Sales and monetary policy

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March 2008

Abstract

This paper analyses the effect of monetary policy in a model where sales occur in equilibrium. Some consumers are loyal to a particular brand of a good, others are bargain hunters. As a result, producers find it optimal to sell their goods at a “normal” price through some retailers and at a “sale” price through others. The margin of adjusting the amount sold on sale is potentially very important: if producers can adjust the fraction sold on sale, with sticky prices and wages, money is approximately neutral if we ignore the reasons behind sales. But that result changes dramatically when we consider that producers optimally choose to sell some of their goods on sale: owing to strategic substitutabilities in the sales decision, money has strong procyclical effects.

Keywords: sales; monetary policy.

Jel Classification: E3, E5.

1 Introduction

A striking fact about prices is that shifts between a “normal” and a “sales” price are the most common observed price changes for many classes of goods. Figure 1 shows weekly retail and wholesale prices for Bass Ale at Dominick’s supermarkets.1 We see that prices change frequently, but usually return to their original level after a price cut. This behaviour is the rule, not the exception.2 The figure also shows that sales are often coordinated between producers and retailers: retail and wholesale prices frequently move in the same direction and at the same time. Indeed, deals between a producer and a retailer to offer a good at a “sale” price are frequent and account for a large part of the marketing effort of producers.3

This pattern of price changes could potentially make a big difference to how we think about the effects of monetary policy. Monetary policy is of course not the principal

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1Prices are for a single six-pack, taken from Goldberg and Hellerstein (2007).
2For a comprehensive analysis of retail prices, see Hosken and Reiffen (2004).
3See Walters (1989) and Dreze and Bell (2003).
driving force behind those sales. However, we expect that decisions about whether to put a product on sale and about the duration of the sale react to demand fluctuations. And this means that those decisions must also react to monetary policy shocks to the extent that these impact on aggregate demand.

So in order to understand the effects of monetary shocks, we need to understand sales — why are the sales there to begin with? What explains such peculiar pricing behaviour? We need a macroeconomic model which is able to account for this pattern of prices: but such a model is currently absent from the literature. To fill this gap we build a tractable general equilibrium model where sales occur as a result of optimizing behaviour. A model of this kind necessarily requires much heterogeneity at the micro level, but we show how the aggregation problems can be overcome so as to be able to analyse the effects of monetary policy at the macro level.

Recently, much attention has been devoted to sales because attempts to measure the frequency of price adjustment using micro data have found that how sales are treated is of tremendous importance. With no distinction drawn between sales and regular price changes, Bils and Klenow (2004) estimate the median duration of a price spell to be between 4 and 4.5 months. On the other hand, Nakamura and Steinsson (2007), excluding sales from their sample, obtain estimates of between 8 and 11 months. As the real effects of monetary policy depend on how sticky prices are, that is, how long it takes for firms to react to monetary shocks by readjusting their prices, it is of the utmost importance to decide whether sales should be counted as price changes for the purposes of monetary policy analysis.

The estimates of price stickiness in the empirical literature consider each retailer and each good individually, but that may be misleading. Producers sell their goods through many retailers, and price promotion deals are typically not offered to all of them at the same time. Hence, even if prices at a given retailer are reasonably sticky, by adjusting the fraction of retailers that buy the good at the “normal” price as opposed to the “sales” price, producers can mimic some price adjustment. But how much effective price adjustment does this margin allow for? To what extent is this margin really a substitute for price adjustment per se? What would the impact of monetary policy be in a world like this?

\[\text{4Varian (1980) and Sobel (1984) present models with a price-discrimination motive for sales, but their models are not suitable for macroeconomic analysis. Our model is very different from theirs but the rationale for sales is similar. Kehoe and Midrigan (2007) incorporate sales in a macroeconomic model by assuming a different menu cost for transitory price changes, but we regard that as a reduced form representation of the phenomenon of sales, as there is no mention as that as a reason for sales in the micro literature.}\]
The sales adjustment margin has received little attention in the monetary policy literature. In our model, the margin of adjusting how much is on sale plays a key role. We consider an environment where a continuum of producers sell their goods through retailers and the price of a particular good may not be the same across all retailers. Producers are therefore allowed to sell their good at a “normal” price through some retailers and at a “sale” price through others.

To begin with, we consider a benchmark case when producers have two exogenously fixed prices. When the monetary shock is revealed, they can adjust the proportion of sales they offer. In this economy, when wages are sticky, as long as the fraction of sales is strictly between zero and one, monetary policy has no effect on the quantity produced by any firm. Following an increase in the money supply, producers offer a smaller fraction of their goods on sale, so the price level increases. Strikingly, this margin wipes out any possible effect on output in spite of both price and wage rigidity. One could argue that changing the fraction of goods sold at sales price could be costly, but notice that the adjustment of sales means that the firm does not then need to change its production plans, and it is not obvious that changing the proportion of sales is inherently more costly than changing the total quantity produced.

This result indicates the potential significance of the sales adjustment margin. But the previous exercise presupposes that firms have chosen two prices at some earlier stage. But why would firms actually want to have multiple prices for the same good? We have so far not explained why they would want this, and the implications for monetary policy are likely to depend on the rationale for having two prices in equilibrium. So we build a model with an important new feature: customers can now choose between a range of brands for each type of product they buy.

The model assumes some consumers are loyal to a particular brand of a good, while others are bargain hunters. This generates a demand curve where marginal revenue is non-monotonic. In this environment, there can be an equilibrium where producers sell their goods through different retailers at different prices. We focus on economies where there is an equilibrium in two prices: a normal price (high) and a sale price (low). We develop a tractable general equilibrium model of sales with predictions that allow it to be calibrated to match the micro evidence on pricing that has become available in recent years.

One important implication of the model is that there are decreasing returns to sales.

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5By price-level we mean the effective cost of the basket of goods purchased by consumers. This may or may not correspond to official statistical indices depending on how sales are treated when these are compiled.
When many brands of the same type of product are offered on sale in a supermarket, other producers of this type of product are more reluctant to put their own brand on sale. But if no-one is selling on sale, producers have much more incentive to sell their own brand at a lower price and capture more of the bargain-hunting customers.

This finding has important consequences for the effects of monetary policy. Following a positive monetary policy shock, producers reduce their fraction of sales. But as they do this, it becomes more attractive for others to target the bargain hunters by offering more sales. The net effect is that sales fall by a smaller amount leading to a smaller increase in the price level, so monetary policy has a relatively larger effect on output than otherwise.

Our calibrated model can be used to assess the quantitative impact of a monetary policy shock. In our baseline specification, with sticky wages, around 78% of the effects of a monetary shock fall on output and 22% on prices. We run robustness checks and find that the breakdown of the effects of the monetary shock has between 65% and 80% of the effect falling on output. This is in sharp contrast to the neutrality result obtained when a normal and a sales price are assumed, but no rationale for their presence is considered.

After analysing the effect of a monetary shock in a static model, we consider a dynamic environment where firms are free to adjust the fraction sold on sale in every period, but wages and prices are sticky (Calvo pricing and a similar assumption for wages). We then contrast the impulse responses to monetary shocks in this model to those of a model without sales where consumers are homogeneous, but which is otherwise identical. The results show that the response of output is more persistent in the model with sales than in the standard model. So, not only is the instantaneous effect of the monetary shock on output larger than in the standard model, but also the effect becomes more marked the longer after the original shock.

These results demonstrate the importance of modelling the reasons behind producers’ sales decisions, even in the analysis of monetary policy to which the conventional wisdom suggests they should be essentially orthogonal. The margin of adjusting the fraction of sales for a good is potentially very important. However, owing to the strategic substitutabilities inherent in sales, that is, due to competition for consumers with more elastic demand, it is not wise for firms to use the sales margins as much as might be expected when responding to aggregate shocks. Thus, the real effects of monetary policy remain, even in a world of apparent price flexibility.
2 The model with exogenous sales

There is a continuum of consumers (measure 1); a continuum of producers (measure 1); a continuum of retailers (measure 1).

Consumers maximize utility that depends on consumption and labour supply. Each consumer shops at a single retailer. In equilibrium, the consumer is indifferent about which retailer she goes.

Each producer produces one good and goods are imperfect substitutes.

Producers sell their goods to retailers, from whom consumers buy. We abstract from the potentially complicated bargaining between retailers and producers, which would complicate the model without necessarily adding new insights. We assume that the bargaining is efficient: prices are set to maximize the joint surplus. We consider that a good assumption because prices of transactions between producers and retailers can be conditional on quantities and so in principle they can choose an optimal mix of final price and quantities.\(^6\)

Crucially, the producer can sell its good through some retailers at price \(P_N\) and through some others at the sales price, \(P_S\). In this section, we will not explain why the producer would do so, we will take it as given. It will be endogeneized in the next section.

We add the assumption that producers set their prices in advance.

2.1 The consumer’s problem

Consumers have Dixit-Stiglitz preferences. Their utility is given by:

\[
U(C,L) = \frac{C^{1-\gamma} - 1}{1-\gamma} - \frac{L^{\omega+1}}{\omega + 1}
\]

\[
C = \left( \int_\tau C(\tau)^{(\varepsilon-1)/\varepsilon} \, d\tau \right)^{\varepsilon/(\varepsilon-1)}
\]

\(\varepsilon > 1, \gamma > 0, \omega > 0. \quad C(i) \) denotes an agent’s consumption of good \(\tau\).

If the price paid for good \(\tau\) is \(P(\tau)\), we have that the price index \(P\) faced by that consumer is:

\[
P = \left( \int_{\tau} P(\tau)^{1-\varepsilon} \, d\tau \right)^{1/(1-\varepsilon)}
\]

The budget constraint can be written as:

\[
PC \leq WL + D
\]

\(^6\)In reality, there is evidence that retailers do not always pass all discount obtained from producers to consumers (Walters, 1989, Blattberg et al, 1995). However, the availability of scanners at supermarkets has allowed producers to get better control of retailers’ sales, so deals have been getting more efficient deals (Dreze and Bell, 2003).
where $W$ is the nominal wage, $D$ is dividends.

Denoting by $M$ the money holdings of an agent, a cash in advance constraint implies:

$$C = \frac{M}{P}$$

So demand for good $\tau$ is given by:

$$C(\tau) = \left( \frac{P(\tau)}{P} \right)^{-\varepsilon} C$$

There is no investment nor government expenditures in the model, so goods market equilibrium implies $C = Y$. So

$$Y = \frac{M}{P} \text{ and } C(\tau) = \left( \frac{P(\tau)}{P} \right)^{-\varepsilon} Y$$

### 2.2 The producer’s problem

In order to produce and distribute $q(\tau)$ units of good $\tau$, labour is needed. More specifically

$$q(\tau) = L(\tau)^{\alpha}$$

Which implies:

$$L(\tau) = q(\tau)^{1/\alpha}$$

Labour is homogeneous.

It is assumed that, for some exogenous reason, the producer sells a proportion $s$ of goods at price $P_S$ through some retailers and a proportion $(1-s)$ at price $P_N$ through other retailers. So:

$$q(\tau) = s \left( \frac{P_S}{P} \right)^{-\varepsilon} Y + (1-s) \left( \frac{P_N}{P} \right)^{-\varepsilon} Y$$

The real profits associated with production of good $i$ are given by:

$$\pi = s \left( \frac{P_S}{P} \right)^{1-\varepsilon} Y + (1-s) \left( \frac{P_N}{P} \right)^{1-\varepsilon} Y - \frac{W}{P} \left[ s \left( \frac{P_S}{P} \right)^{-\varepsilon} Y + (1-s) \left( \frac{P_N}{P} \right)^{-\varepsilon} Y \right]^{1/\alpha}$$

(1)

In a symmetric equilibrium, where all firms have the same $P_S$ and $P_N$, all firm choose the same $s$ and $q$. However, the quantity produced by the firm ($q$) is not exactly the same as the aggregate output ($Y$). The relationship between $q$ and $Y$ is given by:

$$q = \delta Y \text{ where } \delta = s \left[ \frac{P_S}{P} \right]^{-\varepsilon} + (1-s) \left[ \frac{P_N}{P} \right]^{-\varepsilon}$$

For $s = 0$ and $s = 1$, $\delta = 1$.  

2.3 The impact of monetary policy

After the producer has chosen \( P_N \) and \( P_S \), and wages are set at \( W \), the money supply (\( M \)) is revealed.

The producer can then adjust the amount (\( s \)) sold at sale price.

The following proposition establishes the main result of this section:

**Proposition 1** Given prices \( P_N \) and \( P_S \), as long as \( s \) is between 0 and 1, the firm’s output \( q(\tau) \) is not affected by monetary shocks.

**Proof.** The first order condition with respect to \( s \) implies:

\[
\left( \frac{P_S}{P} \right)^{1-\varepsilon} Y - \left( \frac{P_N}{P} \right)^{1-\varepsilon} Y = \frac{W}{\alpha P} q(\tau)^{\frac{1-\alpha}{\alpha}} \left[ \left( \frac{P_S}{P} \right)^{-\varepsilon} Y - \left( \frac{P_N}{P} \right)^{-\varepsilon} Y \right]
\]

Which yields:

\[
q(\tau) = \left[ \frac{\alpha}{W} \left( \frac{P_S^{1-\varepsilon} - P_N^{1-\varepsilon}}{P_S^{-\varepsilon} - P_N^{-\varepsilon}} \right) \right]^{\alpha/(1-\alpha)}
\]

The equation shows that the quantity produced, \( q(\tau) \), does not depend on any nominal variable (\( P \) or \( Y \)), which proves the claim.  

Monetary policy causes small changes in \( Y \) because of the way \( Y \) is aggregated to form the consumption basket. But because \( q(i) \) and \( Y \) are very similar, \( Y \) is approximately constant — indeed the value of \( Y \) is the same in the cases \( s \to 0^+ \) and \( s \to 1^- \).

A positive shock to \( M \) leads producers to sell fewer of their goods on sale. As \( Y \) is approximately constant, an increase in \( M \) has to be followed by a corresponding increase in \( P \). The prices \( P_S \) and \( P_N \) are sticky; the proportion sold on sale (\( s \)) is responsible for the adjustment.

Notice that the above result holds for any \( P_S \) and \( P_N \) as long as \( s \in (0,1) \). The producer can mimic any price increase by adjusting \( s \).

The intuition is the following: higher \( s \) means that: (i) revenues are higher because at the sale price the quantity sold is higher and the price is lower, but as \( \varepsilon > 1 \), the quantity effect dominates; (ii) costs are higher because quantity sold is higher and, on the top of that, (iii) as marginal cost is increasing, the marginal cost of production also increases. The producer chooses \( s \) so that the marginal increment in revenue from increasing sales compensates the marginal increase in cost due to higher quantity plus the marginal increment in cost.

Now, if prices and wages are fixed, an increase in the price level multiplies the demand for goods at both prices by the same factor — \( P^{-\varepsilon} \) — and an increase in \( Y \) also multiplies
the demand for goods at both prices by the same factor — $Y$. So, as long as the marginal cost of production is constant, changes in $P$ and $Y$ do not affect the optimal choice of $s$, because such changes do not affect the ratio of the difference between the marginal revenues and the difference between marginal costs at prices $P_S$ and $P_N$, respectively.

But changes in the marginal cost of production affect the balance between these two effects. If marginal cost is too high, it is worth while for the producer to cut production and reduce $s$ (the opposite case is marginal cost being too low). With monetary shocks, the optimal decision of the producer is to keep marginal costs constant. Any change in $P$ or $Y$ affects demand and the producer adjusts $s$ to keep the quantity produced constant.

In this model with two exogenously fixed prices, the possibility of adjusting the amount on sale is enough to translate an increase in $M$ into an increase in $P$ with no effect on the quantity produced by the firm.

3 The model with sales in equilibrium

The above model consider sales but does not explain why there are sales in the first place. In this section, we will provide a rationale for sales. As we’ll see, that makes a big difference.

There is a continuum of types of goods and, for each type, there is a continuum of brands. In equilibrium, consumers buy the whole range of different types of goods, but buy only one brand for each type. Each produces manufactures one particular brand for a given type. Producers face two types of consumers: the loyal customers, who always buy their preferred brand, and the bargain-hunters, who will seek out the best deals, taking into account their own preferences. In equilibrium, producers may choose to sell at different prices.

The model is set up so that although different consumers are loyal to brands of different goods, in the aggregate everything is symmetric. So, all brands have the same fraction of loyal costumers and all costumers are loyal to a brand for the same fraction of goods.

Except for the myriad of available brands, the model is exactly as before.

3.1 The consumer’s problem

There is a continuum of types of goods (indexed by $\tau$) and, for each type, there is a continuum of brands (indexed by $b$).
There is a continuum of consumers with measure 1. Their utility function is:

\[ C = \left( \int_{\Lambda} C_L(\tau)^{(e-1)/\varepsilon} d\tau + \int_{\Omega/\Lambda} C_B(\tau)^{(e-1)/\varepsilon} d\tau \right)^{\varepsilon/(e-1)} \]

where

\[ C_L(\tau) = C(\tau, b_L(\tau)) \quad \text{and} \quad C_B(\tau) = \left( \int_0^1 C(\tau, b)^{(\eta-1)/\eta} db \right)^{\eta/(\eta-1)} \]

\( \Lambda \) is the set of goods for which the consumer is loyal to a particular brand. Its measure is \( \lambda \). For each good \( \tau \in \Lambda \), the consumer chooses brand \( b_L(\tau) \). To make everything symmetric, so that the model aggregates fine, we assume that goods that belong to the set \( \Lambda \) and the brands to which the consumer are loyal are randomly assigned to consumers. So, given the law of large numbers, every brand of every good has a measure \( \lambda \) of loyal consumers.

As before, the demand for good \( \tau \in \Lambda \) is given by:

\[ C(\tau, b_L(\tau)) = \left( \frac{P(\tau, b_L(\tau))}{P} \right)^{-\varepsilon} C \]

For the goods the consumer is a bargain-hunter (measure \( (1 - \lambda) \)), we get:

\[ C(\tau, b) = \left( \frac{P(\tau, b)}{P_B(\tau)} \right)^{-\eta} C_B(\tau) \]

where

\[ P_B(\tau) = \left( \int_0^1 P(\tau, b)^{1-\eta} db \right)^{1/(1-\eta)} \]

and the demand for the composite good \( \tau \in \Omega/\Lambda \) is given by:

\[ C^B(\tau) = \left( \frac{P^B(\tau)}{P} \right)^{-\varepsilon} C \]

### 3.2 The producer’s problem

For some parameter values, the producer faces a demand function with non-monotonic marginal revenue. As marginal costs are increasing, for some parameter values, in equilibrium all producers will choose to have a fraction \( s \) of their goods sold at price \( P_S \) through some retailers, and a fraction \( (1 - s) \) of their goods sold at price \( P_N \) through other retailers.

Here, we focus on those equilibria. We later show that is indeed an equilibrium for some parameter values.

In equilibrium, given our assumptions about the randomness of preference factors and tastes for goods and brands and the law of large numbers, all consumers have the same
consumption ($C$) and face the same price level ($P$). Good markets equilibrium implies $C = Y$.

We now consider the problem of one producer when all other producers sell their goods at price $P_S$ through $s$ retailers and at price $P_N$ through $(1 - s)$ retailers.

A producer faces a measure $\lambda$ of loyal consumers. Moreover, there is a measure $(1 - \lambda) \times 1$ that may buy its brand depending on prices and preferences.

The following proposition characterizes the demand faced by a producer.

**Proposition 2** Suppose all other producers sell their goods at price $P_S$ through $s$ retailers and at price $P_N$ through $(1 - s)$ retailers. A producer selling its good at price $P_1$ faces the following demand function:

$$[\lambda + (1 - \lambda)v(P_1)] \left(\frac{P_1}{P}\right)^{-\varepsilon} Y$$

where $v(P_1)$ is given by:

$$v(P_1) = \left(\frac{P_B}{P_1}\right)^{\eta-\varepsilon} = \left(s \left(\frac{P_1}{P_S}\right)^{\eta-1} + (1 - s) \left(\frac{P_1}{P_N}\right)^{\eta-1}\right)^{-(\eta-\varepsilon)/(\eta-1)}$$

(3)

Note that $v(P_1)$ depends the price changed by the producer ($P_1$) and the other producers’ choices of $s$, $P_S$ and $P_N$.

Equation (3) shows that for any price $P_1$, $v(P_1)$ depends positively on $P_S$ and $P_N$. If the other producers charge a higher price, the measure of bargain hunters that choose the producer’s brand is larger.

Moreover, if $P_N > P_S$, then $v(P_1)$ is decreasing in $s$. A higher proportion of brands offered at discount implies a smaller measure of bargain hunters choosing the producer’s brand.

Thus, $v(P_1)$ depends not only on the producer’s decision but also on the decision of the other producers in the economy. This is the key feature of this model of endogenous sales.

### 3.2.1 The producer’s choice

Now suppose the producer sells his good at price $P_S(\tau, b)$ through $s(\tau, b)$ retailers and at price $P_N(\tau, b)$ through $(1 - s(\tau, b))$ retailers, while all other producers sell their goods at price $P_S$ through $s$ retailers and at price $P_N$ through $(1 - s)$ retailers.

Define $v_S = v(P_S(\tau, b))$ and $v_N = v(P_N(\tau, b))$. The values of $v_S$ and $v_N$ depend on the individual producer’s prices, prices of the others brands and the aggregate $s$, but does not depend on the producer’s own choice of $s(\tau, b)$. 

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The profit of a producer can thus be written as:

\[
\pi = \zeta_S \left( \frac{P_S(\tau, b)}{P} \right)^{1-\varepsilon} Y + \zeta_N \left( \frac{P_N(\tau, b)}{P} \right)^{1-\varepsilon} Y - W \left[ \zeta_S \left( \frac{P_S(\tau, b)}{P} \right)^{-\varepsilon} Y + \zeta_N \left( \frac{P_N(\tau, b)}{P} \right)^{1-\varepsilon} Y \right]^{1/\alpha}
\]

(4)

where \(\zeta_S\) and \(\zeta_N\) are defined as:

\[
\zeta_S = s(\tau, b) [\lambda + (1 - \lambda) v_S]
\]

\[
\zeta_N = (1 - s(\tau, b)) [\lambda + (1 - \lambda) v_N]
\]

Equation (4) is similar to Equation (1), but instead of \(s\) and \((1 - s)\), we have \(\zeta_S\) and \(\zeta_N\), respectively. As we will see, that difference will play a key role in our analysis.

Taking derivative of Equation (4) with respect to \(s(\tau, b)\), we get the following first order condition:

\[
q = \left[ \frac{\alpha}{W} \left( \frac{(\lambda + (1 - \lambda) v_S) P_S^{1-\varepsilon} - (\lambda + (1 - \lambda) v_N) P_N^{1-\varepsilon}}{(\lambda + (1 - \lambda) v_S) P_S^{\varepsilon} - (\lambda + (1 - \lambda) v_N) P_N^{\varepsilon}} \right) \right]^{\alpha/(1-\alpha)}
\]

(5)

which is similar to the first order condition obtained in the case with exogenous sales (Equation (2)). The difference is that the terms \((\lambda + (1 - \lambda) v_S)\) and \((\lambda + (1 - \lambda) v_N)\) multiply prices. Here, the amount produced is not independent of the others’ choice regarding sales.

Define the mark-ups \(\mu_S\) and \(\mu_N\) as

\[
\mu_S = \frac{P_S(\tau, b)}{xP}, \quad \mu_N = \frac{P_N(\tau, b)}{xP}
\]

where \(x\) is the real marginal cost. Manipulating the first order conditions with respect to \(P_N\) and \(P_S\), we get:

\[
\mu_S = \frac{\lambda \varepsilon + \eta (1 - \lambda) v_S}{\lambda (\varepsilon - 1) + (\eta - 1)(1 - \lambda) v_S}
\]

\[
\mu_N = \frac{\lambda \varepsilon + \eta (1 - \lambda) v_N}{\lambda (\varepsilon - 1) + (\eta - 1)(1 - \lambda) v_N}
\]

The above equations yield the equilibrium values of \(\mu_S\), \(\mu_N\) and \(s\).

3.2.2 Equilibrium

For some parameter values, the producer charges a unique price \((P_S = P_N)\). In other cases, in equilibrium, producers find it optimal to sell their goods at the normal price through some retailers and at the sales price through others. Then

\[
P^B(\tau) = \left( s P_S^{1-\eta} + (1 - s) P_N^{1-\eta} \right)^{1/(1-\eta)}
\]
Figure 2 shows marginal revenue for an equilibrium where producers charge $P_S = 0.9022$ and $P_N = 1.2111$. It is non-monotonic, so the profit function is not concave and the firm may be better off charging different prices. Figure 3 plots the marginal revenue as a function of the price. We see that, at the margin, a producer is indifferent between selling one unit at price $P_S$ and $P_N$, they yield the same marginal revenue. But marginal costs are increasing, so the producer strictly prefer that mix than charging only one price.

In equilibrium, $P_S(\tau, b) = P_S$, $P_N(\tau, b) = P_N$ and $s(\tau, b) = s$.

The other equilibrium conditions are:

\[
x = \left[ s(\lambda + (1 - \lambda)v_S)\mu_S^{1-\epsilon} + (1 - s) (\lambda + (1 - \lambda)v_N)\mu_N^{1-\epsilon} \right]^{1/(\epsilon-1)}
\]

\[
\delta = \left[ s(\lambda + (1 - \lambda)v_S)\mu_S^{-\epsilon} + (1 - s) (\lambda + (1 - \lambda)v_N)\mu_N^{-\epsilon} \right] x^{-\epsilon}
\]

\[
Y = \left( \frac{(\alpha x)^{\alpha}}{\delta^{1-\alpha+\omega}} \right)^{1/(1-\alpha+\alpha\gamma+\omega)}
\]

\[
q = \delta Y
\]

\[
W = Y^{\gamma} q^{\omega/\alpha}
\]

\[
P = \frac{M}{Y}
\]

### 3.2.3 Decreasing return to sales

In contrast to the case of exogenous sales, here the individual decision about production depends on $s$, the amount of sales, through $v_S$ and $v_N$. A higher $s$ implies that it is more difficult for a producer to sell to a bargain hunter. That provides more incentives for them to target the loyal customers, thus reducing sales of their own goods.

In this model, as the motivation for sales is considered, money is pro-cyclical.

**Proposition 3** With sticky prices and wages, $q$ is increasing in $M$.

**Proof.** Taking the implicit derivative of $q$ with respect to $s$, and doing loads of algebra, we get that $q$ is decreasing in $s$. So a positive monetary shock leads to an increases in $q$, for suppose not: then, a positive shock to $M$ increases $P$. If prices are sticky, that can only be obtained by a decrease in $s$. But that leads to an increase in $q$, which contradicts the assertion.

As seen earlier, a monetary shock induces companies to reduce the amount sold at the lower price ($P_S$). But with a lower $s$, $v_S$ and $v_N$ increase: the chances of selling to a bargain hunter is higher. In other words, as producers reduce their sales, incentives to target bargain hunters increase. So the reduction in sales (increase in $P$) following the monetary shock is less pronounced.
4 The impact of monetary policy

The impact of monetary policy is analysed by studying the effect of small perturbations in $M$ after $P_S$, $P_N$ and $W$ have already been chosen. We calibrate the model and calculate the impact on output and prices of an unexpected small increase in the money supply.

4.1 Numerical solution

There are 6 parameters to calibrate: $\gamma, \omega, \alpha, \lambda, \varepsilon, \eta$.

The calibration of the first 3 parameters follow the literature. In our baseline calibration, $\gamma = 3$ (relative risk aversion coefficient), $\omega = 1.4$ (inverse of the Frisch elasticity) and $\alpha = 2/3$ (labour share). The effect of monetary policy is not affected at all by changes in $\omega$ and $\gamma$.

The other 3 parameters are chosen to match some facts about sales and price-setting:

- $P_S/P_N$. Nakamura and Steinsson (2007) find that the median difference between $\log(P_S)$ and $\log(P_N)$ is 0.295.

\[
\log(P_N) - \log(P_S) = 0.295 \Rightarrow P_S/P_N = 0.745
\]

and that is the value we choose for $P_S/P_N$.

- Equilibrium $s$: Nakamura and Steinsson (2007) report that the fraction of price quotes with sales, weighted by expenditure, is 7.4%. If weighted by the number of observations, the fraction of price quotes with sales in their sample goes up to 12.1%. As the price level is an expenditure-weighted average, the impact of monetary policy on prices (and therefore on output) is best estimated using the expenditure-weighted number. So, in our baseline calibrations, parameters are set to match $s = 7.4\%$, but we also experiment with $s = 12.1\%$.

- The ratio between the quantity sold in a particular retailer at the sale and normal prices. In our model, this corresponds to $\frac{(\lambda + (1 - \lambda)v_S)}{(\lambda + (1 - \lambda)v_N)}$. Using the elasticities obtained in the comprehensive study of Chakravarthi et al (1996), we get that a price reduction of the magnitude we consider would multiply demand by a factor around 5.\(^7\) We use this number in our baseline calibration. Pesendorfer (2002) finds a quantity ratio of 7 in the market of ketchup, we do sensitivity analysis with this number. The changes in demand are large, and the substitution between

\(^7\)The results depend on how the sale is advertised, 5 is approximately an average.
brands plays a key role: people are not buying more beer, they are buying Corona instead of Budweiser, ice-cream instead of chocolate cake.

4.1.1 Algorithm

We want to find an equilibrium that approximately matches the facts above. So, the algorithm to solve the model chooses parameters that minimize a loss function of the sum of square deviations between equilibrium and target values of those 3 indicators.

For given parameters $\lambda, \varepsilon, \eta$, we use an iterative procedure to solve for the steady-state equilibrium. Starting from an initial guess of $(s, P_S, P_N)$, we calculate the macroeconomic aggregates and relevant equilibrium variables $(Y, P, q, W, x)$. Then we find the best response of a producer (her choices of $s, P_S$ and $P_N$), using the Nelder-Mead (Simplex) algorithm. Iterating on $(s, P_S, P_N)$, we obtain the equilibrium in this economy.\(^8\)

4.2 Results

The results are shown in the attached tables. In all exercises, in order to match the sales facts, we found high values of $\lambda$ (closer to 1, most consumers are loyal to a brand). $\varepsilon$ is around 3 and $\eta$ is around 20. That should not be taken too literally — after all, they depend on the Dixit-Stiglitz utility function.

Tables 1 and 2 show the results according to our baseline calibration. 78% of the effect of monetary policy goes to output, and 22% goes to prices. That is in sharp contrast with the model with exogenous sales where money is neutral.

Tables 3 and 4 show that an increase in the fraction of sales or an increase in the quantity ratio reduce the impact of monetary policy on output. Yet, output absorbs most of its impact (around 70%).

The differences between the effect of monetary policy in our calibration and in the case of exogenous sales stem from the decreasing returns from sales, which is related to the reasons for why producers sell their goods at different prices through different retailers.

5 Dynamics

We now consider a dynamic environment, where the economy is subject to small shocks and study the behavior of the economy around the steady state.

---

\(^8\)One could think that using the first-order conditions would help. Trouble is, the procedure to find conditions that match the first-order conditions were usually to a local minimum of profits. We used the first order conditions just to check the results obtained by the Simplex algorithm.
Each firm chooses two prices for their goods (the normal price, \( P_N \), and the sales price, \( P_S \)). Firms are always free to choose how much they sell at each price, but prices are sticky. In any period, a producer can readjust his prices with probability \( \theta_p \).

We assume all goods are consumed at the period they are purchased. Pesendorfer (2002) and Hendel and Nevo (2006) argue that consumers build inventories of goods on sale, while Gupta (1988) finds that this effect is negligible. The absence of a dip in demand following sales is a general finding in this literature (Blattberg et al, 1995), which contradicts the inventory behavior. For convenience, we assume consumers do not store goods. If the stockpiling is significant, price elasticity is actually smaller than most studies obtain, but the logic of our exercise would still hold.

Wages are also sticky. To include that in the model, we follow the convention in the literature (Erceg, Henderson and Levin (2000)) and assume that labour is heterogeneous. Each household supply \( h_t(j) \) hours of labor. A firms’ production function is \( q_t = h_t^\alpha \), where

\[
h_t = \left( \int_0^1 h_t^{(\varepsilon-1)/\varepsilon} (j) \, dj \right)^{\varepsilon/(\varepsilon-1)}
\]

In equilibrium,

\[
W_t = \left( \int_0^1 W_t^{1-\varepsilon} (j) \, dj \right)^{1/(1-\varepsilon)}
\]

The worker chooses her wage and, in any period, is allowed to readjust it with probability \( \theta_w \).

Generalize \( v \) equation (prove this all in the appendix?).

Victory margins:

\[
v(P_1, D_t) = \int_{-\infty}^{\infty} \frac{n}{\sigma} \phi \left( \frac{z - (\varepsilon - 1)^2}{\sigma} \right) \left[ \sum_{\ell=0}^{\infty} (1 - \theta_p)^{\ell} \left\{ s_{\ell,t} \Phi \left( \frac{z-\log P_1 + \log R_{S,t-\ell}}{\sigma} \right) + (1 - s_{\ell,t}) \Phi \left( \frac{z-\log P_1 + \log R_{N,t-\ell}}{\sigma} \right) \right\} \right]^{n-1} dz
\]

\[
v_{S,\ell,t} = v(R_{S,t-\ell}, D_t) \quad v_{N,\ell,t} = v(R_{N,t-\ell}, D_t)
\]

Demand multipliers:

\[
\zeta_{S,\ell,t} = s_{\ell,t} (\lambda + (1 - \lambda)v_{S,\ell,t}) \quad \zeta_{N,\ell,t} = (1 - s_{\ell,t}) (\lambda + (1 - \lambda)v_{N,\ell,t})
\]

Price index:

\[
P_t = \left( \sum_{\ell=0}^{\infty} (1 - \theta_p)^{\ell} \left\{ \zeta_{S,\ell,t} R_{S,t-\ell}^{1-\varepsilon} + \zeta_{N,\ell,t} R_{N,t-\ell}^{1-\varepsilon} \right\} \right)^{\frac{1}{1-\varepsilon}}
\]

Wage index:

\[
W_t = \left( \sum_{\ell=0}^{\infty} (1 - \theta_w)^{\ell} B_{\ell,t}^{1-\xi} \right)^{\frac{1}{1-\xi}}
\]
First-order condition w.r.t. sales:

\[
x_{\ell,t} = \frac{(\lambda + (1 - \lambda)\nu_{S,\ell,t})g_{S,\ell,t}^{1-\varepsilon} - (\lambda + (1 - \lambda)\nu_{N,\ell,t})g_{N,\ell,t}^{1-\varepsilon}}{(\lambda + (1 - \lambda)\nu_{S,\ell,t})g_{S,\ell,t}^{\varepsilon} - (\lambda + (1 - \lambda)\nu_{N,\ell,t})g_{N,\ell,t}^{\varepsilon}}
\]

Marginal cost:

\[
x_{\ell,t} = \frac{w_{\ell}}{q_{\ell,t}}\varepsilon
\]

First-order condition for sales prices:

\[
\sum_{\ell=0}^{\infty} (\beta \theta_p)^t \mathbb{E}_t \left[ s_{t,\ell+t} x_{t,\ell+t} P_{t,\ell}^c Y_{t,\ell+t} \right] \left\{ \begin{array}{c}
(\varepsilon - 1)(\lambda + (1 - \lambda)\nu_{S,\ell+t}) + (1 - \lambda)\nu_{S,\ell+t} + \eta_{S,\ell+t} \\
- (\varepsilon(\lambda + (1 - \lambda)\nu_{S,\ell+t}) + (1 - \lambda)\nu_{S,\ell+t} + \eta_{S,\ell+t})
\end{array} \right\} = 0
\]

First order condition for the wage:

\[
\sum_{\ell=0}^{\infty} (\beta \theta_w)^t \mathbb{E}_t \left[ W_{t,\ell+t} L_{t,\ell} \right] \left\{ \begin{array}{c}
\frac{B_t}{P_{t,\ell}^c} Y_{t,\ell+t}^\gamma - \frac{\xi}{\xi - 1} \left( \frac{B_t}{W_{t,\ell+t}} \right)^{-\varepsilon\omega} h_{t,\ell+t} \\
- (\varepsilon(\lambda + (1 - \lambda)\nu_{N,\ell+t}) + (1 - \lambda)\nu_{N,\ell+t} + \eta_{N,\ell+t})
\end{array} \right\} = 0
\]

5.1 Log-linearized equations

Results:

Quantity produced (firm demand function):

\[
q_t = Y_t - \frac{\varepsilon_S}{\delta} (P_{S,t} - P_t) - \frac{\varepsilon_N}{\delta} (P_{N,t} - P_t) + \frac{\bar{\theta}_S}{\delta} \tilde{\nu}_{S,t} + \frac{\bar{\theta}_N}{\delta} \tilde{e}_{N,t}
\]

Optimal sales first-order condition:

\[
\left[ (\lambda + (1 - \lambda)\bar{v}_S)g_{S}^{\varepsilon} - (\lambda + (1 - \lambda)\bar{v}_N)g_{N}^{\varepsilon} \right] x_t = ((1 - \varepsilon)\tilde{\mu}_S + \varepsilon) (\lambda + (1 - \lambda)\bar{v}_S)g_{S}^{\varepsilon} (P_{S,t} - P_t)
\]

\[
- ((1 - \varepsilon)\tilde{\mu}_N + \varepsilon) (\lambda + (1 - \lambda)\bar{v}_N)g_{N}^{\varepsilon} (P_{N,t} - P_t)
\]

\[
+ (1 - \lambda)(\mu_N - 1)g_{N}^{\varepsilon} v_{S,t} - (1 - \lambda)(\mu_N - 1)g_{N}^{\varepsilon} v_{N,t}
\]

General price index:

\[
P_t = \tilde{\nu}_{S}g_{S}^{1-\varepsilon}P_{S,t} + \tilde{\nu}_{N}g_{N}^{1-\varepsilon}P_{N,t} - \frac{\bar{\theta}_S}{\varepsilon - 1} \lambda_{S,t} - \frac{\bar{\theta}_N}{\varepsilon - 1} \lambda_{N,t}
\]

Demand multipliers:

\[
\xi_{S,t} = (\lambda + (1 - \lambda)\bar{v}_S)s_t + \bar{s}(1 - \lambda)v_{S,t}
\]
Victory margins:

\[ v_{S,t} = (1 - \bar{s}) c_{vSN} (P_{N,t} - P_{S,t}) - c_{vSf} s_t \]

Production function/real marginal cost:

\[ x_t = w_t + \frac{1 - \alpha}{\alpha} q_t \]

5.2 Numerical results

We obtain impulse response functions using our baseline calibration and: one period is one month, \( \xi = 6, \beta = 0.9975 \) (which implies interest rates of around 3% a year), \( \theta_P = 1/9 \) (as Nakamura and Steinsson (2007) find a median price spell of 9 months disregarding sales) and \( \theta_W = 1/12 \).

Figure 4 brings the impulse responses of output and inflation to a unit shock to \( M \), compared to a benchmark model where consumers have no choice of brands, but otherwise identical to the one with sales. The elasticity of demand for goods in the benchmark model is calculated to generate the same average markup as the model with sales generate.

The impulse response to output peaks at 18 months in both cases. In the first 18 months, the initial effect of output in the model with sales is around 78% of the effect in the benchmark model. From then on, this proportion starts to increase (it is 83% after 3 years, 87% after 4 years and 92% after 5 years).

The strategic substitutabilities regarding sales generate extra persistence on the real effects of monetary policy.

6 Extensions

6.1 Flexible wages

6.1.1 The model with exogenous sales

Now consider what happens when wages adjust following the monetary shock. In equilibrium they are equal to:

\[ W = PY^\gamma q^{\omega/\alpha} \]  

The real wage in equal to ratio between marginal disutility of work and marginal utility of consumption.

The following proposition establishes the main result of this section:

**Proposition 4** Given prices \( P_N \) and \( P_S \), as long as \( s \) is between 0 and 1, neglecting changes in \( \delta \) (which are quantitatively small) the effect of monetary policy on \( Y \) and \( P \) is
given by:
\[
\frac{\partial P}{\partial M} M = \gamma \alpha + 1 + \omega - \alpha \quad \frac{\partial Y}{\partial M} Y = -\alpha
\]

\[
\frac{\partial P}{\partial M} M = \gamma \alpha + 1 + \omega - 2\alpha
\]

\[
\frac{\partial Y}{\partial M} M = \omega - \alpha
\]

Proof. The first order condition (Equation 2) and the equation for the wage (Equation 6) yield:
\[
PY^{\gamma + \omega/\alpha} y^{(1-\alpha)/\alpha} = \alpha \left( \frac{P^1 - \varepsilon_1}{P^\varepsilon - \varepsilon_1} \right)
\]

\[
\log(P) + \left( \gamma + \frac{\omega + 1 - \alpha}{\alpha} \right) \log(Y) + \left( \frac{\omega + 1 - \alpha}{\alpha} \right) \log(\delta) = \log \left[ \alpha \left( \frac{P^1 - \varepsilon_1}{P^\varepsilon - \varepsilon_1} \right) \right]
\]

Differentiating with respect to \( \log(M) \):
\[
\frac{\partial P}{\partial M} M = - \left( \frac{\gamma \alpha + \omega + 1 - \alpha}{\alpha} \right) \frac{\partial Y}{\partial M} Y - \left( \frac{\omega + 1 - \alpha}{\alpha} \right) \frac{\partial \delta}{\partial M} M
\]

Neglecting changes in \( \delta \) and using the fact that the elasticities of \( Y \) and \( P \) must sum to 1, the claim is obtained.

As before, with monetary shocks, the optimal decision of the producer is to keep marginal cost constant. But now marginal cost changes with changes in \( P \) or \( Y \) because of their effect on wages. An increase in \( Y \) raises marginal costs via higher wages (elasticity \( \gamma + \omega/\alpha \)) and via higher output, hence lower marginal production (elasticity \( (1 - \alpha)/\alpha \)). It also increases \( P \), via higher wage (elasticity 1). The optimal decision of the producer implies quantities and prices such that nominal marginal cost is kept constant.

For any reasonable calibration, the effect of \( Y \) on marginal costs is higher than the effect of \( Y \) on prices, which implies that a positive monetary shock increases prices and decreases output. A positive shock to the money supply is followed by a decrease in sales, which increases \( P \). But if the producer acts as before, keeping fixed the quantity it produces, the increase in \( P \) means its nominal marginal cost will actually increase. To keep marginal cost constant, the producer makes an additional reduction in sales, leading to a further increase in prices and a, consequently a decline in output. In equilibrium, the impact of the decrease in output on marginal cost compensates the impact of the increase in price. Surprisingly, money is counter-cyclical.

Calibration The impact of monetary policy depends on \( \gamma \), \( \omega \) and \( \alpha \). Setting \( \gamma = 3 \) (relative risk aversion coefficient), \( \omega = 1.4 \) (inverse of the Frisch elasticity) and \( \alpha = 2/3 \) (the elasticity of output with respect to labor), we obtain:
\[
\frac{\partial Y}{\partial M} Y = -21.7\%
\]
Sensitivity with respect to $\gamma$: with log utility ($\gamma = 1$), the effect on output equals $-38.5\%$, and with $\gamma = 5$, the effect on output is: $-15.2\%$.

Sensitivity with respect to $\omega$: with $\omega = 0.5$, the effect on output equals $-30.8\%$. With $\omega = 3$, the effect on output is: $-18.2\%$.

Summing up, in this model, monetary policy is significantly countercyclical: a 1% increase in $M$ leads to a contraction of output of around 0.2%.

6.1.2 The model with sales in equilibrium

As before, in the model with sales in equilibrium, money has a more positive impact on output. Using parameters of our baseline calibration, but allowing wages to adjust at every period, we obtain that 17.5% of the effect of a money increase goes to output and 82.5% goes to prices. This contrasts with the counter-cyclical effects obtained above.

7 Concluding remarks

TBW

References


Figure 1: Weekly retail and wholesale prices for Bass Ale. Prices are for a single six-pack. From Goldberg and Hellerstein (2007).
Figure 2: Non-monotonic marginal revenue.
Parameters: $\lambda = 0.901, \varepsilon = 3.01, \eta = 19.73; \gamma = 3, \alpha = 2/3, \omega = 1.4$.
In equilibrium: $s = 0.074, P_S = 0.9022, P_N = 1.2111$. 
Figure 3: Marginal Revenue x Price.
In equilibrium, $P_S = 0.9022$, $P_N = 1.2111$. Marginal revenue is the same at both prices.
Table 1: Baseline calibration

<table>
<thead>
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<th>(\varepsilon)</th>
<th>(\eta)</th>
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</thead>
<tbody>
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<td>PS/PN</td>
<td>0.745</td>
<td>0.901</td>
<td>3.01</td>
<td>19.7</td>
</tr>
<tr>
<td>Quantity ratio</td>
<td>5.00</td>
<td>19.7</td>
<td>5.00</td>
<td>19.7</td>
</tr>
</tbody>
</table>

\[
\frac{\Delta Y}{Y} = 77.6\% \quad, \quad \frac{\Delta P}{P} = 22.4\%
\]

Table 2: Robustness: \(s = 12.1\%\)

<table>
<thead>
<tr>
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<th>(s)</th>
<th>(\lambda)</th>
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<td>5.00</td>
<td>19.7</td>
<td>5.00</td>
<td>19.7</td>
</tr>
</tbody>
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\[
\frac{\Delta Y}{Y} = 69.8\% \quad, \quad \frac{\Delta P}{P} = 30.2\%
\]

Table 3: Robustness: \(q = 7\).

<table>
<thead>
<tr>
<th></th>
<th>(s)</th>
<th>(\lambda)</th>
<th>(\varepsilon)</th>
<th>(\eta)</th>
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<tbody>
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<td>Quantity ratio</td>
<td>7.00</td>
<td>26.2</td>
<td>7.00</td>
<td>26.2</td>
</tr>
</tbody>
</table>

\[
\frac{\Delta Y}{Y} = 72.4\% \quad, \quad \frac{\Delta P}{P} = 27.6\%
\]
Figure 4: Impulse responses to a money shock