Abstract

There are plenty of economic studies pointing out some requirements, like the inexistence of fiscal dominance, for inflation targeting framework be implemented in successful (credible) way. Essays on how public targets could be used in the absence of such requirements are unusual. In this paper we appraise how central banks could use inflation targeting before soundness economic fundamentals have been achieved.

First, based on concise framework, where confidence crises and imperfect information are neglected, we conclude that less ambitious (greater) target for inflation increases the credibility in the precommitment. Optimal target is higher than the one obtained using the Cukierman-Liviatan [7] model, where increasing credibility effect is not considered. Second, extending the model to make confidence crises possible, multiple equilibria solutions becomes possible too. In this case, to set greater targets for inflation may stimulate confidence crises and reduce the policymaker credibility. On the other hand, multiple (bad) equilibria may be avoided. The optimal target depends on the likelihood of each equilibrium be selected.

Finally, when perturbing common knowledge uniqueness is restored even considering confidence crises, as in Morris-Shin[14]. The first result, i.e. less ambitious target for inflation increases credibility in precommitment, is also recovered. Adding a precise public signal, coordinated self-fulfilling actions and equilibrium multiplicity may still exist for some lack of common knowledge (as in Angeleto and Werning[1]). In this case, to set greater targets for inflation may stimulate confidence crisis again, reducing the policymaker credibility. From another aspect, multiple (bad) equilibria may be avoided. Optimal policy prescriptions depend on the likelihood of each equilibrium be selected. Results also indicate that more precise public information may open the door for bad equilibrium, contrary to the conventional wisdom that more central bank transparency is always good when considering inflation targeting framework.
1 Introduction

Thinking economy as a large group of individuals, each one of them granted with some kind of good and prone to consume many types of goods, the determination of the aggregate price level can be viewed as a result from some noncooperative game where seller decisions about setting its own *ask-price* are *non trivial* strategies. Large group means that it is difficult for each player to predict the other actions and the equilibrium aggregate price level. As each individual price has no other benchmark but the other prices, setting its own price appears to be an unpleasant task for each seller if there is asymmetric beliefs and no-coordination signal. Even supposing that each player knows the relative equilibrium prices, some confusion may come from what should be the nominal price level. As long as the actual inflation depends on the expected inflation (aggregated), private agents try to predict the beliefs of others when forming their own (see Simonsen [18]).

In this way, expectations may play a crucial role in the determination of the equilibrium nominal price. In an extreme case, they may induce multiple equilibria of self fulfilling type. Another reason for inflation be viewed as a self fulfilling equilibrium comes from the fact that the *ex-post* government behavior is not immune to the public *ex-ante* expectations. Government may need short run effects from inflation to smooth some real shock associated with an adverse expectation\(^1\). Such confidence crises bring “non fundamental” volatility to the economy\(^2\).

Inflation targeting regime may be viewed as a good set-up to by pass coordination failure and to reduce price volatility. On the other hand, to be implemented in successful way, the announcement must be listened by the public and some policy credibility is required. It is well known that trade-offs faced by policymaker change before, and after private expectations have been set. Before, there are incentives for announcing the social optimal rate of inflation. After, there are incentives for picking higher inflation levels. Although inflation is bad news, since it always implies some welfare

\(^1\)As argued by Calvo in [4], when the government debt is auctioned off to the public, and there is no attempt to manage expectations or to peg interest rates on the government debt, self fulfilling equilibria for inflation are possible. Greater inflation is used to reduce the tax requirement to pay maturing debt. In Araujo, Leon and Santos [2], inflation may reduce default risk during external confidence crisis.

\(^2\)Obstfeld [17] was one of the pioneers in studying models with such feature (self fulfilling currency crises).
cost to the economy, it may be implemented and even be desirable during crisis times. When its value exceeds the public expectations, one can think on positive welfare effects such as expansionary results on employment in the presence of nominal contracts; seigniorage revenues and reductions in the real value of outstanding government debt as response to political shocks (as reelection intention, or popular pressure for more public expenditure); and balance-of-payment considerations (a fall in the commodity price that intensively takes part in the exports or a speculative attack on the external debt may turn big devaluation of the local currency required).

1.1 Literature

As already shown in many papers by comparing rules versus discretion (see Kydland and Prescott[12] and Barro and Gordon, [3]), the total lack of the ability in precommit brings to the economy more inflation and less welfare. For this reason policymakers use to be concerned about their own credibility. Other studies have pointed out “the requirements” to inflation targeting be implemented in successful (credible) way, such as central bank independence and transparency; absence of fiscal dominance; not to target other indicators such as wages, the level of employment, or the exchange rate (see Debelle, Masson, Savastano and Sharma[8] and Mishkin [13]). All of these recommendations may help the policymaker in improving the target credibility and should be viewed as pre requisite for the ideal inflation targeting framework.

From another aspect, when a policymaker is subject to fiscal dominance and to shocks that make bigger inflation desired, no precommit at all may be a bad solution. Although soundness commitment must be the final target, some commitment should be adopted even under low credibility. Obviously, for a very strong policymaker or for a very weak policymaker, commitment to social optimal inflation rate and no precommitment are, respectively, the best equilibria candidates. But for intermediate cases, what should be the policy prescription? Is it possible to set less ambitious (greater) targets for inflation in order to get more credibility and a better coordination instrument? We show that it may be appropriate for some emerging-market economies to set modest targets while soundness
fundamentals and credibility have not been achieved. Perhaps, this is the reason why Chile’s central bank has adopted a very gradualist approach to lowering its inflation objectives (see Mishkin[13]).

1.2 A Glance at the Empirical Evidence

In Figures 1 and 2 we present results from inflation targeting adoption in twenty countries, twelve emerging economies and eight developed economies. As shown, the adoption reduced all big inflation (greater than 10% in year basis) fast and significantly. Inflation path like the one observed in Peru, where after the target announcement the inflation falls from 50% to 10% in year basis (see Figure 1), suggests that expectations play an important role in the determination of the actual inflation rate. In such case, it is difficult to explain the decreasing in the inflation rate using only some “fundamental path”.

On the other hand, especially for low inflation cases (lesser than 10% in year basis), the strength of the fundamentals seems to be quite important to determine the actual inflation rate. For example, during the adoption of the inflation targeting framework, Iceland experienced an economic expansion that evolved into domestic expenditure growth well in excess of national income resulting in a widening external current account deficit. This fact seems to be one of the reasons why the actual inflation has not fallen after the inflation targeting adoption (see Figure 2). The current account deficit also increased the inflation rate in Australia in the beginning of the 90’s (see Figure 2). In South Korea, the current account improvement and the sharp appreciation of the won-dollar exchange rate helps to keep inflation much lower than the first target announced (see Figure 1).

Obviously, the strength of the economic fundamentals is more than the current account balance and its effects on the exchange rate. The measure for this strength is somewhat difficult and arbitrary.

Considering the inflation targeting context, the freedom from the dominance of fiscal policy is frequently mentioned as a basic and important requirement for credible commitment to be implemented. Since reliance on seigniorage is perhaps the simplest and most common indication of fiscal dominance, we selected the share of seigniorage in public expenditure as a proxy for the
ability of the central bank in precommit. One can think of this measure of reliance on seigniorage as a *simplified* measure for the strength of the status quo (target for inflation), since many other factors like the target level itself and current account considerations should be computed to measure strength in a more precise way.

We plot this rough proxy for strength in Figure 3, where the “breaking-commitment country-year” sample is selected. “Breaking-commitment” means that actual inflation was out of the target band. We also plot the target level and the relative target band deviation$^3$. Based on the positive correlation obtained (60%) between the target level and the measure of reliance on seigniorage, one may guess that a policymaker with weaker ability in setting the status quo tends to chose greater target level for inflation. On the other hand, this correlation falls from (60%) to (3.8%) if we consider only country-year observations where the target level was less than (5%). Considering this same restricted sample, where seigniorage and the target level are not correlated, the correlation between the measure of reliance on seigniorage and the relative deviation from the target is (55%). One may guess that weaker policymaker tends to break its commitment with more “surprise inflation” than the stronger one.

Perhaps, greater target and lesser reliance on seigniorage helps to achieve “more credible” commitment. Another way to get “more credible” commitment may be to hide the weakness. To check if a policymaker with weak ability in precommit tends to be less transparent is very difficult since a good measure for transparency needs hard work and also because less transparency itself should be viewed as weaker ability. In spite of such difficulty, we try to appraise this issue and plot in Figure 4 the proxy for strength of the status quo and the central bank transparency index (obtained at [9]). Based on it we do not find evidence for positive correlation between the proxy for strength of the status quo and the transparency level, although we believe that private information in the policymaker office has some value, especially when the policymaker is exposed to eventual speculative forces.

$^3 (\frac{\pi - \bar{\pi}}{\pi})$ if $\pi > \bar{\pi}$ or $\left(\frac{\pi - \bar{\pi}}{\pi}\right)$ if $\pi < \bar{\pi}$; where $\pi$ is the actual inflation and $[\bar{\pi}, \bar{\pi}]$ is the target band.
Possibly, if there was some transparency index available for emerging economies too, the results would be different.

Next, we propose some models to appraise how targets should be settled in order to achieve credibility and welfare improvement in “building credibility economies”, where central bank is neither too strong nor too weak.

2 Inflation Targeting Models

The framework used as starting point to our analyses is based on the Cukierman-Liviatan model [7] and is similar to the one presented in Barro [3]. It has some appropriate features such as to consider hearer’s uncertainty about the commitment enforcement and to suggest that the optimal target should be decreasing in the policymaker credibility. During this section, we gradually extend this framework to compute not only the effect of the credibility on the target, but also the effect of the target choice on the credibility, based on the assumption that less ambitious (greater) targets are fulfilled more likely. We also present different approaches for uncertainty. First, the Cukierman-Liviatan model with two policymaker types; the “strong” one which always adheres to the announced policy and the “weak” one which does only if it is ex-post expedient; rules out the possibility of more realistic policymaker preferences, namely: to perform the target whenever the economy is not hit by an adverse shock, but to accept more inflation when crisis times arise after the target announcement. Second, as the value of keeping the promise tends to be increasing in the policymaker credibility for multi-period economies, credibility itself may affects the policymaker decision about respecting (or not) its commitment. Finally, individuals usually disagree about their guesses for future inflation. One possible reason is the presence of imperfect information that may be relevant to policy prescription.

2.1 Basic Model

The economy is described as one-shot game where the policymaker announces the target for inflation ($\pi_a$), expectations are formed ($\pi_e$) and actual inflation is picked up ($\pi$). There are two policymaker
types $i \in \{1, 2\}$ with different abilities to precommit. The first type (“strong”) always fulfills its commitment while the second one (“weak”) does only if it is ex post expedient. Their objective function is positively related to surprise inflation and negatively related to actual inflation, as follows:\footnote{We add the “cost of cheating the public” function $c^i(\pi_a, \pi)$ to the original Cukierman-Liviatan model \cite{Cukierman1992} to formalize that the strong type always fulfills the target pre-announced while the weak type are not concerned about the previous announcement.}

\[
v^i(\pi_e, \pi_a) = \max_{\pi \geq 0} A[\pi - \pi_e] - \frac{\pi^2}{2} - c^i(\pi_a, \pi) \text{ where } c^i(\pi_a, \pi) = \{K^i, \pi_a \neq \pi \}, i \in \{1, 2\}\]

\[
K^1 = A^2, \quad K^2 = 0, \quad A > 0, \quad \pi_a \geq 0, \text{ and } \pi_e \geq 0
\]

Note that the policymaker best response for actual inflation ($\pi^*$) is either the target level ($\pi_a$) pre-announced or the discretionary inflation rate level ($A$). We can compute the welfare gain ($w^i_a$) of type (i) keeping the target ($\pi_a$) as follows:\footnote{From now on, the welfare gain from keeping actual inflation on the target will be denoted by ($w_a$).

\[
K^1 = A^2, \quad K^2 = 0, \quad A > 0, \quad \pi_a \geq 0, \text{ and } \pi_e \geq 0
\]

Since the goal of the inflation targeting regime is to coordinate expectations from the discretionary inflation rate level ($A$) to socially optimal level (0), it is easy to check that ($\pi_a \leq A$), ($w^1_a > 0$) and ($w^2_a \leq 0$), and both types of ability are justified for any target level.

The private agent is not a strategic player. Its role is to process information, to form beliefs concerning the policymaker’s type, and to compute inflationary expectations. It is assumed that the private uncertainty about the policymaker type is formed based on the exogenous probability ($\alpha$) of the type being strong ($i = 1$), which is the same for all private agents. This probability measures the policymaker credibility. The public’s inflation expectation is given by:

\[
E[\pi|\alpha, \pi_a] = \pi_e = \alpha \pi_a + (1 - \alpha)A
\]

Based on this framework Cukierman and Liviatan \cite{Cukierman1992} answered the following question: “what should be the optimal announcement $\pi^*_a$ for each type (i)”? For ($\alpha = 1$), the announcement and the expectations have the same value. Then, the policymaker type 1 promises and delivers zero inflation
rate. If we consider $\alpha \in (0,1)$, the policymaker 1 promises and delivers $A(1-\alpha)$ inflation rate. As $\alpha$ tends to zero the announcement effect on the expectations vanishes and the policymaker 1, who always keeps its promises, tends to pre-announce the discretionary inflation rate level. Although type 2 ends up inflating at the discretionary rate, it has interest to keep itself indistinguishable at the announcement stage in order to stimulate lower expectations ($\pi_e < A$). It follows that $\pi_{a}^{*i} = A(1-\alpha)$ for both types ($i$). Accordingly, full credibility ($\alpha = 1$) is not required for inflation targeting be implemented in successful way. In the absence of precommitment the result leads to an inflationary bias ($A$) that can be reduced whenever policymakers are able to precommit with some credibility ($\alpha > 0$). This bias reduction improves welfare. To totally eliminate the inflationary bias and to achieve the best social inflation rate (zero), the ability to commit must not be only present but must also be recognized beyond any doubts by the public. Otherwise, a lower inflationary bias reappears.

Next, we gradually extend this framework to argue that there are some other reasons for inflation target be higher than the socially optimal level.

2.1.1 Endogenous Credibility

Less ambitious (greater) targets are fulfilled more likely when monetary policy is subordinated to fiscal financing requirements and economy is subject to shocks that make short-run effects from inflation desirable. In this sense, $(\alpha)$ should not be an exogenous variable because when $(\pi_a)$ is selected $(\alpha)$ should be affected. Then, we consider the doubt about the ability to precommit coming not from some private suspicion related to the central bank type being either very “serious” or very “cheater”. Perhaps, the cost of being over the target vary as from the announcement stage if the economy is hit by an adverse shock and the doubt may be present in the policymaker office too.

The model set-up considered here is the same as the previous one, but with only one type of policymaker, which is common knowledge. Instead of being a real number, the cost of cheating the public $(k)$ is now uniformed distributed on the support $[K,\bar{K}]$ and is drawn after the public expectations have been formed. Actual inflation is picked up at the end of the period. A low realization
of \( k \) can be viewed as a shock that decreases the value of keeping the commitment without using inflation short run effects. If we set \( (\bar{K} = 0) \) or \( (K = A^2) \) the equilibria are trivial: with \( (\alpha = 0) \) and discretionary inflation, or with \( (\alpha = 1) \) and full credibility for zero-inflation-commitment, respectively. To keep attention on the intermediate case where \( \alpha \in (0, 1) \), we assume that \( k \) is drawn from \( U[0, B] \), with \( B > 0 \). Depending on the values of \( k \) and \( \pi_a \) the commitment is fulfilled. The credibility is given by:

\[
\alpha(\pi_a) = \left[ \text{prob}(w_a(k; \pi_a)) > 0 \right]
\]

\[
\alpha(\pi_a) = 1 - \frac{1}{B} \left[ A(A - \pi_a) - \frac{A^2}{2} + \frac{\pi_a^2}{2} \right]
\]

When choosing the target, the policymaker understands that the greater is its level, more credible is its policy since \( \frac{d\alpha}{d\pi_a} \geq 0 \). In particular, only \( A \)-inflation commitment is full credible.

As in the previous model, because the possibility of the cost of cheating the public being positive, the commitment is listened, drives expectations and adds value to the economy. But now we have a different answer to the following question: “\textit{what should be the optimal target } \pi_a^* \text{?}”. On the one hand, for a given \( (\alpha) \), the closer to zero the target announcement is, the lesser is the expected inflation since \( (\pi_a) \) drives it. This fact increases welfare for any fixed \( (\alpha \neq 0) \). But on the other hand, as closer is the target announcement to zero, closer to the zero the credibility \( (\alpha) \) is.

With this background in mind we define the economy with only one type of policymaker; given by the parameter \( (A) \) and the common knowledge distribution of \( k \sim U \ [0, B] \); with two positive parameters \( (A, B) \). The following proposition characterizes the equilibrium:

**Proposition 1** For any economy \( (A > 0, B > 0) \), the equilibrium target \( (\pi_a^*) \) exists, is unique, and is in the interior of the set \( [0, A] \). If we solve the original Cukierman-Liviatan model for \( \alpha^* (\pi_a^*) \), its solution \( (\pi_a^{**}) \) is lower than \( (\pi_a^*) \). \textit{Proof: Appendix.}

A less ambitious target improves credibility in the announcement and induces positive welfare effect for economies where the policymaker is not able to set “full-credible” commitment. Then, \( ^6 K = 0 \) and \( K = B > 0 \).
when setting targets the policymaker must be aware that the announcement effect on expectations is reduced by credibility and that the announcement itself affects credibility. We present in Figure 5 the optimal target announced when considering the credibility effect versus the one announced when this effect is neglected. Note that, the weaker is the ability in precommitment (lesser $B$) the greater is the difference between the two announcement values and closer to zero credibility ($\alpha$) is.

### 2.2 Credibility Cost for Repeated Game and Self-fulfilling Inflation

One possible reason to assume some cost for cheating the public may come from the fact that the previous game has many periods in fact, and not just one. To cheat may be punished by credibility loss in the subsequent period. In this sense, credibility ($\alpha$) may affect the policymaker’s incentive to defend the target. To show this we use the same model again, but considering infinite periods, discount factor equals to $\beta \in (0, 1)$ and $B = 0$. In the first movement of the game the policymaker decides one target for inflation. Given the target, the expectations are formed in the beginning of the period. Then, actual inflation is picked-up by the policymaker at the end of the period. This timing-actions are the same for all periods. We also assume that the initial policymaker credibility is exogenous and equal to ($\alpha$). Next period credibility is ($\alpha$) again if the inflation matches the target, or falls permanently from ($\alpha$) to (0) otherwise. This fall is the penalty for cheating and an explicit positive cost ($k$) is not required for the target to be fulfilled anymore.

The initial payoff and the payoff for the deviation from the target, given ($\pi_a, \alpha$), equals to:

$$W_a(\pi_a, \alpha) = \max_{\{\pi_t\}_t} \sum_{t=0}^{\infty} \beta^t \left\{ A [\pi_t - \pi_{t,e}(\alpha, \pi_{t-1}, \pi_a)] - \frac{\pi_t^2}{2} \right\}$$

$$D(\pi_a, \alpha) = A\alpha [A - \pi_a] - \frac{A^2}{2(1 - \beta)}$$

The payoff for keep the target is given by:

$$W(\pi_a, \alpha) = \frac{\beta(1 - \alpha)D - A(1 - \alpha) [A - \pi_a] - \frac{\pi_a^2}{2}}{[1 - \beta\alpha]}$$

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7On the other hand, as ($B$) gets bigger the credibility ($\alpha$) becomes closer to one and the smaller is the difference between $\pi_a^*$ and $\pi_a^{**}$. 

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It follows that:

\[ w_a(\pi_a, \alpha) = \frac{A}{(1 - \beta \alpha)} \left\{ \frac{A}{2} - \frac{\pi_a^2}{2A} - (A - \pi_a)(1 - \beta \alpha) \right\} \]

It is easy to check that if the policymaker is sufficiently concerned about the future \((\beta > \frac{1}{2})\), then \(w_a(\pi_a, 1)\) is not negative and \(w_a(\pi_a, 0)\) is negative for any \(\pi_a \in [0; A]\). In other words, if this economy is populated by very optimistic agents \((\alpha = 1)\), the policymaker announces and implements the zero inflation rate. If this economy is populated by very pessimistic agents \((\alpha = 0)\), the unique equilibrium for inflation is the discretionary one. This result is quite similar with that obtained in the first model.

The optimal target here, i.e. \(A(1 - \alpha)(1 - \beta \alpha)\), is also decreasing in the exogenous credibility. But there is one important difference. In the first model, the doubt about the commitment enforcement comes from the uncertainty about the policymaker type (incentive). Now, the policymaker type is unique and deterministic. The commitment enforcement depends on which credibility will be selected. There is one set for fundamentals but it is easy to find two possible self-fulfilling equilibria for zero target announcement:

- Everybody believes in the target. The target is fulfilled (perfect commitment).
- Nobody believes in the target. The target is abandoned (discretionary).

Although we have made a strong hypothesis that the policymaker can not improve the credibility path and that to cheat is punished with zero credibility, this simple framework shows that the relative value of respecting the target is increasing in credibility for a repeated game. This feature opens the door for multiple equilibria and self-fulfilling inflations.

Back to the one shot-game framework to appraise this equilibrium property, we define the economy \((\xi)\) with the following set of parameters \(\{(A > 0); (\pi_a \in [0; A]); (\epsilon \geq 0); (n \in \mathbb{R})\}\) plus the increasing function \(h(\alpha)\), which are all common knowledge. The policymaker type is unique and is given by the cost of cheating \(K = h(\alpha) + \tilde{k}\); where \(\tilde{k}\) is a random variable distributed with \(U[n - \epsilon, n]\), \((\alpha)\) is the endogenous credibility, and \(h(.)^8\) gives the strength of speculative forces.

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8In order to reach a simple characterization of the equilibrium we also assume that \(h : [0, 1] \rightarrow \mathbb{R}\) is linear.
With such assumptions, both fundamental and expectations shocks may be important to compute policymaker incentives in choosing actual inflation.

It is always possible to reach an equilibrium for any economy \( (\xi) \), and it may be possible to reach more than one, as follows:

**Proposition 2** The economy \( \xi \) always admits equilibrium. Defining \( x \equiv -f(A, \pi_\alpha) \), for \( (\epsilon > h(1) - h(0)) \), it is possible that: \( (i) \ x < (n - \epsilon + h(1)) \), \( (ii) \ x \in [n - \epsilon + h(1), n + h(0)] \), \( (iii) \ x > n + h(0) \). In the first case only perfect commitment \( (\alpha = 1) \) equilibrium is possible. In the second case only imperfect commitment is possible \( (\alpha \in (0, 1)) \). In the last case only discretionary \( (\alpha = 0) \) equilibria is possible. When \( (\epsilon \leq h(1) - h(0)) \) it is possible that: \( (i) \ x < (n - \epsilon + h(1)) \), \( (ii) \ x \in [n + h(0), n - \epsilon + h(1)] \), \( (iii) \ x \geq n + h(0) \). In the first case, perfect commitment equilibrium is possible. In the second case perfect commitment, imperfect commitment and discretionary equilibria are possible. In the last case discretionary equilibria is possible. Proof: Appendix.

According to this proposition, if there are too much doubt concerned to the future policymaker incentives \( (\epsilon > h(1) - h(0)) \), then the equilibrium is unique: perfect commitment for very strong policymaker, discretionary for very weak policymaker and imperfect commitment otherwise. When \( \epsilon \) is sufficient large, there is no room for self-fulfilling inflation.

On the other hand, if the region for possible policymaker incentives \( (\epsilon \text{ decreases}) \), the uniqueness remains only for a very strong or a very weak policymaker. The intuition is that some economies may be subject to multiple equilibria when the strength of the commitment depends much on the credibility \( (\alpha) \) before the \( k \)'s assortment. In such case, to relax (to increase) the target for inflation may have two welfare effects. First, and the new one, it is possible that only perfect commitment equilibrium remains when the target is increased, since \( \left( \frac{dx}{\pi_\alpha} \right) \leq 0 \). Second, as we have \( h(\alpha(k^*)) > \epsilon^9 \), the critical \( k^* \) tends to be greater and the state region for good expectations shrinks

\[ 9k^* \text{ solves: } k = x - h \left( \frac{x}{\epsilon} \right). \]
when the target is increased. Then, the announcement credibility may be increasing in the target or not.

As we have shown in the Figure 6, when \( h(\alpha) \equiv \rho \alpha \) and multiple equilibria are possible, to increase the target may be a good deal if it avoids multiplicity. But this decision also depends on the probability of each equilibrium be selected. Because of the coordination failure anyone of them could be the one and, unfortunately, this model can not help to compute their likelihood.

Such difficulty is usually by-passed by the definition of an arbitrary sunspot variable\(^{10}\) that would permit us to compute expected welfare for each target. Obviously, the policy recommendations would be very different depending on the assumptions about the sunspot distribution. In the following section we opt to use another approach, considering the public as a strategic player. Up to here its only role was to process information, to form beliefs concerning the policymaker’s incentive, and to compute inflationary expectations. Adding strategies and payoff structure to private agents (public) and assuming an exogenous information structure, we can appraise the coordination aspect in a more realistic way. The coordination motive arise from strategic complementarity in public actions.

2.3 Self-Fulfilling Inflation with Imperfect Information

The economy (\( \xi \)) is defined again as one-shot game with the function \( h(\alpha) = \alpha \rho \) plus the following set of parameters \( \{ (A > 0), (\rho > 0), (\pi_a \in [0, A]), (\sigma > 0), (\sigma_p > 0) \} \). The last two of them define information structure as we describe next. The policymaker chooses the actual inflation after expectations being formed \( (\pi_e) \) by the public, keeping the inflation equal to the target \( (\pi_a) \) if and only if \( w_a \equiv \tilde{k} + \rho \alpha - x(\pi_a) \geq 0 \). Otherwise it inflates at level \( A \). \( \tilde{k} \) is drawn in the beginning of the game from the support of the improper uniform (over the entire real line). The population of private agents is continuous and normalized to unit. Each private agent \( (j) \) may set \( \alpha^j \) equal to one or zero. If he sets \( \alpha^j \) equal to zero he believes that target will not be matched. With some cost, he can speculate based on his beliefs (buying foreign currency, for example). If he sets \( \alpha^j \) equal to one

he believes that target will be matched. In this case, he does not bet against the policymaker (keep savings denominated in local currency, for example). Then, the aggregate credibility \((\alpha)\) is given by \(\text{prob}(\alpha^j = 1)\). Each \((j)\) payoff is defined as being equal to \((1 - \alpha^j)(g_s - c)\). The speculative gain \(g_s\) depends on the policymaker’s response. If the target is sustained, then \(g_s = g_a\), otherwise \(g_s = g_A\), where \((g_A > c > g_a)\). With this payoff structure, to speculate is a good deal only when the target is abandoned since \((g_A - c > 0)\) and \((g_a - c < 0)\). The incentive to attack is increasing in the size of the attack. As we have argued, in some economies, the target may be abandoned when credibility is low (i.e. sufficiently large group of speculators is present).

To keep our framework as close as possible to the one proposed in Angeletos and Werning [1] we define \(g_A \equiv 1\) and \(g_a \equiv 0\) and consider that the strength of the status-quo \((\tilde{k})\) is not common knowledge. Instead observing the realization of the \(k\)-value, each player \((j)\) observes public signal \((s^p)\) and private signal \((s^j)\),

\[
s^j = k + \sigma \varepsilon_j; \sigma > 0 \text{ and } \varepsilon_j \sim \mathcal{N}(0, 1)
\]

\[
s^p = k + \sigma_p \varepsilon_p; \sigma_p > 0 \text{ and } \varepsilon_p \sim \mathcal{N}(0, 1)
\]

\((\varepsilon_j)\) is assumed to be independent of \((k)\) and \((\varepsilon_{j'})\) for all \(j' \neq j\). \((\varepsilon_p)\) is also assumed to be independent of \((k)\) and \((\varepsilon_j)\). \(\mathcal{N}(0, 1)\) denotes the standard normal distribution.

### 2.3.1 The Equilibrium

We focus on monotone equilibria defined as perfect Bayesian. For each public signal, the agent \((j)\) attacks if and only if its private signal \((s^j)\) is less than some threshold \(s^* (s^p)\). The mass of agents that ends up attacking is given by:

\[
\text{prob}(s^j < s^* (s^p) | s^p, k) = \Phi\left(\frac{s^* (s^p) - k}{\sigma}\right) = 1 - \alpha
\]

where \(\Phi(.)\) denotes the cumulative distribution function for the standard normal. The policymaker will sustain the target if and only if \(k\) is greater than \(k^*\), which is given by:

\[
k^*(s^p) = x + \rho \cdot \Phi\left(\frac{s^* (s^p) - k^* (s^p)}{\sigma}\right) - \rho
\]
The expected payoff from attacking must be equal to zero whenever $s^j = s^* (s^p)^{11}$, which implies the following indifference condition:

$$\sqrt{\tau}.\Phi^{-1} (c) = k^* (s^p) - \frac{\tau s^* (s^p)}{\sigma^2} - \frac{\tau s^p}{\sigma^2_p}$$

where $\tau = \frac{\sigma^2_p \sigma^2}{\sigma^2_p + \sigma^2}$

Which after replacing $s^* (k^* (s^p))$ becomes:

$$\Phi^{-1} \left( \frac{k^* (s^p) + \rho - x}{\rho} \right) = \frac{\sigma}{\sigma^2_p} [k^* - s^p] + \frac{\sigma}{\sqrt{\tau}} \Phi^{-1} (1 - c)$$

It is always possible to find at least one $k^* \in [x - \rho, x]$ that solves this equation, and this solution will be unique for every public signal $(s^p)$ if and only if $\sigma \in (0, \frac{\sigma^2_p \sqrt{2\pi}}{\rho})$.

For any (positive) doubt related to public signal, $\sigma_p$, uniqueness is ensured by a sufficiently small (positive) doubt related to private signal, $\sigma$. That is the first proposition from Angeletos-Werning [1] and states that multiplicity may vanish when the common knowledge is perturbed, as in Morris and Shin [14]. This result always holds for some exogenous information because precise private information anchors individual behavior and makes difficult to predict the actions of others. Under the reasonable assumption that improvement in the private signal implies improvement in the public signal, it is possible that public information becomes more precise faster than the private one, and so multiplicity may still exist even for very small common knowledge perturbation ($\sigma \rightarrow 0$). In this case, the public signal drives individual behavior more than the private signal, motivating mass movements.

Keeping the exogenous information structure it is possible to set multiple-equilibria economies ($\xi^m$) assuming that $\sigma > \frac{\sigma^2_p \sqrt{2\pi}}{\rho}$ and unique-equilibrium economies ($\xi^u$) assuming that $\sigma \in (0, \frac{\sigma^2_p \sqrt{2\pi}}{\rho})$.

Proposition 3 For ($\xi^u$) type economy, to relax the target will increase the commitment credibility.

For ($\xi^m$) type economy, to relax the target may turn the commitment more enforceable, or not. It will depend on the equilibrium $k^* (s^p)$ that have been selected. The effect on the credibility will depend on the likelihood of each equilibrium. Proof: Appendix.

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11 $s^* (s^p) = \sigma.\Phi^{-1} \left( \frac{k^* (s^p) + \rho - x}{\rho} \right) + k^* (s^p)$
The key intuition is that when the target is increased the policymaker becomes stronger and the shock required for the commitment to be abandoned becomes greater (smaller $\tilde{k}$—realization), for a given ($\alpha$). This fact inhibits attacks and adds credibility. On the other hand, as the policymaker sets a greater target for inflation, stronger attack-strategies may be settled, decreasing the credibility. The first effect is always preponderant for $(\xi_n)$ type economy.

For $(\xi^m)$ type economy the first effect tends to be preponderant when extreme equilibria are selected (the $k^*(s^p)$ closest to $x$ (or to $x - \rho$)). In both cases, the enlarge in the attack size induced by more aggressive strategies tends to be canceled by the first effect. When strategies are extremes, too optimistic or too pessimistic, the size of an attack is closer to zero or to one, respectively. For more pondered strategies, based on the not-extreme equilibrium $k^*(s^p)$, the attack-mass and the no-attack-mass are both significant. Then, the enlarge in the attack size induced by more aggressive strategies is preponderant and critical $k$ becomes greater for greater targets (see Figure 7). If pondered equilibrium tends to be the selected one in $(\xi^m)$ type economy, to relax the target in order to get more credibility is a bad idea. The result would be more strength to speculative movement, and the commitment enforcement would be reduced.

2.3.2 Central Bank Transparency

According to the results, when the policymaker does not have strong ability to precommit, more precise public information may open the door for bad equilibrium, contrary to the conventional wisdom that more transparency is always good news. Some other papers have argued in the same direction. In Metz [5], more precise public information increases the likelihood of currency crises in case of bad fundamentals. Morris and Shin ( [15] and [16] ) have pointed out that welfare effect of increased public disclosures is ambiguous and that there is a dilemma between managing market prices and learning from market prices. They also conclude that when a Central Bank cannot actually control inflation, the inflation targeting regime could fail and undermine credibility. In this sense, it would be better for the Central Bank simply to forecast inflation and point out the extent to which its
forecasts are contingent on fiscal policy. Our results suggest that inflation targeting may be a good set-up based on the assumption that \textit{rule} is better than \textit{discretion}. Target framework may be used whenever the policymaker can actually control \textit{some} level of inflation in \textit{some} states of nature.

3 Concluding Remarks

Inflation is bad news but sometimes it takes place as a consequence of economic crisis, specially in emerging economies where local currency use to be weaker. Such crisis may be just confidence crises or may be based on more fundamental economics reasons. Using different theoretical approaches (models) we appraise how target for inflation should be considered in such economies.

First, ruling out confidence crises, we conclude that less ambitious target for inflation increases credibility in precommitment. This effect makes optimal target bigger than the one for economies that are equal, except for the fact that the credibility effect is not computed\textsuperscript{12}. We reach this result for a large range of economies (subject to different shocks distribution and driven by policymakers with different preferences).

When confidence crises are possible, multiple equilibria may come up. In this case, to set greater targets for inflation stimulate confidence crisis and may reduce the policymaker credibility. On the other hand, multiple (bad) equilibria may be avoided. The optimal policy will depend on the likelihood of each equilibrium be selected.

To avoid “sunspot solution” for multiple equilibria economy we restore uniqueness breaking common knowledge with exogenous information structure. In this case, it is possible to ensure again that less ambitious target for inflation increases credibility in precommitment.

Finally, adding \textit{precise} public signal, self-fulfilling actions and equilibrium multiplicity may still exist even for a small lack of common knowledge. In this case, to set greater targets for inflation may stimulate confidence crisis and reduce the policymaker credibility. Multiple (bad) equilibria may be avoided for some given public signal. The optimal policy will depend on the likelihood of each

\textsuperscript{12}As in Cukierman and Liviatan ([7]).
equilibrium be selected, again. In this way, more precise public information may open the door for bad equilibrium, contrary to the conventional wisdom that more Central Bank transparency is always good when considering inflation targeting framework.
References


[2] Araujo, Aloisio; Leon, Marcia; and Rafael Santos. “Speculative Attacks, Openness and Crises”.


4 Figures

![Figure 1: Emerging Economies and Inflation Targeting Adoption (Source: Central Banks and [11])]
Figure 2: Developed Economies and Inflation Targeting Adoption. (Source: Central Banks and [11])
Figure 3: Target and Reliance on Seignorage in Breaking Commitment Sample (Source: Central banks, IMF and [10])
Figure 4: Reliance on Seignorage and Transparency (Source: IMF and [9])

Figure 5: Credibility Effect on the Target Announcement
Figure 6: Self-fullfilling Equilibria ($A = 15\% ; \rho = 2.5\%$)

Figure 7: No-Common Knowledge Equilibrium $k^* \left( s^p = 0 \right)$ \left( A = 15\%, \frac{\sigma}{\sigma_p} = 15, c = .5, \rho = 7.5\% \right)
Proof. Proof of proposition 1: The policymaker from economy \((A, B)\) solves the following problem:

\[
\pi_a^* = \arg \max_{\pi_a \in [0, A]} E \left[ v(\pi_a, k) \right] ; k \sim U [0, B]
\]

\[
v(\pi_a, k) = \max_{\pi \geq 0} \left[ A(\pi - \pi_a(\pi)) - \frac{\pi^2}{2} - c(\pi_a, \pi, k) \right]
\]

\[
c(\pi_a, \pi, k) = \begin{cases} 0 & \text{if } \pi_a = \pi \\ k & \text{if } \pi_a \neq \pi \end{cases}
\]

and it is easy to check that,

\[
\pi_e = \alpha \pi_a + (1 - \alpha) A
\]

\[
\alpha(\pi_a) = 1 - \frac{1}{B} \left[ A(A - \pi_a) - \frac{A^2}{2} + \frac{\pi_a^2}{2} \right]
\]

\[
E[k|\pi_a \neq \pi] = \frac{A(A - \pi_a)}{2} + \frac{\pi_a^2}{4} - \frac{A^2}{4}
\]

It follows that:

\[
\pi_a^* = \arg \max_{\pi_a} \frac{1}{B} \left[ B + A\pi_a - \frac{\pi_a^2}{2} - \frac{A^2}{2} \right] \left( \frac{3A^2}{4} - \frac{A\pi_a}{2} - \frac{\pi_a^2}{4} \right) - \frac{3A^2}{4} - \frac{\pi_a^2}{4} + \frac{A\pi_a}{2}
\]

The equilibrium, \(\pi_a^*\), must solve:

\[
u(\pi_a^*) = v(\pi_a^*)
\]

\[
u(\pi_a) = \frac{B}{2} (A - \pi_a)
\]

\[
v(\pi_a) = \left( \frac{A + \pi_a}{2} \right) \left( B + A\pi_a - \frac{\pi_a^2}{2} - \frac{A^2}{2} \right) - \left( \frac{3A^2}{4} - \frac{A\pi_a}{2} - \frac{\pi_a^2}{4} \right) (A - \pi_a)
\]

Since \([\frac{AB}{2} - A^3 = v(0) < u(0) = \frac{AB}{2}]\), \([AB = v(A) > u(A) = 0]\) and \([uv(\cdot) < 0; uv(\cdot) > 0 \forall (\pi_a \leq A)]\), then the equilibrium \(\pi_a^* \in (0, A)\) exists and is unique for any \((A > 0, B > 0)\). Now, defining \(D(\pi_a^*) \equiv \pi_a^* - A(1 - \alpha(\pi_a^*))\), it is easy to check that \(\text{sign}(D(\pi_a^*)) = \text{sign}(2B\pi_a^* + 2A^2\pi_a^* - A^3 - A(\pi_a^*)^2)\). \(D(\cdot)\) is positive for \(\pi_a = A\) and negative for \(\pi_a = 0\). Next we will show that \(\text{sign}(D(\pi_a^*))\) is always positive, since \(\pi_a^*\) is lower bounded by some positive value and \(D(\cdot)\) is positive for \(\pi_a \in [0, A]\). From \(\text{sign}(u(\pi_a) - v(\pi_a)) = \text{sign}\left( (\pi_a)^3 + 2A^3 - \pi_a (3A^2 + 2B) \right)\) we conclude that \(\pi_a^*\) solves
\[(\pi^*_a)^3 + 2A^3 - \pi^*_a (3A^2 + 2B) = 0.\] Then, we can set \(\pi^*_a = \frac{2A^3 + (\pi^*_a)^3}{(3A^2 + 2B)}\). Since \(D(\pi^*_a)\) is positive whenever \(\pi^*_a \geq \frac{A^3}{2B + A^2}\), we must check if \(\left(\frac{2A^3 + (\pi^*_a)^3}{(3A^2 + 2B)} \geq \frac{A^3}{2B + A^2}\right)\) holds. It is easy to check that it holds for any \(B \geq \frac{A^2}{2}\). Now, if we set \(\pi_a = \frac{A^3}{B}\), we have \(sign \left( u(\pi_a) - v(\pi_a) \right) = sign \left( \frac{A^4}{B^2} - 3 \right)\). We conclude that \(\frac{A^3}{B}\) is a lower bound to \(\pi^*_a\) whenever \(B < \frac{A^2}{\sqrt{3}}\). In this case, \(\pi^*_a \geq \frac{A^3}{B} \geq \frac{A^3}{2B + A^2}\) and \(D(\pi^*_a) > 0\) again. Since \(\left(\frac{A^2}{\sqrt{3}} > \frac{A^2}{2}\right)\), we conclude that \(\pi^*_a \geq A (1 - \alpha (\pi^*_a))\) for any \((A > 0, B > 0)\).

Proof. Proof of proposition 2: The target is fulfilled whenever \(w_a = k + h(\alpha) - x \geq 0\), with \(x(\pi_a) \equiv \left[A(A - \pi_a) + \frac{\pi_a^2}{2} - \frac{A^2}{2}\right]\). The region for which the target \((\pi_a)\) may induce multiple equilibria expectations is given by the interval \([K^d, K^u]\), where:

\[
K^u(\pi_a, \alpha) = \inf \{k \in \mathbb{R} \mid (-x + k + h(\alpha)) \geq 0\} = x(\pi_a) - h(\alpha)
\]

\[
K^d(\pi_a, \alpha) = \sup \{k \in \mathbb{R} \mid (-x + k + h(\alpha)) \leq 0\} = x(\pi_a) - h(\alpha)
\]

\[
\bar{\alpha} = \min \left\{ \frac{n - K^d}{\epsilon}; 1 \right\} \quad \text{if } [K^d, K^u] \cap [n - \epsilon, n] \neq \phi
\]

\[
\alpha = \max \left\{ \frac{n - K^u}{\epsilon}; 0 \right\} \quad \text{if } [K^d, K^u] \cap [n - \epsilon, n] \neq \phi
\]

\[
\alpha = 0 \text{ if } K^d > n
\]

\[
\alpha = 1 \text{ if } K^u < n - \epsilon
\]

There are five possible cases for the “\([K^d, K^u]\)-position” related to the support \([n - \epsilon, n]\), as follows:

<table>
<thead>
<tr>
<th>Case</th>
<th>(\alpha \in [0, 1])</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(x \in [n + h(0), n - \epsilon + h(1)])</td>
</tr>
<tr>
<td>2</td>
<td>(x \in [n + h(0), n - \epsilon + h(1)])</td>
</tr>
<tr>
<td>3</td>
<td>(x \in [n + h(0), n - \epsilon + h(1)])</td>
</tr>
<tr>
<td>4</td>
<td>(n - \epsilon + h(1) &gt; x)</td>
</tr>
<tr>
<td>5</td>
<td>(n + h(0) &lt; x)</td>
</tr>
</tbody>
</table>

Considering \((h(1) - h(0) \geq \epsilon)\). Otherwise, cases 2 and 3 does not exist and for cases 1, 4 and 5 we set \(x \in [n - \epsilon + h(1), n + h(0)]\) instead of setting \(x \in [n + h(0), n - \epsilon + h(1)]\).

Proof. Proof of proposition 3:

Since \((\frac{dx}{d\pi_a}) = \pi_a - A\), to relax (to increase) target means to reduce \(x\). From \(\Psi(k^*, x) \equiv \Phi^{-1}\left(\frac{k^* + \rho - x}{\rho}\right) - \frac{\sigma}{\sigma_p} [k^*] = -\frac{\sigma}{\sigma_p} s^p + \Phi^{-1}(1 - c) \frac{\sigma}{\sqrt{\pi}}\) we conclude that \(\Psi(., x)\) is increasing in \(k^*\).
for every $s^p$ if $\frac{\sigma_p}{\sigma_p^2} \leq \sqrt{2\pi}$. Reduction in $x$ must be compensated by reduction in $k^*$ in order to keep $\left[ -\frac{\sigma_p}{\sigma_p^2} s^p + \Phi^{-1} (1 - c) \frac{\sigma_p}{\sqrt{\pi}} = \Psi(k^*, x) \right]$ valid. The region over the $\tilde{k}$—support where the target is fulfilled increases for all $(s^p)$ and the size of attack decreases with the decreasing in $s^*(s^p)$.

$\Psi(., x)$ will be decreasing in $k^*$ for some possible equilibrium $\bar{k}^*(s^p)$ whenever $\frac{\sigma_p}{\sigma_p^2} > \sqrt{2\pi}$.

In this case, reduction in $x$ must be compensated by an increasing in $\bar{k}^*$ in order to keep $\left[ -\frac{\sigma_p}{\sigma_p^2} s^p + \Phi^{-1} (1 - c) \frac{\sigma_p}{\sqrt{\pi}} = \Psi(\bar{k}^*, x) \right]$ valid. So, an increase in the target may imply an increasing in $\bar{k}^*$, $s^*(\bar{k}^*)$, and an increase in the size of attack. □