Nominal Portfolio Choice and Wealth Redistribution Effects of Inflation Surprises*

Tiago C. Berriel
Princeton University
tberriel@princeton.edu
www.princeton.edu/~tberriel

November 18, 2009

Abstract

Different ratios of nominal assets to total wealth imply that unexpected changes in price level impact relative financial wealth. Nominal asset transfers are, however, an incomplete account of the impact on total wealth, if shocks that change inflation also affect household’s current and future disposable income or the return of the real assets. In this paper, I measure the impact of unanticipated inflation on consumption, a proxy for total wealth, rather than on portfolio revaluation. I find that inflation surprise correlated positively with total wealth in the US since 1980. Moreover, the consumption adjustment is similar in magnitude for different ages and income, without relative winners and losers predicted by portfolio revaluation effects. This fact can be explained in a general equilibrium model with endogenous portfolio choice and different aggregate risks in agents’ disposable income. Analytically, I show that transfers in the asset markets can be a misleading measure of total wealth transfers, given the insurance nature of portfolio decisions. If working-age agents are more exposed to aggregate risks than agents above 65 years old, a calibrated OLG model matches the US data on both consumption impact and nominal asset positions over the life cycle. Moreover, the difference in nominal asset positions across income groups can be explained: on average the poor bear more business cycle risk on their uninsurable income than middle-class agents, while rich bear less.

*I thank without implicating Ricardo Reis and Chris Sims for invaluable advice, and Alan Blinder, Marco Cagetti, Carlos Carvalho, Gauti Egertson, Chris Erceg, Luca Guerrieri, Diogo Guillen, Oleg Itskhoki, Nobu Kyiotaki, Cecilia Machado, Sam Schulhofer-Wohl, Felipe Schwartzman, Satoru Shimizu, Antonella Tutino, Mark Watson, Adam Zawadowski; and conference seminars participants at the Federal Reserve Board, Princeton University macro workshop and Princeton Macro/International Macro Seminar for useful comments. I thank the Federal Reserve Board for the hospitality while part of this research was undertaken.
1 Introduction

Do unanticipated movements in the price level lead to wealth redistribution? This long-standing question in macroeconomics has received attention on both policy and academic circles. On the policy side, significant redistribution can lead to political support for inflation control.\(^1\) On the academic literature, relevant wealth redistribution of inflation would call for models that explicitly take redistribution into account when considering the welfare costs of inflation. Whether and how total wealth is redistributed in case of a surprise to inflation is, however, a non-trivial question. A thorough answer has to address the impact of the aggregate shocks that affect inflation on both disposable income and on portfolio valuation of the agents. This paper addresses this question for the US by quantifying the impact of inflation shocks on consumption of different groups of households, which is a proxy for total wealth.

Empirically, I find that inflation surprises correlate positively with consumption adjustments, even for agents suffering losses through nominal portfolio revaluations. Moreover, consumption adjustment is roughly similar for households with different characteristics such as age and income, despite the documented heterogeneity in nominal asset positions. I reconcile this evidence on consumption with the documented heterogeneity in nominal assets in a model with two key features: (i) heterogeneous aggregate risk on uninsurable income and (ii) optimal choice of nominal assets holdings as part of their portfolio decision.

There are two large branches of literature on the redistributive effects of inflation surprises: the first focus on the implications for income inequality and the second on the effects for portfolio revaluation.\(^2\) The first branch of the literature documented that in the sixties and seventies inflation worked as a progressive tax, decreasing income inequality. Blinder and Esaki (1977) and Brimmer (1971) point out that the lower quintiles of income distribution benefit from inflation surprises, while Bach and Stephenson (1974) show higher inflation came with an increase in labor income.

On the portfolio valuation analysis, Modigliani and Papademos (1978) and Fisher and Modigliani (1978) argue that inflation surprises would transfer resources from net nominal creditors (retired or old agents) to net nominal borrowers (young and active).\(^3\) More recently, Doepke and Schneider (2006a) have quantified the portfolio revaluation effects due to unanticipated movements in the price level. They report that different groups of households hold heterogeneous amounts of net nominal assets in the US: richer and older agents hold a larger fraction of their total wealth in nominal terms, while middle-class and younger hold relatively less (even negative) nominal positions. If the price level is the only variable unexpectedly moving up, returns on nominal assets would be relatively lower and would benefit the agents with less exposure to nominal assets.\(^4\) Importantly,

---

\(^1\) As quoted in Palmer (1973), Arthur Burns has asserted that: "there can be little doubt that poor people... are the chief surfer of inflation."

\(^2\) Fisher and Modigliani (1978) section 4 presents a review of the literature from the fifties to the late seventies.

\(^3\) Other studies explore the impact of inflation on disposable income through unequal tax indexation. Feldstein (1982), for example, points out that a nominal tax system might lead to an increase in the tax burden of real capital income, reducing the returns of savings on capital, with potential implication to wealth distribution.

\(^4\) They also take into account differences in duration of nominal exposure, making it theoretically possible for agents with larger nominal positions but shorter duration to lose less than other agents with smaller nominal positions and
rich and old are the relative losers and middle-class and young are the relative winners of a shock that affects only the price level.\footnote{Mei and Terajima (2008) perform a similar exercise for Canada.}

This paper brings this two branches of literature together by allowing that both agents' lifetime disposable income and the value of their portfolio to be correlated with inflation surprises. There are several reasons this might be the case: positive inflation surprises can be associated with economic expansions that increase agents' contemporaneous or expected income; or, since the government is a net nominal borrower, inflation reduces real government debt, which leads to lower future taxes. Moreover, portfolio decisions should reflect the correlation marginal utility of the agents (a function of their life-time disposable income) and the differential of returns of the assets. The choice of nominal assets in the portfolio is no exception. Therefore, I approach this questions in three ways: (i) empirically, computing the impact of inflation surprises in total wealth; (ii) in a simple model, where I show analytically that endogenous portfolio choice can explain the different pattern of both portfolio valuation and the total wealth given an inflation surprise; (iii) in a calibrated OLG model, where I show that an arguably reasonable heterogeneity in aggregate risks on uninsurable income and complementarity between labor and consumption allow the model to match the US data for different age and income groups on two dimensions: on the average nominal asset holdings and on the impact of inflation surprises on consumption.

In order to go beyond financial market transfers, this paper drops the assumption on the recent literature on portfolio valuation effects that a shock to the price level does not impact other variables. I consider, instead, historical unpredictable movements in inflation in the US. This is crucial if one is interested on redistributive effects of realistic inflation surprises episodes, like the one in the early eighties and the recent deflation of 2008. By observing these events, it is natural to believe that high inflationary (or deflationary) episodes were characterized by shocks that did not affect exclusively the price level. The structural shocks in action impact also other aggregate variables such as output, government policies like current or future taxation and the return of real assets in the economy. Moreover, inflation surprises might affect agents differently, depending on their position on the life-cycle or on their human capital. Younger agents might face more uncertain future outcomes in a recession, while high-income agents might foresee relatively larger future taxes hikes in a deflationary recession. Castaneda et al (1997), Gomme et al (2004) and Jaimovich and Siu (2006) are examples that document heterogeneity in labor income fluctuations in business cycles frequencies.

Instead of financial asset positions, I focus on the impact of agents' total life-time wealth. If the decision on nominal portfolio holdings is endogenous, wealth transfers in the asset market can be misleading. As an example, consider a complete market economy where agents’ labor income correlate heterogeneously with shocks that affect the price level. Agents will optimally decide to hold different amounts of nominal debt, so there will be transfers in the asset markets, in case of inflation surprises. But, with complete markets, no relative wealth transfers occur and no relative...

\footnote{Mei and Terajima (2008) perform a similar exercise for Canada.}
consumption adjustment or welfare consequences take place.

I formalize these ideas in a simple model, where I show that asset transfers is an incomplete account of welfare given a inflation shock. If agents’ life-time wealth innovations correlate heterogeneously with inflation surprises and nominal asset holdings is optimally decided by the households, they will hold different nominal positions and the response of total wealth (or consumption adjustment) to an innovation in inflation will not track their relative nominal exposure.

Lastly, in a calibrated DSGE with overlapping generations and endogenous portfolio decisions, I show that, if there is heterogeneity on exposure to aggregate risk in uninsurable income over the life cycle and complementarity between labor and consumption, the model matches the data with respect to (i) the distribution of ratios of nominal assets to total wealth and (ii) the pattern of consumption changes accompanying inflation surprises. Moreover, the heterogeneity necessary to match the data is consistent with evidence that there is a sharp decrease in aggregate risk on uninsurable income by the time of retirement. More specifically, the model requires uninsurable income of agents below 65 years old to be more affected by the business cycle than agents above 65. When sorting the agents’ in different income groups, a calibrated version of the model also matches the evidence on nominal positions and does a fair job in accounting for the consumption reaction to inflation surprises.

Section 2 comprises the empirical part of the paper. There I show how to calculate different measures of unanticipated inflation movements and describe the data used as consumption for households. I also present the results on how total wealth is affected by unanticipated movements to inflation. In section 3, I present a simple model where heterogeneity in the correlation of inflation shocks can lead to different nominal portfolio holdings and show that transfers in the asset market do not track total wealth transfers. Section 4 presents numerical simulations on extensions of the simple model. In section 5, I describe the overlapping-generations DSGE model and its calibration, while section 6 presents the results and compare it to the US evidence. Section 7 concludes.

**Related Literature:**

In addition to the classic literature and the recent work of Doepke and Schneider (2006a) and Meh and Terajima (2008) mentioned above, there is a large empirical literature on redistributive effects of inflation, part of which I will briefly discuss here. On wealth effects of inflation episodes, Bach and Ando (1957), Budd and Seiders (1971) compare the wealth holdings of different groups of agents before and after an inflationary episode. They conclude that inflation tend to reduce wealth inequality. As for the income inequality in the US, there is evidence that inflation tends to work as progressive tax. Blank and Blinder (1986), Cutler and Katz (1991), Jäntti (1994), Romer and Romer (1999) and Mocan (1999) show that different measures of inequality falls in inflationary episode. This paper differs from this literature by considering the effects of inflation surprises on total wealth, using consumption as proxy.\footnote{The evidence is less clear for other countries: Bulir and Gulde (1995). Romer and Romer (1998), Dollar and Kraay (2000) and Easterly and Fischer (2001) find a positive impact on income inequality in a panel of countries.}

\footnote{Curtler and Katz (1991) calculate the effect of inflation on consumption inequality, but does not focus on inflation surprises. Romer and Romer (1998) and Mocan (1999) consider the effects on unexpected inflation on income}
In the theory part, this paper is closely related to Meh et al (2008) and Doepke and Schneider (2006b). Meh et al (2008) calibrate their model using nominal holdings data for Canada to analyze the gains in welfare of changing from an inflation targeting to a price targeting regime. Doepke and Schneider (2006b) calibrate their model using the US evidence and study the welfare and macroeconomics effects of different fiscal policies. In this paper, I depart from these studies by taking portfolio allocation as an endogenous decision and focusing on the insurance role of net nominal positions in this context.

Erosa and Ventura (2002), Heer and Sussmuth (2007) and Albanesi (2007) show that the combination of imperfections in the asset markets and nominal assets (mainly, cash) necessary in transactions lead to regressive effects of inflation in wealth redistribution. This paper differs from this literature by focusing on unanticipated inflation and by the absence of transaction costs or cash in advance constraints.

Pescatori (2007) and Lee (2007) extend the nominal frictions model to incomplete markets case. The first show that optimal policy accounts for relative consumption across agents, while the second argues that incomplete markets might increase real rigidity and real effects of monetary shocks. This paper differs in three aspects: there is portfolio choice in a richer asset structure, allowing agents trade equity, nominal bonds and real bonds; capital is considered in production; and there are no nominal frictions.

2 Surprise Inflation Effect on Consumption

In order to assess how unanticipated movements in inflation correlate with consumption, one needs three key elements: (i) a measure of consumption for different agents; (ii) a series for inflation surprises and (iii) an empirical strategy.

2.1 Measure of Consumption

I use the Consumer Expenditure Interview Survey (CEX) as my source of data on consumption. The CEX is a rotating panel of households that are selected to be representative of the U.S. population. Continuous data are available since the first semester of 1980. Each quarter the survey contains detailed information on quarterly consumption expenditures for all households interviewed during that quarter.

2.1.1 Benchmark consumption measure

The measure of consumption is meant to capture the flow of consumption services that accrue to a household in a given period. Ideally, I would like to capture all the service flows from nondurables and the services provided by durables. I follow Krueger and Perri (2007) and incorporate services flows on consumption. I, then, consider service flows from housing as the rent paid by the households distribution.
who indeed rent their home and the self-reported hypothetical rent by households who own a house. For service flows of cars, I take a constant fraction (1/32) of the value of the stock of vehicles owned by the household. Since CEX does not provide direct information on the value of the stock of cars, I follow the procedure used by Cutler and Katz (1991) and use information from households who currently purchase vehicles (and for which we therefore observe the value of the purchase) to impute the value of the stock of vehicles for all households. I call nondurables expanded the resulting measure of nondurables including services flows.

Also, since one can use the number of adult equivalents in the household using the census equivalence scale (Dalaker and Naifeh, 1998), it is possible to consider measures of consumption per capita. Each expenditure component is deflated by each expenditure-specific consumer price index.

### 2.1.2 Alternatives to benchmark

I consider as a first alternative measure of consumption simply nondurable goods, disregarding rent payments or car services. This quantity is called nondurables. This series has the advantage of being directly reported in the CEX and the disadvantage of being a narrower measure of nondurables, when compared to the benchmark case. As additional robustness exercises, I also consider total food consumption (which include expenditure with food outside home) and total expenditure (which include expenditure in durable goods).

I also present the results without adjusting by the number of adult equivalents in the household.

### 2.1.3 Age and Income Groups

Using the CEX information, I divided the households by age and wealth groups. Concerning age, I follow Doepke and Schneider (2006a) and define six groups in the sample: younger than 35, from 35 to 45 years old, from 45 to 55, from 55 to 65, from 65 to 75 and older than 75. These age groups allow for a direct comparison with the evidence on average nominal asset holdings in agents’ financial positions.

For income groups, I follow Parker and Vissing-Jorgensen (2009) by separating the sample based on their distribution of consumption expenditure. For each age, I define rich the top 25% in consumption expenditure distribution and poor the bottom 25%. The households not defined as poor or rich are called middle-class.\(^8\)

### 2.1.4 Sample Selection

I exclude households that report, at least in one quarter, zero food expenditures or only food expenditures, and those who report positive labor income but no hours worked. I also excluded

---

\(^8\)An alternative definition of income groups follows from Doepke and Schneider (2006a) and is based on income and financial wealth information. The trade-off is the low quality of income and wealth data in the CEX (see Heathcote, 2009, for a discussion) and the possible problems of splitting the sample by the dependent variable. The results under this alternative income groups’ definition allow the same conclusions of the analysis below and are available upon request.
households whose wages of the reference person was below half of the minimum wage.\textsuperscript{9} Also in the benchmark case, I exclude households in which the head of household age changes to another category bin of age, education, gender or race.

2.2 Measure of Inflation Surprise

The measure of inflation surprises consists of one-period forecast errors of inflation in reduced-form Bayesian VARs models. The decision to use Bayesian VARs to identify innovations to inflation comes from the evidence of their good forecasting properties.\textsuperscript{10}

It is important to emphasize that the inflation surprises extracted here do not have any structural interpretation. The forecast errors from the BVAR capture the unidentified combination of realized structural shocks that affects inflation.

The benchmark inflation surprise comes from the forecast errors of inflation in a Bayesian VAR containing four variables: inflation (CPI for all urban consumers, seasonally adjusted), nominal interest rates (Fed Fund rates), output growth (Real Gross Domestic Output) and commodities inflation (CRB Spot Index). These series comprise the usual minimum set of variables used in either macro forecast exercises or structural VAR estimations. The data is from 1978 first quarter until 2008 fourth quarter, which covers the period of the consumption data.

For this Bayesian VAR, I followed Litterman’s prior that a random-walk model is the initial belief for the series stochastic processes. After trying different priors on the weight and persistence of this initial belief, I chose the model that fits the data best, by the marginal data density criterion.\textsuperscript{11} The results are robust to different specifications of the Litterman’s prior hyper-parameters. I also used 4, 6 and 8 lags and the results do not change significantly. Here, I report only the results for the case with 4 lags to conserve space.

As alternatives to the benchmark case, I also present the results using forecast errors from two other BVAR models: (i) excluding commodities inflation series from the estimation and (ii) including M2 and unemployment series, a set of variables often used in the empirical macro literature, as in Sims and Zha (2006).

2.3 Empirical Strategy

In the benchmark equation for different groups, I estimate how consumption adjusts to inflationary shocks, using the regression equation below:

\[
\log(C_i^t) = \alpha_0 + \alpha_q D_q + \alpha_1 t + \alpha_s D_{i \in s} + D_{i \in s} \gamma_{i} \pi_{i} + X_{i} \beta + \epsilon_i^t
\]  

\textsuperscript{9}These sample selections are usual in the empirical literature using CEX. It attempts to exclude clear reporting errors. (see Krueger and Perri, 2006, for similar sample selection).

\textsuperscript{10}The forecast errors for inflation show very little persistence in the estimates presented here. Litterman (1986) and Sims (1993) are classic references.

\textsuperscript{11}The tightness parameter is set to 3, the weight on the variance covariance dummies and the decay parameters are both set to 0.5.
where $\pi^j_t$ is the measure of inflation surprise $j$ described in section 2.2; $X_t$ contains characteristics of the head of the household that could affect consumption such as race, sex, education attainment, if the household is rural and the US region in which they live; and $D_{i\in s}$ is a dummy variable that assumes value one if agent is in groups $s$. As mentioned in section 2.1.2, groups are defined either by age or by age and income. I also allow for quarterly dummies $D_q$, a time trend and a constant.

The intuition for the estimation strategy above comes from standard consumption savings problem as in Hall (1978), where consumption growth should react only to news. The BVAR forecast errors are expected to capture the contemporaneous realization of the structural shocks that affect inflation, and are, this way, unexpected to the consumer. By regressing these forecast errors on different household groups’ consumption, one measures the different exposure of each of them to the aggregate shocks that affect inflation.

Naturally, if the innovations identified in the BVAR affect only the price level, it would affect negatively the return of nominal assets, while not affecting other sources of household income. In this case, a permanent-income consumer would experiment larger consumption reductions, the larger their holdings of nominal assets. Thus, consumption adjustment should follow a similar pattern as the household’s nominal asset positions. If the shocks that affect inflation also affect non-tradable disposable income or the value of other assets held by the agents, it is less clear how the pattern of consumption should look like: different exposure to aggregate shocks might be insured by different positions in the asset market.

As an alternative to this benchmark equation, I present a fixed-effects version of equation (1):

$$\log(C^i_t) = \alpha_0 + \alpha_q D_q + \alpha_1 t + \alpha_s D_{i\in s} + D_{i\in s} \gamma_s \pi^j_t + X^i_t + \pi^j_t + \alpha^i$$

(2)

This alternative to this benchmark equation has advantages. First, it allows one to explore the panel component of the CEX. One can observe up to four quarters of consumption data for a single household. This approach would take into consideration individual-specific (but not time-specific) component of consumption reaction to inflation shocks that could bias the estimation presented in the benchmark case. However, in order to estimate these fixed-effects equations one needs variation within households. This seems to be absent in some of the groups of interest as will be clear in the discussion below.

I also consider how consumption growth is affected by different groups, which is the estimation strategy closest to the idea that consumption growth should only respond to news. Here, the disadvantages are the same as for the fixed-effects. If, however, the unpredictable movements in inflation are well extracted it should not affect past consumption, implying that (1) to be a close substitute to (3).

$$\log(C^i_t/C^i_{t-1}) = \alpha_0 + \alpha_q D_q + \alpha_1 t + \alpha_s D_{i\in s} + D_{i\in s} \gamma_s \pi^j_t + X^i_t + \pi^j_t + \alpha^i$$

(3)

If a price level shock leads to either a revaluation of agents’ nominal positions or is correlated with persistent shifts on their life-time income, one would expect that past realizations of inflation...
shocks to impact current consumption. I, then, estimate equation (1) using lagged realization of inflation surprises as shown in equation (4). This would also capture delays in consumption adjustments due to information frictions or habit.

\[
\log(C_i^t) = \alpha_0 + \alpha_q D_q + \alpha_t t + \alpha_s D_{i\epsilon s} + \gamma_s D_{i\epsilon s} \pi_{t-j}^i + X_i^t \beta + \varepsilon_i^t
\] (4)

2.4 Results

2.4.1 Nominal Net Positions

Table 1, which is taken from Doepke and Schneider (2006a) (henceforth DS), reports the net nominal positions as a percentage of each groups’ total wealth for the year of 1989, using the Survey of Consumers Finance and Flows of Fund data. They calculate these positions for different age groups, as well as dividing the sample among rich, middle-class and poor. There is a clear pattern in age for both the entire sample and when the sample is divided in income groups: as households get old, they increase their net nominal positions. On average, households below 45 years old are net nominal borrowers, while those above this threshold are net nominal lenders.

For the rich, only the youngest group, with agents 35 years-old or younger, are net nominal borrowers, while any older group shows on average positive nominal exposure that increases smoothly with age, reaching a 30.6% nominal share of total net wealth for the group above 75 years old. In the middle class group, the difference in net nominal positions in the life-cycle is the most extreme. While the youngest has more than 100% of their average net worth in net nominal liabilities, middle-class agents over 65 were on average the largest net nominal lenders as a fraction of their total wealth. For the poor, agents below 55 are net borrowers and above this age net lenders. Doepke and Schneider (2006a) also find that the population as a whole lends in nominal terms to the government.\(^{12}\)

[TABLE 1 HERE]

In case of an unanticipated inflation shock, portfolio revaluations would imply transfer of resources in financial markets. The winners and losers would follow closely the net nominal exposures in Table 1, where the old would lose relatively to the young and the rich compared to the poor.\(^{13}\) Also, given that the government is a net borrower, there are transfers of resources from the population to the government.

\(^{12}\) Foreign holdings on nominal assets were small in 1989, increasing afterwards in the nineties.

\(^{13}\) Considering duration of nominal liabilities and nominal assets, Doepke and Schneider (2006a) show that the relative winners and losers in asset markets transfers track well their nominal positions. They also considered the case for persistent unpredictable movements in inflation, which relies on slow information acquisition by the agents of the inflationary process and are ignored here.
2.4.2 Consumption Adjustment - Benchmark case

Contrary to the evidence on transfers through nominal holdings to the government, inflation surprises lead to an increase in consumption of the entire population, as seen in Table 2 column (1-A) and (1-B): in the OLS estimations 1% inflation surprise leads to an average consumption adjust of 1.25%, while the fixed-effects estimates point to a response of 0.7% in consumption, both statistically significant at 1% significance level.

For different age groups, Table 2 column (2) shows that estimations of equation (1). The results show that consumption adjustment is also significantly positive for all age groups, even for groups where a price level shock has a permanent negative valuation effect on their portfolio. Moreover, the impact on consumption does not show monotonicity on age, as opposed to the losses in nominal portfolio revaluations.

There is a clear positive impact on the two oldest groups, despite the fact that these are precisely the groups that suffer the biggest hit on their financial wealth in an inflationary episode. Also, the impact on consumption of the youngest group is either equal or smaller than the average adjustment of the 36-45 and 46-55 groups, despite the fact that the youngest group benefits the most from an inflation shock being the largest net borrowers. As a conclusion, (i) shocks that affects positively inflation seems also to impact positively consumption, independently of the agents’ nominal portfolio holdings and (ii) these positive impact are similar in magnitude for different age groups, ranging from 1% to 1.6% in the OLS estimations (one cannot reject the null that the coefficients are the same across different age groups, p-value equals 0.9524).

The controls of the column (1) regression show expected signs in the estimate consumption equation, when one does not control for income: white, male and urban and more educated households consume more than their counterparts, while the age-consumption profile reaches a peak at the 65-75, decreasing for older ages.

Table 2 column (3) presents the estimates of equation (2) for the different age groups. Qualitatively, the results are the same: there is evidence of a positive impact of inflation surprises on consumption and the magnitude of this impact is not monotone on age. Quantitatively, the point estimates are smaller than the OLS estimations: consumption adjustment ranges from 0.6% to 1.1% for the households below 75 years old. For the eldest group of 75 or older, the fixed-effects estimates are very close to zero and statistically insignificant. Here again, one cannot reject that all coefficients are the same (p-value=0.34).

In column (4), I show the results for the estimation of equation (3). The estimates are very close to the fixed-effects estimates, with positive consumption adjustment (between 0.7% to 1.05%) to agents below 75 years old. Again in this case the estimated correlation of inflation surprises and consumption for the eldest group is not statistically different from zero.

The estimation of equation (1) for the different income groups are presented in columns (5)-(7)

\footnote{This is the group with the smallest number of consumption observations and with the highest fraction of households whose consumption is computed just once. Since the fixed-effects estimate needs at least two observations in each household, it is not clear if the zero result comes from small sample problems.}
of Table 2. A similar pattern for the consumption adjustment is again clear: a general positive impact and non-monotonicity in age.

For the rich, all age groups have positive consumption adjustment in case of inflation surprise, with the largest magnitude for the 36-45 and 56-65 year-old groups, above 1.7%. The two groups above 65 years old also show a positive adjustment in consumption, about 1.2% increase in consumption for a 1% surprise increase in inflation. Moreover, the youngest group, despite being the only group short in nominal terms, shows an impact in the consumption similar to the other age groups.

In the case of middle-class households, there is again a positive point estimate for all age groups. Agents older than 65 show on average strongly positive and statistically significant consumption adjustment, despite losses in valuation of their portfolios: a 1% inflation shock leads to 1.3% adjustment in consumption. The non-monotonicity is clear in this case as well, since agents younger than 65 adjust their consumption by slightly less than 1%.

For the poor there are two age groups where the estimates are close to zero and are not statistically significant, those younger than 35 and older than 75. Surprisingly, the youngest group is, among the poor, benefiting the most from portfolio revaluation in an inflationary episode. The other age groups show a similar positive consumption adjustment of above 1%, given a 1% inflation innovation.

The controls estimates for each income groups are generally similar to the age-group-only case.\footnote{Except for the fact that, among the rich, consumption does not seem to increase with education.}

I also estimated eqn. (2) and (3) for different income groups. The results for middle-class follow closely the full sample case fixed-effects estimates: point estimates are smaller and we can still sign a positive impact for all groups, except the oldest which is very close to zero with a p-value of 0.02. The estimates for the rich and poor subsamples are imprecise for the two eldest groups, which have smaller samples and a lower of fraction of agents show more than once. For these groups, the results are no longer robust across different measures of consumption and inflation innovation.

Finally, column (6) presents the estimates of equation (4) for the first lag of inflation surprises. The results show similar features to the contemporaneous impact of inflation shock: positive statistically significant impact and no monotonicity on age. Moreover, it shows an even larger consumption adjustment than the contemporaneous shock for some groups: around 2% of impact on consumption for the age groups from 36-55.

\[\text{[TABLE 2 HERE]}\]

2.4.3 Different Measures of Consumption

Table 3 shows estimates of equation (1) for alternative measures of consumption. The bottom-line conclusion of the section above persists here: consumption commoves positively with inflation surprises and this correlation does not decrease with age.

In column (1), the independent variable still incorporates service flows but the adjustment by
the number of adult equivalent in the household is absent. The point estimates are roughly the same and are statistically significant for all groups, except for the 46-55 year-old group. As result, the co-movement of inflation surprises and consumption is not driven by the size of the household, or by errors in constructing the per capita series.

In column (2), the consumption measure excludes all service flows from rent and cars. This series has the advantage of being directly observed on the CEX, but comes at the cost of a restricted definition of non-durables. In this case, the results are very similar to the benchmark case for the age groups below 75. For the eldest group, the point estimate is smaller and not statistically significant.

Total expenditure including expenditure with durables is presented in column (3). Besides confirming the previous conclusions, the estimates imply that inflation surprises coincide with increases in durables purchases, since the point estimates in this case are larger than the benchmark case, ranging from 1.3% to 2.1%.

Results for food expenditure are presented in column (4). For this case, the impact of inflation surprise on consumption is not significant for most of the age groups. Besides being a noisier series, food, being in many cases a necessity is not a natural candidate for proxying wealth.

Table 4 reports the estimates of equation (1) for different income groups, using alternative measures of consumption (all described in table 3, except food expenditure). The results are again similar to the ones presented in table 1, except that for the rich consumption adjustment for agents older than 65 years old are no longer significant. Grouping the agents by income and age leads to relatively very few observations in the oldest two groups. This might explain the positive and economically significant point estimates that are not statistically different from zero.

2.4.4 Alternative Inflation Surprises

Table 5 presents the estimates of equations (1), (2) and (4) using one-step forecast errors of two alternative BVAR models as inflation surprises: BVAR 3, with three variables (inflation, output growth and interest rates) and BVAR 6, with 6 variables (inflation, output growth, interest rates, commodities inflation, M2 and unemployment rate).

The results are similar: there is a positive co-movement between inflation surprise and consumption measure. The effects are also strong for the impact of first-lag inflation surprises on current consumption. The fixed-effect estimates for the eldest group is again zero, while the other age groups present consumption adjustment estimates slightly smaller than the benchmark case.
2.4.5 Discussion

As a conclusion, the fact that unexpected movements in the price level correlates positively with consumption implies that the return of the nominal assets is not the only variable adjusting given an inflation surprise. If one takes consumption as a proxy for total wealth, it implies that other components of agents’ life-time disposable income or the value of agents’ real assets correlate positively with inflation surprises.

Moreover, the similar point estimates suggest that there might be insurance against inflation surprises. In sections 3 to 6, this paper argues that portfolio choice helps to implement this insurance.

Another possible interpretation that does not rely on the permanent income hypothesis is that the structural shocks that change inflation, not only affect other sources agents’ wealth, but also their marginal utilities, driving their consumption decisions.\(^{16}\) In any case, shocks to the price level have effects on other variables relevant to agents’ welfare beyond the valuation of nominal assets.

For some agents, the positive consumption adjustment given an inflation could be explained by the easing of borrowing constraints, rather than impact their wealth or life-time disposable income. This is potentially relevant for young and poor agents, as the reduction of nominal debt would allow them to borrow more and increase consumption.

However, the borrowing constraints should be less relevant for richer and older agents, since they have, on average, positive holdings of both financial wealth and nominal assets. For these groups of agents, inflation surprises lead to average losses in nominal positions, which would hardly soften an eventual borrowing constraint. Since I find that this group also adjusts positively consumption, borrowing constraints cannot address the entire pattern of consumption adjustments given inflation surprises.

3 A Simple Model

I start with a simple model for two reasons. First, it allows for exact analytical solutions, providing clear intuition on the mechanisms that generate heterogeneous nominal holdings for hedging motives. Second, this set-up highlights which assumptions are needed to generate asset allocation and consumption adjustment in response to inflation innovations that are in line with the data.

This simplicity comes at the cost of ignoring the life-cycle of households, making it impossible to match the age groups evidence of section 2. Section 4 provides a OLG model that address this issue. Here, two kinds of heterogeneity are introduced: (i) agents’ uninsurable income shows different loads on aggregate risk and (ii) agents’ taxes react differently to innovations on real debt.

\(^{16}\)In section 5, there are preference shocks of this kind when I assume complementarity between consumption and labor.
3.1 Setup

Consider a two-period \((t = 0; 1)\) endowment economy. There are \(N\) types of agents, which differ on their endowment process and taxes paid. Each agent \(i = \{1 : N\}\) is endowed with two types of trees, producing the quantities \(X^i\) and \(Y^i\) of the single good of the economy in period 1. The endowment process \(X^i\) is interpreted as dividends from tradable companies endowed to individual \(i\) and, therefore, trading contingent on its realization is allowed. The second stochastic endowment is interpreted as labor income or dividends from illiquid assets. This way, \(Y^i\) represents non-tradable income.

Independently of their type, all households share the same CRRA preferences; they start with the same initial wealth, make portfolio decisions in period 0 and consume only in period 1. I allow for trade in nominal bonds and claims on the realization of insurable part of the endowments, which is interpreted as equity trading.

There is a government that can tax each consumer according to her type and is subject to a budget constraint. The government issues nominal debt in period 0, and taxes and retires all debt in period 1. It consumes the single good of the economy in period 1, and its expenditure can be stochastic.

3.1.1 Consumers

The representative agent of type \(i\) maximizes expected utility in period zero:

\[
E_0 \left[ \log C^i \right],
\]

where \(C^i\) is her consumption level of the single good in period 1 and the inverse of the intertemporal elasticity of substitution is assumed to be one. It is clear from (5) that consumption is sufficient to evaluate agent’s welfare. She is subject to the following budget constraints in period 0 and 1, respectively.

\[
W^i = B^i + \sum_{j=1}^{N} q^j A(j, i) \quad \text{and}
\]

\[
C^i + \tau^i = X^i + Y^i + \frac{RB^i}{P} + \sum_{j=1}^{N} X^j A(j, i)
\]

\(A(j, i)\) is agent \(i\)'s net purchases of claims of endowment \(X^j\) in time 0 and \(B^i\) is agent \(i\)'s holding of nominal bonds, \(q^j\) is the price of a claim on endowment \(X^j\), and \(R\) is the nominal interest rate. The amount of wealth \(W^i\) is an initial condition and has to sum up the total outstanding government debt in the economy, \(\sum_j W^j = B\). Without loss of generality, I assume that the initial wealth is the same across agents, \(W^i = W^j, \forall i, j\). Qualitatively, this symmetry does not affect the results and keeps the interpretation of the main results simple. As implicitly assumed in (7),
agent \(i\) will receive the fruit endowments \(X^i\) and \(Y^i\). \(P\) is the price index of the economy, i. e., the price of the single good and \(\tau^i\) is the real amount of lump-sum taxes paid by households \(i\). Notice that, since this is a two-period model, an equivalent to a no-Ponzi condition implies that the total bond holdings of the consumer \(i\) in period 1 should be greater or equal than zero. In writing (7), I have already imposed the optimality condition that agent’s wealth in the end of period 1 is equal to zero.

As equation (6) makes clear, there is no consumption at \(t = 0\), but, at this date, agents are allowed to trade assets. The agent starts with initial wealth, \(W^i\), which is equal to the value of her asset holdings in period \(t = 0\). In period 1, after the endowment and price level uncertainty are realized, households consume using their resulting financial income less taxes, as seen in equation (7).

The optimality conditions of the consumer are

\[
E_0 \left[ \frac{1}{C^i} R^i \right] = E_0 \left[ \frac{1}{C^j} X^j/q^j \right] \quad \forall i, j
\] (8)

This set of equations show the standard non-arbitrage conditions for the \(N + 1\) assets available.

### 3.1.2 Government

The government starts in period 0 with a stock of nominal debt \(B\) and collects tax on all agents in period 1 (\(\tau^i\) for agent of type \(i\)). In period 1, the government consumes \(G\) goods and, thus, is subject to the following budget constraint

\[
\sum_{j=1}^{N} \tau^j = R^i B/P + G.
\] (9)

The transversality conditions of the agents lead to \(B_1 = 0\), that is, the total debt of the government has to be paid in period 1.

The government issues only nominal bonds and taxes its citizen heterogeneously. These are mild assumptions, since there is plenty of evidence that most governments in advanced countries issue primarily fiat debt, and taxes are different according to characteristics, such as income. More precisely, I assume that expected taxes are the same for all agents, but the tax adjustment to innovations in government real liabilities is heterogeneous, as can be seen in the rule bellow:

\[
\tau^i = \frac{1}{N} \left( \frac{RB}{P} + \bar{G} \right) + a^i \left( \left( \frac{1}{P} - \frac{1}{P} \right) RB + (G - \bar{G}) \right)
\] (10)

\[
\sum_{j=1}^{N} a^j = 1
\] (11)

where \(P\) and \(G\) are the expected values for price-level and government expenditure that differ from the realized price and government expenditure by shocks defined in section 3.1.4 below and (11) is necessary to fulfill (9).
The exogeneity of the taxes rule is clearly a simplification of the model. In the section 5, the quantitative model presents taxes as function of agents’ current income while the intertemporal adjustment of taxes given shocks to real government debt is contingent on the agents’ position in the income distribution. In this section, the heterogeneity on how taxes respond to shocks is just a simplest way to introduce different exposure to aggregate risks in agents’ disposable income.

### 3.1.3 Market Clearing

Markets should clear for each of the assets and for the good in the economy. Goods market clearing implies that agents’ consumption plus government consumption equals the sum of endowments, as in the following condition:

\[
\sum_{j=1}^{N} C^j + G = \sum_{j=1}^{N} Y^j + \sum_{j=1}^{N} X^j
\]

In the market for assets note that the claims on endowments are inside assets, while there is an outside supply of nominal bonds by the government. Market-clearing conditions are, then,

\[
\sum_{i=1}^{N} A(j, i) = 0
\]

\[
\sum_{j=1}^{N} B^j = B
\]

### 3.1.4 Uncertainty

There are \(2N + 1\) exogenous variables in this economy: both tradable and non-tradable endowments of each agents and the price level.

There is a key assumption on the distribution of these exogenous variables: price level and uninsurable income are affected by a common underlying aggregate shock. The foundations for this assumptions are discussed in detail in the quantitative section. Thus, the price level can be written as:

\[
\frac{1}{P} = \frac{1}{\bar{P}} + \varepsilon^\pi + \varepsilon^a
\]

where \(\varepsilon^\pi\) and \(\varepsilon^a\) represent, respectively, a pure shock to the price level and an aggregate shock that also affects the endowment processes. The first shock performs a role similar to the thought experiment in the recent empirical literature on nominal portfolio holdings: the resulting price change would redistribute wealth by different nominal exposure in the asset markets.\(^{17}\) The second shock adds an additional channel: a price change is correlated with relative movements in agent’s uninsurable income. The exogeneity in the price level is a clear simplification of this simple model and allows for a simple way to introduce correlation in the innovations to the price level and to

\(^{17}\) In this model, however, there is a feedback effect through taxes.
aggregate output. In a quantitative model of section 5, price is endogenous and the correlation between price level and output arises endogenously.

The endowment process are

\[ Y^i = \bar{Y} + \varepsilon^{Yi} + c_i^2 \varepsilon^a \]
\[ X^i = \bar{X} + \varepsilon^{Xi} \]

where \( c_i^2 \) is known, while \( \varepsilon^{Xi} \) and \( \varepsilon^{Yi} \) are agent-specific endowment shocks. Notice that, the uninsurable endowment loads on the aggregate risk, \( c_i^2 \), are agent-specific. The interpretation is that agents labour income have different exposure to the business cycles.

Government expenditure is also stochastic, \( G_1 = \kappa \varepsilon^G \).

3.1.5 Competitive Equilibrium

A competitive equilibrium is a set of endogenous variables that satisfies the consumer’s maximization problem, government behavior and market-clearing conditions, given initial conditions and exogenous processes.

The endogenous variable set comprises of all consumption allocations, asset allocations, prices, and taxes. In this simple model it is \( \{C^i, A(j, i), B^i, q^i, \tau^i\} \).

The set of initial conditions is given by the initial amount of government outstanding bonds, pre-determined interests on these bonds and the initial wealth of each agent. In the simple model, it is \( \{R, B, W^i\} \).

The exogenous variable set encompasses the variables that are assumed to be direct functions of the shocks of the economy of the economy. In the simple model, it is \( \{Y^i, X^i, G, P\} \).

3.2 Results

3.2.1 Completes Markets

I begin by presenting the results under complete markets. The reason to start here is twofold. First, this simple case allows for analytical exact solutions. Second, it is the simplest way to gain intuition on the relation of nominal portfolios and total wealth exposure to shocks to inflation.

I assume that there are two states in the economy \( s = \{\vartheta_1, \vartheta_2\} \) in the economy with \( \text{Prob}(s = \vartheta_1) = \pi \) and \( \text{Prob}(s = \vartheta_2) = 1 - \pi \). I also assume that:

\[ \varepsilon^G = \varepsilon^{Yi} = \varepsilon^{Xi} = 0, \forall s. \]  

The only uncertainty in this economy is, then, over the realization of \( \varepsilon^a \), the shock that affects both
uninsurable income and the price level. I assume that that\footnote{In order to simplify notation, I also assumed w.l.o.g. throughout this section that $\bar{P}=1$.}

\begin{align}
\varepsilon^a &= \varepsilon^{a,\vartheta_1}, \text{if } s = \vartheta_1 \\
\text{and } \varepsilon^a &= \varepsilon^{a,\vartheta_2}, \text{if } s = \vartheta_2, \varepsilon^{a,\vartheta_1} \neq \varepsilon^{a,\vartheta_2}.
\end{align}

The proposition below summarizes the result.

**Proposition 1** If taxation is heterogeneous as in eqns.\,(10) and \,(11), there are shocks to the price level and the endowment process that satisfy equations \,(15), \,(16), \,(17), \,(18), \,(19) and \,(20), then (i) relative nominal portfolio holdings are determined by taxes and uninsurable endowment process, i.e.,

\begin{align}
B^i - B^j &= (\lambda^*_i - \lambda^*_j) \left( \sum_{k=1}^{N} c^k_2 \right) + (a^i - a^j) B + \left( -c^i_2 + c^j_2 \right) \\
\frac{\partial (B^i - B^j)}{\partial c^i} &< 0 \text{ and } \frac{\partial (B^i - B^j)}{\partial a^i} > 0
\end{align}

where $\lambda^*_k, \forall k$, are functions of the parameters defined in the proof; and (ii) realized inflation has no effect on relative consumption (or relative welfare).

All the proofs are presented in the appendix.

This result makes clear that either heterogeneous taxation or different correlation between uninsurable income and the price level leads to differences in nominal portfolios holdings across agents. An agent with a larger share of taxes is benefiting the most in case of higher realized inflation that leads to lower real government debt. Hedging motives makes this agent willing to hold more of the asset with relative lower return when there is higher inflation, the nominal bond. A similar reasoning goes through in uninsurable income: a relative large load on aggregate risk (large $c^i_2$) implies significant increase in uninsurable income given a lower inflation level (higher $\varepsilon^a$). This way, one would be willing to hold more of the asset with relative lower return in a deflationary event, i.e., equities.

Moreover, complete markets ensure that there are no relative movements on total wealth or consumption, i.e., there are no relative winners or losers in different price level realizations. This result, thus, highlights the lack of connection between transfers in the asset markets and life-time wealth and welfare.

### 3.2.2 Incomplete Markets

Contrary to the previous section, I assume that all shocks $\varepsilon^k_1$ have independent normal distribution with variance $\sigma^2_k$. I also assume that the agent specific idiosyncratic shocks have the same variance, i.e., $\sigma^2_{X_i} = \sigma^2_X$ and $\sigma^2_{Y_i} = \sigma^2_Y, \forall i$. As a consequence, markets are now incomplete even up to a first-order of approximation. Following a recent literature on international finance, I compute optimal
or zero-order portfolios by first log-linearizing the model around a non-stochastic solution. Then, I compute first-order solution of the models, as functions of exogenous shocks and portfolio holdings and then solving for the portfolio that satisfy a second-order accurate solution for the difference in the Euler equation of the agents.\textsuperscript{19} The proposition below states the main results under incomplete markets.\textsuperscript{20}

**Proposition 2** If taxation is heterogeneous as in (10) and (11), there are shocks to the price level and the endowment process that satisfy eqns. (15), (16), and (17), then, (i) optimal zero-order nominal bond holdings are given by

\[
B^i = \frac{N}{N} \left( \frac{\sigma^2_\pi + \sigma^2_a}{\sigma^2_\pi + \sigma^2_a + 2\sigma^2_X} \right) a^i B + \frac{\sigma^2_a}{N (\sigma^2_\pi + \sigma^2_a + 2\sigma^2_X)} \left( -N c^i_2 + \sum_{j=1}^{N} c^j_2 \right) + \frac{1}{N} \frac{2\sigma^2_X}{N (\sigma^2_\pi + \sigma^2_a + 2\sigma^2_X)} B
\]

and (ii) consumption impact of innovations to inflation do not follow the nominal portfolios positions.

Notice that the intuition in complete markets goes through the same way. Agents whose taxes adjust more to innovations on government real debt, tend to hold more nominal assets as an insurance and those whose uninsurable endowment has larger loads on the aggregate shock hold less nominal bonds.

Differently from the case of complete markets, the presence of other shocks can change the relative importance of insurance due to heterogeneous taxes or heterogeneous uninsurable income. In this example, taxes and the return of nominal bonds are affected by the two different shocks to the price level, $\pi^\pi$ and $\pi^a$, while uninsurable income is affected only by $\pi^a$. This way the larger $\sigma^2_\pi$ is relative to $\sigma^2_a$ the more important is the hedging motives from taxes with respect to uninsurable income. Notice also that the larger is $\sigma^2_X$, the variance of the tradable endowment, the more worried agents are with equity diversification, making either taxes and uninsurable income motives for heterogeneous nominal holdings less important.

The table below shows how relative consumption change given a unit shock of each of the shocks that change the price-level, $\pi^\pi$ and $\pi^a$. The impact of these shocks on relative consumption makes clear the second part of proposition 2.

\textsuperscript{19}The solution method is developed in Devereux and Sutherland (2008). For applications in international finance, see Berriel and Bhattarai (2009) and Coeurdacier et al (2009).

\textsuperscript{20}I also assume w.l.o.g. that the agents have symmetric non-stochastic endowments, i. e., $X=Y$. 
For clarity, I discuss the impact on relative consumption in two cases: only heterogeneity in disposable income and only heterogeneous tax. In the first case, taxes are the same for all agents, i.e. $a_i = a_j$; also assume without loss of generosity that $c_2 > c_1$. In this case, by (23), $B^i > B^j$. In the first line of the table above, one can see that an unit shock $\varepsilon^\pi$, a deflation shock, increases the relative consumption of agent $j$. This makes sense: the agent holding more bonds would experience higher return on their portfolio in this deflation episode and the value of his equity holdings or his disposable income would remain unchanged. In the case of a unit shock to $\varepsilon^a$, also a deflation shock, agent $i$ experiences an increase in relative consumption, even though he suffers relative losses in his nominal asset positions. Markets here provide only partial insurance: the losses in portfolio are smaller than the gains in uninsurable income.

In the case of heterogeneity only in taxes, equal income process for the agents imply that $c_2 = c_1$ and, without loss of generality, I assume that $a_i > a_j$. By (23), $B^i > B^j$. A unit shock to either $\varepsilon^\pi$ and $\varepsilon^a$ (again a deflationary shock) would impact positively agent $j$’s relative consumption. The intuition is that agents end up with only partial insurance and the impacts of taxes reduction are larger than the losses in portfolio valuations.

Given that welfare is monotone on consumption these results imply that net nominal net positions are not good predictors of relative winners and losers on unanticipated inflation episodes. Corollary 3, brings home this point by establishing the exact conditions under which larger nominal positions comes with expected welfare losses, given a positive inflation shock.

**Corollary 3** An agent $i$, with larger optimal nominal portfolio positions than agent $j$, i.e., $B^i > B^j$, will be expected to benefit from an surprise movement in inflation, if the condition below is satisfied

\[
B(\sigma_i^2 + \sigma_j^2)(a(i) - a(j)) + \sigma_j^2(c_i^2 - c_j^2) > 0
\]  

(24)

One should bear in mind that either (i) $a(i) > a(j)$ and $c_2 \geq c_1^j$ or (ii) $a(i) \geq a(j)$ and $c_2 > c_1^j$ are sufficient for $B^i > B^j$. Condition (24) is fulfilled for all the cases where taxes and uninsurable endowment risks push on the same direction in allocation nominal portfolio, i.e., when the agents that pay most taxes are the ones increasing by the most their uninsurable income in case of $\varepsilon^a$ shock.

The table below provides a numerical example where condition (24) holds. In this example an inflationary episode is considered. For that, I assume that inflation is generated by $\varepsilon^\pi = \varepsilon^a = -1$ and that these two shocks have the same variance. There are three agents, the parameters are set

<table>
<thead>
<tr>
<th>Unit shock to:</th>
<th>$\frac{\Delta(C^i - C^j)}{\Delta c^j}$, $j = \pi, a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon^\pi$</td>
<td>$\frac{N\sigma_i^2}{2\sigma_i^2 + \sigma_j^2 + N\sigma_j^2}(-c_i^j + c_j^j) + \frac{2B_i\sigma_i^2}{2\sigma_i^2 + \sigma_j^2 + N\sigma_j^2}(a^j - a^i)$</td>
</tr>
<tr>
<td>$\varepsilon^a$</td>
<td>$\frac{N\sigma_i^2 + \sigma_j^2}{2\sigma_i^2 + \sigma_j^2 + N\sigma_j^2}(c_i^j - c_j^j) + \frac{2B_i\sigma_j^2}{2\sigma_i^2 + \sigma_j^2 + N\sigma_j^2}(a^j - a^i)$</td>
</tr>
</tbody>
</table>
to $a(1) = 1/6$, $a(2) = 1/3$, $a(3) = 1/2$, $c_1 = 1/2$, $c_2 = 0$, $c_3 = -1/2$, and $B = 1$. Agent 1, with relatively less nominal holdings, adjusts negatively its consumption as a result of the total inflation shock, as shown in column (4). Columns (2) and (3) show that $\varepsilon^\pi$ and $\varepsilon^a$ affect relative consumption in opposite ways, as argued in the discussion of Prop. 2 (ii) for heterogeneous uninsurable income.

<table>
<thead>
<tr>
<th></th>
<th>(1) Nominal Holdings</th>
<th>(2) $\varepsilon^\pi = -1$ on $C^i$</th>
<th>(3) $\varepsilon^a = -1$ on $C^i$</th>
<th>(4) Total $C^i$ Adjustment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agent 1</td>
<td>0.0834</td>
<td>0.1667</td>
<td>−0.3333</td>
<td>−0.1667</td>
</tr>
<tr>
<td>Agent 2</td>
<td>0.3333</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Agent 3</td>
<td>0.5834</td>
<td>−0.1667</td>
<td>+0.3333</td>
<td>+0.1667</td>
</tr>
</tbody>
</table>

As a conclusion, I highlight that the results of the simple model imply that nominal asset revaluation is a bad predictor of winners and losers of inflationary episodes, when portfolio decisions are endogenous and shocks to inflation also affect other sources of agents’ life-time permanent wealth. Other results discussed above are less general. In particular, agents whose uninsurable income are more exposed to aggregate shocks hold less bonds in this economy because the relative return of the nominal bond is higher than the return of alternative riskless asset, in case of a positive aggregate shock. If the relative return of the alternative asset were higher, this specific prediction would be reverted and still agents would use their portfolio as insurance.

## 4 Dynamic Simple Model

In this section I present numerical solutions for a dynamic extension of the simple model. The intuition in Proposition 1 and 2 goes through in this less simple (and yet relevant) environment. In the dynamic economy, I allow for lagged effects of shocks that affect inflation on both taxes and uninsurable income. This last channel sheds light on the importance of timing impact of shocks that affect inflation on disposable income. Since this section is illustrative, the parameter values for the income or taxes processes or the number of agents were chosen for easy of exposition and are not meant to be realistic.\textsuperscript{21}

Here, I show that contemporaneous co-movement between disposable income and inflation innovations is not necessary, in order to generate the relation between consumption adjustment to inflation shocks and nominal portfolio holdings in the previous section. The intuition for this result is simple: agents take into account the impact of an inflation shock over their life-time wealth, not the impact on today’s flows, when adjusting consumption.

The dynamic setup allows the model to incorporate the effects of the delayed impact on disposable income of innovations that affect current inflation. This way, one can address empirically

\textsuperscript{21}In particular, I assume that all shocks are i.i.d. with unit standard deviation.
relevant fiscal policy setups such as deflationary recessions where poor households receive immediate and low-persistent tax relief, while wealthier agents expect taxes to increase in the future.

4.1 Consumers

The representative agent of type $i$ maximizes the following expected utility by choosing the amount consumed as well as her portfolio composition:

$$E_0 \left[ \sum_{t=0}^{\infty} \beta^t \left( C_t^i \right)^{1-\sigma} \right]$$

where $C_t^i$ is his consumption level of the single good, and is subject to the following intertemporal budget constraint:

$$C_t^i + \frac{B_t^i}{P_t} + \sum_{j=1}^{N} q_t^i E_t(j, i) + \tau_t^i = Y_t^i + X_t^i$$

$$+ \frac{R_{t-1} B_{t-1}^i}{P_t} + \sum_{j=1}^{N} (q_t^j + X_t^j) A_t(j, i)$$

where the notation of section 3.2 is maintained.

4.2 Government

There are simple rules as descriptions of government policy. The government conducts monetary policy using a interest rate rule given by

$$R_t = \gamma_0 (P_t/P_{t-1})^\gamma \exp(\varepsilon_{r,t})$$

The government faces the following period budget constraint

$$\frac{B_t}{P_t} = \frac{R_{t-1} B_{t-1}}{P_{t-1}} - \sum_{j=1}^{N} \tau_t^j + G_t.$$  

In the present version of the model, the rule for heterogeneous taxation reacts not only to the current amount but can also depend on past values of real government debt.

$$\tau_t^i = \phi_0 \prod_{j=0}^{\infty} \left( \frac{B_{t-j}}{P_{t-j}} \right)^{\zeta_{i+1}}$$

where the coefficients are such that there is always passive fiscal policy, i.e, the price level is not determined by (29) and that steady-state taxes are the same for all agents. With this fiscal rule,
tax adjustments to innovations in real government debt can be heterogeneous both on its initial impact and on its persistence.

4.3 Uncertainty

There are three different types of shocks in this section. First, I assume a stochastic process for $\varepsilon_{r,t}$. It can be interpreted as the monetary policy residual after controlling for inflation. I assume that it has the form $\varepsilon_{r,t} = \varepsilon_{t}^{M} + \varepsilon_{t}^{A}$, where $\varepsilon_{t}^{M}$ is a pure policy i.i.d. shock and $\varepsilon_{t}^{A}$ is also i.i.d but it is can also affect the endowments in the economy.

Second, the endowment processes are stochastic. The endowments against which agents can issue claims is assumed to be completely idiosyncratic, $X_{t}^{i} = \tilde{X} + \varepsilon_{t}^{X}$. By the results discussed in section 4.1, pure i.i.d. shocks on the tradable endowment is not a restrictive assumption and makes interpretation of results clearer. The uninsurable endowment process loads on current and lagged aggregate shocks:

$$Y_{t}^{i} = \tilde{Y}^{i} + \varepsilon_{t}^{Y} + \sum_{j=0}^{\infty} b_{j}^{i} \varepsilon_{t-j}$$

Third, there are also government expenditure shocks $G_{t} = \kappa \varepsilon_{t}^{G}$.

Markets should clear for each of the assets and for the one good in the economy and their expressions are trivial extensions of (12), (13) and (14).

4.4 Results

Analytical solution for portfolio holdings in this stochastic model with incomplete markets is not feasible in this dynamic setup, I, then, turn to numerical solutions. In order to do that, I set the value of $\beta = 0.99$, the steady-state government debt, $\tilde{B} = 1$, $N = 3$ and all standard deviations of stochastic processes equal to one. I also assume that:

$$b_{j}^{i} = k_{i} (\phi_{i})^{-j}$$

where $b_{j}^{i}$ is the impact of $\varepsilon_{t-j}^{A}$ on agent’s $i$ uninsurable endowment. In other words, agents are heterogeneous on initial impact of the inflation shock, as well as on how fast the effects of $\varepsilon_{t}^{A}$ on uninsurable income decay over time. In the following table, I show two numerical examples.

In the first case, aggregate shock has the same long-term impact on uninsurable income for the three agents, but the initial impact and decay are different. More precisely, I assume that $k_{1} = 2$, $k_{2} = 2.9519$ and $k_{3} = 3.9038$, while $\phi_{1} = 0.8$, $\phi_{2} = 0.7$ and $\phi_{3} = 0.6$. Given this parameters values, the long-run impact of $\varepsilon_{t}^{A}$ on uninsurable income is the same for all agents, i.e., $k_{i}^{j} \frac{1}{1-\beta \phi_{i}} = \frac{k_{j}^{j}}{1-\beta \phi_{j}}$, $\forall i$ and $j$.

In the second exercise, I assume values for $\phi_{i}$ ($\phi_{1} = 0.8$, $\phi_{2} = 0.5$ and $\phi_{3} = 0.2$), such that the agents with higher initial adjustment in uninsurable income given a aggregate shock experience lower long-run effects, i.e., $\frac{k_{1}^{j}}{1-\beta \phi_{1}} > \frac{k_{2}^{j}}{1-\beta \phi_{2}} > \frac{k_{3}^{j}}{1-\beta \phi_{3}}$.
<table>
<thead>
<tr>
<th></th>
<th>First Case</th>
<th>Second Case</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$B^i / B$ $E_{t-1}(C^i_t / \pi_t = 1%)$</td>
<td>$B^i / B$ $E_{t-1}(C^i_t / \pi_t = 1)$</td>
</tr>
<tr>
<td>Agent 1</td>
<td>0.333 $-$0.93%</td>
<td>0.896 $-$0.78%</td>
</tr>
<tr>
<td>Agent 2</td>
<td>0.333 $-$0.93%</td>
<td>0.149 $-$0.78%</td>
</tr>
<tr>
<td>Agent 3</td>
<td>0.333 $-$0.93%</td>
<td>$-0.045$ $-$0.78%</td>
</tr>
</tbody>
</table>

In the first column, I report the nominal bond holdings of the agent as a fraction of total bonds and in the second column, the expected movement in consumption given a 1% movement in inflation. As it is clear in the second column and as expected in flexible price endowment economies, there is a negative correlation between price level innovations and output and consumption. This contrasts with section 3.7, where the price level followed an exogenous process. In this flexible prices economy where monetary policy follows a Taylor rule any endowments shocks affects negatively the price level, including the agent-specific i.i.d. shocks in the tradable and non-tradable endowments. Moreover, nominal portfolios take into consideration all the risks that affect the price level. Therefore, decomposing the hedging motives behind optimal nominal portfolio decision for individual shocks, as done in section 3.2.2, becomes impractical.

The first case, shows that, if the total present value of the impact of the aggregate shock on uninsurable endowment is the same across agents, i.e., if $k_i^{1-\beta_i} = k_j^{1-\beta_j}$, $\forall i$ and $j$, then independently of the initial impact, agents choose the same nominal portfolio and there is no expected relative consumption adjustment given an inflationary shock.

Moreover, the second case shows that nominal portfolio positions follow the long-run impact of the aggregate shock, i.e., if $k_i^{1-\beta_i} > k_j^{1-\beta_j}$, then $B^i > B^j$, independently of initial impact of $\varepsilon_i^A$ on uninsurable income. In this numerical illustration, even under incomplete markets, the agents show very similar expected consumption responses to inflation surprises. These results point that inflation surprises effects on life-time wealth and not on contemporaneous income or taxes drive the nominal portfolio choice of the agents and that trading goes a long-way in providing hedge against unanticipated inflation.

A completely analogous exercise can be done for taxes. If one assumes that the functional form $t_{j+1} = (t_i) \tau_i^*$, it follows that the overall future taxes reaction to innovations in real government debt, $\frac{\tau_i^*}{1-\beta_t}$, is what matters for nominal portfolio decisions and not the initial impact $\tau_i^*$.  

22 see Woodford (2003) chapter 2.
These results do not rely on specific functional forms for the lagged effects of aggregate shocks in income. The model was solved with alternative functional forms for $b_j^i$ and $t_{j+1}^i$ and the results confirm the above intuition.

This exercise sheds light on the empirical task of identifying the insurance role of nominal asset positions. A natural first idea would be to regress how contemporaneous taxes or uninsurable income are affected by inflation surprises in different income groups. The results above point out that this can be misleading. Instead, they suggest that measures of long-run impact of inflation surprises in either taxes or uninsurable income should be used. Unfortunately, there is not a straightforward empirical counterpart for these variables.

5 Quantitative model

In the simple model there are key features missing: (i) life-cycle of the agents, (ii) production and capital, (iii) dynamics (except for the extension in 4.2). Without a combination of (i) and (iii), a model cannot address the evidence on different nominal exposure and consumption reaction to inflation surprises presented in section 2. Introduction (ii) allows for a more realistic asset structure and a proper calibration of the exogenous supply of government nominal bonds as a share of total private sector wealth.

5.1 Setup

Agents live for 85 years. They become head of households at the age of 26 and face certain death at 85. By the time of their death, each agent has a 26-year-old heir that will take over the household decisions. As head of the households agents make decisions over consumption, savings and asset allocation. Besides differences in age, agents are divided in income groups: rich, poor and middle-class. As a simplification, agents in each income groups have heirs in the same income group.

There are two assets in the economy: nominal bonds in positive net supply by the government and capital.

5.1.1 Consumers

In period $t$, the age-$k$, type-$h$ agent maximizes her discounted life-time utility (including a weight on the utility of her heir),

$$ E_t \left\{ \sum_{i=k}^{60} \beta^{i-1} U(C_{t+i}^{hi}, N_{t+i}^{hi}) + \beta^{60} \theta \left[ \sum_{i=1}^{60} \beta^{i-1} U(C_{t+60+i}^{hi}, N_{t+60+i}^{hi}) \right] \right\}, \quad (31) $$

where $\beta$ is the intertemporal discount factor, $\theta$ is a intergenerational discount factor, $U$ is his utility function, $C_{t}^{hi}$ is the consumption level at age $i$ of agent of income $h$ in period $t$. and $N_{t}^{hi}$ is the amount of effective labor.
I assume that $U$ has three properties. (i) $\frac{\partial U}{\partial C} > 0$, $\frac{\partial^2 U}{\partial C^2} < 0$ (ii) $\frac{\partial U}{\partial N} < 0$, $\frac{\partial^2 U}{\partial N^2} > 0$ and (iii) $\frac{\partial^2 U}{\partial N\partial C} > 0$. Assumptions (i) and (ii) are standard and imply positive and decreasing marginal utility of consumption and leisure. Assumption (iii) implies complementarity between labor and consumption: an agent working more hours experiments higher marginal utility of consumption and tends to consume more. This complementarity has precedents in the empirical literature of estimation of Euler equations, as in Attanazio and Weber (1995) and rationalizes the similar life-cycle profile observed between labor and consumption.

Equation (32) is a simple functional form that satisfies properties (i), (ii) and (iii) around the steady-state of the economy, for the parameters values discussed in section 5.2: $^{24}$

$$U(C_{hi}^t, N_{hi}^t) = a_{hi} \left( \frac{(N_{hi}^t)^{1+\nu}}{1+\nu} \right) - \lambda \left( \frac{(N_{hi}^t)^{1+\nu}}{1+\nu} \right)$$

where $\sigma$ is the inverse of the elasticity of intertemporal substitution assumed to be the same for different ages and income groups, $\nu$, $\lambda$ are preference parameters for disutility of labor and $a_{hi}$ is a group-specific weight on utility function. The parameter $\varphi$ defines the degree of complementarity between $C_{hi}^t$ and $N_{hi}^t$.

The agent faces the following budget constraint:

$$C_{hi}^t + \tau_{hi}^t + \frac{B_{hi}^t}{P_t} + k_{hi}^{t+1} = \omega_t N_{hi}^t + \Psi_t^h, \text{ and}$$

$$C_{hi}^t + \tau_{hi}^t + \frac{B_{hi}^t}{P_t} + k_{hi}^{t+1} = \omega_t N_{hi}^t + \frac{R_{t-1} B_{hi}^{t-1}}{P_t} + (1+r_t)k_{hi}^t, \text{ if } k = 2 : 60$$

where $B_{hi}^t$ and $k_{hi}^{t+1}$ are respectively, the nominal bond and capital holdings of agent of age $i$ and income $h$. In addition, $\tau_{hi}^t$ are lump-sum taxes paid, while $\omega_t$ are the wages received on unit of effective labor. Also, $\Psi_t = \frac{R_{t-1} B_{hi}^{t-1}}{P_t} + r_{t-1} B_{hi}^{t-1}$ is the bequest received by the youngest agent in $t$, by the oldest deceased in $t - 1$.

The optimality conditions of the consumer imply the standard non-arbitrage conditions between the two assets in the economy.

5.1.2 Firms

Output in this economy is given by a Cobb-Douglas aggregate production function,

$$Y_t = \tilde{A} R_t^{\alpha} N_t^{1-\alpha}$$

$^{23}$ Aguiar and Hurst (2005, 2007) show that the decrease in computed consumption observed in retirement comes due to either substitution towards home production or time searching for the same products with lower prices. This way, computed consumption would reflect expenditure and not consumption of the agents. Here, there is no home production and prices are known, so expenditure and consumption are the same.

$^{24}$ A microfounded alternative to (32) that satisfies the three properties above is to introduce home production as in Benhabib et al (1991).
where $N_t$ and $K_t$ are respectively aggregate effective labor and capital at time $t$. Given prices, firms maximize profits and as a result the first-order conditions imply:

$$r_t + \delta = \alpha \bar{A} \left( \frac{K_t}{N_t} \right)^\alpha$$

$$\omega_t = \bar{A}(1 - \alpha) \left( \frac{K_t}{N_t} \right)^{\alpha - 1}$$

where is the depreciation rate of capital and $\omega_t$ is the wage rate.

### 5.1.3 Government

The government conducts monetary policy using a nominal interest rate rule given by

$$R_t = \gamma_0 (P_t / P_{t-1})^\gamma \exp(\varepsilon_{r,t})$$

where $\varepsilon_{r,t}$ is a standard monetary-policy shock.

The government faces the following period budget constraint:

$$\frac{B_t}{P_t} = \frac{R_{t-1} B_t}{P_t} - \sum_h \sum_{i=1}^{60} \tau_{i}^{hi} + G_t.$$  

(38)

Here again, the government issues only nominal bonds and can tax its citizens heterogeneously. This is a realistic assumption as discussed in section 3.1.2. In this section the steady-state taxes will follow each agent’s income.\textsuperscript{25} For simplicity, I will assume that $G_t$ is exogenous and non-stochastic.

In the present version of the model, the rule for taxation is exogenously given below:

$$\tau_{i}^{hi} = \phi_0^{hi} \left( \frac{B_t}{P_t} \right)^{\phi_1}.$$  

(39)

where the coefficients of (39) are such that there is passive fiscal policy (i.e., the price level is not determined by (38)), i.e., $\phi_1 > 1$.

### 5.1.4 Uncertainty and the labor market

Following Storesletten, Telmer and Yaron (2001) (henceforth STY), I assume that each agent is endowed $N_t^{hi}$ units of effective labor, which they supply inelastically. Again, following STY, I assume that there are idiosyncratic shocks to the amount of effective labor, $A_t^{hi}$. Moreover, I assume, in contrast to previous literature, that there is also an aggregate component $A_t$ to $N_t^{hi}$. Agents, depending on their income group or on their age, can experience different effects of the aggregate productivity shock, $A_t$, on their effective labor endowment, as shown by agent-specific values of $\chi^{hi}$ in equation (40).

\textsuperscript{25}Weinzierl (2009) suggests that, in an environment with imperfect information, taxes contingent on age can be Pareto improving for the US. Although, it could help matching the life-cycle pattern of nominal positions, I do not allow these type of taxes in the model.
\[ N_{it}^{hi} = \chi^{hi} \exp(\chi^{hi} A_t + A_{hi}^t) \] (40)

The interpretation for the introduction of the aggregate shocks and different values of \( \chi^{hi} \) is that for some agents uninsurable income is more affected by business cycles shocks: a recession on average affects more the young than the retirees for example. The simple form of (40) is convenient for two reasons. First, as will be discussed in next section, it will enable me to use previous research using income data for the US, PSID, to calibrate the process for \( A_{hi}^t \). Second, it enables different exposure to business cycles risks keeping the convenience of a competitive labor market with a single market wage.

The three types shocks in the economy (the monetary policy shock, \( \varepsilon_{r,t} \); the aggregate shocks to agent’s effective labor, \( A_t \); and the agent-specific idiosyncratic shocks to the amount of effective labor, \( A_{hi}^t \)) are assumed to be AR(1).

5.1.5 Market Clearing

Markets should clear for each of the assets and for the good. For the good market, we have the following condition:

\[ \sum_h \sum_{j=1}^{60} C_{t}^{hj} + G_t + K_{t+1} - (1 - \delta)K_t = Y_t \] (41)

In the market for assets notice that there is an outside supply of nominal bonds, \( B_t \), by the government and that the total amount of capital holdings should equal aggregate capital. Market-clearing conditions are, then:

\[ \sum_h \sum_{j=1}^{60} B_{t}^{hj} = B_t \] (42)

\[ \sum_h \sum_{j=1}^{60} k_{t}^{hi} = K_t \] (43)

Labor market clearing implies that:

\[ \sum_h \sum_{j=1}^{60} N_{t}^{hj} = N_t \] (44)

5.1.6 Competitive Equilibrium

An equilibrium is a set of quantities and prices, \( \{C_{t}^{hj}, B_{t}^{hj}, \sigma_{t}^{hi}, N_{i}^{hi}, Y_t, N_t, K_t, G_t, P_t, R_t, r_t, \omega_t\} \), and exogenous processes \( \{A_t, A_{hi}^t, \varepsilon_{r,t}\} \), for all \( t \geq 0 \), all age groups, \( i = \{1 : 60\} \), and income groups, \( h = \{Rich, Middle, Poor\} \), such that (i) agents maximize (31) subject to (33); (ii) government obey (38) and follow policies (28) and (39); (iii) markets clear: (41), (42), (43) and (44) hold.
5.2 Solution method

I apply the Devereux and Sutherland (2008) method to solve for international portfolio positions to the case of closed-economy overlapping generation model.\textsuperscript{26}

The detailed derivation is in the appendix and here, I just describe the basic procedure that follows Devereux and Sutherland. First, I log-linearize the system of equations that represents the economy around the non-stochastic steady-state. The first-order system can be written with the steady-state portfolio holdings appearing only multiplied by the differential of returns of the assets. Then, I consider the steady-state portfolio holdings multiplied by the differential in returns as additional exogenous shocks and use standard methods to solve the first-order system. After that, I approximate up to a second-order the Euler equations of the agents and substitute the endogenous variables as a function of the shocks and steady-state portfolio holdings. Lastly, I solve for the steady-state portfolio that satisfy the Euler equations.

5.3 Calibration.

Consumption and Income Steady-States: The steady state consumption ratios for different ages were calibrated using CEX average consumption of nondurable expanded goods, for the years between 1980 and 2004. For the steady-state income ratios across ages, I used the Current Population Survey, (1990), table HINC-02. For different classes, I also assumed income inequality increasing over the life-cycle, which is consistent with the literature on inequality over the life-cycle.\textsuperscript{27} For the income of the rich, I used data on the 20% top income distribution and for the poor the bottom 20% in income distribution. Steady-state taxes are a constant percentage of agents’ income, and the sum of steady-state taxes are set to fulfill the government budget constraint. This leads to a life-cycle pattern for steady-state wealth of the agents consistent with the model. The graph below shows the data on consumption and the resulting steady-state values for wealth.

\textbf{FIGURE 1 HERE}

Preference Parameters: In this model, there is no borrowing constraints and the parameters $a_{hi}$ change for different ages and income groups, in order for the steady-levels of consumption to satisfy the first-order conditions. The value for $\beta$ is standard and equal to 0.96. The value for the inverse of the elasticity of substitution, $\sigma = 2$, is on the range used in the literature. The intergenerational discount rate, $\theta$, is set to 0.9. This parameter value affects the steady-state level of interest rates, but the results are robust to alternative values of $\theta$.

The complementarity between effective labor and consumption plays an important role in the model: it allows for a realistic correlation between output (and thus consumption) and inflation.

\textsuperscript{26}Dedola and Lombardo (2008) explore the equivalence between Devereux and Sutherland (2009)’s method to Judd and Guu (2001) alternative solution and show, that at least for some class of models they lead to the same portfolio solutions.

\textsuperscript{27}In the benchmark calibration the ratio between income of the rich over income of the poor starts at 1.5 and finishes around 3. The results do not change qualitatively if no increase in inequality is imposed or if slightly steeper changes are imposed.
The introduction of this complementarity leads to an increase in demand that coincides with an increase in output allowing for realistic correlations between inflation shocks and consumption innovations.\(^{28}\) Hence, the complementarity parameter, \(\varphi\), is set to 5, in order to achieve the positive correlation of aggregate consumption and inflation shocks that match the evidence of Table 2, column (1-B).

**Government, Interest Rate and Capital:** Government debt over GDP is set on 40% in the benchmark steady-state. This is roughly the level of US debt in the beginning of our sample for consumption, in the early eighties. The results remain similar for \(\frac{B}{Y} = 50\%\) or 60%. In the benchmark case, the policy parameter \(\phi_{h}^{i}\) is set to 2, \(\forall h, i\), making fiscal policy passive in the price level determination. As mentioned before, steady-state taxes are proportional to each agent’s income as follows:

\[
\tau_{hi} = \frac{\bar{N}^{hi}}{\sum_{h} \sum_{i} N_{hi}} \left( \left( \bar{R} - 1 \right) \bar{B} + \bar{G} \right)
\]

I assume that the depreciation rate \(\delta = 0.075\). This parameters value combined with the values for \(\theta\) and \(\beta\) imply the steady-rate one-year real interest rate to be 4.3\%. The capital share of output, \(\alpha\), is equal to 0.4. This implies that aggregate capital over GDP in steady-state, \(\frac{K}{Y}\), equals 3.38. Total net worth of the agents over GDP in this economy is 3.78, in line with estimates from the flow of funds (around 4.1 for 2007).

**Exogenous Process:** To estimate the parameters of the aggregate exogenous processes, I use yearly US data from 1960 to 2008. Using the method outlined in Heathcote and Perri (2008), I computed the parameters of the process for productivity shock \(A_{t}\) from the production function. Using real GDP and total non-farm hours, the persistence parameter is set to 0.9049 and \(\sigma_{\Delta}^{2} = 0.064\%\). Since \(\gamma = 1.5\), one can use (37) to construct a series for \(\varepsilon_{R,t}\) and compute its persistence, 0.48, and standard deviation, \(\sigma_{\varepsilon_{R}}^{2}\), 0.09\%. For the idiosyncratic income shocks, I follow Storelleton, Telmer and Yaron (2002) and set the persistence to 0.91. The variance of the idiosyncratic, \(\sigma_{A_{i}}^{2}\), is set to 5\%, in line with Heaton and Lucas (1996) estimates and between the high and low estimates of \(\sigma_{A_{i}}^{2}\), Storelleton, Telmer and Yaron (2001).

The calibration of \(\chi_{hi}\) will be detailed in the following section.

## 6 Quantitative Model Results

In the results section, I focus on the following questions (i) how much heterogeneity in agents’ disposable income is necessary for the nominal portfolio holdings in the model to match the ones in the data? (ii) Is the resulting impact in consumption of 1% inflation shock, consistent with estimates in section 2?

The two following subsections focus on answering this question for two cases. In the first one, income groups are aggregated and the results are compared to the evidence in different age groups (table 1 -column (1) and table 2 - columns (2)). In the second case, I also consider different income

\(^{28}\)An alternative to this assumption is the introduction of sticky prices and cost-push shocks.
groups and the results are compared to the evidence for different income groups (table 1 - columns (2)-(4) and table 2 - columns (3)-(5)).

6.1 Age groups

In this subsection, the income groups are ignored, as I calibrate the model only for the average life-cycle profiles in the US economy.

6.1.1 Calibration of $\chi^i$ to match net nominal positions

The parameters $\chi^i$ are calibrated in order to achieve the net nominal positions in the data (Table 1 - first column). Intuitively, one would expect that the business-cycle risk on uninsurable income not to be discontinuous on age. I incorporate that intuition and limit the number of free parameters in the calibration by assuming that $\chi^i$ is a fourth-order polynomial on age:

$$
\chi^i = \mu_0 + \mu_1 i + \mu_2 i^2 + \mu_3 i^3 + \mu_4 i^4
$$

and the $\mu$ parameters are chosen to minimize the mean square difference of the average net nominal positions over wealth in the data and the model. This procedure results in the age-profile for $\chi^i$ in figure 2. There is a similar load on the aggregate risks across agents until the age of 65, after this age the impact of $A_t$ on uninsurable income falls sharply. This is in line with the common assumption that uninsurable income risks decrease by the age of retirement (see STY (2000), Rios-Rull (1993), for example), as well as with the evidence that retirees have their income more protected against inflation shocks due to indexation (as argued in Modigliani and Fisher, 1978).

FIGURE 2 HERE

As clear in figure 3, this age profile matches the evidence on net nominal positions very closely. Formal measures of evaluation also indicate that: there is an average difference of 4.4% between the model and the data. A regression where the model NNP/Wealth for age groups explains the data counterpart has a $R^2$ of 0.9598.

FIGURE 3 HERE

Why are young agents short in nominal assets? It is important to remember that in this economy marginal utility depends on both consumption and labor. When $A_t$ shock hits the marginal utility of the agents below 65 increase, pointing to higher adjustment in consumption. These agents hold, then, more of the assets with relatively higher return, equity in this case, in order to finance higher consumption levels. Given the low steady-state levels of wealth and income for agents below 45

\footnote{The order of the polynomial was determined balanced a trade-off of fit of nominal positions of the model against the dimensionality of the maximization procedure. A third-order polynomial for (46) shows a good fit for all age groups analysis in this section. However, in the different income groups case, there are substantial increases in fit by assuming fourth-order polynomial.}
year old, in order to support the consumption adjustment agents need to be short in nominal terms. For agents above 65, the impact $A_t$ on marginal utility decreases, pointing to smaller consumption adjustment when $A_t$ hits. This way, these agents are willing to hold relatively more of the asset with relatively lower returns, nominal bonds in this case. As a conclusion, the complementarity make the working-age agents willing to absorb more of the aggregate fluctuations in productivity. This effect is moderated by the monetary policy shock. This shock only affects the relative return on nominal bonds in this flexible-price model, leading to redistribution across agents with different nominal positions.

Note that the channel that determine agents’ nominal positions in the quantitative model is different from the one in simple model. In the latter, agents with more exposure to aggregate shocks ended up holding less bonds, because the relative return of the bonds (in relation with the riskless asset) was positively correlated with uninsurable income. In the model in this section, however, the relative return of bond with respect to capital is negatively correlated with $A_t$ and, thus, with labor income. As discussed in the previous paragraph, the complementarity between labor and consumption in the quantitative model plays a key role in altering the role of asset holdings in smoothing marginal utility. The two models share, however, a common feature: insurance (in terms of smoothing marginal consumption) plays a crucial role in determining nominal positions as a fraction of total assets and the winners and losers of unexpected inflationary episodes.

### 6.1.2 Impact on Consumption

Given that the nominal positions are matched, can this model deliver consumption reaction to inflation shocks similar to the estimates in section 2? The first thing to point out is that in this flexible-price framework, all shocks in the model affect the price level on impact. So, it is natural to present the expected consumption adjustment, given a 1% inflation realization, $E_{t-1}(C_t^i/\pi_t = 1%)$.

Figure 4 plots the estimates of the consumption adjustment (nondurables expanded) for different ages given a 1% inflation shock from table 2 - columns (2), as well as mean of $E_{t-1}(C_t^i/\pi_t = 1%)$ of the model. The consumption adjustment in the model tracks really closely the ones on the data and is almost always in the center of the confidence interval band. This is considered a strength of the model, since there was no free-parameter calibrated to generate this consumption adjustment profile through the life cycle.

![FIGURE 4 HERE](image)

In the model, the young agents, that on average benefit the most from nominal asset revaluation, adjust their consumption less than the 36-65 years old. And even the older groups that lose in their nominal position in case of inflation adjust positively their consumption. The explanation is clear and is similar to the one in section 3: unpredictable movements in inflation coincide with positive

---

30 In the model solution both $C_t^i$ and $\pi_t$ can be written as linear function of contemporaneous and past shocks of the model. Assuming that the shocks are normal, a simple normal update is needed to calculate $C_t^i/\pi_t = 1%$.

31 Note, however, that the complementarity between labor and consumption was set to match aggregate consumption adjustment given a 1% inflation surprise, as mentioned in section 5.3.
chances in other components of agent wealth: uninsurable income and equity returns. Also, there is an additional channel, the complementarity between labor and consumption, leading to higher levels of consumption for agents with higher loads on aggregate risk.

6.2 Different Income groups

In this subsection, the model is disaggregated in the different income groups described in section 5. Here the questions are similar to those of section 6.1: (i) how should the uninsurable income of different income groups be impacted by the aggregate shock, in order to match the evidence on Table 1, columns (2)-(4)? (ii) Which income groups have uninsurable income more exposed to the aggregate risk $A_t$? Are there differences on the life-cycle profile of aggregate risk across income groups? (iii) Lastly, is the model able to match the evidence on consumption adjustment for different income groups (Table 2, columns (3)-(5))? 

6.2.1 Calibration of $\chi^{hi}$ to match net nominal positions

The procedure of calibration for $\chi^{hi}$ is exactly the same as described in 6.1.1, with three fourth-order polynomials (one for each income group) in place of equation (46). Each income group has the same weight on the calibration, therefore $\mu^h_k$, $k = 0 : 4$ are chosen in order to minimize the sum across income and age groups of the square deviation between the average nominal holdings over groups’ wealth on the model and the data. The implied age-profile of aggregate risk on uninsurable income is presented in figure 5.

FIGURE 5 HERE

If one considers an average over the life cycle, the exposure of the uninsurable income decreases with income. The interpretation is that low income jobs are more affected by the business cycles shocks than high skill jobs, which is in line with the evidence in Caneda et al (1997). All groups decrease their exposure to aggregate risk by the age of retirement as in the previous section. The middle-class needs a steeper profile on $\chi^{middle,i}$ over the life cycle, in order to match the extreme nominal positions that goes from -114% to +38%, from young to old.

In this case, the exercise of matching the nominal holdings calibrating the $\mu^h_k$ is more involved, since there is now trade across income groups. The intuition is, however, the same as the one discussed in section 6.1.1: the productivity shock, $A_t$, and the complementarity between labor and consumption imply that agents more exposed to the aggregate productivity shock should hold less of the nominal bonds and be more exposed to fluctuations in aggregate productivity. This should be weighted by the risk associated with the monetary policy shock that affect the return of the nominal assets.

Again, as shown in Figure 6, the model is close to the data. The average difference between the model and the data for the poor, middle-class are, respectively 3.6% (with net nominal assets in the data on the range from -36.6 to 26.4%), 12.9% (data values from -114% to 38.1%) and
3.8% (data values from -4% to 27.5%). Moreover, regressions explaining the nominal positions over wealth from the data using the model counterpart lead to $R^2$ of 0.9657 for the poor, 0.9329 for the middle-class and 0.8838 for the rich.

**FIGURE 6 HERE**

### 6.2.2 Impact on Consumption

In this section, I replicate the exercise of section 6.2 for the different income groups. Figure 7 compares $E_{t-1}(C_i^t/\pi_t = 1\%)$ in the model with the estimates of table 2.

**FIGURE 7 HERE**

The poor and rich show a non-monotone consumption adjustment that clearly does not follow the transfers due to revaluation of nominal portfolio. Moreover, for these two groups the impact on consumption in the model resembles the estimates in table 2. All rich agents above 35 suffer losses in their nominal positions and still enjoy positive consumption adjustment. Poor below 35 years-old shows smaller consumption adjustments than the poor between 45-65 years old, clearly not following the transfers through asset positions.

For the old middle-class, however, the expected impact on consumption is lower in the model than the estimates. The explanation is that in order to get such extreme change in nominal positions through the life-time, the model requires a steep decreasing life-time profile of $\chi_{middle,i}$. This way, the productivity shocks affects the marginal utility of the agents very differently: positively for lower ages and negatively for retired.

Even not matching the middle-class consumption adjustments, this quantitative exercise makes some points clear. First, if agents are heterogeneous on their uninsurable income, there is a hedging role for the relative amount of nominal assets on their portfolio. Second, a calibrated model can generate the life-cycle pattern of nominal portfolio. Third, consumption adjustment does not follow nominal portfolio revaluation in case of inflation. It tends to be positive and similar across agents, as in the data.

### 6.2.3 Different Tax Rules

Here I allow taxes to respond to innovations in real government debt, accordingly to income. More precisely, I separate the agents in three income brackets $j = \{Low, Median, High\}$\(^{32}\) and define different tax rules for each group.

$$
\tau_{hi}^t = \phi_0^h \prod_{i=0}^{N} \left( \frac{B_{t-i}}{P_{t-i}} \right) \phi_1^i,
$$

\(^{32}\)Note that these groups do not coincide with the income groups. A middle-age poor agents are for example $j = median$, since at the age they are not at the bottom third of income distribution. Nonetheless, they are poor, since they have lower life-time labor income.
where $\phi_0^{hi}$ is defined to keep the steady-state values defined in (45). Taxes also respond to innovations to contemporaneous and past N-1 periods real government debt. Keeping the loads on aggregate risk described in section 6.2.1, I calibrate $\phi_1^{ji}$ for the three income brackets in order to minimize the sum across income and age groups of the square deviation between the average nominal holdings over groups’ wealth on the model and the data. The results for the nominal asset holdings are shown in figure 8. Naturally, it improves the fit of the model, however, with the exception of the rich group, is quantitatively less relevant.\footnote{In this example, I set N to 5.}

**FIGURE 8 HERE**

The calibrated total tax adjustment that results for this experiment implies the largest overall tax adjustment for the rich, while the poor bear the smallest adjustment. This is results seems in accordance with tax adjustments for the wealthiest in recent US history.\footnote{An example is the signal of the Obama administration that future taxes for households that earn more than $250,000 will increase, as a reaction to recent innovations on real government debt.}

<table>
<thead>
<tr>
<th></th>
<th>Total Tax Adjustment to $\Delta \left( \sum_i \frac{B_{t-i}}{P_{t-i}} \right)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>1.2721</td>
</tr>
<tr>
<td>Median</td>
<td>1.5018</td>
</tr>
<tr>
<td>High</td>
<td>2.6757</td>
</tr>
</tbody>
</table>

### 6.3 Robustness of the model

I tried two alternative asset structures in this model. First, I added real government bonds to capital and nominal bonds. With this asset structure, the main results remain quantitative and qualitative similar. Second, I re-wrote the model in the case without capital and a ratio of real to nominal bonds supplied exogenously by the government to match the US data. In this case, there is still heterogeneous nominal assets through the life cycle and smooth consumption responses to inflation surprises. However, the levels of nominal assets holdings depend on the fraction of nominal assets to total wealth, too low in this case.

Finally, I conduct exhaustive checks on the calibration of the parameters governing the shocks. First, I allowed for the life-time profile of the variance of idiosyncratic labor income shocks, $\sigma_{Ai}^2$. Second, instead of $\sigma_{Ai}^2 = 5\%$, I experiment with either the upper and lower estimates of this parameter in STY, 3.7% and 18.1%. Third, I re-calibrate the shocks in different time periods (i) 1980 - 2004, (ii)1960 -1990. None of these alternative calibrations change significantly the conclusions of the paper.
7 Conclusion

This paper presents evidence that unanticipated inflation correlates positively with agents’ consumption. In addition, consumption adjustments are similar across agents in different groups. If one takes consumption as a proxy for total wealth, this evidence suggests that agents were, on average, insured against past inflation surprises.

In addition to implications to inequality, measuring redistributive effects of inflation shocks are crucial to evaluate monetary policy models. The workhorse new Keynesian models, for example, ignore redistributive effects of inflation focusing on inefficient price dispersion as the main source of welfare loss associated with shocks to inflation. In these models, the existence of complete markets eliminates all redistribution effects of aggregate and idiosyncratic shocks. Evidence of large redistribution would undermine the complete-market/representative-agent assumption, as well as point to models where shocks to inflation impact wealth redistribution and, consequently, welfare. This paper suggests that, recently, this has not been the case for aggregates shocks that affect inflation in the US.

A model reconciles this evidence with heterogeneity in net nominal asset positions in the US data, where nominal assets play an insurance role against shocks that affect the price level. Two key features of the model drives this results: nominal asset holdings as result of portfolio decisions and heterogeneous aggregate risks on agents’ disposable income. Moreover, it suggests that systematically heterogeneous fiscal policy leads to heterogeneity in nominal holdings.

I plan to extend the analysis in this paper for the open-economy case. This way, it will be possible to address the implications of the recent accumulation of US nominal assets by foreigners. Another interesting research avenue is the implication of redistribution to optimal policy in the presence of nominal rigidities and endogenous portfolio decisions. This extension of the new Keynesian literature may bring interesting insights on the role of policy in redistributive effects of aggregate shocks. A third extension is to incorporate a term structure in the model, allowing for trading in nominal bonds with different maturities.
8 Tables and Figures

<table>
<thead>
<tr>
<th>Age Group</th>
<th>All income</th>
<th>Rich</th>
<th>Middle</th>
<th>Poor</th>
</tr>
</thead>
<tbody>
<tr>
<td>less 35</td>
<td>-0.426</td>
<td>-0.04</td>
<td>-1.14</td>
<td>-0.366</td>
</tr>
<tr>
<td>36-45</td>
<td>-0.101</td>
<td>0.038</td>
<td>-0.316</td>
<td>-0.338</td>
</tr>
<tr>
<td>46-55</td>
<td>0.023</td>
<td>0.066</td>
<td>-0.048</td>
<td>-0.055</td>
</tr>
<tr>
<td>55-65</td>
<td>0.152</td>
<td>0.163</td>
<td>0.14</td>
<td>0.075</td>
</tr>
<tr>
<td>65-75</td>
<td>0.194</td>
<td>0.167</td>
<td>0.252</td>
<td>0.175</td>
</tr>
<tr>
<td>more 75</td>
<td>0.306</td>
<td>0.275</td>
<td>0.381</td>
<td>0.264</td>
</tr>
</tbody>
</table>

Table 1: Net Nominal Positions
<table>
<thead>
<tr>
<th>π surprise</th>
<th>1.254*** (0.190)</th>
<th>0.708*** (0.116)</th>
</tr>
</thead>
<tbody>
<tr>
<td>π surprise, less 36</td>
<td>1.187*** (0.348)</td>
<td>0.747*** (0.205)</td>
</tr>
<tr>
<td>π surprise, 36 - 45</td>
<td>1.368*** (0.408)</td>
<td>0.780*** (0.260)</td>
</tr>
<tr>
<td>π surprise, 46 - 55</td>
<td>1.654*** (0.486)</td>
<td>1.084*** (0.300)</td>
</tr>
<tr>
<td>π surprise, 56 - 65</td>
<td>1.046*** (0.501)</td>
<td>0.699*** (0.315)</td>
</tr>
<tr>
<td>π surprise, 66 - 75</td>
<td>1.075*** (0.522)</td>
<td>0.556* (0.332)</td>
</tr>
<tr>
<td>π surprise, 76 plus</td>
<td>1.128* (0.646)</td>
<td>-0.099 (0.413)</td>
</tr>
<tr>
<td>white</td>
<td>0.237*** (0.002)</td>
<td>0.237*** (0.001)</td>
</tr>
<tr>
<td>male</td>
<td>0.147*** (0.002)</td>
<td>0.147*** (0.002)</td>
</tr>
<tr>
<td>urban</td>
<td>0.062*** (0.004)</td>
<td>0.062*** (0.004)</td>
</tr>
<tr>
<td>elementary school</td>
<td>0.205*** (0.014)</td>
<td>0.205*** (0.014)</td>
</tr>
<tr>
<td>some high school</td>
<td>0.347*** (0.013)</td>
<td>0.347*** (0.013)</td>
</tr>
<tr>
<td>high school grad</td>
<td>0.592*** (0.013)</td>
<td>0.592*** (0.013)</td>
</tr>
<tr>
<td>some college</td>
<td>0.707*** (0.013)</td>
<td>0.707*** (0.013)</td>
</tr>
<tr>
<td>college grad</td>
<td>0.963*** (0.013)</td>
<td>0.963*** (0.013)</td>
</tr>
<tr>
<td>more than college</td>
<td>1.047*** (0.014)</td>
<td>1.047*** (0.014)</td>
</tr>
<tr>
<td>less than 36</td>
<td>-0.306*** (0.003)</td>
<td>-0.306*** (0.003)</td>
</tr>
<tr>
<td>36-45</td>
<td>-0.116*** (0.003)</td>
<td>-0.116*** (0.003)</td>
</tr>
<tr>
<td>46-55</td>
<td>0.026*** (0.003)</td>
<td>0.026*** (0.003)</td>
</tr>
<tr>
<td>56-65</td>
<td>0.06*** (0.003)</td>
<td>0.06*** (0.003)</td>
</tr>
<tr>
<td>66-75</td>
<td>0.090*** (0.003)</td>
<td>0.090*** (0.003)</td>
</tr>
<tr>
<td>cons</td>
<td>6.967*** (0.014)</td>
<td>6.967*** (0.014)</td>
</tr>
</tbody>
</table>

**0.10 ***0.05 ****0.01
US region and quarter dummies and time trend includes, not reported
CEX - 1980Q1 - 2004Q4
Huber-White Estimator of Standard Errors in Parenthesis; F. E. = Fixed Effects

Table 2: Benchmark regressions
### Table 3: 1% Inflation Surprise on Different Consumption Measures

<table>
<thead>
<tr>
<th></th>
<th>(0)</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Benchmark</td>
<td>Non-Durables</td>
<td>Non-Durables per capita</td>
<td>Total Expenditure per capita</td>
<td>Food Expenditure per capita</td>
</tr>
<tr>
<td>π surprise, less 36</td>
<td>1.187***</td>
<td>1.007***</td>
<td>1.452***</td>
<td>1.665***</td>
<td>0.425</td>
</tr>
<tr>
<td></td>
<td>(0.348)</td>
<td>(0.389)</td>
<td>(0.335)</td>
<td>(0.392)</td>
<td>(0.392)</td>
</tr>
<tr>
<td>π surprise, 36 - 45</td>
<td>1.368***</td>
<td>1.019***</td>
<td>1.179***</td>
<td>2.156***</td>
<td>0.631</td>
</tr>
<tr>
<td></td>
<td>(0.408)</td>
<td>(0.434)</td>
<td>(0.413)</td>
<td>(0.482)</td>
<td>(0.447)</td>
</tr>
<tr>
<td>π surprise, 46 - 55</td>
<td>1.654***</td>
<td>1.212***</td>
<td>1.148***</td>
<td>1.977***</td>
<td>1.236**</td>
</tr>
<tr>
<td></td>
<td>(0.486)</td>
<td>(0.521)</td>
<td>(0.501)</td>
<td>(0.577)</td>
<td>(0.547)</td>
</tr>
<tr>
<td>π surprise, 56 - 65</td>
<td>1.046**</td>
<td>0.831</td>
<td>0.927*</td>
<td>1.556***</td>
<td>-0.336</td>
</tr>
<tr>
<td></td>
<td>(0.501)</td>
<td>(0.544)</td>
<td>(0.533)</td>
<td>(0.607)</td>
<td>(0.596)</td>
</tr>
<tr>
<td>π surprise, 66 - 75</td>
<td>1.075**</td>
<td>1.207**</td>
<td>1.496***</td>
<td>1.450**</td>
<td>1.216*</td>
</tr>
<tr>
<td></td>
<td>(0.522)</td>
<td>(0.567)</td>
<td>(0.565)</td>
<td>(0.634)</td>
<td>(0.649)</td>
</tr>
<tr>
<td>π surprise, 76 plus</td>
<td>1.128*</td>
<td>1.285*</td>
<td>0.628</td>
<td>1.355*</td>
<td>1.194</td>
</tr>
<tr>
<td></td>
<td>(0.646)</td>
<td>(0.719)</td>
<td>(0.785)</td>
<td>(0.796)</td>
<td>(0.825)</td>
</tr>
<tr>
<td>white</td>
<td>0.237***</td>
<td>0.187***</td>
<td>0.198***</td>
<td>0.243***</td>
<td>0.187***</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.003)</td>
<td>(0.002)</td>
<td>(0.003)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>male</td>
<td>0.147***</td>
<td>0.244***</td>
<td>0.144***</td>
<td>0.172***</td>
<td>0.190***</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.002)</td>
<td>(0.002)</td>
<td>(0.002)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>urban</td>
<td>0.062***</td>
<td>0.446***</td>
<td>0.056***</td>
<td>0.125***</td>
<td>0.106***</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.004)</td>
<td>(0.004)</td>
<td>(0.005)</td>
<td>(0.005)</td>
</tr>
<tr>
<td>elementary school</td>
<td>0.205***</td>
<td>0.146***</td>
<td>0.186***</td>
<td>0.182***</td>
<td>0.055**</td>
</tr>
<tr>
<td></td>
<td>(0.014)</td>
<td>(0.015)</td>
<td>(0.014)</td>
<td>(0.015)</td>
<td>(0.015)</td>
</tr>
<tr>
<td>some high school</td>
<td>0.347***</td>
<td>0.258**</td>
<td>0.317***</td>
<td>0.324***</td>
<td>0.130***</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.015)</td>
<td>(0.014)</td>
<td>(0.015)</td>
<td>(0.015)</td>
</tr>
<tr>
<td>high school grad</td>
<td>0.592***</td>
<td>0.480**</td>
<td>0.508***</td>
<td>0.566***</td>
<td>0.260***</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.015)</td>
<td>(0.014)</td>
<td>(0.015)</td>
<td>(0.014)</td>
</tr>
<tr>
<td>some college</td>
<td>0.707***</td>
<td>0.532**</td>
<td>0.615***</td>
<td>0.690***</td>
<td>0.301***</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.015)</td>
<td>(0.014)</td>
<td>(0.015)</td>
<td>(0.015)</td>
</tr>
<tr>
<td>college grad</td>
<td>0.963***</td>
<td>0.785**</td>
<td>0.810***</td>
<td>0.946***</td>
<td>0.475***</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.015)</td>
<td>(0.014)</td>
<td>(0.015)</td>
<td>(0.015)</td>
</tr>
<tr>
<td>more than college</td>
<td>1.047***</td>
<td>0.870***</td>
<td>0.892***</td>
<td>1.033***</td>
<td>0.529***</td>
</tr>
<tr>
<td></td>
<td>(0.014)</td>
<td>(0.015)</td>
<td>(0.014)</td>
<td>(0.015)</td>
<td>(0.015)</td>
</tr>
<tr>
<td>less than 36</td>
<td>-0.306***</td>
<td>-0.306***</td>
<td>-0.200***</td>
<td>-0.481***</td>
<td>-0.056***</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.003)</td>
<td>(0.003)</td>
<td>(0.004)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>36-45</td>
<td>-0.116***</td>
<td>-0.116***</td>
<td>-0.028***</td>
<td>-0.031***</td>
<td>0.146***</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.003)</td>
<td>(0.004)</td>
<td>(0.004)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>46-55</td>
<td>0.026***</td>
<td>0.026**</td>
<td>0.093***</td>
<td>0.121***</td>
<td>0.223**</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.003)</td>
<td>(0.004)</td>
<td>(0.004)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>56-65</td>
<td>0.06***</td>
<td>0.06**</td>
<td>0.105***</td>
<td>0.221***</td>
<td>0.186**</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.003)</td>
<td>(0.004)</td>
<td>(0.004)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>66-75</td>
<td>0.090***</td>
<td>0.090***</td>
<td>0.130***</td>
<td>0.186***</td>
<td>0.205***</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.003)</td>
<td>(0.004)</td>
<td>(0.004)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>_cons</td>
<td>6.969***</td>
<td>6.969***</td>
<td>6.493***</td>
<td>8.597***</td>
<td>7.639***</td>
</tr>
<tr>
<td></td>
<td>(0.014)</td>
<td>(0.014)</td>
<td>(0.014)</td>
<td>(0.092)</td>
<td>(0.099)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.2754</td>
<td>0.2754</td>
<td>0.2061</td>
<td>0.2221</td>
<td>0.1435</td>
</tr>
<tr>
<td>Observations</td>
<td>425797</td>
<td>425797</td>
<td>425797</td>
<td>425797</td>
<td>212901</td>
</tr>
</tbody>
</table>

*0.10 ** 0.05 *** 0.01
US region and quarter dummies and time trend includes, not reported
CEX - 1980Q1 - 2004Q4
Huber-White Estimator of Standard Errors in Parenthesis

Table 3: Robustness to consumption measures
### Table 4: Robustness to different consumption measures - Income groups

<table>
<thead>
<tr>
<th></th>
<th>(1 - A) Non-Durables Expanded</th>
<th>(1 - B) Non-Durables per capita</th>
<th>(1 - C) Total Expenditure per capita</th>
<th>(2 - A) Non-Durables Expanded</th>
<th>(2 - B) Non-Durables per capita</th>
<th>(2 - C) Total Expenditure per capita</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>π surprise, less 36</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rich</td>
<td>1.712***</td>
<td>0.906***</td>
<td>0.05</td>
<td>0.929***</td>
<td>0.635***</td>
<td>-0.296</td>
</tr>
<tr>
<td>Middle-class</td>
<td></td>
<td></td>
<td></td>
<td>1.739***</td>
<td>1.027***</td>
<td>-0.247</td>
</tr>
<tr>
<td>Poor</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.322)</td>
<td>(0.189)</td>
<td>(0.639)</td>
<td>(0.350)</td>
<td>(0.171)</td>
<td>(0.484)</td>
</tr>
<tr>
<td></td>
<td>(0.322)</td>
<td>(0.189)</td>
<td>(0.639)</td>
<td>(0.350)</td>
<td>(0.171)</td>
<td>(0.484)</td>
</tr>
<tr>
<td><strong>π surprise, 36 - 45</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rich</td>
<td>0.843**</td>
<td>0.900***</td>
<td>1.280**</td>
<td>0.840*</td>
<td>1.057***</td>
<td>1.820***</td>
</tr>
<tr>
<td>Middle-class</td>
<td></td>
<td></td>
<td></td>
<td>2.047***</td>
<td>2.011***</td>
<td>1.544***</td>
</tr>
<tr>
<td>Poor</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.459)</td>
<td>(0.228)</td>
<td>(0.640)</td>
<td>(0.482)</td>
<td>(0.219)</td>
<td>(0.553)</td>
</tr>
<tr>
<td></td>
<td>(0.459)</td>
<td>(0.228)</td>
<td>(0.640)</td>
<td>(0.482)</td>
<td>(0.219)</td>
<td>(0.553)</td>
</tr>
<tr>
<td><strong>π surprise, 46 - 55</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rich</td>
<td>0.546</td>
<td>0.767***</td>
<td>1.813**</td>
<td>0.52</td>
<td>0.41</td>
<td>1.220*</td>
</tr>
<tr>
<td>Middle-class</td>
<td></td>
<td></td>
<td></td>
<td>2.628***</td>
<td>1.116***</td>
<td>0.761</td>
</tr>
<tr>
<td>Poor</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.556)</td>
<td>(0.275)</td>
<td>(0.799)</td>
<td>(0.594)</td>
<td>(0.266)</td>
<td>(0.678)</td>
</tr>
<tr>
<td></td>
<td>(0.556)</td>
<td>(0.275)</td>
<td>(0.799)</td>
<td>(0.594)</td>
<td>(0.266)</td>
<td>(0.678)</td>
</tr>
<tr>
<td><strong>π surprise, 56 - 65</strong></td>
<td></td>
<td></td>
<td></td>
<td>1.141*</td>
<td>0.464</td>
<td>-0.211</td>
</tr>
<tr>
<td>Rich</td>
<td></td>
<td></td>
<td></td>
<td>1.777**</td>
<td>0.311</td>
<td>0.917</td>
</tr>
<tr>
<td>Middle-class</td>
<td></td>
<td></td>
<td></td>
<td>3.482***</td>
<td>1.076***</td>
<td>1.676**</td>
</tr>
<tr>
<td>Poor</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.608)</td>
<td>(0.286)</td>
<td>(0.789)</td>
<td>(0.627)</td>
<td>(0.280)</td>
<td>(0.726)</td>
</tr>
<tr>
<td></td>
<td>(0.608)</td>
<td>(0.286)</td>
<td>(0.789)</td>
<td>(0.627)</td>
<td>(0.280)</td>
<td>(0.726)</td>
</tr>
<tr>
<td><strong>π surprise, 66 - 75</strong></td>
<td></td>
<td></td>
<td></td>
<td>0.386</td>
<td>0.808***</td>
<td>2.112**</td>
</tr>
<tr>
<td>Rich</td>
<td></td>
<td></td>
<td></td>
<td>0.86</td>
<td>1.214***</td>
<td>2.083**</td>
</tr>
<tr>
<td>Middle-class</td>
<td></td>
<td></td>
<td></td>
<td>1.19</td>
<td>1.524***</td>
<td>1.803**</td>
</tr>
<tr>
<td>Poor</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.638)</td>
<td>(0.308)</td>
<td>(0.833)</td>
<td>(0.707)</td>
<td>(0.308)</td>
<td>(0.717)</td>
</tr>
<tr>
<td></td>
<td>(0.638)</td>
<td>(0.308)</td>
<td>(0.833)</td>
<td>(0.707)</td>
<td>(0.308)</td>
<td>(0.717)</td>
</tr>
<tr>
<td><strong>π surprise, 76 plus</strong></td>
<td></td>
<td></td>
<td></td>
<td>0.973</td>
<td>1.213***</td>
<td>1.432</td>
</tr>
<tr>
<td>Rich</td>
<td></td>
<td></td>
<td></td>
<td>1.45</td>
<td>1.213***</td>
<td>-0.125</td>
</tr>
<tr>
<td>Middle-class</td>
<td></td>
<td></td>
<td></td>
<td>0.772</td>
<td>1.200***</td>
<td>0.328</td>
</tr>
<tr>
<td>Poor</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.917)</td>
<td>(0.422)</td>
<td>(0.994)</td>
<td>(1.108)</td>
<td>(0.422)</td>
<td>(0.950)</td>
</tr>
<tr>
<td></td>
<td>(0.917)</td>
<td>(0.422)</td>
<td>(0.994)</td>
<td>(1.108)</td>
<td>(0.422)</td>
<td>(0.950)</td>
</tr>
</tbody>
</table>

**Notes:**
- US region and quarter dummies and time trend included, not reported
- CEX - 1980Q1 - 2004Q4
- Huber-White Estimator of Standard Errors in Parentheses
- R-squared: 0.3307, 0.4887, 0.2932, 0.1293, 0.0093, 0.2409, 0.1829
- Observations: 106373, 212895, 106529, 106373, 212905, 106512, 106373, 212901, 106517

Table 4: 1% Inflation Surprise on Different Consumption Measures - Income Groups
Table 5: 1% Inflation Surprise on Non-Durables Expanded per Capita - Alternative Inflation Surprises

<table>
<thead>
<tr>
<th></th>
<th>OLS Var 3 Full Sample</th>
<th>OLS Var 6 Full Sample</th>
<th>Fixed Effects Var 3 Full Sample</th>
<th>Fixed Effects Var 6 Full Sample</th>
<th>OLS - Growth - Var 3 Full Sample</th>
<th>OLS - Growth - Var 6 Full Sample</th>
<th>OLS - First Lag - Var 3 Full Sample</th>
<th>OLS - First Lag - Var 6 Full Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi ) surprise, less 36</td>
<td>1.365***</td>
<td>1.129***</td>
<td>0.691***</td>
<td>0.768***</td>
<td>0.587*</td>
<td>0.904***</td>
<td>0.845**</td>
<td>0.620*</td>
</tr>
<tr>
<td></td>
<td>(0.339)</td>
<td>(0.366)</td>
<td>(0.203)</td>
<td>(0.203)</td>
<td>(0.318)</td>
<td>(0.343)</td>
<td>(0.323)</td>
<td>(0.323)</td>
</tr>
<tr>
<td>( \pi ) surprise, 36 - 45</td>
<td>1.452***</td>
<td>0.880**</td>
<td>0.821***</td>
<td>0.869***</td>
<td>1.228***</td>
<td>2.158***</td>
<td>2.136**</td>
<td>1.764**</td>
</tr>
<tr>
<td></td>
<td>(0.400)</td>
<td>(0.422)</td>
<td>(0.260)</td>
<td>(0.278)</td>
<td>(0.328)</td>
<td>(0.347)</td>
<td>(0.381)</td>
<td>(0.381)</td>
</tr>
<tr>
<td>( \pi ) surprise, 46 - 55</td>
<td>1.436***</td>
<td>1.673***</td>
<td>1.036***</td>
<td>0.889***</td>
<td>1.105***</td>
<td>0.963**</td>
<td>1.592***</td>
<td>1.855***</td>
</tr>
<tr>
<td></td>
<td>(0.477)</td>
<td>(0.506)</td>
<td>(0.300)</td>
<td>(0.321)</td>
<td>(0.394)</td>
<td>(0.422)</td>
<td>(0.454)</td>
<td>(0.454)</td>
</tr>
<tr>
<td>( \pi ) surprise, 56 - 65</td>
<td>1.045*</td>
<td>0.921*</td>
<td>0.701**</td>
<td>0.708**</td>
<td>0.708*</td>
<td>0.818*</td>
<td>1.264***</td>
<td>1.179**</td>
</tr>
<tr>
<td></td>
<td>(0.489)</td>
<td>(0.524)</td>
<td>(0.312)</td>
<td>(0.337)</td>
<td>(0.414)</td>
<td>(0.447)</td>
<td>(0.457)</td>
<td>(0.457)</td>
</tr>
<tr>
<td>( \pi ) surprise, 66 - 75</td>
<td>0.833</td>
<td>0.4267</td>
<td>0.591*</td>
<td>0.391</td>
<td>0.761*</td>
<td>0.728</td>
<td>0.439</td>
<td>0.369</td>
</tr>
<tr>
<td></td>
<td>(0.514)</td>
<td>(0.541)</td>
<td>(0.333)</td>
<td>(0.357)</td>
<td>(0.426)</td>
<td>(0.454)</td>
<td>(0.485)</td>
<td>(0.485)</td>
</tr>
<tr>
<td>( \pi ) surprise, 76 plus</td>
<td>0.628</td>
<td>1.663***</td>
<td>-0.012</td>
<td>-0.133</td>
<td>-0.445</td>
<td>-0.225</td>
<td>0.827</td>
<td>1.606**</td>
</tr>
<tr>
<td></td>
<td>(0.664)</td>
<td>(0.697)</td>
<td>(0.414)</td>
<td>(0.414)</td>
<td>(0.536)</td>
<td>(0.567)</td>
<td>(0.638)</td>
<td>(0.638)</td>
</tr>
<tr>
<td>white</td>
<td>0.237***</td>
<td>0.237***</td>
<td>0.237***</td>
<td>0.237***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.002)</td>
<td>(0.002)</td>
<td>(0.002)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>male</td>
<td>0.147***</td>
<td>0.147***</td>
<td>0.147***</td>
<td>0.147***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.002)</td>
<td>(0.002)</td>
<td>(0.002)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>urban</td>
<td>0.062***</td>
<td>0.062***</td>
<td>0.062***</td>
<td>0.062***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.004)</td>
<td>(0.004)</td>
<td>(0.004)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>elementary school</td>
<td>0.205***</td>
<td>0.205***</td>
<td>0.205***</td>
<td>0.205***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.014)</td>
<td>(0.014)</td>
<td>(0.014)</td>
<td>(0.014)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>some high school</td>
<td>0.347***</td>
<td>0.347***</td>
<td>0.347***</td>
<td>0.347***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.013)</td>
<td>(0.013)</td>
<td>(0.013)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>high school grad</td>
<td>0.592***</td>
<td>0.592***</td>
<td>0.592***</td>
<td>0.591***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.013)</td>
<td>(0.013)</td>
<td>(0.013)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>some college</td>
<td>0.707***</td>
<td>0.707***</td>
<td>0.707***</td>
<td>0.707***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.013)</td>
<td>(0.013)</td>
<td>(0.013)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>college grad</td>
<td>0.963***</td>
<td>0.923***</td>
<td>0.963***</td>
<td>0.963***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.013)</td>
<td>(0.013)</td>
<td>(0.013)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>more than college</td>
<td>1.047***</td>
<td>1.047***</td>
<td>1.047***</td>
<td>1.047***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.014)</td>
<td>(0.014)</td>
<td>(0.013)</td>
<td>(0.013)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>less than 36</td>
<td>-0.306***</td>
<td>-0.306***</td>
<td>-0.306***</td>
<td>-0.306***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.003)</td>
<td>(0.003)</td>
<td>(0.003)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>36-45</td>
<td>-0.116***</td>
<td>-0.116***</td>
<td>-0.116***</td>
<td>-0.116***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.003)</td>
<td>(0.003)</td>
<td>(0.003)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>46-55</td>
<td>0.026***</td>
<td>0.026***</td>
<td>0.026***</td>
<td>0.026***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.003)</td>
<td>(0.003)</td>
<td>(0.003)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>56-65</td>
<td>0.06***</td>
<td>0.063***</td>
<td>0.06***</td>
<td>0.06***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.003)</td>
<td>(0.003)</td>
<td>(0.003)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>66-75</td>
<td>0.090***</td>
<td>0.090***</td>
<td>0.090***</td>
<td>0.090***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.003)</td>
<td>(0.003)</td>
<td>(0.003)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>cons</td>
<td>6.969***</td>
<td>6.969***</td>
<td>6.969***</td>
<td>6.969***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.014)</td>
<td>(0.014)</td>
<td>(0.014)</td>
<td>(0.014)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

R-squared 0.2754 0.2753 0.001 0.001 0.003 0.003 0.2754 0.2754

Observations Groups
425797 425797
159820 159820

*0.10 **0.05 ***0.01

US region and quarter dummies and time trend includes, not reported
CEX - 1980Q1 - 2004Q4
Huber-White Estimator of Standard Errors in Parenthesis
Var 3 means inflation shock comes from benchmark BVAR without Commodities inflation
Var 6 means inflation shock comes from BVAR adding M2 and unemployment series

Table 5: Robustness to alternative measures of inflation surprise
Figure 1: Consumption and Wealth in Steady-State

Figure 2: Calibration of the loads on aggregate risk
Figure 3: Net Nominal Positions over Wealth - Model vs. Data

Figure 4: Consumption Adjustment given 1% inflation surprise
Figure 5: Loads on Aggregate Risks - Income Groups

Figure 6: Net Nominal Positions over Wealth - Income Groups - Model vs. Data
Figure 7: Consumption Adjustment given 1% Inflation Surprise - Income Groups

Figure 8: Nominal Positions over Wealth - Income Groups - Heterogenous Loads on Aggregate Risks and Heterogenous Tax Rule

8.1 Proof of Proposition 1:

Markets are complete. From the central planners’ problem (assuming weight $\lambda_k$ for consumer $k$), one gets that:

$$\frac{C^i}{C^j} = \frac{\lambda_i}{\lambda_j}, \quad \forall i, j$$  \hspace{1cm} (48)
From the resource constraint in state $s$, we have:

$$C^{i,s} = \lambda_i \left( N\bar{X} + N\bar{Y} + \left( \sum_{k=1}^{N} c^k \right) \varepsilon^{a,s} \right) \tag{49}$$

The price of the Arrow-Debreu securities are:

$$\mu_{\vartheta_1} = \frac{\pi}{N\bar{X} + N\bar{Y} + \left( \sum_{k=1}^{N} c^k \right) \varepsilon^{a,\vartheta_1}} \tag{50}$$

$$\mu_{\vartheta_2} = \frac{1 - \pi}{N\bar{X} + N\bar{Y} + \left( \sum_{k=1}^{N} c^k \right) \varepsilon^{a,\vartheta_2}} \tag{51}$$

In order to find the central planer solution that satisfies each agent budget constraint, one needs that:

$$\lambda_i = \lambda_i^* = \frac{1}{N} + \left( \mu_{\vartheta_1} \varepsilon^{a,\vartheta_1} + \mu_{\vartheta_2} \varepsilon^{a,\vartheta_2} \right) \left( c^i - \frac{1}{N} \sum_{k=1}^{N} c^k - \left( R\frac{B}{N} - Ra_iB \right) \right) \tag{52}$$

Note that $\sum_{i=1}^{N} \left( R\frac{B}{N} - Ra_iB \right) = 0$. Using, (49), (50) and (52), one gets the the consumption allocation of each agent in the competitive equilibrium. Notice that all claims $A(j, i)$ are identical and riskless in this setup and any prices that satisfies:

$$Rq^i = \frac{\pi (1 + \varepsilon^{a,\vartheta_1}) + (1 - \pi) (1 + \varepsilon^{a,\vartheta_2}) \bar{Y}_1}{\bar{X} (\pi + (1 - \pi) \bar{Y}_1 \bar{Y}_2)}, \forall i$$

where $\bar{Y}_s = N\bar{X} + N\bar{Y} + \left( \sum_{k=1}^{N} c^k \right) \varepsilon^{a,s}$. Here $R$ is normalized to 1. With the asset structure described in section 3.1. There is one portfolio allocation that replicates complete markets allocation.

$$B^i = \lambda_i^* \left( \sum_{k=1}^{N} c^k \right) + a^iB - c^i \tag{53}$$

The difference of (53) for $i$ and $j$ gives (21). The fact that $\frac{\left( \sum_{k=1}^{N} c^k \right) \varepsilon^{a,s}}{\bar{Y}_s} < 1$ makes (22) straightforward. Moreover, (49) and (52) gives (ii).
8.2 Proof of Proposition 2:
The log-linearized (7), (9) and (10), government budget constraint and tax rule are respectively, (assuming $\hat{y} = \frac{y - \bar{y}}{\bar{y}}$)\(^{35}\):

$$\dot{C}^i + \ddot{\tau}^i = \frac{1}{2} \dot{X}^i + \frac{1}{2} \dot{\hat{y}}^i - RB^i \dot{\hat{P}} + \sum_{j=1}^{N} \frac{1}{2} \dot{X}^i \epsilon(j, i) \tag{54}$$

$$\sum_{j=1}^{N} \dot{\tau}^j = - RB \dot{\hat{P}} + G \tag{55}$$

$$\dot{\tau}^i = - a^i RB \dot{\hat{P}} + a^i G \tag{56}$$

The exogenous process (15), (16) and (17)

$$- \dot{P} = \epsilon^\pi + \epsilon^a \tag{57}$$

$$\frac{1}{2} \dot{Y}^i = c_1 \epsilon^Y + \epsilon^X \tag{58}$$

and

$$\frac{1}{2} \dot{X}^i = \epsilon^X \tag{59}$$

Combining (54)-(59), one can write the consumption of each agent as a function of the shocks and his portfolio holdings.

Using Devereux Sutherland, one gets by approximating up to second order the Euler equation:

$$E \left[ \left( \dot{C}^i - \dot{\hat{C}}^j \right) R^k \right] = 0, \quad \forall i, j, k \tag{60}$$

where $R^k$ is the difference of first-order approximations of the equity $k$ returns and the return on nominal bonds. In matrix form, $Cd\Sigma Rx = 0$, where $\Sigma$ is the diagonal matrix with the variances of the shocks $\epsilon$, $Cd$ is the N-1 by N+4 matrix with the coefficients multiplying $\epsilon$ of all $\dot{C}^i - \dot{\hat{C}}^j$ and $Rx$ is the N+4 by N matrix of the coefficients multiplying $\epsilon$ of the differential returns in all $R^k$. Combining these N-1 equations and (14), gives us part (i) of proposition 2. Note that $R$ can be normalized. With the asset allocations, one can plug them back in (54), this way part (ii) of proposition 2, as well as the discussion and tables that follow are straightforward.

8.3 OLG Portfolio Solution

After log-linearization, the agents’ budget constraint is given by

$$c_t^{hi} \dot{C}_t^{hi} + \bar{W}_t^{hi} \dot{W}_t^{hi} + \dot{\hat{p}}^h = R \bar{w}_t^{hi} \dot{W}_t^{hi} + \bar{r}_t^{hi} \dot{\hat{P}}_t + \omega N_t^{hi} \dot{\omega}_t + \omega \dot{\hat{N}}^{hi} N_t^{hi} + \xi_t^{hi} \tag{61}$$

where $\xi_t^{hi} = \Gamma_k^{hi} (\dot{K}_t - \dot{\hat{K}}_t) = \Gamma_k^{hi} (\dot{R}_t - \dot{\hat{R}}_t - \dot{Q}_t - \Theta_1 \dot{Q}_t - \Theta_2 \dot{\hat{Q}}_t)$, i.e., up to a constant it is

$^{35}$For easy of notation, I assume that $\bar{X} = \bar{Y} = \frac{1}{2}$. This leads to $\dot{\bar{C}}^i = 1$ and to $\frac{\bar{R}}{\bar{q}} = \frac{1}{2}$. It also follows that $\ddot{\tau}^i = \frac{\bar{R}B}{\bar{N}}$
the steady-state capital holdings of the agent times the differential of return between capital and nominal bonds. Following Devereux-Sutherland, $\xi_t^{hi}$ is taken as an extra exogenous shock and solve the system up to a first-order. Notice that, since $\xi_t^{hi}$ only shows up in the agents budget constraint. This gives the difference of the first-order approximation of the marginal utilities, $Cd_t$, for different agents and differential of returns, $\hat{R}_t^B - \hat{R}_t^K$, as a function of the original shocks of the economy and $\xi_t^{hi}$.

$$Cd_t = D_1\varepsilon_t + D_2\xi_t$$  \hspace{1cm} (62)

$$\hat{R}_t^B - \hat{R}_t^K = R_1\varepsilon_t + R_2\xi_t$$  \hspace{1cm} (63)

where $\varepsilon_t$ is a vector with all the aggregate and idiosyncratic shocks in the economy and $\xi_t$ is a vector with all $\xi_t^{hi}$.

However, $\xi_t$ is a function of the steady-state portfolio and differential returns:

$$\xi_{t2} = \Gamma\bar{k}(\hat{R}_t^B - \hat{R}_t^K)$$

where $\bar{k}$ is a vector with all steady-state $k^{hi}$. This gives:

$$\hat{R}_t^B - \hat{R}_t^K = (1 - R_2\Gamma\bar{k})^{-1}R_1\varepsilon_t$$  \hspace{1cm} (64)

$$Cd_t = (D_1 + D_2\Gamma\bar{k}^{hi}(1 - R_2\Gamma\bar{k})^{-1}R_1)\varepsilon_t$$  \hspace{1cm} (65)

Here again second-order approximation of the Euler equations imply that:

$$E_t\left[Cd_{t+1}(\hat{R}_{t+1}^B - \hat{R}_{t+1}^K)\right] = 0$$  \hspace{1cm} (66)

$$(D_1 + D_2\Gamma\bar{k}^{hi}(1 - R_2\Gamma\bar{k})^{-1}R_1)\Sigma(1 - R_2\Gamma\bar{k})^{-1}R_1 = 0$$  \hspace{1cm} (67)

The expression in (67) are used to numerically get the optimal values for $k^{hi}$. Market clearing conditions and the fact that each agents steady-state wealth equals their steady-state portfolio holdings imply the optimal steady-state nominal portfolio.

References


