Regulating collateral-requirements when markets are incomplete

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The possibility of **default** creates a need for the introduction of “incentives” (enforcement mechanisms) for agents to keep their **promises**.

It is certainly **not** possible in practice to devise a strong enough mechanism that ensures that all promises will be kept in **all** circumstances.

It is often the case that this is also not desirable economically.

In fact, the main effect of such may be indistinguishable a simple restrictions on trade which prevent efficient risk sharing.
On the other hand, the all too obvious repercussions of default, particularly in modern economies where promises greatly exceed physical endowments, highlight the importance of the careful design of any mechanism which proposes to be an improvement.

Whether better results may be obtained by a purely endogenous approach or by one involving government regulation, is often a quantitative issue.

Numerical investigation, even of simplified models, may give us a better understanding of what to expect, and hope for, in actual economies.
While several enforcement mechanisms may be present in an actual economy, a major role is clearly played by collateral.

In fact, the vast majority of debt is guaranteed by tangible assets, and the failure to fulfill promises has often as main consequence the seizing of collateral.

An important example is given by mortgages, when residential homes serve as collateral for loans to households.

Similarly, corporate bonds are often backed by equipment and plants.

In financial markets, investors can borrow money to establish a position in stocks, using these as collateral.
In our work, we consider the general equilibrium model introduced in Geanakoplos, J. and Zame, W.R. (2007) “Collateralized Asset Markets”.

It describes a two-period economy where collateral constitutes the only enforcement mechanism.

Individuals have to put up durable goods as collateral when they want to take short positions in financial markets.

Agents are allowed to default on their promises without any punishment (in particular no reputation effects), but in the case of default, the collateral is seized and distributed among creditors.
A key feature of the model is that scarcity and an unequal distribution of collateralizable durable goods affects risk-sharing and welfare.

If the durable good is plentiful and satisfy the collateral constraints of the agent, the model is equivalent to a standard Arrow-Debreu model (and competitive equilibrium allocations are Pareto-optimal).

If, on the other hand, the collateralizable durable good is scarce, most assets are not traded in equilibrium and markets appear to be incomplete.
Can regulation improve welfare?

- In the presence of scarcity, a most interesting question is whether welfare improvements might be achieved through government regulation.
- It is a quantitative question who in the economy gains and who loses through a regulation of collateral-requirements.
- We provide a series of examples, some of them illustrative and some realistically calibrated, in order to address this question.
- The numerical examples illustrate that regulation of margin requirements generally does not lead to Pareto-improvements. However often a majority of agents would favor a regulation since it is welfare improving for them.
In our model, we can interpret the assets with low collateral-requirement as a ‘subprime loan’. In particular they carry higher interest rates, and tend to be bought by agents who lack collateralizable durable goods in the present.

Should one banish subprime loans?

We find out that restricting trade in the subprime assets tends to hurt all agents.

This was a robust feature in our numerical investigations of the model.

On the other hand, in some cases, both rich and poor agents gain if only subprime loans can be traded (and markets for prime loans are shut down). However, the middle-class loses if only subprime loans can be traded and it is therefore not Pareto-improving.
We consider a pure exchange economy over two periods $t = 0, 1$.

$S$ states;
$L = 2$ where good 1 is the perishable good and good 2 is the durable good without depreciation;
$H$ agents;
$e^h$ initial endowment;
$J$ assets;
$A_j$ promise; we assume that $A_j = (1, 0)^T$ in period 1 in each state.
$C_j$ borrower collateral requirement.

We will assume that: $S = J$
Utility maximization problem

Given $p \in \mathbb{R}^{2(S+1)}_+$, and $q \in \mathbb{R}^J_+$ the agent $h$ aims to maximize his utility function under the budget constraints:

$$\max_{x \geq 0, \theta \geq 0, \varphi \geq 0} u^h(x^h)$$

s.t.

$$p(0) \cdot (x^h(0) - e^h(0)) + q \cdot (\theta^h - \varphi^h) \leq 0;$$

$$p(s) \cdot (x^h(s) - e^h(s)) - p_2(s)x_2^h(0) - \sum_{j \in \mathcal{J}} (\theta^h_j - \varphi^h_j) \min\{1, p_2(s)C_j\} \leq 0;$$

$$x_2^h(0) - \sum_{j \in \mathcal{J}} \varphi^h_j C_j \geq 0$$  \hspace{1cm} (1)

- A GEIC equilibrium is a list $\langle \bar{p}, \bar{q}, (\bar{x}^h, \bar{\theta}^h, \bar{\varphi}^h)_{h \in \mathcal{H}} \rangle$ such that: i) all agents optimize and ii) commodity and asset markets to clear.
S assets are enough

- Generically for each GEIC equilibrium we have $p_2(s) \neq p_2(s')$ (with $p_1(s)$ normalized to 1 for all $s$) for each $s, s' \in S$.
- We assume throughout that each asset $j$ promises one unit of good 1 in each state $s = 1, ..., S$, so the assets distinguish themselves by collateral requirement $C_j$. We write $(1, C_j)_{j \in J}$ to characterize all assets.
- We denote by $J^{CC}$ (CC standing for complete set of collateral requirements) a set of assets satisfying
  \[
  \{C_j, j \in J^{CC}\} = \{1/p_2(s), s \in S\},
  \]
  hence $J^{CC}$ has exactly $S$ elements.
- We denote by GEICC a GEIC equilibrium in which the set of assets available for trade contains $J^{CC}$.
- We denote by GEIRC a GEIC equilibrium with an exogenously fixed set of collateral requirements.
Proposition (Araujo, Kubler and Schommer)

Given an economy \( ((u^h)_{h \in H}, (e^h)_{h \in H}, (1, C_j)_{j \in J}) \) and a GEIC equilibrium \( [(\bar{x}, \bar{\theta}, \bar{\varphi}); (\bar{p}, \bar{q})] \), suppose that for each \( s \) there is a \( j \in J \) with \( C_j p_2(s) = 1 \), then \( \bar{x} \) and \( \bar{p}, \bar{q} \) are GEIC equilibrium consumptions and prices for any economy \( ((u^h)_{h \in H}, (e^h)_{h \in H}, (1, C_j)_{j \in J}) \) if \( J \subset \tilde{J} \).

**Sketch of the proof.** Assume that \( i, j \in J \), \( C_i < C_j \) and that there is no \( k \in J \) with \( C_i < C_k < C_j \). If an individual holds an asset \( \tilde{j} \in \tilde{J} \) with \( C_i < C_{\tilde{j}} < C_j \), there obviously exists a \( \eta > 0 \) such that \( \eta C_i + (1 - \eta) C_j = C_{\tilde{j}} \). The individual can then simply hold \( \eta \) units of asset \( i \) and \( (1 - \eta) \) units of asset \( j \), obtaining the same payoff in the second period and holding the exact same collateral.
Complete markets

- It is difficult to give a formal condition on endowments and preferences for the collateralizable good being 'abundant' and equilibrium allocations being optimal.
- Geanakoplos and Zame (2007) in Theorem 4 give a necessary and sufficient condition for the trivial case of certainty but show themselves that this does not apply to uncertainty.
- We consider the case with uncertainty.
- Given an Arrow-Debreu equilibrium \((\rho, (x^h)_{h \in H})\), assume without loss of generality that
  \[
  \frac{\rho_2(1)}{\rho_1(1)} > \frac{\rho_2(2)}{\rho_1(2)} > \ldots > \frac{\rho_2(S')}{\rho_1(S')}.\]
Complete markets

There is a GEICC equilibrium with identical allocation if and only if portfolios \( \zeta^h_j = -\min(0, \eta^h_j) \) with \( \eta^h \in \mathbb{R}^S \) defined by

\[
\eta^h_j = A \cdot B \quad \text{where}
\]

\[
A = \begin{pmatrix}
1 & 1 & \ldots & 1 & 1 \\
1 & 1 & \ldots & 1 & \rho_2(2)/\rho_1(2) \\
1 & 1 & \ldots & \rho_2(3)/\rho_1(3) & \rho_2(3)/\rho_1(3) \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
1 & \rho_2(S)/\rho_1(S) & \ldots & \rho_2(S)/\rho_1(S) & \rho_2(S)/\rho_1(S)
\end{pmatrix}^{-1}
\]

\[
B = \begin{pmatrix}
x_1(1) - e_1^h(1) + \frac{\rho_2(1)}{\rho_1(1)}(x_2(1) - e_2^h(1) - x_2(0)) \\
\vdots \\
x_1(S) - e_1^h(S) + \frac{\rho_2(S)}{\rho_1(S)}(x_2(S) - e_2^h(S) - x_2(0))
\end{pmatrix}
\]

satisfy the collateral constraints of the agent, given Arrow-Debreu prices and consumptions of the durable good.
Theorem (Araujo, Kubler and Schommer)

Given an Arrow-Debreu equilibrium \((\rho, (x^h)_{h \in \mathcal{H}})\), a necessary condition for there to exist a GEICC equilibrium with the same allocation is that for all agents \(h\) and all pairs of states \(s\) and \(s'\) with \(\frac{\rho_2(s)}{\rho_1(s)} > \frac{\rho_2(s')}{\rho_1(s')}\) we have

\[
\begin{align*}
&x_1^h(s) - e_1^h(s) + \frac{\rho_2(s)}{\rho_1(s)}(x_2^h(s) - e_2^h(s)) \geq \\
x_1^h(s') - e_1^h(s') + \frac{\rho_2(s')}{\rho_1(s')}(x_2^h(s') - e_2^h(s')) \geq 0
\end{align*}
\]

Proof. Define \(\tilde{\rho}_s = \frac{\rho_2(s)}{\rho_1(s)}\) and suppose that

\[
x_1^h(s) - e_1^h(s) + \tilde{\rho}_s(x_2^h(s) - e_2^h(s)) < x_1^h(s') - e_1^h(s') + \tilde{\rho}_{s'}(x_2^h(s') - e_2^h(s')).
\]

Then in any GEICC equilibrium, the portfolio of agent \(h\), \(\eta^h \in \mathbb{R}^{S-1}\) together with the durable good holding \(x_2^h(0)\) must satisfy

\[
\sum_{j=1}^{S-1} \eta^h_j \min(1, C_j \tilde{\rho}_s) + \tilde{\rho}_s x_2^h(0) < \sum_{j=1}^{S-1} \eta^h_j \min(1, C_j \tilde{\rho}_{s'}) + \tilde{\rho}_{s'} x_2^h(0).
\]
Necessary condition for markets to be complete

Setting $\varphi^h_j = -\min(0, \eta^h_j)$ and $\theta^h_j = \max(0, \eta^h_j)$, this can be written as

$$
\left(- \sum_j \varphi^h_j \min(1, \tilde{\rho}_s C_j) + \tilde{\rho}_s x^h_2(0)\right) + \sum_j \theta^h_j \min(1, \tilde{\rho}_s C_j) < \tag{2}
$$

$$
\left(- \sum_j \varphi^h_j \min(1, \tilde{\rho}_s' C_j) + \tilde{\rho}_s' x^h_2(0)\right) + \sum_j \theta^h_j \min(1, \tilde{\rho}_s' C_j)
$$

However, if $\frac{\rho_2(s)}{\rho_1(s)} > \frac{\rho_2(s')}{\rho_1(s')}$, clearly

$$
\sum_j \theta^h_j \min(1, \tilde{\rho}_s C_j) \geq \sum_j \theta^h_j \min(1, \tilde{\rho}_s' C_j).
$$
Moreover, we have

\[- \sum_j \varphi_j^h \min(1, \tilde{\rho}_s C_j) \geq - \sum_{j: \tilde{\rho}_s C_j < 1} \varphi_j^h \tilde{\rho}_s C_j - \sum_{j: \tilde{\rho}_s C_j \geq 1} \varphi_j^h\]

and

\[- \sum_j \varphi_j^h \min(1, \tilde{\rho}_s' C_j) = - \sum_{j: \tilde{\rho}_s' C_j < 1} \varphi_j^h \tilde{\rho}_s' C_j - \sum_{j: \tilde{\rho}_s' C_j \geq 1} \varphi_j^h\]

Since by collateral constraint \(- \sum_j \varphi_j^h C_j + x_2^h(0) \geq 0\), we therefore obtain

\[- \sum_j \varphi_j^h \min(1, \tilde{\rho}_s C_j) + \tilde{\rho}_s x_2^h(0) \geq - \sum_j \varphi_j^h \min(1, \tilde{\rho}_s' C_j) + \tilde{\rho}_s' x_2^h(0)\]

and hence (2) cannot possibly hold. \(\Box\)
Example 1: High preferences for the durable good

- Two states in period 1;
- Two agents with utility function:

\[ u^h(x) = \alpha \log(x_1(0)) + (1 - \alpha) \log(x_2(0)) \]
\[ + \frac{1}{2} \sum_{s=1}^{2} (\alpha \log(x_1(s)) + (1 - \alpha) \log(x_2(s))) \]

- Endowments are:

\[ e^1 = (e^1_1(0), e^1_2(0); e^1_1(1), e^1_2(1); e^1_1(2), e^1_2(2)) = (4, 2; 4, 0; 4, 0); \]
\[ e^2 = (e^2_1(0), e^2_2(0); e^2_1(1), e^2_2(1); e^2_1(2), e^2_2(2)) = (2, 2; 6, 0; 2, 0). \]

- The parameter \( \alpha \) (preferences for the perishable good) will determine the Arrow-Debreu price of the durable good and whether complete markets are feasible in this example.
Example 1: High preferences for the durable good

- The necessary condition of theorem is defined by
  \[ v_s^h = x_1^h(s) - e_1^h(s) + \frac{\rho_2(s)}{\rho_1(s)}(x_2^h(s) - e_2^h(s)), \quad s = 1, \ldots, S \, . \]

- We first consider \((1 - \alpha) = 0.2\) (low preferences for the durable good), in this case \(\frac{\rho_2(1)}{\rho_1(1)} = 0.625 > 0.375 = \frac{\rho_2(2)}{\rho_1(2)}\).

- Direct computation shows that \(v_1^1 = 3.25 > 0.35 = v_1^1\) but that \(v_2^2 = -0.75 < 1.15 = v_2^2\) and that therefore the necessary condition fails to hold for agent 2 and the GEICC equilibrium **is not Pareto-efficient**.

- Now, we consider \((1 - \alpha) = 0.67\) (high preferences for the durable good) we obtain \(\frac{\rho_2(1)}{\rho_1(1)} = 5 > 3 = \frac{\rho_2(2)}{\rho_1(2)}\) and \(v_1^1 = 12 > 5.6 = v_1^1\) and \(v_2^2 = 8 > 6.4 = v_2^2\)  

- Necessary condition holds for both agents and the GEICC equilibrium **is efficient**.
Example 2: Even distribution of the durable good among agents

- Even if the value of the durable good is high, it is important that each agent owns a sufficient amount of the durable good.

- We fix \((1 - \alpha) = 0.67\) (high preferences for the durable good) but vary endowments of the durable good as follows:

  \[ e^1 = (4, (4 - \omega); 4, 0; 4, 0); \quad e^2 = (2, \omega; 6, 0; 2, 0). \]

- We consider \(\omega = 0\), in this case \(\frac{\rho_2(1)}{\rho_1(1)} = 5 > 3 = \frac{\rho_2(2)}{\rho_1(2)}\), but \(v_1^1 = 22 > 11.6 = v_2^1\) and \(v_1^2 = -2 < 0.4 = v_2^2\) so and the necessary condition for agent 2 fails (is not Pareto-efficient).

- For \(\omega = 1.5\) we obtain \(\frac{\rho_2(1)}{\rho_1(2)} = 5 > 3 = \frac{\rho_2(1)}{\rho_1(2)}\) and \(v_1^1 = 14.5 > 7.1 = v_2^1\) and \(v_1^2 = 5.5 > 4.9 = v_2^2\) that necessary condition holds for both agents and the GEICC equilibrium is efficient.
Constrained Pareto optimality

- By conventional theorem about constrained Pareto optimality, if prices do not change in the second period, the market chooses the asset structure efficiently.

**Theorem (Araujo, Kubler and Schommer)**

If all agents have identical homothetic utility, given a GEICC equilibrium with actively traded assets $\mathcal{J}^{CC}$, there is no other set of assets $\mathcal{J}'$ such that in the resulting GEIRC equilibrium (with an exogenously fixed set of collateral) all agents are better off.

**Proof.** Identical homothetic utility implies that at all states $s = 1, \ldots, S$ spot prices are independent of the assets traded since this only affects the distribution of wealth across agents. But then, the standard argument shows that there cannot be a GEIRC equilibrium that is Pareto-better than the GEICC equilibrium.
In this section, we describe three numerical examples.

The first example illustrates how scarce durable good leads to a situation where only very few assets are traded and welfare losses due to imperfect risk-sharing are large.

The second example is calibrated to match real world data.

We use the algorithm described in Schommer, S. (2008). “Computing general equilibrium with incomplete markets and default” to approximate GEIC equilibrium numerically.
Example 1: With scarce collateralizable goods only few assets are traded

- Four states in period 1.
- Two agents, each with identical utility,

\[ u^h(x) = \log(x_1(0)) + \log(x_2(0)) + \frac{1}{4} \sum_{s=1}^{4} (\log(x_1(s)) + \log(x_2(s))) \]

We consider a variety of profiles of endowments, differing by distribution of durable (collateralizable) good in the first period:

- \( e^1(0) = (4, \eta), \quad e^1(1) = e^1(2) = (1, 0), \quad e^1(3) = e^1(4) = (2, 0); \)
- \( e^2(0) = (1, (1 - \eta)), \quad e^2(1) = e^2(3) = (1, 0), \quad e^2(2) = e^2(4) = (2, 0.2). \)

- We consider \( \eta \geq 1/2 \)
Since we assume identical homothetic utility, spot-prices do not depend on $\eta$ (distribution of durable good in the first period).

The set $\{C_j, j \in J^{CC}\} = \{p_1(s)/p_2(s), s \in S\}$ consists of the four assets with collateral requirements $C_1 = 0.5$, $C_2 = 0.4$, $C_3 = 0.333$ and $C_4 = 0.3$.

The assets’ payment in the states is defined by $\frac{\min\{p_1(s), p_2(s)C_j\}}{p_1(s)}$ follows in Table below:

**Table:** Assets’ payment in the states

<table>
<thead>
<tr>
<th>Assets</th>
<th>state 1</th>
<th>state 2</th>
<th>state 3</th>
<th>state 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$j=1$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$j=2$</td>
<td>0.8</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$j=3$</td>
<td>0.667</td>
<td>0.833</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$j=4$</td>
<td>0.6</td>
<td>0.75</td>
<td>0.9</td>
<td>1</td>
</tr>
</tbody>
</table>
Example 1: Only few assets are traded

Table: Portfolio agent 1 for different $\eta$ (agent 1’s durable good in t=0)

<table>
<thead>
<tr>
<th>$\eta$</th>
<th>asset 1</th>
<th>asset 2</th>
<th>asset 3</th>
<th>asset 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.95</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.51</td>
</tr>
<tr>
<td>0.85</td>
<td>0</td>
<td>0.15</td>
<td>0</td>
<td>0.62</td>
</tr>
<tr>
<td>0.81</td>
<td>0</td>
<td>0.68</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.8</td>
<td>0</td>
<td>0.70</td>
<td>-0.01</td>
<td>0</td>
</tr>
<tr>
<td>0.5</td>
<td>-0.48</td>
<td>1.08</td>
<td>-0.81</td>
<td>0</td>
</tr>
</tbody>
</table>

- $\eta = 0.95$: the only asset traded is the one with the lowest margin requirement.
- $\eta = 0.85$: asset 2 is traded for risk-sharing in the second period.
- $\eta = 0.81$: agent 2 has sufficient collateral to sell only asset 2.
- $\eta = 0.8$: buying asset 3 is a way for the borrower to insure.
- $\eta = 0.5$: both agents have sufficient collateral to establish short positions.
Example 1: How large are the welfare losses?

Table: Welfare rate for distribution of durable good: agent 1 and 2

<table>
<thead>
<tr>
<th>$\eta$</th>
<th>Lender (agent 1)</th>
<th>Borrower (agent 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.95</td>
<td>0.988</td>
<td>0.933</td>
</tr>
<tr>
<td>0.9</td>
<td>0.989</td>
<td>0.969</td>
</tr>
<tr>
<td>0.85</td>
<td>0.991</td>
<td>0.978</td>
</tr>
<tr>
<td>0.8</td>
<td>0.992</td>
<td>0.982</td>
</tr>
<tr>
<td>0.75</td>
<td>0.992</td>
<td>0.989</td>
</tr>
<tr>
<td>0.5</td>
<td>0.993</td>
<td>0.996</td>
</tr>
</tbody>
</table>

- Between $\eta = 0.95$ and $\eta = 0.85$ a donation could so improve risk-sharing and benefit donor (transfer paradox).
- Between $\eta = 0.8$ and $\eta = 0.5$ the welfare losses remain more or less constant for agent 1, while there are still substantial improvements for agent 2.
Example 1: Exogenously selecting margin requirements can make both agents better off?

- In this example all agents have identical homothetic utility.
- According to the identical homothetic utility Theorem is impossible to make both agents better off, by exogenously selecting margin requirements.
- This obviously does not imply, however, that all possible margin-requirements are Pareto-ranked.
- First we assume $\eta = 0.95$, i.e,

\[
e_1(0) = 0.95, \quad e_2(0) = 0.05\]

In this case, it seems likely that GEIC equilibrium allocations are in fact Pareto-ranked.
The highest utility for both agents is with low collateral (red point), this is true for all $\eta \geq 0.88$. 

**Figure:** Welfare rate for several exogenous collateral (C). For all points (equilibria) only one asset is traded. Durable good (agent 1): $\eta = 0.95$
**Introduction**

**The model**

**Theoretical results**

**Numerical examples**

**Example 1:** Scarce durable goods markets ‘appear’ incomplete

**Example 2:** A calibrated example

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**Figure:** Welfare rate for several exogenous collateral (C).

For yellow points two assets are traded and the reaming points only one asset is traded.

Durable good (agent 1):
\[ \eta = 0.85 \]

- Now low collateral is good only lender (red point), this is true for all \( 0.82 \leq \eta \leq 0.87 \).
We want to investigate the role of sub-prime loans for risk-sharing as well as who in the economy gains and who loses through regulation;

We assume that there are 4 types of agents whose endowments we calibrated to match the income and wealth distribution in US data;

We interpret endowments in the non-durable as income while endowments in the durable good are interpreted as wealth;

Four states in period 1.

Four types of agents.

Agents have heterogeneous utility:

\[ u^h(x) = \alpha^h \log(x_1(0)) + (1 - \alpha^h) \log(x_2(0)) + \sum_{s=1}^{4} \varepsilon_s (\alpha^h \log(x_1(s)) + (1 - \alpha^h) \log(x_2(s))) \]

We choose \( \alpha^h \) to roughly match a relative price of durable to non-durable good of 1/2

\[ \alpha^1 = 0.5, \quad \alpha^2 = 0.4, \quad \alpha^3 = 0.3, \quad \alpha^4 = 0.6 \]
The four agents’ endowments are:

\[ e_1^1(0) = (0.61, 0.84), \quad e_1^1(1) = e_1^1(3) = (0.63, 0), \quad e_1^1(2) = e_1^1(4) = (0.21, 0); \]
\[ e_2^2(0) = (0.22, 0.12), \quad e_2^2(1) = e_2^2(3) = (0.21, 0), \quad e_2^2(2) = e_2^2(4) = (0.63, 0); \]
\[ e_3^3(0) = (0.12, 0.04), \quad e_3^3(1) = e_3^3(2) = (0.11, 0), \quad e_3^3(3) = e_3^3(4) = (0.05, 0); \]
\[ e_4^4(0) = (0.05, 0.00), \quad e_4^3(1) = e_4^3(2) = (0.05, 0), \quad e_4^3(3) = e_4^3(4) = (0.11, 0). \]

Probabilities are:

\[ \varepsilon_s = (0.60, 0.18, 0.18, 0.04). \]

- Since preferences are heterogeneous, we search for GEIRC equilibria that could be Pareto better.
In the GEICC equilibria two assets are traded.

- GEIR C1: regulation that maximize the agents 1 and 4’s utilities (there is full default for all traded assets).
- GEIR C2: regulation that forbids sub-prime loans.
- GEIR C3: regulation that forbids default.

Table: Portfolios Agents 1, 2, 3 and 4

<table>
<thead>
<tr>
<th>GEIC</th>
<th>Agent 1</th>
<th>Agent 2</th>
<th>Agent 3</th>
<th>Agent 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>CC</td>
<td>(0.037, -0.16, 0)</td>
<td>(0.24, 0.11, 0)</td>
<td>(0.11, 0.05, 0)</td>
<td>(0.02, 0)</td>
</tr>
<tr>
<td>RC1</td>
<td>(0.043, -0.09, 0)</td>
<td>(0.21, 0)</td>
<td>(0.19, 0.09, 0)</td>
<td>(0.03, 0)</td>
</tr>
<tr>
<td>RC2</td>
<td>(0.16, 0.34)</td>
<td>(0.11, -0.23)</td>
<td>(0.05, -0.10)</td>
<td>(0, -0.01)</td>
</tr>
<tr>
<td>RC3</td>
<td>(0.11, 0.06, 0)</td>
<td>(-0.11, 0, 0)</td>
<td>(0.05, 0)</td>
<td>(0.01, 0)</td>
</tr>
</tbody>
</table>
In the GEICC equilibria two assets are traded. The rich agent 1 lends in the sub-prime asset (0.37 units) and borrows (0.16 units) in the safe asset. Agents 2-4 borrow exclusively sub-prime, while agents 2 and 3 (the middle-class) actually saves some money in the safe mortgage.

- GEIRC1 corresponds to the equilibrium where there is full default for all traded assets. The rich agent 1 benefits from lending more units in the subprime asset (0.43 units), due to higher interest rate, in relation to GEICC equilibrium.
Agents 2 and 3 cannot be made better off through any regulation (GEICC point).

Agents 1 and 4 gain simultaneously if only subprime borrowing is allowed (GEIRC1 point).
Example 2: Robustness analysis

- In order to verify if the previous specification are robust we consider more specifications for preferences.


- They estimate the housing preference \((1 - \alpha^h)\) as 0.2 for US in 2001 based on the average proportion of household housing expenditure according to the Bureau of Labor Statistics (BLS) of the US Department of Labor.

- Here we estimate the durable good preference \((1 - \alpha^h)\) for each type of agent, based on the proportions of housing, furniture and vehicle purchases according to the BLS for 2004.
Example 2: Robustness analysis

- In this case the preference are:
  \[ \alpha^1 = 0.74, \quad \alpha^2 = 0.73, \quad \alpha^3 = 0.72, \quad \alpha^4 = 0.71 \]

- As in the above example, both agents 1 and 4 can be made better off when trade is restricted to be in the sub-prime asset.

- The following values for \( \alpha \) also give this result:
  \[ \alpha^1 = 0.7, \quad \alpha^2 = 0.6, \quad \alpha^3 = 0.5, \quad \alpha^4 = 0.4 \]
  \[ \alpha^1 = 0.4, \quad \alpha^2 = 0.3, \quad \alpha^3 = 0.5, \quad \alpha^4 = 0.7 \]
  \[ \alpha^1 = 0.5, \quad \alpha^2 = 0.4, \quad \alpha^3 = 0.3, \quad \alpha^4 = 0.2 \]

indicating that what seems to be the most robust case is a case as in the above example.
Conclusion

- Show that in the GEIC equilibrium generally only few of the possible contracts are traded with scarce and unequal distribution of durable (collateralizable) good;

- A regulation of collateral requirements never leads to a Pareto-improvement for all agents. However, we show that equilibria corresponding to different regulated collateral requirements are often not Pareto-ranked, and some agents can be benefited with regulation;

- In our numerical examples, it is never optimal to regulate the market for subprime loans (middle-class loses from such a regulation). However, in all cases the subprime asset is traded actively and provide the only possibility for the poor agent to purchase the durable good.