the ones presented in CMS and CCW. Aggregating household and government budget constraints, and substituting for the equations of external financing and optimality conditions of firms, we obtain the aggregate resource constraint of the economy:
\[ P_{Y,I} = P_{C,J} \cdot Q^C_I + P_{J,J} \cdot Q^J_I + P_{G,J} \cdot Q^G_I + S_I \cdot P_{X,J} \cdot X_I - P_{M,J} \cdot IM_I \quad (59) \]
which, using the price indices derived above, can also be restated as
\[ P_{Y,I} = P_{H,J} \cdot H^C_I + P_{H,J} \cdot H^I_I + P_{H,J} \cdot H^G_I + S_I \cdot P_{X,J} \cdot X_I \quad (60) \]

Despite the fact that these representations are standard for national accounts, they differ from the respective equations derived in CMS and CCW, as we detail in appendix G.

3. Model Transformation and Steady State Calibration

In this section we describe the transformation of variables that render the model stationary, and detail the steady state calibration.

As we assume a technology shock that permanently shifts the productivity of labor, all real variables, with the exception of hours worked, share a common stochastic

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3 Equation (38) in CMS.
trend. Besides, as the monetary authority aims at stabilizing inflation, rather than the price level, all nominal variables share a nominal stochastic trend.

The strategy consists of three main types of transformation. Real variables are divided by aggregate output \((Y_t)\), nominal variables are divided by the price of aggregate output \((P_{t,t}Y_t)\) and the variables expressed in monetary terms are divided by \(P_{t,t}Y_t\).

Although most transformations are straightforward, some are not trivial. Predetermined variables, such as capital, are scaled by dividing their lead values by \(Y_t\), wages, domestic bonds, and internationally traded bonds are scaled by \(P_{t,t}Y_t\). In addition, in order to make the Lagrange multipliers compatible with the adopted scaling strategy, we multiply them by \(Y_t^\sigma\), resulting in \(Y_t^\sigma\Lambda_{t,t}^I\) and \(Y_t^\sigma\Lambda_{t,t}^J\), for households \(I\) and \(J\), respectively.

The permanent technology shock, \(zn_t\), should also be divided by the aggregate output. Re-scaling the production function for the intermediate goods results in:

\[
\left( \frac{zn_t}{Y_t} \right)^{-1} = z_t \left( \frac{u_{t,t} \cdot K_t}{Y_{t-1}} \right)^\alpha \left( \frac{N^D}{Y_t} \right)^{-\alpha} \left( \frac{Y_t}{Y_{t-1}} \right)^{-\alpha} \left( \frac{zn_t}{Y_t} \right)^{-\psi}
\]

From the above, we can conclude that \(\frac{zn_t}{Y_t}\) is a stationary variable whenever the ratios \(\frac{K_t}{Y_{t-1}}\) and \(\frac{Y_t}{Y_{t-1}}\) are both stationary.

We now turn to the steady state calibration. For the domestic economy, we calibrate the model to reproduce historical averages of the Brazilian economy during the inflation targeting regime (Table 1). The rest of the world is calibrated using an average of the values presented in CMS for the United States and the Euro Area.

**Table 1: Steady State Ratios**

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Brazil</td>
<td>Rest of the World</td>
</tr>
<tr>
<td>TB/P_tY</td>
<td>0.012</td>
<td>0.00</td>
</tr>
<tr>
<td>X/Y</td>
<td>0.128</td>
<td>0.00</td>
</tr>
<tr>
<td>IM/Y</td>
<td>0.122</td>
<td>0.00</td>
</tr>
<tr>
<td>M/P_tY</td>
<td>0.205</td>
<td>1.24</td>
</tr>
<tr>
<td>ROG/P_tY</td>
<td>0.000</td>
<td>0.00</td>
</tr>
<tr>
<td>( \frac{P_I G}{P_Y} )</td>
<td>0.019</td>
<td>0.02</td>
</tr>
<tr>
<td>( T/P_Y )</td>
<td>0.000</td>
<td>0.00</td>
</tr>
<tr>
<td>( B/P_Y )</td>
<td>2.121</td>
<td>2.79</td>
</tr>
<tr>
<td>( SP/P_Y )</td>
<td>0.036</td>
<td>-0.005</td>
</tr>
<tr>
<td>( D/P_Y )</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>( \frac{P_I H}{P_Y} )</td>
<td>0.162</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Calibration and simulations are performed under the assumption of log-linear utility (\( \sigma = 1 \)). The steady state calibration starts by normalizing the stationary prices of intermediate goods at 1. This normalization ensures that the steady state values of some variables will be one, as is the case of final goods prices and Lagrange multipliers associated with the optimization problem of final goods firms. The steady state rate of capital utilization is also fixed at one for both economies. The remaining steady state ratios are calibrated accordingly, as shown in appendix H.

We calibrate the population size using LABORSTA\(^4\) data for the year 2007. The size of household’s group \( J \) in the domestic economy was set to equal the share of households in Brazil that earn less than two minimum wages according to the PNAD 2007 survey. Also according to this survey, relative wages for household group \( I \) were set in our calibrations at 2.86.

The share of fixed costs in total production was set so as to guarantee zero profits in the steady state. The labor demand bias, \( \nu_w \), was calibrated to ensure that households’ groups \( I \) and \( J \) work the same amount of hours. For the stationary labor productivity growth rate, we set 2% for Brazil and the rest of the world using data on GDP growth from the World Bank for the period 2000-2007.

For Brazil, we calibrated the price elasticity \( \mu_c = 0.33 \) according to Araújo et. al. (2006). For the price elasticity \( \mu_I \), we repeated the value set for \( \mu_c \). The home biases \( \nu_c \) and \( \nu_I \) are obtained from the demand equations of imported goods using the steady state value for the supply of consumption and investment goods, and the import quantum. The steady state primary surplus to output ratio, \( sp \), was calibrated as the mean value of the primary surplus in the period 1999-2008. For the rest of the world, the value for \( sp \) was obtained implicitly from the NAWM calibration. The public debt ratio \( B_y \) was set to be consistent with \( sp \).

\(^4\) http://laborsta.ilo.org/
Government expenditures, $g$, for both Brazil and the rest of the world were set residually from the aggregate resource constraint. Government transfers, $tr$, for both Brazil and the rest of the world, were obtained so that household budget constraints close.

With the exception of consumption taxes, $\tau^c$, which were calibrated following Siqueira et. al. (2001), Brazilian tax rates were calibrated based on the current tax law. The lump-sum tax bias, $\nu_p$, which is active only for the foreign economy, was set to one, whilst the transfer bias, $\nu_g$, was implicitly calculated from households $I$ and $J$ budget constraints.

We calibrated the elasticity of substitution between government and private investment goods, $\eta_g$, to a value that is close to 1, and which enabled us to implicitly calibrate $\nu_g$ from other parameters. The inflation target and the respective steady state nominal interest rate in the domestic economy were set according to historical Brazilian averages.

The parameter $\gamma_{c,2}$ that appears in the functional form of the consumption transaction for the domestic economy was set at the same value calibrated in CMS. The parameter $\gamma_{c,1}$ follows from the equation that defines the consumption transaction cost, the calibrated values for money and consumption and the equation that defines the money velocity. Finally, some autoregressive coefficients $(\rho_m, \rho_x, \rho_g)$ were set at 0.9 following the NAWM calibration for $\rho_z$. For autoregressive coefficients referring to government consumption and transfers, $\rho_g$ and $\rho_r$, we used estimated coefficients obtained from isolated econometric regressions.

4. Simulation and policy analysis

In this session, we show impulse responses for shocks to: monetary policy, primary surplus, government transfers and investment. We also compare the model’s predictions for alternative types of primary surplus and monetary policy rules. All simulations were done using the function “stoch_simul” of DYNARE at MATLAB.
4.1. Impulse responses of the calibrated model

Figure 1 shows the impulse responses of a 1 p.p. shock to the interest rate. The shock initially brings consumer price inflation down by roughly 0.10 p.p., with a drop in output levels and growth rates, the latter by more than 1 p.p., and a slight exchange rate appreciation. The interest rate remains above its steady state value for about one year. Consumer price inflation returns to its steady state in 3 quarters, slightly oscillating below the steady state from the sixth to the twelfth quarter. Output growth stays below the steady state in the first two quarters, after what it rises up to 0.5 p.p. above the steady state from the third to the sixth quarter, returning to the steady state afterwards.

Despite the fact that each policy rule responds to a different set of variables, in equilibrium the fiscal response intertwines with monetary conditions, the key linking element being the public debt. The interest rate hike puts pressure on the public debt, which initially rises by about 1 p.p. of the quarterly GDP. However, the anti-cyclic component of the fiscal rule forces the primary surplus to react to the economic downturn in the initial periods, and the fiscal rule is loosened through a reduction in the primary surplus of about 0.05 p.p. of GPD from its steady state. This reaction is enabled by an increase in government consumption. In the third quarter, public debt reaches a peak, and the output growth surpasses its stationary rate. This development puts pressure on the fiscal rule for a rise in the primary surplus of up to 0.10 p.p. of GPD, through a reduction in government consumption. Consequently, the debt initiates a downward path, yet still above its steady state for a long time afterwards.

The economy decelerates in the aftermath of a monetary policy shock. Capital utilization is below the steady state and firms pay lower wages to households. The amount of labor and consumption also drops. The distributional effects are less favorable to less specialized and more constrained households.

The dynamics of endogenous variables after the shock affects GDP composition. Although private consumption to GDP falls in the first quarter, it immediately bounces upwards after the second quarter mostly to replace investment and public consumption.

Figure 1: Impulse responses to a contractionist shock to monetary policy
Figure 2 shows the impulse responses of a 1 p.p. reduction in the primary surplus. The shock initially increases government consumption by about 0.4 p.p. of GDP and raises public investment by 1% from its steady state. Such expansionist effect, however, is not enough to sustain output growth above its steady state. It also does not significantly impact private consumption or labor. The monetary effects of the fiscal shock comprise an increase of up to 0.2 p.p. in consumer price inflation, and, in spite of the contractionist stance of monetary policy, inflation remains above its steady state for a prolonged period.
Figure 2: Impulse responses to an expansionist fiscal shock

- Interest rate (% annualized)
- Output (% of stationary GDP)
- Public Debt (% of stationary GDP)
- Consumer price inflation (% annualized)
- Output growth (% annualized)
- Primary surplus (% of GDP)
- Capital utilization
- Government consumption (% of GDP)
- Wages (% of stationary wages)
- Hours worked
- Private consumption (% of stationary consumption)
- Total investment (% of stationary investment)
- Government investment (% of stationary investment)
- Private investment (% of stationary investment)
- Private consumption (% of GDP)
- Government consumption (% of GDP)
- Investment (% of GDP)
- Exports (% of GDP)
- Imports (% of GDP)

steady state
1 p.p. shock to the primary surplus/GDP
To account for the fact that transfers are usually an instrument used for income distribution, the shock to government transfers (Figure 3) is biased towards less specialized and more constrained households. The hike in government transfers is enabled by debt issuance and by a reduction in government consumption and public investment. These actions initially result in a downturn in economic activity. Monetary policy reacts to poor economic conditions and to the drop in inflation by keeping interest rates slightly below the steady state. The tightening in government consumption and investment necessary to allow for new expenditures with transfers counterbalances the positive effects of the shock on private consumption. As a result, private consumption grows inexpressively. The distributional effect of the shock vanishes after about 5 quarters.
Figure 3: Impulse responses to a shock to government transfers

- **Interest rates**: % annualized
  - 14.72
  - 14.70
  - 14.68
  - 14.66
  - 14.64

- **Output**: % of stationary GDP
  - 100.00%
  - 99.80%
  - 99.60%
  - 99.40%
  - 99.20%

- **Public debt**: % of quarterly GDP
  - 214.0%
  - 213.0%
  - 212.0%
  - 211.0%
  - 210.0%

- **Consumer price inflation**: % annualized
  - 3.35%
  - 3.45%
  - 3.55%
  - 3.65%
  - 3.75%

- **Primary surplus**: % of GDP
  - 211.0%
  - 212.0%
  - 213.0%
  - 214.0%
  - 215.0%

- **Public debt**: % of quarterly GDP
  - 19.00%
  - 19.40%
  - 19.80%
  - 20.20%
  - 20.60%

- **Private consumption**: % of stationary consumption
  - 19.20%
  - 19.40%
  - 19.60%
  - 19.80%
  - 20.00%

- **Government consumption**: % of GDP
  - 17.94%
  - 18.00%
  - 18.06%
  - 18.12%
  - 18.18%

- **Total investment**: % of GDP
  - 12.70%
  - 12.80%
  - 12.90%
  - 13.00%
  - 13.10%

- **Government transfers**: % of GDP
  - 12.55%
  - 12.60%
  - 12.65%
  - 12.70%
  - 12.75%

- **Exports**: % of GDP
  - 12.04%
  - 12.08%
  - 12.12%
  - 12.16%
  - 12.20%

- **Imports**: % of GDP
  - 12.05%
  - 12.10%
  - 12.15%
  - 12.20%
  - 12.25%

- **Nominal exchange rate**: % of the stationary exchange rate
  - 100.00%
  - 99.80%
  - 99.60%
  - 99.40%
  - 99.20%

- **Private consumption**: % of stationary consumption
  - 100.00%
  - 100.05%
  - 100.10%
  - 100.15%
  - 100.20%

- **Government consumption**: % of GDP
  - 12.00%
  - 12.05%
  - 12.10%
  - 12.15%
  - 12.20%

- **Output**: % of stationary GDP
  - 100.00%
  - 99.80%
  - 99.60%
  - 99.40%
  - 99.20%

- **Capital utilization**: %
  - 100.00%
  - 99.80%
  - 99.60%
  - 99.40%
  - 99.20%

- **Government investment**: % of stationary investment
  - 100.00%
  - 99.95%
  - 99.90%
  - 99.85%
  - 99.80%

- **Exports**: % of GDP
  - 100.00%
  - 99.80%
  - 99.60%
  - 99.40%
  - 99.20%

- **Imports**: % of GDP
  - 100.00%
  - 99.80%
  - 99.60%
  - 99.40%
  - 99.20%
A shock to government investment (Figure 4), of about 1 p.p. of GDP, crowds out private investment, as the rental rate of public capital is cheaper. The rise in government expenditures with investment is financed through cuts in consumption expenditures, driving the primary surplus down to levels below the steady state, and through debt issuance. Afterwards, the rise in public debt exerts a contractionist pressure on the fiscal rule, and the primary surplus rises after the third quarter. The initial inflationary spike results in a contractionist monetary policy reaction, and the final outcome is a drop in economic dynamism, with output below its steady state path for 5 quarters. Afterwards, it reverts back up to surpass the steady state. The strong increase in investment does not result in relevant changes to the labor market or to private consumption.
Figure 4: Impulse responses to a shock to government investment
4.2 – Policy analysis

To understand how the interaction of fiscal and monetary policy affects the model’s predictions, we analyze impulse responses, variances and variance decompositions after policy shocks under a number of different specifications for the policy rules.

4.2.1 – Small departures from the calibrated model

Figure 5 shows the impulse responses of a monetary policy shock with varying degrees of fiscal commitment with the stationary path of public debt. Increasing the fiscal commitment allows the same monetary policy shock to have a much stronger impact on inflation and on the output growth. In addition, it amplifies the distributive impact of the monetary policy shock, in favor of the group of more specialized households (group $I$) who also have more investment alternatives.

Table 3 shows variances and variance-decomposition when only the fiscal and monetary policy shocks are active. Under varying degrees of commitment to the stationary level of the debt, an increase in the coefficient of the fiscal rule associated with the deviation of the debt from its steady state increases the volatility of consumer price inflation and the correlation between inflation and output growth. As to the volatility of the output growth, the effects are non-linear. The shock decomposition shows that the influence of the monetary shock on output growth variance attains its least value with a coefficient of 0.18, a level that also grants the least variance of output growth. On the other hand, the greatest influence of the monetary policy shock onto inflation variance obtains with a coefficient of 0.31.
Figure 5: Fiscal commitment to the steady state level of the public debt: impulse responses of a monetary policy shock

Table 3: Higher commitment with the stationary path of the public debt in the fiscal rule

<table>
<thead>
<tr>
<th>Moments of the shocks (in p.p.)</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>SD of the monetary policy shock</td>
<td>SD = 1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SD of the fiscal shock</td>
<td>= 1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corr between shocks</td>
<td>Corr = 0.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fiscal commitment to the public debt</th>
<th>Coefficient in the fiscal rule</th>
<th>0.04²</th>
<th>0.18</th>
<th>0.31</th>
<th>0.50</th>
</tr>
</thead>
<tbody>
<tr>
<td>SD of cons. price inflation</td>
<td>0.10</td>
<td>0.20</td>
<td>0.44</td>
<td>1.04</td>
<td></td>
</tr>
<tr>
<td>SD of GDP growth</td>
<td>1.30</td>
<td>1.28</td>
<td>1.37</td>
<td>1.93</td>
<td></td>
</tr>
<tr>
<td>Corr between variables</td>
<td>4.78</td>
<td>9.68</td>
<td>29.41</td>
<td>58.85</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Moments of endogenous variables (in p.p.)</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Steady state</td>
<td>MS³</td>
<td>FS³</td>
<td>MS</td>
<td>FS</td>
<td>MS</td>
<td>FS</td>
<td>MS</td>
<td>FS</td>
<td></td>
</tr>
<tr>
<td>Consumer price inflation</td>
<td>15.63</td>
<td>84.37</td>
<td>47.98</td>
<td>52.02</td>
<td>58.48</td>
<td>41.52</td>
<td>45.16</td>
<td>54.84</td>
<td></td>
</tr>
<tr>
<td>GDP growth</td>
<td>7.86</td>
<td>92.14</td>
<td>5.22</td>
<td>94.78</td>
<td>10.85</td>
<td>89.15</td>
<td>25.53</td>
<td>74.47</td>
<td></td>
</tr>
</tbody>
</table>

¹ SD = standard deviation / Corr = correlation
² calibrated value
³ MS = monetary shock / FS = fiscal shock (to the primary surplus)
Greater rigor in the conduct of the fiscal policy (Table 4), characterized here by a lower standard deviation of the shock to the fiscal rule, significantly increases the influence of monetary policy on the variance of inflation. Notwithstanding, the variance of output growth is mostly attributed to the monetary policy shock. Greater rigor implies a drop in the variance of inflation and output growth.

**Table 4: Greater rigor in implementation of the primary surplus rule**

<table>
<thead>
<tr>
<th>Moments of the shocks (in p.p.)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>SD of the monetary policy shock $\sigma_1$ = 1.00</td>
<td></td>
</tr>
<tr>
<td>SD of the fiscal shock $\sigma_2$ = 0.47</td>
<td></td>
</tr>
<tr>
<td>Corr between shocks $\rho_{1,2}$ = 0.00</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fiscal commitment to the public debt</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient in the fiscal rule</td>
<td>0.04, 0.18, 0.31, 0.50</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Moments of endogenous variables (in p.p.)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>SD of cons. price inflation</td>
<td>0.06, 0.16, 0.36, 0.79</td>
</tr>
<tr>
<td>SD of GDP growth</td>
<td>0.69, 0.66, 0.76, 1.25</td>
</tr>
<tr>
<td>Corr between variables</td>
<td>24.41, 14.81, 39.12, 65.23</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variance decomposition (%)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>↓variability / → shock</td>
<td>MS, FS</td>
</tr>
<tr>
<td>Consumer price inflation</td>
<td>45.12, 54.88, 80.36, 19.64, 86.21, <strong>13.79</strong>, 78.51, 21.49</td>
</tr>
<tr>
<td>GDP growth</td>
<td>27.45, 72.55, <strong>19.64</strong>, 80.36, 35.06, 64.94, 60.34, 39.66</td>
</tr>
</tbody>
</table>

$/1$ SD = standard deviation / Corr = correlation
$/2$ calibrated value
$/3$ MS = monetary shock / FS = fiscal shock (to the primary surplus)

Figure 6 shows the impulse responses to a combination of a contractionist monetary policy shock and an expansionist fiscal policy shock. Increasing the magnitude of the fiscal shock reduces the contractionist impact of the monetary policy shock upon inflation and output growth. Public debt shows a strong increase, accompanied by a higher persistence to revert back to the steady state.
Figure 6: Correlation between policy shocks: Impulse responses to a monetary policy shock

Table 5 shows the effects on the variances, co-variances and variance decompositions of different degrees of correlation between policy shocks. In this exercise we adopt the specification of a fiscal rule that reacts strongly to deviations of the debt from its steady state (coefficient of 0.18), and that is tightly managed (0.47 standard deviation of the fiscal shock). Positive correlations refer to contractionist monetary and fiscal policies, whereas negative correlations imply that the contractionist monetary policy shock is partially counterbalanced by a loosening fiscal shock. Negative correlations increase the volatility of inflation and output growth, and increase the impact of the monetary policy shock upon inflation and output growth.
Table 5: Varying the correlation between monetary and fiscal policy (primary surplus) shocks

<table>
<thead>
<tr>
<th>Moments of the shocks (in p.p.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SD of the monetary policy shock $\uparrow$ = 1.00</td>
</tr>
<tr>
<td>SD between fiscal shocks = <strong>0.47</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Corr between policy shocks</th>
<th>0.80</th>
<th>0.50</th>
<th>0.00</th>
<th>-0.50</th>
<th>-0.80</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fiscal commitment to the public debt</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coefficient in the fiscal rule = <strong>0.18</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Moments of the variables (in p.p.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SD of cons. price inflation</td>
</tr>
<tr>
<td>SD of output growth</td>
</tr>
<tr>
<td>Corr between variables</td>
</tr>
</tbody>
</table>

Variance decomposition (%) - when the 1st shock is in monetary policy

<table>
<thead>
<tr>
<th>Down variance / → shock</th>
<th>MS $^3$</th>
<th>FS $^3$</th>
<th>MS</th>
<th>FS</th>
<th>MS</th>
<th>FS</th>
<th>MS</th>
<th>FS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumer price inflation</td>
<td>95.27</td>
<td>4.73</td>
<td>88.74</td>
<td>11.26</td>
<td>80.36</td>
<td>19.64</td>
<td>78.70</td>
<td>21.30</td>
</tr>
<tr>
<td>GDP growth</td>
<td>80.49</td>
<td>19.51</td>
<td>53.70</td>
<td>46.30</td>
<td>19.64</td>
<td>80.36</td>
<td><strong>13.68</strong></td>
<td>86.32</td>
</tr>
</tbody>
</table>

Variance decomposition (%) - when the 1st shock is in the fiscal rule

<table>
<thead>
<tr>
<th>Down variance / → shock</th>
<th>MS $^3$</th>
<th>FS $^3$</th>
<th>MS</th>
<th>FS</th>
<th>MS</th>
<th>FS</th>
<th>MS</th>
<th>FS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumer price inflation</td>
<td>80.63</td>
<td>19.37</td>
<td>53.94</td>
<td>46.06</td>
<td>19.64</td>
<td>80.74</td>
<td><strong>12.83</strong></td>
<td>87.17</td>
</tr>
<tr>
<td>GDP growth</td>
<td>95.23</td>
<td>4.77</td>
<td>88.68</td>
<td>11.32</td>
<td>80.36</td>
<td>19.64</td>
<td>78.90</td>
<td>21.10</td>
</tr>
</tbody>
</table>

/1 SD = standard deviation / Corr = correlation
/2 calibrated value
/3 MS = monetary shock / FS = fiscal shock (to the primary surplus)

Table 6 shows the impact of monetary policy rules that react more to deviations of expected inflation from the target. The values are obtained after setting the coefficient of the public debt in the fiscal rule at 0.18 and fiscal rigor at 0.47. Increasing monetary policy response to deviations from the inflation target results in a drop in the variances of inflation and output growth, and also in the correlation between them. However, the variance of consumer price inflation that is attributed to the fiscal shock increases. When the coefficient attached to inflation targets is set at 2.44, the monetary policy shock has the smallest influence on the variance of the output growth.
Table 6: Varying the monetary policy commitment to the inflation target

<table>
<thead>
<tr>
<th>Moments of the shocks (in p.p.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SD of the monetary policy shock (^{1}) = 1.00</td>
</tr>
<tr>
<td>SD of the fiscal shock = 0.47</td>
</tr>
<tr>
<td>Corr between shocks (^{1}) = 0.00</td>
</tr>
<tr>
<td>Fiscal commitment to the public debt</td>
</tr>
<tr>
<td>Coefficient in the fiscal rule = 0.18</td>
</tr>
<tr>
<td>Monetary policy commitment to the inflation target</td>
</tr>
<tr>
<td>Coefficient in the mon.policy rule</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Moments of endogenous variables (in p.p.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SD of cons. price inflation</td>
</tr>
<tr>
<td>SD of GDP growth</td>
</tr>
<tr>
<td>Corr between variables</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variance decomposition (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>↓variance / → shock</td>
</tr>
<tr>
<td>Consumer price inflation</td>
</tr>
<tr>
<td>GDP growth</td>
</tr>
</tbody>
</table>

\(^{1}\) SD = standard deviation / Corr = correlation
\(^{2}\) calibrated value
\(^{3}\) MS = monetary shock / FS = fiscal shock (to the primary surplus)

We find an specific combination of monetary and fiscal commitment that grants the lowest volatility in output growth, for a variance of inflation equal to the one obtained under the calibrated model. Such combination is shown in the second column of Table 7. Such combination increases the share of inflation variance that is attributed to the monetary policy shock, although the highest stake is still with the fiscal shock.

4.2.2 – Fiscal and monetary policy activeness

In Dynare, the model shows a unique solution for time paths of endogenous variables under two regions of policy activeness (Figure 7), maintaining the remaining parameters as they were originally calibrated. Under active monetary policy (\(\phi_n > 1.1\)), the response of the fiscal rule to deviations of the public debt to its steady state ratio may range in the positive interval of \((0.03, \infty)\), where the original calibrated parameter belongs, or from \((-\infty, -1.21)\). The curiosity about the second interval of convergence is the strong cyclicality that obtains for important macroeconomic variables (Figure 8).
The model also shows a unique solution (in Dynare) in regions where monetary policy is passive. The cyclicality that results is strong in general, but for practically null values of fiscal responsiveness to the debt and monetary responsiveness to deviation from the inflation target, the model reestablishes lower cyclicalities.

Figure 7: Regions where the model converges to unique solution in Dynare

1/ The region was plotted only for the interval and . The colored regions continue for the regions beyond the plotted limits.
Figure 8: Some plots of impulse responses to a monetary policy shock under distinct combinations of policy parameters in the region where the model converges to a unique solution in Dynare

- **$\phi_M = 0.04$**
  - Nominal interest rate
  - Consumer price inflation
  - Output
  - Public debt/GDP
  - Primary surplus/GDP
  - Government consumption/GDP

- **$\phi_M = 0.30$**
  - Nominal interest rate
  - Consumer price inflation
  - Output
  - Public debt/GDP
  - Primary surplus/GDP
  - Government consumption/GDP

- **$\phi_M = 1.00$**
  - Nominal interest rate
  - Consumer price inflation
  - Output
  - Public debt/GDP
  - Primary surplus/GDP
  - Government consumption/GDP
$\phi_{\text{IR}} = -1.25$  $\phi_{\text{IR}} = 1.00$  $\phi_{\text{IR}} = -1.00$  $\phi_{\text{IR}} = 0.00$  $\phi_{\text{IR}} = 0.00$

$\phi_{\omega} = 1.57$  $\phi_{\omega} = 0.00$  $\phi_{\omega} = 0.90$  $\phi_{\omega} = 0.00$  $\phi_{\omega} = 0.90$
Table 7: An exercise of superior combinations of fiscal and policy rules

<table>
<thead>
<tr>
<th>Moments of the shocks (in p.p.)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>SD of the monetary policy shock</td>
<td>1.00</td>
</tr>
<tr>
<td>SD of the fiscal policy shock</td>
<td>1.00</td>
</tr>
<tr>
<td>Corr between shocks</td>
<td>0.00</td>
</tr>
</tbody>
</table>

| Fiscal commitment to the public debt         |          |
| Coefficient in the fiscal rule              | 0.04 ²   |
| Monetary policy commitment to the inflation target | 0.27 |

| Moments of endogenous variables (in p.p.)   |          |
| SD of cons. price inflation                 | 0.10     |
| SD of output growth                         | 1.30     |
| Corr between variables                      | 4.78     |

| Variance decomposition (%)                  |          |
| ↓ variance / → shock                       | MS       |
|                                           | FS       |
| Consumer price inflation                   | 15.63    | 84.37 |
| GDP growth                                 | 3.88     | 96.12 |

¹ SD = standard deviation / Corr = correlation
² calibrated value
³ MS = monetary shock / FS = fiscal shock (to the primary surplus)

4.2.3 – Alternative types of monetary policy rules

The model can also be used to analyze the effects of adopting a distinct monetary policy rule. Table 8 compares the moments and shows a variance decomposition of key endogenous variables under alternative types of monetary policy rules. Augmenting the monetary policy rule to include an explicit component of reaction to exchange rate volatility or to the gap in output growth, the drop in inflation after a monetary policy shock is more persistent, and the initial impact on economic activity is milder. If the monetary policy rule reacts to the exchange rate variability, the volatility of inflation slightly reduces, albeit with an increase in the variance of output growth. The absolute magnitude of the correlation between economic growth and inflation reduces, but changes sign. If the monetary policy rule reacts to the gap in output growth, the variance in output growth reduces, albeit with an increase in the variance of consumer price inflation and the exchange rate. The monetary policy shock also contributes less to the variances of inflation, output growth and the exchange rate.
Table 8: Alternative monetary policy rules

<table>
<thead>
<tr>
<th>Moments of the shocks (in p.p.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SD of the monetary policy shock = 1.00</td>
</tr>
<tr>
<td>SD of the fiscal policy shock = 1.00</td>
</tr>
<tr>
<td>Corr between shocks = 0.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Monetary policy rules</th>
<th>calibrated model</th>
<th>calibrated rule + reaction to the exchange rate</th>
<th>calibrated rule + reaction to the output growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>SD of inflation</td>
<td>0.10</td>
<td>0.04</td>
<td>0.41</td>
</tr>
<tr>
<td>SD of GDP growth</td>
<td>1.30</td>
<td>1.27</td>
<td>0.85</td>
</tr>
<tr>
<td>SD of exchange rate variation</td>
<td>0.68</td>
<td>0.22</td>
<td>1.28</td>
</tr>
<tr>
<td>Corr between consumer price inflation and GDP growth</td>
<td>4.78</td>
<td>0.46</td>
<td>-7.51</td>
</tr>
<tr>
<td>Corr between consumer price inflation and exchange rate variation</td>
<td>48.84</td>
<td>40.25</td>
<td>46.36</td>
</tr>
<tr>
<td>Corr between GDP growth and exchange rate variation</td>
<td>8.58</td>
<td>-25.58</td>
<td>-78.61</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Moments of endogenous variables (in p.p.)</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Variance decomposition (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MS</td>
</tr>
<tr>
<td>---------------------------</td>
</tr>
<tr>
<td>Consumer price inflation</td>
</tr>
<tr>
<td>GDP growth</td>
</tr>
<tr>
<td>Exchange rate variation</td>
</tr>
</tbody>
</table>

/1 SD = standard deviation / Corr = correlation
/2 calibrated value
/3 MS = monetary shock / FS = fiscal shock (to the primary surplus)

Backward looking rules do not substantially alter the dynamics of the main macroeconomic variables after a monetary policy shock. However, the responses after a fiscal policy shock are significantly different from those obtained under forward looking rules. For one thing, monetary policy reaction after a fiscal shock is slower. Inflation deviates a little less from the steady state, and the reversal of output to its steady trend is quicker.
5. Conclusion

In this paper we revised the work in CMS and CCW, correcting important equations relating to prices, wages and the aggregate resource constraint of the economy. In order to better approximate the modeled economy to the current practice of fiscal policy in a number of countries, we introduced a different modeling strategy to account for the role of the fiscal sector in the economy, and its interaction with monetary policy. We allowed the government to invest and the private sector to decide upon the utilization of public or private capital. For the same purpose, we introduced labor specialization to allow for wage heterogeneity amongst households that supply the same amount of worked hours.

The model responses to monetary policy shocks are the standard ones obtained in the literature, although more short-lived under the adopted calibration strategy. The simulations show an important endogenous interaction of monetary policy conditions with fiscal policy responses, although policy rules are not directly responsive to one another. Expansionist primary surplus shocks can boost economic activity, yet with significant implications to inflation. However, the simulations show that fiscal transfer shocks, aimed at redistributing income, and shocks to public investment negatively affect general economic conditions as consequence of the fiscal rule.

Different specifications for the rule significantly affect the results implied by the model. The simulations with different degrees of fiscal commitment to the stationary path of the public debt and with greater rigor in the implementation of the primary surplus rule make explicit the tradeoff that the shocks imply on the volatility of inflation and output growth. Increasing commitment enhances the contractionist impact of a monetary policy shock upon inflation, albeit at the cost of a higher impact in output growth. The volatility of inflation and output growth increases, as does the correlation between them. A more rigorous implementation of the primary surplus rule, on the other hand, implies lower variance of inflation and output growth, but the correlation between them increases with the degree of rigor.

Simultaneous shocks to the primary surplus rule and to monetary policy make explicit the contrasting objectives of these policies. Primary surplus shocks dampen the contractionist effect of the monetary policy shock onto inflation and output, and also reduce the variance of inflation and output growth.
A higher commitment to the inflation target in the monetary policy rule reduces the variance of inflation and output growth, and their correlation, with the drawback that the fiscal shock gains importance in affecting the variance of inflation.

Different specifications of monetary policy rules also yield qualitatively distinct predictions. After a monetary policy shock, rules that include an explicit reaction either to the exchange rate variability or to the output growth show more sluggishness at the reversal of inflation to the steady state. The initial impact of the shock onto the economic activity is milder, yet more persistent.

Both alternative monetary policy rules reduce the variance of output growth. However, the reaction to the output growth increases the variance of inflation. Although the paper does not show optimal policy exercises, a monetary policy reaction to the exchange rate holds favorable outcomes: the variance of inflation and the correlation between inflation and output growth reduce, and the monetary policy shock gains a much greater stake at the variance of inflation.

References


APPENDIX

A. The model

We describe below the domestic economy. The foreign economy is modeled symmetrically, except for some distinct parameters and the modeling of the fiscal and monetary policy rules.

A.1 Households

A.1.1. Group I

Households are distributed into two groups. Every period each individual \( i \in I = [0, 1 - \omega] \) in group I chooses consumption of a final private good \( C_{i,t} \) and wage \( W_{i,t} \) for its labor services \( N_{i,t} \) to maximize the intertemporal utility function

\[
E_t \left\{ \sum_{k=0}^{\infty} \beta^k \left[ \frac{1}{1-\sigma} \left( C_{i,t+k} - \kappa C_{i,t+k-1} \right)^{-\sigma} - \frac{1}{1+\xi} \left( N_{i,t+k}^{1+\xi} \right)^{1+\xi} \right] \right\}^{1+\xi} \]

(A.1)

where \( \kappa \) is an external habit persistence parameter, \( \beta \) is the intertemporal discount factor, \( \frac{1}{\sigma} \) is the intertemporal elasticity of consumption substitution, and \( \frac{1}{\xi} \) is the elasticity of labor effort relative to the real wage.

Group I has access to complete financial markets, and allocates its total income in consumption, investment \( I_{i,t} \) in capital goods, domestic government bonds \( B_{i,t+1} \), money \( M_{i,t} \), and foreign private bonds \( B_{i,t+1}^F \). Transactions with foreign bonds are subject to a risk premium \( \Gamma_{i} \left( B_{i,t}^F \right) \), where \( B_{i,t}^F := \frac{1}{1-\omega} \int_0^{1-\omega} B_{i,t}^F di \). Consumption expenditures are taxed at the rate \( \tau^C_i \) and are also subject to a transaction cost \( \Gamma_{i} \left( v_{i,t} \right) \).

Cost functions are detailed in Appendix B.

Labor services, capital rents, and profits \( D_{i,t} \) are also taxed. Households own the private capital stock \( K_{i,t+1} \) and decide on firms’ capital utilization \( u_{i,t} \), subject to a cost \( \Gamma_{u} \left( u_{i,t} \right) \), earning a gross rate of return \( R_{K,i,t+1} \). Households also receive transfers \( TR_{i,t} \).
from the government and, only in the case of the foreign economy, pay a lump sum tax \( T_{ij} \). The intertemporal budget constraint is

\[
\left( 1 + \tau_t^C + \Gamma_t (v_{t,i}) \right) P_{C,i} C_{i,t} + P_{i,t} I_{i,H,t} + R_t^{-1} B_{i,t+1} \]

\[+ \left( \left( 1 - \Gamma_{t}^{F} (B_{i,t+1}^{F}) \right) p_t R_{F,t} \right) S_t B_{i,t+1}^{F} + M_{i,t} + \Xi_{i,t} + \Phi_{i,t} \]

\[= \left( 1 - \tau_t^N - \tau_t^k \right) W_{t,i} N_t + \left( 1 - \tau_t^k \right) [u_{t,i} R_{K,H,t} - \Gamma_t (u_{t,i})] K_{i,H,t} + \tau_t^k \delta P_{t,i} K_{i,H,t} \]

\[+ \left( 1 - \tau_t^N \right) D_{t,i} + TR_{t,i} - T_{t,i} + B_{t,i} + S_t B_{i,t+1}^{F} + M_{i,t-1} \]

where \( v_{t,i} \) is consumption velocity, with \( v_{t,i} := \frac{\left( 1 + \tau_t^C \right) P_{C,i} C_{i,t}}{M_{i,t}} \), \( \Xi_{i,t} \) is a lump sum rebate on the risk premium introduced in the negotiation of international bonds, and \( \Phi_{i,t} \) is the stock of contingent securities negotiated within group \( I \), which act as an insurance against risks on labor income.

We assume that private capital \( K_{i,H,t+1} \) accumulated by each household follows the transition rule:

\[
K_{i,H,t+1} = (1 - \delta) K_{i,H,t} + \left( 1 - \Gamma_t \left( \frac{I_{i,H,t}}{I_{i,H,t-1}} \right) \right) I_{i,H,t} \]

(A.3)

Setting \( \Lambda_{i,t} \) and \( \Lambda_{i,t} Q_{i,t} \) respectively as the Lagrange multipliers for the budget constraint and the capital accumulation function, maximization of the utility function with respect to \( C_{i,t} , I_{i,t} , K_{i,H,t+1} , u_{t,i} , B_{t,i+1} , B_{i,t}^{F} \), and \( M_{i,t} \) yield the following first order conditions:

\[
\Lambda_{i,t} = \frac{(C_{i,t} - \kappa C_{i,t-1})^{-\sigma}}{1 + \tau_t^C + \Gamma_t (v_{t,i}) + \Gamma_t (v_{t,i}) v_{t,i}} \]

(A.4)

\[
P_{i,t} = Q_{i,t} \left( 1 - \Gamma_t \left( \frac{I_{i,H,t}}{I_{i,H,t-1}} \right) - \Gamma_t \left( \frac{I_{i,H,t}}{I_{i,H,t-1}} \right) I_{i,H,t} \right) \]

\[+ \beta E_t \left[ \Lambda_{i,t+1} \gamma_t \left( \frac{I_{i,H,t+1}}{I_{i,H,t}} \right) \right] \]

\[+ \beta E_t \left[ \Lambda_{i,t+1} \gamma_t \left( \frac{B_{i,t+1}^{F}}{B_{i,t}^{F}} \right) \right] \]

\[+ \beta E_t \left[ \Lambda_{i,t+1} \gamma_t \left( \frac{M_{i,t+1}}{M_{i,t}} \right) \right] \]

\[= \beta E_t \left[ \Lambda_{i,t+1} \gamma_t \left( \frac{(1 - \delta) Q_{i,t+1} + (1 - \tau_t^k) R_{K,H,t+1} u_{t,i+1}}{P_{C,i+1}} \right) \right] \]

(A.5)

\[Q_{i,t} = \beta E_t \left[ \Lambda_{i,t+1} \gamma_t \left( \frac{(1 - \delta) Q_{i,t+1} + (1 - \tau_t^k) R_{K,H,t+1} u_{t,i+1}}{P_{C,i+1}} \right) \right] \]
(1 − τi^E_0) \cdot \frac{P_{L,i,0}}{P_{C,i,0}} + \Gamma_{i}^\prime(u_{i,i,t}) \left( \frac{\tau_i^E}{P_{C,i,0}} - Q_{i,0} \right) = 0 \quad \text{(A.7)}

\beta R_{i} \left[ \frac{\Lambda_{i,t+1}}{\Lambda_{i,t}} \frac{P_{C,i,t}}{P_{C,i,t+1}} \right] = 1 \quad \text{(A.8)}

\beta(1 − \Gamma^p_{i} (B_{i,i,t})) r P_{i} R_{i} E_{i} \left[ \frac{\Lambda_{i,t+1}}{\Lambda_{i,t}} \frac{P_{C,i,t}}{P_{C,i,t+1}} \frac{S_{i,t+1}}{S_{i,t}} \right] = 1 \quad \text{(A.9)}

\beta E_{i} \left[ \frac{\Lambda_{i,t+1}}{\Lambda_{i,t}} \frac{P_{C,i,t}}{P_{C,i,t+1}} \right] = 1 - \Gamma_{i}^\prime(v_{i,i,t}) \frac{\nu_{i,i,t}^2}{(1 + \tau^E_i)} \quad \text{(A.10)}

Within each group, households compete in a monopolistic competitive labor market. By setting wage \( W_{i,i,t} \), household \( i \) commits to meeting any labor demand \( N_{i,i,t} \).

Wages are set à la Calvo, with a probability \( (1 − \xi_i) \) of optimizing each period. Households that do not optimize readjust their wages based on a geometric average of realized and steady state inflation \( \bar{W}_{i,i,t} := \left( \frac{P_{C,i,t-1}}{P_{C,i,t-2}} \right)^{\chi_{i,t}} \pi_{C,i}^{1-\chi_{i,t}} W_{i,i,t-1} \). Every optimizing household chooses \( \tilde{W}_{i,i,t} = \bar{W}_{i,i,t} \).

Household’s optimization with respect to \( \tilde{W}_{i,i,t} \) yields the first order condition:

\[
\left( \sum_{k=0}^{\infty} (\xi_{i,t} \beta)^k N_{i,i+k,t} \right) \left[ \left( \frac{\Lambda_{i,t+k} - \tau^{N}_{i+k} - \tau^{W}_{i+k}}{\Lambda_{i,t+k}} \tilde{W}_{i,i,t} \left( \frac{P_{C,i,t+k-1}}{P_{C,i,t-k}} \right)^{\chi_{i,t}} \pi_{C,i}^{1-\chi_{i,t}} \right)^{\eta_{i,t}} - \frac{\eta_{i,t}}{\eta_{i,t}-1} N_{i,i+k,t} \right] = 0 \quad \text{(A.11)}
\]

where \( \eta_{i,t}/(\eta_{i,t}-1) \) is the after-tax real wage markup, in the absence of wage rigidity (when \( \xi_{i,t} \to 0 \)), with respect to the marginal rate of substitution between consumption and leisure. The markup results from the worker’s market power to set wages. Equation (A.11) can be expressed in the following recursive form, detailed in appendix C:

\[
(1 - \omega)^{\xi_{i,t}} \left( \frac{\tilde{W}_{i,i,t}}{P_{C,i,t}} \right)^{1 + \eta_{i,t} \xi_{i,t}} = \frac{\eta_{i,t}}{\eta_{i,t}-1} \cdot \frac{F_{i,i,t}}{G_{i,i,t}} \quad \text{(A.12)}
\]

where
A.1.2. Group $J$

Households in group $J$ can smooth consumption only through money holdings. Their decision is to choose consumption $C_{j,t}$ and money $M_{j,t}$ to maximize

$$E_t \left\{ \sum_{k=0}^{\infty} \beta^k \left( \frac{1}{1-\sigma} \left( C_{j,t+k} - \kappa C_{j,t+k-1} \right)^{1-\sigma} - \frac{1}{1+\xi} (N_{j,t+k}^{1+\xi}) \right) \right\}$$

subject to the budget constraint

$$\left( 1+\tau_t^e + \Gamma \left( v_{j,t} \right) \right) P_{C,j} C_{j,t} + M_{j,t} = \left( 1-\tau_t^n - \tau_t^w \right) W_{j,t} N_{j,t} + TR_{j,t} - T_{j,t} + M_{j,t-1} + \phi_{j,t}$$

First order conditions yield:

$$\Lambda_{j,t} = \frac{\left( C_{j,t} - \kappa C_{j,t-1} \right)^{-\sigma}}{1+\tau_t^e + \Gamma \left( v_{j,t} \right) + \Gamma \left( v_{j,t} \right) v_{j,t}}$$

and

$$\beta E_t \left[ \frac{\Lambda_{j,t+1} P_{C,j}}{\Lambda_{j,t} P_{C,j+1}} \right] = 1 - \Gamma \left( v_{j,t} \right) \frac{\nu_{j,t}^2}{(1+\tau_t^e)}$$

where $\Lambda_{j,t} / P_{C,j}$ is the Lagrange multiplier associated with the budget constraint.

Household $j$ sets wages in a way that is symmetric to household $i$, differing only as to the probability of being chosen to maximize $\left( 1-\xi_j \right)$, which is group-specific.

A.2 Firms

There are two types of firms in the model: producers of tradable intermediate groups and producers of non-tradable final goods. Firms producing intermediate goods are indexed by $f \in [0,1]$. All of final goods producers, except for the one producing public
consumption goods, combine domestic and foreign intermediate goods in the production.

A.2.1 Intermediate goods firms
A continuum of firms, indexed by $f \in [0,1]$, produce tradable intermediate goods $Y_{f,t}$ under monopolistic competition. To this end, they competitively rent capital $K_{f,t}$ from households in group $I$, and hire labor $N_{f,t}$ from households in both $I$ and $J$ groups using the technology.

$$Y_{f,t} = z_t \left( u_{f,t} K_{f,t} \right)^{\alpha} \left( z_{n_t} N_{f,t} \right)^{1-\alpha} - \psi z_t$$  \hspace{1cm} (A.17)

where $\psi$ is a fixed cost chosen to ensure zero profit in the steady state, and $z_t$ and $z_{n_t}$ are respectively temporary and permanent shocks that follow the process:

$$\ln(z_t) = (1 - \rho_z) \ln(z) + \rho_z \ln(z_{t-1}) + \varepsilon_{z,t}$$  \hspace{1cm} (A.18)

and

$$\frac{z_{n_t}}{z_{n_{t-1}}} = (1 - \rho_{zn}) g_Y + \rho_{zn} \frac{z_{n_{t-1}}}{z_{n_{t-2}}} + \varepsilon_{zn,t}$$  \hspace{1cm} (A.19)

where $z$ is the stationary level of total factor productivity, $g_Y$ is the steady state growth rate of labor productivity, $\rho_z$ and $\rho_{zn}$ are parameters, and $\varepsilon_{z,t}$ and $\varepsilon_{zn,t}$ are white noise shocks.

For a given level of production, firms take the cost of capital $R_{K,t}$, the average per capita wage $W_{t}$, and social security contribution $\tau_t W_{t}$ as given to minimize $R_{K,t} K_{f,t} + (1 + \tau_t W_{t}) W_{t} N_{f,t}$ subject to the technology in (A.18). Setting $MC_{f,t}$ as the Lagrange multiplier associated with the technology constraint, the first order conditions to this problem are

$$MC_{f,t} \frac{\alpha (Y_{f,t} + \psi)}{u_{f,t} K_{f,t}} = R_{K,t}$$  \hspace{1cm} (A.20)

$$MC_{f,t} \frac{(1 - \alpha) (Y_{f,t} + \psi)}{N_{f,t}} = (1 + \tau_t W_{t}) W_{t}$$  \hspace{1cm} (A.21)

Conditions (A.20) and (A.21) associated with technology (A.17) imply that $MC_{f,t}$ represents the firm’s marginal cost:
\[ MC_{f,j} = \frac{R_{K,f}K_{f,j} + (1 + \tau_t^W_t)W_tN_{f,j}}{Y_{f,j} + \psi} \]  
(A.22)

which can also be expressed as a function of wages and capital remuneration

\[ MC_r = \frac{1}{z, \alpha^\tau (1 - \alpha)^{1-\alpha}} \left( R_{K,f} \right)^{1-\alpha} \left( (1 + \tau_t^W_t)W_t \right)^{\alpha} \]  
(A.23)

which in turn implies that the marginal cost is equal across firms, i.e., \( MC_{f,j} = MC_r \).

Let \( K_{f,j}^S \) denote usable capital demanded by the intermediate goods’ firm \( f \). We assume that private \( K_{f,j}^H \) and public \( K_{f,j}^G \) capital goods transform into usable capital through the following CES technology:

\[ K_{f,j}^S = \left[ (1 - \omega_g)^{\eta_g} \left( K_{f,j}^H \right)^{\eta_g-1} + (\omega_g)^{\eta_g} \left( K_{f,j}^G \right)^{\eta_g-1} \right]^{1-\eta_g} \]  
(A.24)

where \( \omega_g \) represents the economy’s degree of dependence on government investment, and \( \eta_g \) stands for the elasticity of substitution between private and public goods, and also relates to the sensitivity of demand to the cost variation in each type of capital.

For a given total demand for capital, the intermediate firm minimizes total cost of private and public capital, solving:

\[ \min_{K_{f,j}^H, K_{f,j}^G} R_{K,f}^H K_{f,j}^H + R_{K,f}^G K_{f,j}^G \]  
(A.25)

subject to the technology constraint (A.24).

First order conditions to this problem yield

\[ K_{f,j}^H = (1 - \omega_g) \left( \frac{R_{K,j}^H}{R_{K,f}} \right)^{-\eta_g} K_{f,j}^S \]  
(A.26)

\[ K_{f,j}^G = \omega_g \left( \frac{R_{K,j}^G}{R_{K,f}} \right)^{-\eta_g} K_{f,j}^S \]  
(A.27)

which can be combined to yield the average rate of return on capital

\[ R_{K,f} = \left( (1 - \omega_g) \left( R_{K,j}^H \right)^{-\eta_g} + \omega_g \left( R_{K,j}^G \right)^{-\eta_g} \right)^{-1} \]  
(A.28)

Aggregating the distinct types of capital across firms, using (A.28), yields aggregate capital rented to intermediate goods firms.
\[
K^S_t = \left( (1 - \omega_g)\frac{1}{\eta_g} \left( K^H_t \right)^{\eta_g - 1} + \omega_g \frac{1}{\eta_g} \left( K^G_t \right)^{\eta_g - 1} \right)^{\frac{\eta_g}{\eta_g - 1}}
\]

and the aggregate demand functions for each type of capital good are:

\[
K^G_t = \omega_g \left( \frac{R^G_{K,t}}{R_{K,t}} \right)^{-\eta_g} K^S_t
\]

\[
K^H_t = (1 - \omega_g) \left( \frac{R^H_{K,t}}{R_{K,t}} \right)^{-\eta_g} K^S_t
\]

Labor demanded by firm \( f \) from both types of households is aggregated with a CES technology

\[
N_{f,t} := \left( (1 - \nu_{\omega} \omega)^{1/\eta} \left( N_{f,t}^I \right)^{-1/\eta} + \nu_{\omega} \omega \left( N_{f,t}^J \right)^{-1/\eta} \right)^{\eta/(\eta - 1)}
\]

where

\[
N_{f,t}^I := \left[ \left( \frac{1}{1 - \omega} \right)^{1/\eta_I} \int_0^{N_{f,t}} \left( N_{f,t}^I \right)^{-1/\eta_I} \, di \right]^{\eta_I/(\eta_I - 1)}
\]

\[
N_{f,t}^J := \left[ \frac{1}{\omega} \int_{1 - \omega}^{N_{f,t}} \left( N_{f,t}^J \right)^{-1/\eta_J} \, dj \right]^{\eta_J/(\eta_J - 1)}
\]

where \( \eta \) is the elasticity of substitution between labor from households in group \( I \) and \( J \), \( \eta_I \) is the inverse-elasticity of substitution between members of group \( I \), and \( \eta_J \) is the inverse-elasticity of substitution between members of group \( J \).

Taking average wages \( (W_{f,t}^I, W_{f,t}^J) \) in both groups as given, firms choose how much to hire from both groups of households by minimizing total labor cost \( W_{f,t} N_{f,t}^I + W_{f,t} N_{f,t}^J \) subject to (A.32). It follows from first order conditions that

\[
N_{f,t}^I = (1 - \nu_{\omega} \omega) \left( \frac{W_{f,t}^I}{W_t} \right)^{-\eta} N_{f,t}
\]

\[
N_{f,t}^J = \nu_{\omega} \omega \left( \frac{W_{f,t}^J}{W_t} \right)^{-\eta} N_{f,t}
\]

where the aggregate wage is:

\[
W_t = \left[ (1 - \nu_{\omega} \omega) W_{f,t}^I^{-\eta} + \nu_{\omega} \omega W_{f,t}^J^{-\eta} \right]^{1/\eta}
\]
For a given total demand for labor, conditions (A.35) and (A.36) imply that the demand for labor from each group of households is increasing in the size of the group.

Aggregating labor demand across firms, and using $N_f^D = \int_0^1 N_f df$, yields the following aggregate demand functions for each group of households:

$$N_i^I = (1 - \nu_i \omega) \left( \frac{W_{i,j}}{W_i} \right)^{-\eta_i} N_i^D$$ \hspace{1cm} \text{(A.38)}

$$N_j^J = \nu_j \omega \left( \frac{W_{j,i}}{W_j} \right)^{-\eta_j} N_i^D$$ \hspace{1cm} \text{(A.39)}

The firm demands labor $N_{f,i}^I$ and $N_{f,j}^J$ from each individual in groups $I$ and $J$ taking individual wages $W_{i,j}$ and $W_{j,i}$ as given to minimize the average cost

$$\int_0^{1-\omega} W_{i,j} N_{j,i}^j d_i + \int_{1-\omega}^1 W_{j,i} N_{j,i}^j d_j$$ subject to aggregation constraints (A.38) and (A.39). First order conditions yield:

$$N_{f,i}^I = \frac{1}{1 - \omega} \left( \frac{W_{i,j}}{W_{i,j}} \right)^{-\eta_i} N_{f,i}^I$$ \hspace{1cm} \text{(A.40)}

$$N_{f,j}^J = \frac{1}{\omega} \left( \frac{W_{j,i}}{W_{j,i}} \right)^{-\eta_j} N_{f,j}^J$$ \hspace{1cm} \text{(A.41)}

where wages for each group of households are

$$W_{i,j} = \left[ \frac{1}{1 - \omega} \int_0^{1-\omega} W_{i,j}^{1-\eta_i} d_i \right]^{1/(1-\eta_i)}$$ \hspace{1cm} \text{(A.42)}

$$W_{j,i} = \left[ \frac{1}{\omega} \int_{1-\omega}^1 W_{j,i}^{1-\eta_j} d_i \right]^{1/(1-\eta_j)}$$ \hspace{1cm} \text{(A.43)}

Firms’ labor demand and wage setting conditions combine into aggregate wages for each household group as a function of optimal and mechanically readjusted wages:

$$W_{i,j} = \left[(1 - \xi_i)(\bar{W}_{i,j})^{1-\eta_i} + \bar{\xi}_i (\bar{W}_{i,j})^{-\eta_i} \right]^{1/(1-\eta_i)}$$ \hspace{1cm} \text{(A.44)}

$$W_{j,i} = \left[(1 - \xi_j)(\bar{W}_{j,i})^{1-\eta_j} + \bar{\xi}_j (\bar{W}_{j,i})^{-\eta_j} \right]^{1/(1-\eta_j)}$$ \hspace{1cm} \text{(A.45)}
Prices are set under monopolistic competition, with Calvo-type price rigidities. We assume local currency pricing. Let $P_{H,f,t}$ and $P_{X,f,t}$ be the prices for goods sold by firm $f$ in the domestic and foreign markets, with $\xi_H$ and $\xi_X$ denoting the probability that the firm will not optimize prices in each of these markets. Non-optimizing domestic and foreign firms mechanically adjust their prices according to the rules

$$
\bar{P}_{H,f,t} := \left( P_{H,f,t-1} \right)^{x_H} \left( \pi_H \right)^{-x_H} P_{H,f,t-1}
$$

(A.46)

$$
\bar{P}_{X,f,t} := \left( P_{X,f,t-1} \right)^{x_X} \left( \pi_X \right)^{-x_X} P_{X,f,t-1}
$$

(A.47)

where $\pi_H$ and $\pi_X$ are domestic and foreign intermediate goods’ steady state inflation rates.

Optimizing firms choose the prices $\bar{P}_{H,f,t}$ and $\bar{P}_{X,f,t}$ to maximize the expected discounted sum of nominal profits:

$$
E_t \left[ \sum_{k=0}^{\infty} \Lambda_{t,t+k} \left( (\xi_H)^k D_{H,f,t+k} + (\xi_X)^k D_{X,f,t+k} \right) \right]
$$

(A.48)

where $\Lambda_{t,t+k}$ is household $t$’s average discount factor, given by

$$
\Lambda_{t,t+k} = \frac{1}{1-\omega} \sum_{i=0}^{t+k} \beta^i \frac{\Lambda_{i,t+k}}{\Lambda_{i,t}} \frac{P_{C,t}}{P_{C,t+k}}
$$

(A.49)

and nominal profits, net of fixed costs, are defined as

$$
D_{H,f,t} = \left( P_{H,f,t} - MC_t \right) H_{f,t}
$$

(A.50)

$$
D_{X,f,t} = \left( S_t, P_{X,f,t} - MC_t \right) X_{f,t}
$$

(A.51)

Optimization is subject to the price indexation rule, to domestic and foreign demand for firm $f$’s goods, $H_{f,t}$ and $X_{f,t}$, taking as given the marginal cost, the exchange rate and aggregate demand.

First order conditions for the pricing decisions yield

$$
E_t \left[ \sum_{k=0}^{\infty} (\xi_H)^k \Lambda_{t,t+k} \left( \bar{P}_{H,t} \left( \frac{P_{H,t+k}}{P_{H,t-1}} \right)^{x_H} \left( \pi_H \right)^{-x_H} \frac{\theta}{\theta-1} MC_{t+k} \right) H_{f,t+k} \right] = 0
$$

(A.52)

and
\[
E_i \left[ \sum_{k=0}^{\infty} (\xi X)^k \Lambda_{I,i,k} \left( S_{i+k} \bar{p}_{X,i} + \frac{P_{X,i+k-1}}{P_{X,i-1}} \right) X_i \right] \left( \pi_{i} \right)^{1-X_i} \left( \pi_{X,i} \right)^{X_i} \left( \frac{\theta}{\theta-1} MC_{i,k} \right) X_{i+k} = 0 \quad (A.53)
\]

As firms are identical, they face the same optimization problem, choosing the same optimal price \( \bar{P}_{H,i} = \bar{P}_{H,i} \) and \( \bar{P}_{X,i} = \bar{P}_{X,i} \).

Pricing equations (A.52) and (A.53) can be restated recursively as

\[
\bar{P}_{H,i} = \frac{\theta}{\theta-1} F_{H,i} \quad (A.54)
\]
\[
\bar{P}_{X,i} = \frac{\theta}{\theta-1} F_{X,i} \quad (A.55)
\]

where

\[
F_{H,i} := MC_{H,i} + \xi H \beta E_i \left\{ \frac{N_{I,i,k+1}}{N_{I,i}} \left( \frac{\pi_{H,i+1}}{\pi_{H,i} \pi_{H,i}} \right) \right\} \theta \quad (A.56)
\]
\[
G_{H,i} := P_{H,i} + \xi H \beta E_i \left\{ \frac{N_{I,i,k+1}}{N_{I,i}} \left( \frac{\pi_{H,i+1}}{\pi_{H,i} \pi_{H,i}} \right) \right\} \theta^{-1} \quad (A.57)
\]
\[
F_{X,i} = MC_{X,i} + \xi X \beta E_i \left\{ \frac{\pi_{X,i+1}}{\pi_{C,i+1}} \left( \frac{\pi_{X,i+1}}{\pi_{X,i} \pi_{X,i}} \right) \right\} \theta \quad (A.58)
\]
\[
G_{X,i} = S_i P_{X,i} + \xi X \beta E_i \left\{ \frac{\pi_{X,i+1}}{\pi_{C,i+1}} \left( \frac{\pi_{X,i+1}}{\pi_{X,i} \pi_{X,i}} \right) \right\} \theta^{-1} \quad (A.59)
\]

The terms \( \theta/(1-\theta) \) and \( \theta^*/(1-\theta^*) \) denote the domestic and export price markups over nominal marginal costs, in the absence of price rigidities, where \( \theta \) is the elasticity of substitution between domestic intermediate goods and \( \theta^* \) is the analogue for export goods.

Aggregating over firms, domestic and export intermediate goods prices are

\[
P_{H,i} = \left[ (1-\xi H) (\bar{P}_{H,i})^{-\theta} + \xi H (\bar{P}_{H,i})^{-\theta} \right]^{1/(1-\theta)} \quad (A.60)
\]
\[
P_{X,i} = \left[ (1-\xi X) (\bar{P}_{X,i})^{-\theta} + \xi X (\bar{P}_{X,i})^{-\theta} \right]^{1/(1-\theta)} \quad (A.61)
\]

A.2.2 Final goods firms
Each one of three firms produces a distinct non-tradable final good for investment, and for private and public consumption. Except for the public consumption good, the production of final goods combines both foreign and domestic intermediate goods using a CES-type technology.

A.2.2.a. Private consumption goods

To produce private consumption goods, $Q^C$, the firm purchases bundles of domestic $H^C_t$ and foreign $IM^C_t$ intermediate goods. To adjust its imported share of inputs, the firm faces a cost $\Gamma_{IM^C_t} (IM^C_t / Q^C)$, detailed in Appendix B. Letting $v_C$ denote the bias towards domestic intermediate goods, the technology to produce private consumption goods is

$$Q^C_t = \left\{ (v_C)^{\frac{1}{\mu_C}} \left[ H^C_t \right]^{\frac{1-\mu_C}{\mu_C}} + \left\{ (1-v_C)^{\frac{1}{\mu_C}} \left[ \Gamma_{IM^C_t} (IM^C_t / Q^C) IM^C_t \right]^{\frac{1-\mu_C}{\mu_C}} \right\} \right\}^{\mu_C/(\mu_C-1)} \tag{A.58}$$

where

$$H^C_t = \left\{ \int (H^C_{t,x})^{1-\theta} d\theta \right\}^{\theta/(\theta-1)}$$

$$IM^C_t = \left\{ \int (IM^C_{t,x})^{1-\theta'} d\theta' \right\}^{\theta'/(\theta'-1)}$$

The firm will minimize total input costs

$$\min_{H^C_t, IM^C_t} P_H H^C_t + P_{IM^C} IM^C_t \tag{A.59}$$

subject to the technology constraint (A.58) taking intermediate goods prices as given.

The corresponding Lagrange problem is

$$\min_{H^C_t, IM^C_t, \lambda} P_H H^C_t + P_{IM^C} IM^C_t + \lambda_t \left\{ \left( (v_C)^{\frac{1}{\mu_C}} \left[ H^C_t \right]^{\frac{1-\mu_C}{\mu_C}} + \left\{ (1-v_C)^{\frac{1}{\mu_C}} \left[ \Gamma_{IM^C_t} (IM^C_t / Q^C) IM^C_t \right]^{\frac{1-\mu_C}{\mu_C}} \right\} \right\}^{\mu_C/(\mu_C-1)} \right\} \tag{A.60}$$

and the first order conditions associated with the choice of $H^C_t$ yield:
which is the demand for intermediate domestic goods for the production of consumption goods. Multiplying this by \( P_{H,i} \) yields nominal costs with intermediate domestic goods

\[
P_{H,i}H_i^C = v_c \left( \frac{P_{H,i}}{\lambda_i^C} \right)^{-\mu_c} Q_i^C \tag{A.62}
\]

The first order condition of the Lagrangean problem with respect to \( IM_i^C \) yields

\[
IM_i^C = (1-v_c) \left( P_{IM,i} / \Gamma_i^IM_i^C (IM_i^C / Q_i^C) \right)^{-\mu_c} \left( \frac{\Gamma_i^IM_i^C (IM_i^C / Q_i^C)}{1 - \Gamma_i IM_i^C (IM_i^C / Q_i^C)} \right) \lambda_i^C Q_i^C \tag{A.63}
\]

where \( \Gamma_i^IM_i^C \) is detailed in the appendix.

Multiplying (A.63) by \( P_{IM,i} \) yields the nominal cost to use imported intermediate goods

\[
P_{IM,i}IM_i^C = (1-v_c) \left( P_{IM,i} / \Gamma_i^IM_i^C (IM_i^C / Q_i^C) \right)^{-\mu_c} \left( \frac{\Gamma_i^IM_i^C (IM_i^C / Q_i^C)}{1 - \Gamma_i IM_i^C (IM_i^C / Q_i^C)} \right) \lambda_i^C Q_i^C \tag{A.64}
\]

The first order condition to the Lagrangean problem associated with the choice of the Lagrange multiplier \( \lambda_i^C \) is

\[
\lambda_i^C = \left[ v_c P_{H,i}^{1-\mu_c} + (1-v_c) \left( P_{IM,i} / \Gamma_i^IM_i^C (IM_i^C / Q_i^C) \right)^{-\mu_c} \right]^{1/\mu_c} \tag{A.65}
\]

To see that \( \lambda_i^C \) is not a price index in this context, notice that the nominal cost of inputs to the final goods firm can be expressed as

\[
P_{H,i}H_i^C + P_{IM,i}IM_i^C
\]

Substituting \( \lambda_i^C \) in the expression above, using (A.65), results in the optimal cost being a function of prices and the proportion of imports to total production, \( IM_i^C / Q_i^C \):
If final goods firms yield zero profits, we can define the corresponding price index for one unit of final good as

$$P_{c,t} = \frac{P_{H,t} + P_{IM,t} IM_{t}^{C}}{Q_{t}^{C}}.$$  \hspace{1cm} (A.66)

Defining the variable $\Omega_{t}^{C}$ as

$$\Omega_{t}^{C} = \left\{ v_{C} (P_{H,t})^{-\mu c} + (1-v_{C}) \left( P_{IM,t} / \Gamma^{3 \ IM_{t}^{C}} (IM_{t}^{C} / Q_{t}^{C}) \right) \right\}^{-\mu c} \left( \frac{P_{IM,t}}{P_{c,t}} \right)^{-\mu c} \left( \frac{1}{1-\Gamma^{3 \ IM_{t}^{C}} (IM_{t}^{C} / Q_{t}^{C})} \right)^{-\mu c},$$  \hspace{1cm} (A.67)

we obtain from (A.64) – (A.66) that the correct price index in this framework is

$$P_{c,t} = \left( \Omega_{t}^{C} \right)^{-\mu c} \left( \lambda_{t}^{C} \right)^{\mu c}.$$  \hspace{1cm} (A.68)

Notice that only when $\Omega_{t}^{C} = \lambda_{t}^{C}$ do we obtain $P_{c,t} = \lambda_{t}^{C} = \Omega_{t}^{C}$.

However, in general, the demand equations as a function of the price index (using equations (A.60), (A.62) and (A.67)) are

$$H_{t}^{C} = v_{C} \left( \frac{P_{H,t}}{\Omega_{t}^{C}} \right)^{-\mu c} \left( \frac{P_{H,t}}{P_{c,t}} \right)^{-\mu c} Q_{t}^{C},$$  \hspace{1cm} (A.69)

$$IM_{t}^{C} = (1-v_{C}) \left( \frac{P_{IM,t}}{\Omega_{t}^{C}} \right)^{-\mu c} \left( \frac{P_{IM,t} / \Gamma^{3 \ IM_{t}^{C}} (IM_{t}^{C} / Q_{t}^{C})}{P_{c,t}} \right)^{-\mu c} Q_{t}^{C} \left( \frac{1-\Gamma^{3 \ IM_{t}^{C}} (IM_{t}^{C} / Q_{t}^{C})}{1} \right)^{-\mu c}$$  \hspace{1cm} (A.70)

A.2.2.b. Investment goods

The firm producing investment goods $Q_{t}^{I}$ combines domestic $H_{t}^{I}$ and foreign $IM_{t}^{I}$ intermediate goods using the technology:

$$Q_{t}^{I} = \left\{ (v_{I})^{\mu_{I} \left( H_{t}^{I} \right)^{-\mu_{I}}} \left( 1-\Gamma_{IM_{t}^{I}} (IM_{t}^{I} / Q_{t}^{I}) \right) IM_{t}^{I} \right\}^{-\mu_{I} \left( H_{t}^{I} \right)^{-\mu_{I}}},$$  \hspace{1cm} (A.71)

where
and $\Gamma_{IM}^I \left(\frac{IM_i^I}{Q_i^f}\right)$ is an adjustment cost in the use of imported goods in the production of investment goods and $v_i$ stands for the bias towards domestic goods.

The cost minimization problem for the investment goods firm is exactly analogous to the one for the consumption good. The demand for domestic and imported intermediate goods is

$$H_i^I = \left(\int_0^1 (H_{f,i}^I)^{-1/\theta} \ d\theta \right)^{\theta/(\theta-1)}$$

$$IM_i^I = \left(\int_0^1 (IM_{f,i}^I)^{-1/\theta} \ d\theta \right)^{\theta'/\theta-1}$$

and the investment goods price index is

$$P_{I,i} = \left[ \Omega_i \left(\frac{P_{H,i}}{Q_i^f}\right)^{1-\mu_i} \left(\frac{P_{H,i}}{P_{f,i}}\right)^{-\mu_i} \right]^{1-\mu_i}$$

where

$$\Omega_i = \left[ v_i \left(\frac{P_{H,i}}{Q_i^f}\right)^{1-\mu_i} + (1-v_i) \left(\frac{\Gamma_{IM}^I (IM_i^I / Q_i^f)}{\left[1-\Gamma_{IM}^I (IM_i^I / Q_i^f)\right]}\right)^{-\mu_i} \right]^{1-\mu_i}$$

and

$$\lambda_i = \left[ v_i P_{H,i}^{-\mu_i} + (1-v_i) \left(\frac{\Gamma_{IM}^I (IM_i^I / Q_i^f)}{\left[1-\Gamma_{IM}^I (IM_i^I / Q_i^f)\right]}\right)^{1-\mu_i} \right]^{1-\mu_i}$$

A.2.2.c Public consumption goods

Public goods $Q_i^G$ are produced only from domestic intermediate goods using the technology

$$Q_i^G = H_i^G = \left(\int_0^1 (H_{f,i}^G)^{-1/\theta} \ d\theta \right)^{\theta/(\theta-1)}$$

The first order condition for the cost minimization problem is
\[ H_i^G = Q_i^G \]  
which yields the public consumption goods price index:
\[ P_{G,i} = P_{H,i} \]  

To build on the amount \( H_i^G \) of domestic intermediate goods to produce public consumption goods, the firm demands \( H_{f,i}^G \) from each of the domestic intermediate goods firms, following the cost minimization first order conditions:
\[ H_{f,i}^G = \left( \frac{P_{H,f,i}}{P_{H,i}} \right)^{-\theta} H_i^G \]  

A.2.3. Aggregation

Aggregating the demand for intermediate goods from the final goods firms results in the following demands for each domestic and foreign intermediate goods’ firms:
\[ H_{f,i} := H_{f,i}^C + H_{f,i}^I + H_{f,i}^G = \left( \frac{P_{H,f,i}}{P_{H,i}} \right)^{-\theta} H_i \]  
\[ IM_{f,i} := IM_{f,i}^C + IM_{f,i}^I = \left( \frac{P_{IM,f,i}}{P_{IM,i}} \right)^{-\theta} IM_i \]  

The total demand for domestic and foreign intermediate products is:
\[ H_i := H_i^C + H_i^I + H_i^G \]  
\[ IM_i := IM_i^C + IM_i^I \]  

The demand for intermediate goods imported from foreign firm \( f^* \) directly determines firm \( f^* \)'s exports adjusted by the countries’ sizes
\[ (1-s)X_{f^*,i} = sIM_{f^*,i} \]  

The local pricing assumption can be restated as
\[ P_{IM,f,i} = P_{X,f,i}^* \]  
and therefore the aggregate prices of imported goods should equal the aggregate prices of goods exported by the foreign producer
\[ P_{IM,i} = \left[ \int_0^1 (P_{IM,f,i}^*)^{1-\theta} df^* \right]^{1-\theta} = \left[ \int_0^1 (P_{X,f,i}^*)^{1-\theta} df^* \right]^{1-\theta} = P_{X,i}^* \]
Demand for firm \( f \)'s goods by foreign firms determines firm \( f \)'s export quantum, adjusted by the countries' sizes:

\[
sX_{f,t} = (1-s)IM^*_f, \quad (A.88)
\]

Similarly, prices of goods imported from domestic firm \( f \) by the foreign importer should equal the export price set by firm \( f \) in foreign currency:

\[
P^*_M, f,t = P_{X,f,t} \quad (A.89)
\]

Therefore, the aggregate export price \( P_{X,t} \) should equal the aggregate import price in the foreign economy:

\[
P_{X,t} = \int_0^1 \left( \frac{1}{1-\theta} \right)^{1-\theta} df = \int_0^1 \left( \frac{1}{1-\theta} \right)^{1-\theta} df \equiv P^*_M, t \quad (A.90)
\]

### A.3. Market clearing

Any aggregated model variable \( Z_t \) denoted in per capita terms results from the aggregation \( Z_t = \int_0^1 Z_{h,t} dh = (1-\omega)Z_{I,t} + \omega Z_{J,t} \) where \( Z_{I,t} \) and \( Z_{J,t} \) are the respective per capital values of \( Z_t \) for families \( I \) and \( J \).

Therefore, we define

\[
M_{I,t} = \frac{1}{1-\omega} \int_0^{1-\omega} M_{I,t} di \quad (A.91)
\]

\[
M_{J,t} = \frac{1}{\omega} \int_0^{1-\omega} M_{J,t} dj \quad (A.92)
\]

\[
TR_{I,t} = \frac{1}{1-\omega} \int_0^{1-\omega} TR_{I,t} di \quad (A.93)
\]

\[
TR_{J,t} = \frac{1}{\omega} \int_0^{1-\omega} TR_{J,t} dj \quad (A.94)
\]

\[
T_{I,t} = \frac{1}{1-\omega} \int_0^{1-\omega} T_{I,t} di \quad (A.95)
\]

\[
T_{J,t} = \frac{1}{\omega} \int_0^{1-\omega} T_{J,t} dj \quad (A.96)
\]
\[ B_{i,t+1} := \frac{1}{1-\omega} \int_0^{1-\omega} B_{i,t+1} \, di \quad (A.97) \]

\[ I_{i} := \frac{1}{1-\omega} \int_0^{1-\omega} I_{i} \, di \quad (A.98) \]

\[ K_{i,t} := \frac{1}{1-\omega} \int_0^{1-\omega} K_{i,t} \, di \quad (A.99) \]

\[ D_{i,t} := \frac{1}{1-\omega} \int_0^{1-\omega} D_{i,t} \, di \quad (A.100) \]

and, aggregating over household groups:

\[ C_t := (1-\omega)C_t + \omega C_{t,t} \quad (A.101) \]

\[ M_t := (1-\omega)M_t + \omega M_{t,t} \quad (A.102) \]

\[ TR_t := (1-\omega)TR_t + \omega TR_{t,t} \quad (A.103) \]

\[ T_t := (1-\omega)T_{t,t} + \omega T_{t,t} \quad (A.104) \]

\[ B_{t+1} := (1-\omega)B_{t+1} \quad (A.105) \]

\[ B^F_{t+1} := (1-\omega)B^F_{t+1} \quad (A.106) \]

\[ I_t := (1-\omega)I_t \quad (A.107) \]

\[ K_t := (1-\omega)K_t \quad (A.108) \]

\[ D_t := (1-\omega)D_t \quad (A.109) \]

The equilibrium between supply and demand for labor occurs at the individual level:

\[ N_{i,t} = N'_{i,t} := \int_0^{1} N'_{i,t} \, df \quad (A.110) \]

\[ N_{j,t} = N'_{j,t} := \int_0^{1} N'_{j,t} \, df \quad (A.111) \]

which, aggregating the demand of all firms in equations (A.40) and (A.41), yields

\[ N_{i,t} = \frac{1}{1-\omega} \left( \frac{W_{i,t}}{W_{i,t}} \right)^{-\eta_t} N'_{i,t} \quad (A.112) \]

\[ N_{j,t} = \frac{1}{\omega} \left( \frac{W_{j,t}}{W_{j,t}} \right)^{-\eta_t} N'_{i,t} \quad (A.113) \]

and can also be represented, using equations (A.38) and (A.39), as a function of total demand for labor by firms:
Aggregate supply by each household group is defined as \( N_{i,s} \) and \( N_{j,s} \), and we define \( N_{s,s} \) as the total supply of labor. Aggregating the supply of labor using equations (A.112) and (A.113) yields

\[
N_{i,s} = \left( \frac{W_{i,s}}{W_i} \right)^{\eta_i} \cdot N_i^D
\]

\[
N_{j,s} = \left( \frac{W_{j,s}}{W_j} \right)^{\eta_j} \cdot N_i^D
\]

Aggregating the demand for labor from household groups \( I \) and \( J \) yields:

\[
N_{i,d} := \frac{1}{1 - \omega} \int_0^{1 - \omega} N_{i,d} \, di = \frac{1}{1 - \omega} \int_0^{1 - \omega} \left( \frac{W_{i,s}}{W_i} \right)^{-\eta_i} \cdot N_i^D \, di;
\]

\[
N_{j,d} := \frac{1}{1 - \omega} \int_0^{1 - \omega} N_{j,d} \, dj = \frac{1}{1 - \omega} \int_0^{1 - \omega} \left( \frac{W_{j,s}}{W_j} \right)^{-\eta_j} \cdot N_i^D \, dj;
\]

Therefore, the relation between aggregate supply and aggregate demand depends on wage dispersion:

\[
N_{i,s} = \frac{\Psi_{i,s}}{1 - \omega} \cdot N_i^l
\]

\[
N_{j,s} = \frac{\Psi_{j,s}}{\omega} \cdot N_i^l
\]

where the wage dispersion for households \( I \) and \( J \) is represented by:

\[
\Psi_{i,s} := (1 - \xi_i) \left( \frac{\tilde{W}_{i,s}}{W_i} \right)^{-\eta_i} + \xi_i \left( \frac{\tilde{\pi}_{C,i-1} \tilde{\pi}_{C,i}^{-\xi_i}}{\pi_{W,i}} \right) \cdot \Psi_{i,-1}
\]

\[
\Psi_{j,s} := (1 - \xi_j) \left( \frac{\tilde{W}_{j,s}}{P_{j,s}Y_j} \right)^{-\eta_j} \left( \frac{W_{j,s}}{P_{j,s}Y_j} \right)^{\xi_j} \left( \frac{\tilde{\pi}_{C,j-1} \tilde{\pi}_{C,j}^{-\xi_j}}{\pi_{W,j}} \right) \cdot \Psi_{j,-1}
\]

and \( \pi_{W,i} \) and \( \pi_{W,j} \) stand for household \( I \) and \( J \) wage inflation rates, detailed in appendix F.

Aggregating the demand for labor from household groups \( I \) and \( J \) yields:

\[
N_{s,d} := \int_0^{1 - \omega} N_{i,d} \, di + \int_0^{1 - \omega} N_{j,d} \, dj = (1 - \omega) \cdot N_{i,d} + \omega \cdot N_{j,d} = \Psi_{i,d} \cdot N_i^l + \Psi_{j,d} \cdot N_i^l
\]

\[
N_{s,s} := \Psi_{i,s} \cdot N_i^l + \Psi_{j,s} \cdot N_i^l
\]
which results in a relation between total aggregate supply and demand that depends on the total wage dispersion index:

\[ N_{S,t} = \psi_t N^D_t \]  \hspace{1cm} (A.120)

where total wage dispersion is \( \psi_t := \left\{ (1 - \omega) \left( \frac{W_{J,t}}{W_t} \right)^{-\eta} \psi_{J,t} + \omega \left( \frac{W_{I,t}}{W_t} \right)^{-\eta} \psi_{I,t} \right\} \).

Total production of domestic intermediate firm \( f \) fulfills:

\[ Y_{f,t} = H_{f,t} + X_{f,t} \]  \hspace{1cm} (A.121)

Let \( Y_t \) be the total supply of intermediate goods in the domestic economy, and \( X_t \) be the total demand for export goods produced in the domestic economy. We thus obtain

\[ Y_t := \int Y_{f,t} df = \int H_{f,t} df + \int X_{f,t} df = \int \left( \frac{P_{H,f,t}}{P_{H,t}} \right)^{-\theta} H_t df + \int \left( \frac{P_{X,f,t}}{P_{X,t}} \right)^{-\theta} X_t df \]

which results in

\[ Y_t = \psi_{H,t} H_t + \psi_{X,t} X_t \]  \hspace{1cm} (A.122)

where price dispersion in the domestic and export markets for intermediate goods is:

\[ \psi_{H,t} := (1 - \xi_H) \left( \frac{P_{H,t}}{P_{H,t}} \right)^{-\theta} + \xi_H \left( \frac{\pi_{H,t-1} - \pi_{H,t}}{\pi_{H,t}} \right)^{-\theta} \psi_{H,t-1} \]

\[ \psi_{X,t} := (1 - \xi_X) \left( \frac{P_{X,t}}{P_{X,t}} \right)^{-\theta} + \xi_X \left( \frac{\pi_{X,t-1} - \pi_{X,t}}{\pi_{X,t}} \right)^{-\theta} \psi_{X,t-1} \]

Aggregate demand for export goods fulfills \( sX_t := \int sX_{f,t} df = (1 - s) \int IM^*_X df \),

which results in

\[ sX_t = (1 - s)IM^*_t \]  \hspace{1cm} (A.123)

Let \( P_{Y,t} \) denote the intermediate goods price index, which satisfies

\[ P_{Y,t} = \int P_{H,f,t} H_{f,t} df + \int P_{X,f,t} X_{f,t} df = H_t \int \left( \frac{P_{H,f,t}}{P_{H,t}} \right)^{-\theta} df + X_t \int \left( \frac{P_{X,f,t}}{P_{X,t}} \right)^{-\theta} df \]

We thus obtain that
In the competitive market for final goods, equilibrium requires that the following relations be satisfied

\[ Q^C_t = C_t + (1 - \omega) \Gamma_v(v_{i,j})C_{i,j} + \omega \Gamma_v(v_{j,j})C_{j,j} = C_t + \Gamma_{v,t} \]  

(A.125)

\[ Q^{I}_t = I_t + \Gamma_v(u_{i,t})P_{t,i}K_t \]  

(A.126)

\[ Q^{G}_t = G_t \]  

(A.127)

where \( \Gamma_{v,t} \) is the aggregate real transaction cost, \( \Gamma_{v,t} := \int_0^{1-\omega} \Gamma_v(v_{j,j})C_{j,j} \, dj + \int_0^1 \Gamma_v(v_{i,j})C_{i,j} \, df \)

In the capital market, \( u_t \) is the average capital utilization, which satisfies

\[ u_t := \frac{1}{K_t} \int_0^1 K_{j,t} \, df \]  

(A.128)

Profit distribution fulfills

\[ (1 - \omega)D_{H,t} := \int_0^1 D_{H,t,i} \, df + \int_0^1 D_{X,t,i} \, df \]  

(A.129)

To obtain the aggregate resource constraint of the economy, we use households and government budget constraints. Aggregating households’ budget constraints into the budget constraint for group \( I \) and \( J \), we obtain:

\[ P_{C,t}C_t + (1 - \omega) \Gamma_v(v_{i,j})P_{C,t}C_{i,j} + \omega \Gamma_v(v_{j,j})P_{C,t}C_{j,j} + P_{I,t}I_t \]  

\[ + \tau_t^C P_{C,t}C_t + (\tau_t^N + \tau_t^{i,i})W_t.N_t^D + \tau_t^K [u_{i,t}.R_{K,t} - \delta^i + \Gamma_u(u_{i,t})].P_{t,i}K_t \]  

\[ + \tau_t^D.D_t + T_t + (M_t - M_{t-1}) - TR_t - (B_t - R_{t-1}^i.B_{t-1}) \]  

\[ = W_t.N_t^D + u_{j,t}.R_{K,t} - \Gamma_u(u_{j,t}).P_{j,t}K_t + D_t \]  

\[ + S_t \left[ B_t^e - \left( [1 - \Gamma_{e^t}(B_{e^t})]R_{e^t} \right)^{-1} \right] B_{e^t} \]  

(A.130)

We can rewrite the government budget constraint as:

\[ P_{G,t}G_t - \tau_t^W W_t.N_t^D = \tau_t^C P_{C,t}C_t + (\tau_t^N + \tau_t^{i,i})W_t.N_t^D \]  

\[ + \tau_t^K [u_{i,t}.R_{K,t} - \delta^i + \Gamma_u(u_{i,t})].P_{t,i}K_t \]  

\[ + \tau_t^D.D_t + T_t + (M_t - M_{t-1}) - TR_t - (B_t - R_{t-1}^i.B_{t-1}) \]  

and plug it into households aggregate constraint, to obtain the economy’s aggregate budget constraint.
Substitution of supply and demand equilibrium conditions in final goods markets (A.125)-(A.127) into the equation above yields

\[ P_{c,t}C_t + (1 - \omega)\Gamma_{v} v_{i,t} C_{i,t} + \omega \Gamma_{v} v_{i,t} C_{i,t} + [P_{i,t} I_t + P_{g,t} G_t] = (1 + \tau^w r) W_i N_{i,t}^D + u_{i,t} R_{K, i} K_{i} + D_i + S_i \left\{ B^F_t - \left[ (1 - \Gamma_{B} (B^F_{i,t}^{-1})) R_{F, i} \right]^{-1} B^F_{i,t+1} \right\} \]

Aggregating (A.20) and (A.21) across firms results in

\[ u_{i,t} R_{K, i} K_{i} = \alpha M C_i (Y_i + \psi) \]  
\[ (1 + \tau^w r) W_i N_{i,t}^D = (1 - \alpha) M C_i (Y_i + \psi) \]

Plugging (A.134) and (A.135) into the equilibrium condition

\[ D_i + M C_i (Y_i + \psi) = P_{Y, i} Y_i \]

The equation above, coupled with the trade balance financing equation

\[ S_i \left\{ B^F_t - \left[ (1 - \Gamma_{B} (B^F_{i,t}^{-1})) R_{F, i} \right]^{-1} B^F_{i,t+1} \right\} = P_{i,m, i} M I_t - S_i P_{X, i} X_t \]

results in the economy’s resource constraint

\[ P_{Y, i} Y_t = P_{c, t} Q_t^C + P_{i, t} Q_t^I + P_{g, t} Q_t^G + S_i P_{X, i} X_t - P_{i, m, i} M I_t \]

Consider the demand for domestic and intermediate goods to produce final consumption goods (equations (A.69) and (A.70)). Multiplying the first by \( P_{H, i} \) and the latter by \( P_{i, m, i} / \Gamma_{IM}^C \), and adding them up yields

\[ P_{H, i} H_t^C + P_{i, m, i} M I_t^C = \left\{ v_c \left( \frac{P_{H, i}}{P_c} \right) \right\}^{\mu_C} + \left( 1 - v_c \right) \left( \frac{\Gamma_{M}^{IM} P_{i, m, i} / \Gamma_{C}^{IM} P_c}{1 - \Gamma_{IM}^{C} (IM_t^C / Q_t^C)} \right) \left( \frac{P_{i,m,i}}{\Gamma_{IM}^C} \right) \]

\[ P_{H, i} H_t^C + P_{i, m, i} M I_t^C = \left\{ v_c \left( \frac{P_{H, i}}{P_c} \right) \right\}^{\mu_C} + \left( 1 - v_c \right) \left( \frac{\Gamma_{M}^{IM} P_{i, m, i} / \Gamma_{C}^{IM} P_c}{1 - \Gamma_{IM}^{C} (IM_t^C / Q_t^C)} \right) \left( \frac{P_{i,m,i}}{\Gamma_{IM}^C} \right) \]

From the definition of \( \Omega_{Ct} \) in (A.67)
\[
\Omega_{c,t} = \left\{ \nu_t \cdot (P_{H,t})^{1-\mu_C} + (1 - \nu_t) \cdot \left( \frac{\Gamma^F (I^C_t / Q^C_t)}{1 - \Gamma (I^C_t / Q^C_t)} \right) \cdot \left( \frac{P_{IM,t}}{\Gamma^F (I^C_t / Q^C_t)} \right)^{1-\mu_C} \right\}^{1-\mu_C}
\]

we obtain:

\[
P_{H,t} H^C_t + P_{IM,t} IM^C_t = \left\{ \nu_t \cdot \left( P_{H,t} \right)^{1-\mu_C} + (1 - \nu_t) \cdot \left( \frac{\Gamma^F (I^C_t / Q^C_t)}{1 - \Gamma (I^C_t / Q^C_t)} \right) \cdot \left( \frac{P_{IM,t}}{\Gamma^F (I^C_t / Q^C_t)} \right)^{1-\mu_C} \right\} \cdot \left( \frac{1}{\Omega_{c,t}} \right)^{1-\mu_C} \cdot P_{c,t}
\]

and thus

\[
P_{c,t} Q^C_t = P_{H,t} H^C_t + P_{IM,t} IM^C_t
\]

and the consumption price index is

\[
P_{c,t} = \Omega_{c,t}^{1-\mu} \cdot \left\{ \nu_t \cdot (P_{H,t})^{1-\mu_c} + (1 - \nu_t) \cdot \left( \frac{P_{IM,t}}{\Gamma^F (I^C_t / Q^C_t)} \right)^{1-\mu_c} \right\}^{1-\mu_c}
\]

We can obtain a similar expression for the expenditures with investment goods. Considering the equations below

\[
H^I_t = \nu_t \left( P_{I,t} \right)^{1-\mu_I} \cdot \left( \frac{P_{H,t}}{P_{I,t}} \right)^{-\mu_I} \cdot Q^I_t
\]

\[
IM^I_t = (1 - \nu_t) \left( P_{I,t} \right)^{1-\mu_I} \cdot \left( \frac{P_{IM,t}}{P_{I,t}} \right)^{-\mu_I} \cdot \left( \frac{Q^I_t}{1 - \Gamma (I^I_t / Q^I_t)} \right)
\]

\[
P_{c,t} = \Omega_{I,t}^{1-\mu} \cdot \left\{ \nu_t \cdot (P_{H,t})^{1-\mu_I} + (1 - \nu_t) \cdot \left( \frac{P_{IM,t}}{\Gamma^F (I^I_t / Q^I_t)} \right)^{1-\mu_I} \right\}^{1-\mu_I}
\]

\[
\Omega_{I,t} = \left\{ \nu_t \cdot (P_{H,t})^{1-\mu_I} + (1 - \nu_t) \cdot \left( \frac{\Gamma^F (I^I_t / Q^I_t)}{1 - \Gamma (I^I_t / Q^I_t)} \right) \cdot \left( \frac{P_{IM,t}}{\Gamma^F (I^I_t / Q^I_t)} \right)^{1-\mu_I} \right\}^{1-\mu_I}
\]

we obtain

\[
P_{I,t} Q^I_t = P_{H,t} H^I_t + P_{IM,t} IM^I_t
\]

For government final goods, we use (A.78) and (A.79) to obtain

\[
P_{G,t} Q^G_t = P_{H,t} H^G_t .
\]

Substituting these results into the aggregate budget constraint of the economy yields the resource constraint of the economy

\[
P_{Y,t} Y_t = \left( P_{H,t} H^C_t + P_{IM,t} IM^C_t + P_{H,t} H^I_t + P_{IM,t} IM^I_t + P_{H,t} H^G_t + S_t P_{X,t} X_t - P_{IM,t} IM_t \right)
\]

As aggregate demand for domestic and imported intermediate goods are

\[
H_t := H^C_t + H^I_t + H^G_t
\]
Substituting into (A.139) and rearranging terms yields
\[ P_{Y,t}Y_t := P_{H,t}H_t + S_t,P_{X,t}X_t \tag{A.140} \]

Market clearing requires
\[ P_{Y,t}Y_t = P_{H,t}H_t^C + P_{H,t}H_t^I + P_{H,t}H_t^G + S_t,P_{X,t}X_t \tag{A.141} \]

International bond markets are in equilibrium when
\[ s_zB_{t+1}^F + (1-s_z)B_{t+1}^{F'} = 0 \tag{A.142} \]

and the balance of payments fulfills
\[
\left\{ \left( 1 - \Gamma B^F \left( \frac{S_{t-1},R_{t-1}^{-1},B_{t-1}^F}{P_{Y,t-1}Y_{t-1}} \right) \right) \right\}^{-1} B_{t+1}^F = B_t^F + \frac{TB_t}{S_t} \tag{A.143} 
\]
where the trade balance is defined as
\[ TB_t := S_t,P_{X,t}X_t - P_{IM,t}IM_t \tag{A.144} \]

Domestic terms of trade are defined as \( ToT_t = \frac{P_{IM,t}}{S_t,P_{X,t}} \). Contingent bonds add to zero:
\[ \int_{0}^{1-\phi} \Phi_{i,t} \, di = 0 \tag{A.145} \]
and so do individual rebates:
\[ \int_{0}^{1-\phi} \Xi_{i,t} \, di = 0 \tag{A.145} \]
B. Cost functions

We describe below the functional form for each of the cost functions in the paper.

Consumption transactions cost:
\[
\Gamma_t(v_{h,t}) := \gamma_{v,1} \cdot v_{h,t} + \gamma_{v,2} \cdot v_{h,t}^{-1} - 2\sqrt{\gamma_{v,1} \cdot \gamma_{v,2}} \tag{B.1}
\]

Cost on the transaction with international bonds:
\[
\Gamma_{B}^r(B_{t,t}^F) := \gamma_{h}^r \left( \exp \left( \frac{S_t(1-\omega)B_{t,t}^F}{P_{Y,t}Y_t} \right) - 1 \right) \tag{B.2}
\]

Cost on the utilization of capital:
\[
\Gamma_s(u_{t,i}) := \gamma_{u,1} (u_{t,i} - 1) + \frac{\gamma_{u,2}}{2} (u_{t,i} - 1)^2 \tag{B.3}
\]

Cost on the adjustment of the level of investment:
\[
\Gamma_i \left( \frac{I_{t,i}}{I_{t,i-1}} \right) := \gamma_i \left( \frac{I_{t,i}}{I_{t,i-1}} - g_Y \right)^2 \tag{B.4}
\]

Cost on the adjustment of the import share in the production of final consumption goods:
\[
\Gamma_{IM^C} \left( \frac{IM_t^C}{Q_t^C} \right) := \gamma_{IM^C} \left( \frac{IM_t^C}{IM_{t-1}^C} \frac{Q_{t-1}^C}{Q_t^C} - 1 \right)^2 \tag{B.5}
\]

Cost on the adjustment of the import share in the production of investment goods:
\[
\Gamma_{IM^I} \left( \frac{IM_t^I}{Q_t^I} \right) := \gamma_{IM^I} \left( \frac{IM_t^I}{IM_{t-1}^I} \frac{Q_{t-1}^I}{Q_t^I} - 1 \right)^2 \tag{B.6}
\]

\[
\Gamma_{IM^C}^3 \left( \frac{IM_t^C}{Q_t^C} \right) := 1 - \Gamma_{IM^C} \left( \frac{IM_t^C}{Q_t^C} \right) - \Gamma_{IM^C} \left( \frac{IM_t^C}{Q_t^C} \right) \left( \frac{IM_t^C}{Q_t^C} \right) \tag{B.7}
\]

\[
\Gamma_{IM^I}^3 \left( \frac{IM_t^I}{Q_t^I} \right) := 1 - \Gamma_{IM^I} \left( \frac{IM_t^I}{Q_t^I} \right) - \Gamma_{IM^I} \left( \frac{IM_t^I}{Q_t^I} \right) \left( \frac{IM_t^I}{Q_t^I} \right) \tag{B.8}
\]
C. Derivation of the recursive form for wage setting

The first order condition in wage setting is

\[ E_i \left[ \sum_{k=0}^{\infty} \left( \xi_{i,t} \beta \right) \cdot N_{i,t+k} \left\{ \Lambda_{i,t+k} \left( 1 - \tau_{i,t+k}^N - \tau_{i,t+k}^W \right) \tilde{W}_{i,j} \left( \frac{P_{C,i,t+k-1}}{P_{C,i,t}} \right)^{\chi_{i}} \frac{\pi_{C}^{(1-\chi_{i})}}{\eta_{i} - 1 \left( N_{i,t+k} \right)^{\zeta}} \right\} \right] = 0 \]  

\[ \text{(C.1)} \]

where \( N_{i,t+k} = \frac{1}{1 - \omega} \left( \frac{\tilde{W}_{i,j} \left( \frac{P_{C,i,t+k-1}}{P_{C,i,t}} \right)^{\chi_{i}} \frac{\pi_{C}^{(1-\chi_{i})}}{\eta_{i}}}{W_{i,j}} \right)^{\gamma_{i}} . N_{i} \)

We will show that the recursive formula below (C.2) is equivalent to the first order condition in (C.1):

\[ (1 - \omega)^{\zeta} \left( \frac{\tilde{W}_{i,j}}{P_{C,j}} \right)^{1+\eta_{i} / \zeta} = \frac{\eta_{i} F_{i,t}}{\eta_{i} - 1 G_{i,t}} \]  

\[ \text{(C.2)} \]

where

\[ F_{i,t} := \left( \frac{W_{i,t}}{P_{C,j}} \right)^{\eta_{i}} N_{t} + \tilde{\xi}_{i} \cdot \beta E_{i} \left( \frac{\pi_{C,i,t+1}}{\pi_{C,t}^{1-\chi_{i}}} \right)^{\eta_{i} \cdot (1+\zeta)} \left( \frac{W_{i,t+1}}{P_{C,i,t+1}} \right)^{\eta_{i}} . F_{i,t+1} \right\} ; \]

\[ G_{i,t} := \Lambda_{i,j} \left( 1 - \tau_{i}^N - \tau_{i}^W \right) \left( \frac{W_{i,t}}{P_{C,j}} \right)^{\eta_{i}} N_{i} + \tilde{\xi}_{i} \cdot \beta E_{i} \left( \frac{\pi_{C,i,t+1}}{\pi_{C,t}^{1-\chi_{i}}} \right)^{\eta_{i} \cdot (1-1)} . G_{i,t+1} \right\} ; \]

Notice that \( F_{i,t} \) can be rewritten as

\[ F_{i,t} := \left( \frac{W_{i,t}}{P_{C,j+k}} \right)^{\eta_{i}} N_{t}^{1+\zeta} + \tilde{\xi}_{i} \cdot \beta E_{i} \left( \frac{\pi_{C,i,t+1}}{\pi_{C,t}^{1-\chi_{i}}} \right)^{\eta_{i} \cdot (1+\zeta)} \left( \frac{W_{i,t+1}}{P_{C,i,t+1}} \right)^{\eta_{i}} N_{t+1}^{1+\zeta} \]

\[ + \tilde{\xi}_{i} \cdot \beta E_{i+1} \left( \frac{\pi_{C,i,t+1}}{\pi_{C,t}^{1-\chi_{i}}} \right)^{\eta_{i} \cdot (1+\zeta)} \left( \frac{\pi_{C,i,t+1}}{\pi_{C,i,t+1}^{1-\chi_{i}}} \right)^{\eta_{i} \cdot (1+\zeta)} F_{i,t+2} \]

and thus
Assuming the transversality conditions

\[ F_{i,k} := E_i \left( \sum_{k=0}^{\infty} \xi_k \beta^k \left( \frac{P_{C,k+1}}{P_{C,k} \pi_{C}^{k(1-x_i)}} \right)^{\eta_i(1+\xi)} \sum_{j=0}^{\infty} \xi_j \beta^j \left( \frac{W_{i,k}}{P_{C,k} \pi_{C}^{j(1-x_i)}} \right)^{\eta_j(1+\xi)} \right) \]

we obtain

\[ F_{i,k} := E_i \left( \sum_{k=0}^{\infty} \xi_k \beta^k \left( \frac{P_{C,k+1}}{P_{C,k} \pi_{C}^{k(1-x_i)}} \right)^{\eta_i(1+\xi)} \sum_{j=0}^{\infty} \xi_j \beta^j \left( \frac{W_{i,k}}{P_{C,k} \pi_{C}^{j(1-x_i)}} \right)^{\eta_j(1+\xi)} \right) \]

and substituting it into the recursive formula (C.2), we obtain:

\[ E_i \left( \sum_{k=0}^{\infty} \xi_k \beta^k \left( \frac{P_{C,k+1}}{P_{C,k} \pi_{C}^{k(1-x_i)}} \right)^{\eta_i(1+\xi)} \sum_{j=0}^{\infty} \xi_j \beta^j \left( \frac{W_{i,k}}{P_{C,k} \pi_{C}^{j(1-x_i)}} \right)^{\eta_j(1+\xi)} \right) \]

\[ \Lambda_{i,k+1} \left( 1 - \tau_{i,k}^{N} - \tau_{i,k}^{W} \right) N_{i,k+1} \]

\[ = \eta_i \left( \sum_{k=0}^{\infty} \xi_k \beta^k \left( \frac{P_{C,k+1}}{P_{C,k} \pi_{C}^{k(1-x_i)}} \right)^{\eta_i(1+\xi)} \sum_{j=0}^{\infty} \xi_j \beta^j \left( \frac{W_{i,k}}{P_{C,k} \pi_{C}^{j(1-x_i)}} \right)^{\eta_j(1+\xi)} \right) \]

\[ \left( \frac{W_{i,k}}{P_{C,k}} \right)^{\eta_i(1+\xi)} \left( \frac{W_{i,k}}{P_{C,k}} \right)^{\eta_j(1+\xi)} \left( \frac{W_{i,k}}{P_{C,k}} \right)^{\eta_j(1+\xi)} \}

\[ \left( N_{i,k+1} \right)^{\eta_i(1+\xi)} \]
After some algebraic manipulation, we obtain

\[
E_t \sum_{k=0}^\infty (\xi_t, \beta)^k \left\{ \frac{\lambda_{j,zk}(1-\tau_{r_{zk}} - \tau_{w_{zk}})}{h_k} \left( \frac{P_{C,zk}/P_{e,k}}{(P_{C,zk-1}/P_{e,k})^{\gamma_j}} \right)^{t_j-1} \left( \frac{W_{I_{zk}}}{P_{C,j}} \right) \eta_j \left( \frac{\tilde{W}_{I_{zk}}}{P_{C,j}} \right)^{\frac{\gamma_j}{1 - \omega}} \left( \frac{1}{1 - \omega} \right) \right\} = 0
\]

Multiplying by the positive constant

\[
E_t \sum_{k=0}^\infty (\xi_t, \beta)^k \left\{ \frac{\lambda_{j,zk}(1-\tau_{r_{zk}} - \tau_{w_{zk}})}{h_k} \left( \frac{P_{C,zk}/P_{e,k}}{(P_{C,zk-1}/P_{e,k})^{\gamma_j}} \right)^{t_j-1} \left( \frac{W_{I_{zk}}}{P_{C,j}} \right) \eta_j \left( \frac{\tilde{W}_{I_{zk}}}{P_{C,j}} \right)^{\frac{\gamma_j}{1 - \omega}} \left( \frac{1}{1 - \omega} \right) \right\} = 0
\]

After some algebraic manipulation, we obtain

\[
E_t \sum_{k=0}^\infty (\xi_t, \beta)^k \left\{ \frac{\lambda_{j,zk}(1-\tau_{r_{zk}} - \tau_{w_{zk}})}{h_k} \left( \frac{P_{C,zk}/P_{e,k}}{(P_{C,zk-1}/P_{e,k})^{\gamma_j}} \right)^{t_j-1} \left( \frac{W_{I_{zk}}}{P_{C,j}} \right) \eta_j \left( \frac{\tilde{W}_{I_{zk}}}{P_{C,j}} \right)^{\frac{\gamma_j}{1 - \omega}} \left( \frac{1}{1 - \omega} \right) \right\} = 0
\]

which yields the first order condition

\[
E_t \sum_{k=0}^\infty (\xi_t, \beta)^k \left\{ \frac{\lambda_{j,zk}(1-\tau_{r_{zk}} - \tau_{w_{zk}})}{h_k} \left( \frac{P_{C,zk}/P_{e,k}}{(P_{C,zk-1}/P_{e,k})^{\gamma_j}} \right)^{t_j-1} \left( \frac{W_{I_{zk}}}{P_{C,j}} \right) \eta_j \left( \frac{\tilde{W}_{I_{zk}}}{P_{C,j}} \right)^{\frac{\gamma_j}{1 - \omega}} \left( \frac{1}{1 - \omega} \right) \right\} = 0
\]
D. Derivation of the recursive form for the price setting rule

The first order condition for the export prices is analogous to the one for intermediate goods:

\[
E_i \left[ \sum_{t=1}^{\infty} (\xi_t H_t) \Lambda_{i,s,t+k} \left( \frac{P_{H,t+k-1}}{P_{H,t}} \right)^{x_u} \left( \frac{\pi_{H,t}}{\pi_{H,t-1}} \right)^{1-x_u} \left( \frac{\theta}{\theta-1} \right)^{x_u} \right] = 0 \tag{D.1}
\]

where

\[
\Lambda_{i,s,t+k} = \frac{1}{1-\omega} \sum_{i=0}^{\infty} \left[ \frac{P_{C,i}}{P_{C,i+s}} \right] \text{ and}
\]

\[
H_{i,s,t+k} = \left( \frac{P_{H,i,s,t+k}}{P_{H,i,s,t+k-1}} \right)^{x_u} \left( \frac{\pi_{H,t}}{\pi_{H,t-1}} \right)^{1-x_u} \left( \frac{\theta}{\theta-1} \right)^{x_u} .
\]

Consider the recursive formula below:

\[
\tilde{P}_{H,i} = \frac{\theta}{\theta-1} F_{H,i} G_{H,i} \tag{D.2}
\]

where

\[
F_{H,i} \equiv MC_i, H_i + \xi_t H_i \beta E_i \left\{ \frac{\Lambda_{i,s,t+1}}{\Lambda_{i,t}} \left( \frac{\pi_{H,t+1}}{\pi_{H,t}} \right)^{x_u} \right\} ;
\]

\[
G_{H,i} \equiv P_{H,i}, H_i + \xi_t H_i \beta E_i \left\{ \frac{\Lambda_{i,s,t+1}}{\Lambda_{i,t}} \left( \frac{\pi_{H,t+1}}{\pi_{H,t}} \right)^{x_u} \right\} ;
\]

We will show below that the recursive formula in (D.2) is equivalent to the first order condition in (D.1). Solving \( F_{H,i} \) recursively, we obtain:

\[
F_{H,i} \equiv MC_i, H_i + \xi_t H_i \beta E_i \left\{ \frac{\Lambda_{i,s,t+1}}{\Lambda_{i,t}} \left( \frac{\pi_{H,t+1}}{\pi_{H,t}} \right)^{x_u} \right\} .
\]

Assuming the transversality conditions below:
We obtain

\[
\lim_{k \to \infty} E \left\{ \frac{g_k}{s^k} \beta^k \cdot \frac{\Lambda_{i,t+k}}{\Lambda_{i,t+k-1}} \cdot \left( \frac{P_{H,2+t}}{P_{H,2+t-1}} \right)^{\chi \lambda_{i,t+k}^{1-X_{H,i}^{k}}} \right\} = 0
\]

and

\[
\lim_{k \to \infty} E \left\{ \frac{g_k}{s^k} \beta^k \cdot \frac{\Lambda_{i,t+k}}{\Lambda_{i,t+k-1}} \cdot \left( \frac{P_{H,2+t}}{P_{H,2+t-1}} \right)^{\theta-1} \cdot \left( \frac{P_{H,2+t}}{P_{H,2+t-1}} \right)^{\chi \lambda_{i,t+k}^{1-X_{H,i}^{k}}} \right\} = 0
\]

we obtain

\[
F_{H,t} := E \left\{ \sum_{k=0}^{\infty} \frac{\tilde{g}_k}{s^k} \beta^k \cdot \frac{\Lambda_{i,t+k}}{\Lambda_{i,t}} \cdot \left( \frac{P_{H,2+t}}{P_{H,2+t-1}} \right)^{\chi \lambda_{i,t+k}^{1-X_{H,i}^{k}}} \cdot P_{H,2} \cdot \frac{\Lambda_{i,t+k}}{\Lambda_{i,t+k-1}} \cdot \left( \frac{P_{H,2+t}}{P_{H,2+t-1}} \right)^{\chi \lambda_{i,t+k}^{1-X_{H,i}^{k}}} \right\} H_{i,t+k} = 0
\]

As \( G_{H,t} \cdot \tilde{p}_{H,t} = \frac{\theta}{\theta-1} F_{H,t} \), then

\[
E \left\{ \sum_{k=0}^{\infty} \frac{\tilde{g}_k}{s^k} \beta^k \cdot \frac{\Lambda_{i,t+k}}{\Lambda_{i,t}} \cdot \left( \frac{P_{H,2+t}}{P_{H,2+t-1}} \right)^{\chi \lambda_{i,t+k}^{1-X_{H,i}^{k}}} \cdot \tilde{p}_{H,t} \cdot \left( \frac{P_{H,2+t}}{P_{H,2+t-1}} \right)^{\chi \lambda_{i,t+k}^{1-X_{H,i}^{k}}} \cdot \frac{\Lambda_{i,t+k}}{\Lambda_{i,t+k-1}} \cdot \left( \frac{P_{H,2+t}}{P_{H,2+t-1}} \right)^{\chi \lambda_{i,t+k}^{1-X_{H,i}^{k}}} \cdot \frac{\Lambda_{i,t+k}}{\Lambda_{i,t+k-1}} \cdot \left( \frac{P_{H,2+t}}{P_{H,2+t-1}} \right)^{\chi \lambda_{i,t+k}^{1-X_{H,i}^{k}}} \right\} H_{i,t+k} = 0
\]

Multiplying both sides by \( \left( \frac{\tilde{p}_{H,t}}{P_{H,t}} \right)^{\theta} \neq 0 \), we obtain

\[
E \left\{ \sum_{k=0}^{\infty} \frac{\tilde{g}_k}{s^k} \beta^k \cdot \frac{\Lambda_{i,t+k}}{\Lambda_{i,t}} \cdot \tilde{p}_{H,t} \cdot \left( \frac{P_{H,2+t}}{P_{H,2+t-1}} \right)^{\chi \lambda_{i,t+k}^{1-X_{H,i}^{k}}} \cdot \frac{\Lambda_{i,t+k}}{\Lambda_{i,t+k-1}} \cdot \left( \frac{P_{H,2+t}}{P_{H,2+t-1}} \right)^{\chi \lambda_{i,t+k}^{1-X_{H,i}^{k}}} \right\} H_{i,t+k} = 0
\]

To obtain the equivalence of the recursive form to the first order condition, we need to have

\[
\beta^k \cdot \frac{\Lambda_{i,t+k}}{\Lambda_{i,t}} = \lambda_{i,t+k} = \frac{1}{1-\omega} \int_{0}^{1} \beta^k \cdot \frac{\Lambda_{i,t+k}}{\Lambda_{i,t}} \cdot \frac{P_{i,t}}{P_{i,t+k}} \cdot di
\]
In particular, when consumption decisions are the same across households within group $I$, we have:

$$\Lambda^{I,t+k}_{t+k} = \frac{\Lambda_{I,t+k}}{P_{C,t+k}}$$

E. Derivation of the price indices for final goods

The consumption price index that results from solving the problem in (39) is not the one CMS obtain. The corresponding Lagrange problem is

$$\min_{H_i^c, IM^c_t, \lambda_i} P_{H,i} H_i^c + P_{IM,i} IM^c_t$$

$$+ \lambda_i^c \left\{ \left( \frac{(V_C)^{1/\mu_c}}{H_i^c} \right)^{1/\mu_c} + \left( (1-V_C)^{1/\mu_c} \left( 1 - \Gamma_{it} (IM^c_t/Q_i^c) \right) \right)^{1/\mu_c} \right\}$$

and the first order conditions associated with the choice of $H_i^c$ yields:

$$H_i^c = V_C \left( \frac{P_{H,i}}{\lambda_i^c} \right)^{-\mu_c} Q_i^c$$

which is the demand for intermediate domestic goods for the production of consumption goods. Multiplying this by $P_{H,i}$ yields nominal costs with intermediate domestic goods

$$P_{H,i} H_i^c = V_C \left( \frac{P_{H,i}}{\lambda_i^c} \right)^{-\mu_c} \lambda_i^c Q_i^c$$

(E.3)

The first order condition of the Lagrangean problem with respect to $IM^c_t$ yields demand for imported intermediate goods to produce final consumption goods:

$$IM^c_t = (1-V_C) \left( \frac{P_{IM,i} \Gamma^{-3}_{IM^c} (IM^c_t/Q_i^c)}{\lambda_i^c} \right)^{-\mu_c} Q_i^c$$

(E.4)

where $\Gamma^{-3}_{IM^c}$ is detailed in appendix B.

Multiplying (E.4) by $P_{IM,i}$ yields the nominal cost to use imported intermediate goods

$$P_{IM,i} IM^c_t = (1-V_C) \left( \frac{P_{IM,i} \Gamma^{-3}_{IM^c} (IM^c_t/Q_i^c)}{\lambda_i^c} \right)^{-\mu_c} \left( \frac{\Gamma^{-3}_{IM^c} (IM^c_t/Q_i^c)}{1 - \Gamma_{it} (IM^c_t/Q_i^c)} \right) \lambda_i^c Q_i^c$$

(E.5)
The first order condition to the Lagrangean problem associated with the choice of the Lagrange multiplier $\lambda^C_t$ is

$$\lambda^C_t = \left[ v_c P_{H,t}^{1-\mu_c} + (1-v_c) \left( \frac{P_{IM,t}}{\Gamma^3 IM^C (IM^C_t/Q^C_t)} \right)^{1-\mu_c} \right]^{-\frac{1}{1-\mu_c}}$$

(E.6)

In CMS, this multiplier is assumed to be the price index for one unit of the consumption good. However, this result is not compatible with their assumption that final goods firms operate with zero profits, as we show next.

To see that $\lambda^C_t$ is not a price index in this context, first notice that the nominal cost of inputs to the final goods firm can be expressed as

$$P_{H,t} H^C_t + P_{IM,t} IM^C_t$$

Substituting $\lambda^C_t$ in the expression above, using (45), results in the optimal cost being a function of prices and the proportion of imports to total production, $IM^C_t/Q^C_t$:

$$P_{H,t} H^C_t + P_{IM,t} IM^C_t$$

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Substituting $\lambda^C_t$ in the expression above, using (45), results in the optimal cost being a function of prices and the proportion of imports to total production, $IM^C_t/Q^C_t$:

$$P_{H,t} H^C_t + P_{IM,t} IM^C_t$$

(E.7)

If final goods firms yield zero profits, we can define the corresponding price index for one unit of final good as $P_{C,t} = \frac{P_{H,t} H^C_t + P_{IM,t} IM^C_t}{Q^C_t}$. Defining the variable $\Omega^C_t$ as

$$\Omega^C_t = \left[ v_c \left( P_{H,t} \right)^{1-\mu_c} + (1-v_c) \left( \frac{P_{IM,t}}{\Gamma^3 IM^C (IM^C_t/Q^C_t)} \right)^{1-\mu_c} \right]^{-\frac{1}{1-\mu_c}}$$

(E.8)

we obtain from (E.5) – (E.7) that the correct price index in this framework is
\[ P_{c,t} = \left( \Omega_t^c \right)^{\mu_c} \left( \lambda_t^c \right)^{\nu_c} \]  
(E.9)  

Only when \( \Omega_t^c = \lambda_t^c \) do we obtain \( P_{c,t} = \lambda_t^c = \Omega_t^c \). This requires  
\[
\left( \frac{\Gamma^3_{IM} (IM_t^c/Q_t^c)}{1 - \Gamma^3_{IM} (IM_t^c/Q_t^c)} \right) = 1, \text{ a very specific case.}
\]

In general, when this equality does not hold, the demand equations, as a function of the price index (using equations (E.2), (E.4) and (E.9)), should be  
\[
H_t^c = V_c \left( \frac{P_{H,t}}{\Omega_t^c} \right)^{-\mu_c} \left( \frac{P_{H,t}}{P_{c,t}} \right)^{-\mu_c} Q_t^c
\]  
(E.10)  
\[
IM_t^c = (1 - V_c) \left( \frac{P_{c,t}}{\Omega_t^c} \right)^{-\mu_c} \left( \frac{P_{IM,t} \Gamma^3_{IM} (IM_t^c/Q_t^c)}{P_{c,t}} \right)^{-\mu_c} \frac{Q_t^c}{1 - \Gamma^3_{IM} (IM_t^c/Q_t^c)}
\]  
(E.11)  

F. Derivation of the wage distortion index

Consider the set \( H = [0,1] \) representing the households in the economy. This set is divided into two disjoint groups, \( I \) and \( J \), i.e., \( H = I \cup J \), where \( \omega \in [0,1] \) represents the relative amount of members of group \( J \) over the total amount of households in \( H \). Therefore, \( \omega = \int_I dh \) and \( 1 - \omega = \int_J dh \).

At every time \( t \) a (Calvo) lottery occurs to decide which households will re-optimize their wage decisions. We can thus fix the set \( V_t := \{ h \in H : h \text{ does not optimize her wage at time } t \} \), and its complementary set of optimizing households \( V_t^c \). Should each household \( i \in I \) have a probability \( \xi_i \in [0,1] \) of not optimizing, we obtain \( \xi_i(1 - \omega) = \int_{I \setminus V_t} dh \) and \( (1 - \xi_i)(1 - \omega) = \int_{I \setminus V_t^c} dh \), for every \( t \).

Assume that each household \( i \in I \) sets its wage \( W_{i,t} \) according to this lottery, where \( \tilde{W}_{i,t} \) is the optimized wage and \( \bar{W}_{i,t} \) is the non-optimized wage. In particular, the model implies that all households that optimize do so identically and choose the same optimal wage \( \tilde{W}_{i,t} \).

Furthermore, assume that when a household does not optimize, it readjusts its wage using a geometric average of past inflation and the inflation target. In other words,  
\[
\tilde{W}_{i,t} = \tilde{W}_{i,t} \text{ and } \bar{W}_{i,t} = \pi_{c,t}^{\chi_t} \pi_{t-1}^{\chi_t} \bar{W}_{i,t-1}, \text{ where } \chi_t \in [0,1] \text{ is a constant weight.}
Consider the wage dispersion index defined as $\psi_{t,i} := \frac{1}{1-\omega} \int \left( \frac{W_{i,t}}{W_{I,t}} \right)^{-\eta_i} \, d\omega$. To obtain a recursive representation of this index, we shall assume that the following equality holds

$$\frac{1}{\xi_i (1-\omega)} \int_{\tau \in \mathcal{N}_i} \left( \frac{W_{i,t}}{W_{I,t}} \right)^{-\eta_i} \, d\omega = \frac{1}{1-\omega} \int \left( \frac{W_{i,t-1}}{W_{I,t-1}} \right)^{-\eta_i} \, d\omega$$

In words, we assume that the wage dispersion, at time $t-1$, of households in group $I$ who do not optimize at time $t$ (left-hand side) is equal to the wage dispersion of all members of group $I$ at time $t-1$ (right-hand side). This is a very important and stringent assumption, which is implicit in the “Calvo scheme”.

Substitution of the equations above into the wage dispersion equation yields

$$\psi_{t,i} = \frac{1}{1-\omega} \int \left( \frac{W_{i,t}}{W_{I,t}} \right)^{-\eta_i} \, d\omega = \frac{1}{1-\omega} \int \left( \frac{W_{i,t}}{W_{I,t}} \right)^{-\eta_i} \, d\omega + \frac{1}{1-\omega} \int \left( \frac{W_{i,t-1}}{W_{I,t-1}} \right)^{-\eta_i} \, d\omega$$

$$= \frac{1}{1-\omega} \int \left( \frac{\tilde{W}_{i,t}}{W_{I,t}} \right)^{-\eta_i} \, d\omega + \frac{1}{1-\omega} \int \left( \frac{\tilde{W}_{i,t}}{W_{I,t}} \right)^{-\eta_i} \, d\omega$$

$$= \frac{1}{1-\omega} \int \left( \frac{\tilde{W}_{i,t}}{W_{I,t}} \right)^{-\eta_i} \, d\omega + \frac{1}{1-\omega} \int \left( \frac{\tilde{W}_{i,t-1}}{W_{I,t-1}} \right)^{-\eta_i} \, d\omega$$

This result can be restated recursively as:

$$\psi_{t,i} = (1-\xi_i) \left( \frac{\tilde{W}_{i,t}}{W_{I,t}} \right)^{-\eta_i} + \xi_i \left( \frac{\tilde{W}_{i,t-1}}{W_{I,t-1}} \right)^{-\eta_i} \psi_{t-1,i}$$

Analogous reasoning can be applied to obtain the corresponding recursive representation of the wage dispersion index for households in group $J$. 
G. Derivation of the aggregate resource constraint

To obtain the aggregate resource constraint of the economy, we use households and government budget constraints. Aggregating households’ budget constraints into the budget constraint for group \( I \) and \( J \), we obtain:

\[
P_{C,t}C_t + (1 - \omega)\Gamma_t(v_{t,t})P_{C,t}C_{t,t} + \omega\Gamma_t(v_{t,t})P_{C,t}C_{t,t} + P_{I,t}I_t + \tau^C_{t}P_{C,t}C_t + (\tau^W_{t} + \tau^V_{t})W_tN_t^D + \tau^K_tu_{t,t}R_{K,t} - \tau^K_t(\delta + \Gamma_t(u_{t,t}))P_{I,t}K_t
\]

\[
+ \tau^D_{t}D_t + T_t + (M - M_{t-1}) - TR_t - (B_t - R^{-1}B_{t+1})
\]

\[
= W_tN_t^D + u_{t,t}R_{K,t}K_t - \Gamma_t(u_{t,t})P_{I,t}K_t + D_t
\]

\[
+ S_t \left\{ B^F_t - \left[ (1 - \Gamma_t (F_{t+1}^F))R_{F,t} \right]^{-1}B^F_{t+1} \right\}
\]

We can rewrite the government budget constraint as:

\[
P_{G,t}G_t - \tau^W_tW_tN_t^D = \tau^C_tP_{C,t}C_t + (\tau^W_t + \tau^V_t)W_tN_t^D
\]

\[
+ \tau^K_tu_{t,t}R_{K,t} - (\delta + \Gamma_t(u_{t,t}))P_{I,t}K_t
\]

\[
+ \tau^D_{t}D_t + T_t + (M - M_{t-1}) - TR_t - (B_t - R^{-1}B_{t+1})
\]

and plug it into households aggregate constraint, to obtain the economy’s aggregate budget constraint

\[
P_{C,t}C_t + (1 - \omega)\Gamma_t(v_{t,t})C_{t,t} + \omega\Gamma_t(v_{t,t})C_{t,t} + [P_{I,t}I_t + \Gamma_t(u_{t,t})P_{I,t}K_t] + P_{G,t}G_t
\]

\[
= (1 + \tau^W_t)W_tN_t^D + u_{t,t}R_{K,t}K_t + D_t
\]

\[
+ S_t \left\{ B^F_t - \left[ (1 - \Gamma_t (F_{t+1}^F))R_{F,t} \right]^{-1}B^F_{t+1} \right\}
\]

Substitution of supply and demand equilibrium conditions in final goods markets (A.125)-(A.127) into the equation above yields

\[
P_{C,t}Q^C_t + P_{I,t}Q^I_t + P_{G,t}Q^G_t = (1 + \tau^W_t)W_tN_t^D + u_{t,t}R_{K,t}K_t + D_t
\]

\[
+ S_t \left\{ B^F_t - \left[ (1 - \Gamma_t (F_{t+1}^F))R_{F,t} \right]^{-1}B^F_{t+1} \right\}
\]

Aggregating (A.20) and (A.21) across firms results in

\[
u_{t,t}R_{K,t}K_t = \alpha.MC_t(Y_t + \psi)
\]

\[
(1 + \tau^W_t)W_tN_t^D = (1 - \alpha).MC_t(Y_t + \psi)
\]

Plugging (A.134) and (A.135) into the equilibrium condition

\[
D_t + MC_t(Y_t + \psi) = P_{F,t}Y_t
\]

yields

\[
(1 + \tau^W_t)W_tN_t^D + u_{t,t}R_{K,t}K_tMC_t(Y_t + \psi) + D_t = P_{F,t}Y_t
\]
The equation above, coupled with the trade balance financing equation
\[ S_t \left[ B_{t+1}^F - \left( \left[ 1 - \Gamma \right] B_{t+1}^F \right) R_{t+1} \right] ^{-1} B_{t+1}^F = P_{IM,t} IM_t - S_t P_{X,t} X_t \]
results in the economy’s resource constraint
\[
P_{Y,t} Y_t = P_{C,t} Q_t^C + \frac{I_t}{P_{i} / \bar{c}} + P_{G,t} Q_t^G + S_t P_{X,t} X_t - P_{IM,t} IM_t \quad (A.134)
\]

Consider the demand for domestic and intermediate goods to produce final consumption goods (equations (A.69) and (A.70)). Multiplying the first by \( \frac{H_t}{P_{i}} \), and the latter by \( \frac{PM_{IM,t}}{\Gamma_{IM,t}^3} \), and adding them up yields
\[
P_{H,t} H_t^C + P_{IM,t} IM_t^C = \left( \frac{P_{H,t}}{P_{C,t}} \right)^{\gamma_{PC}} \left( \frac{\Gamma_{IM,t}^3}{1 - \Gamma_{IM,t}^3} \left( \frac{P_{IM,t}}{\Gamma_{IM,t}^3} \right) \right)^{\gamma_{PC}} \left[ P_{C,t} Q_t^C \right]^\mu_{PC} \quad (A.135)
\]
\[
P_{H,t} H_t^C + P_{IM,t} IM_t^C = \left( \frac{P_{H,t}}{P_{C,t}} \right)^{\gamma_{PC}} \left( \frac{\Gamma_{IM,t}^3}{1 - \Gamma_{IM,t}^3} \left( \frac{P_{IM,t}}{\Gamma_{IM,t}^3} \right) \right)^{\gamma_{PC}} \left[ P_{C,t} Q_t^C \right]^\mu_{PC} \quad (A.136)
\]
From the definition of \( \Omega_{C,t} \) in (A.67) we obtain:
\[
P_{H,t} H_t^C + P_{IM,t} IM_t^C = \left( \frac{P_{H,t}}{P_{C,t}} \right)^{\gamma_{PC}} \left( \frac{\Gamma_{IM,t}^3}{1 - \Gamma_{IM,t}^3} \left( \frac{P_{IM,t}}{\Gamma_{IM,t}^3} \right) \right)^{\gamma_{PC}} \left[ P_{C,t} Q_t^C \right]^\mu_{PC} \quad (A.137)
\]
and thus
\[
P_{C,t} Q_t^C = P_{H,t} H_t^C + P_{IM,t} IM_t^C \quad (A.138)
\]
We can obtain a similar expression for the expenditures with investment goods.

Considering the equations below and the price indices
\[
H_t^I = V_t \left( \frac{P_{I,t}}{\Omega_{I,t}} \right)^{\gamma_{IP}} \left( \frac{P_{H,t}}{P_{I,t}} \right)^{-\gamma_{IP}} \cdot Q_t^I
\]
\[
IM_t^I = (1 - V_t) \left( \frac{P_{I,t}}{\Omega_{I,t}} \right)^{\gamma_{IP}} \left( \frac{P_{IM,t}}{P_{I,t}} \right)^{-\gamma_{IP}} \cdot \left( 1 - \Gamma_{IM,t} \right) \left( \frac{IM_t^I}{Q_t^I} \right) \cdot Q_t^I
\]
we obtain
\[
P_{C,t} Q_t^I = P_{H,t} H_t^I + P_{IM,t} IM_t^I
\]
For government final goods, we use (A.78) and (A.79) to obtain 
\[ p_{G,t} \cdot q_{G,t} = \pi_{H,t} \cdot H_{i}^{G}. \] Substituting these results into the aggregate budget constraint of the economy yields the resource constraint of the economy

\[ \pi_{Y,t} \cdot Y_{t} = \pi_{H,t} \cdot H_{i}^{C} + \pi_{IM,t} \cdot IM_{i}^{C} + \pi_{H,t} \cdot H_{i}^{I} + \pi_{IM,t} \cdot IM_{i}^{I} + \pi_{H,t} \cdot H_{i}^{G} + S_{t} \cdot p_{X,t} \cdot X_{t} - P_{IM,t} \cdot IM_{t}, \quad (A.139) \]

As aggregate demand for domestic and imported intermediate goods are

\[ H_{i} := H_{i}^{C} + H_{i}^{I} + H_{i}^{G} \]

\[ IM_{i} := IM_{i}^{C} + IM_{i}^{I} \]

Substituting into (A.139) and rearranging terms yields

\[ \pi_{Y,t} \cdot Y_{t} := \pi_{H,t} \cdot H_{i} + S_{t} \cdot p_{X,t} \cdot X_{t}, \quad (A.140) \]

Market clearing requires

\[ \pi_{Y,t} \cdot Y_{t} = \pi_{H,t} \cdot H_{i}^{C} + \pi_{H,t} \cdot H_{i}^{I} + \pi_{H,t} \cdot H_{i}^{G} + S_{t} \cdot p_{X,t} \cdot X_{t}, \quad (A.141) \]
### H. Table of parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A. Households</strong></td>
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<td><strong>B. Intermediate-good firms</strong></td>
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C. Final-good firms

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D. Fiscal authority
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<td><strong>E. Monetary Authority</strong></td>
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<tr>
<td>( \Pi )</td>
<td>1.04500</td>
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Notes
NAWM: *
Areosa, Areosa e Lago (2006): †
Silveira (2006): ‡