The Price Theory of Two-Sided Markets

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Definition of a two-sided market

1. Two groups of consumers
2. Value from connecting (proportional to partners)
3. Price balance matters (Coase/price neutrality fails)

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Goal: How are two-sided markets the same and different? How do effects of policies differ?

Rochet and Tirole canonical model: multiplicative demand $D^A(p^A)D^B(p^B)$

Problem: Topsy-turvy stop full analysis

Strategy: Divide and conquer

Positive analysis: vulnerability

Normative analysis: average surplus
The “vulnerability” of demand

- Standard monopolist problem \((p, D(\cdot), c)\)
- Familiar FOC:

\[
m \equiv p - c = \frac{p}{\epsilon(p)} \equiv \gamma(p)
\]

- To get sufficiency: \(\gamma' < 0 \iff \) log-concave demand, assume this
- Margin = “exploitation”, \(\gamma\) is consumer’s “vulnerability” of demand
Why does competition lower price?

When two symmetric firms \( D^1(p_1, p_2) = D^2(p_2, p_1) \), two notions of elasticity/vulnerability:

- Total vulnerability \( \gamma = \frac{p}{\epsilon} \) vs. own-price vulnerability \( \gamma_o = \frac{p}{\epsilon_o} \)
- \( m = \gamma_o \) for Bertrand eq, analogous to monopoly
- \( \gamma_o < \gamma \) as \( \epsilon_o > \epsilon \)
- See graph

Standard intuition through vulnerability: under competition raising price drives away more consumers

- So optimal to set lower price

The argument parallel in two-sided markets

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Starting point of my analysis

- Compare monopoly to duopoly ownership of two platforms

Start with Rochet-Tirole (2003) foc’s:

Monopoly:
\[ m \equiv p^A + p^B - c = \gamma^A(p^A) = \gamma^B(p^B) \]

Duopoly:
\[ m \equiv p^A + p^B - c = \gamma^A_o(p^A, p^B) = \gamma^B_o(p^B, p^A) \]

- Analogous to standard market, but “balance" vulnerabilities (total or own-price) between “buyers" and “sellers"
- Crucial that \( \gamma^i \) decline; log-concavity
- And substitutability \( \gamma^i_o(p^i, p^j) < \gamma^i(p^i), \forall p^i, p^j, i \)
  (multi-homing or own-price elasticity)
- \( \gamma^i_o \) less constrained than \( \gamma \)
Proposition 1

- Price level $\bar{p} \equiv p^A + p^B$
- Price level under monopoly ownership $\bar{p}_M^*$
- Price level at duopoly equilibrium $\bar{p}_C^*$

$$\bar{p}_M^* > \bar{p}_C^*$$

Many Bertrand eq., holds for all.
Proof strategy

- General approach: Separate competition pushing down prices from topsy-turvy (price balance)
- For this, construct vulnerability *level*
  
  \[
  \gamma(p) \equiv \gamma^A(p^A(p)) ; \quad p^A(p) \text{ solves } \gamma^A(p^A) = \gamma^B(p - p^A)
  \]
  given \( p \)
- Same for competition: \( \gamma_o(p) \)
- Then invoke standard market strategy above
Balance of competition and individual prices

- Not just interested in price level, but also in individual prices
- Difficult to say much generally, but to illustrative extreme cases
If competition completely unbalanced:

\[ \gamma^A_0(p^A, p^B) = \gamma^A(p^A), \quad \gamma^B_0(p^B, p^A) < \gamma^B(p^B), \quad \forall p^A, p^B \]

\[ p^B_M > p^B_C \text{ but } p^A_M < p^A_C \]

- Price level falls \(\rightarrow\) equilibrium vulnerability falls
- \(\gamma^A\) stable and declining \(\rightarrow\) \(p^A\) rises

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Proposition 3

Opposite extreme; if competition perfectly balanced:
\[ \gamma^A_0(p^A, p^B) = \alpha \gamma^A(p^A), \gamma^B_0(p^B, p^A) = \alpha \gamma^B(p^B) \forall p^A, p^B, \alpha \in (0, 1) \]

\[ p^B_M > p^B_C \quad p^A_M > p^A_C \]
Analysis of balanced competition

Note that $p^A_\star(\bar{p})$ solving $\gamma^A(p^A) = \gamma^B(\bar{p} - p^A)$ also solves
$\gamma^A_0(p^A, \bar{p} - p^A) = \gamma^B_0(\bar{p} - p^A, p^A)$

Thus, given price level falls, only need that
$0 < p^A_\star'(\bar{p}) < 1$, $\forall \bar{p}$, implicitly differentiate:

$$0 < p^A_\star'(\bar{p}) = \frac{\gamma^B'}{\gamma^A' + \gamma^B'} < 1$$
Price controls

- **Unilateral price control** is $p^A \leq p^A_{\text{max}}$, $p^B$ unregulated
  - Proposition 4: Same as completely unbalanced competition; pressure on one side, none on the other

- **Price level control** is $\bar{p} \leq \bar{p}_{\text{max}}$
  - Proposition 5: Same as perfectly balanced competition; unchanged dynamics of price balance, no topsy-turvy

Subsidies

- All subsidies equivalent: reducing cost from \( c \) to \( c - \sigma \)
  same as decreasing effective price from \( p^A \) to \( p^A - \sigma \)
- Balance unchanged
- Subsidies reduce (effective) price level, simpler formula
  using vulnerability, applies (simplifies) in standard markets:

\[
\frac{d\bar{p}}{d\sigma} = -\frac{1}{1 - \gamma'}
\]
A framework for welfare analysis

- Multiplicative demand/externality form: \( D^A(p^A)D^B(p^B) \) under monopoly
- Profit: \( (\bar{p} - c)D^A(p^A)D^B(p^B) \)
- Surplus side \( i \):
  \[ V^i(p^i) = \int_{p^i}^{\infty} D^i(p)dp \]
- Log-concavity of surplus
- Average surplus:
  \[ \bar{V}^i(p^i) = \frac{V^i(p^i)}{D^i(p^i)} \]
Welfare criteria

1. Social surplus (consider profits):

\[ \pi^{soc} = D^A V^B + D^B V^A + (\bar{p} - c) D^A D^B \]

2. (Tax-augmented) Consumer surplus (with subsidy \( \sigma \)):

\[ \pi^{tax} = D^A V^B + D^B V^A - \sigma D^A D^B \]
Linear vulnerability class

- Demand has linear vulnerability iff:

\[
D(p) = \begin{cases} 
\frac{(a-p)^\alpha}{b} & p \leq a \\
0 & p > a 
\end{cases}
\]

- Log-concavity and positivity imply \( b, \alpha > 0 \)
- Only interesting if \( p \leq a \), then:

\[
\gamma(p) = -\frac{D(p)}{D'(p)} = \frac{a-p}{\alpha}
\]

- Average surplus:

\[
\bar{V}(p) = \frac{V(p)}{D(p)} = \frac{a-p}{1+\alpha}
\]

- \( \alpha \) measures relative curvature; \( \alpha > 1 \) convex; \( \alpha < 1 \) concave
Again, separate balance from level
Like Rochet-Tirole, first consider balance given level
They ask, does monopolist choose optimal balance?
Yes in bi-linear case, not clear how general
Everything given $\bar{p} > c$, easily extended to $\bar{p} \leq c$:

1. **Monopolist profit=volume maximizing** (RT2003):

   $$\gamma^A(p^A) = \gamma^B(\bar{p} - p^A)$$

2. **Consumer surplus maximizing** (RT2003):

   $$\bar{V}^A(p^A)\gamma^A(p^A) = \bar{V}^B(\bar{p} - p^A)\gamma^B(\bar{p} - p^A)$$

3. **Social surplus maximizing** ($\lambda \equiv \frac{1}{1 + \bar{p} - c}$):

   $$\left[1 - \lambda + \lambda\bar{V}^A(p^A)\right] \gamma^A(p^A) = \gamma^B(\bar{p} - p^A) \left[1 - \lambda + \lambda\bar{V}^B(\bar{p} - p^A)\right]$$
Corollary 1

- Do these often agree?
- Consider linear vulnerability class:
  \[ \gamma = \frac{1 + \alpha}{\alpha} V \]
- Only agreement when \( \alpha_A = \alpha_B \)
- Measure 0 under Lebesgue measure over \((\alpha_A, \alpha_B)\)
- Rochet-Tirole coincidence extremely special
- Disagreement proportional to difference in relative curvature on two sides
Transfers

- Monopolist chooses wrong balance
- How do we identify improvements to welfare?

Suppose we start at monopoly prices:

1. If \( \overline{V}^i > \overline{V}^j \), transfer from \( i \) to \( j \) good for social/consumer welfare (opposite bad)

2. If \( \overline{V}^i > \gamma^i = \gamma^j \), transfer from \( i \) to \( j \) benefits average \( i \), reverse hurts

3. Result: some transfers (price balance controls?) benefit both sides’ average consumers
Optimal price level

- Assume both price balance: both prices declining in price level
  - Price level regulation
  - Subsidies
  - Balanced competition

1. $\bar{p}^* < c$
2. Value of reducing price level $> 0$ whenever $\bar{p} \geq c$
Proof that optimal price level is below cost

Let $p^i(\bar{p})$ to be price on side $i$ with price level $\bar{p}$

Differentiating social surplus with respect to price level yields:

$$D^A' p^A' V^B + D^B' p^B' V^A + (\bar{p} - c) \left[ D^A' p^A' D^B + D^B' p^B' D^A \right]$$

$D^i'' < 0$, $p^i'' > 0$ and $\bar{p} - c \geq 0$ for $\bar{p} \geq c$ so last term (associated with traditional monopoly distortions) is non-positive. So expression $\leq$:

$$D^A' p^A' V^B + D^B' p^B' V^A$$

These are the externalities…clearly this is strictly negative
To get formula for socially optimal price level, we need to take a stand on balance

- Natural choice: socially optimal price balance

**Natural Ramsey pricing form:**

\[ \overline{p}^{**} = c - \overline{V}^i \left( p^{j**} (\overline{p}^{**}) \right) \]

In linear vulnerability case this is:

\[ \overline{p}^{**} = a_i + a_j - \frac{a_i + a_j - c}{1 - \nu} \]

\[ \nu \equiv \frac{1}{2 + \alpha_i + \alpha_j} \]

The more concave demand, the great should be \( c - \overline{p} \)
How should we correct if monopoly governs price balance? "Socially optimal subsidies"

General formula complex, but in linear vulnerability:

\[ \bar{p}^* = a_i + a_j - \frac{a_i + a_j - c}{1 - \eta} \]

\[ \eta \equiv \frac{\alpha_i \alpha_j}{(\alpha_i + \alpha_j)^2} \left[ \frac{1}{1 + \alpha_i} + \frac{1}{1 + \alpha_j} \right] \]

- \( \bar{p}^* > \bar{p}^{**} \) if \( \alpha_i \neq \alpha_j \)
- Ramsey pricing + adjustment
- Give less subsidies because monopolist does not optimally allocate them
Proposition 11 and Corollary 5

- In standard market, subsidies are big transfer to firm; bad for tax-augmented consumer welfare
- Is this still true in two-sided markets?
- General conditional complex
- But in linear vulnerability, subsidy increases tax-augmented consumer surplus if both demands convex
Competition has two effects: price level and balance

- When competition is perfectly (sufficiently) balanced or when price level control imposed, both prices fall
- Thus only level effect
- So by Proposition 8, these are welfare-enhancing
Unbalanced competition

- What about unbalanced competition and regulation?
- Level effect always good
- Balance effect may be positive or negative (in a strong sense) by Lemma 1
- Balance may dominate level effect
- Sufficiently small, sufficiently unbalanced fall in price level may *harm both sides* (on average)!
- Sufficiently small, sufficiently unbalanced fall in price level may *benefit both sides more than any* balanced reduction in price level
- Anything goes?

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A related project

- Some two-sided markets have intermediary between platform and consumers on one side: debit card clearing network (Star, Interlink) and issuing banks.
- Is it better that these be integrated (Interlink by Visa) or separate (Star)?
Results on vertical integration

- Proofs available in longer version of paper (separate paper soon)
- Price level, buyers price always lower under integration
- Seller’s price depends on curvature of buyer’s vulnerability
- Quite robust (competition, strategic set-up)
- Two-sided double marginalization problem
- Bonus: new characterization of standard double marginalization problem using vulnerability
Is policy important in two-sided markets?

- Monopoly *may* not distort too much in two-sided markets (Rochet and Tirole 2003?, Wright 2003, 2004)
- Policy too unpredictable to intervene (Wright 2006)
- No difference between importance of policy in two-sided versus standard markets (Evans 2003)
- My results: even more important than in standard markets
- First-order harm from market power + second order
- Two dimensions of distortion (relative to social optimum)

Insights for antitrust

1. Cannot infer “anti-competitive behavior” or collusion from individual prices; instead use price level.
2. Price level better surrogate for welfare than individual prices...but not sufficient.
3. Competition may cause harm, so if authority know this is the situation, forbearance may be advisable.
4. *But* ex-ante probably balance effects neutral, so competition policy important.
5. Vertical integration probably good.

Insights for regulation (and subsidies)

1. Unilateral price controls (net neutrality, interchange fee regulation) raises prices to other side of the market.
2. Price level controls an interesting alternative (at least we know direction).
3. Price balance controls can help in theory.
4. *But* strategic issues emerge in identifying which way to move.
5. Subsidies very attractive, relative to standard market.

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