Finance and Misallocation: Evidence from Plant-Level Data

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Our goal

- Measure effect of finance frictions on resource (mis)allocation
  - TFP losses
Mechanism we study

- An agent’s entrepreneurial ability fluctuates over time

- Ask:
  - Do frictions distort re-allocation $K, L$ across entrepreneurs?
  - Do frictions distort entry entrepreneurship?

- Narrow focus
  - Can model *endogenously* generate large dispersion $MP_K$?
  - Abstract differences i-rates due to government etc.
Our approach

• Model of establishment dynamics with borrowing constraints

• Require model accounts plant-level facts (Korea, Colombia):

  1. Size distribution of establishments
  2. Variability and persistence of output
  3. Difference growth rates young vs. old plants
  4. Difference returns to capital young vs. old
Our findings

• Find mechanism is weak:
  • 7% TFP losses in economy without external finance
  • Much weaker (up to 40% losses) than previously found:
    • Jeong-Townsend’06, Buera-Kaboski-Shin’09, Amaral-Quintin’05, Moll’09, Greenwood-Sanchez-Wang’09

• Plant-level facts key to this result
  • TFP losses much larger if ignore facts 2-4
Finance vs. TFP our model

Figure 1: TFP vs. External Finance

External Finance to GDP
Outline

- Model with no exit-entry
- Model with exit-entry and occupational choice
Model Overview

- Small open economy
- Continuum of entrepreneurs differ in (time-varying) productivity
- Plant only source of income. Risk not diversifiable

Finance [Evans-Jovanovic (1989)]:

- Save risk-free asset, \( r: \beta(1 + r) < 1 \)
- Must pay labor, capital before production.
- Debt subject to collateral constraint
Technology

• Production function:

\[ Y_{it} = A_{it}^{1-\eta} \left( L_{it}^\alpha K_{it}^{1-\alpha} \right)^\eta \]

• \( \eta < 1 \), span of control

• Productivity : \( \Phi(A_{it+1} | A_{it}) \)
Problem of entrepreneur

\[ \sum_{t=0}^{\infty} \beta^t \frac{C_{it}^{1-\gamma}}{1 - \gamma} \]

- \( B_{it} \): assets

- spend \( WL_{it} + K_{it} \) before producing
  - borrow \( D_{it} = WL_{it} + K_{it} - B_{it} \) from bank
  - collateral constraint: \( D_{it} \leq (\lambda - 1)B_{it} \)

\[ C_{it} + B_{it+1} = Y_{it} + (1 - \delta) K_{it} + (1 + r) [B_{it} - WL_{it} - K_{it}] \]
Timing

\[ A_t^{1-\eta} (L_t^\alpha K_t^{1-\alpha})^\eta + (1 - \delta) K_t \]

borrow
\[ B_t \rightarrow W_t L_t + K_t - B_t \]
repay \((1 + r) (W_t L_t + K_t - B_t) \rightarrow B_{t+1}\)

consume \(C_t\)

\[ t \rightarrow t + 1 \]
Problem of entrepreneur

Reduces to

$$\max \sum_{t=0}^{\infty} \beta^t \frac{C_{it}^{1-\gamma}}{1-\gamma}$$

s.t.

$$C_{it} = (1 + r) B_{it} + \pi (B_{it}, A_{it}) - B_{it+1}$$

where

$$\Pi (B_{it}, A_{it}) = \max_{K_{it}, L_{it}} A_{it}^{1-\eta} \left( L_{it}^{\alpha} K_{it}^{1-\alpha} \right)^{\eta} - (1 + r) W_t L_{it} - (r + \delta) K_{it}$$

s.t. $$W L_{it} + K_{it} \leq \lambda B_{it}$$
Solution to static problem

- Homogeneity: \( b = B/A, k = K/A, \pi = \Pi/A \)

\[
\pi (b) = \max_{k,l} \left(l^\alpha k^{1-\alpha}\right)^\eta - (1 + r) Wl - (r + \delta) k
\]
\[
\text{s.t. } Wl + k \leq \lambda b
\]

- Solution:

\[
fl (l, k) = [1 + \tilde{r}(b)] W
\]
\[
fk (l, k) = \tilde{r}(b) + \delta
\]

- \( \tilde{r}(b) \): shadow cost of funds, \( \tilde{r}(b) = r + \mu(b) \)
Dynamic program

\[
V(b, a) = \max_{b'} \frac{c^{1-\gamma}}{1-\gamma} + \beta \int \exp(a' - a)^{1-\gamma} V\left(\frac{b'}{\exp(a' - a)}, a'\right) d\Phi(a'|a)
\]

where

\[
c = (1 + r) b + \pi(b) - b'.
\]

- Solution:

\[
c^{-\gamma} = \beta \int (1 + r + \mu') \exp(a' - a)^{-\gamma} c'^{-\gamma} d\Phi(a'|a)
\]
Decision rules

A. Shadow cost of funds

B. Savings, $b'/b$
Response to productivity shock

A. Productivity, $a$

B. Shadow cost of funds, $r$

C. Capital Stock, $K$, log

D. Assets, $B$, log

Figure 3: Impulse response to a productivity shock
Summarize

• Absent $\Delta a$ ergodic distribution $b$ degenerate
  
  • $\tilde{r}$ equal across entrepreneurs
  
  • No misallocation
  
  • Banerjee-Moll ’09

• Misallocation requires large dispersion $b$: large shocks to $a$
TFP losses due to misallocation

- Efficient allocations:

\[
\max_{K_i, L_i} Y = \int_0^1 A_i^{1-\eta} \left( L_i^{\alpha} K_i^{1-\alpha} \right)^\eta \, di
\]

s.t. \( \int_0^1 K_i \, di = K, \quad \int_0^1 L_i \, di = L \)

- Solution:

\[
L_i = \frac{A_i}{\int_0^1 A_i \, di} L, \quad K_i = \frac{A_i}{\int_0^1 A_i \, di} K
\]

\[
Y = A \left( L^{\alpha} K^{1-\alpha} \right)^\eta, \quad A = \int_0^1 A_i \, di
\]
TFP losses due to misallocation

- Allocations with frictions:

\[
L_i = \omega_i^l \frac{A_i}{\int_0^1 A_i \, di} L, \quad K_i = \omega_i^k \frac{A_i}{\int_0^1 A_i \, di} K
\]

\[
Y = A (L^\alpha K^{1-\alpha})^\eta, \quad A = \int_0^1 \omega_i A_i^{1-\eta} \, di
\]

- ‘Worst-case’: \( \omega_i = 1 \) (\( L_i = L \), \( K_i = K \)).

- Efficient: \( \omega_i = A_i^\eta \)
Data: Korea (’91-’96) and Colombia (’81-’91)

- Manufacturing plants
- All establishments 5+ (Korea), 10+ (Colombia) workers
- Balanced panel, 31500 plants Korea, 5000 Colombia
- Revenue, labor, intermediate inputs, investment, capital
- \( Y = \text{value added} = \text{revenue} - \text{intermediate inputs} \)
Fact 1: size distribution of establishments

- Output (value-added) concentrated largest establishments

<table>
<thead>
<tr>
<th>Fraction of largest establishments</th>
<th>Korea</th>
<th>Colombia</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>0.57</td>
<td>0.30</td>
</tr>
<tr>
<td>5%</td>
<td>0.77</td>
<td>0.61</td>
</tr>
<tr>
<td>10%</td>
<td>0.84</td>
<td>0.75</td>
</tr>
<tr>
<td>20%</td>
<td>0.91</td>
<td>0.88</td>
</tr>
</tbody>
</table>
**Fact 2: distribution of output growth rates**

- $\Delta y_{it}$ volatile, fat-tailed

<table>
<thead>
<tr>
<th></th>
<th>Korea</th>
<th>Colombia</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(\Delta y_{it})$</td>
<td>0.54</td>
<td>0.49</td>
</tr>
<tr>
<td>kurtosis($\Delta y_{it}$)</td>
<td>12.9</td>
<td>20.8</td>
</tr>
<tr>
<td>iqr($\Delta y_{it}$)</td>
<td>0.49</td>
<td>0.36</td>
</tr>
</tbody>
</table>
**Fact 3: persistence of output**

- \( y_{it} \) persistent, autocorrelation decays slowly

<table>
<thead>
<tr>
<th>( \text{corr}(y_{it}, y_{it-1}) )</th>
<th>Korea</th>
<th>Colombia</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.93</td>
<td></td>
<td>0.96</td>
</tr>
<tr>
<td>( \text{corr}(y_{it}, y_{it-3}) )</td>
<td>0.89</td>
<td>0.93</td>
</tr>
<tr>
<td>( \text{corr}(y_{it}, y_{it-5}) )</td>
<td>0.86</td>
<td>0.90</td>
</tr>
</tbody>
</table>
Fact 4: Debt-to-GDP


<table>
<thead>
<tr>
<th></th>
<th>Korea</th>
<th>Colombia</th>
</tr>
</thead>
<tbody>
<tr>
<td>Debt-to-GDP</td>
<td>1.2</td>
<td>0.3</td>
</tr>
</tbody>
</table>
Parameterization

- Assigned parameters
  - $\gamma = 1$ (CRRA)
  - $\beta = 0.92$ (discount factor)
  - $r = 0.04$ (risk-free rate)
  - $\delta = 0.06$ (depreciation rate)
  - $\eta = 0.85$ (span of control)
  - $\alpha = 0.67$ (labor share)

- Calibrate rest to minimize distance moments model-data
Calibration

- Productivity:
  \[ \ln(A_{it}) = a_{it} = Z_i + \tilde{a}_{it} \]

- \( Z_i \): permanent component. Bounded Pareto.
  \[ \Pr[\exp(Z_i) \leq x] = \frac{1 - x^{-\mu}}{1 - H^{-\mu}}. \]

- \( \tilde{a}_{it} \): variable component. Fat-tailed shocks.
  \[ \tilde{a}_{it} = \rho \tilde{a}_{it-1} + \varepsilon_{it} \]
  \[ \varepsilon_{it} \sim \begin{cases} 
  N \left(0, \sigma_1^2, \varepsilon\right) & \text{with prob. } 1 - \kappa \\
  N \left(0, \sigma_2^2, \varepsilon\right) & \text{with prob. } \kappa 
\end{cases} \]
Calibration

- Calibrate $\theta = \{\lambda, \rho, \sigma_1, \sigma_2, \kappa, \mu, H\}$

- Match plant-level moments and debt-to-GDP for Korea

$$\min_{\theta} \left[ \Gamma(\theta) - \Gamma^d \right]' W \left[ \Gamma(\theta) - \Gamma^d \right]$$

- $W = var(\Gamma^d)^{-1}$, boostrap.

- Standard errors:

$$V = \frac{1}{N} \left[ \frac{\partial \Gamma(\theta)}{\partial \theta'} W \frac{\partial \Gamma(\theta)}{\partial \theta} \right]^{-1}$$
### Parameter values

<table>
<thead>
<tr>
<th>Estimate</th>
<th>(s.e.)</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>2.58 (0.01)</td>
<td>collateral constraint</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.74 (0.01)</td>
<td>AR(1) productivity</td>
</tr>
<tr>
<td>$\sigma_{1,\varepsilon}$</td>
<td>0.09 (0.00)</td>
<td>s.d. shocks</td>
</tr>
<tr>
<td>$\sigma_{2,\varepsilon}$</td>
<td>0.31 (0.00)</td>
<td>s.d. shocks</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.07 (0.00)</td>
<td>probab. 2</td>
</tr>
<tr>
<td>$\mu$</td>
<td>3.64 (0.02)</td>
<td>Pareto exponent</td>
</tr>
<tr>
<td>$H$</td>
<td>4.91 (0.02)</td>
<td>upper bound $Z_i$</td>
</tr>
</tbody>
</table>

- Implies $Z_i$ accounts 2/3 variance $a_i$
## Fit

<table>
<thead>
<tr>
<th></th>
<th>Korea Data</th>
<th>Model</th>
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<tbody>
<tr>
<td>$\sigma(\Delta y_{it})$</td>
<td>0.54</td>
<td>0.51</td>
</tr>
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<td>kurtosis($\Delta y_{it}$)</td>
<td>12.9</td>
<td>12.9</td>
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<td>0.49</td>
<td>0.47</td>
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<tr>
<td>$corr(y_{it}, y_{it-1})$</td>
<td>0.93</td>
<td>0.95</td>
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<td>0.86</td>
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</tr>
<tr>
<td>fraction $Y$ largest 1%</td>
<td>0.57</td>
<td>0.59</td>
</tr>
<tr>
<td>fraction $Y$ largest 5%</td>
<td>0.77</td>
<td>0.83</td>
</tr>
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<td>0.90</td>
</tr>
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<td>0.95</td>
</tr>
<tr>
<td>Debt-to-GDP</td>
<td>1.2</td>
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</tbody>
</table>
Role of permanent component

- Eliminate $Z_i$ and re-calibrate

<table>
<thead>
<tr>
<th></th>
<th>Korea Data</th>
<th>No $Z_i$</th>
</tr>
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<tbody>
<tr>
<td>$\sigma(\Delta y_{it})$</td>
<td>0.54</td>
<td>0.51</td>
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<td>12.9</td>
<td>18.2</td>
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<tr>
<td>iqr($\Delta y_{it}$)</td>
<td>0.49</td>
<td>0.43</td>
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<tr>
<td>$corr(y_{it}, y_{it−1})$</td>
<td>0.93</td>
<td>0.95</td>
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<td>0.87</td>
</tr>
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<td>0.86</td>
<td>0.78</td>
</tr>
<tr>
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</tr>
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<td>fraction $Y$ largest 5%</td>
<td>0.77</td>
<td>0.53</td>
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<td>0.66</td>
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<td>0.79</td>
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<td>1.2</td>
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</tbody>
</table>
Model predictions

- Report results for Korean calibration

- Also for US, Colombia, No external debt.
  - Same $a_i$ process. Vary $\lambda$ to match Debt-to-GDP.
## Model predictions

<table>
<thead>
<tr>
<th></th>
<th>US</th>
<th>Korea</th>
<th>Colombia</th>
<th>No Debt</th>
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<tbody>
<tr>
<td>$\lambda$</td>
<td>50</td>
<td>2.6</td>
<td>1.2</td>
<td>1</td>
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<td>Debt-to-GDP</td>
<td>2.3</td>
<td>1.2</td>
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<td>0</td>
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<tr>
<td>Fract. constrained</td>
<td>0.04</td>
<td>0.54</td>
<td>0.80</td>
<td>0.86</td>
</tr>
<tr>
<td>Median $\tilde{r} - r$</td>
<td>0.01</td>
<td>0.03</td>
<td>0.04</td>
<td>0.05</td>
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<tr>
<td>iqr $\tilde{r} - r$</td>
<td>0.02</td>
<td>0.03</td>
<td>0.04</td>
<td>0.05</td>
</tr>
<tr>
<td>99% $\tilde{r} - r$</td>
<td>0.16</td>
<td>0.15</td>
<td>0.19</td>
<td>0.20</td>
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<tr>
<td>$\sigma(\Delta y_{it})$</td>
<td>0.70</td>
<td>0.51</td>
<td>0.37</td>
<td>0.35</td>
</tr>
</tbody>
</table>
Model predictions

Figure 4: Productivity vs. shadow cost of funds

Shadows cost of funds, $\tilde{r}$

Productivity, $a$
TFP losses

- $\Delta TFP$ if eliminate finance frictions, $\lambda = \infty$:

<table>
<thead>
<tr>
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</tr>
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<tbody>
<tr>
<td>TFP losses, %</td>
<td>1.0</td>
<td>3.9</td>
<td>5.5</td>
<td>6.9</td>
</tr>
</tbody>
</table>

Bottomline: TFP losses small
Finance vs. TFP

Figure 1: TFP vs. External Finance

External Finance to GDP
Why are TFP losses small?

- Recall if \( \ln(A_{it}) = Z_i + \tilde{a}_{it} \) constant: no TFP losses

- Too little time-series variation \( \ln(A_{it}) \)

- TFP losses if \( K, L \) do not move at all with \( \tilde{a}_{it} \):
  - \( A = \int_0^1 \exp(\tilde{a}_{it}) \, di \) vs. \( A = \int_0^1 \exp(\tilde{a}_{it})^{1-\eta} \, di \)

<table>
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<td>3.9</td>
<td>5.5</td>
<td>6.9</td>
</tr>
<tr>
<td>’Worst-case’ losses, %</td>
<td>8.6</td>
<td>8.6</td>
<td>8.6</td>
<td>8.6</td>
</tr>
</tbody>
</table>
Summarize

- Finance frictions: small TFP losses from misallocation
- Too little time-series variability output (productivity)
- Losses small even if $K, L$ do not react at all $\Delta$ productivity
- Show next TFP losses much larger if ignore data $\Delta y_{it}$
Counterfactual experiments

- Illustrate role of micro facts for TFP numbers

- Set $Z_i = 0$

- $a_{it} = \rho a_{it-1} + \epsilon_{it}, \epsilon_{it} \sim N(0, \sigma^2)$

- Three experiments
  1. Choose $\rho, \sigma^2$ to match $corr(y_{it}, y_{it-1})$, size distribution
  2. Lower $\rho = 0.80$ (Moll’09)
     - raise $\sigma^2$ to match size distribution
     - keep $\sigma^2$ constant
# Counterfactual experiments

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<tr>
<th></th>
<th>Our</th>
<th>1</th>
<th>2</th>
<th>3</th>
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<tbody>
<tr>
<td>$\sigma(\Delta y_{it})$</td>
<td>0.51</td>
<td>1.05</td>
<td>2.17</td>
<td>1.03</td>
</tr>
<tr>
<td>$\text{corr}(y_{it}, y_{it-1})$</td>
<td>0.95</td>
<td>0.93</td>
<td>0.80</td>
<td>0.77</td>
</tr>
<tr>
<td>$\text{corr}(y_{it}, y_{it-3})$</td>
<td>0.89</td>
<td>0.80</td>
<td>0.52</td>
<td>0.47</td>
</tr>
<tr>
<td>$\text{corr}(y_{it}, y_{it-5})$</td>
<td>0.85</td>
<td>0.69</td>
<td>0.34</td>
<td>0.30</td>
</tr>
<tr>
<td>Fraction Y top 1%</td>
<td>0.59</td>
<td>0.52</td>
<td>0.44</td>
<td>0.13</td>
</tr>
<tr>
<td>Fraction Y top 10%</td>
<td>0.90</td>
<td>0.88</td>
<td>0.88</td>
<td>0.48</td>
</tr>
<tr>
<td>TFP loss Korea</td>
<td>3.9</td>
<td>10.5</td>
<td>18.3</td>
<td>6.6</td>
</tr>
<tr>
<td>TFP loss Colombia</td>
<td>5.5</td>
<td>18.1</td>
<td>29.5</td>
<td>10.9</td>
</tr>
<tr>
<td>’Worst-case’ loss</td>
<td>8.6</td>
<td>54.3</td>
<td>69.9</td>
<td>20.2</td>
</tr>
</tbody>
</table>
Summarize

- Model can easily generate much larger TFP losses
- But only if $\Delta y_{it}$ much more volatile than data
- Lower $\rho$ lowers TFP losses if hold $\sigma(\Delta y_{it})$ constant
**Alternative Parameterizations**

- Lower $\beta = 0.85$: less internal accumulation
- Higher $\eta = 0.95$: greater losses misallocation
- Lower elast. subst $K, L$: $\theta = 0.5$

$$Y_i = A_i^{1-\eta} \left[ \alpha L_i^{\theta-1} + (1 - \alpha) K_i^{\theta-1} \right]^{\frac{\theta}{\theta-1}} \eta$$

- Recalibrate to match moments
## Alternative Parameterizations

<table>
<thead>
<tr>
<th></th>
<th>Benchmark</th>
<th>Low $\beta$</th>
<th>High $\eta$</th>
<th>Low $\theta$</th>
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</thead>
<tbody>
<tr>
<td>TFP loss US</td>
<td>1.0</td>
<td>2.2</td>
<td>0.7</td>
<td>2.9</td>
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<tr>
<td>TFP loss Korea</td>
<td>3.9</td>
<td>6.5</td>
<td>3.2</td>
<td>5.6</td>
</tr>
<tr>
<td>TFP loss Colombia</td>
<td>5.4</td>
<td>8.8</td>
<td>5.4</td>
<td>6.9</td>
</tr>
<tr>
<td>’Worst-case’ loss</td>
<td>8.6</td>
<td>12.6</td>
<td>8.3</td>
<td>8.4</td>
</tr>
</tbody>
</table>
Do entrepreneurs save?

- TFP losses small because internal accumulation

- Question: do agents save in the data?

- Answer by studying how $K/Y$ varies with $Debt/Y$
  - $K =$ debt + internal funds
  - If do not save ($\beta$ low), $K/Y$ low in low $\lambda$ economies
**K/Y vs. Debt-to-GDP Data**

Figure 5: K/Y vs. External Finance

Data (elast. = 0.51)
Model predictions

Figure 5: K/Y vs. External Finance

\[ \beta = 0.85 \text{ (elast. } = 0.56) \]

\[ \beta = 0.92 \text{ (elast. } = 0.36) \]

Data (elast. = 0.51)
Model with entry/exit

- Do finance frictions distort entry/exit decision?
  - Inefficient selection into entrepreneurship

- Do finance frictions distort entry?
  - $Z_i$ source of TFP losses now
Model Overview

- Small open economy

- Continuum of agents. Each period decide whether
  - Work: supply 1 unit labor. Earn $W$
  - Entrepreneur: $Y_{it} = A_{it}^{1-\eta} (L^{\alpha} K^{1-\alpha})^\eta$

- Entrepreneurs can borrow subject to collateral constraint

- Agents die with prob. $1 - p$

- Replaced $1 - p$ newly born.
  - Draw $Z_i$, $(a_i = 0)$, receive endowment $B_0(Z_i)$
Dynamic program

\begin{equation}
\pi(b) = \max_{k,l} \left( l^\alpha k^{1-\alpha} \right)^\eta - (1 + r) Wl - (r + \delta) k
\end{equation}

\begin{equation}
\text{s.t. } Wl + k \leq \lambda b
\end{equation}

\begin{equation}
V(b,a) = \max_{b'} \frac{c^{1-\gamma}}{1-\gamma} + \beta \int \exp{(a' - a)^{1-\gamma}} V\left( \frac{b'}{\exp(a' - a)}, a' \right) d\Phi(a'|a)
\end{equation}

where

\begin{equation}
c = (1 + r) b + \max \left[ \pi(b), \frac{W}{\exp(a)} \right] - b'.
\end{equation}
Occupational choice

Entrepreneur

Efficient

Worker

A

B

0 10 20 30 40 50 60 70 80 90 100
Equilibrium

- $\mu(B, A)$: stationary distribution
- $I(B, A) = W > \Pi(B, A)$: work
- $W$ solves:

$$\int I(B, A) \, d\mu(B, A) = \int L(B, A) (1 - I(B, A)) \, d\mu(B, A)$$
Parametrization

• Two economies:
  
  • No initial endowment: \( B_0(Z_i) = 0 \)
  
  • With initial endowment: \( B_0(Z_i) = \phi(WL(Z_i) + K(Z_i)) \)

• Additional parameters: \( p, \phi \)
  
  • Same set of moments as earlier (use entire panel)
  
  • Add exit hazards, by age, share output by exiting plants
## Economy with no initial endowment

<table>
<thead>
<tr>
<th></th>
<th>Korea Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(\Delta y_{it})$</td>
<td>0.56</td>
<td>0.57</td>
</tr>
<tr>
<td>$\text{corr}(y_{it}, y_{it-1})$</td>
<td>0.92</td>
<td>0.93</td>
</tr>
<tr>
<td>$\text{corr}(y_{it}, y_{it-5})$</td>
<td>0.86</td>
<td>0.74</td>
</tr>
<tr>
<td>fraction $Y$ largest 5%</td>
<td>0.72</td>
<td>0.66</td>
</tr>
<tr>
<td>fraction $Y$ largest 20%</td>
<td>0.87</td>
<td>0.89</td>
</tr>
<tr>
<td>fraction age 1-5</td>
<td>0.51</td>
<td>0.62</td>
</tr>
<tr>
<td>fraction age 6-10</td>
<td>0.26</td>
<td>0.16</td>
</tr>
<tr>
<td>fraction age $&gt;10$</td>
<td>0.23</td>
<td>0.21</td>
</tr>
<tr>
<td>exit hazard</td>
<td>0.33</td>
<td>0.25</td>
</tr>
<tr>
<td>output share if exit</td>
<td>0.07</td>
<td>0.07</td>
</tr>
<tr>
<td>Debt-to-GDP</td>
<td>1.2</td>
<td>1.2</td>
</tr>
</tbody>
</table>
## Model predictions (Korea)

<table>
<thead>
<tr>
<th></th>
<th>No exit/entry</th>
<th>With exit/entry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fract. constrained</td>
<td>0.54</td>
<td>0.83</td>
</tr>
<tr>
<td>Median $\tilde{r} - r$</td>
<td>constr</td>
<td>0.03</td>
</tr>
<tr>
<td>iqr $\tilde{r} - r$</td>
<td>constr</td>
<td>0.03</td>
</tr>
<tr>
<td>99% $\tilde{r} - r$</td>
<td>constr</td>
<td>0.15</td>
</tr>
<tr>
<td>TFP losses</td>
<td>3.9</td>
<td>10.6</td>
</tr>
<tr>
<td>Due occup. choice</td>
<td>-</td>
<td>0.2</td>
</tr>
</tbody>
</table>

**Bottomline: TFP losses larger**
Why TFP losses larger?

• Equilibrium $W$ low.
  • Talented entrepreneurs enter almost immediately.
  • Initially very constrained.

• Question: are entering plants constrained data?

• Compare $\Delta y$, returns to capital young vs. old plants
Young vs. Old plants

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<td>$\Delta y$ ages 1-5 vs. 10+</td>
<td>0.05</td>
<td>0.20</td>
</tr>
<tr>
<td>$\Delta y$ ages 6-10 vs. 10+</td>
<td>0.02</td>
<td>0.06</td>
</tr>
<tr>
<td>$Y/K$ ages 1-5 vs. 10+</td>
<td>0.04</td>
<td>0.30</td>
</tr>
<tr>
<td>$Y/K$ ages 1-5 vs. 10+</td>
<td>0.06</td>
<td>0.17</td>
</tr>
</tbody>
</table>

Young much more constrained in the model
Economy with initial endowment

- At birth, all agents receive $B_0(Z_i) = \phi(WL(Z_i) + K(Z_i))$

- Choose $\phi$ to match relative growth rates, $Y/K$ young
Growth rate by age

Figure 5: Growth rates vs. age

- Model without initial endowment
- Model with initial endowment
- Korean data
## Economy with initial endowment

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</tr>
<tr>
<td>TFP losses US</td>
<td>-</td>
<td>1.5 %</td>
</tr>
<tr>
<td>TFP losses Korea</td>
<td>-</td>
<td>5.1 %</td>
</tr>
<tr>
<td>TFP losses Colombia</td>
<td>-</td>
<td>6.7 %</td>
</tr>
</tbody>
</table>
Role of fixed costs

• Do fixed costs prevent internal accumulation?

• Assume \( Y = A^{1-\eta} \left( (L - \bar{L})^\alpha K^{1-\alpha} \right)^\eta \)

• Choose \( \bar{L} \) to match \( L \) vs. \( Y \) relationship data
  
  • Regress \( \ln\left( \frac{Y}{L} \right) \) on \( \ln(Y) \): 0.13 (vs. \( \approx 0 \) absent fixed costs)

• Need \( \bar{L} = 0.06 \) of aggregate labor used.
Role of fixed costs

- Increase TFP losses, but not much:

<table>
<thead>
<tr>
<th></th>
<th>No fixed cost</th>
<th>Fixed Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δy ages 1-5 vs. 10+</td>
<td>0.06</td>
<td>0.08</td>
</tr>
<tr>
<td>Δy ages 6-10 vs. 10+</td>
<td>0.01</td>
<td>0.03</td>
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<td>0.10</td>
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TFP losses US: 1.5% vs. 1.6%
TFP losses Korea: 5.1% vs. 5.7%
TFP losses Colombia: 6.7% vs. 7.1%
Conclusions

- Model that accounts plant-level data:
  - Small TFP losses from misallocation


- Need large shocks, constrained entrants to break correlation assets/productivity
  - Inconsistent with plant-level data