Optimal provision of implicit and explicit incentives in asset management contracts*

Samy Osamu Abud Yoshima
EPGE-FGV/RJ - Advisor: Professor Luis Henrique B. Braido

July 13, 2005

Abstract

This paper investigates the importance of the flow of funds as an implicit incentive provided by investors to portfolio managers. We build a two-period binomial moral hazard model to explain the main trade-offs in the relationship between flow, fees and performance. The main assumption is that effort depends on the combination of implicit and explicit incentives while the probability distribution function of returns depends on effort. In the case of full commitment, the investor’s relevant trade-off is to give up expected return in the second period vis-à-vis to induce effort in the first period. The more concerned the investor is with today’s payoff, the more willing he will be to give up expected return in the following periods. Without commitment, we consider that the investor also learns some symmetric and imperfect information about the ability of the manager to generate positive excess return. In this case, observed returns reveal ability as well as effort choices. We show that powerful implicit incentives can explain the flow-performance relationship and we provide a numerical solution in Matlab that characterize these results.

*This paper is part of my dissertation thesis to obtain the title of Master in Economics at EPGE-FGV/RJ. I would like to thank Angelo Polydoro and Enrico Vasconcelos for computational and Matlab assistance; Gustavo C. Branco and Guilherme Bragança from Mellon Brascan for the funds’ data. Special thanks to Amaury Fonseca Jr., Carlos Eugênio da Costa, Delano Franco, Fabiana D’Atri, Fernando H. Barbosa, Genaro Lins, Jaime Jesus Filho, Jair Koiller, Luis H. B. Braido, Marcelo Fernandez, and M. Cristina T. Terra for guidance and fruitful discussions during the development of this project. Many other people have contributed to the development of this project and I am thankful and indebted with them all.
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1 Introduction

In the asset management industry, the flow of funds of most portfolio managers respond to past observed performance. This paper address this behavior as a response to incentives which, in its turn, may be compatible with the idea that returns may not be persistent. In fact, in our model, expected returns depend on the total incentives provided by the investor.

We build a dynamic binomial moral hazard model to describe the interaction between the two relevant forms of incentives present in asset management contracts. Explicit and implicit incentives are used in this relationship as to induce the agent to exert higher levels of effort. Explicit incentives are those written in a contract and enforceable by a court of law. They usually depend on the actual excess return of the performance evaluation period, affecting players’ utilities in this same period. On the other hand, implicit incentives are not written in any enforceable contract. They depend on the history of excess returns once this set of information reveals ability and/or effort exerted by the portfolio manager. This incentive only affects utility in the following periods.

Explicit incentive typical clauses found in these contracts are linear in excess return with the intercept ($\alpha$) and slope ($\beta$) of the contract fixed during the whole life of the contract. Most contracts for the delegation of investment decisions are: 1) low-powered contracts with $\alpha > 0$, $\beta > 0$ and limited liability over a high-water mark$^2$; 2) fixed management fee contracts with $\alpha > 0$ and $\beta = 0$ which resembles a salary contract. The limited liability and the high-water feature turns the linear contract into a convex one, resembling the payoff structure of a call option on the asset value of the fund.

Besides the explicit incentives, investor provide implicit incentives that are represented by the flow of funds. Loosely speaking, we should expect the flow of fund to vary positively when the observed past excess return is positive while past negative excess return should be followed by withdrawals from the fund. Meanwhile the flow of funds is part of an intertemporal allocation of the investor’s wealth, we show that it also plays the important role of a powerful implicit component in the optimal provision of incentives under a moral hazard framework.

$^1$The term implicit refers to informal incentives like reputation building, career concerns (in our case, flow concerns), other informal rewards and quid pro quos. It is not written in the contract but it complements the design and specification of long-term contracting.

$^2$See subsection 4 of the Appendix
Given the algebraic difficulties of the problem, we are only able to offer a numerical solution. We consider two problems in this paper. First, we assume that only one type of manager exist. Some dynamic inconsistency concerns arise and we solve another problem with two types of managers. In the last section of the Appendix, we describe the typical contract found in the marketplace and explore the possible incentive problems that arise in this contract.

The paper is organized as follows. In this section, we describe some literature and show some data on the flow of funds. Section 2 describe the basic model and its numerical solution. Section 3 shows the model with heterogeneous managers with a Bayesian adjustment of the posterior probability distribution of returns. A numerical solution is also offered. Section 4 concludes.

1.1 The literature

Stiglitz (1974) reasoned the existence of low powered explicit contracts as a consequence of the trade-off between efficiency and insurance. Ghatak and Pandey (2002) explained low power of linear explicit with limited liability contracts under a multi-task moral hazard model in the effort to produce agricultural goods and the risk implied in various possible techniques of production. They used data of sharecropping contracts in rural areas from Northern India. The principal recover the first-best solution by writing a contract to induce the agent to exert maximum effort and minimum risk. Then, the power of the contract is lower than suggested by earlier results of the literature. The intuitive reason for lowering the power of the contract is to reduce the marginal benefit of the outliers of the probability distribution of returns. Hence, the tenant has the appropriate incentives to reduce risk increasing choices of production.

Implicit incentives usually appear in literature as career concerns and periodical bonus payments. The general problem of combined implicit and explicit incentive provision is present in Pearce and Stacchetti (1998) who show that implicit incentives contracts’ efficiency is increased if short-term explicit contracts are written in the context of a repeated principal-agent model. Besides, risk averse agents prefer implicit incentives that vary negatively with explicit incentives.

Holmström (1982) was the first to introduce incomplete and symmetric information to model career concerns. Gibbons and Murphy (1992) study the importance of career concerns
and show that the optimal contract optimizes total incentives. They show that the greater is the importance of implicit incentives, the least powerful is the explicit component of the contract in an executive compensation contract.

Levin (2003) studies relation incentive contracts and show the conditions under which stationary explicit contracts are optimal and how incentives interact in the trade-off between efficiency, screening and dynamic enforcement in the case of hidden information. In the moral hazard case, enforcement compress the information obtained from the noisy signal and leads to only two levels of performance while poor performance is followed by a termination of the relationship even if the performance measure is subjective.

This general question applied to the asset management case is found in Heinkel and Stoughton (1994). They assume the existence of a linear contract and derives the optimal contract structure and retention policy. Using a different and simplified approach, we also find that the explicit incentive is less powerful in a two-period economic setting. Any contract only elicits partial information about the portfolio manager and, hence, \textit{ex-post} performance measurement becomes crucial in defining the optimal retention policy.

Several papers study the importance of flow as a response to incentives in the asset management industry. Chevalier and Ellison (1997) analyze the importance of the flow of funds by studying risk-taking choices of different periods in comparison with the preceding performance. They show that the variance of returns is greater after low performance as a risky alternative to recuperate lost ground in the competition for flow of funds. Berk and Green (2004) build a parsimonious model to explain their findings. They use a Bayesian approach without any information asymmetry to model the flow of funds dependence on observed past returns.

Finally, Basak \textit{et al} (2003) show that the flow of funds play a very significant role in altering risk exposure given its importance as an implicit incentive. In a dynamic asset allocation framework, they show that the risk exposure and consequent volatility of returns depends on the current excess return of the fund. They demonstrate that the effects of departing from investor’s desired exposure is greater as year-end approaches.

1.2 Data and stylized facts

We analyze the performance-flow relationships for over 100 open and closed ended mutual funds in Brazil. We run regressions of the percentage accumulated flow of funds daily
variation over different periods of time on different sizes of lags of cumulative excess returns over the CDI and their respective lagged variance of returns, controlling for the fund’s age and size. We control for other omitted variables that could explain flow of funds by adjusting the standard deviation. The econometric model is

\[
\text{Flow}_{t,s}^i = \beta_1 r_{t,s}^i + \beta_2 \sigma^2 (r_{t,s}^i) + \beta_3 \text{Age}_t + \beta_4 \text{NAV}_t + \varepsilon_t
\]

Empirical regularities found in this work shows that the percentage accumulated flow respond positively to excess return for all periods and lags. This effect is stronger for long-dated lags of past cumulative excess return. Since it seems reasonable to consider variance and return to be positively correlated, the sign of the coefficient of variance of returns is less conclusive. When the sign is negative, it is expected that the flow of funds would be better explained by their Sharpe ratios. As in the rest of the literature, old funds’s flow has a smaller sensitivity to lagged excess return as well as large funds.

<table>
<thead>
<tr>
<th>Accum. Flow</th>
<th>Excess return (252)</th>
<th>Var. of returns (252)</th>
<th>Age</th>
<th>NAV</th>
<th>Adj. R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>63</td>
<td>0.3460 (0.0175) *</td>
<td>896.97 (206, 35) *</td>
<td>-0.0001 (0) *</td>
<td>0 (0) *</td>
<td>3.00%</td>
</tr>
<tr>
<td>126</td>
<td>0.8541 (0, 0548) *</td>
<td>3870.96 (1030, 51) *</td>
<td>-0.0002 (0) *</td>
<td>0 (0) *</td>
<td>5.86%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Accum. Flow</th>
<th>Excess return (378)</th>
<th>Var. of returns (378)</th>
<th>Age</th>
<th>NAV</th>
<th>Adj. R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>252</td>
<td>2.11 (0.07) *</td>
<td>1000.45 (652, 98) +</td>
<td>-0.0005 (0, 0001) *</td>
<td>0 (0) *</td>
<td>33.35%</td>
</tr>
<tr>
<td>315</td>
<td>3.35 (0, 10) *</td>
<td>-2500.68 (141, 18) *</td>
<td>-0.0010 (0, 0001) *</td>
<td>0 (0) *</td>
<td>60.47%</td>
</tr>
<tr>
<td>378</td>
<td>18.08 (3, 71) *</td>
<td>-12780.7 (1840, 41) *</td>
<td>-</td>
<td>0 (0) *</td>
<td>5.69%</td>
</tr>
</tbody>
</table>

* indicates that the null hypothesis is rejected at 1% significance
** indicates that the null hypothesis is rejected at 5% significance
*** indicates that the null hypothesis is rejected at 10% significance
+ indicates that the null hypothesis is rejected at 15% significance
2 The model under full commitment

Consider a risk neutral investor\(^3\) who hires a risk averse portfolio manager to invest a share of his assets\(^4\) in an economy that lasts for two periods. We build a binomial model, i.e., there are two possible states of nature in each period. At the beginning of each period for each node of the decision tree, the investor decides the percentage of that share to be allocated with the portfolio manager, \(\Omega_0, \Omega_H\) and \(\Omega_L\). Besides, the contract that regulates this relationship describes the portion of return, \(\omega_H\) and \(\omega_L\), that is paid to the manager in each node of the decision tree. We still make two assumptions regarding the compensation schedule: 1) the explicit incentive is stationary; i.e., they do not vary during the life of the contract and 2) the explicit incentives are multiplied by the implicit incentives. These assumptions bring a lot of realism to the model since this schedule is the one frequently observed in the industry.

The portfolio manager has a time-separable utility function with impatient parameter, \(\delta\), and he is free to decide how to allocate the assets under his management in any possible investment alternative available in the economy. In order to make these decisions, the manager exerts costly and unobservable effort. Portfolio manager’s effort decisions represent his set of feasible investment strategies and appear as more intense access to information, increased leverage, greater duration of fixed income instruments, open gap and credit risks, active day-trading, foreign exchange risks, etc. Thus, effort decisions affect the probability distribution of excess return \(\text{ex ante}\), \(\pi_{t,s}\) and they are considered to be non-negative and assume continuous values on the unit interval such that \(e_{t,s} \in [0, 1]\).

We still assume that the cost function of effort is monotonically increasing and twice continuously differentiable in effort; such that we have \(\Psi(0) = 0, \lim_{e_s \to \infty} \Psi'(e_{t,s}) = \infty, \Psi'(\cdot) > 0, \Psi''(\cdot) > 0\) and \(\Psi'''(\cdot) \geq 0\) which guarantees sufficiency conditions for interior solution and easy calculation of several static comparisons. In order to simplify the algebraic calculations, we define a quadratic time-separable cost function

\[
\Psi(e_{t,s}) = \frac{k}{2} (e_{t,s})^2
\]  

(1)

The asymmetric information aspect of the model relies in the fact that \(e_{t,s}\) is unobservable by the investor. In each period, the two states of nature are associated with two levels of

\(^3\)The risk neutrality assumption is due to the standard justification that investors can diversify managers’ specific risks away while each manager may not.

\(^4\)We do not make any consideration about these assets or their associated markets.
excess return. The return of the investments made by the portfolio manager are compared to a pre-defined benchmark return, $r_b$. The investor, then, is not capable to know with certainty if excess return is due to effort or good fortune (luck). Indeed, the excess return, $r_s$, is a noisy signal of $e_{t,s}$ and the portfolio manager is rewarded only on the basis of this noisy signal.

In the binomial model, high effort is associated with a higher excess return, $r_H$, and a particular compensation for the manager, $\omega_H$. On the other hand, low effort is associated with a lower level of excess return, $r_L$, and a different compensation for the manager, $\omega_L$.

The function that describes the probability of obtaining a particular value of excess return is linear in effort and it is given by

$$\pi_{t,s} = a + be_{t,s} \quad , \quad \forall \begin{cases} a + b < 1 \\ a, b > 0 \\ 0 \leq e_{t,s} \leq 1 \end{cases}$$

The effort and parametric restrictions are necessary to avoid negative, equal or greater than one values of probabilities of return. The coefficients of the function that transforms effort into the probability of occurrence of a particular state of nature are exogenously given in our model and they determine the level of informativeness of the noisy signal, $r_s$. The intercept, $a$, can be seen as a parameter that only depends on specifics characteristics of each portfolio manager while the slope, $b$, represents the shift in the distribution of return derived from variations in effort. The greater is the value of $b$, the more dependent of effort is probability distribution of return.

If $a \approx 1$ and $b \approx 0$, then return is a sufficient statistic for highly skilled managers or, we may say, for specific features of the instruments and markets traded by the manager. Hence, return will only allow the investor to infer about manager’s ability or about the implied risks of the portfolio; for example, given a particular investment regulation. In this case, moral hazard would not be an issue. In the case $a \approx \frac{1}{2}$ and $b \approx \frac{1}{2}$, the variance of return will be high and its level of informativeness will be very low. Therefore, it should neither be regarded as a good measure to evaluate idiosyncratic manager’s profiles nor a good proxy of high levels of effort. On the other hand, when $a \approx 0$ and $b \approx 1$, then return is a sufficient statistic for high levels of effort decisions executed by the manager and, thus, it should be used as a proxy of the manager compensation structure in our model. Moral hazard plays an important role in the maximization problem of the investor and inducing optimal effort

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increase the value of the relationship.

In the model, expected return as well as variance depend explicitly on effort and are given by

\[
E[r_{t,s}] = (a + be_{t,s})r_H - (1 - a - be_{t,s})r_L
\]  

(3)

and

\[
Var[r_{t,s}] = (a + be_{t,s}) \cdot (1 - a - be_{t,s}) \cdot (r_H - r_L)^2
\]  

(4)

thus, expected return and variance are endogenous to effort decisions in our model.

The binomial distribution has an interesting relationship between the expected excess return and its variance. Low effort leads to low expected return and to low variance of returns as well. As \(e_{t,s}\) increases and approaches half, both variance and expected return go up while variance attains its maximum at \(e_{t,s} = 0.5\). So, medium effort is related to a greater average return but maximum variance. As \(e_{t,s}\) goes to one, expected return reaches its maximum and variance is at its minimum again, that is, 0. In this model, the distribution of excess returns conditional on high effort, \(\pi_{t,s}\), stochastically dominates in first-order the distribution of excess returns conditional on low effort, \((1 - \pi_{t,s})\). However, in a second-order stochastic dominance sense, the distribution of excess return conditional to low effort dominates the one conditional to high effort for \(0 \leq \pi_{t,s} < \frac{1}{2}\). On the other hand, for \(\frac{1}{2} \leq \pi_{t,s} \leq 1\), the distribution of excess return conditional to high effort dominates stochastically in a second-order sense the distribution of excess return conditional to low effort.

In economic terms, effort choice represents the reduced form of two tasks: effort choices increase expected return and risk choices shift variance of returns. In the interval \(\pi_{t,s} \in [0; \frac{1}{2}]\), they are substitutes tasks. Only for higher than half effort choices becomes complementary tasks. Remember that it is less costly to induce complementary tasks than two substitute tasks in a second best environment since there are economies of scope when these tasks entails moral hazard. Then, these economies of scope only appear for levels of effort greater than half.

### 2.1 The timing of the model and the decision tree

The timing of the two period model is explained as follows. At the beginning of the first period, the investor simultaneously offers a contract \(\{\omega_H, \omega_L, \Omega_0\}\) to the portfolio manager.
that pays fees $\omega$ for an initial investment $\Omega_0$. The manager makes an effort decision to define and execute a particular asset allocation strategy. Then, nature moves and a particular value of excess return, $r_{1,s}$, is realized.

At the end of the first period, the investor and the portfolio manager observe $r_0$ and, then, the investor changes $\Omega_0$ to $\Omega_H$ or $\Omega_L$, according to $r_{1,s}$. In the beginning of the second period, the manager chooses an state-dependent effort and nature will move again such that a particular value of excess return, $r_{2,s}$, is realized.

Time table goes here
Decision tree graph goes here

2.2 The portfolio manager problem: optimal choice of effort

The portfolio manager maximizes expected utility by choosing the optimal levels of effort

$$
\max_{e_0,e_1,e_2} U_M = \sum_{t=0}^{2} \delta^t \left[ \sum_{s_t=\ell} \sum_{e_t} P(r_t|e_t) u(\Omega_{t,s} \omega_{t,s}) - \frac{k}{2} (e_{t,s}) \right]
$$

where the utility function of the portfolio manager presents the usual properties of concavity:

$$
u_{\Omega} (\Omega = 0, \omega = 0) = u_{\omega} (\Omega = 0, \omega = 0) = \infty \quad \lim_{\Omega,\omega \to \infty} u_{\Omega} (\cdot, \cdot) = u_{\omega} (\cdot, \cdot) = 0, \quad u_{\Omega} (\Omega, \omega) > 0, \quad u_{\omega} (\Omega, \omega) > 0, \quad u_{\Omega\omega} (\Omega, \omega) < 0, \quad u_{\omega\omega} (\Omega, \omega) < 0. \quad \text{Risk aversion creates inefficiencies in the provision of effort due to the effects of moral hazard and, in this case, the risk neutral investor should pay a premium for a risk averse manager to participate.}
$$

The reservation utility of the portfolio manager is exogenously given and is equal to $U_M$.

The investor has all bargaining power and can make take-it-or-leave-it offers to the portfolio manager subject to providing him with an expected payoff which yields at least $U_M$. 

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Normalizing $\Omega_0 = 1$, we obtain

\[
e^*_0 = \frac{b[u(\omega_H) - u(\omega_L)]}{k} + \delta b \left\{ \begin{array}{c}
ka [u(\Omega_H \omega_H) - u(\Omega_L \omega_H)] \\
+k(1-a) [u(\Omega_H \omega_L) - u(\Omega_L \omega_L)] \\
+b^2 \left( \frac{2k-1}{2k} \right) \left[ (u(\Omega_H \omega_H) - u(\Omega_H \omega_L))^2 - (u(\Omega_L \omega_H) - u(\Omega_L \omega_L))^2 \right] \end{array} \right\} = A(\omega, \Omega)
\]

(6)

\[
e^*_1 = \frac{b[u(\Omega_H \omega_H) - u(\Omega_H \omega_L)]}{k} = E(\omega, \Omega)
\]

(7)

\[
e^*_2 = \frac{b[u(\Omega_L \omega_H) - u(\Omega_L \omega_L)]}{k} = I(\omega, \Omega)
\]

(8)

Observe that the optimal effort choice in the first period has a dynamic component represented by the present value of the difference in utility that the manager derive in each of the two possible states of nature in the second period. There are three terms: the first two terms show the difference in utility given distorted incentives weighted by the exogenous factor representing the manager’s specific characteristics, $a$ and $k$. The greater is $a$, the greater is $\Omega_H - \Omega_L$ and the greater is $\omega_H$, the greater is $e^*_o$. The third term contains two quadratic terms representing the difference in utility given distorted incentives weighted by the level of informativeness of return, $b$, as well as the cost function parameter, $k$. That is, optimal choice in the hidden action problem contains all elements of the compensation schedule, revealing the power of the implicit incentive in the dynamic moral hazard problem.

For a given $a$ and $b$, when $\omega_H > \omega_L$ and $\Omega_H > \Omega_L$, the optimal choice of unobservable effort in the first period is higher in the dynamic problem than in the static version. With enough dynamic incentive $\Omega_H > \Omega_0$ and enough dynamic penalization $\Omega_L < \Omega_0$, it is possible to reduce the cost of implementing second-best solutions with a smaller distortion between $\omega_H$ and $\omega_L$, i.e., the power of the explicit contract will be lower than in the static version. The importance of the implicit dynamic incentive is raised when the limited liability constraint binds.

In the second and last period of the relationship, the dynamic component vanishes and only the distortion in the explicit incentives matter for the manager, a solution that is similar to the static version of the hidden action problem. In fact, memory plays an important role by differentiating the compensation in each node of the second period. Memory appears while the implicit incentive depends on the return observed in the first period. Then, the
optimal effort solution will obey
\[ e_0^*(\omega, \Omega) \geq e_1^*(\omega, \Omega) \geq e_2^*(\omega, \Omega) \]  

(9)

2.3 The investor problem: optimal provision of incentives

The risk-neutral investor maximizes expected profit by choosing the optimal levels of incentives

\[
\max_{\omega_H, \omega_L, \Omega_H, \Omega_L} V_I = \sum_{t=0}^{2} \left[ \sum_{s_t=r_t} P(r_t | e_t) \right. \\
\left. \Omega_{t,s} (r_{t,s} - \omega_{t,s}) - (\Omega_{t,s}) \overline{r}_b \right] 
\]

(10)

where \( \overline{r}_b \) is the return of the outside investment alternative of the investor - the benchmark return can be obtained without any effort and incentive provision. When \((\Omega_H - 1) > 0\), the investor is borrowing at this benchmark rate and investing the resources in the fund. While \((\Omega_H - 1) < 0\), the investor is withdrawing resources from the fund and re-investing them in benchmark return-linked instruments. The investor observes excess return at the end of every period and decides to change the implicit incentive based on the history of excess returns. Excess return represents a noisy signal of effort with mean and variance respectively given by (3) and (4).

In equilibrium, the investor anticipates the optimal choice of actions taken by the portfolio manager and design an incentive compatible contract. When \( \overline{r}_b = 0 \), the problem of the investor becomes

\[
\max_{\omega_H, \omega_L, \Omega_H, \Omega_L} V_I = (a + be_0^*(w, \Omega)) (r_H - \omega_H) + (1 - a - be_0^*(w, \Omega)) (r_L - \omega_L) \\
+ \delta (a + be_0^*(w, \Omega)) [(a + be_1^*(w, \Omega)) \Omega_H (r_H - \omega_H) + (1 - a - be_1^*(w, \Omega)) \Omega_H (r_L - \omega_L)] \\
+ \delta (1 - a - be_0^*(w, \Omega)) [(a + be_2^*(w, \Omega)) \Omega_L (r_H - \omega_H) + (1 - a - be_2^*(w, \Omega)) \Omega_L (r_L - \omega_L)] 
\]

subject to the following participation constraints. We normalize the reservation utility to
zero in each node and write

\[
\left\{ \begin{array}{l}
\{(a + be_0^* (\omega, \Omega)) u (\omega_H) + (1 - a - be_0^* (\omega, \Omega)) u (\omega_L) - \frac{k}{2} (e_0^* (\omega, \Omega))^2 \} \\
+ \delta (a + be_0^* (\omega, \Omega)) \{ (a + be_1^* (\omega, \Omega)) u (\Omega_H \omega_H) - \frac{k}{2} (e_1^* (\omega, \Omega))^2 \} \\
+ \delta (1 - a - be_0^* (\omega, \Omega)) \{ (a + be_2^* (\omega, \Omega)) u (\Omega_L \omega_H) - \frac{k}{2} (e_2^* (\omega, \Omega))^2 \} \\
\end{array} \right\} \geq 0
\] (11)

It is necessary to write two limited responsibility constraints for the explicit incentives since the manager has limited liability in excess return and, thus, can only be penalized for exerting low levels of effort through the implicit incentive.

\[
\omega_H \geq 0
\] (12)

\[
\omega_L \geq 0
\] (13)

Since it is neither possible to borrow resources from the manager’s fund nor to leverage positions in the fund by borrowing at the benchmark rate, there are also two short-selling and two borrowing constraints for the implicit incentives such that

\[
0 \leq \Omega_H \leq 1
\] (14)

\[
0 \leq \Omega_L \leq 1
\] (15)

besides these non-negativity constraints, effort choices executed by the manager in each period must be in the unit interval

\[
0 \leq e_0^* (\omega, \Omega) \leq 1
\] (16)

\[
0 \leq e_1^* (\omega, \Omega) \leq 1
\] (17)

\[
0 \leq e_2^* (\omega, \Omega) \leq 1
\] (18)

All first-order conditions are shown in subsection 1 of the Appendix. The equilibrium solution \{\omega_H^*, \omega_L^*, \Omega_H^*, \Omega_L^*\} is algebraically intractable and can only have a numerical solution. The MatLab code and its results are shown, respectively, in subsection 2 and 3 of the Appendix.
2.4 Characterization of the optimal incentive contract

In equilibrium, the investor offers an incentive compatible contract \( \{ \omega^*_H, \omega^*_L, \Omega^*_H, \Omega^*_L \} \) that satisfies all the constraints of his problem. The investor provides total incentives that equalize the marginal excess expected return and the implied costs of effort induction. He does so by simultaneously combining and distorting both the implicit and the explicit incentive’s compensation structure as to maximize the intertemporal excess expected return.

The explicit incentive reduces net excess expected return. When \( r_L < 0 \), (13) binds and the investor offers \( \omega^*_L = 0 \), because of limited liability. In equilibrium, the investor sets \( \omega^*_H \geq 0 \) as to increase the probability of high return in each node of the decision tree. This result is natural since setting \( \omega^*_H > \omega^*_L = 0 \), induces positive effort, increases the probability of high return in all nodes of the tree and, thus, increases excess expected return. For a given solution \( \{ \Omega^*_H, \Omega^*_L \} \), the optimal level of performance fee, \( \omega^*_H \), equalizes marginal excess expected return due to shifts in the probability distribution of return to the marginal cost of exerting effort in all nodes of the tree. Then, the optimal contract is a combination of \( \{ \omega^*_H, 0, \Omega^*_H, \Omega^*_L \} \).

Since we are imposing the first-order approach (\textit{FOA}) - by substituting the portfolio managers’ first-order conditions into the investor’s objective function - it needs to be checked if the second-order conditions (\textit{SOC}) satisfy the necessary and sufficient conditions for a local maxima. That is, we verify if at the solution found numerically, \( \{ \omega^*_H, \omega^*_L, \Omega^*_H, \Omega^*_L \} \), the Hessian matrix of the portfolio manager’s maximization problem is negative semi-definite.

2.4.1 Comparison with the case where \( \Omega^*_H = \Omega^*_L = 1 \)

The flow of funds serves two purposes. First, it determines the investor’s asset allocation strategy. From a finance and portfolio allocation perspective, we know that the risk neutral investor should choose \( \Omega^* = 1 \) if excess expected return is positive. On the other side, when net excess expected return is negative, the investor sets \( \Omega^* = 0 \).

Due to hidden action considerations, the flow of funds also plays the role of an implicit incentive as to avoid moral hazard in the execution of effort. In the dynamic model, the investor desires to induce greater effort in the first period while its benefits are greater than the ones generated by effort executed in the second period. That is, investor faces a intertemporal trade-off between inducing effort in the first period - \textit{vis-à-vis} inducing effort in second period - increasing expected expected return in the first period - \textit{vis-à-vis} inducing effort in second period - increasing expected return.
return in the second period. Then, the investor distort the implicit incentive equilibrium allocations that may differ from the natural and trivial solution described above. Then, the flow of funds modify the allocation classical rule such that

\[ E[r_{t,s}] - (\bar{r}_b + \omega_H^*) > 0 \Rightarrow 0 < \Omega_{t,s}^* \leq 1 \]

and

\[ E[r_{t,s}] - (\bar{r}_b + \omega_H^*) \leq 0 \Rightarrow 0 \leq \Omega_{t,s}^* < 1 \]

Moreover, since expected return is endogenous in this model and given \((9)\), we have

\[ E[r_{1,0}^*] > \delta E[r_{1,1}^*] \geq \delta E[r_{2,1}^*] \]

Indeed, there is economic value in providing distorted implicit incentives at the cost of destroying the relationship in the second period whenever one observes negative excess return in the first period. To maximize expected utility, the investor decides how much endogenous expected return to give up in the second period in order to obtain endogenous expected return derived from higher induced effort in the first period.

Let’s consider three cases. In the first one, there is no distortion in the implicit incentive such that \(\Omega_H^* = \Omega_L^* = 1\). Then, effort choices will be given by

\[ e_0^* = e_1^* = e_2^* = \frac{bH(\omega_H)}{k} \leq 1 \quad (19) \]

In this case, \(\pi_0^* = \pi_1^* = \pi_2^* = a + be^* = \left(a + \frac{b^2u(\omega_H)}{k}\right) < 1\) and the manager earns

\[ U_M = (1 + \delta)u(\omega_H) \left[a + \frac{b^2}{2k}u(\omega_H)\right] \geq 0 \]

In this case, only the explicit incentive, \(\omega_H\), affects effort choices. From \((12)\), the risk averse manager participation constraints is always greater than zero for all \(\omega_H > 0\) and, hence, the constraint is not binding \((\lambda = 0)\). On his side, the risk neutral investor earns net excess expected return

\[ V_I = (1 + \delta) \left(a + \frac{b^2u(\omega_H)}{k}\right)(r_H - r_L - \omega_H) + (1 + \delta)r_L > 0 \quad (20) \]

From \((20)\), the investor problem reduces to choosing \(\omega_H^*\). Then, the Lagrangian becomes

\[ L_1 = (1 + \delta) \left(a + \frac{b^2u(\omega_H)}{k}\right)(r_H - r_L - \omega_H) + (1 + \delta)r_L \]
Optimal choice of $\omega_H^*$ will satisfy first-order conditions such that

$$
\omega_H^* + \frac{1}{u'(\omega_H^*)} \left( u(\omega_H^*) + \frac{ak}{b^2} \right) = (r_H - r_L)
$$

When we use the CRRA utility function, $u(\omega) = \frac{(\omega\rho)^{\rho-1}}{\rho-1}$, we have

$$
b^2 \rho \omega_H^* + ak (\rho - 1) (\omega_H^*)^{2-\rho} = b^2 (\rho - 1) (r_H - r_L)
$$

(22)

Now, consider a second extreme case. Suppose that the investor offers full implicit incentive distortion. Then, we have $\Omega_H^* = 1$ and $\Omega_L^* = 0$. In this case, effort choices will be equal to

$$
e_0^* = \frac{b u(\omega_H)}{k} \left( 1 + \delta \left( ak^2 + \frac{b^2}{2} (2k - 1) u(\omega_H) \right) \right)
$$

(23)

$$
e_1^* = \frac{b u(\omega_H)}{k}
$$

(24)

Then, we have $e_0^* > e_1^* > e_2^* = 0$ for $k \geq \frac{1}{2}$.

In this case, the participation constraint is given by

$$
U_M = \left[ u(\omega_H) \left( 1 + \delta \left( a + \frac{b^2 u(\omega_H)}{2} \right) \right) \left( a + \frac{b^2 u(\omega_H)}{k} \left( 1 + \delta \left( ak^2 + \frac{b^2 u(\omega_H)(2k-1)}{2} \right) \right) \right) \right. \\
- \frac{u(\omega_H)^2 b^2}{2k} \left( \delta + \left( 1 + \delta \left( ak^2 + \frac{b^2 u(\omega_H)(2k-1)}{2} \right) \right)^2 \right)
\geq 0
$$

(25)

From (25), we can approximately the minimum value of $\omega_H^*$ that binds the participation constraint. If $3b^6 \delta^2 - 8b^6 k \delta^2 + 4b^6 k^2 \delta^2 = 0$ and $-b^4 \delta + ab^4 \delta^2 - 2ab^4 k \delta^2 - 2ab^4 k^2 \delta^2 + 2ab^4 k^3 \delta^2 \neq 0$, 


then we can write one possible solution

\[
\omega_H^* > u^{-1} \left( \frac{1}{+ab^4\delta^2 (a\delta - 1)} \right) > 0 \quad (26)
\]

Due to the fact that it reduces the net expected return in all nodes of the tree, we have \( \omega_H^* < r_H \). The investor earns an expected return equal to

\[
V_I = \left( a + \frac{b^2u(\omega_H)}{k} \left( 1 + \delta \left( \frac{ak^2}{(2k-1)u(\omega_H)b^2} \right) \right) \right) \left( 1 + \delta \left( \frac{a}{+b^2u(\omega_H)} \right) \right) (r_H - r_L - (47))
\]

+ \( (1 + \delta) r_L \)

If \((27) > (20)\) for the same explicit contract, \( \omega_H^* \), then it is optimal (in comparison with the first case described above) for the investor to fully distort the contract and offer a compensation scheme with maximum powered implicit incentives. This results follows from the fact that the marginal benefit obtained in the first period would be greater in module than the excess expected return given up in the second period. This occurs when \( \delta \neq 0, -b^6\delta + 2b^4k\delta = 0 \) and \( 2b^4 - b^4k - ab^4k\delta + 2b^4k^2 + 4ab^4k^2\delta \neq 0 \) such that we can write one possible solution
\[ \omega_H^* > u^{-1} \left( \frac{1}{(2b^4 - b^4k - ab^4k\delta)} \left( \frac{b^2k}{2b^4k^2 + 4ab^4k^2\delta} \right) \right) > 0 \] (28)

From (27), the investor problem reduces to choosing \( \omega_H^* \). Then, the Lagrangian becomes

\[
\mathcal{L}_2 = \left( a + \frac{b^2u(\omega_H)}{k} \right) \left( 1 + \delta \left( 1 + \frac{ak^2}{2(2k-1)u(\omega_H)b^2} \right) \right) \left( 1 + \delta \left( a + \frac{b^2u(\omega_H)}{k} \right) \right) \left( r_H - r_L - \omega_H \right) + (1 + \delta) r_L - \lambda
\]

Optimal choice of \( \omega_H^* \) will satisfy first-order conditions of the Lagrangian above. The algebraic solution here is also intractable and we compare the numerical solution in this case with the numerical solution in case 1. Please, refer to the next section.

A third possible case is algebraically intractable and it has two possibilities. Either \( \Omega_H^* = 1 \) and \( 0 < \Omega_L^* < 1 \) or \( 0 < \Omega_H^* < 1 \) and \( \Omega_L^* = 0 \). In this two cases, \( \Omega_H^* > \Omega_L^* \) and, then, \( e_0^* > e_1^* > e_2^* \). It often occurs when the investor prefers to offer implicit incentive at a higher cost in terms of giving up positive excess expected returns or seizing negative excess expected returns. The implicit incentive not only complements the explicit incentive, but it also substitutes it in inducing effort whenever \( \omega_H^* \approx 0 \). On the other hand, variations in the implicit incentive that are very costly in terms of excess expected return are less intense and they are compensated by more distortion in the explicit incentive. These results are shown in the next subsection.

Nevertheless, these results may present some dynamic inconsistency concerns, from the perspective of the beginning of the second period, since the investor may change his decision
and not give up positive excess expected return once effort induced in the first period was already executed and the probability distribution function of returns of the first period does not influence the one of the second period. Therefore, implicit incentives distortion would represent a non-credible threat. We can adopt several strategies to solve this problem. For example, we may assume that the repeated game is played infinitely or that reputation concerns would force the investor to choose this costly allocation strategy.

Another possibility is that this problem may explain the little presence of complex long term explicit contracts in the asset management industry. That is, the investor writes a simple long term explicit contract and allows the powerful implicit incentive depend on his beliefs at each node of the decision tree in each period of the relationship. In other words, possible agency problems derived from simple and/or incomplete explicit incentives may be partially solved by delegating extreme power to the implicit incentive. Yet, flow of funds concerns complements the performance fee and may correct some wrong incentives and risk incongruities that may arise with the design of a simple explicit mechanism.

2.5 Numerical results

We assume that the risk aversion manager’s preferences are represented by a constant relative risk aversion utility function (CRRA) and that the parameter of risk aversion of the function is \( \rho \). The utility function is of the form

\[
u (\omega) = \frac{(\omega \Omega)^{\rho-1}}{\rho - 1}
\]

The coefficient of relative risk aversion is given by

\[
R_R = 2 - \rho
\]

The parameters values of the model are the coefficients \( a \) and \( b \) of the linear function that defines the probability distribution function of return in each node of the tree. The parameter of impatience is \( \delta \) and the cost function coefficient is \( k \). The high and low return depend on the level of the benchmark rate and the number of days as well as on the benchmark percentage variation of the benchmark return obtained in each state of nature; respectively, \( r_B \), \( days \),
$\text{cdiperch}_H$ and $\text{cdipercl}_L$. We calculate $r_H$ and $r_L$ in the following way

$$r_H = \left( (1 + \text{cdi})^{(\text{days}/252)} - 1 \right) \times \text{cdiperch}_H$$

$$r_L = \left( (1 + \text{cdi})^{(\text{days}/252)} - 1 \right) \times \text{cdipercl}_L$$

$$r_B = \left( (1 + \text{cdi})^{(\text{days}/252)} - 1 \right)$$

The table shows all parameter values of the basic scenario

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<th>b</th>
<th>$\delta$</th>
<th>$\text{om}_0$</th>
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We use the fmincom function of the Optimization toolbox from Matlab. The MatLab code files are presented in subsection 2 of the Appendix 2. We make some adjustments in our problem to be able to solve it numerically. Since the fmincom function is a non-linear constrained minimization function, we multiply the investor objective function by minus one. We also have to multiply the participation constraint by minus one since the fmincom function read inequality constraints as functions smaller or equal than zero.

Given the nonlinearity existent in the problem, the global optimal solution depends on the initial guess values provided in the computational program. So, we run the code for several starting points and select the best result that satisfy all the constraint in the model.

Besides, we run the model for several set of parameters. The best result of each set of parameters is presented in the tables below and in the subsection 3 of the Appendix. The first column indicates the parameter that varies and the respective values of the analysis.

When the cost parameter is lower ($k = 4$), the effort upper bound restriction binds with only the implicit incentives full distortion; then, there is no more room for extra explicit incentive
distortion and its power is reduced.

As expected return goes up, the implicit incentive grows faster than the explicit incentive. Observe that, given first period incentive considerations, $\Omega_H > 0$ even when $E [r_1] < 100\%$ of the benchmark return. As $cdiperch$ goes up, both the implicit incentive and the explicit incentives are offered. When, the implicit incentive upper bound constraint becomes active, the explicit incentive becomes more powerful and the performance fee is higher.

For greater levels of the parameter $a$, the probability distribution function of return is less dependent on effort execution; then, the investor’s marginal return obtained with distortion in the incentive structure is smaller. The implicit incentive distortion with the objective to induce effort in the first period still plays a major role and it is used, even when it is costly as seen in the two last cases of the table above. The explicit incentive is clearly less and less significant in the induction of effort as $a$ increases.

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<td>0.85</td>
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<td>0.00%</td>
<td>100.00%</td>
<td>0.00%</td>
<td>0.460</td>
<td>0.326</td>
<td>0.021</td>
<td>1.5836</td>
<td>0.1443</td>
<td>5.99%</td>
<td>114.341%</td>
<td>106.407%</td>
<td>88.225%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.90</td>
<td>1.3441%</td>
<td>0.00%</td>
<td>100.00%</td>
<td>0.00%</td>
<td>0.468</td>
<td>0.326</td>
<td>0.021</td>
<td>1.6125</td>
<td>0.1526</td>
<td>5.97%</td>
<td>114.830%</td>
<td>106.422%</td>
<td>88.228%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.95</td>
<td>1.3886%</td>
<td>0.00%</td>
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<td>0.00%</td>
<td>0.478</td>
<td>0.328</td>
<td>0.021</td>
<td>1.6512</td>
<td>0.1608</td>
<td>6.17%</td>
<td>115.469%</td>
<td>106.530%</td>
<td>88.232%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

21
Patient players, represented by greater levels of \( \delta \) (delta), are less willing to trade-off expected return in the first period vis-à-vis expected return in the second period. Then, more explicit incentive is used to induce higher effort. On the other hand, the portfolio manager is also more interested in smoothing consumption between the two periods.

<table>
<thead>
<tr>
<th>Rho</th>
<th>wh</th>
<th>wl</th>
<th>omh</th>
<th>oml</th>
<th>( e_0 )</th>
<th>( e_1 )</th>
<th>( e_2 )</th>
<th>Man Exp Util</th>
<th>Inv Exp ExcRet</th>
<th>Perf Fee (%aa)</th>
<th>E [r0exc] (%cdi)</th>
<th>E [r1exc] (%cdi)</th>
<th>E [r2exc] (%cdi)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>100.00%</td>
<td>2.02%</td>
<td>1.000</td>
<td>0.730</td>
<td>0.600</td>
<td>20.0904</td>
<td>0.2398</td>
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<td>146.500%</td>
<td>130.429%</td>
<td>122.729%</td>
</tr>
<tr>
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<td>0.00%</td>
<td>100.00%</td>
<td>0.00%</td>
<td>0.871</td>
<td>0.517</td>
<td>0.082</td>
<td>5.0894</td>
<td>0.1978</td>
<td>5.57%</td>
<td>103.800%</td>
<td>117.760%</td>
<td>91.878%</td>
</tr>
<tr>
<td>1.15</td>
<td>1.3441%</td>
<td>0.00%</td>
<td>100.00%</td>
<td>0.00%</td>
<td>0.468</td>
<td>0.326</td>
<td>0.021</td>
<td>1.6125</td>
<td>0.1526</td>
<td>5.97%</td>
<td>114.830%</td>
<td>106.420%</td>
<td>88.225%</td>
</tr>
<tr>
<td>1.20</td>
<td>1.3666%</td>
<td>0.00%</td>
<td>100.00%</td>
<td>0.00%</td>
<td>0.274</td>
<td>0.212</td>
<td>0.005</td>
<td>0.6441</td>
<td>0.1341</td>
<td>6.07%</td>
<td>103.316%</td>
<td>99.601%</td>
<td>87.317%</td>
</tr>
<tr>
<td>1.25</td>
<td>1.3288%</td>
<td>0.00%</td>
<td>99.85%</td>
<td>0.00%</td>
<td>0.170</td>
<td>0.140</td>
<td>0.001</td>
<td>0.3029</td>
<td>0.1253</td>
<td>5.91%</td>
<td>97.136%</td>
<td>95.329%</td>
<td>87.083%</td>
</tr>
</tbody>
</table>

As rho approaches 2, the manager becomes less risk averse. The more risk averse is the manager, the more efficient is the usage of the implicit incentive, that is, the flow of fund’s power is much greater as the portfolio manager becomes more risk averse. The riskier averse is the manager, less distortion is needed to induce effort in the first period, reducing the impact of the intertemporal trade-off.

<table>
<thead>
<tr>
<th>cdi</th>
<th>wh</th>
<th>wl</th>
<th>omh</th>
<th>oml</th>
<th>( e_0 )</th>
<th>( e_1 )</th>
<th>( e_2 )</th>
<th>Man Exp Util</th>
<th>Inv Exp ExcRet</th>
<th>Perf Fee (%aa)</th>
<th>E [r0exc] (%cdi)</th>
<th>E [r1exc] (%cdi)</th>
<th>E [r2exc] (%cdi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.00%</td>
<td>0.1515%</td>
<td>0.00%</td>
<td>100.00%</td>
<td>0.00%</td>
<td>0.315</td>
<td>0.223</td>
<td>0.014</td>
<td>1.0923</td>
<td>0.0185</td>
<td>5.08%</td>
<td>105.725%</td>
<td>100.255%</td>
<td>87.836%</td>
</tr>
<tr>
<td>6.00%</td>
<td>0.4729%</td>
<td>0.00%</td>
<td>100.00%</td>
<td>0.00%</td>
<td>0.386</td>
<td>0.273</td>
<td>0.017</td>
<td>1.3210</td>
<td>0.0581</td>
<td>5.25%</td>
<td>109.988%</td>
<td>103.219%</td>
<td>88.023%</td>
</tr>
<tr>
<td>10.00%</td>
<td>0.8129%</td>
<td>0.00%</td>
<td>100.00%</td>
<td>0.00%</td>
<td>0.426</td>
<td>0.299</td>
<td>0.019</td>
<td>1.4605</td>
<td>0.0994</td>
<td>5.42%</td>
<td>112.373%</td>
<td>104.817%</td>
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</tr>
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<td>15.00%</td>
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<td>100.00%</td>
<td>0.00%</td>
<td>0.468</td>
<td>0.326</td>
<td>0.021</td>
<td>1.6125</td>
<td>0.1526</td>
<td>5.97%</td>
<td>114.830%</td>
<td>106.420%</td>
<td>88.225%</td>
</tr>
<tr>
<td>17.50%</td>
<td>1.5847%</td>
<td>0.00%</td>
<td>100.00%</td>
<td>0.00%</td>
<td>0.482</td>
<td>0.336</td>
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<td>1.6677</td>
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<td>6.04%</td>
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<td>88.260%</td>
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<tr>
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<td>107.465%</td>
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<td>0.00%</td>
<td>0.509</td>
<td>0.353</td>
<td>0.022</td>
<td>1.7732</td>
<td>0.2402</td>
<td>6.16%</td>
<td>117.294%</td>
<td>107.992%</td>
<td>88.324%</td>
</tr>
</tbody>
</table>

Here, an interesting result appears. Since the percentage excess return is greater in nominal terms for greater levels of benchmark rates of return, the investor has more room to distort the explicit incentive. This in an intuitive result once it makes sense to believe that it is harder to obtain percentage of the benchmark excess returns when the benchmark return is greater. For example, one hundred and fifty percent of a twenty percent benchmark rate is a three percent excess return and for a two percent benchmark rate of return, we are talking about one percent excess return. Then, greater benchmark returns are related with more explicitly powerful contracts, or we may say, greater performance fee.

In general terms, the optimal solution shows that the implicit incentive varies significantly in the second period depending on the expected return in each node. The investor provides powerful implicit incentives after the observation of high return in the first period and the
upper bound (14) binds. On the other hand, after low return is observed in the first period, the investor penalizes the manager, withdrawing all or almost all resources from the manager and (15) binds. The dynamic implicit incentive component is so strong in this model that this result occurs even if the expected return is positive in the bad state of nature of the second period.

The intuition behind it is that since both the total payoff and the explicit incentives depend on the amount of implicit incentives, effort induction by using the flow of funds is more efficient under a moral hazard framework.

When we compare the results above where the implicit incentive is very powerful with the results where there are no implicit incentives, $\Omega^*_H = \Omega^*_L = 1$, we observe that the participation constraint is also not binding, but the portfolio manager utility is lower, i.e., he extracts less rent Second, we observe that, in most cases, the investor is better off with powerful implicit incentives provision. Exceptions occur when expected excess return is not very dependent on effort. A third result is counter-intuitive since we would expect that these two incentives to be substitutes when, in fact, we observe that they complement each other as seen in the table below.

<table>
<thead>
<tr>
<th>Case 1</th>
<th>General results</th>
<th>Differences (General case - case 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>omh = oml = 1</td>
<td>omh</td>
<td>oml</td>
</tr>
<tr>
<td></td>
<td>wh</td>
<td>omh</td>
</tr>
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<td>4</td>
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</tr>
<tr>
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<td>1,200</td>
<td>0.9047%</td>
</tr>
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<td>1,300</td>
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<td>0.9750%</td>
</tr>
<tr>
<td>4</td>
<td>1,500</td>
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</tr>
<tr>
<td>4</td>
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</tr>
<tr>
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</tr>
<tr>
<td>8</td>
<td>1,400</td>
<td>1.1018%</td>
</tr>
<tr>
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</tr>
<tr>
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<td>1,750</td>
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<td>2,000</td>
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### General results

<table>
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<th>a</th>
<th>Case 1 wh</th>
<th>Case 1 omh</th>
<th>Case 1 oml</th>
<th>Differences</th>
<th>Case 1 Utility</th>
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### General results

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<th>Case 1 oml</th>
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### General results

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<td>0.88%</td>
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</tr>
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### General results

<table>
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<th>Case 1 omh</th>
<th>Case 1 oml</th>
<th>Differences</th>
<th>Case 1 Utility</th>
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<th>Case 1 Ret</th>
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<td>0.7701%</td>
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</tr>
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</table>
An exception for this third result occurs when we have a corner effort solution in the first period. In this case, explicit incentives provision is restricted since the implicit incentive is already providing all possible effort. However, when we impose no implicit incentive provision, more explicit incentive provision is allowed and necessary to maximize effort exertion.

3 The model with two types of portfolio managers

Suppose that there two types of portfolio managers in the economy and that they are heterogeneous in the ability to generate positive excess return at each period and each node of the decision tree, $\eta_1$ and $\eta_2$. The high ability portfolio manager, $\eta_1$, produce positive excess return with positive probability in the good state of nature of the binomial model while the low ability portfolio manager, $\eta_2$, always produces negative excess return and never adds any value to the relationship. In this case, $P(r_H|\eta_1) > 0$ and $P(r_H|\eta_2) = 0$. We adopt a key simplification in the model and make the level of ability unknown to everyone in the economy, whether the investor or the manager. Therefore, the portfolio manager’s type is an incomplete and symmetric information. Only the prior distribution over $\eta$ is commonly known and shared by all contracting parties ex ante.\(^5\) Since the information is symmetric, there is no need for investors to offer menus of contracts in order to induce workers to self-select.

We further assume that the proportion of $\eta$ in the economy is $\lambda$ and the percentage of $\eta$ is $(1 - \lambda)$. In the first period, the investor has to infer the probability of return based on his belief of $\lambda$. In the second period, the investor uses his belief and the information derived from the the excess return observed in the first period to infer about the portfolio manager’s type, $\eta$. Now, return is a noisy signal of effort and ability.

All the assumptions and notation remain the same unless for a new superscript in each effort function which indicates the type of the manager. Then, we describe effort as $e_{t,s}^m \in [0, 1]$, $\eta = \eta_1, \eta_2$. In the first period, the probability of high return is given by the probability distribution of return conditional to the portfolio manager’s expected level of ability

\[
\pi_{1,0} = P(\eta_1) \cdot P(r_H|\eta_1) + P(\eta_2) \cdot P(r_H|\eta_2) \\
= \lambda \left( a_{1,0}^\eta + b_{1,0}^\eta e_{1,0}^\eta \right) + (1 - \lambda) \left( a_{1,0}^\eta + b_{1,0}^\eta e_{1,0}^\eta \right)
\]

The investor and the manager observe the realized return in the first period and learn about the manager’s ability. Then, the investor adjusts the posterior distribution of return in

\(^5\)This idea was first introduced by Holmstrom (1982a).
a Bayesian way to obtain the probabilities of high return in each node of the second period.

\[
\pi_{2,2} = P(r_H|\eta) P(\eta|r_L) + P(r_H|\eta) P(\eta|r_L) \\
= (a_{1,0}^{\eta} + b_{1,0}^{\eta} e_{1,0}^{\eta}) P(\eta|r_L) + (a_{1,0}^{\eta} + b_{1,0}^{\eta} e_{1,0}^{\eta}) P(\eta|r_L)
\]

while the probability that the manager is of the \( \eta \) or \( \eta \) type given the return observed in the first period are respectively given by

\[
P(\eta|r_H) = \frac{P(\eta)P(r_H|\eta)}{P(\eta)P(r_H|\eta)+P(\eta)P(r_H|\eta)} , \quad P(\eta|r_L) = \frac{P(\eta)P(r_L|\eta)}{P(\eta)P(r_L|\eta)+P(\eta)P(r_L|\eta)}
\]

### 3.1 The portfolio manager problem: optimal choice of effort

When the observation of return in the first period reveals hiss type, the portfolio manager solves

\[
\max_{e_0, e_1, e_2} U_M = \sum_{t=0}^{2} \delta^t \left[ \sum_{s_t=\tau_t} P(r_t|e_t) u(\Omega_{t,s} \omega_{t,s}) - \frac{k}{2} (e_{t,s}) \right] 
\]

\[
= \pi_{1,0} u(\Omega_0 \omega_H) + (1 - \pi_{1,0}) u(\Omega_0 \omega_L) - \frac{k}{2} (e_{1,0})^2 \\
+ \delta \left\{ \begin{array}{l}
\pi_{1,0} \left[ \pi_{2,1} u(\Omega_H \omega_H) + (1 - \pi_{2,1}) u(\Omega_H \omega_L) - \frac{k}{2} (e_{2,1})^2 \right] \\
+ (1 - \pi_{1,0}) \left[ \pi_{2,2} u(\Omega_L \omega_H) + (1 - \pi_{2,2}) u(\Omega_L \omega_L) - \frac{k}{2} (e_{2,2})^2 \right] \end{array} \right\}
\]

since we assume that the bad manager never generate positive excess return, \( P(r_H|\eta) = 0 \) and \( P(r_L|\eta) = 1 \). Then, the probabilities of high return in each node are given by

\[
\pi_{1,0} = \lambda (a + b e_{1,0}^i) \\
\pi_{2,1} = (a + b e_{2,1}^i) P(\eta|r_H) = a + b e_{2,1}^i \\
\pi_{2,2} = (a + b e_{2,2}^i) P(\eta|r_L) = (a + b e_{2,2}^i) \frac{\lambda (1 - \pi_{1,0})}{1 - \lambda \pi_{1,0}}
\]

The reservation utility of the portfolio manager is exogenously given and is equal to \( U_M \). Again, the investor has all bargaining power and can make take-it-or-leave-it offers to the portfolio manager subject to providing him with an expected payoff which yields at least \( U_M \). Normalizing \( \Omega_0 = 1 \), the first-order conditions of the manager’s problem are given by

\[
e^*_1 = \frac{\lambda b [u(\omega_H) - u(\omega_L)]}{k} + \frac{\delta \lambda b}{k} \left\{ \begin{array}{l}
\pi_{2,1} u(\Omega_H \omega_H) + (1 - \pi_{2,1}) u(\Omega_H \omega_L) - \frac{k}{2} (e_{2,1})^2 \\
- \pi_{2,2} u(\Omega_L \omega_H) - (1 - \pi_{2,2}) u(\Omega_L \omega_L) + \frac{k}{2} (e_{2,2})^2 \end{array} \right\}
\]

\[
= A(\omega_H, \omega_L, \Omega_H, \Omega_L)
\]
Observe that the optimal effort choice in the bad state of nature in the second period depend on the optimal effort choice in the first period. Calculating explicit expressions for $e_{1,0}$ and $e_{2,2}$ becomes algebraically intractable and the numerical solution are also provided for effort choices. Given the Bayesian adjustment of posteriors, we know that $e_{1,0} > e_{2,2}$ and that $e_{2,1} > e_{2,2}$. However, we can not say anything about the relationship between $e_{1,0}$ and $e_{2,1}$. As the numerical results show, depending on the parameter values, the difference between them may have any sign.

### 3.2 The investor problem: optimal provision of incentives

Now, the risk-neutral investor solves

$$\max_{\omega_H, \omega_L, \Omega_H, \Omega_L} U_I = \pi_0 \Omega_0 (r_H - \omega_H) + (1 - \pi_0) \Omega_0 (r_L - \omega_L) + \delta \pi_0 [\pi_1 \Omega_H (r_H - \omega_H) + (1 - \pi_1) \Omega_H (r_L - \omega_L) - (\Omega_H - \Omega_0) \bar{r}] + \delta (1 - \pi_0) [\pi_2 \Omega_L (r_H - \omega_H) + (1 - \pi_2) \Omega_L (r_L - \omega_L) - (\Omega_L - \Omega_0) \bar{r}]$$

subject to the following constraints. We normalize the reservation utility to zero in each node and write the participation constraint as

$$\frac{(a + be_{0}^* (\omega, \Omega)) u (\omega_H) + (1 - a - be_{0}^* (\omega, \Omega)) u (\omega_L) - \frac{(e_{0}^* (\omega, \Omega))^2}{2}}{\sum_{t=0}^{2} \sum_{s=1}^{2} P(r_t | e_t) u (\Omega_{t,s} \omega_{t,s}) - \frac{k (e_{t,s})}{2}} \geq 0$$

An incentive compatible contract offered by the investor also satisfies the incentive compatibility constraints

$e_0, e_1, e_2 \in \arg \max \sum_{t=0}^{2} \delta^t \left[ \sum_{s=1}^{2} P(r_t | e_t) u (\Omega_{t,s} \omega_{t,s}) - \frac{k (e_{t,s})}{2} \right]$
The manager has limited liability in excess return and can only be penalized for exerting low levels of effort through the implicit incentive, reducing the total compensation in the second-period. Then, it is necessary to write two limited responsibility constraints for the explicit incentives such that

$$\omega_H \geq 0$$

$$\omega_L \geq 0$$ (37)

$$\omega_H \geq 0$$

$$\omega_L \geq 0$$ (38)

Since it is neither possible to borrow resources from the manager’s fund nor to leverage positions in the fund by borrowing at the benchmark rate, there are also two short-selling constraints for the implicit incentives such that

$$0 \leq \Omega_H \leq 1$$

$$0 \leq \Omega_L \leq 1$$ (39)

$$0 \leq \Omega_H \leq 1$$

$$0 \leq \Omega_L \leq 1$$ (40)

besides these non-negativity constraints, effort choices executed by the manager in each period must be in the unit interval to avoid probabilities greater than one

$$0 \leq e^*_0 (\omega, \Omega) \leq 1$$

$$0 \leq e^*_1 (\omega, \Omega) \leq 1$$ (41)

$$0 \leq e^*_2 (\omega, \Omega) \leq 1$$

$$0 \leq e^*_1 (\omega, \Omega) \leq 1$$ (42)

$$0 \leq e^*_2 (\omega, \Omega) \leq 1$$ (43)

The equilibrium solution \( \{ \omega^*_H, \omega^*_L, \Omega^*_H, \Omega^*_L, e^*_0 (\omega, \Omega), e^*_1 (\omega, \Omega), e^*_2 (\omega, \Omega) \} \) is algebraically intractable and can only have a numerical solution.

Observe that the investor provides incentives in order to maximize expected utility as he learns about the manager’s type. For all \( \lambda < \lambda \), it is optimal to offer full distortion in the implicit incentive structure. That is, for a particular belief about the percentage of bad managers in the economy and below this level, there is no cost in providing full distortion in the implicit incentive, i.e., when performance is poor in the first period, withdrawing all resources from the fund can be done without any cost.

### 3.3 Characterization of the optimal incentive contract

In equilibrium, the investor offers an incentive compatible contract \( \{ \omega^*_H, \omega^*_L, \Omega^*_H, \Omega^*_L, e^*_0 (\omega, \Omega) \} \) that satisfies all the constraints of his problem. He also chooses \( \{ e^*_0 (\omega, \Omega); e^*_1 (\omega, \Omega); e^*_2 (\omega, \Omega) \} \)
that satisfy the incentive constraints. The investor provides total incentives that equalize the marginal excess return and the implied costs of effort induction. He does so by simultaneously combining and distorting both the implicit and the explicit incentive’s compensation structure as to maximize the intertemporal excess expected return.

### 3.4 Numerical results

The table shows all parameter values of the basic scenario

<table>
<thead>
<tr>
<th>$k$</th>
<th>$a$</th>
<th>$b$</th>
<th>$\delta$</th>
<th>$\omega_0$</th>
<th>$cdi$</th>
<th>$days$</th>
<th>$cdiperc_H$</th>
<th>$cdiperc_L$</th>
<th>$\rho$</th>
<th>$\lambda$</th>
<th>$r_H$</th>
<th>$r_L$</th>
<th>$r_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>0.10</td>
<td>0.85</td>
<td>0.90</td>
<td>1.00</td>
<td>15%</td>
<td>252</td>
<td>80%</td>
<td>1.15</td>
<td>0.8</td>
<td>22.5%</td>
<td>12.0</td>
<td>15%</td>
<td></td>
</tr>
</tbody>
</table>

The computer codes are presented in subsection 4 of the Appendix. We execute the same procedure than the one described in the first model of this paper. However, due to the difficulties in finding an explicit solution for effort that we would be able to substitute in the investor function, the investor now solves the problem for all effort choices as well, having to respect the incentive constraints from the application of the FOA. Due to calculations restrictions of the Matlab, we use inequalities near zero for each incentive constraint of effort choice of the portfolio manager.

The first column indicates the parameter that varies and the respective values of the analysis.

<table>
<thead>
<tr>
<th>$cdi$</th>
<th>$wh$</th>
<th>$wl$</th>
<th>$omh$</th>
<th>$oml$</th>
<th>$e0$</th>
<th>$e1$</th>
<th>$e2$</th>
<th>Man Exp Util</th>
<th>Inv Exp ExcRet</th>
<th>Perf Fee (% aa)</th>
<th>E [r0 exc] (% cdi)</th>
<th>E [r1 exc] (% cdi)</th>
<th>E [r2 exc] (% cdi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.300</td>
<td>0.6931</td>
<td>0.00%</td>
<td>98.77%</td>
<td>0.00%</td>
<td>7.16%</td>
<td>29.08%</td>
<td>3.01%</td>
<td>0.17</td>
<td>(0.0007)</td>
<td>3.55%</td>
<td>86.43%</td>
<td>97.36%</td>
<td>84.88%</td>
</tr>
<tr>
<td>1.400</td>
<td>0.9980</td>
<td>0.00%</td>
<td>100.00%</td>
<td>0.00%</td>
<td>32.87%</td>
<td>31.02%</td>
<td>1.44%</td>
<td>1.20</td>
<td>0.1290</td>
<td>4.75%</td>
<td>98.21%</td>
<td>101.82%</td>
<td>84.96%</td>
</tr>
<tr>
<td>1.500</td>
<td>1.2171</td>
<td>0.00%</td>
<td>100.00%</td>
<td>0.00%</td>
<td>34.01%</td>
<td>32.09%</td>
<td>1.49%</td>
<td>1.24</td>
<td>0.1352</td>
<td>5.41%</td>
<td>101.79%</td>
<td>106.10%</td>
<td>85.78%</td>
</tr>
<tr>
<td>1.750</td>
<td>1.7833</td>
<td>0.00%</td>
<td>100.00%</td>
<td>0.00%</td>
<td>36.31%</td>
<td>34.25%</td>
<td>1.58%</td>
<td>1.33</td>
<td>0.1513</td>
<td>6.79%</td>
<td>111.06%</td>
<td>117.16%</td>
<td>87.86%</td>
</tr>
<tr>
<td>2.000</td>
<td>2.3401</td>
<td>0.00%</td>
<td>100.00%</td>
<td>0.00%</td>
<td>38.06%</td>
<td>35.86%</td>
<td>1.64%</td>
<td>1.40</td>
<td>0.1682</td>
<td>7.80%</td>
<td>120.85%</td>
<td>128.58%</td>
<td>89.92%</td>
</tr>
</tbody>
</table>

In the first line of the table, $\Omega_H^* \approx 1$ because we set $\Omega_0^* = 1$ and hence effort induction pays
off in the first period as to minimize expected negative excess return.

As $a$ goes up, effort needs not to be induced since the marginal benefit of incentive provision is a decreasing function of $a$. For high levels, $a > 0.50$, both the implicit and the explicit incentive distortion are unnecessary once effort induction is expensive.

The greater in the percentage of bad portfolio managers (lower $\lambda$), less explicit incentive is offered. When $\lambda = 1$, the solution is similar to the first model.

The cost parameter affects the marginal cost due to the provision of incentives. For lower levels of $k$, the marginal cost is low and the implicit incentive increases effort intensively. Observe that $c_0^* = 1$, for $k \leq 2$. In this case, there is no more need to provide anymore explicit incentive. Since, implicit incentives are fully distorted we would like to conclude
that, in this case, the implicit incentive is more powerful than the explicit component.

<table>
<thead>
<tr>
<th>Rho</th>
<th>wh</th>
<th>wl</th>
<th>omh</th>
<th>oml</th>
<th>e0</th>
<th>e1</th>
<th>e2</th>
<th>Man Exp Util</th>
<th>Inv Exp ExcRet</th>
<th>Perf Fee (% aa)</th>
<th>E [r0 exc] (% cdi)</th>
<th>E [r1exc] (% cdi)</th>
<th>E [r2exc] (% cdi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.05</td>
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<td>100.00%</td>
<td>0.00%</td>
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<td>62.10%</td>
<td>12.28%</td>
<td>16.94</td>
<td>0.2087</td>
<td>9.27%</td>
<td>132.20%</td>
<td>123.95%</td>
<td>87.01%</td>
</tr>
<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.10</td>
<td>0.1404%</td>
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<td>62.07%</td>
</tr>
<tr>
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<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.15</td>
<td>0.2171%</td>
<td>0.00%</td>
<td>100.00%</td>
<td>0.00%</td>
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</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.20</td>
<td>0.2229%</td>
<td>0.00%</td>
<td>100.00%</td>
<td>0.00%</td>
</tr>
</tbody>
</table>

Again, riskier aversion to risk makes the implicit incentive more efficient in inducing maximum effort and, hence, less performance fee is needed.

<table>
<thead>
<tr>
<th>delta</th>
<th>wh</th>
<th>wl</th>
<th>omh</th>
<th>oml</th>
<th>e0</th>
<th>e1</th>
<th>e2</th>
<th>Man Exp Util</th>
<th>Inv Exp ExcRet</th>
<th>Perf Fee (% aa)</th>
<th>E [r0 exc] (% cdi)</th>
<th>E [r1exc] (% cdi)</th>
<th>E [r2exc] (% cdi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.50</td>
<td>1.1807%</td>
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<td>100.00%</td>
<td>0.00%</td>
<td>30.15%</td>
<td>31.93%</td>
<td>1.49%</td>
<td>1.09</td>
<td>0.0722</td>
<td>5.25%</td>
<td>99.95%</td>
<td>106.00%</td>
<td>85.84%</td>
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<tr>
<td>0.60</td>
<td>1.2233%</td>
<td>0.00%</td>
<td>100.00%</td>
<td>0.00%</td>
<td>30.80%</td>
<td>31.65%</td>
<td>1.48%</td>
<td>1.12</td>
<td>0.0879</td>
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<td>100.26%</td>
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<td>1.16</td>
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<td>106.04%</td>
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<tr>
<td>0.90</td>
<td>1.2171%</td>
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<td>100.00%</td>
<td>0.00%</td>
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<td>32.09%</td>
<td>1.49%</td>
<td>1.24</td>
<td>0.1352</td>
<td>5.41%</td>
<td>101.79%</td>
<td>106.10%</td>
<td>85.78%</td>
</tr>
<tr>
<td>0.95</td>
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<td>0.1431</td>
<td>5.43%</td>
<td>102.02%</td>
<td>106.11%</td>
<td>85.76%</td>
</tr>
</tbody>
</table>

Patient players will accept more explicit incentive since they are more willing to smooth utility and expected return between the two periods.

<table>
<thead>
<tr>
<th>cdI</th>
<th>wh</th>
<th>wl</th>
<th>omh</th>
<th>oml</th>
<th>e0</th>
<th>e1</th>
<th>e2</th>
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<th>Inv Exp ExcRet</th>
<th>Perf Fee (% aa)</th>
<th>E [r0 exc] (% cdi)</th>
<th>E [r1exc] (% cdi)</th>
<th>E [r2exc] (% cdi)</th>
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</thead>
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<td>1.06</td>
<td>0.0524</td>
<td>5.07%</td>
<td>99.29%</td>
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<td>104.72%</td>
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<td>0.00%</td>
<td>34.01%</td>
<td>32.09%</td>
<td>1.49%</td>
<td>1.24</td>
<td>0.1352</td>
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<td>33.82%</td>
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<td>36.79%</td>
<td>34.69%</td>
<td>1.59%</td>
<td>1.35</td>
<td>0.2108</td>
<td>5.57%</td>
<td>103.11%</td>
<td>107.64%</td>
<td>85.79%</td>
</tr>
</tbody>
</table>

The results are similar to those presented above and now the power of the implicit incentive also reflects learning about the manager’s type, avoiding dynamic inconsistency problems when the intertemporal trade-off of the implicit incentive is too expensive.

4 Conclusions

The existence of optimal contracts with powerful implicit incentives is the most relevant result presented here. This power arises from the fact that, given the typical contract’s characteristics, the flow of funds represents a less conflictive mechanism designed by the investor.
to induce the portfolio manager to exert higher levels of effort. While the implied cost of using explicit incentives reduce net expected return directly, the implicit incentive only affects the investor objective function when the benchmark return is greater than the endogenous expected return obtained by the portfolio manager. The power of the flow of funds might also be an explanation for low powered explicit incentives. Indeed, implicit incentives’ power might complement simple explicit incentives, given the general conditions encountered in the marketplace. Or we may say, powerful implicit incentives may correct some nuisances created by simple and incomplete linear explicit incentives that are detrimental to efficient risk choices executed by the portfolio manager.

However, it does not arise as an important incentive response without a relevant implied cost. First, expected returns are endogenous to effort provision. Second, the trade-off between incentives and performance may be so costly that it even represents a non-credible threat when the portfolio allocation decision is different than the usual solution without any intertemporal incentives consideration.

More importantly, powerful implicit incentives may negatively affect the portfolio manager’s ability to take risks since the implied uncertainty of highly volatile flow of funds creates incentives to myopic investing. This greater income uncertainty reduces the utility of a risk averse manager and may lead to an increase in the likelihood of "closet indexing" of the fund when past excess return is positive and asset under management grows. On the other hand, it may also increase the likelihood of excessive risk-taking when past excess return is negative and the flow of fund’s expected punishment may lead to all-or-nothing bets.

Rational investors should be "forward looking" decision makers. Since the investor can not observe effort executed by the manager, moral hazard issues arise and, hence, "backward looking strategies" maximize expected return. This result may explain an empirical regularity found in the asset management industry that seems to be unreasonable and inconsistent, once past return may not be indicative of future return.

Indeed, if powerful implicit incentives raise flow concerns that are detrimental to optimal effort and risk-taking behavior, it would be desirable to spend time and resources in the designing of somewhat complex explicit incentives clauses that internalize the history of returns as well as pre-defined variables like investor and portfolio manager’s investment profiles and objectives.

For instance, it might make sense to build a compensation structure that depend less on the total volume under management and design a mechanism in which total incentives
are more dependent on performance with a more powerful explicit incentive. The investor should compensate future performance rather than past performance to guarantee that he seizes all the possible benefits of the dynamic relationship in an asset management contract.
5 Appendix

5.1 CPO’s and equations of model in Section 2

Considering that $\lambda$, $\mu_{w_H}$, $\mu_{w_L}$, $\mu_{\Omega_H}$, $\mu_{\Omega_L}$, $\mu^1_{\Omega_H}$, $\mu^1_{\Omega_L}$, $\mu_0$, $\mu_1$, $\mu_2$, $\mu_4$, $\mu_5$, $\mu_6$ are non-negative multipliers of the Kuhn-Tucker Lagrangian, the first-order conditions for the explicit incentives are given by

$$
\lambda = - \left\{ \begin{array}{l}
\left[ A F_{w_H} + \delta (A E H_{w_H} + (1 - A) I M_{w_H}) \right] + \\
+ [A_{w_H} \Gamma_A + \delta (E_{w_H} \Gamma_E + I_{w_H} \Gamma_I)] (\mu_0 - \mu_4) A_{w_H} + (\mu_1 - \mu_5) E_{w_H} + (\mu_2 - \mu_6) I_{w_H} \\
\end{array} \right\}
$$

Total exp cost of effort induction given a variation in $w_H$

while the implicit incentives first-order conditions are given by

$$
\lambda = - \left\{ \begin{array}{l}
[(1 - A) G_{w_L} + \delta [A (1 - E) K_{w_L} + (1 - A) (1 - I) N_{w_L}]] + \\
+ [A_{w_L} \Gamma_A + \delta (E_{w_L} \Gamma_E + I_{w_L} \Gamma_I)] + [A_{w_L} \Gamma_{AA} + \delta (E_{w_L} \Gamma_{EE} + I_{w_L} \Gamma_{II})] \\
\end{array} \right\}
$$

Total exp mrgl product of effort given a variation in $w_L$

and

$$
\lambda = - \left\{ \begin{array}{l}
\delta [A E H_{\Omega_H} + A (1 - E) K_{\Omega_H}] + [A_{\Omega_H} \Gamma_A + \delta E_{\Omega_H} \Gamma_E] + (\mu_0 - \mu_4) A_{\Omega_H} + (\mu_1 - \mu_5) E_{\Omega_H} + (\mu_2 - \mu_6) I_{\Omega_H} \\
\delta [A E S_{\Omega_H} + A (1 - E) T_{\Omega_H}] + [A_{\Omega_H} \Gamma_{AA} + \delta E_{\Omega_H} \Gamma_{EE}] \\
\end{array} \right\}
$$

Total exp cost of effort induction given a variation in $\Omega_H$

and

$$
\lambda = - \left\{ \begin{array}{l}
\mu_{\Omega_L} + (\mu_0 - \mu_4) A_{\Omega_L} + (\mu_2 - \mu_6) I_{\Omega_L} + \\
+ \delta [(1 - A) I M_{\Omega_L} + (1 - A) (1 - I) N_{\Omega_L}] + [A_{\Omega_L} \Gamma_A + \delta I_{\Omega_L} \Gamma_I] \\
\end{array} \right\}
$$

Total exp mrgl product of effort given a variation in $\Omega_L$
The participation constraint and its multipliers first-order conditions are described by

\[
\lambda \left\{ \begin{array}{l}
AP + (1 - A)Q - \frac{k}{2}(e_0^*)^2 + \delta A \left\{ ES + (1 - E)T - \frac{k}{2}(e_1^*)^2 \right\} \\
+ \delta (1 - A) \left\{ IV + (1 - I)W - \frac{k}{2}(e_2^*)^2 \right\}
\end{array} \right\} = 0
\] (44)

and the slackness conditions

\[
\mu_{wH}: \begin{bmatrix} w_H \end{bmatrix} \geq 0
\] (45)

\[
\mu_{wL}: \begin{bmatrix} w_L \end{bmatrix} \geq 0
\] (46)

\[
\mu_{\Omega_H}: \begin{bmatrix} \Omega_H \end{bmatrix} \geq 0
\] (47)

\[
\mu_{\Omega_L}: \begin{bmatrix} \Omega_L \end{bmatrix} \geq 0
\] (48)

\[
\mu_{e_0}^*: e_0^* - 1 = 0
\] (51)

\[
\mu_{e_1}^*: e_1^* - 1 = 0
\] (52)

\[
\mu_{e_2}^*: e_2^* - 1 = 0
\] (53)

\[
\mu_0, \mu_1, \mu_2, \mu_3, \mu_4, \mu_5, \mu_6 \geq 0
\] (56)

where implicitly differentiating the first-order conditions of the portfolio manager gives us

\[
A = a + b.e_0^*
\]

\[
A_{wH} = \frac{\partial e_0^*}{\partial w_H} = b^2 \left\{ \frac{u_{wH} (w_H)}{k} + \delta \left[ \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} k a (u_{wH} (\Omega_H w_H) \Omega_H - u_{wH} (\Omega_L w_H) \Omega_L) \\ b^2 \left( \frac{2k-1}{k} \right) \left[ (u (\Omega_H w_H) - u (\Omega_L w_H)) u_{wH} (\Omega_H w_H) \Omega_H - (u (\Omega_L w_H) - u (\Omega_L w_H)) u_{wH} (\Omega_L w_H) \Omega_L \right] \end{bmatrix} \right] \right] \right\} > 0
\]

\[
A_{wL} = \frac{\partial e_0^*}{\partial w_L} = -b^2 \left\{ \frac{u_{wL} (w_L)}{k} - \delta \left[ \begin{bmatrix} \begin{bmatrix} k (1 - a) (u_{wL} (\Omega_H w_L) \Omega_H - u_{wL} (\Omega_L w_L) \Omega_L) \\ b^2 \left( \frac{2k-1}{k} \right) \left[ (u (\Omega_H w_H) - u (\Omega_L w_H)) u_{wL} (\Omega_H w_L) \Omega_H - (u (\Omega_L w_H) - u (\Omega_L w_H)) u_{wL} (\Omega_L w_L) \Omega_L \right] \end{bmatrix} \right] \right\} < 0
\]
\[ A_{\Omega_H} = \frac{\partial e_0^*}{\partial \Omega_H} = b^2 \delta \left\{ \begin{array}{l} ka. u_{\Omega_H} (\Omega_H w_H) w_H + k (1 - a) u_{\Omega_H} (\Omega_H w_L) w_L \\ + b^2 \left( \frac{2k-1}{k} \right) \left[ \left( u_{\Omega_H} (\Omega_H w_H) w_H - u_{\Omega_H} (\Omega_H w_L) w_L \right) \cdot \left( u (\Omega_H w_H) - u (\Omega_H w_L) \right) \right] \end{array} \right\} > 0 \]

\[ A_{\Omega_L} = \frac{\partial e_0^*}{\partial \Omega_L} = -b^2 \delta \left\{ \begin{array}{l} a. u_{\Omega_L} (\Omega_L w_H) w_H + (1 - a) u_{\Omega_L} (\Omega_L w_L) w_L \\ + b^2 \left( \frac{2k-1}{k} \right) \left[ \left( u_{\Omega_L} (\Omega_L w_H) w_H - u_{\Omega_L} (\Omega_L w_L) w_L \right) \cdot \left( u (\Omega_L w_H) - u (\Omega_L w_L) \right) \right] \end{array} \right\} < 0 \]

\[ E = a + b e_1^* \]

\[ E_{wH} = \frac{\partial e_1^*}{\partial w_H} = \frac{b^2. u_{wH} (\Omega_H \cdot w_H) \cdot \Omega_H}{k} > 0 \]

\[ E_{wL} = \frac{\partial e_1^*}{\partial w_L} = -\frac{b^2. u_{wL} (\Omega_L \cdot w_L) \cdot \Omega_L}{k} < 0 \]

\[ E_{\Omega_H} = \frac{\partial e_1^*}{\partial \Omega_H} = \frac{b^2. [u_{\Omega_H} (\Omega_H \cdot w_H) \cdot w_H - u_{\Omega_H} (\Omega_H \cdot w_L) \cdot w_L]}{k} > 0 \]

\[ I = a + b e_2^* \]

\[ I_{wH} = \frac{\partial e_2^*}{\partial w_H} = \frac{b^2. u_{wH} (\Omega_L \cdot w_H) \cdot \Omega_L}{k} > 0 \]

\[ I_{wL} = \frac{\partial e_2^*}{\partial w_L} = -\frac{b^2. u_{wL} (\Omega_L \cdot w_L) \cdot \Omega_L}{k} < 0 \]

\[ I_{\Omega_L} = \frac{\partial e_2^*}{\partial \Omega_L} = \frac{b^2. [u_{\Omega_L} (\Omega_L \cdot w_H) \cdot w_H - u_{\Omega_L} (\Omega_L \cdot w_L) \cdot w_L]}{k} > 0 \]

and

\[ \Gamma_A = F - G + \delta \cdot [E.H + (1 - E).K - I.M - (1 - I).N] \]

\[ \Gamma_E = A \cdot (H - K) \]

\[ \Gamma_I = (1 - A) \cdot (M - N) \]

\[ \Gamma_{AA} = \left\{ P - Q - \frac{(A - a)}{b^2} \right\} + \delta \left[ \left( ES + (1 - E) T - \frac{k}{2} (e_1^*)^2 \right) - \left( IV + (1 - I) W - \frac{k}{2} (e_2^*)^2 \right) \right] \]

\[ \Gamma_{EE} = A \cdot \left( S - T - \frac{(E - a)}{b^2} \right) \]

\[ \Gamma_{II} = (1 - A) \cdot \left( V - W - \frac{(I - a)}{b^2} \right) \]
The utility function of the investor and its respective derivatives to all arguments in each node of the model are given by

\[ F = (r_H - w_H) \implies F_{w_H} = -1 < 0 \]
\[ G = (r_L - w_L) \implies G_{w_L} = -1 < 0 \]
\[ H = \Omega_H \cdot (r_H - w_H) \implies \begin{cases} H_{w_H} = -\Omega_H < 0 \\ H_{\Omega_H} = (r_H - w_H) \begin{cases} > 0, \text{ if } r_H > w_H \\ < 0, \text{ if } r_H < w_H \end{cases} \end{cases} \]
\[ K = \Omega_L \cdot (r_L - w_L) \implies \begin{cases} K_{w_L} = -\Omega_H < 0 \\ K_{\Omega_H} = (r_L - w_L) \begin{cases} > 0, \text{ if } r_L > w_L \\ < 0, \text{ if } r_L < w_L \end{cases} \end{cases} \]
\[ M = \Omega_L \cdot (r_H - w_H) \implies \begin{cases} M_{w_H} = -\Omega_H < 0 \\ M_{\Omega_H} = (r_H - w_H) \begin{cases} > 0, \text{ if } r_H > w_H \\ < 0, \text{ if } r_H < w_H \end{cases} \end{cases} \]
\[ N = \Omega_H \cdot (r_L - w_L) \implies \begin{cases} N_{w_L} = -\Omega_L < 0 \\ N_{\Omega_H} = (r_L - w_L) \begin{cases} > 0, \text{ if } r_L > w_L \\ < 0, \text{ if } r_L < w_L \end{cases} \end{cases} \]

while the utility function of the manager and its respective derivatives to all arguments in each node of the model are given by

\[ P = u(w_H) \implies P_{w_H} = u_{w_H}(w_H) > 0 \]
\[ Q = u(w_L) \implies Q_{w_L} = u_{w_L}(w_L) > 0 \]
\[ S = u(\Omega_H \cdot w_H) \implies \begin{cases} S_{w_H} = u_{w_H}(\Omega_H \cdot w_H) \cdot \Omega_H > 0 \\ S_{\Omega_H} = u_{\Omega_H}(\Omega_H \cdot w_H) \cdot w_H > 0 \end{cases} \]
\[ T = u(\Omega_H \cdot w_L) \implies \begin{cases} T_{w_L} = u_{w_L}(\Omega_H \cdot w_L) \cdot \Omega_H > 0 \\ T_{\Omega_H} = u_{\Omega_H}(\Omega_H \cdot w_L) \cdot w_L > 0 \end{cases} \]
\[ V = u(\Omega_L \cdot w_H) \implies \begin{cases} V_{w_H} = u_{w_H}(\Omega_L \cdot w_H) \cdot \Omega_L > 0 \\ V_{\Omega_L} = u_{\Omega_L}(\Omega_L \cdot w_H) \cdot w_H > 0 \end{cases} \]
\[ W = u(\Omega_L \cdot w_L) \implies \begin{cases} W_{w_L} = u_{w_L}(\Omega_L \cdot w_L) \cdot \Omega_L > 0 \\ W_{\Omega_L} = u_{\Omega_L}(\Omega_L \cdot w_L) \cdot w_L > 0 \end{cases} \]
5.2 Computer code for the binomial model

See Matlab prints at the end of this document.

5.3 Computer code for the binomial model with learning

See Matlab prints at the end of this document.

5.4 The typical compensation - linear schedule with limited liability and high-water mark

In this section we describe the most common asset management contracts found in the market place. This section serves the purpose to describe possible problems arising from the use of simple explicit compensation schemes.

Typical explicit clauses of contracts are linear with fixed coefficients during the life of the relationship and the payoff of the manager depends on the excess return of the fund, \( r_t \). Long-term contracts are unusual. When they do exist, their maturities are defined in terms of the number of days from the withdrawal request to the delivery of the resources back to the investor. This period is rollover everyday after the lockup period. Then, we will assume that such contract do not exist in our economic environment.

Actually, given some features in the explicit compensation structure, the contract is not linear. First, it presents limited liability in the excess return - calculated as the return of the fund, \( R_t \), in excess of the return of a pre-determined benchmark, \( R^0_t \). Second, performance fee is calculated over the high-water mark of the benchmark; that is, performance fee is only paid if the return exceeds the greater of the two benchmarks - the benchmark itself or the highest historical quota value of the fund which is also always indexed by the benchmark as well. Then, the explicit incentive is convex in excess return and the payoff can be written as

\[
w_t(r_t) = \Omega_{t-1} (r_t^{t-1}) \cdot \{\alpha + \beta \cdot \max [r_t; 0]\} \quad (57)
\]

where \( \Omega_{t-1} (r^{t-1}) \) is the total amount of assets under management and \( r^{t-1} = (r^1, ..., r^{t-1}) \) represents the history of cumulative excess return of the fund over the high-water mark.

We call the total amount of asset under management, in period \( t \), of Net Asset Value (NAV) and write it as

\[
\Omega_{t-1} (r^{t-1}) = q_{t-1} (r_t^{t-1}) \cdot p_{t-1} \quad (58)
\]
where $q_{t-1}(r^{t-1})$ is the outstanding number of quotas of the fund and $p_{t-1}(r^{t-1})$ is the marked-to-market quota value of the fund net of taxes and transactions costs. In period $t$, it is given by

$$p_t = p_0 \cdot \prod_{s=1}^{t} (1 + R_s)$$

(59)

The excess return of the fund in period $t$ is given by

$$r_t = \frac{p_t}{\hat{p}_t} - 1$$

(60)

where the denominator is the high-water quota price. This extra feature of the contract is given by

$$\hat{p}_t = (1 + R_t^0) \cdot \max (p_{t-1}; \hat{p}_{t-1})$$

(61)

Therefore, the high-water mark is given by

$$\hat{r}_{t-1} = \max \left[ 0; \left(1 - \frac{p_{t-1}}{\hat{p}_{t-1}}\right) \right]$$

(62)

The manager’s static payoff consists of a fixed, $\alpha$ - the management fee - and an option on the value of the fund due to the existence of limited liability - the performance fee, $\beta$. From a finance theory perspective, this payoff is always greater than zero and it synthesizes an European call option on the fund’s quota mark-to-market price that the portfolio manager holds against the investor$^6$.

The high-water mark, $\hat{r}_{t-1}$, determines the strike price of the option. Because of the high-water mark feature and the growth rate of the benchmark, this option has a variable strike price$^7$. The high-water mark guarantees that the option is almost certainly out-of-the-money since $\hat{p}_{t-1} \geq p_{t-1}$. The option is, at maximum, at-the-money when $\hat{p}_{t-1} = p_{t-1}$, i.e., $\hat{r}_{t-1} = 0$. The distance between $p_{t-1}$ and $\hat{p}_{t-1}$ determines how much the option is out-of-the-money. So, the greater is $\hat{p}_{t-1}$, the higher is $\hat{r}_{t-1}$ and this implies that $r_t = \hat{r}_{t-1} > 0$ is the minimum rate of excess return that the manager need to achieve from his investments decisions in order to start deriving any positive marginal utility from the option.

From Braido and Ferreira (2003), we learn that options may robustly induce risk-taking, regardless of the specific functional form of the utility function. Higher strike prices transform a riskier portfolio selection that is a second-order stochastically dominated cumulative distribution function. See Goetzman, Ingersoll and Ross (2000)

$^6$Even when the benchmark is zero, the high-water mark feature incorporates all the variability of the fund’s history of return.

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distribution of excess return into a lottery that first-order stochastically dominates all other portfolio choices, even if the excess return probability joint distribution is unknown to the manager/investor. It means that the likelihood of the portfolio manager to choose riskier strategies is greater when his compensation includes an option whose strike price is high enough.

From an incentives theory approach, this out-of-the-money option\(^8\) represents a compensation structure in which the manager derives higher marginal benefits of exerting effort and taking risks from high levels of excess return. The manager has incentives to take more risks, if \(\kappa_t(\Omega_{t-1}(r^{t-1}))\) represents a mean preserving spread of the distribution of cumulative excess return. That is, the manager has incentives to make portfolio choices whose joint prior distribution of excess returns has heavier tails.

Nevertheless, the high-water mark feature is designed to protect the investor from paying excessive performance fees. Suppose the manager performs very well during a certain period of time and the value of the fund hits a record value. Now, imagine that the fund has negative performance in some subsequent periods. In this case, all positive performance that follows the poor performance period will only pay performance fee after the record high-water mark is broken again. Nevertheless, due the option-like nature of the compensation schedule, this contract feature ends up creating more incongruities in risk preferences between managers and investors. Benchmarks with high growth rates only enhance this one problem once the high-water mark will also grow at this rate.

Ghatak and Pandey (2000) build a multi-task model in which the choice of effort moves the average of the distribution of excess returns, in a first-order stochastic dominance sense, and that the risk choice is a mean preserving spread of this distribution, in a second-order stochastic dominance sense. Then, the incentive implications of risk-taking choice reduces the optimal power of the static contract, especially in the presence of limited liability. This reduction in the explicit incentive, \(\beta\), objectives to diminish the marginal utility of the manager from high levels of return, inducing him to choose less risky investment alternatives. In their model, the optimal linear contract \((\alpha^*, \beta^*)\) recovers the first-best solution, that is, manager’s actions are equal to the optimal combination of these weakly substitutes tasks in the case they are contractible.

We would expect that, depending on the strike price of the option, the optimal power of

\(^{8}\)And, we may say, increasingly outer-of-the-money if performance is poor or if the growth rate of the benchmark is high.
the contract, $\beta$, would change as the value of fund is closer or outer-of-the-money. Moreover, $(\alpha^*_t, \beta^*_t)$ also should be a function of the history of performance. Rogerson (1985), in a repeated moral hazard model, shows that memory plays a crucial role in determining future incentives if the distribution of today’s return affect current incentives. However, in the asset management industry, we know that $\alpha$ and $\beta$ are fixed at the start of the fund. This fact amplifies perverse incentives on risk choices, forcing the investor to use implicit features of the contract in order to recover an optimal compensation schedule and, hence, optimal effort and risk choices.

As a consequence, the investor has to monitor the performance of each manager and constantly revise the total amount of assets under management allocated at each portfolio manager. This is done by adjusting the flow of funds $f_t$. This flow is endogenous in the model and we build it as a function of the history of cumulative excess return, $f_t(r^t)$. As investors decide to let cash resources flow in, $f_t > 0$, or out, $f_t < 0$, of the fund, quotas are respectively created, $\Delta q_t > 0$, or redeemed, $\Delta q_t < 0$, at current quota marked-to-market prices, $p_t$.

Then, the total number of quotas in period $t-1$ is given by

$$q_{t-1}(r^{t-1}) = \sum_{s=1}^{t-1} \frac{f_s(r^s)}{p_s}$$

(63)

We obtain the flow of funds in each period $t$, $f_t(r^t)$, as a function of the cumulative return of the fund and the variation in the number of quotas

$$f_t(r^t) = p_t \cdot (q_t(r^t) - q_{t-1}(r^{t-1}))$$

(64)

Normalizing $p_0 = 1$ and after substituting (57) in (64), we obtain

$$f_t(r^t) = \prod_{s=1}^{t} (1 + R_s) \cdot (q_t(r^t) - q_{t-1}(r^{t-1}))$$

(65)

The fixed fee in the contract, the management fee $\alpha$, is a factor expressed in annual percentage terms of the net asset value, $\Omega_{t-1}(r^{t-1})$, being accrued in a pro rata temporis form. It is related to the fixed and some variable costs of managing the fund, including the marginal cost of using the manager’s time9 and/or ability. Once $f_t(r^t)$ is a function of the

9If we consider leisure in the model. However, we do not do so here
history of cumulative excess returns, even in the absence of any performance fee, \( \beta = 0 \), the manager would still have incentives to make effort and risky choices, trying to influence the perception of the market about his level of ability. Besides, \( \Omega_{t-1} (r^{t-1}) \) also multiplies the option-like component of the manager’s wage, affecting more intensively the effort and risk choices of the manager in each period. Then, the manager has great incentives to attract a high volume of assets under management.

Indeed, we argue that the flow of funds is the most important incentive feature of the compensation schedule. This dynamic implicit incentive depends on the history of cumulative excess returns, \( f_t (r^t) \), and we call it flow concern.

This function determine the optimal choices of effort and risk as well as the optimal incentives, taking into consideration reputation effects that arise from the observed history of excess returns. Fama (1980) argues that this dynamic concern may recover first-best solutions removing moral hazard issues in risk-taking. Holmström (1982) demonstrated that risk-aversion and discounting play an important role in confirming Fama’s previsions.

If we substitute (59) and (61) in (60), we can rewrite the excess return of the fund in period \( t \) as

\[
\begin{align*}
    r_t &= \frac{\prod_{s=1}^{t} (1 + R_s)}{(1 + R_0^0) \cdot \max \left( \prod_{s=1}^{t-1} (1 + R_s) ; \hat{p}_{t-1} \right)} - 1
\end{align*}
\]

(66)

Observe that \( \hat{p}_t \) is calculated recursively based on the history of cumulative excess return, \( r^t \).

We may write the total payoff of the manager in each period \( t \) as

\[
\begin{align*}
    u_M (w_t (r_t)) &= \Omega_{t-1} (r^{t-1}) \cdot \left( \alpha + \beta \cdot \max \left[ \frac{p_0 \cdot \prod_{s=1}^{t} (1 + R_s)}{(1 + R_0^0) \cdot \max (p_{t-1} ; \hat{p}_{t-1})} - 1; 0 \right] \right)
\end{align*}
\]

Since the main objective of this paper is to address the relative importance of implicit incentives compared to explicit incentives, we assumed a general form of explicit incentive, \( \omega_H \) and \( \omega_L \), in the model developed below.
References


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