Wealth Composition, 
Endogenous Fertility and 
the Dynamics of Income Inequality

Fernando A. Veloso*

Department of Economics
Ibmec

November 14, 2002

Abstract

This paper analyzes how differences in the composition of wealth between human and physical capital among families affect fertility choices. These in turn influence the dynamics of wealth and income inequality across generations through a tradeoff between quantity and quality of children. Wealth composition affects fertility because physical capital has only a wealth effect on number of children, whereas human capital increases the time cost of child-rearing in addition to the wealth effect. I construct a model combining endogenous fertility with borrowing constraints in human capital investments, in which wealth composition is determined endogenously. The model is calibrated to the PNAD, a Brazilian household survey, and the main findings of the paper can be summarized as follows. First, the model implies that the cross-section relationship between fertility and wealth typically displays a U-shaped pattern, reflecting differences in wealth composition between poor and rich families. Also, the quantity-quality tradeoff implies a concave cross-section relationship between investments per child and wealth. Second, as the economy develops and families overcome their borrowing constraints, the negative effect of wealth on fertility becomes smaller, and persistence of inequality declines accordingly. The empirical evidence presented in this paper is consistent with both implications.

* I am grateful to Gary Becker, Nancy Stokey, and especially Robert Lucas, for invaluable advice, discussions and suggestions. I thank Rubens Cysne, Matthias Doepke, participants at the 8th World Congress of the Econometric Society, the NBER Summer Institute Economic Fluctuations and Growth Working Group on Income Distribution and Macroeconomics, the XXIII Meeting of the Brazilian Econometric Society, the Fifth Annual Meeting of LACEA and seminar participants at the University of Chicago Money and Banking workshop and PUC-RJ for helpful comments. I also thank Casey Mulligan for providing me with fertility data from the Survey of Consumer Finances, and Rosângela Almeida, Vandeli Guerra, David Lam, Rosane Mendonça and Ricardo Paes de Barros for discussions and technical support related to the PNAD data. I gratefully acknowledge financial support from CAPES and the Bradley Foundation. All errors are mine. E-mail: fveloso@ibmecrj.br
1. Introduction

This paper analyzes the implications of the interaction between wealth composition, fertility and investments in children for the dynamic behavior of the income distribution among families. Wealth composition affects fertility because physical capital has only a wealth effect on the optimal number of children, whereas human capital increases the time cost of child-rearing in addition to the wealth effect. Fertility choices in turn influence the dynamics of income inequality across generations due to a tradeoff between quantity and quality of children.

Several studies suggest that human and physical capital have different qualitative effects on fertility and investments in children. Becker and Barro (1988), Benhabib and Nishimura (1993) and Alvarez (1999) analyze fertility models in which families are heterogeneous in their physical capital stocks. In these models, rich families dilute their wealth by having more children than poor families, and fertility behavior leads to long-run equality among families.


The main contribution of this paper is to analyze a model of fertility and investment decisions in which the wealth and income composition are determined endogenously from the allocation of investments in children between human and physical capital. The endogeneity of wealth composition allows for a characterization of the conditions under which fertility is an equalizing force, and when it creates inequality.

effects of the composition of wealth on income distribution, but they do not model fertility
decisions. Lucas (2002) presents a representative agent model of endogenous fertility with
human and physical capital, but its implications for income inequality are not analyzed.

In order to analyze the interaction between wealth composition, fertility and the dynamics
of inequality, I combine a borrowing-constraints model of human and physical capital
investments with a model of fertility behavior.

I assume that families may invest in children's human and physical capital, but cannot
borrow to finance their human capital investments. In any period, families are divided into two
groups. Richer families typically are unconstrained, and make efficient human capital
investments. Additional investments are made in the form of physical capital. Since the children
of unconstrained families are also unconstrained, this group tends to have the same level of
(efficient) human capital, and different physical capital stocks.

Poorer families are typically constrained, and make investments in children only in the
form of human capital. If the children of poor families are also constrained, they will have the
same stock of physical capital, equal to zero, and will be heterogeneous in their human capital
stocks.

Since the source of wealth variations differs for constrained and unconstrained families,
their fertility behavior differs as well. In particular, the negative effect of wealth on fertility is
larger for constrained than for unconstrained families, and the cross-section fertility-wealth
profile typically displays a U-shaped pattern at any point in time. Because of the tradeoff
between quantity and quality of children, this implies a concave cross-section relationship
between investments per child and parental wealth.

The model is calibrated to data from the 1996 Pesquisa Nacional de Amostra Domiciliar
(PNAD), a Brazilian household survey. The PNAD is a series of annual representative cross-
sections of the Brazilian population collected by the Instituto Brasileiro de Geografia e
Estatistica (IBGE). The choice of this data set is motivated by the fact that developing countries
in general, and Brazil in particular, are typically characterized by significant cross-section variation in fertility across education and income classes.\footnote{Kremer and Chen (2000) present evidence that developing countries typically display larger differences in fertility among educated and uneducated families. Lam (1986) provides evidence of a strong negative correlation between fertility and family income in Brazil.}

The numerical simulations suggest three qualitatively different long-run patterns of wealth and income distribution. First, if the degree of intergenerational altruism, the time cost of children and the productivity of human capital investments are high relative to the fertility preference parameter, all families eventually overcome their borrowing constraints and there is long-run income equality. The negative effect of wealth on fertility declines across generations, which reduces persistence of inequality.

Another pattern generated by the model is one in which the degree of altruism, the time cost of children and the productivity of human capital investments are not high enough relative to preferences for numbers of children to allow families to overcome their borrowing constraints, but are large enough to generate convergence among borrowing-constrained dynasties.

The third possible outcome is a situation in which the degree of altruism, the time cost of children and the productivity of human capital investments are too low relative to preferences for numbers of children, so that numbers of children are sufficiently high to generate long-run inequality.

I provide empirical evidence on the main implications of the model based on an empirical analysis of the PNAD and results from other studies. The evidence shows that the fertility-income cross-section profile typically displays a U-shaped pattern, consistent with the model. The interaction between quantity and quality of children tends in turn to generate a concave investment-income cross-section profile for the PNAD data, even though the evidence is mixed for other studies.

I also explore the implications of the model regarding the dynamic behavior of the degree of intergenerational persistence of inequality. The simulations suggest that, given the calibrated parameters, the negative effect of wealth on fertility tends to become smaller across generations, reflecting a weaker cross-section association between wealth and labor income over time.
Moreover, the quantity-quality tradeoff will tend to reduce the degree of persistence in inequality over time.

In order to verify this implication of the model, I compare the degrees of persistence in wages and the coefficients obtained from a regression of fertility on wages for Brazil and the United States. The results suggest that a larger negative effect of wages on fertility increases the persistence of inequality in Brazil.

This paper is organized as follows. Section 2 presents the model. Section 3 calibrates the model to the 1996 PNAD, and illustrates the possible outcomes with the aid of simulations. Section 4 presents empirical evidence supporting the implications of the model. Section 5 concludes and provides directions for future research.

2. The Model

Consider an economy inhabited by $M$ dynasties, indexed by $i = 1, \ldots, M$. Each dynasty is defined by a parent and her descendants. Each person lives for two periods: childhood and adulthood. Parental variables are indexed by $t$, while children variables are indexed by $t + 1$. Families have two sources of wealth: human capital, $h$, and physical capital, $k$.

First-generation parents are heterogeneous in their stocks of human and physical capital, \( \{h_0^i, k_0^i \}_{i=1,\ldots,M} \). Parents are assumed to have identical preferences over their consumption, $c_t$, number of children, $n_t$, and utility per child, $u_{t+1}$, described by

\[
  u_t = \alpha \log c_t + \gamma \log n_t + \beta u_{t+1}
\]

where $\alpha > 0$, $\gamma > 0$ and $0 < \beta < 1$.

---

2 I will drop the dynasty superscripts unless in cases in which they are necessary for the understanding of the model.
Parents are endowed with one unit of time, and spend \( \lambda \) units of time per child and \( \phi \) units of the consumption good rearing their children. Hence, parents work \( 1 - \lambda n \) units of time, earning a wage rate \( w \), described by the following functional specification:  

\[
w = Ah^{1-\epsilon}
\]

where \( A > 0 \) and \( 0 < \epsilon < 1 \).

I will assume that the rental price of capital is determined exogenously, at the constant level \( r \). Parents choose investments in each child's human capital, \( h_{t+1} \), and physical capital, \( k_{t+1} \), but they cannot borrow against their children's earnings in order to finance human capital investments.

The recursive formulation of the decision problem of a typical parent can be described as

\[
v(h, k) = \max_{c, n, h, k} \left\{ \alpha \log c + \gamma \log n + \beta v(h', k') \right\}
\]

s.t.

\[
k' \geq 0
\]

\[
0 \leq n \leq \frac{1}{\lambda}
\]

\[
c + n(\phi + h + k^{'}) = (1 - \lambda n) Ah^{1-\epsilon} + (1 + r) k
\]

where \( h' \) and \( k' \) denote human and physical capital per child, respectively.

The budget constraint captures the interaction between the quantity and quality of children through the term \( n_i (h_{t+1} + k_{t+1}) \). Borrowing constraints are captured by the restriction \( k' \geq 0 \).

It is convenient to rewrite (1) in terms of full-time wealth, \( y \equiv Ah^{1-\epsilon} + (1 + r) k \), and full-time labor income, \( y^L \equiv Ah^{1-\epsilon} \):

---

3 See Loury (1981), Becker and Tomes (1986) and Mulligan and Song Han (2000) for similar specifications of the wage function.
\[ v(h,k) = \max_{c,n,h,k} \left\{ \alpha \log c + \gamma \log n + \beta v(h',k') \right\} \]

s.t.
\[ k' \geq 0 \]
\[ 0 \leq n \leq \frac{1}{\lambda} \]
\[ c + n(\phi + h' + k' + \lambda Ah^{1-\varepsilon}) = Ah^{1-\varepsilon} + (1+r)k \]

The formulation in (2) makes explicit the fact that human capital affects the price of children through the term \( \lambda Ah^{1-\varepsilon} \), while physical capital has only a wealth effect on number of children. This asymmetry between human and physical capital, combined with the interaction between number of children and investments in the budget constraint, is crucial in generating a link between wealth composition and investments per child.

The Euler equation for unconstrained families, that is, families making positive physical capital investments per child, \( k_{t+1} > 0 \), is given by

\[ \frac{c_{t+1}}{c_t} = \frac{\beta (1+r)}{n_t} \frac{1}{n_t} \]  

(3)

The Euler equation for constrained families, that is, families for which optimal physical capital investments in the absence of constraints satisfy \( k_{t+1} < 0 \), is given by

\[ \frac{c_{t+1}}{c_t} = \frac{\beta A(1-\varepsilon)(1-\lambda n_{t+1})}{n_t} h_{t+1}^{1-\varepsilon} \]  

(4)

Notice that the rate of return on investments for both constrained and unconstrained families depend on the optimal fertility rate, \( n_t \). Unconstrained families equalize the rate of return between human and physical capital investments, which implies:

\[ h_{t+1} = \left[ \frac{A(1-\varepsilon)(1-\lambda n_{t+1})}{1+r} \right]^{\frac{1}{\varepsilon}} \]  

(5)
Since the fertility rate of the next generation affects the amount of time they allocate to the marketplace, optimal human capital investments, \( h_{t+1} \), depend on \( n_{t+1} \), as shown in (5). This dependence of current investments on the fertility behavior of future generations makes it very difficult to characterize the model analytically, and makes it necessary to compute numerical simulations to understand the behavior of the model.\(^4\)

3. Model Simulations

3.1. Calibration

The baseline parameters will be calibrated to data from the 1996 Pesquisa Nacional de Amostra Domiciliar (PNAD), a Brazilian household survey. The PNAD is a series of representative cross-sections of the Brazilian population which have been collected annually (except for 1980, 1990, 1991 and 2000) since 1973 by the Instituto Brasileiro de Geografia e Estatistica (IBGE).\(^5\)

I assume that a generation takes 25 years. The annual rate of return on physical capital is chosen to match the average rate of return on 30-year U.S. government bonds, which is about 5.7%. This implies that the net rate of return on physical capital over a generation, \( r \), is chosen to be \( r = 0.03 \). These numbers were taken from Mulligan and Song Han (2000).

I require that \( \beta (1+r) = 1 \), under the assumption that \( r \) corresponds to the steady state rate of return for the U.S.\(^6\) Hence \( \beta = 0.25 \), which corresponds to an annual altruism rate of 0.946.

\(^4\) Kremer and Chen (2000) were faced with the same difficulty, and they dealt with it by assuming that market wages depend on the size of the labor force, rather than the amount of man-hours employed in production.

\(^5\) The PNAD is close to a nationally representative sample, though it is not fully representative of rural areas, especially in the remote frontier regions.

\(^6\) Laitner (1992) and Navarro-Zermeno (1993) suggest that consumption does not grow across generations among non-borrowing-constrained families. I thank Casey Mulligan for this observation.
The elasticity of wages with respect to human capital is $1-\varepsilon$. If we take a log-linear approximation to the first-order condition for human capital investments in children, we obtain the following equation:

$$\log h_{t+1} = \text{cons} + \left[ \left( \frac{\lambda y^T}{\phi + \lambda y^T} \right) (1-\varepsilon) \right] \log h_t$$

(6)

where $\frac{\lambda y^T}{\phi + \lambda y^T}$ is the sample average of the share of time costs in total fertility costs and \text{cons} denotes a constant.

I was not able to compute this share from the Brazilian data, so I used the share of time costs for American women with only elementary education, obtained from Espenshade (1977). This implies a share of fifty-percent. Since the average woman in the sample used to estimate (6) had on average four years of schooling, this may provide a reasonable approximation for Brazil. I then estimated (6) using years of schooling of the mother and the oldest adult child, and obtained a coefficient on $\log h_t$ equal to 0.4. This implies that $1-\varepsilon = 0.8$, so I set the baseline value of $\varepsilon$ to 0.2.$^7$

In order to provide an estimate of the fraction of time allocated to child-rearing, $\lambda$, I use the restriction imposed by the model on the maximum fertility rate:

$$n_{\text{max}} = \frac{1}{\lambda}$$

where $n_{\text{max}}$ is the maximum number of children and the total time endowment is normalized to 1. This restriction has to be satisfied in order for the head of household to supply positive labor hours.

I computed $n_{\text{max}}$ by choosing the highest number of children currently alive for married women aged 40-60 currently employed, and then used the restriction above to estimate $\lambda$. I

$^7$ Mulligan and Song Han (2000) estimate a value of $\varepsilon = 0.4$ for the U.S.
considered only women because there is considerable evidence that women allocate a higher fraction of their time to child-rearing than men.\textsuperscript{8} I used number of children currently alive as a measure of fertility, rather than number of children-ever-born, to reduce the effect of child mortality on the empirical measure of fertility. The chosen age bracket for married women was chosen in order to obtain a measure of completed fertility. For this sample, $n_{\text{max}}$ was found to equal 7, which implies $\lambda = 0.14$.

I order to estimate the parameter $A$ in the wage function, I used the estimated values of $\varepsilon$, $\lambda$, and $r$ and the expression for the optimal human capital per child given by (5). To compute the optimal human capital investment per child, I computed the average schooling among children aged 14-21 still living with their parents. Moreover, this measure was calculated for parents in the highest income decile, because they are more likely to be making optimal human capital investments in children. The restriction that children live with their parents is necessary for one to be able to link data on parents and their children. I also assumed that these families are in steady state, so that $n_{t+1} = n_t$. These calculations lead to an estimate of $A$ equal to 5.

In order to estimate the preference parameters $\alpha$ and $\gamma$, I assume that $\alpha + \gamma = 1$, and use the fact that $\alpha$ and $\gamma$ are the shares of parental consumption and total expenditures in children on family income, respectively. These shares were obtained from the 1987/88 Pesquisa de Orçamento Familiar (POF), a Brazilian household survey that provides detailed data on family expenditures for 9 metropolitan regions. The POF data set implies a share of total expenditures in children of approximately one third, which implies that $\alpha = 0.67$ and $\gamma = 0.33$.\textsuperscript{9}

I estimated the goods cost of fertility per child, $\phi$, by using $\frac{\lambda y^L}{\phi + \lambda y^L} = 0.5$, the estimated value of $\lambda$ and the average value of labor income (computed in model units, see below), which yields $\phi = 0.5$.\textsuperscript{10}

I also chose the range and the initial distribution of the state variables, human and physical capital, in order to match the corresponding values for the PNAD 96. In particular, the

\begin{itemize}
\item \textsuperscript{8} See Leibowitz (1974).
\item \textsuperscript{9} Espenshade (1984) estimates that expenditures on children account for between 30 and 50 percent of total family expenditures in the U.S.
\item \textsuperscript{10} Tamura (1988) uses $\phi = 0.25$.
\end{itemize}
initial distribution consists of 10 pairs of human and physical capital combinations, chosen to fit the distribution of schooling and financial wealth across income deciles among married couples with women aged 40-60 in 1996.

These human-physical capital initial pairs were constructed as follows. I used the wife's average schooling by deciles as a measure of the distribution of human capital among families.\textsuperscript{11} I then used the wage function of the model, calibrated as described above, to compute the lifetime labor income of the household head. From the PNAD 96, I computed the average ratio between capital income (interest, dividends and rent) and labor income for each decile. Finally, I applied these ratios to the constructed measures of lifetime labor income, and used the calibrated interest rate, $r$, to construct measures of the capital stock for each decile. The resulting pairs of human and physical capital were used to compute all time series simulated from the model.

Table 1 presents the baseline parameter values. They will be used in all simulations, unless noted otherwise.

3.2. Simulations

In this subsection, I will simulate the model presented in section 2 for three different parameterizations, intended to illustrate the different long-run patterns that may arise. In section 4, I will estimate the empirical counterparts to the simulations and will assess to what extent the model is able to match the Brazilian data.

One possibility is that the degree of altruism, the time cost of children and the productivity of human capital investments are not high enough relative to preferences for numbers of children, but are large enough to generate convergence among borrowing-constrained families. This possibility will be analyzed in the first simulation.

\textsuperscript{11} Since husbands' and wives' education are highly correlated, the results are similar when we use the former as a measure of the household's human capital. Conceptually, it seems preferable to use mother's education since fertility studies suggest that it is a better proxy for the value of time allocated to children. See Willis (1973).
A second possibility corresponds to the case in which all families eventually overcome their borrowing constraints and there is long-run wealth and income equality among families. This possibility will be analyzed in the second simulation.

A third possibility is that the degree of altruism, the time cost of children and the productivity of human capital investments are too low relative to preferences for numbers of children, so that the number of children is sufficiently high to generate long-run inequality. This possibility will be considered in the third simulation.

a) Baseline case

Figures 1 and 2 present simulation results for the baseline parameterization. Figure 1 displays the policy function for fertility as a function of the state variables, human and physical capital. The policy function for fertility shows that fertility decreases with human capital, and increases with physical capital. This asymmetry of fertility behavior with respect to the source of wealth is particularly pronounced at low human and physical capital levels.

The policy function for physical capital investments for this parameter configuration is trivial, since all families are borrowing-constrained and thus do not leave any bequests in the form of physical capital. This is consistent with the 1996 PNAD data for Brazil, which shows that for a large majority of the population the share of asset income in total income is very small.

The first step in the analysis is to study how the cross-section relationship between fertility and parental wealth affects the cross-section investment profile through the quantity-quality tradeoff, and how both cross-section relationships are affected by the cross-section correlation between parental wealth and labor income. All simulated cross-section relationships were obtained by simulating (2) using the baseline parameter values (unless noted otherwise) and the initial distribution of pairs of human and physical capital, \( \{ h_0^i, k_0^i, i = 1, ..., N \} \), constructed in the way described in the previous subsection.

The three plots in Figure 2 illustrate the relationships between wealth composition, fertility behavior, and investments per child at \( t = 0 \). Time subscripts denote the parents'
The labor income-share, \( \frac{A_{h_t^{1-e}}}{A_{h_t^{1-e}} + (1 + r) k_t} \), increases slightly at low-wealth levels, \( A_{h_t^{1-e}} + (1 + r) k_t \), and declines for higher levels of wealth. Fertility, \( n_t \), declines at low levels of wealth, then becomes constant, and increases slightly at the highest end of the wealth distribution.

The reduction in fertility among poor and middle-income families tends to increase persistence of inequality, as captured by the slope of the graph that relates adult child's wealth, \( A_{h_{t+1}}^{1-e} + (1 + r) k_{t+1} \), to parental wealth, \( A_{h_t}^{1-e} + (1 + r) k_t \). Further increases in parental wealth are associated with constant and rising fertility, which reduces persistence of inequality among richer families.

Table 2 displays the dynamic behavior of the fertility-wealth (labor income) relationship. These estimates, denoted by \( \theta_t \) and \( \theta_t^L \), respectively, are ordinary least-squares coefficients obtained by regressing fertility, \( n_t \), on parental log wealth (labor income), \( \log y_t \left( \log y_t^L \right) \), where the observations were generated by simulating time series of wealth and labor income for all families.

Table 2 shows that the negative effect of wealth (labor income) on fertility in the cross-section of families declines across generations. At \( t = 4 \), all families have the same fertility rate (\( \theta_4 = 0 \)), and in the long-run the fertility regression coefficient cannot be calculated, since there is long-run equality (\( \theta_\infty = . \)).

Table 3 displays the dynamic behavior of the degrees of intergenerational persistence of inequality of wealth and labor income, denoted \( \rho_t \) and \( \rho_t^L \), respectively. These estimates are ordinary least-squares coefficients obtained by regressing total adult child's log wealth (labor income), \( \log y_{t+1} \left( \log y_{t+1}^L \right) \), on parental log wealth (labor income), \( \log y_t \left( \log y_t^L \right) \), where the observations were generated by simulating time series of wealth and labor income for all families.\(^\text{12}\)

\(^{12}\) Mulligan (1997) analyzes how the OLS coefficient obtained from a regression of adult child's log income on parental log income may be interpreted as capturing the degree of intergenerational persistence of inequality.
Three features of Table 3 deserve comment. First, both degrees of persistence of inequality are always less than one, which implies convergence. Second, both persistence coefficients decline monotonically over time. Third, the decline in persistence of inequality is very slow. In fact, as can be observed from Table 3, only at \( t = 5 \) persistence is zero for both wealth and labor income, implying convergence after six generations.

The baseline parameterization leads to policy functions and time series that resemble the equilibrium family behavior in Becker, Murphy and Tamura (1990). Families derive all their income from labor, and fertility is negatively related to income, at least at low-income levels. The dynamic behavior of income distribution for the baseline parameterization differs from the one implied by Becker, Murphy and Tamura (1990), however, in two important aspects. First, the human capital technology in the model presented in this paper is concave. Second, because of the concavity assumption, the model tends to generate long-run equality in labor shares across families, which in turn tends to reduce the negative correlation between fertility and parental wealth, which reinforces the underlying tendency for long-run equality.

**b) Increase in time spent in child-rearing** \((\lambda = 0.4)\)

Figures 3 and 4 present simulation results for the case in which \( \lambda = 0.4 \) (as opposed to \( \lambda = 0.14 \) in the baseline case). The parameter \( \lambda \) is the fraction of time devoted to child-rearing. An increase in \( \lambda \) may be interpreted as a reduction in the productivity of child-rearing or, conversely, as an increase in the cost of fertility. All other parameters and the initial distribution of state variables remain the same.

Figure 3 displays the policy functions for fertility and physical capital investments, respectively, as functions of the state variables, human and physical capital. As observed in the previous example, for any given level of physical capital, fertility declines when human capital increases. Also, for any given level of human capital, fertility increases with physical capital.

One interesting feature of Figure 3 is that, as opposed to the baseline case, a significant fraction of the population is not borrowing-constrained, as can be observed from the fact that
many families leave positive bequests to their children in the form of physical capital. This result illustrates the fact that the extent to which borrowing constraints are binding may be significantly affected by fertility decisions, since the only difference between the policy functions displayed in Figures 1 (baseline case) and 3 is that the latter are computed for a higher $\lambda$.

The three plots in Figure 4 illustrate the relationship between wealth composition, fertility behavior, and investments per child at $t = 0$. The cross-section relationship between fertility and parental wealth displays a U-shaped pattern, which mirrors the inverse U-shaped relationship between labor share and parental wealth. The positive relationship between number of children and parental wealth at higher levels of wealth contributes significantly for the very low degree of persistence in wealth inequality among rich families.

Table 4 shows that the effect of wealth (labor income) on fertility in the cross-section of families becomes positive at $t = 1$. At $t = 4$, all families have the same fertility rate ($\theta_4 = 0$), and in the long-run the fertility regression coefficient cannot be calculated, since there is long-run equality ($\theta_\infty = \ldots$).

Table 5 displays the dynamic behavior of the degrees of intergenerational persistence of inequality of wealth and labor income. From $t = 5$ on, there is wealth and income equality among families. The degree of persistence of labor income declines monotonically, as families overcome their borrowing constraints across generations. At $t = 2$ all families are unconstrained, and this is reflected in a persistence coefficient for labor income equal to zero. There is still some persistence in wealth, however, as parents make investments in children in the form of physical capital. At $t = 3$, all families have the same efficient human capital levels, and this leads to equality of total investments in children, as in Becker and Barro (1988).

The case in which $\lambda = 0.4$ leads to a dynamic behavior that combines elements from Becker and Barro (1988) and Becker, Murphy and Tamura (1990). Initially, most families derive all their income from labor, and fertility is negatively related to income at low-income levels. At high-income levels, fertility is positively related to income, as these families derive most of their

---

13 See Becker and Tomes (1976) for a theoretical analysis that also generates a U-shaped fertility-income cross-section profile.
income from physical capital. As the economy develops, constrained families eventually become unconstrained, and the relationship between labor share and wealth becomes negative. Because of the change in the cross-section wealth composition, the relationship between fertility and wealth becomes positive, which leads to equality in investments across families through the quantity-quality tradeoff.

c) Reduction in the rate of return to human capital ($\varepsilon = 0.5$).

Figures 5 and 6 present simulation results for the case in which $\varepsilon = 0.5$ (as opposed to $\varepsilon = 0.2$ in the baseline case). The parameter $\varepsilon$ affects the rate of return of human capital investments. It can be interpreted as a technological parameter, or as capturing the effect of institutions and government policy on the efficiency of investments in education. All other parameters and the initial distribution of state variables remain the same.

Figure 5 displays the policy function for fertility. For any given level of physical capital, fertility declines when human capital increases. For any given level of human capital, fertility increases with physical capital. This asymmetry of fertility behavior with respect to the source of wealth is qualitatively similar to the one observed for the higher productivity case (Figure 1), but is considerably sharper.

The policy function for physical capital investments for this parameter configuration is trivial, since all families are borrowing-constrained and thus do not leave any bequests in the form of physical capital.

The three plots in Figure 6 illustrate the relationship between wealth composition, fertility behavior, and investments per child at $t = 0$. The qualitative patterns are similar to the ones observed for the higher rate of return case (Figure 2). The main difference is that total investments in children are smaller when $\varepsilon = 0.5$. It should also be noted that, despite the lower productivity, fertility levels at $t = 0$ are higher than the ones observed for $\varepsilon = 0.2$.

---

14 Hall and Jones (1999) interpret productivity parameters as functions of the economic infrastructure, including institutions and government policy.
Table 6 shows that the negative effect of wealth (labor income) on fertility persists for several generations. If we compare Table 6 to Table 2, we can observe that the negative effect of wealth on fertility is larger in every period when the rate of return is smaller.

Table 7 displays the dynamic behavior of the degrees of intergenerational persistence of inequality of wealth and labor income. The dynamic pattern of the coefficients is considerably different from the one displayed when the rate of return of human capital investments is higher (Table 2). When the rate of return is low, both degrees of persistence increase over time, and from $t = 4$ on the relative distance in wealth and labor income among dynasties remains constant (degree of persistence equals 1). If we compare Table 7 to Table 2, we can observe that persistence of inequality is higher in every period when the rate of return is smaller, which is consistent with the fertility pattern displayed in Table 6.

This exercise suggests that persistence of inequality is affected not only by the differences in fertility rates across income classes, but also by fertility levels. Long-run inequality in this example thus arises because of endogenous fertility decisions and, particularly, by the fact that poor families tend to have a number of children excessively high relative to the level of technology.

4. Empirical Evidence

4.1. Evidence on U-shaped fertility-income cross-section profile.

Both the theoretical analysis and the simulations generate a few interesting implications, which I will assess in this section using data from the 1996 PNAD and by providing evidence collected by other researchers. One implication of the model is that the cross-section fertility-income profile tends to produce a U-shaped pattern, reflecting differences in wealth composition between constrained and unconstrained families.
The interaction between quantity and quality of children tends in turn to generate a concave quality-income cross-section profile, where quality is captured by either schooling or income of the adult child.

Table 8 presents ordinary least-squares (OLS) regressions of fertility, adult child's schooling and adult child's log income on parental log income and log income squared, using data from the 1996 PNAD.\textsuperscript{15} Parental and adult child's family income are used as proxies for parental and child's wealth.\textsuperscript{16} I also tried to fit polynomials of higher order to the data, but only the coefficients on father's log income and log income squared were found to be statistically significant.\textsuperscript{17}

Table 8 shows that the coefficient on log parental income in the fertility regression is negative, while the coefficient on log income squared is positive. All coefficients are significant at the one-percent level. The estimated fertility-income profile thus displays a U-shaped pattern, consistent with the model.

Also, in the quality regressions (adult child's schooling and income), the coefficient on log parental income is positive, while the coefficient on log income squared is negative. Thus, the adult child's schooling and income cross-section profiles display a concave pattern, which is also consistent with the model.

Figure 7 uses the regression coefficients displayed in Table 8 to plot the estimated relationship between fertility and parental income, computed for sample means of the other explanatory variables. The range of parental income in Figure 7 is the same as the one used in the simulations, to make the estimated and simulated plots comparable.

The cross-section relationship between fertility and parental income simulated by the model for the baseline parameterization (Figure 2) has a pattern similar to the one estimated from

\textsuperscript{15} Fertility is defined as the number of children currently alive to married women aged 40-60. Only families with nonzero fertility are included in the regressions, since childless adults cannot affect intergenerational mobility. See Mulligan (1997) for a discussion of differences in the fertility-income cross-section relationship when childless families are included.

\textsuperscript{16} I measure family income as the average family income for men who are household heads and who work 40 hours per week on average. Different income averages are calculated for each possible combination of education category, state of residence and whether the individual lives in a urban or rural area, and assigned to all men in each category.

\textsuperscript{17} All regressions use sample weights provided by IBGE to produce a representative sample of individuals for the Brazilian population.
the 1996 PNAD data based on the same income range (Figure 7). The main difference is that the simulated fertility-income profile underestimates the fertility differential found in the 1996 PNAD.

One important reason for this discrepancy between the simulated and actual fertility differential may be the omission of controls for child mortality in the regressions displayed in Table 8. Richer families typically face lower child mortality, which tends to reduce the number of children necessary to achieve a desired fertility rate, and increase the fertility differential between rich and poor families.\footnote{See Meltzer (1992) for a discussion of the effects of child mortality on fertility.}

Figure 7 also presents analogous results for the cross-section relationship between adult child's income and parental income, which may be compared to the simulated plot in Figure 2. As in the model, the empirical investment cross-section profile displays a concave pattern. However, since the model underestimates the fertility differential in the data, it also underestimates the investment differential between poor and rich families arising from the quantity-quality tradeoff.

In addition to the evidence for Brazil provided in this paper, there is some evidence of a U-shaped fertility-income profile for other countries. Mulligan (1997) presents evidence for the United States, using data from the 1990 U.S. Census and the 1989 Survey of Consumer Finances.

Figure 8 displays the fertility-income cross-section relationship estimated in Mulligan (1997) from the 1989 Survey of Consumer Finances. Mulligan uses number of children of household heads between the ages of forty-one and eighty-four as his fertility measure. Both plots displayed in Figure 8 were computed by fitting a fifth-order polynomial in log annual family income to the number of children. The top figure excludes childless families, as is preferable in a study of persistence of inequality, and corresponds to the Brazilian fertility-income relationship presented in Figure 7. The bottom figure includes childless families.
Fertility declines with income for family incomes up to US$ 300,000, and rises with income for incomes between US$ 300,000 and US$ 3,000,000. Fertility declines with income after US$ 3,000,000, but this is based on only thirty-three observations.


The evidence on a concave relationship between adult child's income and father's income, however, is mixed. In addition to the evidence provided in this paper, Mulligan (1993) also found evidence of an inverse U-shaped relationship, but the regression coefficient on the quadratic term is not significant. Both Behrman and Taubman (1990) and Solon (1992) found evidence in the opposite direction, indicating a U-shaped relationship between adult child's and parental income, but the regression coefficients also tend to be insignificant.

The mixed evidence on the shape of the cross-section investment-income profile may be attributed to at least two reasons. First, with the exception of this paper, all studies cited above use small samples, which are not particularly suitable for a study of income nonlinearities. Second, from a theoretical standpoint the implication that the investment-income profile has a concave pattern is less robust than the analogous implication for the fertility profile, because investment is affected by other variables in addition to fertility.

The empirical evidence thus strongly supports the implications of the model regarding the shape of the cross-section relationship between fertility and parental income, but is not conclusive with respect the shape of the investment-income cross-section profile.

4.2. Evidence on the dynamic relationship between fertility and persistence of inequality.

Another implication of the model is related to the dynamic behavior of the fertility-wealth relationship, and its effect on the degree of intergenerational persistence of inequality. The simulations suggest that, given the calibrated parameters, the negative effect of wealth on
fertility tends to become smaller as the country develops, reflecting a weaker cross-section association between wealth and labor income over time. Moreover, the quantity-quality tradeoff will tend to reduce the degree of persistence in inequality as the country develops.

In order to verify this implication of the model, I compare the degrees of persistence in full-time wages and the coefficients obtained from a regression of fertility on wages for the 1976 PNAD and the Panel Study of Income Dynamics (PSID), where the U.S. regressions are taken from Mulligan (1993). Mulligan uses information on father's wage rate and fertility taken from the period 1967-72, which makes the U.S. and Brazilian samples approximately contemporary.

Table 9 presents simple OLS regressions of fertility and adult child's wage on father's wage, both for Brazil and the U.S. Both the fertility and persistence coefficients obtained from the PNAD 76 are roughly the double (in absolute value) of their PSID counterparts. The results for the 1996 PNAD are not shown in Table 9, but they confirm the differences between Brazil and the U.S. For the 1996 sample, the coefficient on father's wage in the fertility regression is -2.10, whereas the wage coefficient in the mobility regression is 0.67.

Table 10 uses a procedure similar to the one used in Mulligan (1993) to estimate the quantitative importance of fertility decisions for the persistence of inequality across generations. These regressions include as an explanatory variable the fitted value obtained from the regression of mother's schooling on father's fertility, father's wage, age variables, a gender dummy and agriculture's share of personal income in the county where the son grew up. The idea is that the fitted value of mother's schooling controls for variations in the price of fertility among families. The change in the coefficient on father's wage when the cost of children is controlled for may then be interpreted as the contribution of fertility behavior for the persistence of inequality.

Becker and Tomes (1986) present evidence from a dozen samples for the period 1960 through 1982 drawn from five countries (U.S., England, Sweden, Switzerland, and Norway). They generally found low intergenerational persistence of inequality, averaging about 0.25. These low estimates may be downward biased, due to sample homogeneity and measurement error in parental income, but revised estimates for the U.S. earnings estimates are in general around 0.4, which is considerably smaller than the estimates for Brazil presented in the text.

Because of data limitations, I use mother's labor income instead of parental fertility as an instrument. Also, for the same reason, I use a dummy variable indicating whether the father lives in a urban area, rather than agriculture share of personal income in the county where the son grew up.
If we compare Tables 9 and 10, we can observe that the coefficient on father's wage in 1976 declines fifty-percent (from 0.76 to 0.38) when we control for the cost of children. The corresponding decline in 1996 is of thirty-four percent (from 0.67 to 0.44). This is consistent with the prediction that fertility becomes relatively less important as a source of inequality as the country develops.

Table 10 also shows that the coefficient on father's wage for the U.S. declines forty-four percent (from 0.36 to 0.20) when one controls for the cost of children. The relative importance of fertility for persistence of inequality in the U.S. (around 1967-71) is thus slightly smaller than the contemporaneous figure for Brazil. Since it is likely that other sources of inequality persistence differ between Brazil and the U.S., it is probably more accurate to compare the absolute values of the effect of fertility on inequality, rather than the percentage change. In this case, the impact of fertility on persistence is much higher for Brazil (0.38) than the U.S. (0.16), which is consistent with the model.

5. Conclusion

This paper analyzed the interaction between wealth composition, endogenous fertility and the dynamics of wealth and income inequality, combining numerical simulations and empirical evidence. The major findings of the paper may be summarized as follows.

The first set of findings relates cross-section relationships between parental wealth, on the one hand, and wealth composition, fertility and investments per child, on the other. The cross-section relationship between fertility and wealth tends to display a U-shaped pattern, reflecting differences in the correlation between wealth and labor income among constrained and unconstrained families. The interaction between quantity and quality of children implies a concave cross-section relationship between investments per child and parental wealth.

The second set of findings is related to the dynamic behavior of the wealth and income distributions. If the degree of intergenerational altruism, the time cost of children and the
productivity of human capital investments are high relative to preferences for numbers of children, all families eventually overcome their borrowing constraints and there is long-run equality among families. The negative effect of wealth on fertility declines as physical capital becomes relatively more important as a source of wealth variations among families. The quantity-quality tradeoff implies that the degree of persistence in income inequality tends to decrease as the economy develops and families become unconstrained.

I provided empirical evidence on these two sets of implications using data from the PNADs 76 and 96 and evidence taken from other studies. First, the evidence shows that the fertility-income cross-section relationship tends to display a U-shaped pattern, as implied by the model. The cross-section relationship between adult child's income and parental income displays a concave pattern, reflecting the interaction between quantity and quality of children.

Second, I provided evidence on the dynamic behavior of persistence of inequality by comparing fertility behavior and persistence of income inequality between Brazil and the United States. I found that the negative effect of income on fertility is larger in Brazil, which tends to raise persistence of inequality in Brazil relative to the U.S. I also provided evidence that the quantitative importance of fertility as a source of persistence of inequality declined in Brazil between 1976 and 1996, which is consistent with the model.

The model constructed in this paper abstracts from several issues, in order to focus on the mechanism relating wealth composition, fertility and investments per child. In particular, the analysis does not consider the relationship between differences in fertility among skilled and unskilled workers and the dynamic evolution of the wage premium, as in Kremer and Chen (2000) and Doepke (2002). One interesting extension of the model would be to consider the general equilibrium interaction between wealth composition, fertility and factor prices.

Another possible extension of the model would be to assume that investments per child also require parental time, in addition to expenditures in goods. In this case, wealth composition would affect investments both directly and through changes in fertility.

Also, Veloso (2002) provides evidence that differences in the allocation of time to child-rearing between husbands and wives is quantitatively important for fertility and schooling
investments in children. This suggests that, in addition to the composition of income between labor and nonlabor sources, the composition of household income between mother's and father's income has important effects on fertility and investments in children.
References


Figure 1- Fertility Policy Functions (baseline parameters)
Figure 2-Cross-Section Profiles at t=0 (baseline parameters)
Figure 3 - Fertility Policy Functions (\(\lambda=0.4\))
Figure 4-Cross-Section Profiles at $t=0$ (lambda=0.4)
Figure 5: Fertility Policy Functions (\(\epsilon=0.5\))
Figure 6-Cross-Section Profiles at $t=0$ (epsilon=0.5)
Figure 7-Cross-Section Profiles (PNAD 76)
Fertility-Income Profile without Childless (SCF89)

Figure 8-Fertility-Income Cross-Section Profiles (SCF 89)
Table 1: Baseline parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.25</td>
</tr>
<tr>
<td>$r$</td>
<td>3</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>0.2</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.14</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.67</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.33</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.5</td>
</tr>
</tbody>
</table>
Table 2: Fertility regression coefficients
(baseline parameters)

<table>
<thead>
<tr>
<th></th>
<th>wealth</th>
<th>labor income</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_0$</td>
<td>$-2.8$</td>
<td>$\theta_0^L = -2.6$</td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>$-2.2$</td>
<td>$\theta_1^L = -2.2$</td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>$-1.6$</td>
<td>$\theta_2^L = -1.6$</td>
</tr>
<tr>
<td>$\theta_3$</td>
<td>$-0.8$</td>
<td>$\theta_3^L = -0.8$</td>
</tr>
<tr>
<td>$\theta_4$</td>
<td>$0$</td>
<td>$\theta_4^L = 0$</td>
</tr>
<tr>
<td>$\theta_\infty$</td>
<td>$.$</td>
<td>$\theta_\infty^L = .$</td>
</tr>
</tbody>
</table>

Table 3: Persistence of inequality
(baseline parameters)

<table>
<thead>
<tr>
<th></th>
<th>persistence of wealth</th>
<th>persistence of labor income</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_0$</td>
<td>$0.82$</td>
<td>$\rho_0^L = 0.79$</td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>$0.67$</td>
<td>$\rho_1^L = 0.67$</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>$0.51$</td>
<td>$\rho_2^L = 0.51$</td>
</tr>
<tr>
<td>$\rho_3$</td>
<td>$0.32$</td>
<td>$\rho_3^L = 0.32$</td>
</tr>
<tr>
<td>$\rho_4$</td>
<td>$0.13$</td>
<td>$\rho_4^L = 0.13$</td>
</tr>
<tr>
<td>$\rho_5$</td>
<td>$0$</td>
<td>$\rho_5^L = 0$</td>
</tr>
<tr>
<td>$\rho_\infty$</td>
<td>$.$</td>
<td>$\rho_\infty^L = .$</td>
</tr>
</tbody>
</table>
Table 4: Fertility regression coefficients  
\( (\lambda = 0.4) \)

<table>
<thead>
<tr>
<th>wealth</th>
<th>labor income</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_0 = -1.25 )</td>
<td>( \theta_0^L = -1.22 )</td>
</tr>
<tr>
<td>( \theta_1 = 0.01 )</td>
<td>( \theta_1^L = 0.01 )</td>
</tr>
<tr>
<td>( \theta_2 = 0.22 )</td>
<td>( \theta_2^L = 0.22 )</td>
</tr>
<tr>
<td>( \theta_3 = 0.89 )</td>
<td>( \theta_3^L = . )</td>
</tr>
<tr>
<td>( \theta_4 = 0 )</td>
<td>( \theta_4^L = . )</td>
</tr>
<tr>
<td>( \theta_\infty = . )</td>
<td>( \theta_\infty^L = . )</td>
</tr>
</tbody>
</table>

Table 5: Persistence of inequality  
\( (\lambda = 0.4) \)

<table>
<thead>
<tr>
<th>persistence of wealth</th>
<th>persistence of labor income</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho_0 = 0.52 )</td>
<td>( \rho_0^L = 0.45 )</td>
</tr>
<tr>
<td>( \rho_1 = 0.35 )</td>
<td>( \rho_1^L = 0.28 )</td>
</tr>
<tr>
<td>( \rho_2 = 0.19 )</td>
<td>( \rho_2^L = 0 )</td>
</tr>
<tr>
<td>( \rho_3 = 0.16 )</td>
<td>( \rho_3^L = . )</td>
</tr>
<tr>
<td>( \rho_4 = 0 )</td>
<td>( \rho_4^L = . )</td>
</tr>
<tr>
<td>( \rho_\infty = . )</td>
<td>( \rho_\infty^L = . )</td>
</tr>
</tbody>
</table>
Table 6: Fertility regression coefficients

(\(\varepsilon = 0.5\) )

<table>
<thead>
<tr>
<th></th>
<th>wealth</th>
<th>labor income</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\theta_0)</td>
<td>-2.95</td>
<td>-2.80</td>
</tr>
<tr>
<td>(\theta_1)</td>
<td>-2.92</td>
<td>-2.92</td>
</tr>
<tr>
<td>(\theta_2)</td>
<td>-1.88</td>
<td>-1.88</td>
</tr>
<tr>
<td>(\theta_3)</td>
<td>-0.92</td>
<td>-0.92</td>
</tr>
<tr>
<td>(\theta_4)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(\theta_\infty)</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 7: Persistence of inequality

(\(\varepsilon = 0.5\) )

<table>
<thead>
<tr>
<th></th>
<th>persistence of wealth</th>
<th>persistence of labor income</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\rho_0)</td>
<td>0.93</td>
<td>(\rho_0^L) = 0.91</td>
</tr>
<tr>
<td>(\rho_1)</td>
<td>0.92</td>
<td>(\rho_1^L) = 0.92</td>
</tr>
<tr>
<td>(\rho_2)</td>
<td>0.95</td>
<td>(\rho_2^L) = 0.95</td>
</tr>
<tr>
<td>(\rho_3)</td>
<td>0.98</td>
<td>(\rho_3^L) = 0.98</td>
</tr>
<tr>
<td>(\rho_4)</td>
<td>1</td>
<td>(\rho_4^L) = 1</td>
</tr>
<tr>
<td>(\rho_\infty)</td>
<td>1</td>
<td>(\rho_\infty^L) = 1</td>
</tr>
<tr>
<td>independent variables</td>
<td>fertility</td>
<td>adult child's schooling</td>
</tr>
<tr>
<td>------------------------</td>
<td>-----------</td>
<td>-------------------------</td>
</tr>
<tr>
<td>parental income</td>
<td>-3.18 *</td>
<td>10.17 *</td>
</tr>
<tr>
<td></td>
<td>(0.76)</td>
<td>(0.58)</td>
</tr>
<tr>
<td>parental income squared</td>
<td>0.17 *</td>
<td>-0.86 *</td>
</tr>
<tr>
<td></td>
<td>(0.05)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>adjusted R squared</td>
<td>0.17</td>
<td>0.48</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>18227</td>
<td>17658</td>
</tr>
</tbody>
</table>

Notes: (a) All income variables are measured in logs. Standard errors in parentheses. The regressions use sample weights provided by IBGE. N refers to the unweighted number of observations. (b) The full-time concept of full income is defined as average family income for men who are household heads and work 40 hours per week on average. Different income averages are calculated for each possible combination of education category, state of residence and whether the individual lives in a urban or rural area. (c) A constant, the mother’s age, its age squared, the father's age, its age squared, the oldest child age, its age squared, the oldest child's sex, a dummy variable for urban areas and a dummy variable for the state in which the family resides are included in each regression. (d) * significant at the one-percent level
Table 9: OLS Regression of fertility and adult child’s wage on father’s wage (full-time)- PNAD 76 and Mulligan (1993)

<table>
<thead>
<tr>
<th>independent variables</th>
<th>fertility (PNAD 76)</th>
<th>adult child's wage (PNAD 76)</th>
<th>fertility (Mulligan (1993))</th>
<th>adult child's wage (Mulligan (1993))</th>
</tr>
</thead>
<tbody>
<tr>
<td>father's wage</td>
<td>-2.18 * (0.04)</td>
<td>0.76 * (0.006)</td>
<td>-1.17 * (0.16)</td>
<td>0.36 * (0.04)</td>
</tr>
<tr>
<td>adjusted R squared</td>
<td>0.18</td>
<td>0.46</td>
<td>0.1</td>
<td>0.21</td>
</tr>
<tr>
<td>N</td>
<td>13384</td>
<td>20408</td>
<td>648</td>
<td>648</td>
</tr>
</tbody>
</table>

Notes: (a) All wage variables are measured in logs. Standard errors in parentheses. The PNAD regressions use sample weights provided by IBGE. N refers to the unweighted number of observations.  
(b) The full-time wage measure for the PNAD is defined as average hourly wage for men who are household heads and work 40 hours per week on average. Different wage averages are calculated for each possible combination of education category, state of residence and whether the individual lives in a urban or rural area.  
(c) A constant, the father's age, its age squared, the child's age, its age squared and the child's sex are included in each regression, and a dummy variable indicating whether the father lives in a urban area (Brazil).  
(d) * significant at the one-percent level
Table 10: 2SLS Regression of fertility and adult child's wage on father's wage (full-time)- PNAD 76, PNAD 96 and Mulligan (1993)

<table>
<thead>
<tr>
<th>independent variables</th>
<th>adult child's wage (PNAD 76)</th>
<th>adult child's wage (PNAD 96)</th>
<th>adult child's wage (Mulligan 1993)</th>
</tr>
</thead>
<tbody>
<tr>
<td>father's wage</td>
<td>0.38 * (0.01)</td>
<td>0.44* (0.01)</td>
<td>0.20 ** (0.11)</td>
</tr>
<tr>
<td>mother's schooling</td>
<td>0.07 * (0.003)</td>
<td>0.03* (0.001)</td>
<td>0.06 (0.05)</td>
</tr>
<tr>
<td>adjusted R squared</td>
<td>0.46</td>
<td>0.45</td>
<td>0.21</td>
</tr>
<tr>
<td>N</td>
<td>16380</td>
<td>34889</td>
<td>648</td>
</tr>
</tbody>
</table>

Notes: (a) All wage variables are measured in logs. Standard errors in parentheses. The PNAD regressions use sample weights provided by IBGE. N refers to the unweighted number of observations.
(b) The full-time wage measure for the PNAD is defined as average hourly wage for men who are household heads and work 40 hours per week on average. Different wage averages are calculated for each possible combination of education category, state of residence and whether the individual lives in a urban or rural area.
(c) For the Mulligan (1993) PSID data mother's schooling is the fitted value from a regression of mother's schooling on the age variables, father's wage, fertility, a gender dummy and agriculture's share of personal income in the county where the son grew up. For the PNAD, mother's schooling is the fitted value from a regression of mother's schooling on the age variables, father's wage, mother's full-time labor income, a gender dummy, and a dummy variable indicating whether the father lives in a urban area.
(d) A constant, the father's age, its age squared, the child's age, its age squared and th
(e) * significant at the one-percent level
    ** significant at the five-percent level